

PHY-563-PROJECT

Title- “Checking mechanical stability for crystal symmetries and Calculation of mechanical properties of materials”

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Aim of the program:

Given the space group or lattice vector matrix of a compound , and the elasticity coefficient matrix of the compound , the program does the following :

1. Identifies the crystal lattice/symmetry type of the compound.
2. Checks mechanical stability of the compound.
3. The Reuss bound, Voigt bounds of the Bulk modulus and Shear modulus of the compound are calculated and displayed. Two different sets of formulae are used to find the bounds of these moduli, and the results are compared.
4. Using the Reuss-Voigt-Hill approximations, values of the Bulk modulus , Shear modulus, Young's modulus and Poisons ratio of the compound are calculated and displayed

Theory:

Given the lattice vector array matrix of a compound, one can find the crystal symmetry of the compound. Each crystal symmetry has a distinct general lattice vector array matrix, which can be used to identify the crystal symmetry.

Each of 230 space groups (in 3 dimensions) fall into one of the 7 crystal systems (triclinic , Monoclinic , Orthorhombic , Tetragonal , Trigonal , Hexagonal , Cubic).

Mechanical stability conditions: These are a set of conditions which the elements of the C matrix of a compound have to satisfy in order for the compound to be mechanically stable. These conditions vary for different crystal systems. The conditions for cubic phase system are shown below as an example. The conditions are taken from- [2]

$$C_{11} > 0, \quad C_{44} > 0, \quad C_{11} > |C_{12}|, \quad (C_{11} + 2C_{12}) > 0.$$

Voigt bound : It is the upper limit of the effective moduli. It is obtained by the average polycrystalline moduli based on assumption of uniform strain throughout a polycrystal.

Reuss bound: It is the lower limit of the effective moduli. It is based on the assumption of uniform stress throughout the polycrystal.

One set of formulae used to calculate the reuss and voigt bounds of the moduli are the following. These formulae are valid for all types of crystal systems. Formulae from – [4]

$$B_R = [S_{11} + S_{22} + S_{33} + 2(S_{12} + S_{13} + S_{23})]^{-1}$$

$$B_V = \frac{1}{9}[C_{11} + C_{22} + C_{33} - 2(C_{12} + C_{13} + C_{23})]$$

$$G_R = 15[4(S_{11} + S_{22} + S_{33}) - 4(S_{12} + S_{13} + S_{23}) + 3(S_{44} + S_{55} + S_{66})]^{-1}$$

$$G_V = \frac{1}{15}[C_{11} + C_{22} + C_{33} - (C_{12} + C_{13} + C_{23}) + 3(C_{44} + C_{55} + C_{66})]$$

The second set of the formulae used vary according to the type of crystal system. The formulae for orthorhombic systems are shown below as an example. The formulae are taken from – [2]

$$B_V = (1/9)[C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})],$$

$$G_V = (1/15)[C_{11} + C_{22} + C_{33} + 3(C_{44} + C_{55} + C_{66}) - (C_{12} + C_{13} + C_{23})],$$

$$B_R = \Delta[C_{11}(C_{22} + C_{33} - 2C_{23}) + C_{22}(C_{33} - 2C_{13}) - 2C_{33}C_{12} + C_{12}(2C_{23} - C_{12}) + C_{13}(2C_{12} - C_{13}) + C_{23}(2C_{13} - C_{23})]^{-1},$$

$$G_R = 15\{4[C_{11}(C_{22} + C_{33} + C_{23}) + C_{22}(C_{33} + C_{13}) + C_{33}C_{12} - C_{12}(C_{23} + C_{12}) - C_{13}(C_{12} + C_{13}) - C_{23}(C_{13} + C_{23})]/\Delta + 3[(1/C_{44}) + (1/C_{55}) + (1/C_{66})]\}^{-1},$$

$$\Delta = C_{13}(C_{12}C_{23} - C_{13}C_{22}) + C_{23}(C_{12}C_{13} - C_{23}C_{11}) + C_{33}(C_{11}C_{22} - C_{12}^2).$$

Voigt-Reuss-Hill approximations: The average of Voigt and Reuss bound. This approximation is used to obtain the B (Bulk modulus) , G (Shear modulus) . Using B and G obtained , E (Young's modulus) and ν (Poisson's ratio) are calculated.

In terms of the Voigt-Reuss-Hill approximations,⁵² $M_H = (1/2)(M_R + M_V)$, $M=B$, G . Young's modulus E and Poisson's ratio ν are obtained by the following formulas:

$$E = 9BG/(3B + G), \quad \nu = (3B - 2G)/[2(3B + G)].$$

Code explanation:

Subroutine initialize: Fills in matrix C, the values of the elasticity coefficients for different compounds

Subroutine symmetryidentifier : Takes input of the lattice vector array matrix from a text file, identifies the crystal system and outputs an integer which represents the type of crystal system , using if-else statements.

Subroutine spacegroup : Takes input of the space group of the compound from the user , and identifies the crystal system to which the particular space group belongs. A text file contains all 230 space groups in matrix form (46 into 5). The input taken from the user is compared with every space group in the text file and if a match is found, the position of the matched space group is filled in h, k . If else statements are then used to match the h,k to the correct crystal system. Subroutine outputs an integer representing the crystal system to which the space group belongs.

