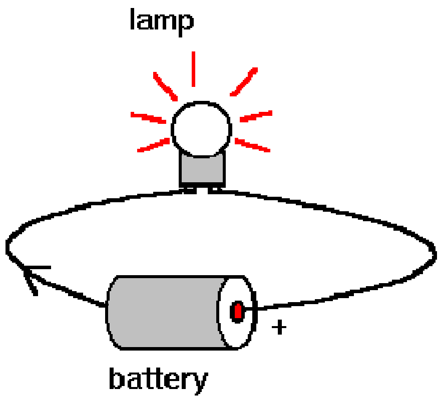
University of New England

Faculty of Arts and Sciences

School of Science and Technology



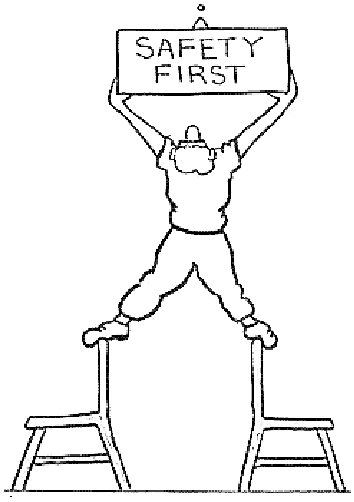
PHYS131

Applied Physics

Lab Manual

CONTENTS

SAFETY INSTRUCTIONS FOR LABORATORY WORK



General

1. Do not conduct experimental work of any sort unless a demonstrator is present.

2. All accidents, regardless of severity, must be reported to a demonstrator **immediately**. All eye injuries must be considered as serious.

3. Closed shoes must be worn.

4 Smoking, eating and drinking are not permitted in the laboratory.

Specific Activities

1. If the fire alarm rings continuously for more than 15 seconds, leave the building immediately in an orderly manner and assemble at the point indicated by the person in charge of evacuation.

2. Note the location of first aid boxes.

3. Note the location of electrical power isolation switches.

4. Handle electrical equipment carefully and according to instructions.

5. Report any faulty apparatus to your demonstrator immediately.

6. Personal protective and monitoring equipment must be worn as directed.

7. Radiation-warning signs must be displayed when radioactive isotopes are in use.

8. Laser-warning signs must be displayed when lasers are in use.

INTRODUCTION

Welcome to the laboratory component of the Physics 131 unit. We hope that you find the laboratory sessions interesting and that they add to your knowledge base in physics. The laboratory work is designed with several aims in mind but the main one is to act as an extension to lectures in order to help you understand and see for yourself that the theories do work.

Some extra aims of the laboratory sessions include:

- To teach you some of the techniques used in making measurements.
- To develop your skill in observations and in the interpretation of measurements.
- To develop your ability to assess uncertainty in measurement.
- To give you a first-hand acquaintance with some natural phenomena.
- To teach you good practice in recording the results to measurements and how to maintain a laboratory log book.
- To teach you how to write a technical report.

For internal students the laboratory sessions commence at 2.00 pm and should finish by 5.00 pm. They take place once per week on a designated day (say Thursday afternoons) and attendance at all laboratory sessions is **compulsory.** If you miss a session due to illness then a doctor’s certificate must be shown to the unit coordinator and you may be asked to make up the session at a later date. Laboratory sessions may not be scheduled for every week (eg week 1 does not have laboratory sessions) and so you must refer to the laboratory timetable or check with the unit coordinator for details. External students complete the laboratory part of the unit at the mandatory residential school.

You should bring the following to each laboratory session:

- A pen, a sharp pencil, a ruler, a protractor and a compass.
- A calculator. This should be able to calculate trigonometric functions, the exponential function and natural logarithms.
- This laboratory manual, your laboratory log book and the unit text book.

Assessment will be based as follows: up to 50% for the effort put in and the ability demonstrated during the practical, and up to 50% for the written work in laboratory log books or presentations.

