AMTH250
Assignment 4

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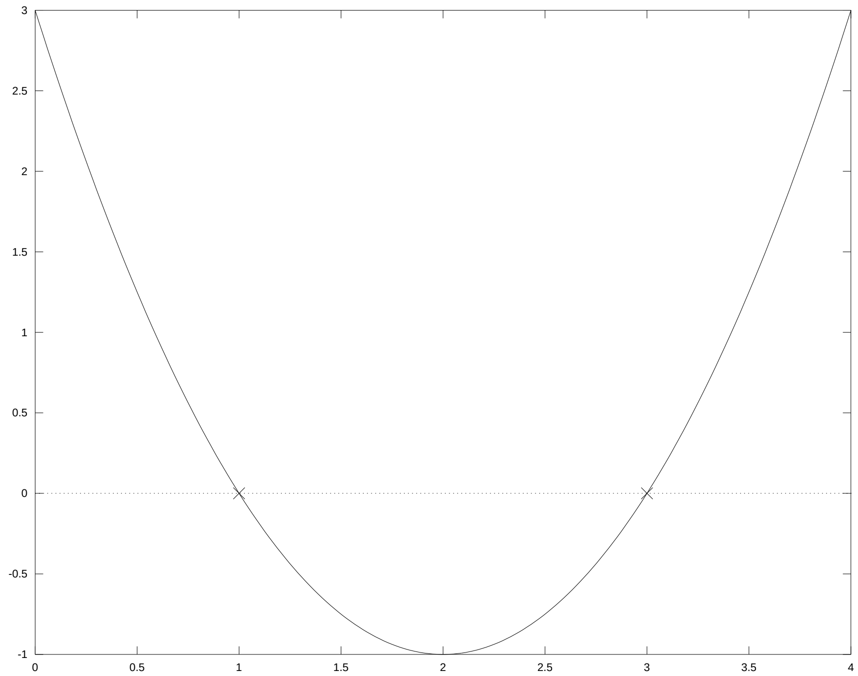
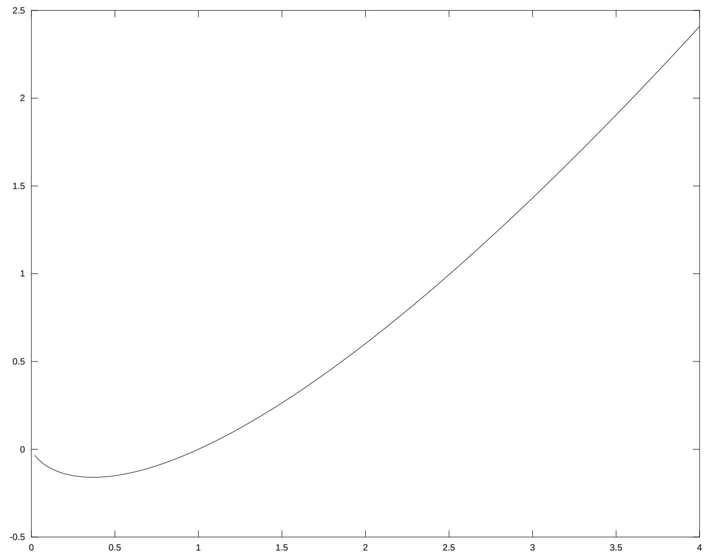
Question 1

Graph of $f(x) = (x-1)(x-3)$ over the interval $x \in [0, 4]$, displaying the $x-$axis and its roots.



Question 2

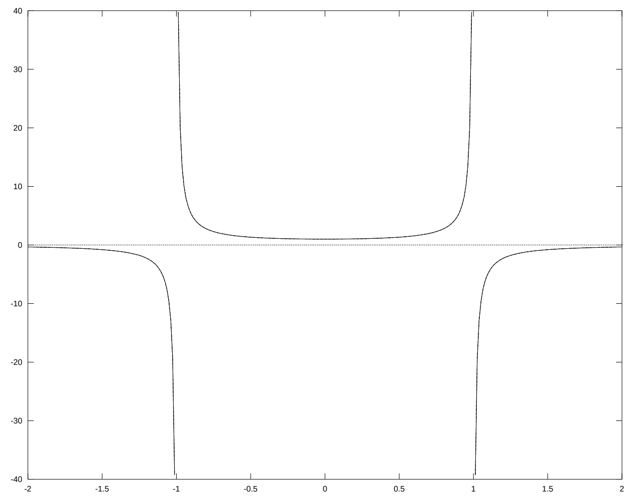
(a) Graph of $f(x) = x \ln x$ over the interval $x \in [0, 2]$.



