MATH102 ASSIGNMENT 2

MARK VILLAR

(1) (a) $f(x) = x^2+1$ has a stationary point when $f'(x) = 2x = 0$, i.e. at $x = 0$. Since f and f' are continuous, minimum and maximum values of f on the intervals $[0,1],[1,2]$ and $[2,3]$ will occur at the endpoints of these intervals or at the stationary point.
As f is increasing over the interval $[0,3]$, the smallest Riemann sum occurs when we take $x_1^* = 0, x_2^* = 1, x_3^* = 2$, giving

$$\begin{aligned}\sum_{k=1}^{3} f(x_k^*)\Delta x_k &= (0^2+1)(1) + (1^2+1)(1) + (2^2+1)(1)\\ &= 1+2+5 = 8\end{aligned}$$

The biggest Riemann sum occurs when we take $x_1^* = 1, x_2^* = 2, x_3^* = 3$, giving

$$\begin{aligned}\sum_{k=1}^{3} f(x_k^*)\Delta x_k &= (1^2+1)(1) + (2^2+1)(1) + (3^2+1)(1)\\ &= 2+5+10 = 17\end{aligned}$$

(b)

$$\int_0^3 (x^2+1)\ dx = \left[\frac{x^3}{3}+x\right]_0^3 = \left(\frac{27}{3}+3\right) - \left(\frac{0}{3}+0\right) = 9+3 = 12$$

(2) (a)

$$\int (x^{27} - 3x^4 + x)\ dx = \frac{1}{28}x^{28} - \frac{3}{5}x^5 + \frac{1}{2}x^2 + C$$

(b)

$$\int \sqrt[5]{x^3}\ dx = \int (x^3)^{\frac{1}{5}} = \int x^{\frac{3}{5}}\ dx = \frac{5}{8}x^{\frac{8}{5}} + C$$

(c)

$$\int \frac{dx}{x\sqrt[3]{x}} = \int x^{-1}x^{-\frac{1}{3}}\ dx = \int x^{-\frac{4}{3}}\ dx = -3x^{-\frac{1}{3}} + C = -\frac{3}{\sqrt[3]{x}} + C$$

(d)

$$\int (7+x)(1+7x)\ dx = \int (7x^2+50x+7)\ dx = \frac{7}{3}x^3 + 25x^2 + 7x + C$$

(3)

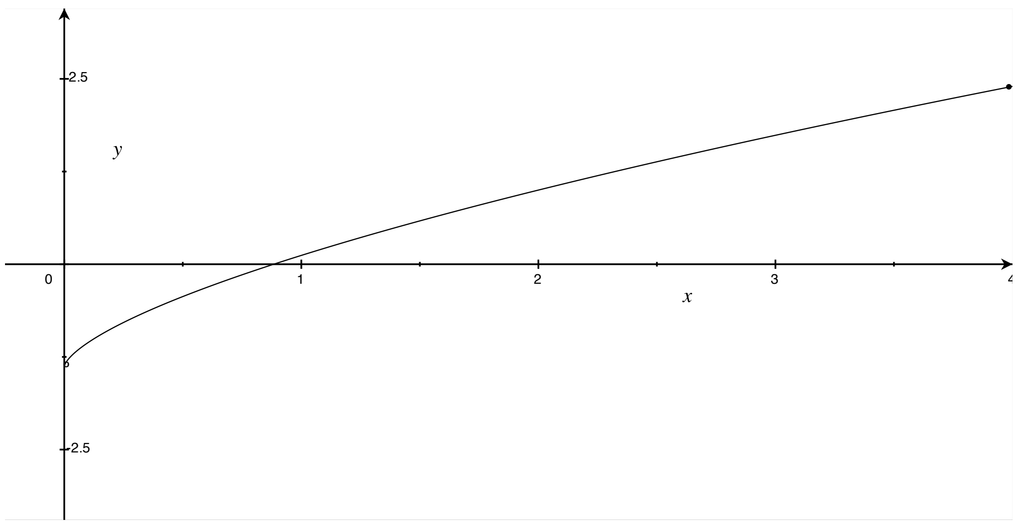
$$\frac{dy}{dx} = \frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}} \Rightarrow y = \int x^{-\frac{1}{3}}\, dx = \frac{3}{2}x^{\frac{2}{3}} + C$$

Since the curve passes through (2,1) then

$$1 = \frac{3}{2}(2)^{\frac{2}{3}} + C \Rightarrow C = 1 - 3 \cdot 2^{-1} \cdot 2^{\frac{2}{3}} = 1 - 3 \cdot 2^{-\frac{1}{3}} = 1 - \frac{3}{\sqrt[3]{2}}$$

Hence, the equation of the curve is given by

$$y = \frac{3}{2}x^{\frac{2}{3}} - \frac{3}{\sqrt[3]{2}} + 1$$



(4)

$$v = \frac{ds}{dt} = 3t^2 + 2t \Rightarrow s = \int (3t^2 + 2t)\, dt = t^3 + t^2 + C$$

Since $s = 10$ when $t = 2$ then

$$10 = 2^3 + 2^2 + C \Rightarrow C = 10 - 8 - 4 = -2$$

Hence, the distance travelled by the moving body is given by

$$s = t^3 + t^2 - 2$$