PURE MATHEMATICS 212

MULTIVARIABLE CALCULUS

SOLUTIONS TO ASSIGNMENT 1.

Question 1. [3 marks]

Denote $P_1 = (2,1,6), P_2 = (4,7,9), P_3 = (8,5,-6)$. Then $v = \overrightarrow{P_1P_2} = \langle 2,6,3\rangle$ and $w = \overrightarrow{P_1P_3} = \langle 6,4,-12\rangle$.

Since $v \cdot w = 2 \cdot 6 + 6 \cdot 4 - 3 \cdot 12 = 0$ the sides are orthogonal and hence the triangle is a right one.

Question 2. [3 marks]

(a) The distance from $(5,2,3)$ to the xy-plane is 3, the length of the z-component;

(b) The distance from $(5,2,3)$ to the xz-plane is 2, the length of the y-component;

(c) The distance from $(5,2,3)$ to the y-axis is $\sqrt{5^2+3^2} = \sqrt{34}$, the length of the hypotenuse of the right triangle the legs of which are the x- and z-components.

Question 3. [2 marks]

Denote the terminal point by (x,y,z). We have

$$\begin{aligned} \langle x-(-2), y-1, z-4\rangle = \langle 1,2,-3\rangle, &\quad \text{i.e.} \\ x+2 = 1, y-1 = 2, z-4 = -3, &\quad \text{i.e.} \quad x = -1, y = 3, z = 1. \end{aligned}$$

Question 4. [3 marks] The plane is spanned by the vectors $v = \overrightarrow{AB} = \langle 1,1,-3\rangle$ and $w = \overrightarrow{AC} = \langle -1,3,-1\rangle$. The vector $n = v \times w = \langle 8,4,4\rangle$ is a normal vector.

Question 5. [3 marks]

$$x = 1+t, y = 1-2t$$

The direction vector $\langle 1,-2\rangle$ is the same as for the parallel line. For $t=0$ it passes through $(1,1)$.

Question 6. [2 marks]

We check if the normal vectors are perpendicular:

a) $n_1 = \langle 1,-1,3\rangle$, $n_2 = \langle 2,0,1\rangle$. $n_1 \cdot n_2 = 5 \neq 0$. Not perpendicular.

b) $n_1 = \langle 3,-2,1\rangle$, $n_2 = \langle 4,5,-2\rangle$. $n_1 \cdot n_2 = 0$. Perpendicular.

Question 7. [4 marks] a) $1(x-(-1))+2(y-4)-(z-(-3))=0$. The coefficients are the coordinates of the directing vector of the line. The resulting equation is $x+2y-z-10=0$.
b) The normal vector is perpendicular to the normal vectors $v=\langle 2,-1,1\rangle$ and $w=\langle 1,1,-2\rangle$ of the given planes. We choose $n=v\times w=\langle 1,5,3\rangle$. The resulting equation is $x+1+5(y-2)+3(z+5)=0$ or $x+5y+3z+6=0$.