MATH102 ASSIGNMENT 3

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(1) (a) Let $u = 2x^3 + 3$, then $\frac{du}{dx} = 6x^2 \Rightarrow dx = \frac{du}{6x^2}$

$$\int x^2 \sqrt[3]{2x^3+3}\ dx = \frac{1}{6}\int u^{\frac{1}{3}}\ du = \frac{1}{6}\cdot\frac{3}{4}\ u^{\frac{4}{3}} + C = \frac{1}{8}(2x^3+3)^{\frac{4}{3}} + C$$

(b) Let $u = a^2 + x^2$, then $\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$$\int \frac{x}{a^2+x^2}\ dx = \frac{1}{2}\int \frac{1}{u}\ du = \frac{1}{2}\log u + C = \frac{1}{2}\log(a^2+x^2) + C$$

(2) (a)

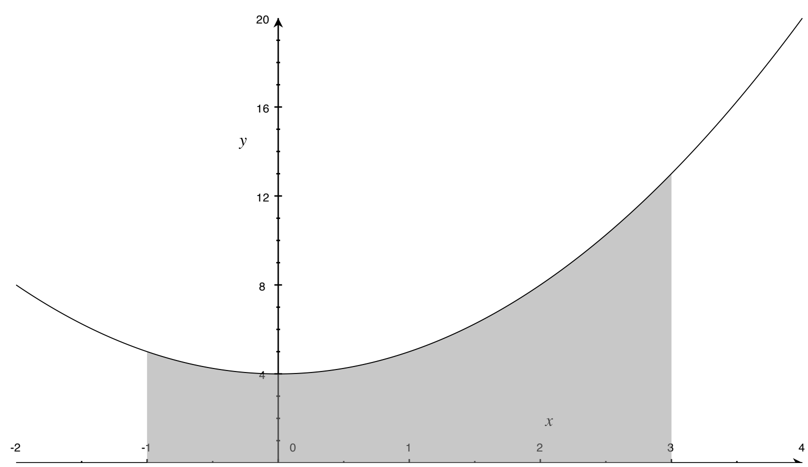
$$\int x^4\sqrt{x^5-1}\ dx = \frac{1}{5}\int 5x^4(x^5-1)^{\frac{1}{2}}\ dx = \frac{1}{5}\cdot\frac{2}{3}(x^5-1)^{\frac{3}{2}} + C$$
$$= \frac{2}{15}(x^5-1)^{\frac{3}{2}} + C$$

(b)

$$\int x\cos(3x^2+1)\ dx = \frac{1}{6}\int 6x\cos(3x^2+1)\ dx = \frac{1}{6}\sin(3x^2+1) + C$$

(3)

$$A = \int_{-1}^{3}(x^2+4)\ dx = \left[\frac{x^3}{3} + 4x\right]_{-1}^{3} = \left(\frac{27}{3} + 12\right) - \left(-\frac{1}{3} - 4\right) = \frac{76}{3}$$



(4) Since $y = x^{\frac{3}{2}}$, then

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$$

The length of the arc of the curve from $(1,1)$ to $(4,8)$ is therefore

$$L = \int_1^4 \sqrt{1 + \left(\frac{3\sqrt{x}}{2}\right)^2}\ dx = \int_1^4 \sqrt{1 + \frac{9x}{4}}\ dx = \int_1^4 \sqrt{\frac{4+9x}{4}}\ dx$$
$$= \frac{1}{2}\int_1^4 (4+9x)^{\frac{1}{2}}\ dx = \frac{1}{2}\cdot\frac{1}{9}\int_1^4 9(4+9x)^{\frac{1}{2}}\ dx$$
$$= \frac{1}{18}\left[\frac{2}{3}(4+9x)^{\frac{3}{2}}\right]_1^4 = \frac{1}{27}\left[(4+36)^{\frac{3}{2}} - (4+9)^{\frac{3}{2}}\right]$$
$$= \frac{1}{27}\big(40^{\frac{3}{2}} - 13^{\frac{3}{2}}\big) = \frac{1}{27}\big(80\sqrt{10} - 13\sqrt{13}\big)$$

(5) (a) Let $u = -x^2$, then $\frac{du}{dx} = -2x \Rightarrow dx = -\frac{du}{2x}$

$$\int xe^{-x^2}\ dx = -\frac{1}{2}\int e^u du = -\frac{1}{2}\ e^u + C = -\frac{1}{2}\ e^{-x^2} + C$$

(b) Let $u = e^{2x} + 1$, then $\frac{du}{dx} = 2e^{2x} \Rightarrow dx = \frac{du}{2e^{2x}}$

$$\int e^{2x}\sqrt{e^{2x}+1}\ dx = \frac{1}{2}\int u^{\frac{1}{2}}\ du = \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \frac{1}{3}(e^{2x}+1)^{\frac{3}{2}} + C$$

(c) Let $u = x^3 + x^2 + 1$, then $\frac{du}{dx} = 3x^2 + 2x \Rightarrow dx = \frac{du}{3x^2+2x}$

$$\int \frac{3x^2+2x}{x^3+x^2+1}\ dx = \int \frac{1}{u}\ du = \log u + C = \log(1 + x^2 + x^3) + C$$

(d) Let $u = \cos x$, then $\frac{du}{dx} = -\sin x \Rightarrow dx = -\frac{du}{\sin x}$

$$\int \tan x\ dx = \int \frac{\sin x}{\cos x}\ dx = -\int \frac{1}{u}\ du = -\log u + C = -\log(\cos x) + C$$