Subroutine findinverse: Takes C , and dimension of C as input and outputs its inverse matrix (elastic compliance coefficient matrix). Online resources were used to understand L-U decomposition and fix some errors present in the subroutine.

Subroutine stability: The purpose of this subroutine was to check whether the given lattice was in a stable configuration or not. We used the conditions given in [2] for this. These include stability conditions for the following 5 lattices: *Cubic*, *Hexagonal*, *Tetragonal*, *Orthorhombic* and *Monoclinic*.

We inputted an integer value 'st' which denotes the lattice type and printed if the lattice is stable or not. We initialized a flag variable to zero which was changed to one only if a stable condition was satisfied. At the end of the subroutine, a flag zero indicated unstable compound whereas a flag one indicated a stable compound.

Subroutine reuss: A simple subroutine which followed [4] and used the simple formula for Bulk modulus and Shear modulus via Reuss method. It doesn't have different equations for different types of lattice.

Subroutine voigt: Like the previous subroutine, it is also a simple subroutine which has one equation each for Bulk and Shear modulus using Voigt method.

Subroutine reussphysrev: This used the equations given in [2]. There were different equations for Bulk and Shear modulus for different lattice type (the 5 types for which stability conditions have been calculated).

Subroutine voigtphysrev: This also used the equation given in [2] for the Voigt method to calculate Bulk and Shear modulus of different lattice type (the 5 types for which stability conditions have been calculated).

In the main program we average the values of Bulk and shear modulus obtained via Reuss and Voigt method. We do this separately for paper [2] and [4]. Then we calculate the values of Young's modulus and Poisson's ratio separately for both papers.

Results

The results were as we expected them to be. We checked for various Li-Sn compounds using data given in [4]. The results matched and the percentage difference between the values obtained by following equations given in [2] and [4] was of the order of 10^{-6} or less. Thus we were able to successfully code a program using multiple subroutines to fulfill the aim of detecting the lattice type of given compound either by lattice symmetry or space group entered by user. We identified if the compound was stable and output the values of 4 properties of the compound namely- Bulk modulus, shear modulus, Young's modulus and Poisson's ratio.

Sources / References :

- [1]- **Necessary and sufficient elastic stability conditions in various crystal systems** by F'elix Mouhat and Francois-Xavier Coudert.
- [2]- **Crystal structures and elastic properties of superhard IrN₂ and IrN₃ from first principles** by Zhi-jian Wu.
- [3]- <https://www.youtube.com/watch?v=m3EojSAgIao> ,
<https://ww2.odu.edu/~agodunov/computing/programs/book2/Ch06/Inverse>
- [4]- **Understanding the Lithiation of Sn Anode for High Performance Li-ion Batteries with Exploration of Novel Li-Sn Compounds at Ambient and Moderately High Pressure** by Raja Sen and Priya Johari.

Additional:

LU decomposition method used to find inverse of a test 3 by 3 matrix :

(From next page)

• Find Inverse of matrix A using LU decomposition method.

• Finding the L and U matrices:

$$A = \begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ - & 1 & 0 \\ - & - & 1 \end{pmatrix}, \quad \begin{matrix} L_{21}, L_{31}, L_{32} \\ \text{we will find and} \\ \text{fill} \end{matrix}$$

U should be of the form:

$$U = \begin{pmatrix} - & - & - \\ 0 & - & - \\ 0 & 0 & - \end{pmatrix}$$

Now for getting U,

$$\begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} \begin{matrix} R_2 \rightarrow R_2 - \text{Cf. } R_1, \\ \text{Cf such that element (2,1)} \\ \text{is 0} \end{matrix}$$
$$= \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 144 & 12 & 1 \end{pmatrix}, \quad \begin{matrix} \text{Cf} = \frac{64}{25} = 2.56 \\ L_{21} = 2.56 \end{matrix}$$

$$\begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 144 & 12 & 1 \end{pmatrix}$$

$R_3 \rightarrow R_3 - C_f \cdot R_1$ such that element $(3,1)$ is 0.

$$\rightarrow \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{pmatrix}$$

$$C_f = \frac{144}{25} = 5.76$$

$$L_{31} = 5.76$$

$$\rightarrow \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{pmatrix}$$

$R_3 \rightarrow R_3 - C_f \cdot R_2$ s.t element $(3,2)$ is 0

$$\rightarrow \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{pmatrix}$$

$$C_f = \frac{-16.8}{-4.8} = 3.5$$

$$L_{32} = 3.5$$

$$\therefore U = \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{pmatrix}$$

Let B is inverse matrix of A ,

$$B = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix}$$

Now, we know

$$\begin{pmatrix} A \end{pmatrix} \begin{pmatrix} B_{11} \\ B_{21} \\ B_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} \begin{pmatrix} B_{11} \\ B_{21} \\ B_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\uparrow \quad \uparrow \quad \uparrow$
 $A = LU \quad (X) \cdot (Y)$

$$LUX = Y$$

Taking $UX = Z$

$$LUX = Y \rightarrow LZ = Y$$

$$LZ = Y \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(forward substitution)

$$z_1 = 1$$

$$2.56z_1 + z_2 = 0 \rightarrow z_2 = -2.56$$

$$5.76z_1 + 3.5z_2 + z_3 = 0$$

$$\rightarrow 5.76 + 3.5(-2.56) + z_3 = 0$$

$$\rightarrow z_3 = 3.2$$

$$\rightarrow \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2.56 \\ 3.2 \end{pmatrix}$$

$$UX = Z$$

$$\rightarrow \begin{pmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2.56 \\ 3.2 \end{pmatrix}$$

