AMTH140 ASSIGNMENT 5

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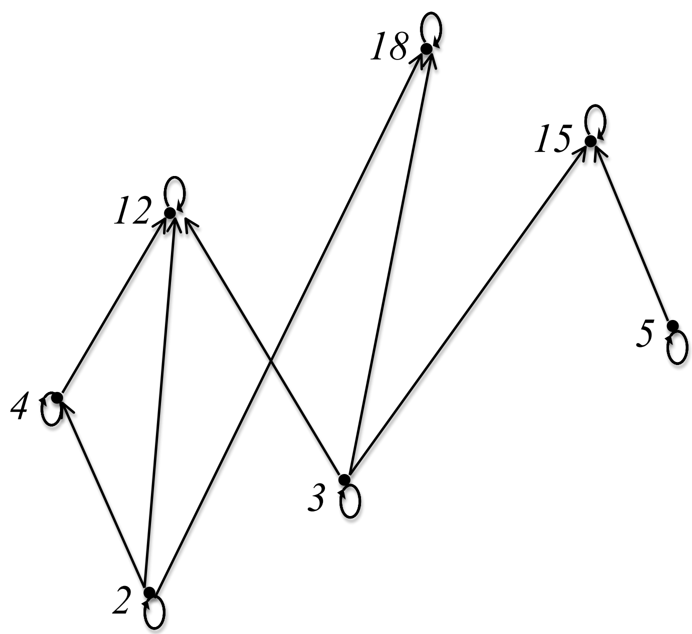
(1) $R = \{(2,2),(2,4),(2,12),(2,18),(3,3),(3,12),(3,15),(3,18),(4,4),(4,12),$
$(5,5),(5,15),(12,12),(15,15),(18,18)\}$

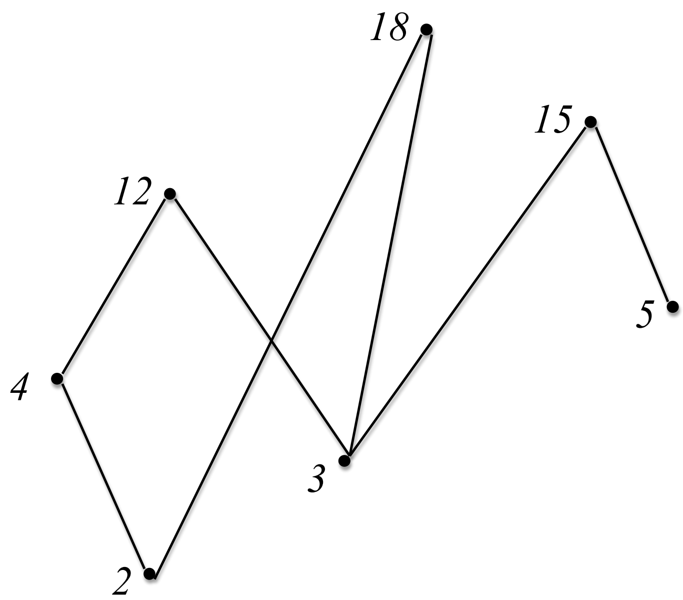
(a) Digraph



(b) R is a partial order relation since the digraph implies

- *Reflexivity:* There are loops on every vertex.
- *Transitivity:* If travelling from vertex v to vertex w along consecutive arrows of the same direction is possible, then there is also a single arrow pointing from v to w. In our case, $(2,4) \in R, (4,12) \in R, (2,12) \in R \Rightarrow R$ is transitive.
- *Antisymmetry:*: There are no arrows in opposite directions joining a pair of different vertices.

(c) Hasse diagram



(d) There are 3 maximal elements $(12, 15, 18)$ and 3 minimal elements $(2, 3, 5)$.

(e) A does not have a greatest element under R since there are finite vertices and there is more than 1 local top vertices. Likewise there is no least element since there is more than 1 local bottom vertices.

(2) (a) $a + a' = 1$

a	a'	$a + a'$
0	1	1
1	0	1

The first B5 property is verified since the third column values equal 1.

(b) $a \cdot a = 0$

a	a'	$a \cdot a'$
0	1	0
1	0	0

The second B5 property is verified since the third column values equal 0.

(c) $a + a \cdot b = a$

a	a'	$a \cdot b$	$a + a \cdot b$
0	1	0	0
0	1	0	0
1	0	0	1
1	1	1	1

The P4 property is verified since the fourth column values equal a.

