AMTH250 Assignment 4

Due: 29th August

Question 1 *[2 marks]*

(a) Plot the graph of $f(x) = (x-1)(x-3)$ over the interval $x \in [0,4]$.

(b) Display the x-axis and mark the roots with a $\times$.

Question 2 *[2 marks]*

(a) Plot the graph of $f(x) = x \ln x$ over the interval $x \in [0,2]$.

(b) What has happened at $x = 0$? How do you fix it?

Question 3 *[3 marks]*

(a) Plot the graph of $f(x) = \dfrac{1}{1-x^2}$ over the interval $x \in [-2,2]$.

(b) Rescale the y-axis to give a better graph.

(c) The function $f(x)$ has singularities at $x = \pm 1$. Depending on the values you chose for x, your graph in (b) may or may not give the appearance of showing the vertical asymptotes at the singularities. Can you produce graphs where these asymptotes do and do not appear?

Question 4 *[2 marks]*

In floating point arithmetic, which of the following operations on two positive floating point numbers can produce an overflow?

(a) Addition

(b) Subtraction

(c) Multiplication

(d) Division

Question 5 *[4 marks]*

In the notes on floating point arithmetic we examined the error in the forward difference approximation to the derivative

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

(a) Repeat the example in the notes, this time using the central difference approximation

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

(b) How do

(i) the minimum error, and

(ii) the value of h at which the minimum error occurs

compare for the two methods?

(c) Produce log-log plots of error against h for the two methods on the one figure.

(d) Using the plot in (c), relate your answer to (b) to the way the truncation and cancellation errors behave for each method.

Question 6 *[3 marks]*

Consider the function

$$f(x) = \frac{e^x - 1}{x}$$

(a) Use l'Hopital's rule to show that

$$\lim_{x \to 0} f(x) = 1$$

(b) Compute $f(x)$ for $x = 10^{-k}$, $k = 1, \ldots 15$. Do your results agree with theoretical expectations? Explain why.

Doing Assignment 4

Make sure you have successfully completed Assignment 3 before you attempt Assignment 4.

1 What to Submit

1. Include the Octave commands you used in answering questions in your assignment. For example, when you include a graph, make sure you include the Octave commands to produce the graph.

2. If you used a script file for some question, you may chose to put it in an appendix to the assignment rather than in the body of the question.

3. There is no need to submit your scripts as seperate files, but your `.tex` file should be submitted. I can, if necessary, easily reproduce your calculations by copying from your `.tex` file into Octave.

2 Getting Help

Not everything you need to do this assignment has been spelled out explicitly. Make sure you know how to use Octave's `help` and access Octave/Matlab documentation. An important part of becoming proficient in Octave is knowing how to look things up for yourself. On the other hand, some things can be puzzling for beginners (and experts!) so make use of the Octave Discussion Forum on Moodle.

3 The Dot Operator

To graph the function $f(x) = x^2$ we might try the following:

```
octave:> x=linspace(-1,1,100);
octave:> f = x^2;
error: for A^b, A must be square
```

We forgot the dot operator:

```
octave:> f = x.^2;
octave:> plot(x, f)
```

and now it works.

To understand the role of the dot operator it needs to kept in mind that vectors and matrices – and vectors are just matrices with one row or one column – are used in two ways: (a) as vectors and matrices in the mathematical sense you learn in linear algebra, and (b) as arrays to store data. Arithmetic operations on matrices (and vectors) can take two forms corresponding to these two contexts, i.e. as mathematical matrices or as data arrays.

For example:

```
octave:> a= [1 2; 3 4]
a =
   1   2
   3   4

octave:> a*a
ans =
    7   10
   15   22

octave:> a.*a
ans =
    1    4
    9   16
```

The operation `a*a` is matrix multiplication, the operation `a.*a` performs element-by-element multiplication.

The dot operator `.` is used in conjunction with the operators `*`, `/` and `^` to perform element by element multiplication, division and exponentiation on vectors and matrices. The plain operators `*` and `^` are matrix multiplication and matrix exponentiation. The operators `/` and `\` applied to matrices are used to solve solve systems of linear equations. Addition and subtraction are the same for data arrays and matrices, so there is no need for a corresponding dot operator.

3.1 Dot Operators and Graphs

To graph a function $f(x)$ in Octave we usually begin by creating a vector or array of x values, e.g. `x = linspace(-10, 10, 200)`. We then evaluate the function $f(x)$ at the array of x values to get an array of values of $f(x)$ and use these arrays to plot our graph. Because the vectors are being used as data arrays, the dot operators should be used for arithmetic operations in constructing the vector of function values. For example; to graph the function $f(x) = x \sin x$ we proceed as as follows:

```
octave:> x = linspace(-10, 10, 200);
octave:> y = x.*sin(x);
octave:> plot(x,y)
```

[Recall that functions such as `sin` act element-by-element when applied to vectors and matrices.]

4 Script Files

Script file are *plain text* files containing a sequence of Octave commands and provide an alternative to entering Octave commands interactively.

A script file, called say `plotsin.m`, to graph $\sin x$ might contain

```
% plot sin(x) over the interval[-10,10]
x = linspace(-10, 10, 200);
y = x.*sin(x);
plot(x,y)
```

Then, at the Octave prompt, the command

```
octave:> plotsin
```

will execute the commands in the script and plot the graph. The comment in script is used by `help`

```
octave:> help plotsin
'plotsin' is a script from the file /home/gbunting/amth250/...

 plot sin(x) over the interval[-10,10]

......
```

Remember:

1. Script files should have the `.m` extension and the name should not have spaces in it.
2. Use a plain text editor such as Notepad to create and edit script files.
3. Make sure script files are saved to a folder where Octave knows where to find them (c.f. Notes to Assignment 3).

5 Hints for Assignment 4

Make sure you understand dot operators.

Question 1

Use `hold on` to add the x axis and marks to the graph. If you are unsure how to draw the $\times$'s, try `help plot`.

Question 2

To do part (b) first work out how has Octave evaluated $f(x)$ at $x = 0$ and what `plot` has done with this value.

Question 3

The Octave Tutorial §5.4 shows how to rescale axes. You may well find part (c) difficult; if so don't waste too much time on it. Your answer to Question 2(b) may provide a hint.

Question 4

A Yes or No answer for each part will do. This question is not difficult or tricky.

Question 5

Here is a script file to perform the forward difference approximation as in the notes on floating point:

```
% Forward difference approximation to f'(x) for f(x) = sin(x) at x = 1.
n = -2:-1:-17;
h = 10 .^n;
exact = cos(1);
approx = (sin(1+h) - sin(1))./h;
err = abs(approx - exact) / exact;
loglog(h, err)
xlabel('ln h')
ylabel('ln Error')
```

For the central difference approximation, only the line defining `approx` needs to be changed.

Giving a good answer to (d) requires some thought about how the error arises as a combination of truncation and cancellation.

Question 6

Besides looking at the computed values of $f(x)$ it is a good idea to also look at $f(x) - 1$.