Tutorial 5

Recall that $\mathbb{N}^* := \mathbb{N} \setminus \{0\}$.

Question 1. Determine whether the following sequences $(u_n)_{n \in \mathbb{N}^*}$ are monotonic.

(a) $u_n = \dfrac{n}{2n+1}$

(b) $u_n = \dfrac{2^n}{n^2}$

(c) $u_n = a + (n-1)d \qquad (a, d \in \mathbb{R})$

(d) $u_n = ar^n \qquad (a, r \in \mathbb{R})$

Question 2. Given the sequence $(u_n)_{n \in \mathbb{N}^*}$, its nth *partial sum*, S_n, is $S_n = \displaystyle\sum_{j=1}^{n} u_j$. Show that

(a) for $u_n = a + (n-1)d$ with $a, d \in \mathbb{R}$, $\;S_n = na + \dfrac{n(n-1)}{2}d$;

(b) $u_n = ar^n$ with $a, r \in \mathbb{R}$, $\;S_n = a\,\dfrac{1 - r^n}{1 - r}\;$ if $r \neq 1$.

Discuss the convergence or otherwise of the corresponding series.

Question 3.

Show that the series $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ converges.