

Tutorial 2

Question 1. Let A and B be two non-empty bounded sets of real numbers. Prove that

$$\begin{aligned}\sup(A \cup B) &= \max\{\sup(A), \sup(B)\}, \\ \inf(A \cap B) &\geq \max\{\inf(A), \inf(B)\}.\end{aligned}$$

Find an example where $\inf(A \cap B) > \max\{\inf(A), \inf(B)\}$.

Question 2. Let a and b be real numbers. Prove that

$$\begin{aligned}\max\{a, b\} &= \frac{a + b + |a - b|}{2} \\ \min\{a, b\} &= \frac{a + b - |a - b|}{2}\end{aligned}$$

Question 3. Determine which of the following sets are bounded above, bounded below, have a supremum, have, an infimum, have a maximum or have a minimum. For those which have a supremum, infimum, maximum or minimum, determine what these are.

- (i) $A := \{x \in \mathbb{R} \mid x = \frac{1}{2^n} \text{ for some } n \in \mathbb{N}\}$
- (ii) $B := \{\cos(n\frac{\pi}{z}) \mid n \in \mathbb{Z}\}$
- (iii) $C := \{x \in \mathbb{R} \mid \frac{x}{1+x} \geq 0\}$
- (iv) $D := \{\frac{x}{1+x} \mid x \in \mathbb{R} \text{ and } x \geq 0\}$

Question 4. Express each of the following complex numbers in the form $x + yi$, with $x, y \in \mathbb{R}$.

- (i) $(2 - i)(2 + i)$
- (ii) $(6 + 5i)(2 - 7i)$
- (iii) $\frac{2 - i}{1 + 2i}$
- (iv) $\frac{1 - 3i}{(2 + i)^2} + \frac{1 + i^3}{1 + i}$

Find the modulus and complex conjugate of each.