MATH102 ASSIGNMENT 5

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(1)

$$\frac{2x+3}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$$

$$2x+3 = A(x+3) + B(x-2)$$

$$x = 2 \to 7 = 5A \Rightarrow A = \frac{7}{5},\ x = -3 \to -3 = -5B \Rightarrow B = \frac{3}{5}$$

$$\begin{aligned}\int \frac{2x+3}{(x-2)(x+3)} &= \frac{7}{5}\int \frac{dx}{x-2} + \frac{3}{5}\int \frac{dx}{x+3} \\ &= \frac{7}{5}\ln|x-2| + \frac{3}{5}\ln|x+3| + C\end{aligned}$$

(2)

$$\frac{4x+3}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2}$$

$$4x+3 = A(x+2) + B = Ax + (2A+B)$$

$$A = 4,\ 2A+B = 3 \Rightarrow B = -5$$

$$\frac{4x+3}{(x+2)^2} = \frac{4}{x+2} - \frac{5}{(x+2)^2}$$

$$\begin{aligned}\int \frac{4x+3}{(x+2)^2} &= 4\int \frac{dx}{x+2} - 5\int \frac{dx}{(x+2)^2} \\ &= 4\ln|x+2| + \frac{5}{x+2} + C\end{aligned}$$

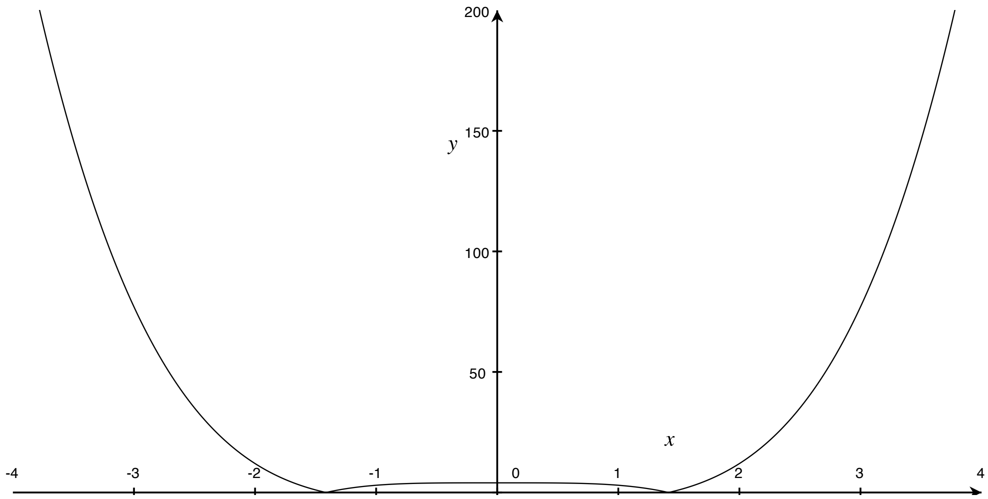
(3)

$$\begin{array}{r} 1 \\ x^2+1\overline{\big)\ \ x^2+5} \\ \underline{-\,x^2-1} \\ 4 \end{array}$$

$$\frac{5+x^2}{1+x^2} = 1 + \frac{4}{x^2+1}$$

$$\int \frac{5+x^2}{1+x^2}\ dx = \int 1\ dx + \int \frac{4}{x^2+1}\ dx$$
$$= x + 4\tan^{-1} x + C$$

(4) $y = |x^4 - 4|$



$$\begin{aligned}
\int_{-3}^{3} |x^4-4|\ dx &= \int_{-3}^{-\sqrt{2}} (x^4-4)\ dx + \int_{-\sqrt{2}}^{\sqrt{2}} (4-x^4)\ dx + \int_{\sqrt{2}}^{3} (x^4-4)\ dx \\
&= 2\int_{\sqrt{2}}^{3} (x^4-4)\ dx + 2\int_{0}^{\sqrt{2}} (4-x^4)\ dx \\
&= 2\left[\frac{x^5}{5} - 4x\right]_{\sqrt{2}}^{3} + 2\left[4x - \frac{x^5}{5}\right]_{0}^{\sqrt{2}} \\
&= 2\left(\frac{243}{5} - 12 - \frac{4\sqrt{2}}{5} + 4\sqrt{2}\right) + 2\left(4\sqrt{2} - \frac{4\sqrt{2}}{5}\right) \\
&= 2\left(\frac{183}{5} + \frac{16\sqrt{2}}{5}\right) + 2\left(\frac{16\sqrt{2}}{5}\right) \\
&= \frac{2}{5}\left(183 + 32\sqrt{2}\right)
\end{aligned}$$

(5) (a) $f(x)$ is odd since it is the product of an even function (x^4) and an odd function ($\csc x$). Hence $\int_{-1}^{1} f(x)\ dx = 0$.

(b) $f(x)$ is even since x is raised to the power of an even number. Hence,

$$\int_{-1}^{1} f(x)\ dx = 2\int_{0}^{1} x^{2010}\ dx = 2\left[\frac{x^{2011}}{2011}\right]_0^1 = \frac{2}{2011}$$

(c) $f(x)$ is odd since it is the product of an odd function $(\tan x)$ and an even function $\left(\frac{1}{1+x^2+x^6}\right)$. Hence $\int_{-1}^{1} f(x)\ dx = 0$.