(Back substitution)

$$0.7x_3 = 3.2 \rightarrow x_3 = 4.571$$

$$-4.8x_2 - 1.56x_3 = -2.56$$

$$\rightarrow x_2 = \frac{-2.56 + 1.56(4.571)}{-4.8} = -0.952$$

$$25x_1 + 5x_2 + x_3 = 1$$

$$\rightarrow x_1 = \frac{1 - 5(-0.952) - 4.571}{25} = 0.0476$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.0476 \\ -0.952 \\ 4.571 \end{pmatrix} = \begin{pmatrix} B_{11} \\ B_{21} \\ B_{31} \end{pmatrix}$$

For 2nd column of B

$$\begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} \begin{pmatrix} B_{12} \\ B_{22} \\ B_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

\uparrow \uparrow
 (X) (Y)

Same process as for column 1 is followed :

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0.0833 \\ 1.417 \\ -5.0 \end{pmatrix} = \begin{pmatrix} B_{12} \\ B_{22} \\ B_{32} \end{pmatrix}$$

For 3rd column of B,

$$\begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} \begin{pmatrix} B_{13} \\ B_{23} \\ B_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

\uparrow \uparrow
 (X) (Y)

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0.0357 \\ -0.4643 \\ 1.429 \end{pmatrix} = \begin{pmatrix} B_{13} \\ B_{23} \\ B_{33} \end{pmatrix}$$

$$B = A^{-1} = \begin{pmatrix} 0.0476 & 0.0833 & 0.0357 \\ -0.952 & 1.417 & -0.4643 \\ 4.571 & -5.0 & 1.429 \end{pmatrix}$$

Code:

```
program Project
```

```
    implicit none
```

```
    integer::ch
```

```
    real,dimension(6,6)::C,S
```

```
    real:: B,G,E,v,Br,Gr,Bv,Gv,Br1,Gr1, Bv1, Gv1, B1,G1, E1, v1,diffb,diffg,diffe,diffv
```

```
    integer::i,j, st,n=6
```

```
    logical::stab !used in stability subroutine
```

```
    C=0
```

```
    S=0
```

```
    call initialize(C)
```

```
    call findinverse(C,S,n)
```

```
    print*,"Enter :"
```

```
    print*,"1. For input of lattice vector array of material"
```

```
    print*,"2. For input of space group of material"
```

```
    read*,ch
```

```
    if(ch==1) then
```

```
        call symmetryidentifier(st)
```

```
        !print*,"Lattice number is ",st
```

```
    else if(ch==2)then
```

```
        call spacegroup(st)
```

```
    end if
```

```

call stability(C,st,stab)

call voigt(C,Bv,Gv)

call voigtphysrev(C,Bv1,Gv1,st)

call reuss(S,Br,Gr)

call reussphysrev(C,Br1,Gr1,st)

```

```

if (stab .eqv. .true.) then
    print*,"Lattice is stable."
else if(stab .eqv. .false.) then
    print*,"Lattice is not stable."
end if

```

```

print*, "Voigt method. Bulk and Shear modulus from Physical review paper Zhi-jian
Wu:",Bv1, Gv1

```

```

print*, "Voigt method. Bulk and Shear modulus from Ma'am's paper:",Bv, Gv

```

```

print*, "Reuss method. Bulk and Shear modulus from Physical review paper Zhi-jian Wu:",
Br1,Gr1

```

```

print*, "Reuss method. Bulk and Shear modulus from Ma'am's paper:", Br,Gr

```

$$B1 = (Bv1 + Br1)/2$$

$$G1 = (Gv1 + Gr1)/2$$

$$E1 = 9*B1*G1/(3*B1 + G1)$$

$$v1 = (3*B1 - 2*G1)/(2*(3*B1 + G1))$$

$$B = (Bv + Br)/2$$

$$G = (Gv + Gr)/2$$

$$E = 9 \cdot B \cdot G / (3 \cdot B + G)$$

$$\nu = (3 \cdot B - 2 \cdot G) / (2 \cdot (3 \cdot B + G))$$

print*, "The four mechanical properties using Physical review paper by Zhi-jian Wu are as follows: "

print*, "Bulk Modulus: ", B1, "GPa"

print*, "Shear Modulus: ", G1, "GPa"

print*, "Young's modulus: ", E1, "GPa"

print*, "Poisson's ratio: ", v1

print*, "The four mechanical properties using Priya ma'am's paper are as follows: "

print*, "Bulk Modulus: ", B, "GPa"

print*, "Shear Modulus: ", G, "GPa"

print*, "Young's modulus: ", E, "GPa"

print*, "Poisson's ratio: ", v

$$\text{diffb} = \text{abs}((B - B1)) \cdot 100 / B1$$

print*, "Percentage difference between Bulk modulus what Ma'am's paper gives vs Zhi-jian Wu's paper gives:", diffb

$$\text{diffg} = \text{abs}((G - G1)) \cdot 100 / G1$$

print*, "Percentage difference between Bulk modulus what Ma'am's paper gives vs Zhi-jian Wu's paper gives:", diffg

