MATH102 ASSIGNMENT 1

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(1) (a) $m(A \cup B) = m(A) + m(B) - m(A \cap B) = 1 + 2 - \frac{1}{2} = \frac{5}{2}$
(b) $m(A \cup B) = m(A) + m(B)$, then $m(A \cap B) = 0$. However, we cannot conclude that $A \cap B = \emptyset$ since (S0) does not postulate $m(A \cap B) = 0 \Rightarrow A \cap B = \emptyset$, with the existence of non-empty sets with measure 0.
(c) An example of a non-empty set with measure 0 is the Cantor set, which is a closed set constructed in the unit interval [0,1] by deleting successive *middle* thirds of intervals. First $(\frac{1}{3}, \frac{2}{3})$ is deleted then $(\frac{1}{9}, \frac{2}{9})$ and $(\frac{7}{9}, \frac{8}{9})$ are removed, and so on. The process is continued indefinitely and the Cantor set is defined as the union of 2^n intervals with total length of $\left(\frac{2}{3}\right)^n$. Since $\left(\frac{2}{3}\right)^n \to 0$ as $n \to \infty$ the Cantor set is measure 0.
Other examples include: any *one-point* set (by definition has length 0), and $\mathbb{Q}_{0,1}$ − the set of all rationals between 0 and 1. The latter consists of countably many points (which all have length 0) so its measure is 0.

(2) (a) C_1 = the event that first circuit fails
C_2 = the event that second circuit fails
C = the event that both circuits fail
$\overline{C}$ = the event that at least one circuit passes
$P(C_1) = P(C_2) = 0.02$ (independent events)
$P(C) = P(C_1) \times P(C_2) = 0.02 \times 0.02 = 0.0004$
$P(\overline{C}) = 1 - P(C) = 1 - 0.0004 = 0.9996$
The probability that the signal passes through any section is 0.9996.
(b) S_i = the event that a signal passes through any section i
S = the event that a signal passes through all sections $i = 1$ to 5
$P(S_i) = 0.9996$ (independent events)
$P(S) = P(S_1) \times ... \times P(S_5) = (0.9996)^5 = 0.9980$
The probability that the missile will function is 0.9980.

(3) (a) A = the event that student learnt material
B = the event that student gets right answer
$P(A) = 0.6, P(\overline{A}) = 0.4, P(B|\overline{A}) = 0.2$
$P(B) = P(B|A)P(A) + P(B|\overline{A})P(\overline{A}) = 1 \times 0.6 + 0.2 \times 0.4 = 0.68$
(b)

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{1 \times 0.6}{0.68} = 0.88$$

(4) $\sum_{n=1}^{4} \cos \frac{\pi n}{2} = \cos \frac{\pi}{2} + \cos \pi + \cos \frac{3\pi}{2} + \cos 2\pi = 0 - 1 + 0 + 1 = 0$