MATH102–Statistics – 2006

Solutions to the Exercises W8

12/9/06

1. If the distribution is symmetric, then the distribution can be reflected or folded about the line of symmetry through the point of symmetry say s on the X–scale. Thus on the scale of $X - s$ the distribution is symmetric about zero. Thus the mean on this scale will be zero, since the negative section of the $X - s$ scale is perfectly balanced with the positive section.

   Thus $E(X - s) = 0$ giving $E(X) = s$.

   Also since the distribution is symmetric about s we have

   $$P(X < s) = P(X > s) = 0.5$$

   and so s is not only the mean, but also the median as well.

2. All questions concerning the normal distribution are best worked out on the scale of the unit normal distribution. Thus we invoke

   $$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, -\infty < z < \infty$$

   where $Z \sim N(0, 1)$.

   Now
   $$f'(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}(-1/2)(2z)$$

   and
   $$f''(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}(-z)(-z) + \frac{1}{\sqrt{2\pi}}(-1)e^{-z^2/2} = 0$$

   if $z^2 - 1 = 0$ ie, $z = \pm 1$. Thus the points of inflection are 1 sd above and below the mean.

3. From 2., we have

$$f'(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}(-1/2)(2z)$$

and so $f'(z) = 0$ gives $z = 0$ as the mode. This is also the mean as

$$E(Z) = \int z f(z) dz = 0$$

since $f(z)$ is an even function centered on 0, while z is odd, leading to $E(Z) = 0$. Thus $z = 0$ is also the mean.

4. The mean is given by

$$E(X) = \int x f(x) dx = \int_0^\infty x\lambda e^{-\lambda x} dx$$

$$= \lambda \int x d\left[\frac{e^{-\lambda x}}{-\lambda}\right] = \left[-xe^{-\lambda x}\right]_0^\infty + \int e^{-\lambda x} dx = 0 + \int e^{-\lambda x} dx$$

$$= \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^\infty = \frac{1}{\lambda}$$

Thus the mean is

$$E(X) = \frac{1}{\lambda}$$

5. The mean is 10 and the sd is 2. Thus we have the following :

   (a) $(10 \pm 2 \times 2) = (6, 14)$, ie, 2 sd above and below the mean. Thus the proportion is 95.5%

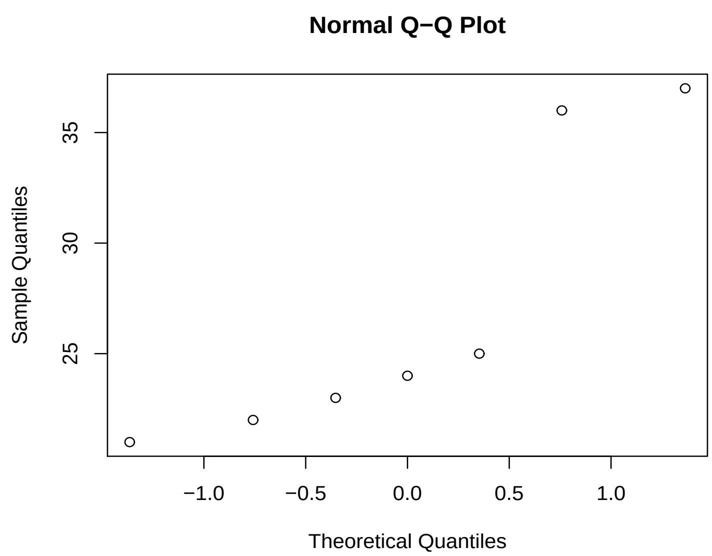
   (b) $(10 \pm 2) = (8, 12)$, ie, 1 sd above and below the mean. Thus the proportion is 68%

   (c) $(10 \pm 3 \times 2) = (4, 16)$, ie, 3 sd above and below the mean. Thus the proportion is 99.7%

6. A plot of Y=Actual and X=Expected will dip to the right, as the data are Right skew.

   See the R output.

```
> x <- c(21,22,23,24,25,36,37)
> qqnorm(x)
```



   So the R graphic dips to the Right, as expected.