UNIVERSITY OF NEW ENGLAND

NAME:________________________________

STUDENT NUMBER:______________________

UNIT NAME PMTH213 – SPECIAL

PAPER TITLE: Linear Algebra

PAPER NUMBER: First and Only

DATE: **Wednesday 1 February 2012** **TIME:** **1:45 PM TO 5:00 PM**

TIME ALLOWED: Three (3) hours and fifteen minutes

NUMBER OF PAGES IN PAPER: THREE (3)

NUMBER OF QUESTIONS ON PAPER: EIGHT (8)

NUMBER OF QUESTIONS TO BE ANSWERED: EIGHT (8)

STATIONERY PER CANDIDATE:						
	0	6 LEAF A4 BOOKS	**0**	ROUGH WORK BOOK	**0**	GENERAL PURPOSE ANSWER SHEET
	1	12 LEAF A4 BOOKS	**0**	GRAPH PAPER SHEETS	**0**	SEE OTHER 'AIDS REQUIRED' BELOW

OTHER AIDS REQUIRED: **NIL**

POCKET CALCULATORS PERMITTED: **NO**

TEXTBOOKS OR NOTES PERMITTED: **FIVE (5) A4 SHEETS OF HANDWRITTEN DOUBLE SIDED NOTES (10 PAGES); NO PHOTOCOPIES; NO PRINTED PAGES. NO SCANNED PAGES**

INSTRUCTIONS FOR CANDIDATES:

- Candidates MAY NOT start writing until instructed to do so by the supervisor.
- Please pay attention to the announcements and read all instructions carefully before commencing the paper.
- Candidates MUST write their name and student number on the top of this page.
- All questions must be answered for full marks
- Questions are not of equal value.
- This examination question paper **MUST BE HANDED IN** with worked scripts. Failure to do so may result in the cancellation of all marks for this examination.

REMEMBER TO WRITE YOUR NAME AND STUDENT NUMBER AT THE TOP OF THIS PAGE

Question 1: [10 *marks*]

Find all linear transformations, $T : \mathbb{R}^3 \to \mathbb{R}^2$, which map the plane determined by the equation $y = x$ onto the line determined by the equation $v = 2u$.

Question 2: [8 *marks*]

Determine whether the real quadratic form

$$Q(x, y, z) = x^2 + 4xy + 8y^2 + 16yz + 10z^2 + 6zx$$

is positive or negative definite, positive or negative semi-definite, or indefinite, justifying your answer.

Question 3: [10 *marks*]

Find all 2×2 rational matrices, $\underline{\mathbf{A}}$, satisfying $\underline{\mathbf{A}}^2 = \underline{\mathbf{0}}$.

Question 4: [10 *marks*]

Let $\underline{\mathbf{A}}^t$ denote the transpose and $\bar{\underline{\mathbf{A}}}$ the complex conjugate of the complex matrix $\underline{\mathbf{A}}$.
The $\underline{\mathbf{A}}$ is *Hermitian* if and only if $\underline{\mathbf{A}}^t = \bar{\underline{\mathbf{A}}}$.
Show that the eigenvalues of a Hermitian matrix must be real numbers.

Question 5: [12 *marks*]

Let $\{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$ be a basis for the vector space V and $T : V \longrightarrow V$ a linear transformation.

(a) Show that if $\mathrm{f}_1 := \mathrm{e}_1 + \mathrm{e}_2,\ \mathrm{f}_2 := \mathrm{e}_1 + \mathrm{e}_3,\ \mathrm{f}_3 := \mathrm{e}_1 + \mathrm{e}_2 + \mathrm{e}_3$, then $\{\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3\}$ is also a basis for V.

(b) Find the matrix, $\underline{\mathbf{B}}$, of T with respect to $\{\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3\}$, given that its matrix with respect to $\{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3\}$ is

$$\underline{\mathbf{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Question 6: [10 *marks*]

Let V be an n-dimensional vector space over the field $\mathbb{F}$.
Prove that the vectors, $\mathrm{v}_1, \ldots, \mathrm{v}_n$, generate V if and only if they are linearly independent.

Question 7: [15 *marks*]

Given the symmetric matrix $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$, find

(a) its eigenvalues,

(b) bases for its eigenspaces,

(c) an orthogonal matrix, P, which diagonalises A, and

(d) $P^{-1}AP$.

Question 8: [25 *marks*]

Let $\mathcal{P}_2$ be the real vector space of all polynomials in t with real coefficients, whose degree is at most 2, so that $V = \{at^2 + bt + c \mid a, b, c \in \mathbb{R}\}$.

(a) Prove that

$$\langle\langle p, q\rangle\rangle := p(-1)q(-1) + 2p(0)q(0) + p(1)q(1) \qquad (p, q \in \mathcal{P}_2)$$

defines an inner product on $\mathcal{P}_2$.

(b) Apply the Gram-Schmidt procedure with respect to this inner product to find an orthonormal basis for the vector subspace of $\mathcal{P}_2$ generated by

$$\mathrm{v}_1 := t^2,\ \mathrm{v}_2 := 1 \ \text{ and } \ \mathrm{v}_3 := t + 1$$