Solutions to the Exercises – Week 5

22/8/06

1. "Verify that the distribution

$$f(x) = 2x,\ 0 < x < 1$$

   is a density."

   If the given distribution function is indeed a density, then

$$\int f(x)dx = 1$$

   Now

$$\int_0^1 2xdx = [x^2]_0^1 = [1^2 - 0^2] = 1$$

   as required, and so the function $f(x)$ is a density.

   Can you check your calculation manually?

   Graphically, the distribution is triangular, base 1 and height 2, and so by the formulae for the area of a right angled triangle the area under the function is $\frac{1}{2} \times 1 \times 2 = 1$ as before.

2. "A form of the *geometric* distribution is

$$P(Y = y) = (1/2)^y,\ y = 1, 2, \ldots, \infty$$

   Verify that this form is a probability function. "

   If the given function is in fact a pf, then

$$\sum_{y=1}^{\infty} P(Y = y) = 1$$

   Now

$$\sum_{y=1}^{\infty} (1/2)^y = 1/2 + 1/4 + 1/8 + \ldots = \frac{1/2}{1 - 1/2} = 1$$

   as the pf forms a geometric progression, hence the name ...

3. "Find the upper and lower quartiles for the distribution in Q1."

   For the lower quartile
   $$\int_0^l 2x dx = 1/4$$
   so
   $$[x^2]_0^l = 1/4, \rightsquigarrow l^2 = 1/4$$
   and so the lower quartile is given by $l = 1/2$.

   To check the corresponding area A of the right angled triangle so formed is
   $$A = \frac{b \times h}{2} = 0.5 \times 2(0.5)/2 = 1/4$$
   as required.

   The upper quartile is given by
   $$\int_0^u 2x dx = 3/4$$
   which gives
   $$[x^2]_0^u = 3/4$$
   ie
   $$u^2 = 3/4 \rightsquigarrow u = \sqrt{3}/2$$
   The corresponding area of the triangle is
   $$bh/2 = \frac{\sqrt{3}}{2} \times 2\frac{\sqrt{3}}{2}/2 = 3/4$$
   as required.

4. "Attempt to find all the quartiles of the pf in Q2."

   For discrete distributions we need to be careful about including or excluding particular values.

   Thus in this case if we define the lower quartile as the value having 1/4 of the population below it, then there is no lower quartile, as $P(Y = 1) = 1/2$. Likewise we might want to take the median as 1, since 1/2 of the distribution is above it, but technically $P(Y < 1) = 0$, as so the median does not exist in the strict definition of having as much of the distribution above as below it.

The upper quartile is 2, since 1/4 of the distribution is above it, since

$$P(Y = 1) + P(Y = 2) = 3/4$$

"Give other ways of describing the distribution using quantiles."

We can form the following Table

x	$P(X = x)$	$P(X \leq x)$	$P(X > x)$	%
1	1/2	1/2	1/2	50
2	1/4	3/4	1/4	25
3	1/8	7/8	1/8	12.5
4	1/16	15/16	1/16	6.25
5	1/32	31/32	1/32	3.125
$\vdots$				

So now if we say wanted to find the 9th decile, this would be given by

$$(1/2)^n = 0.1 \rightsquigarrow 2^n = 10$$

which becomes

$$n = \frac{\ln(10)}{\ln(2)} = \frac{2.3}{0.693} = 3.3$$

in agreement with the Table above.