MATH102 : Statistics – Exercise 3

Solutions

1. Let the event $A = \{$ sex is M $\}$, and event $B = \{$ disease is inherited $\}$.

   Then $P(A) = 0.513 = P(M)$ and $P(\overline{A}) = P(F) = 0.487$,
   with $P(B|A) = P(B|M) = 0.5$ and $P(B|F) = 0$.

   Total probability thm gives
   $$\begin{aligned} P(B) &= P(B|M) \times P(M) + P(B|F) \times P(F) \\ &= 0.5 \times 0.513 + 0 \times 0.487 = 0.2565 \end{aligned}$$
   Thus there is a 26% chance that a child chosen randomly has the disease.

   A tree diagram could be used to verify the calculation of $P(B)$.

2. Let player A have one point and player B two.

   Then
   $$\begin{aligned} P(A \text{ wins } |B \text{ has 2 points}) &= P(A \text{ wins next two points}) \\ &= P(A \text{ wins nextplay}|A \text{ wins play after next}) \\ &= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{aligned}$$
   assuming independence between plays.

   $$\begin{aligned} P(B \text{ wins } |B \text{ has 2 points}) &= 1 - P(A \text{ wins}|B \text{ has 2 points}) \\ &= 1 - \frac{1}{4} = \frac{3}{4} \end{aligned}$$
   So the odds are 3:1 in favour of B winning, and so the stakes should be divided in that ratio. Thus the stakes should be split as per Pascal's suggestion, ie, 3:1, of 3/4 to B and 1/4 to A.

3. Each card has two sides. Thus we can name each side 1 or 2. This gives the total possibilities as

	Face up	Face covered	
1	W_1	W_2	WW
2	W_2	W_1	card
3	W_1	R_2	RW
4	R_2	W_1	card
5	R_1	R_2	RR
6	R_2	R_1	card

Thus if the face up is Red, then the possibilities are reduced to 4,5 and 6. Hence the probability that the face covered is Red is 2 out of 3, ie, 2/3. Hence the bet is not fair.

Since your are the "bookie", the dealer will be paid \$1 twice as often as he should, since P(R) = 2/3 and P(W) = 1/3 .

If the odds were really even, your would pay him a dollar (and return his \$1 bet) as often as you keep his \$1 bet.

So, do no take his bet.

An alternative to the problem uses Bayes' theorem. We have 3 cards

I : RR

II : WW

III : WR

Assume that $P(I) = P(II) = P(III) = 1/3$, ie, that the choice of card is random ...

Let the event $A = \{$ card chosen $\} = \{I,\ II,\ III\}$,

and $B = \{$ colour of face up $\} = \{W,\ R\}$.

The problem requires that we find $P(A = I|B = R)$.

Bayes thm gives

$$P(A_1|B_1) = \frac{P(B_1|A_1) \cdot P(A_1)}{P(B_1)}$$

or in terms of the notation of the problem

$$P(I|R) = \frac{P(R|I) \cdot P(I)}{P(R)}$$

Now $P(R|I) = 1$, $P(R|II) = 0$ and $P(R|III) = 1/2$.

$$P(R) = P(R|I) \cdot P(I) + P(R|II) \cdot P(II) + P(R|III) \cdot P(III)$$

$$= (1 + 0 + 1/2)/3 = 1/2$$

to give $P(I|R) = (1/3)/(1/2) = 2/3$ as before.

Again a tree diagram will verify the value for $P(R) = 1/2$, which is logical as there are many Red faces as White.

4. Let event $D = \{$ cat died $\}$, and $B = \{$ butler is alone with the cat $\}$.

We require $P(B|D)$.

Now we know $P(D|B) = 1/2$, $P(B) = 1/5$, $P(D|\overline{B}) = 1/4$ and $P(\overline{B}) = 4/5$.

$$P(B|D) = \frac{P(D|B) \cdot P(B)}{P(D)} = \frac{1/2 \times 1/5}{P(D)}$$

Now

$$P(D) = P(D|B) \cdot P(B) + P(D|\overline{B}) \cdot P(\overline{B})$$

$$= 1/2 \times 1/5 + 1/4 \times 4/5 = 1/10 + 1/5 = 3/10$$

to give

$$P(B|D) = \frac{1/10}{3/10} = 1/3$$

So there is 1 chance in three that the butler did it.

A tree diagram will aid in the verification of $P(D)$.