MATH102 ASSIGNMENT 6

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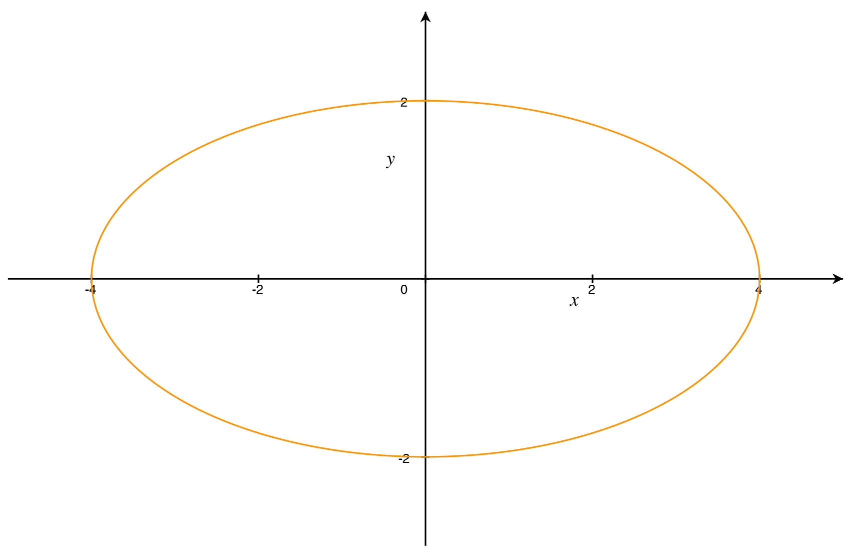
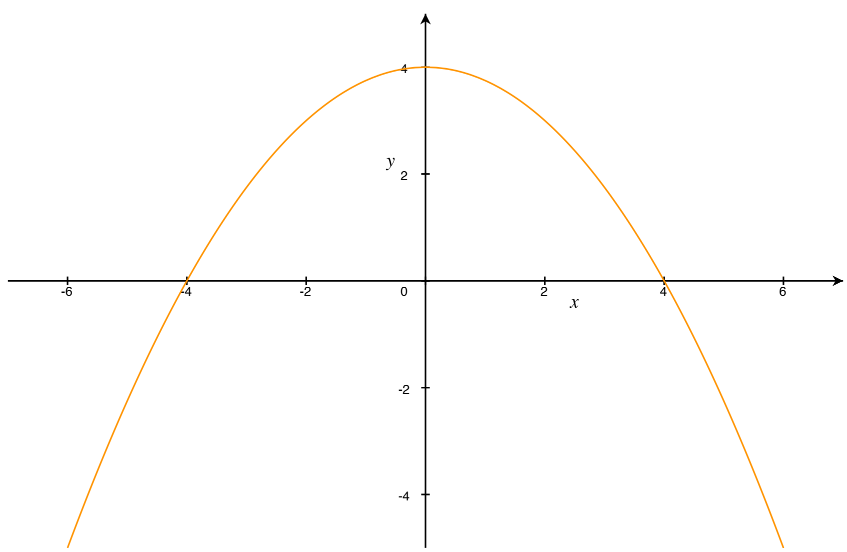
(1) (a) $x^2 + 4y^2 = 16$ is an ellipse. When $y = 0,\ x = \pm 4$ so the curve cuts the x-axis at -4 and 4. When $x = 0,\ y = \pm 2$ so the curve cuts the y-axis at -2 and 2.



