AMTH140 ASSIGNMENT 3

MARK VILLAR

(1) (a) 10 comparisons will be performed using the Insertion Sort algorithm.

Steps	Lists						Extra comparisons
0	$\boxed{\text{E}}$	C	A	D	B	F	0
1	E	$\boxed{\text{C}}$	A	D	B	F	0
2	C	E	$\boxed{\text{A}}$	D	B	F	1
3	A	C	E	$\boxed{\text{D}}$	B	F	2
4	A	C	D	E	$\boxed{\text{B}}$	F	2
5	A	B	C	D	E	$\boxed{\text{F}}$	4
6	A	B	C	D	E	F	1

(b) 9 comparisons will be performed using the Quick Sort algorithm.

Steps	Lists						Extra comparisons
0	$\underline{\text{E}}$	C	A	D	B	F	0
1	$\underline{\text{C}}$	A	D	B	E*	F*	5
2	$\underline{\text{A}}$	B	C*	D*	E*	F*	3
3	A*	B*	C*	D*	E*	F*	1

(2) (a) Using Fleury's algorithm, we start at e since $\delta(e) = 3$ is odd. We then choose d as the next vertex since the removal of $\{e, d\}$ will not result in breaking the remaining subgraph into disconnected components. Repeating this procedure until all the other edges are removed, we find the following Eulerian path.

$$e \to d \to c \to g \to d \to f \to g \to b \to f \to e \to a \to b$$

(b) A circuit with exactly 5 edges that goes through vertex a and edge $\{e, f\}$ is

$$\{a, e\} \to \{e, f\} \to \{f, g\} \to \{g, b\} \to \{b, a\}$$

(c) A Hamiltonian path that goes through the edges $\{c, d\}$ and $\{e, f\}$ is

$$\{a, b\} \to \{b, g\} \to \{g, c\} \to \{c, d\} \to \{d, f\} \to \{f, e\}$$

(d) A Hamiltonian circuit that goes through the edges $\{c, d\}$ and $\{e, f\}$ is

$$\{a, b\} \to \{b, g\} \to \{g, c\} \to \{c, d\} \to \{d, f\} \to \{f, e\} \to \{e, a\}$$

(e) By Fleury's algorithm, a Eulerian circuit does not exist. No walk can be found that starts and ends at the same vertex without going through some repeated edges. Hence the graph is not Eulerian.

(3) (a) G_1, G_3 and G_5 can be grouped into the same isomorphism class while the remaining 3 graphs are not isomorphic to each other and thus are isomorphism classes in their own right.

(b) The isomorphic invariant that distinguishes the group comprising G_1, G_3 and G_5 is the number of edges, all containing 10 edges each. By contrast, G_2 and G_4 have 12 each while G_6 has 13. It follows that G_6 is distinguished by this same invariant property. Meanwhile, G_2 and G_4 are distinguishable because of vertices of a given degree. G_2 is the only graph that contains a vertex of degree 6 while G_4 is the only one that contains a vertex of degree 5.

(c) G_1 and G_5 are isomorphic to each other. The following isomorphism mapping associates the corresponding vertices of the two graphs.

$$V(G_1) \xrightarrow{\text{g}} V(G_5)$$

$$g(v_1) = w_3, \; g(v_2) = w_2, \; g(v_3) = w_6, \; g(v_4) = w_1$$
$$g(v_5) = w_5, \; g(v_6) = w_7, \; g(v_7) = w_4$$

(4) (a)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b)

$$A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 3 & \boxed{5} \\ 5 & 5 & 7 \end{bmatrix}$$

There are 5 walks from v_2 to v_3 which are of length 3.

(5) (a) By the degree-sum formula, such a graph cannot be a tree since

$$N = \sum_{v \in V} \deg(v) = 2 \cdot 1 + 1 \cdot 2 + 5 \cdot 3 = 19 \text{ (odd)} \neq 2|E|$$

where N is the sum of the total degrees of all vertices and E is the total number of edges. Because for any graph, N must always be *even*, the following properties suggest that this is not a graph at all, and therefore is not a tree.

(b) For a graph with n vertices to be a tree it must have exactly $n-1$ edges. A graph with 7 vertices and the following properties cannot be a tree because it doesn't have 6 edges.

$$N = 2 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 14 = 2|E|$$
$$E = 7$$

(c) Such a graph with 6 vertices could be a tree since it has 5 edges.

$$N = 4 \cdot 1 + 2 \cdot 3 = 10 = 2|E|$$
$$E = 5$$

The following tree illustrates these properties.