An Introduction to Complex Networks - Part 3

Malayaja Chutani
(IIT Madras)

Dynamics of Complex Systems 2018

Network Construction

Erdős-Rényi random network:

- Start with N nodes.
  Connect each pair of nodes with probability p.
- This creates a graph with $\frac{pN(N-1)}{2}$ edges, on average.
- In a random graph with connection probability p, the degree k_i of a node i follows a binomial distribution:
  $P(k_i = k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$.
  This probability represents the number of ways in which k edges can be drawn from a node i.

- Let X_k = number of nodes with degree k.
  We want to find the probability that X_k takes on a given value, $P(X_k = r)$.
  Expectation value of number of nodes with degree k:
  $E(X_k) = NP(k_i = k) = N\binom{N-1}{k}p^k(1-p)^{N-1-k} = \lambda_k$.
- For large N, $P(k) \approx e^{-\langle k \rangle}\frac{\langle k \rangle^k}{k!}$

Small-world network:

- Watts and Strogatz (1998) showed a way to construct networks in between regular and random, i.e. with high clustering coefficients and small characteristic path lengths.
- Start with a ring lattice with N nodes and k edges per node.
- Rewire each edge randomly with probability p, excluding self-connections and duplicate edges.
  $p = 0$ corresponds to regularity, $p = 1$ corresponds to disorder.

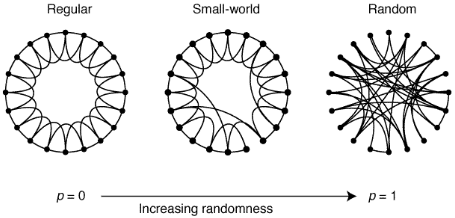


Figure: From Watts and Strogatz, 1998

- Random connections act like shortcuts, and connect distant parts of the graph.
- Small-world networks are highly clustered, like regular networks, and have small characteristic path lengths, like random graphs.

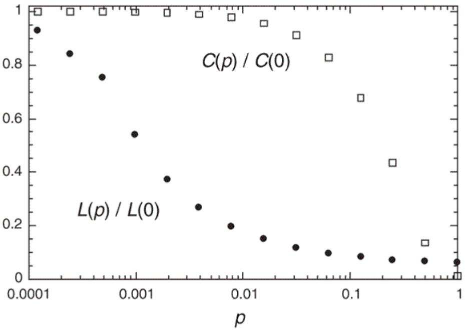


Figure: From Watts and Strogatz, 1998

Scale-free network:

- Most real-world networks grow by the adding new nodes, and likelihood of connecting to a node depends on the node's degree, i.e. preferential attachment (Barabási and Albert, 1999).

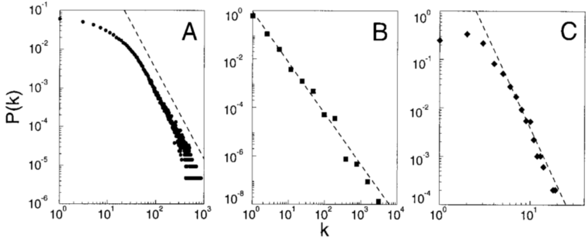
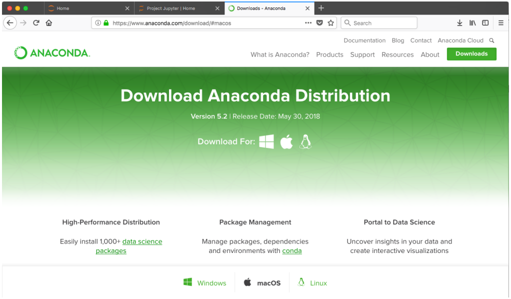


Figure: Degree distributions for (A) actor collaboration graph: $N = 212250$, $\langle k \rangle = 28.78$, $\gamma = 2.3$, (B) WWW: $N = 325729$, $\langle k \rangle = 5.46$, $\gamma = 2.1$, (C) power grid data: $N = 4941$, $\langle k \rangle = 2.67$, $\gamma = 4$. From Barabási and Albert, 1999.

- Start with a small number (m_0) of nodes.
  At every time step, add a new node with $m \leq m_0$ edges.
  A new node will link to an existing node with probability
  $\Pi(k_i) = k_i / \Sigma_j k_j$.
- After t time steps this procedure results in a network with $N = t + m_0$ nodes and mt edges.
- It has been reported that this network evolves such that the degree distribution follows a power law.

Networkx Tutorial



References for Networkx Tutorial

- https://networkx.github.io/documentation/stable/index.html
- http://snap.stanford.edu/class/cs224w-2012/nx_tutorial.pdf
- https://networkx.github.io/documentation/stable/auto_examples/drawing/plot_degree_histogram.html

For more information, see:

- D. J. Watts and S. H. Strogatz, Collective dynamics of 'small-world' networks, Nature 393, 440 (1998).
- A.-L. Barabaśi and R. Albert, Emergence of Scaling in Random Networks, Science, 286, 509 (1999).
- R. Albert and A.-L. Barabási, Statistical mechanics of complex networks, Rev. Mod. Phys. 74, 47 (2002).
- M. E. J. Newman, The Structure and Function of Complex Networks, SIAM Rev. 45, 2 (2003).
- M. E. J. Newman, Networks: An Introduction (Oxford University Press, New York, 2010).
- A.-L. Barabási, Network Science. `http://networksciencebook.com/`