$$\text{diffe} = \text{abs}((E - E1)) \cdot 100 / E1$$

print*, "Percentage difference between Bulk modulus what Ma'am's paper gives vs Zhi-jian Wu's paper gives:", diffe

$$\text{diffv} = \text{abs}((\nu - \nu1)) \cdot 100 / \nu1$$

print*, "Percentage difference between Bulk modulus what Ma'am's paper gives vs Zhi-jian Wu's paper gives:", diffv

end program Project

subroutine spacegroup(t) !returns an integer st indicating which lattice type given
spacegroup belongs to

implicit none

integer::choice

integer,intent(out):: t

character(len=17)::charch

integer::i,j,h,k,flag=0

character(len=17)::c(46,5)

open(1857,file='spacegroupinputvvf.txt',status='old',action='read')

print*,"Enter Space group"

read*,charch

do i=1,46

 read(1857,*)c(i,:)

end do

do i=1,46

 do j=1,5

 if(c(i,j)==charch) then

 h=i

 k=j

```

        end if

    end do

end do

if((h==1).AND.(k.GE.1).AND.(k.LE.2)) then

    print*,"Lattice is Triclinic "

    t=6

    flag=1


else if((h==1).AND.(k.GE.3).AND.(k.LE.5)) then

    print*,"Lattice is Monoclinic "

    t=5

    flag=1

else if((h.GE.2).AND.(h.LE.3).AND.(k.GE.1).AND.(k.LE.5)) then

    print*,"Lattice is Monoclinic "

    t=5

    flag=1

else if((h.GE.4).AND.(h.LE.14).AND.(k.GE.1).AND.(k.LE.5)) then

    print*,"Lattice is Orthorhombic "

    t=4

    flag=1

else if((h==15).AND.(k.GE.1).AND.(k.LE.4)) then

    print*,"Lattice is Orthorhombic "

    t=4

    flag=1

else if((h==15).AND.(k==5)) then

```

```
print*,"Lattice is Tetragonal "

t=3

flag=1

else if((h.GE.16).AND.(h.LE.28).AND.(k.GE.1).AND.(k.LE.5)) then

    print*,"Lattice is Tetragonal "

    t=3

    flag=1

else if((h==29).AND.(k.GE.1).AND.(k.LE.2)) then

    print*,"Lattice is Tetragonal "

    t=3

    flag=1

else if((h==29).AND.(k.GE.3).AND.(k.LE.5)) then

    print*,"Lattice is Trigonal "

    t=7

else if((h.GE.30).AND.(h.LE.33).AND.(k.GE.1).AND.(k.LE.5)) then

    print*,"Lattice is Trigonal "

    t=7

    flag=1

else if((h==34).AND.(k.GE.1).AND.(k.LE.2)) then

    print*,"Lattice is Trigonal "

    t=7

    flag=1

else if((h==34).AND.(k.GE.3).AND.(k.LE.5)) then

    print*,"Lattice is Hexagonal "

    t=2

    flag=1
```

```
else if((h.GE.35).AND.(h.LE.38).AND.(k.GE.1).AND.(k.LE.5)) then
```

```
    print*,"Lattice is Hexagonal "
```

```
    t=2
```

```
    flag=1
```

```
else if((h==39).AND.(k.GE.1).AND.(k.LE.4)) then
```

```
    print*,"Lattice is Hexagonal "
```

```
    t=2
```

```
    flag=1
```

```
else if((h==39).AND.(k==5)) then
```

```
    print*,"Lattice is Cubic "
```

```
    t=1
```

```
    flag=1
```

```
else if((h.GE.40).AND.(h.LE.46).AND.(k.GE.1).AND.(k.LE.5)) then
```

```
    print*,"Lattice is Cubic "
```

```
    t=1
```

```
    flag=1
```

```
end if
```

```
if(flag==0) then
```

```
    print*,"Invalid spacegroup entered."
```

```
    stop
```

```
end if
```

```
end subroutine spacegroup
```

```
subroutine symmetryidentifier(st)
```


implicit none

integer,intent(out)::st

integer::j,i

real,allocatable, dimension(:,:):L

real::v

allocate(L(3,3))

open(193,file='latticevinputf.txt',status='old',action='read')

v=3.0

do j= 1,3

 read(193,*)L(j,:)

end do

print*,"Lattice array:"

do j= 1,3

 print*,L(j,:)

end do

if(L(1,2)==L(1,3).AND.L(1,3)==L(2,1).AND.L(2,1)==L(2,3).AND.L(2,3)==L(3,1).AND.L(3,1)==L(3,2).AND.L(1,2)==0) then

 if(L(1,1)==L(2,2).AND.L(2,2)==L(3,3)) then

 print*,"Lattice is sc"

 i=1

 else if((L(1,1)==L(2,2)).AND.(L(2,2).NE.L(3,3))) then

 print*,"Lattice is tetragonal P"

 i=5

 else if((L(1,1).NE.L(2,2)).AND.(L(2,2).NE.L(3,3)).AND.(L(1,1).NE.L(3,3))) then

 print*,"Lattice is orthorombic P"

