Tutorial 3

Question 1.

Decide which of the following is a function, explaining your answer.

(a) Let X be the set of all current telephone subscribers in Australia. Let Y be the set of all telephone numbers in use in Australia.

Define $f \colon x \longrightarrow Y, \quad x \longmapsto y,$ where y is x's telephone number.

(b) Let X, Y be $\mathbb{R}$, the set of all real numbers.

Define $f \colon X \longrightarrow Y, \quad x \longmapsto y,$ where $y = x^2$.

(c) Let X, Y be $\mathbb{R}$, the set of all real numbers.

Define $f \colon X \longrightarrow Y, \quad x \longmapsto y,$ where $x = y^2$.

(d) Let X, Y be the set of all non-negative real numbers, $\mathbb{R}_0^+ := \{r \in \mathbb{R} \mid r \geq 0\}$.

Define $f \colon X \longrightarrow Y, \quad x \longmapsto y,$ where $x = y^2$.

(e) Let X, Y be $\mathbb{R}$, the set of all real numbers.

Define $f \colon X \longrightarrow Y, \quad x \longmapsto y, \quad$ where $y = \begin{cases} x & \text{if } x \leq 0 \\ -x & \text{if } x \geq 0 \end{cases}$

Question 2. Decide which of the following functions can be composed, and for those which can, what is their composition? Justify your answer.

$$f \colon \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto x^2$$
$$g \colon \mathbb{R} \longrightarrow \mathbb{R}, \quad y \longmapsto 2y$$
$$h \colon \mathbb{R} \longrightarrow \mathbb{R}, \quad z \longmapsto z + 2$$

Question 3. Take $X \subseteq \mathbb{R}$.

When are $f \colon X \to \mathbb{R}, \ x \mapsto \sqrt{x^2 - 3x + 2}$ and $g \colon X \to \mathbb{R}, \ x \mapsto \tan(3x + 2)$ functions? Which of these functions is injective (1–1)?

Question 4. Take functions $f \colon X \to Y$ and $g \colon Y \to Z$.

(a) Show that if f and g are both surjective (onto), then $g \circ \colon X \to Z$ is also surjective.

(b) Show that if $g \circ f$ is surjective, then so is g.

(c) Find an example where $g \circ f$ is surjective, but f is not.