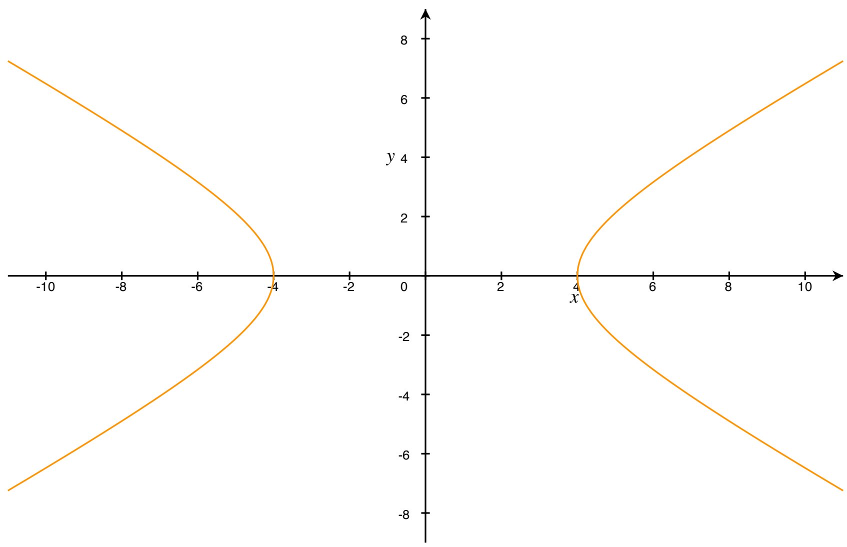
(b) $x^2 + 4y = 16$ is a parabola. The equation can be re-expressed as $y = 4 - \frac{x^2}{4}$. When $y = 0, x = \pm 4$ and when $x = 0,\ y = 4$.



(c) $x^2 - 2y^2 = 16$ is an hyperbola. The equation can be re-expressed as

$$y = \pm\sqrt{\frac{x^2 - 16}{2}}$$

The numerator implies that the curve is undefined for $-4 < x < 4$. Moreover, when $y = 0,\ x = \pm 4$.



(2)

$$x = t - \sin t,\ y = 1 - \cos t,\ 0 \leq t \leq \pi$$

$$\frac{dx}{dt} = 1 - \cos t,\ \frac{dy}{dt} = \sin t$$

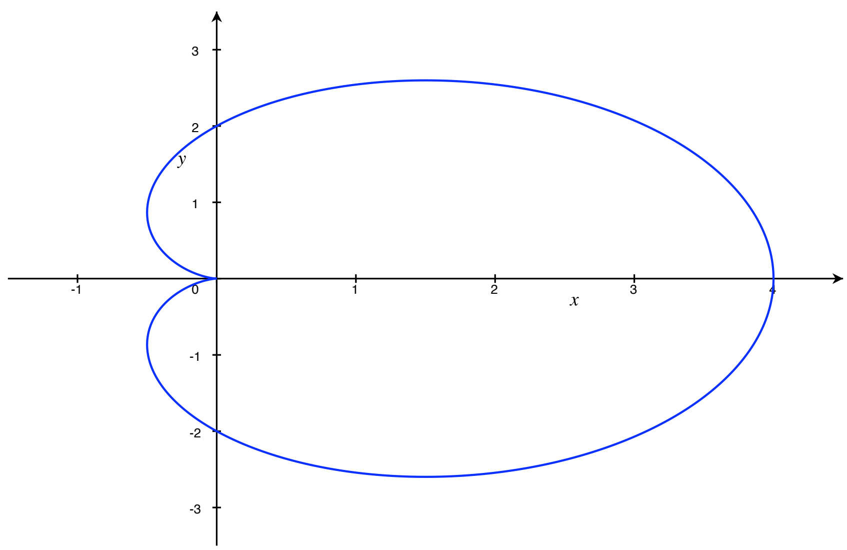
$$\begin{aligned}
L &= \int_0^\pi \sqrt{(1-\cos t)^2 + (\sin t)^2}\ dt \\
&= \int_0^\pi \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\ dt \\
&= \int_0^\pi \sqrt{2 - 2\cos t}\ dt \\
&= \int_0^\pi \sqrt{2 - 2\Big(2\cos^2\left(\frac{t}{2}\right) - 1\Big)}\ dt \\
&= \int_0^\pi \sqrt{4 - 4\cos^2\left(\frac{t}{2}\right)}\ dt \\
&= \int_0^\pi \sqrt{4\Big(1 - \cos^2\left(\frac{t}{2}\right)\Big)}\ dt
\end{aligned}$$

$$= 2\int_0^{\pi} \sqrt{\sin^2\left(\frac{t}{2}\right)}\ dt \ = \ 2\int_0^{\pi} \sin\left(\frac{t}{2}\right)\ dt$$
$$= 2\cdot 2\int_0^{\pi} \frac{1}{2}\sin\left(\frac{t}{2}\right)\ dt \ = \ 4\left[-\cos\left(\frac{t}{2}\right)\right]_0^{\pi}$$
$$= -4\left(\cos\frac{\pi}{2} - \cos 0\right) = -4(0-1) = 4$$

(3) $$r = 2 + 2\cos\theta,\ x = (2+2\cos\theta)\cos\theta,\ y = (2+2\cos\theta)\sin\theta$$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	4	0	0	0	4
y	0	2	0	-2	0

(a) The graph of the cardioid in the xy-plane is given below.



