PMTH212 ASSIGNMENT 1

MARK VILLAR

(1) Let $P = (2, 1, 6)$, $Q = (4, 7, 9)$ and $R = (8, 5, -6)$.

$$\overrightarrow{PQ} = \langle 4-2, 7-1, 9-6\rangle = \langle 2, 6, 3\rangle$$
$$\overrightarrow{PR} = \langle 8-2, 5-1, -6-6\rangle = \langle 6, 4, -12\rangle$$
$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 2 \times 6 + 6 \times 4 + 3 \times (-12) = 12 + 24 - 36 = 0$$
$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{||\overrightarrow{PQ}||\ ||\overrightarrow{PR}||} = 0$$
$$\theta = \frac{\pi}{2} \text{ or } 90^\circ$$

(2) Let $P = (5, 2, 3)$. The orthogonal projection of P to the
 (a) xy-plane is $Q = (5, 2, 0)$ and the distance is $|PQ| = 3$
 (b) xz-plane is $R = (5, 0, 3)$ and the distance is $|PR| = 2$
 (c) y-axis is $S = (0, 2, 0)$ and the distance is $|PS| = \sqrt{5^2 + 3^2} = \sqrt{34}$

(3) If the initial point is $(-2, 1, 4)$ then the terminal point (x_2, y_2, z_2) of $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ is found by

$$\langle 1, 2, -3\rangle = \langle x_2 - (-2), y_2 - 1, z_2 - 4\rangle$$
$$(x_2, y_2, z_2) = (1-2, 2+1, -3+4) = (-1, 3, 1)$$

(4) We form two vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, both lying on the plane. By the properties of the cross product, $\overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$, hence to the plane. Thus we can use $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ as a normal vector. Since

$$\overrightarrow{AB} = \langle 1-0, -1-(-2), -2-1\rangle = \langle 1, 1, -3\rangle$$
$$\overrightarrow{AC} = \langle -1-0, 1-(-2), 0-1\rangle = \langle -1, 3, -1\rangle$$

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = (-1+9)\mathbf{i} - (-1-3)\mathbf{j} + (3+1)\mathbf{k}$$
$$= 8\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$$

(5) To find parametric equations for the line passing through $(1,1)$ and parallel to $x=-5+t,\ y=1-2t$, we use the vector equation $\vec{r}=\vec{r_0}+t\vec{v}$. Since $\vec{u}$ and $\vec{v}$ are parallel iff $\vec{u}=t\vec{v}$ for some scalar t, $\vec{r_0}=\langle 1,1\rangle$ and $\vec{u}=\vec{v}=\langle 1,-2\rangle$ implies

$$x=1+t,\ y=1-2t$$

(6) (a) Let ρ_1 and ρ_2 be the planes $x-y+3z-2=0$ and $2x+z-1=0$ respectively. The corresponding normal vectors are $\vec{n_1}=\langle 1,-1,3\rangle$ and $\vec{n_2}=\langle 2,0,1\rangle$. Since

$$\vec{n_1}\cdot\vec{n_2}=1\times 2+(-1)\times 0+3\times 1=5\neq 0$$

then ρ_1 and ρ_2 are not perpendicular.

(b) Let ρ_3 and ρ_4 be the planes $3x-2y+z-1=0$ and $4x+5y-2z-4=0$ respectively. The corresponding normal vectors are $\vec{n_3}=\langle 3,-2,1\rangle$ and $\vec{n_4}=\langle 4,5,-2\rangle$. Since

$$\vec{n_3}\cdot\vec{n_4}=3\times 4+(-2)\times 5+1\times(-2)=0$$

then ρ_3 and ρ_4 are perpendicular.

(7) (a) The equation of the plane that passes through $(-1,4,-3)$ and is perpendicular to $\vec{n}=\langle 1,2,-1\rangle$ is given by

$$\begin{aligned}&1(x+1)+2(y-2)-1(z+3)=0\\&x+1+2y-4-z-3=0\\&x+2y-z-6=0\end{aligned}$$

(b) The equation of the plane that passes through $(-1,2,-5)$ and is perpendicular to both $\vec{n_1}=\langle 2,-1,1\rangle$ and $\vec{n_2}=\langle 1,1,-2\rangle$ is found by

$$\begin{aligned}\mathbf{n}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&-1&1\\1&1&-2\end{vmatrix}&=(2-1)\mathbf{i}-(-4-1)\mathbf{j}+(2+1)\mathbf{k}\\&=\mathbf{i}+5\mathbf{j}+3\mathbf{k}=\langle 1,5,3\rangle\end{aligned}$$

Hence,

$$\begin{aligned}&1(x+1)+5(y-2)+3(z+5)=0\\&x+1+5y-10+3z+15=0\\&x+5y+3z+6=0\end{aligned}$$