Solutions to the Exercises (W11): MATH102–Statistics

17/10/06

1. Now
$$V(X) = E(X^2) - [E(X)]^2$$
and
$$E(X^2) = \sum_{j=0}^{\infty} j^2 P(X=j) = \sum_{j=0}^{\infty} j^2 \frac{e^{-\lambda}\lambda^j}{j!}$$
$$= \sum_{j=1} j\frac{e^{-\lambda}\lambda^j}{(j-1)!} = \lambda \sum_{j=1}^{\infty} j\frac{e^{-\lambda}\lambda^{j-1}}{(j-1)!}$$
Using $J = j-1$ gives
$$E(X^2) = \lambda \sum_{j-1=0}^{\infty} j\frac{e^{-\lambda}\lambda^{j-1}}{(j-1)!} = \lambda \sum_{J=0}^{\infty}(J+1)\frac{e^{-\lambda}\lambda^J}{J!}$$
$$= \lambda\left[\sum_{J=0}^{\infty} J\frac{e^{-\lambda}\lambda^J}{J!} + \sum_{J=0}^{\infty}\frac{e^{-\lambda}\lambda^J}{J!}\right]$$
$$= \lambda\left[E(X)+1\right] = \lambda\left[\lambda+1\right] = \lambda^2+\lambda$$
Hence $V(X) = \lambda^2 + \lambda - \lambda^2 = \lambda$

as required.

2. To estimate λ calculate the mean, ie, the no. of accidents per machinist, as
$$\bar{x} = (0 + 1\times 74 + 2\times 26 + 3\times 8 + 4\times 4 + 5\times 6)/414 = 0.47343$$
So the expected number of accidents per machinist is 0.47, or about 1 accident for every two machinists. We now use this estimate of λ to estimate the expected frequencies under the Poisson assumption.

x	$P(x)$	$f_e(x) = P(x) \times 414$
0	$e^{-\lambda} = 0.622862$	257.9
1	$P(0) \times \lambda = 0.294882$	122.1
2	$P(1) \times \lambda/2 = 0.069803$	28.9
3	$P(2) \times \lambda/3 = 0.011016$	4.5
4	$P(3) \times \lambda/4 = 0.001304$	0.5
5	$P(4) \times \lambda/5 = 0.000123$	0.1

Categories 3, 4 and 5+ need to be melded as their expected frequencies are below 5, giving

No. of accidents	0	1	2	3+	Total
Observed Frequency	296	74	26	18	414
Expected Frequency	257.9	122.1	28.9	5.1	414

Thus the df = 4 - 1 - 1 = 2.

The observed value if χ^2 is

$$\begin{aligned}\sum(O-E)^2/E &= (296-257.9)^2/257.9 + (74-122.1)^2/122.1 \\ &\quad +(26-28.9)^2/28.9 + (18-5.1)^2/5.1 \\ &= 5.63 + 18.95 + 0.29 + 32.63 = 51.87\end{aligned}$$

Since the $E(\chi^2) =$ df we reject the assumption of the Poisson distribution. Also $\chi^2_{2,5\%} = 5.99$ leading us again to reject the hypothesis of the generating distribution being Poisson.

3. From the previous question we find that

$$\sum_{i=0}^{i=5} P(x_i) = 0.622862 + \ldots + 0.000123 = 0.999990$$

meaning that

$$\sum_{i=6}^{i=\infty} P(x_i) = 0.00001$$

which gives

$$f_e(6) + f_e(7) + \ldots = 0.00414 \text{ (persons)}$$

which can be ignored.