The instructions for each experiment are written in a way that will lead you through the aim and theory of what is being examined. You should read through these and make sure that you understand them. Then stop and think of how you would design an experiment given the aim and theory that you have read. You may wish to do this with the apparatus on the bench in mind or you may wish to design an experiment using apparatus that you have come across elsewhere.

Once you have done this, read through the procedure given for the experiment and take the measurements that are detailed there. For each experiment write up a short Aim, Theory, Method, Results and Conclusion in your laboratory log book as you go. You will be expected to do this in the laboratory time so don't spend hours on this component of the experimental work. It is important however to enter all your measurements in your laboratory log book and note any mistakes that were made. Improvements can be commented on in the conclusion. You will be expected to hand in the write up at the end of each laboratory session so that assessment and feedback can be provided by laboratory demonstrators at the next session. Please make full use of the demonstrators experience during a practical and quickly ask about anything you are stuck on, or unsure of. They have been provided to help guide you through the practical.

Where asked to take measurements you are expected to give a value for the uncertainty in that measurement. Only give uncertainties for **calculated** quantities where asked. The next section describes these uncertainties and which ones we expect you to use and understand in the first year laboratory.

NOTES ON UNCERTAINTY

Measurements cannot be made precisely. There are a number of factors that contribute to this, including faulty apparatus, apparatus with its zero of scale at a nonzero position, fluctuations in environmental conditions (such as temperature and pressure), actual variations that exist in the required physical quantity (eg. when you measure the position of an electron you will get several values around an average value due to the uncertainty principle), and the fact that measuring instruments have a limit to their precision (if you measure a distance of 1 ½ metres with a ruler that has metre markings then this is less precise than for the case where a ruler with cm markings is used).

Every measurement has an uncertainty. The numerical value of the uncertainty is a measure of how far the stated value of the quantity deviates from the true value. Sometimes the word error is used in place of uncertainty. The types of uncertainties you will encounter in first year Physics are:-

1) Systematic or regular uncertainties
2) Statistical or random uncertainties
3) Uncertainties due to limit of reading of instruments.

Systematic or regular uncertainties are produced by well defined causes. They may arise from the instrument or from the observer or they may be due to some environmental condition. Such uncertainties may be constant in magnitude or they may vary in some regular way. Repeated observations and statistical analysis will not necessarily reveal them. For example, if a micrometer does not read zero when the jaws are closed, all readings will be too large or too small; if a barometer vacuum is imperfect, every pressure reading will be too small.

These uncertainties should be eliminated as far as possible, for example, by allowing for the zero error of the micrometer or making a correction for the imperfect vacuum in the barometer. If they cannot be eliminated then a note should be made of possible causes of uncertainty.

Statistical or random uncertainties arise from various unknown sources and may include the actual variations that exist in the magnitude of the measured quantity. The occurrence of this type of uncertainty is usually revealed by repeated observations. If it is possible to repeat a measurement many times, and if the repetitions give different results, then a good estimate of the uncertainty is half of the maximum deviation. About ten repetitions are needed for this to be reliable.

Uncertainties due to limit of reading of instruments The limit of reading of an instrument scale is the smallest quantity which can be reliably measured. For example, the smallest division of a metre rule is 1 mm. The limit of reading of this instrument is generally taken to be ±0.5 mm. Note, however that differences of 0.2 mm may be reliably measured by someone with good eyesight.

NOTE—almost all measurements are in fact two measurements: one to verify the zero of the instrument's scale and one to make the measurement. Both have uncertainty. This means that the net uncertainty is equal to the size of the finest scale division.

In the first year laboratory we expect you to use the uncertainties due to the limit of reading of instruments and to eliminate or comment on the other two. This means that every time you make a measurement we expect you to also quote a value for the uncertainty in this measurement in terms of the smallest quantity that can be reliably measured. Sometimes you will be asked to calculate a value (such as total mass = $m_1 + m_2$) and quote the uncertainty on this calculated value. You would do this using the uncertainties on each measured quantity. There are standard methods for doing this, depending on the calculation. The next section shows you how to calculate the maximum uncertainties when combining measured values for which you know the uncertainty.