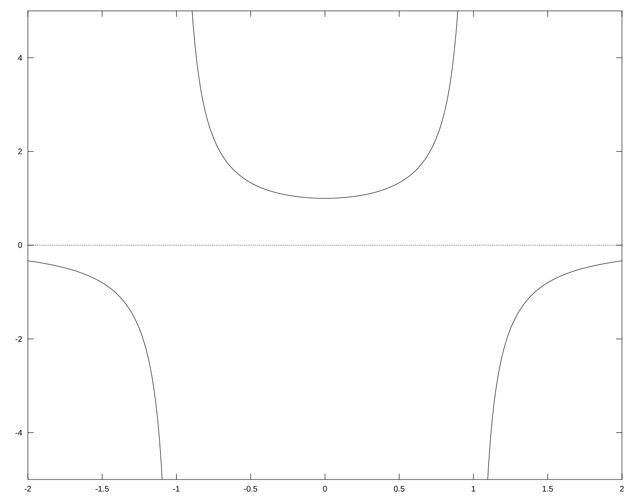
(b) The curve approaches the origin from the right but never reaches it. This is because Octave evaluates $f(x)$ at $x = 0$ as $f(0) = 0 \times (-\text{Inf}) = \text{NaN}$. To fix this, you could plot the graph of $f(x)$ over the interval $x \in (0, 2]$ such that $x = 0$ is excluded.

Question 3

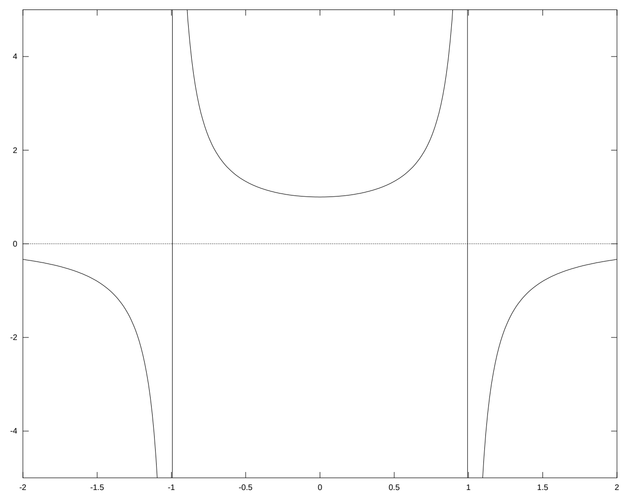
(a) Graph of $f(x) = \frac{1}{1-x^2}$ over the interval $x \in [-2, 2]$ with default scaling.



(b) Graph of $f(x) = \frac{1}{1-x^2}$ with rescaled y-axis.



(c) Graph of $f(x) = \frac{1}{1-x^2}$ with vertical asymptotes.



Question 4

(a) No

(b) No

(c) Yes

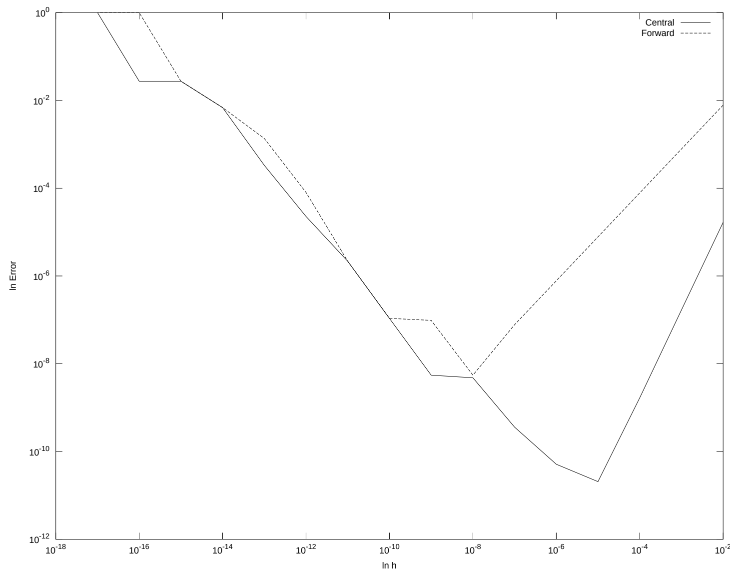
(d) No

Question 5

(a) Please see Appendix.

(b) Central difference approximation (CDA) has a smaller minimum error while forward difference approximation (FDA) has a smaller value of h at which minimum error occurs.

(c) Log-log plots of error against h comparing difference approximation methods.



