AMTH140 ASSIGNMENT 4

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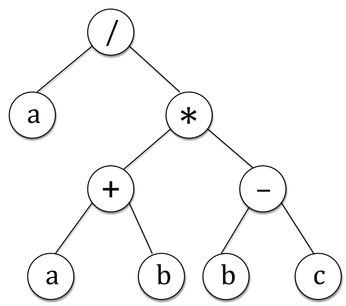
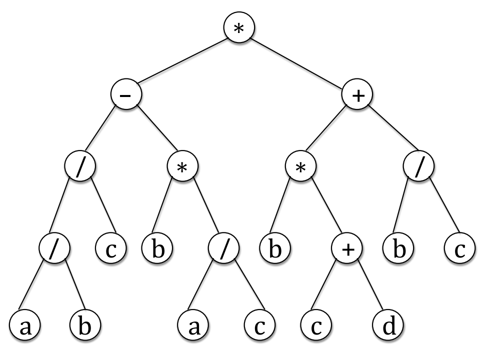
(1) (a) (i) $a/((a+b)*(b-c))$

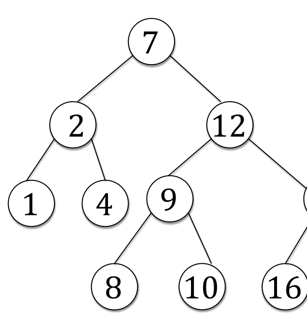


(ii) $(((a/b)/c)-(b*(a/c)))*((b*(c+d))+(b/c))$



(b) $a, b, /, c, /, b, a, c, /, *, -, b, c, d, +, *, b, c, /, +, *$

(2) Binary Tree Sort



(3) We define the network connection between the eight towns by the following weighted edge set.

e_i	$w(e_i)$
$e_1 = \{A, B\}$	5
$e_2 = \{A, D\}$	6
$e_3 = \{B, C\}$	8
$e_4 = \{B, D\}$	11
$e_5 = \{B, E\}$	9
$e_6 = \{C, E\}$	5
$e_7 = \{C, H\}$	5
$e_8 = \{D, E\}$	7
$e_9 = \{E, G\}$	16
$e_{10} = \{E, F\}$	13
$e_{11} = \{F, G\}$	9
$e_{12} = \{G, A\}$	11
$e_{13} = \{G, H\}$	7

Using Kruskal's algorithm we find the minimal spanning tree has weight 62 and the minimum cost of this network is $930,000$.

Steps	N(1)	N(2)	N(3)	N(4)	N(5)	N(6)	N(7)	N(8)	E	W
Initially	1	2	3	4	5	6	7	8	$\emptyset$	0
After 1	1	1	3	4	5	6	7	8	$\{e_1\}$	5
After 2	1	1	3	1	5	6	7	8	$\{e_1, e_2\}$	11
After 3	1	1	1	1	5	6	7	8	$\{e_1, e_2, e_3\}$	19
After 5	1	1	1	1	1	6	7	8	$\{e_1, e_2, e_3, e_5\}$	28
After 7	1	1	1	1	1	6	7	1	$\{e_1, e_2, e_3, e_5, e_7\}$	33
After 9	1	1	1	1	1	6	1	1	$\{e_1, e_2, e_3, e_5, e_7, e_9\}$	49
After 10	1	1	1	1	1	1	1	1	$\{e_1, e_2, e_3, e_5, e_7, e_9, e_{10}\}$	62

(4) (a) (i)

$$22 = 5 \times 4 + \boxed{2}$$
$$5 = 1 \times 4 + \boxed{1}$$
$$1 = 0 \times 4 + \boxed{1} \qquad 22 = 112_4$$

Since $16 = 4^2$ every two consecutive digits from left to right starting from the fractional point in base 4 will correspond to a single digit in hexadecimal representation, such that

$$12_4 = \boxed{6}$$
$$01_4 = \boxed{1} \qquad 112_4 = 16_{16}$$

(ii)

$$321_5 = 3 \times 5^2 + 2 \times 5 + 1 \times 1 = 75 + 10 + 1 = 86$$

$$86 = 21 \times 4 + \boxed{2}$$
$$21 = 5 \times 4 + \boxed{1}$$
$$5 = 1 \times 4 + \boxed{1}$$
$$1 = 0 \times 4 + \boxed{1} \qquad 321_5 = 1112_4$$

$$12_4 = \boxed{6}$$
$$11_4 = \boxed{5} \qquad 1112_4 = 56_{16}$$

(iii) Since $4 = 2^2$ every two consecutive digits from left to right starting from the fractional point in binary will correspond to a single digit in base 4.

$$10_2 = \boxed{2}$$
$$01_2 = \boxed{1}$$
$$\bullet$$
$$11_2 = \boxed{3}$$
$$01_2 = \boxed{1}$$
$$11_2 = \boxed{3}$$
$$01_2 = \boxed{1} \qquad 1110111.011_2 = 1313.12_4$$

$$12^4 = \boxed{6}$$

$$\bullet$$

$$13_4 = \boxed{7}$$

$$13_4 = \boxed{7} \qquad 1313.12_4 = 77.6_{16}$$

(b) (i)