Estimating Uncertainty in Repeated Measurements.

The relevant references regarding calculations of uncertainties can be found on the NIST website (http://physics.nist.gov/cuu/Uncertainty/index.html). If you have measured N times a value of some quantity, x_i, you can estimate the true value of this parameter by calculating the mean:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = \frac{1}{N}\left[x_1 + x_2 + \ldots + x_N\right]$$

The standard uncertainty to be associated with these measurements can be estimated as the standard deviation of the mean:

$$u(\bar{x}) = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$

Then we can write the result of our measurements as $\bar{x} \pm u$. The larger the number N, the closer the estimated values, given by these two equations, to their true quantities.

Combinations of Uncertainties

Rule 1.	The uncertainty in a sum or difference is the **square root** of the **sum** of the **squares** of the uncertainties

Let us assume that $m_1 = 81 \pm 3$ g and $m_2 = 40 \pm 4$ g are statistically independent quantities. Then clearly $\overline{m_1 + m_2} = \overline{m_1} + \overline{m_2} = 121$ g and $\overline{m_1 - m_2} = \overline{m_1} - \overline{m_2} = 41$ g. In both cases the total uncertainty is $u_{total} = \sqrt{u_1^2 + u_2^2} = \sqrt{3^2 + 4^2} = 5$ g.
Note that uncertainty of the result is always larger than the largest uncertainty of the initial quantities.

Rule 2.	The percentage uncertainty in a product or quotient is the **square root** of the **sum** of the **squares** of the individual percentage uncertainties.

In the above example, m_1 has a percentage uncertainty of (3/81) x l00 ~ 4%; and m_2 a percentage uncertainty of (4/40) x l00 = 10%. So:

$m_1 m_2$ = 3240 g^2 with an uncertainty of 11%

and m_1 / m_2 = 2.03 with an uncertainty of 11%

That is,

$$m_1 m_2 = 3240 \pm 360 \text{ g}^2$$
$$m_1 / m_2 = 2.03 \pm 0.22$$

The final case to be considered is that of functions of one variable. You will not be expected to calculate the uncertainties for these in the first year laboratory but we place them here for your general interest.

An example would be to suppose an angle θ is measured to be θ = 0.520 ± 0.010 radians. What is the uncertainty in sin θ?

Rule 3.	The uncertainty in $\boxed{f(x) \text{ is } f'(x)\ \delta x \text{ where } \delta x \text{ is the uncertainty in } x}$.

Rule 3 comes from the definition of a differential. If $y = f(x)$ then

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

Now suppose that δx is small, but not quite zero. We interpret δx as the uncertainty in x. Then

$$f'(x)\delta x \sim f(x + \delta x) - f(x)$$

The right-hand side of this expression is just the uncertainty in the function. That is, if

$$y = f(x)$$

then

$$\delta y = f(x + \delta x) - f(x)$$

This demonstrates rule 3, which can he used to prove rules l and 2.

In the above example, θ = 0.520 ± 0.010 radians, so the uncertainty in sin θ is (cos 0.52) x 0.01 = 0.009 since *d*(sin θ) / *d*θ = cos θ. Therefore sin θ = 0.497 ± 0.009.

It follows from Rule 3 that the **percentage** uncertainty in x^n is *n* times the **percentage** uncertainty in *x*. For example the percentage uncertainty in $\sqrt{x}$ is half that in *x*.

Setting out Uncertainty Calculations

Calculations where several operations are combined may cause confusion. A tabular form of calculation assists considerably and should be used for all uncertainty calculations.

