Tutorial 9

You may use the facts, proven in MATH102, that for all $x \in \mathbb{R}$, $\frac{d}{dx}(e^x) = e^x$ and that if $x > 0$, $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

Question 1.

Determine, if they exist, the following limits.

(i) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

(ii) $\lim_{x \to \infty} x^{\frac{1}{x}}$

(iii) $\lim_{x \to \infty} \frac{x + \sin x}{x}$

(iv) $\lim_{x \to \infty} \frac{\frac{d}{dx}(x + \sin x)}{\frac{d}{dx} x}$

Comment. Question 1(iii) and (iv) illustrate what can occur with indiscriminate use of the Bernoulli-de l'Hôpital Rule.

Question 2.

Given a cylindrical tank of height 10 m., how accurately must the internal radius be measured if we are to calculate its volume within 1% of the true value?

Question 3.

Find the intervals on which the function

$$f : \mathbb{R} \longrightarrow \mathbb{R}, \quad x \longmapsto e^{-x} \cos x$$

is (i) monotonic, (ii) concave. Determine its extrema, and draw the graph of

$$[0, 4\pi] \longrightarrow \mathbb{R}, \quad x \longmapsto f(x)$$