(b)

$$A = \int_0^{2\pi} \frac{1}{2}\big(2+2\cos\theta\big)^2\ d\theta = \frac{1}{2}\int_0^{2\pi} \big[2\big(1+\cos\theta\big)\big]^2\ d\theta$$
$$= \frac{1}{2}\int_0^{2\pi} 4\big(1+\cos\theta\big)^2\ d\theta = 2\int_0^{2\pi} (\cos^2\theta + 2\cos\theta + 1)\ d\theta$$
$$= 2\int_0^{2\pi} \left(\frac{1+\cos 2\theta}{2} + 2\cos\theta + 1\right)\ d\theta$$
$$= 2\int_0^{2\pi} \left(\frac{1}{2}\cos 2\theta + 2\cos\theta + \frac{3}{2}\right)\ d\theta$$

$$
\begin{aligned}
&= 2\int_0^{2\pi} \left(\frac{1}{4}\cdot 2\cos 2\theta + 2\cos\theta + \frac{3}{2}\right)\, d\theta \\
&= 2\left[\frac{1}{4}\sin 2\theta + 2\sin\theta + \frac{3}{2}\theta\right]_0^{2\pi} \\
&= 2\left(\frac{1}{4}\sin 4\pi + 2\sin 2\pi + \frac{3}{2}\cdot 2\pi - 0\right) \\
&= 2\left(\frac{1}{4}\cdot 0 + 2\cdot 0 + 3\pi\right) = 6\pi
\end{aligned}
$$

(4)

$$
\begin{aligned}
\cos x &= S_4(x) + R_4(x),\ |x| < R \\
\cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + R_4(x) \\
\cos(-0.01) &\approx 1 - \frac{(0.01)^2}{2} + \frac{(0.01)^4}{24} \\
&\approx 0.9999500004 \text{ (to 10 decimal pl.)}
\end{aligned}
$$

Since $|f^{(5)}(\xi)|$ is either $\cos(\xi)$ or $\sin(\xi)$ and hence ≤ 1, then

$$
\begin{aligned}
R_4(x) &= \frac{f^{(5)}(\xi)}{5!}\, x^5 \ \Rightarrow\ |R_4(x)| \leq \frac{|x^5|}{5!} \\
|R_4(-0.01)| &\leq \frac{|0.01^5|}{5!} \approx 8.\dot{3}\times 10^{-13}
\end{aligned}
$$

(5) (a) Using the geometric series $a + ax + ax^2 + ax^3 + ... + ax^n = \dfrac{a}{1-x}$ for $|x| < 1$,

$$
\begin{aligned}
0.312312... &= 0.312 + 0.000312 + 0.000000312 + ... \\
&= 0.312 + 0.312(0.001) + 0.312(0.001)^2 + ... \\
&= \frac{0.312}{1-0.001} = \frac{0.312}{0.999} = \frac{312}{999}
\end{aligned}
$$

(b) Using $\dfrac{1}{1+x^2} = 1 - x^2 + x^4 - ... + (-1)^n x^{2n} + ...$ and replacing x^2 by x^4,

$$
\begin{aligned}
\frac{1}{1+x^4} &= 1 - x^4 + x^8 - ... + (-1)^n x^{4n} + ... \\
&= \sum_{n=0}^{\infty} (-1)^n x^{4n}
\end{aligned}
$$

(c)

$$\begin{aligned}
\int_0^x \frac{1}{1+x^4}\ dx &= \int_0^x \left(1 - x^4 + x^8 - ... + (-1)^n x^{4n} + ...\right)\ dx \\
&= \left[x - \frac{x^5}{5} + \frac{x^9}{9} - ... + (-1)^n \frac{x^{4n+1}}{4n+1} + ...\right]_0^x \\
&= x - \frac{x^5}{5} + \frac{x^9}{9} - ... + (-1)^n \frac{x^{4n+1}}{4n+1} + ... \\
&= \sum_{n=0}^{\infty} (-1)^n\ \frac{x^{4n+1}}{4n+1}
\end{aligned}$$