Tutorial 7

You may use the facts, proven in MATH102, that for all $x \in \mathbb{R}$, $\frac{d}{dx}(e^x) = e^x$ and that if $x > 0$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

Question 1.

Differentiate the following functions.

(a) $f\colon \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto x^2 e^x$

(b) $f\colon \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

(c) $f\colon \mathbb{R} \longrightarrow \mathbb{R}, \quad y \longmapsto \ln\left(y + \sqrt{y^2+1}\right)$

Question 2. Suppose that the function $f\colon X \to \mathbb{R}, \quad x \mapsto y$ is differentiable. Find $\frac{dy}{dx}$ where possible and explain any restrictions if, for all $x \in X$,

(a) $x^2 + y^2 = 1$

(b) $xy^2 + y\sin(xy) + e^x = 0$

(c) $e^{2x} - 2ye^x - 1 = 0$

Question 3. Verify that the function

$$f\colon \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto x^3 - 6x^2 + 3x - 7$$

satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 6]$ find the values of c that satisfy the conclusion of the theorem.

Question 4.

Use the Mean Value Theorem to show that if $f\colon \mathbb{R} \to \mathbb{R}$ satisfies $f'(x) = 0$ for all $x \in \mathbb{R}$, then f must be a constant function.
Find an example of a function $f\colon X \to \mathbb{R}$ with $X \subseteq \mathbb{R}$ such that $f'(x) = 0$ for all $x \in X$, but f is not constant.