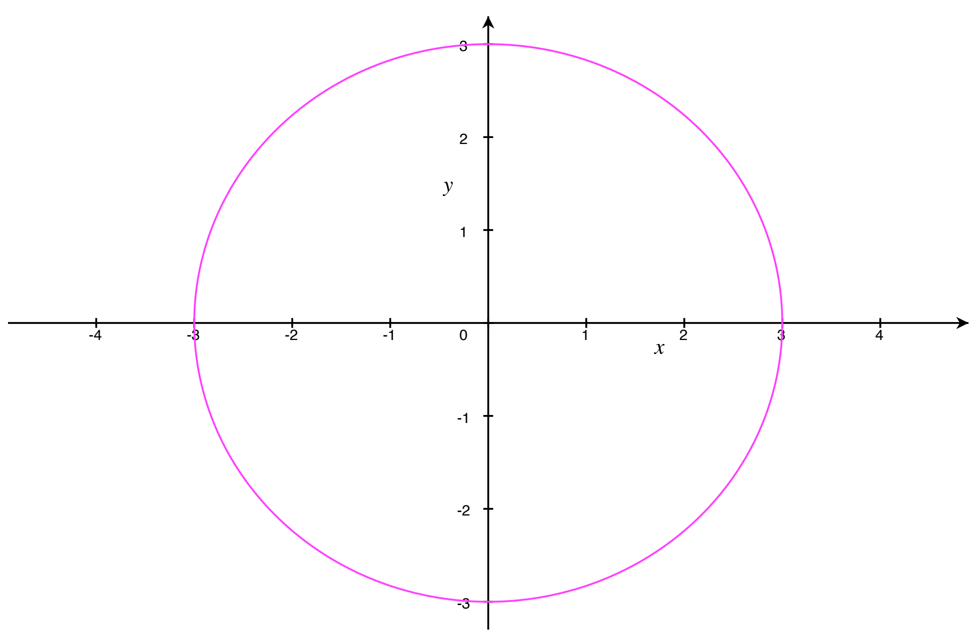
$$6_7 + 1_7 = 7_{10} = 10_7 = 1\boxed{0}$$
$$2_7 + 6_7 + 1_7 = 9_{10} = 12_7 = 1\boxed{2}$$
$$5_7 + 4_7 + 1_7 = 10_{10} = 13_7 = 1\boxed{3}$$
$$3_7 + 1_7 = 4_{10} = 4_7 = \boxed{4} \qquad 3526_7 + 461_7 = 4320_7$$

(ii)

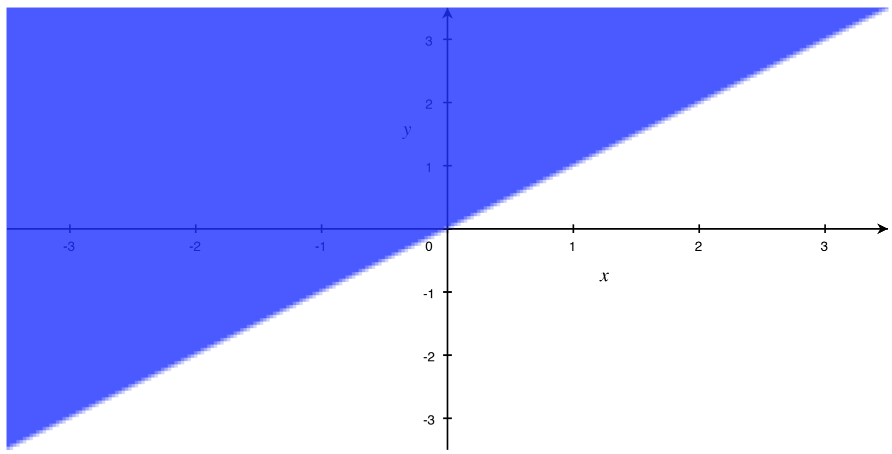
$$6_7 - 1_7 = 5_{10} = 5_7 = \boxed{5}$$
$$2_7 + 10_7 = 2_{10} + 7_{10} = 9_{10} = 12_7 \qquad \text{borrowed 1 from 5}$$
$$12_7 - 6_7 = 9_{10} - 6_{10} = 3_{10} = 3_7 = \boxed{3}$$
$$4_7 - 4_7 = 0_{10} = 0_7 = \boxed{0}$$
$$3_7 - 0_7 = 3_{10} = 3_7 = \boxed{3} \qquad 3526_7 - 461_7 = 3035_7$$

(5)

$$R = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 9\}$$



$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x \leq y\}$$



(6) (a) R is reflexive since there is a loop at each vertex. R is symmetric since there is at least one edge that connects v and w directly and that same edge connects w to v in this undirected graph. However,

$$(v_6, v_7) \in R,\ (v_6, v_8) \in R,\ (v_7, v_8) \notin R \ \Rightarrow\ R \text{ is not transitive}$$

(b) R is not an equivalence relation since it is not transitive.

(c) R' is now an equivalence relation since $[v_6] = [v_7] = [v_8] = \{v_6, v_7, v_8\}$, hence satisfying the transitivity condition. The induced partition of the set V, made up of the three distinct equivalence classes of R' is therefore

$$\{v_1, v_2, v_3, v_4\},\ \{v_5\},\ \{v_6, v_7, v_8\}$$