(3) A Boolean expression corresponding to $f : S \times S \times S \longrightarrow S$ where $S = \{0, 1\}$ is

$$f(x, y, z) = x'y'z' + x'y'z + x'yz' + xyz$$

(4) The voting machine below specifies a Boolean function $f : S \times S \times S \times S \longrightarrow S$ where $S = \{0, 1\}$.

w	x	y	z	For	Against	Circuit condition
0	0	0	0	0	4	0
0	0	0	1	1	3	0
0	0	1	0	1	3	0
0	0	1	1	2	2	0
0	1	0	0	1	3	0
0	1	0	1	2	2	0
0	1	1	0	2	2	0
0	1	1	1	$\boxed{3}$	1	$\boxed{1}$
1	0	0	0	1	3	0
1	0	0	1	2	2	0
1	0	1	0	2	2	0
1	0	1	1	$\boxed{3}$	1	$\boxed{1}$
1	1	0	0	2	2	0
1	1	0	1	$\boxed{3}$	1	$\boxed{1}$
1	1	1	0	$\boxed{3}$	1	$\boxed{1}$
1	1	1	1	$\boxed{4}$	0	$\boxed{1}$

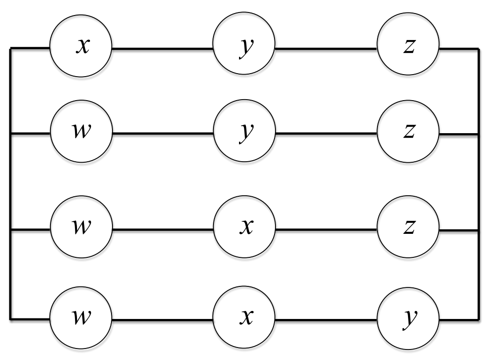
A corresponding Boolean expression would be

$$f(w,x,y,z) = w'xyz + wx'yz + wxy'z + wxyz' + wxyz$$

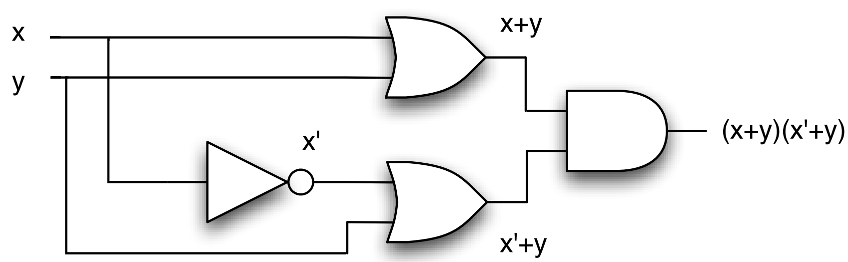
which simplifies to

$$\begin{aligned} &= w'xyz + wxyz + wx'yz + wxyz + wxy'z + wxyz + wxyz' + wxyz \\ &\quad (\text{since } a + a = a) \\ &= (w' + w)xyz + (x' + x)wyz + (y' + y)wxz + (z' + z)wxy \\ &= xyz + wyz + wxz + wxy \end{aligned}$$

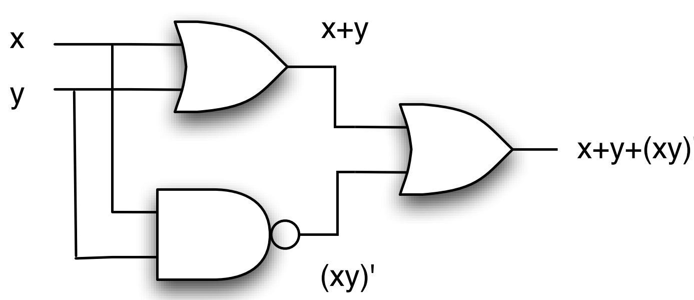
This can be represented by the following switching system.



(5) (a) $(x + y)(y + x')$



(b) $x + y + (xy)'$



(6) We apply the simplification procedure outlined in the Lecture Notes (p.138) on the following Karnaugh maps.

(a) $x'y'z + x'y'z' + x'yz'$

	xy	xy'	$x'y'$	$x'y$
z			1	
z'			1	1

A minimal representation would therefore be $x'z' + x'y'$.

(b) $w'xyz + w'xyz' + wx'yz' + wx'y'z' + wxy'z + wx'y'z + w'xy'z + w'x'y'z$

	wx	wx'	$w'x'$	$w'x$
yz				1
yz'		1		1
$y'z'$		1		
$y'z$	1	1	1	1

A minimal representation would therefore be $wx'z' + w'xy + y'z$.