 i=7

```

end if

else if((L(1,1).NE.L(2,2)).AND.(L(2,2).NE.L(3,3)).AND.(L(1,1).NE.L(3,3))) then

    if(L(1,1)==(-L(2,2)).AND.(L(3,2).NE.0)) then

        print*, "Lattice is fcc"

        i=2

    else if((L(1,2)==0).AND.(L(3,2).NE.0)) then

        print*, "Lattice is orthorombic fc"

        i=9

    else if(L(1,2)==L(2,2)) then

        if(L(2,3)==L(3,3)) then

            print*, "Lattice is orthorombic body centered"

            i=10

        else

            print*, "Lattice is orthorombic base centered"

            i=8

        end if

    else if((L(1,1)==(sqrt(v)/2*L(1,1))).AND.(L(2,1)==(-L(2,2)*sqrt(v)))) then

        print*, "Lattice is Hexagonal"

        i=4

    else if((L(1,2)==L(1,3)).AND.(L(1,3)==L(2,3)).AND.(L(2,3)==0)) then

        if(L(2,1)==0) then

            print*, "Lattice is Monoclinic p , unique axis b "

            i=13

        else if((L(3,1)==0).AND.(L(3,2)==0)) then

            print*, "Lattice is Monoclinic p , unique axis c "

```

```

i=11

else if(L(2,1).NE.0) then

  print*,"Lattice is Triclinic "

  i=14

end if


else if(L(1,1)==L(3,1).AND.(L(1,3)==(-L(3,3)))) then

  print*,"Lattice is Monoclinic base centered "

  i=12

else if(L(1,1)==(-L(3,1)).AND.(L(1,3)==(-L(2,2)/2))) then

  print*,"Lattice is Trigonal R , 3 fold axis c "

  i=15


else

  print*,"Unidentified lattice type"

  i=20


end if


else if((L(1,1)==L(2,2)).AND.(L(2,2)==L(3,3)).AND.(L(1,3).NE.0).AND.(L(2,1).NE.0)) then

  print*,"Lattice is bcc"

  i=3

else if((L(1,2)==L(1,3)).AND.(L(1,3)==L(2,3)).AND.(L(2,3)==0)) then

  if(L(2,1)==0) then

    print*,"Lattice is Monoclinic p , unique axis b "

    i=13

  else if((L(3,1)==0).AND.(L(3,2)==0)) then

```

```

    print*, "Lattice is Monoclinic p , unique axis c "

    i=11

else if(L(2,1).NE.0) then

    print*, "Lattice is Triclinic "

    i=14

else

    print*, "Unidentified lattice"

    i=20

end if

else if(L(1,1)==L(3,1).AND.(L(1,3)==(-L(3,3)))) then

    print*, "Lattice is Monoclinic base centered "

    i=12

else if(L(1,1)==(-L(3,1)).AND.(L(1,3)==(-L(2,2)/2))) then

    print*, "Lattice is Trigonal R , 3 fold axis c "

    i=15

else if((L(1,1)==L(2,2)).AND.(L(1,2)==(-L(2,1)))) then

    print*, "Lattice is Trigonal I, bct "

    i=6

else

    print*, "Unidentified lattice"

    i=20

end if

if((i.GE.1).AND.(i.LE.3)) then

    st=1

else if(i==4) then

```



```

    st=2
else if(i==5) then
    st=3
else if((i.GE.7).AND.(i.LE.10)) then
    st=4
else if((i.GE.11).AND.(i.LE.13)) then
    st=5
else
    st=100
end if

```

end subroutine symmetryidentifier

```

subroutine stability(coeff,st, stab)
    implicit none
    integer,intent(in):: st
    real,dimension(6,6),intent(in):: coeff
    logical,intent(out)::stab
    stab= .false.

```

!Taking a value s. If s=1 then its cubic, s=2 implies hexagonal, s=3 implies tetragonal,s=4 implies orthorhombic,s=5 implies monoclinic

```

    if(st==1) then

        if (((coeff(1,1)-coeff(1,2)) > 0) .and. ((coeff(1,1)+2*coeff(1,2))>0) .and. ((coeff(4,4))>0))
then

```

```

        stab=.true.

    else

        stab=.false.

    end if

    else if (st==2) then

        if ((coeff(1,1)> abs(coeff(1,2))) .and. ((2*(coeff(1,3)**2))
<coeff(3,3)*(coeff(1,1)+2*coeff(1,2))) &

        .and. (coeff(4,4)>0 ) ) then

            stab=.true.

        else

            stab=.false.

        end if

    else if(st==3) then

        if ((coeff(1,1)> abs(coeff(1,2))) .and. ((2*(coeff(1,3)**2))
<coeff(3,3)*(coeff(1,1)+coeff(1,2))) &

        .and. (coeff(4,4)>0 ) .and. (coeff(6,6)>0) ) then

            stab=.true.

        else

            stab=.false.

        end if

    else if(st==4) then

        if ((coeff(1,1)>0) .and. (coeff(2,2)>0) .and. (coeff(3,3)>0) .and. (coeff(4,4)>0) &

