FPYLLL

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- fpylll is a Python library for performing lattice reduction on lattices over the Integers.
- It is based on the fplll
- fpylll also implements a few algorithms beyond fplll and provides some interface niceties

Mission

Make implementing lattice-reduction strategies so easy, we can demand people to publish their code.

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Make implementing lattice-reduction strategies so easy, even I can do it.

Highlevel Interface

First of all, fpylll is a thin wrapper around fplll. In the example below, we first generate an NTRU-like matrix and consider the norm of the first row:

```
from fpylll import IntegerMatrix, LLL
q = 1073741789
A = IntegerMatrix.random(30, "ntrulike", bits=30, q=q)
A[0].norm()
```

3294809651.09

We then call LLL reduction and observe the output:

```
LLL.reduction(A)
A[0].norm()
```

82117.5815888

If LLL reduction isn't strong enough, we can call the BKZ algorithm for some block size k.

```
from fpylll import BKZ
BKZ.reduction(A, o=BKZ.Param(block_size=10))
A[0].norm()
```

71600.8858744

Highlevel Interface

Or SVP directly.

```
from fpylll import SVP
q = 1073741789
B = IntegerMatrix.random(10, "ntrulike", bits=7, q=127)
SVP.shortest_vector(B)
```

(0, 2, -6, -6, 0, 0, 3, 2, -5, 1, 0, 5, -5, -3, 0, -1, -3, 3, -5, -7)

Lowlevel Interface

```
from fpylll import IntegerMatrix, GSO
q = 1073741789
A = IntegerMatrix.random(30, "ntrulike", bits=30, q=q)
M = GSO.Mat(A)
M.update_gso()
```

True

```
M.get_r(0,0)
```

1.26228336805e+19

Lowlevel Interface

```
A[2][3] + 2*A[3][3]
```

2

```
with M.row_ops(2,4):
    M.row_addmul(2,3,2)
```

```
A[2][3]
```

2

Coverage

```
bkz.pyx
enumeration.pyx
gso.pyx
integer_matrix.pyx
lll.pyx
svpcvp.pyx
wrapper.pyx
```

Of course, fpylll being a Python library means you can use your favourite Python libraries with it.

For example, say, we want to LLL reduce many matrices in parallel, using all our cores, and to compute the norm of the shortest vector across all matrices after LLL reduction.

Multicore

We'll make use of Python's multiprocessing:

```
from multiprocessing import Pool
```

For this example, we want dimension 40, four worker processes and 32 matrices:

```
from fpylll import *
q = 1073741789
workers = 4
tasks = 32
A  = []

for i in range(tasks):
    A.append(IntegerMatrix.random(20, "ntrulike", bits=30, q=q))
```

32

Multicore

Let's get to work: we create a pool of workers and kick off the computation:

```
pool = Pool(workers)
A = pool.map(LLL.reduction, A)
```

Finally, we output the minimal norm found:

```
min([A_[0].norm() for A_ in A])
```

7194.54515588

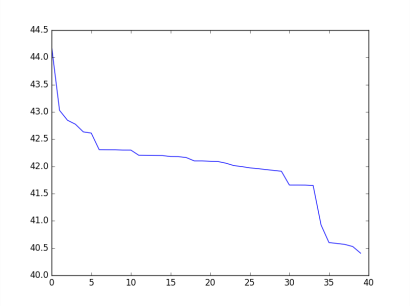
Plotting

```
from fpylll import IntegerMatrix, GSO
from math import log
q = 1073741789
A = IntegerMatrix.random(40, "ntrulike", bits=30, q=q)
M = GSO.Mat(A)
M.update_gso()
gso = [log(M.get_r(i,i)) for i in range(40)]
```

True

Plotting

```
import matplotlib
import matplotlib.pyplot as plt
plt.plot(gso)
plt.savefig('gso.png')
'gso.png' # return this to org-mode
```



Developing Lattice-Reduction Algorithms

- The main objective of fpylll is to make developing and experimenting with the kind of algorithms implemented in fplll easier.
- For example, there are a few variants of the BKZ algorithm in the literature which essentially re-combine the same building blocks — LLL and an SVP oracle — in some way.
- These kind of algorithms should be easy to implement.

The code below is an implementation of the plain BKZ algorithm in 70 lines of Python.

BKZ

```
from fpylll import IntegerMatrix, GSO, LLL, BKZ
from fpylll import Enumeration as Enum
from fpylll import gso
```

BKZ

```
class BKZReduction:
    def __init__(self, A):
        wrapper = LLL.Wrapper(A)
        wrapper()

        self.A = A
        self.M = GSO.Mat(A, flags=gso.GSO.ROW_EXPO)
        self.lll_obj = LLL.Reduction(self.M)
```

BKZ

```
    def __call__(self, block_size):
        self.M.discover_all_rows()

        while True:
            clean = self.tour(block_size, 0, self.A.nrows)
            if clean:
                break
```

BKZ

```
def tour(self, block_size, min_row, max_row):
    clean = True
    for kappa in range(min_row, max_row-1):
        bs = min(block_size, max_row - kappa)
        clean &= self.svp_reduction(kappa, bs)
    return clean
```

BKZ

```
def svp_reduction(self, kappa, block_size):
    clean = True

    self.lll_obj(0, kappa, kappa + block_size)
    if self.lll_obj.nswaps > 0:
        clean = False

    max_dist, expo = self.M.get_r_exp(kappa, kappa)
    delta_max_dist = self.lll_obj.delta * max_dist

    solution, max_dist = Enum.enumerate(self.M, max_dist, expo, \
                                        kappa, kappa + block_size, None)

    if max_dist >= delta_max_dist:
        return clean
```

