

Tutorial 4

Question 1. Decide whether the following functions are monotonic, explaining your answer carefully.

(a) $f: \mathbb{R}_0^- \rightarrow \mathbb{R}, x \mapsto x^2$

(b) $g: \mathbb{R}_0^+ \rightarrow \mathbb{R}, x \mapsto x^2$

(c) $h: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto x^2$

(d) $h: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto x^3$

(Recall that $\mathbb{R}_0^- = \{r \in \mathbb{R} \mid r \leq 0\}$ and $\mathbb{R}_0^+ = \{r \in \mathbb{R} \mid r \geq 0\}$.)

Question 2. Determine which of the following limits exist, and evaluate those which do, carefully justifying your answer.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

(b) $\lim_{x \rightarrow 0} \cos x$

(c) $\lim_{x \rightarrow 0} \frac{|x|}{x}$

(d) $\lim_{x \rightarrow 0} \frac{1}{x^2 + 1}$