AMTH250 Assignment 6

Due: 10th October

Question 1 *[2 marks]*

A function

$$y = a_1 e^{-2x} + a_2 e^{-x} + a_3 + a_4 e^{x} + a_5 e^{2x}$$

has the values

x	-2	-1	0	1	2
y	125.948	7.673	-4.000	-14.493	-103.429

Determine the coefficients a_1, a_2, a_3, a_4, a_5.

Question 2 *[2 marks]*

Interpolate the following values of the function $y = \sqrt{x}$ using **(a)** a cubic spline, **(b)** a pchip cubic and **(c)** a polynomial of degree 5.

x	0	0.01	0.04	0.09	0.16	0.25
y	0	0.1	0.2	0.3	0.4	0.5

Which is most accurate over most of the domain?
Which is most accurate between 0 and 0.01?

Question 3 *[2 marks]*

For the data

x	1.0	2.0	3.0	4.0	5.0	6.0	7.0
y	1.9	2.7	4.8	5.3	7.1	9.4	11.3

(a) Compute and plot the cubic spline and pchip interpolants and their first, second and third derivatives. Use `ppderiv` for the derivatives.

(b) Verify, from the graphs, that the defining conditions for a cubic spline and a pchip cubic are satisfied. How does the "not-a-knot" condition for the cubic spline show up in the graphs?

Question 4 *[2 marks]*

The following problem arises in surveying. Suppose we wish to determine the altitudes x_1, x_2, x_3 and x_4 of four points. As well as measuring each altitude with respect to some reference point, each point is measured with respect to all of the others. The resulting measurements are:

x_1	2.95
x_2	1.74
x_3	-1.45
x_4	1.32
$x_1 - x_2$	1.23
$x_1 - x_3$	4.45
$x_1 - x_4$	1.61
$x_2 - x_3$	3.21
$x_2 - x_4$	0.45
$x_3 - x_4$	-2.75

These form an overdetermined set of linear equations. Find the least-squares solution. How do the computed values compare to the direct measurements?

Question 5 *[2 marks]*

Fit a quadratic polynomial, power function $y = ax^p$ and an exponential function $y = ae^{kx}$ to the data:

x	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
y	27.7	39.3	38.4	57.6	46.3	54.8	108.5	137.6	194.1	281.2

Which do you think gives the best representation of the data.

Question 6 *[6 marks]*

The function `polytrig` fits a combination of a polynomial $p(x)$ and trig functions

$$f(x) = p(x) + \sum_{i=1}^{n} (a_i \sin jkx + b_i \cos jkx)$$

to data. We will refer to n as the 'degree' of the trig functions. Use `polytrig` to experiment with fitting functions to the CO_2 data from the notes.

(a) First experiment with fitting a polynomial.

 (i) What degree polynomial best captures the general trend of the data? How would you justify your choice?

 (ii) If we fit polynomials of successively higher degree are the coefficients of the different polynomials related? Can you explain what you observe?

(b) Now experiment with a combination of polynomial and trig functions.

   (i) What conclusions can you draw about how well the model fits the data for various combinations of polynomial degree and trig function degree?

   (ii) Which combination gives the "best" model of the data. How would you justify your choice?

   (iii) If we fix the polynomial degree and fit models with successively higher trig degree how are the coefficients of the polynomial part related? How are the coefficients of the trig part related? Can you explain what you observe?

(c) Is there any problem with the conditioning of the design matrix? What if we hadn't rescaled the time data?

(d) Comment on any aspect of your experiments that you find interesting.

Notes on the Assignment

Question 3

Given a piecewise polynomial `pp` found by

```
pp = interp1(..., ..., 'spline', 'pp')
```

for example, then

```
pp1 = ppderiv(pp)
```

will compute the derivative of the derivative of `pp` as a piecewise polynomial. Like the original function, `pp1` can then be evaluated with `ppval`.

Question 6

The function

```
[a yy] = polytrig(x, y, k, n1, n2)
```

fits a function

$$f(x) = p_{n_1}(x) + \sum_{i=1}^{n_2} (a_i \sin jkx + b_j \cos ikx)$$

with $p_{n_1}(x)$ a polynomial of degree n_1, to data x and y. Here `k` is the constant $k = 2\pi/T$, T the period, and `n1` and `n2` are the degrees of the polynomial and trigonometric parts of model. The return value `a` is the coefficients appearing in the model, and yy is the model function evaluated at the `x` data values.

To apply this to the CO_2 data first follow the steps in the notes to (i) read in the data, (ii) rescale time and (iii) compute the constant `k`. Now to compute the final model in the notes which has polynomial degree 4 and trigonometric degree 2 proceed as follows:

```
octave:> [a42 c42] = polytrig(tt,conc,k,4,2);
```

This computes the coefficients `a42` of the model function and and evaluates the model function `c42` at the data points

Now we can plot the model

```
octave:> plot(t,c42)
```

compute residuals

```
octave:> r42 = conc-c42;
```

and so on.