```

```

.and. (coeff(5,5)>0) .and. (coeff(6,6)>0) .and. &
(coeff(1,1)+coeff(2,2)+ coeff(3,3) +2*(Coeff(1,2) +Coeff(1,3)+Coeff(2,3))>0) .and. &
((coeff(1,1)+coeff(2,2)-2*coeff(1,2))>0) .and. (Coeff(1,1)+Coeff(3,3)-2*Coeff(1,3)>0) &
.and. (coeff(2,2)+coeff(3,3)-2* coeff(2,3)>0) ) then

stab=.true.

else

stab=.false.

end if

else if(st==5) then !monoclinic

if ( (coeff(1,1)> 0) .and. (Coeff(2,2)>0) .and. (Coeff(3,3)> 0) .and. (Coeff(4,4)>0) &
.and. (Coeff(5,5)> 0) .and. (Coeff(6,6)> 0) .and. &
(Coeff(1,1) + Coeff(2,2) + Coeff(3,3) + 2*(Coeff(1,2) + Coeff(1,3) + Coeff(2,3))>0) .and.
((Coeff(3,3)*Coeff(5,5) &
- Coeff(3,5)**2)>0) .and. ((Coeff(4,4)*Coeff(6,6)- Coeff(4,6)**2)>0) .and. &
((Coeff(2,2) + Coeff(3,3) -2*Coeff(2,3))>0)) then

stab=.true.

else

stab=.false.

end if

else

print*, "Stability condition and formulas for various modulus not given in paper."

stop

end if

```

```
end subroutine stability
```

```
subroutine initialize(C)
```

```
  implicit none
```

```
  real,dimension(6,6),intent(inout)::C
```

```
  integer:: i,j
```

```
  !inputting C matrix for different compounds given in Ma'am's research paper
```

```
  !3 belongs to trigonal which we do not have case for
```

```
  print*,"Enter 1- Sn, 2- Li2Sn5, 3- Li4Sn1, 4- Li, 5-Li5Sn1 , 6-Enter C matrix through text  
file."
```

```
  read*, i
```

```
  if (i==1) then
```

```
    C(1,1)=56.12
```

```
    C(2,2)=56.12
```

```
    C(3,3)=56.12
```

```
    C(4,4)=43.26
```

```
    C(5,5)=43.26
```

```
    C(6,6)=43.26
```

```
    C(1,2)=26.60
```

```
    C(1,3)=26.60
```

```
    C(2,1)=26.60
```

```
    C(2,3)=26.60
```

```
    C(3,1)=26.60
```

```
    C(3,2)=26.60
```

else if (i==2) then

$$C(1,1)=79.03$$

$$C(3,3)=95.24$$

$$C(4,4)=19.85$$

$$C(6,6)=14.93$$

$$C(1,2)=37.11$$

$$C(2,1)=37.11$$

$$C(2,2)=79.03$$

$$C(2,3)=15.45$$

$$C(1,3)=15.45$$

$$C(3,1)=15.45$$

$$C(3,2)=15.45$$

$$C(5,5)=19.85$$

else if(i==3) then

$$C(1,1)=73.59$$

$$C(2,2)=73.59$$

$$C(3,3)=75.37$$

$$C(4,4)=8.40$$

$$C(5,5)=8.40$$

$$C(6,6)=0$$

$$C(1,2)=9.14$$

$$C(1,3)=0.60$$

$$C(1,4)=-9.28$$

$$C(2,1)=9.14$$

$$C(2,3)=0.6$$

$$C(2,4)=9.28$$

$$C(3,1)=0.6$$

$$C(3,2)=0.6$$

$$C(4,1)=-9.28$$

$$C(4,2)=9.28$$

$$C(5,6)=-9.28$$

$$C(6,5)=-9.28$$

else if (i==4) then

$$C(1,1)=15.09$$

$$C(2,2)=15.09$$

$$C(3,3)=15.09$$

$$C(4,4)=7.26$$

$$C(5,5)=7.26$$

$$C(6,6)=7.26$$

$$C(1,2)=13.38$$

$$C(1,3)=13.38$$

$$C(2,1)=13.38$$

$$C(2,3)=13.38$$

$$C(3,1)=13.38$$

$$C(3,2)=13.38$$

else if (i==5) then

$$C(1,1)=58.33$$

$$C(2,2)=65.85$$

$C(3,3)=45.84$

$C(4,4)=16.74$

$C(5,5)=19.63$

$C(6,6)=7.35$

$C(1,2)=-5.87$

$C(1,3)=12.71$

$C(1,5)=1.41$

$C(2,1)=-5.87$

$C(2,3)=6.65$

$C(2,5)=6.22$

$C(3,1)=12.71$

$C(3,2)=6.65$

$C(3,5)=8.10$

$C(5,1)=1.41$

$C(5,2)=6.22$

$C(5,3)=-8.10$

$C(6,4)=8.22$

else if(i==6) then

open(1599,file='Cinputvf.txt',status='old',action='read')

do j=1,6

read(1599,*)C(j,:)

end do

print*, "C matrix from text file is :"

do j=1,6

print*,C(j,:)

end do

end if

end subroutine

subroutine findinverse(C,MI,n) !calculates and returns S matrix

implicit none

integer,intent(in)::n

real,dimension(n,n), intent(in)::C

real,dimension(n,n):: M

real,intent(out)::MI(n,n)

real::L(n,n),U(n,n)

real::x(n),y(n),z(n)

real::fac

integer::i,j,k

M=C

y=0.0

U=0.0

L=0.0

do k=1,n-1

do i=k+1,n

fac=M(i,k)/M(k,k)

