Tutorial 8

You may use the facts, proven in MATH102, that for all $x \in \mathbb{R}$, $\frac{d}{dx}(e^x) = e^x$ and that if $x > 0$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

Question 1.

Use the Mean Value Theorem to show that for all $x > 0$, $\ln(1 + x) < x$.

Question 2.

Determine the intervals on which the function

$$f : \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto \frac{(x+1)^2}{x^2+1}$$

is monotonically increasing and where it is monotonically decreasing. Find where it is concave up and where it is concave down. Find the extrema of f and sketch its graph.

Question 3.

An lidless container is to be fashioned from square metal sheet, whose side is 30 cm long, by cutting out four congruent smaller squares, one from each corner of the sheet (as illustrated below) and bending up the sides.
Let each side of the four small squares be x cm long. What value for x maximises the volume of the container?