Tutorial 1

Question 1.

(a) Write down the negation and the converse of the proposition
   If a positive integer is divisible by 4 and by 6, then it is divisible by 24.

(b) Write down the negation of the propositions
   Every prime number is odd.
   Some clever people do dumb things.

Question 2.

Let a be a real number. Prove that, if, for every real number, b,

$$(a + b)^2 = a^2 + b^2,$$

then $a = 0$.

Question 3.

Use induction to prove that for all counting numbers, n,

$$\sum_{j=1}^{n} j^3 = 1^3 + 2^3 + \ldots + n^3 = \left(\frac{1}{2}n(n+1)\right)^2.$$

Question 4.

Let a be a positive real number. Prove that, for every counting number, n,

$$(1 + a)^n \geq 1 + na.$$

Question 5.

Prove that if n is a counting number, then $3^{2n} - 1$ is divisible by 8.

Question 6.

Given any subset Y of the set X, we write $Y' = X \setminus Y$. Use Venn diagrams to demonstrate that, for all subsets A and B of X,

$$A \setminus B = A \cap B'.$$