MATH102 – Statistics

Exercises W6 - Solutions

29/8/06

1. $$E(aX) = \int xf(x)dx = a\int xf(x)dx = aE(X)$$

2. $$E(aX)^2 = \int (ax)^2 f(x)dx = a^2 \int x^2 f(x)dx = a^2E(X)^2$$

3. $$E[(X_1+X_2)^2] = E(X_1^2+X_2^2+2X_1X_2) = E(X_1)^2+E(X_2)^2+2E(X_1X_2)$$

4. $$V(b) = E[b-E(b)]^2 = E[b-b]^2 = 0$$

5. $$V(aX) = a^2E[X-E(X)]^2 = a^2V(X)$$

6. $$V(aX+b) = E[aX+b-E(aX+b)]^2 = E[aX+b-E(X)-b]^2$$
$$= a^2E[X-E(X)]^2 = a^2V(X)$$

7. The sample mean is $\widehat{\mu} = \bar{x} = 5$.

X	1	2	3	4	5	6	7	8	9
$X-\bar{X}$	-4	-3	-2	-1	0	1	2	3	4
$(X-\bar{X})^2$	16	9	4	1	0	1	4	9	16

Thus
$$s^2 = \frac{\sum_i (x-\bar{x})^2}{n-1} = 60/8 = 7.5$$

8.
$$\mu = E(X) = \int_0^{60} x\frac{1}{60}dx = \left[\frac{x^2}{2} \times 60\right]_0^{60} = \frac{60^2}{120} = 30$$

Thus the mean is 30 seconds.

Now
$$\sigma^2 = V(X) = EX^2 - [EX]^2$$

and
$$EX^2 = \int_0^{60} x^2\frac{1}{60}dx = \left[\frac{x^3}{3 \times 60}\right]_0^{60} = 60^2/3$$

and so
$$V(X) = 60^2/3 - 30^2 = 300$$

Thus the variance is 300 seconds^2, ie, the standard deviation is $\sqrt{300} =$ 17.32 seconds.

An alternative is to use $V(X) = E(X - EX)^2$ and find

$$V(X) = \int_0^{60} (x-30)^2\frac{1}{60}dx = \frac{1}{60}\left[\frac{(x-30)^3}{3}\right]_0^{60}$$
$$= \frac{1}{60}\left[\frac{30^3}{3} - \frac{(-30)^3}{3}\right] = 300$$

as before.

9.
$$E(\sum_{i=1}^{n} X_i) = E(X_1 + \ldots + X_n) = n\mu$$

10.
$$V(\sum_{i=1}^{n} X_i) = V(X_1) + \ldots + V(X_n) = n\sigma^2$$