L(i,k)=fac

do j=k+1,n

M(i,j)=M(i,j)-(fac*M(k,j))

```
        end do
    end do
end do
```

```
do j=1,n
    do i=1,j
         $U(i,j)=M(i,j)$ 
    end do
end do
```

```
end do
do i=1,n
     $L(i,i)=1.0$ 
end do
```

```
do k=1,n
     $y(k)=1.0$ 
     $z(1)=y(1)$ 
    do i=2,n
         $z(i)=y(i)$ 
        do j=1,i-1
             $z(i)=z(i)-(L(i,j)*z(j))$ 
        end do
    end do
     $x(n)=z(n)/U(n,n)$ 
    do i=n-1,1,-1
         $x(i)=z(i)$ 
    end do
end do
```

```

do j=n,i+1,-1
  x(i)=x(i)-(U(i,j)*x(j))
end do
  x(i)=x(i)/U(i,i)
end do

```

```

do i=1,n
  MI(i,k)=x(i)
end do
y(k)=0.0

```

```

end do

```

```

end subroutine findinverse

```

subroutine reuss(S,Br,Gr) !subroutine which return Br and Gr value according to Ma'am's paper

```

implicit none

```

```

real,dimension(6,6),intent(in)::S

```

```

real,intent(inout)::Br,Gr

```

```

real:: st

```

```

integer:: i

```

```

Br = 1/(S(1,1) + S(2,2) + S(3,3) + 2*(S(1,2) + S(1,3) + S(2,3)))

```

```

Gr = 15/(4*(S(1,1) + S(2,2) + S(3,3))- 4*(S(1,2) + S(1,3) + S(2,3)) + 3*(S(4,4) + S(5,5) + S(6,6)))

```

end subroutine

subroutine reussphysrev(C,Br1,Gr1,st) !subroutine which return Br and Gr value according to Zhi's paper

implicit none

real,dimension(6,6)::C

real,intent(inout)::Br1,Gr1

real::M,C2,Bv1,a,b,c1,d,e,f,g, del

integer,intent(in):: st

!If st=1 then its cubic, st=2 implies hexagonal, st=3 implies tetragonal,st=4 implies orthorhombic,st=5 implies monoclinic

if (st==1) then

$$Br1 = (C(1,1) + 2 * C(1,2)) / 3$$

$$Gr1 = (5 * (C(1,1) - C(1,2)) * C(4,4)) / (4 * C(4,4) + 3 * (C(1,1) - C(1,2)))$$

else if (st==2) then

$$C2 = (C(1,1) + C(1,2)) * C(3,3) - 2 * C(1,3) ** 2$$

$$M = C(1,1) + C(1,2) + 2 * C(3,3) - 4 * C(1,3)$$

$$Bv1 = (2 * (C(1,1) + C(1,2)) + 4 * C(1,3) + C(3,3)) / 9$$

$$Br1 = C2 / M$$

$$Gr1 = (5/2) * (C2 * C(4,4) * C(6,6)) / (3 * Bv1 * C(4,4) * C(6,6) + C2 * (C(4,4) + C(6,6)))$$

else if (st==3) then

$$C2 = (C(1,1) + C(1,2)) * C(3,3) - 2 * C(1,3) ** 2$$

$$M = C(1,1) + C(1,2) + 2 * C(3,3) - 4 * C(1,3)$$

$$Bv1 = (2 * (C(1,1) + C(1,2)) + 4 * C(1,3) + C(3,3)) / 9$$

$$Br1 = C2 / M$$

$$Gr1 = 15 / ((18 * Bv1 / C2 + (6 / (C(1,1) - C(1,2)))) + (6 / C(4,4)) + (3 / C(6,6)))$$

else if(st==4) then

$$\text{del} = C(1,3) * (C(1,2) * C(2,3) - C(1,3) * C(2,2) + C(2,3) * (C(1,2) * C(1,3) - C(2,3) * C(1,1))) \\ + C(3,3) * (C(1,1) * C(2,2) - C(1,2)) ** 2)$$

$$\text{Br1} = \text{del} / (C(1,1) * C(2,2) + C(3,3) - 2 * C(2,3)) + C(2,2) * (C(3,3) - 2 * C(1,3)) - 2 * C(3,3) * C(1,2) + \\ C(1,2) * (2 * C(2,3) - C(1,2)) + \&$$

$$C(1,3) * (2 * C(1,2) - C(2,3) * (2 * C(1,3) - C(2,3)))$$

$$\text{Gr1} = 15 / (4 * (C(1,1) * (C(2,2) + C(3,3) + C(2,3)) + C(2,2) * (C(3,3) + C(1,3)) + C(3,3) * C(1,2) - \\ C(1,2) * (C(2,3) + C(1,2)) \&$$

$$- C(1,3) * (C(1,2) + C(1,3)) - C(2,3) * (C(1,3) + C(2,3))) / \text{del} + 3 * ((1 / C(4,4)) + (1 / C(5,5)) \\ + (1 / C(6,6))))$$