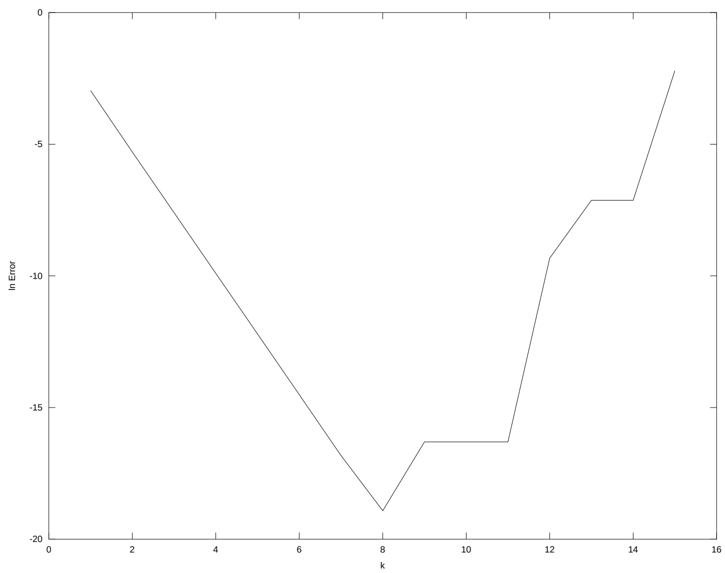
(d) Truncation error under CDA decreases as h decreases to 10^{-5} while FDA's truncation error decreases to even smaller values of h, until 10^{-8}. Cancellation error under CDA begins to increase at $h = 10^{-5}$ until 10^{-17}, while FDA's cancellation error increases from $h = 10^{-8}$ until $h = 10^{-16}$.

Question 6

(a) Let $g(x) = e^x - 1$ and $h(x) = x$. Since $\lim g(x)$ and $\lim h(x)$ are both 0 as x approaches 0, then by l'Hopital's Rule

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x}{1} = e^0 = 1$$

(b) My results do not exactly agree with theoretical expectations since increasing k should improve our approximation's precision by virtue of x approaching 0. However, we see from the graph below that minimum error occurs at $k = 8$. For $k > 8$, error due to cancellation begins to increase and our approximation worsens as k increases.



Appendix

1.
```
%graph of f(x)=(x-3)(x-1)
x1=linspace(0,4,200);
f1=x1.^2-4*x1+3;
plot(x1,f1)
hold on
xaxis=zeros(1,200);
plot(x1,xaxis,'.')
hold on
x1=1;
f1=x1.^2-4*x1+3;
plot(x1,f1,'x')
hold on
x1=3;
f1=x1.^2-4*x1+3;
plot(x1,f1,'x')
```

2. (a)
```
%graph of f(x)=xlnx
x2=linspace(0,2,200);
f2=x2.*log(x2);
plot(x2,f2)
```

3. (a)
```
%graph of f(x)=1/(1-x^2) with default scaling
x3=linspace(-2,2,317);
f3=1./(1-x3.^2);
plot(x3,f3)
xaxis=zeros(1,317);
hold on
plot(x3,xaxis,'.')
```

   (b)
```
%graph of f(x)=1/(1-x^2) with rescaled y-axis
axis([-2,2,-5,5])
```

   (c)
```
%graph of f(x)=1/(1-x^2) with vertical asymptotes
figure(2)
x3=linspace(-2,2,318);
f3=1./(1-x3.^2);
plot(x3,f3)
axis([-2,2,-5,5])
xaxis=zeros(1,318);
hold on
plot(x3,xaxis,'.')
```

5. (a)
```
%central difference approximation to f'(x) for f(x)=sin(x) at x=1
n=-2:-1:-17;
h=10.^n;
exact=cos(1);
approx_c=(sin(1+h)-sin(1-h))./(2*h);
err_c= abs(approx_c-exact)/exact;
```
(b)
```
%forward difference approximation to f'(x) for f(x)=sin(x) at x=1
approx_f=(sin(1+h)-sin(1))./h;
err_f= abs(approx_f-exact)/exact;
```
(c)
```
%log-log plots of error against h comparing central and
% forward difference approximation
loglog(h,err_c,'b',h,err_f,'r')
xlabel('ln h')
ylabel('ln Error')
legend('Central','Forward')
```

6. (b)
```
%approximates f(x)=(e^x-1)/x for x=10^(-k), k=1,...,15 and graphs
%its precision errors
k=1:1:15;
x=10.^(-k);
f=(e.^x-1)./x;
exact=1;
err=abs(f-exact);
plot(k,log(err))
axis([0,16,-20,0])
xlabel('k')
ylabel('ln Error')
```