BKZ

```python
        nonzero_vectors = len([x for x in solution if x])

        if nonzero_vectors == 1:
            first_nonzero_vector = None
            for i in range(block_size):
                if abs(solution[i]) == 1:
                    first_nonzero_vector = i
                    break

            self.M.move_row(kappa + first_nonzero_vector, kappa)
            self.lll_obj.size_reduction(kappa, kappa + 1)
```

BKZ

```
        else:
            d = self.M.d
            self.M.create_row()

            with self.M.row_ops(d, d+1):
                for i in range(block_size):
                    self.M.row_addmul(d, kappa + i, solution[i])

            self.M.move_row(d, kappa)
            self.lll_obj(kappa, kappa, kappa + block_size + 1)
            self.M.move_row(kappa + block_size, d)

            self.M.remove_last_row()

        return False
```

Beyond fplll

- In the meantime fpylll has gained a `contrib` module which implements additional algorithms.
- As of writing, it contains
    - a simple demo implementation of BKZ (see above),
    - a simple implementation of Dual BKZ and a slightly feature enhanced re-implementation of fplll's BKZ which collects additional statistics compared to fplll's implementation of the same algorithm.

Beyond fplll

```
from copy import copy
from fpylll import BKZ
from fpylll.contrib.bkz import BKZReduction

C = copy(A)
b = BKZReduction(C)
b(BKZ.Param(block_size=30, flags=BKZ.AUTO_ABORT|BKZ.VERBOSE))
stats = b.stats; stats
```

{"i": 25, "total": 12.01, "time": 0.40, "preproc": 0.10, "svp": 0.10, "r_0": 7.3483e+09, "slope": -0.0538, "enum nodes": 20.31, "max(kappa)": 6}

Beyond fplll

That output isn't that different from fplll outputs. However, in contrast to fplll (because I didn't bother to implement it over there, yet) we also get access to a `stats` object after the computation finished. Let's use it to inquire how many nodes where visited during enumeration

```
stats.enum_nodes
```

39977944

and how much time we spent in enumeration:

```
stats.svp_time
```

3.119223

Integration with other Projects

fpylll integrates reasonably nicely with Sage: converting back and forth between data types is seamless. For example:

```
sage: A = random_matrix(ZZ, 10, 10)
sage: from fpylll import IntegerMatrix, LLL
sage: B = IntegerMatrix.from_matrix(A)
sage: LLL.reduction(B)
sage: B.to_matrix(A)[0]
```

(-2, 1, 0, -1, 0, 0, 1, -2, 0, 0)

In fact, when installed inside Sage, element access for `IntegerMatrix` accepts and returns `sage.rings.integer.Integer` directly, instead of Python integers.

```
sage: type(B[0,0])
<type 'sage.rings.integer.Integer'>
```

Integration with other Projects

fpylll also integrates somewhat with NumPy.

```
from fpylll import *
A = IntegerMatrix.random(4, "ntrulike", bits=7, q=127)
```

We'd like to do some analysis on its Gram-Schmidt matrix, so let's compute it:

```
sage: M = GSO.Mat(A)
sage: M.update_gso()
```

Let's dump it into a NumPy array and spot check that the result is reasonably close:

```
import numpy
from fpylll.numpy import dump_mu
N = numpy.ndarray(dtype="double", shape=(8,8))
dump_mu(N, M, 0, 8)
N[1,0] - M.get_mu(1,0)
```

0.0

Let's do something more or less useful with our output:

```
numpy.linalg.eigvals(N)
```

[-7.99122854e-40 +0.00000000e+00j -5.65065189e-40 +5.65065189e-40j -5.65065189e-40 -5.65065189e-40j -8.15663058e-56 +7.99122854e-40j -8.15663058e-56 -7.99122854e-40j 7.99122854e-40 +0.00000000e+00j 5.65065189e-40 +5.65065189e-40j 5.65065189e-40 -5.65065189e-40j]

Tests

fpylll runs tests on every check-in for Python 2 and 3. As an added benefit, this extends test coverage for fplll as well, which only has a few highlevel tests.

```
def test_lll_lll():
    for m, n in dimensions:
        A = make_integer_matrix(m, n)
        b00 = []
        for float_type in float_types:
            B = copy(A)
            M = GSO.Mat(B, float_type=float_type)
            lll = LLL.Reduction(M)
            lll()
            if (m, n) == (0, 0):
                continue
            b00.append(B[0, 0])
        for i in range(1, len(b00)):
            assert b00[0] == b00[i]
```

"This is all nice and well", I hear you say, "but I prefer to do my computations in Lisp, so thanks, but not thanks".

https://imgs.xkcd.com/comics/lisp_cycles.png

No worries, Hy has you covered:

```
=> (import [fpylll [*]])
=> (setv q 1073741789)
=> (setv A (.random IntegerMatrix 30 "ntrulike" :bits 30 :q q))
=> (car A)
row 0 of <IntegerMatrix(60, 60) at 0x7f1cbbfbf888>
=> (get A 1)
row 1 of <IntegerMatrix(60, 60) at 0x7f1cbbfbf888>
=> (-> (car A) (.norm))
4019682565.5285482

=> (.reduction LLL A)
=> (.norm (car A))
6937.9845776709535
```

Help Wanted

- fpylll isn't quite done yet.
- Besides testing and documentation, it would be nice if someone would attempt to re-implement fplll's LLL wrapper in pure Python.
- This would serve as a test case to see if everything that's needed really is exposed and as a starting point for others who like to tweak the strategy.
- Speaking of LLL, fpylll is currently somewhat biased towards playing with BKZ, i.e. it would be nice to see how useful it is for trying out tweaks to the LLL algorithm.

Fin

Thank You