else if(st==5) then

$$a = C(3,3) * C(5,5) - C(3,5) ** 2$$

$$b = C(2,3) * C(5,5) - C(2,5) * C(3,5)$$

$$c1 = C(1,3) * C(3,5) - C(1,5) * C(3,3)$$

$$d = C(1,3) * C(5,5) - C(1,5) * C(3,5)$$

$$e = C(1,3) * C(2,5) - C(1,5) * C(2,3)$$

$$f = C(1,1) * (C(2,2) * C(5,5) - C(2,5) ** 2) - C(1,2) * (C(1,2) * C(5,5) - C(1,5) * C(2,5)) + \\ C(1,5) * (C(1,2) * C(2,5) \&$$

$$- C(1,5) * C(2,2)) + C(2,5) * (C(2,3) * C(3,5) - C(2,5) * C(3,3))$$

$$g = C(1,1) * C(2,2) * C(3,3) - C(1,1) * C(2,3) ** 2 - C(2,2) * C(1,3) ** 2 - C(3,3) * C(1,2) ** 2 + \\ 2 * C(1,2) * C(1,3) * C(2,3)$$

$$\text{del} = 2 * (C(1,5) * C(2,5) * (C(3,3) * C(1,2) - C(1,3) * C(2,3))) + C(1,5) * C(3,5) * (C(2,2) * C(1,3) - \\ C(1,2) * C(1,3)) + \&$$

$$C(2,5) * C(3,5) * (C(1,1) * C(2,3) - C(1,2) * C(1,3))) - (C(1,5) ** 2 * (C(2,2) * C(3,3) - C(2,3) ** 2) \&$$

$$+ C(2,5) ** 2 * (C(1,1) * C(3,3) - C(1,3) ** 2) + C(3,5) ** 2 * (C(1,1) * C(1,2) - C(1,2) ** 2)) \\ + g * C(5,5)$$

$$\text{Br1} = \text{del} / (a * (C(1,1) + C(2,2) - 2 * C(1,2)) + b * (2 * C(1,2) - 2 * C(1,1) - C(2,3)) + c1 * (C(1,5) - \\ 2 * C(2,5)) + d * (2 * C(1,2) \&$$

```

+ 2*C(2,3) - C(1,3) - 2*C(2,2)) + 2*e*(C(2,5) - C(1,5))+f)

Gr1 = 15/(4*(a*(C(1,1)+C(2,2)+C(1,2))+b*(C(1,1)-C(1,2)-C(2,3)) +c1*(C(1,5)+C(2,5))
+d*(C(2,2)-C(1,2)-C(2,3)-C(1,3))+ &

e*(C(1,5)- c(1,5)) +f)/del + 3*((g/del) +(C(4,4) +C(6,6))/(C(4,4)*C(6,6)- C(4,6)**2)))

end if

end subroutine

subroutine voigt(C,Bv,Gv) !subroutine which return Bv and Gv value according to Ma'am's
paper

implicit none

real,dimension(6,6),intent(in)::C

real,intent(out)::Bv,Gv

! print*,C(1,1),C(2,2),C(3,3),C(1,2),C(1,3),C(2,3)

Bv = (C(1,1) + C(2,2) + C(3,3)+ 2*(C(1,2) + C(1,3) + C(2,3)))/9

Gv = (C(1,1) + C(2,2) + C(3,3)- (C(1,2) + C(1,3) + C(2,3)) + 3*(C(4,4) + C(5,5) + C(6,6)))/15

end subroutine

subroutine voigtphysrev(C,Bv1,Gv1,st) !subroutine which return Bv and Gv value according
to Zhi's paper

implicit none

real,dimension(6,6)::C

real,intent(inout)::Bv1,Gv1

```

real::M

integer,intent(in):: st

!If st=1 then its cubic, st=2 implies hexagonal,

!st=3 implies tetragonal,st=4 implies orthorhombic,st=5 implies monoclinic

if (st==1) then

$$Bv1 = (C(1,1) + 2*C(1,2))/3$$

$$Gv1 = (C(1,1) - C(1,2) + 3*C(4,4))/5$$

else if (st==2) then

$$M = C(1,1) + C(1,2) + 2*C(3,3) - 4*C(1,3)$$

$$Bv1 = (2*(C(1,1) + C(1,2)) + 4*C(1,3) + C(3,3))/9$$

$$Gv1 = (M + 12*C(4,4) + 12*C(6,6))/30$$

else if (st==3) then

$$M = C(1,1) + C(1,2) + 2*C(3,3) - 4*C(1,3)$$

$$Bv1 = (2*(C(1,1) + C(1,2)) + 4*C(1,3) + C(3,3))/9$$

$$Gv1 = (M + 3*C(1,1) - 3*C(1,2) + 12*C(4,4) + 6*C(6,6))/30$$

else if(st==4) then

$$Bv1 = (C(1,1) + C(2,2) + C(3,3) + 2*(C(1,2) + C(1,3) + C(2,3)))/9$$

$$Gv1 = (C(1,1) + C(2,2) + C(3,3) + 3*(C(4,4) + C(5,5) + C(6,6)) - (C(1,2) + C(1,3) + C(2,3)))/15$$

else if(st==5) then

$$Bv1 = (C(1,1) + C(2,2) + C(3,3) + 2*(C(1,2) + C(1,3) + C(2,3)))/9$$

$$Gv1 = (C(1,1) + C(2,2) + C(3,3) + 3*(C(4,4) + C(5,5) + C(6,6)) - (C(1,2) + C(1,3) + C(2,3)))/15$$

end if

end subroutine