AMTH140 ASSIGNMENT 2

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(1) 3 comparisons are needed to to find the letter H using binary search.

i	1	2	3	4	5	6	7	8	9
$I(i)$	A	B	D	E	$\boxed{\text{G}}$	H	I	J	K

$$\text{Middlemost} : 5 = \left\lfloor \frac{1+9}{2} \right\rfloor$$

New sublist:

i	6	7	8	9
$I(i)$	H	$\boxed{\text{I}}$	J	K

$$\text{Middlemost} : 7 = \left\lfloor \frac{6+9}{2} \right\rfloor$$

Newer sublist:

i	6
$I(i)$	$\boxed{\text{H}}$

(2) (a) Since the last two columns of the following truth table are exactly the same, then $p \to q \vee r \equiv p \wedge (\sim r) \to q$.

p	q	r	$\sim r$	$q \vee r$	$p \wedge (\sim r)$	$p \to (q \vee r)$	$p \wedge (\sim r) \to q$
T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	F	T	F	T	T
F	F	F	T	F	F	T	T

(b) Since the last column is always true, then $p \to p \vee q$ is a tautology.

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

(3) $p \wedge q \to \sim r,\ p \vee (\sim q),\ \sim q \to p,\ \therefore \sim r$ is invalid since the argument form fails at 1 critical row (Row 3), where the premises are all true but the conclusion is false.

p	q	r	$\sim q$	$p \wedge q$	$\sim r$	$p \wedge q \to \sim r$	$p \vee (\sim q)$	$\sim q \to p$
T	T	T	F	T	F	F	T	T
T	T	F	F	T	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
T	F	T	T	F	$\boxed{F}$	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
T	F	F	T	F	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$	$\boxed{T}$
F	T	T	F	F	F	T	F	T
F	T	F	F	F	T	T	F	T
F	F	T	T	F	F	T	T	F
F	F	F	T	F	T	T	T	F

(4) $\sim r \to p,\ (\sim p) \vee q,\ \sim s \to (\sim p) \wedge (\sim r),\ (\sim p) \wedge r \to (\sim s) \vee t,\ \sim q,\ \therefore t$

1. $(\sim p) \vee q$	premise
$\sim q$	premise
$\therefore \sim p$	disjunctive syllogism
2. $\sim r \to p$	premise
$\sim p$	from 1
$\therefore\ r$	modus tollens
3. $\sim p$	from 1
r	from 2
$\therefore \sim p \wedge r$	conjunctive addition
4. $(\sim p) \wedge r \to (\sim s) \vee t$	premise
$\sim p \wedge r$	from 3
$\therefore\ (\sim s) \vee t$	modus ponens
5. $(\sim p) \wedge (\sim r) \equiv \sim (p \vee r)$	De Morgan's law

6. $\sim s \to \sim (p \vee r)$	premise given 5
$\therefore\ (p \vee r) \to s$	contrapositive equivalence
7. r	from 2
$\therefore\ r \vee p \ \equiv\ p \vee r$	disjunctive addition
8. $p \vee r \to s$	from 6
$p \vee r$	from 7
$\therefore\ s$	modus ponens
9. $(\sim s) \vee t$	from 4
s	from 9
$\therefore\ t$	disjunctive syllogism

(5) (a) For any natural number n, there exists a natural number m, such that n is equal to one greater the square of m.

(b) (i) Since $p \to q \equiv \sim p \vee q$ for any statements p and q, the negation of $\forall x \in \mathbb{R},\ (x^2 > 1) \to (x < -1)$ can be written as

$$\begin{aligned} \sim (\forall x \in \mathbb{R},\ (x^2 > 1) \to (x < -1)) &\equiv \sim (\forall x \in \mathbb{R}, \sim (x^2 > 1) \vee (x < -1)) \\ &\equiv \exists x \in \mathbb{R}, \sim (\sim (x^2 > 1) \vee (x < -1)) \\ &\equiv \exists x \in \mathbb{R}, (x^2 > 1) \wedge \sim (x < -1) \\ &\equiv \exists x \in \mathbb{R}, (x^2 > 1) \wedge (x \geq -1) \end{aligned}$$

(ii) Its contrapositive can be written as $\forall x \in \mathbb{R},\ (x \geq -1) \to (x^2 \leq 1)$.