Example:

$$V = xy(p + q)$$

By measurement, we find that *x* = 1.32 + 0.04 m, *y* = 0.470 ± 0.020 m, *p* = 14.1 ± 0.5 m, *q* = 13.2 ± 0.5 m. We want to calculate *V* and its uncertainty.

The uncertainty table below is filled in by the following steps.

1. Insert the experimental data, taking care that all quantities are expressed in SI units.

2. Calculate (*p* + *q*) and its uncertainty from Rule 1.

3. To calculate *xy*(*p* + *q*) and its uncertainty by Rule 2, the percentage uncertainties of (*p* + *q*), *x* and *y* are required.

4. Calculate *xy*(*p* + *q*).

5. Calculate the percentage uncertainty in *xy*(*p* + *q*) from Rule 2 and finally the absolute uncertainty.

Quantity	Value	Uncertainty	% Uncertainty
x	①1.32	①0.04	③3
y	①0.470	①0.020	③4.3
p	①14.1	①0.5	
q	①13.2	①0.5	
p+q	②27.3	②0.7	③2.5
xy(p+q)	④16.9	⑤0.9	⑤5.8

The answer is therefore *V*= 16.9 ± 0.9 m^3

Uncertainties on graphs

You may also be required to plot values on a graph, and for linear graphs calculate a value from the gradient of this graph and quote an uncertainty for this value. You therefore need to know how to calculate the uncertainty in the best-fit line. This is done by first plotting your points on the graph with what are known as error bars. These are lines drawn through the plotted point indicating the span of values that a measured quantity could have. The size of the error bars indicates the uncertainty in that measurement (in both x and y). So if you were plotting a point at $x = 1 \pm 0.2\ m$ and $y = 2 \pm 0.2\ m$ you would plot a point at $x = 1\ m$, $y = 2\ m$ and place a line of length 0.2 m to either side of it in x and in y. You would then draw a best fit line from which to calculate your measured value for the gradient and a worst fit line (that is still contained within the error bars) to calculate the maximum deviation from this value. This is the uncertainty in the gradient.

The Least Squares Method may be used to calculate both the gradient, *m*, and the uncertainty in the gradient, *Δm*, from the raw data. This is available as a program on some programmable calculators or from a web-based calculator, e.g. http://www.dartmouth.edu/~chemlab/info/resources/linear/linear.html

Quoting uncertainties and reporting results

The rules for quoting uncertainties and reporting results are:

- Quote one significant figure in the uncertainty unless that digit is a '1' or '2', in which case quote two significant figures.

- Quote the result to a precision that matches the uncertainty.

Example	35.83 ± 0.06 m	correct
	35.8 ± 0.06 m	incorrect
	35.8294 ± 0.06 m	incorrect

- If a power of ten is needed, use the same power for the uncertainty as for the result. This makes the statement of result easier to read.

Example	$(2.174 \pm 0.011) \times 10^{-7}$ J	correct
	$2.174 \times 10^{-7} \pm 1.1 \times 10^{-9}$ J	incorrect

- ALWAYS include the units.

NOTES ON THE DRAWING OF GRAPHS

1. Every graph must have a title or informative caption. It should be self contained, so that a reader does not have to dig around in the text to discover what the graph is about.
2. Choose a scale which fills the grid as far as possible, but always use multiples of 1, 2 or 5 for the major divisions. Other choices make it difficult to plot points.
3. ALWAYS label the axes with the name of the quantity being plotted. **SHOW THE UNITS**.
4. Data points must be clearly visible. Use small crosses or dots inside circles.
5. The graph should be a smooth curve, almost always a straight line, which lies evenly between the plotted points with some on either side. Do not sacrifice smoothness by forcing the line to pass through every point. Do not attach undue significance to the origin; it is subject to the same experimental uncertainties as any other point.
6. If the points lie approximately on a straight line, and if there is sufficient reason to expect that a straight line is a correct representation of the data, then draw the line of best fit using a ruler and a **sharp** pencil. A clear plastic ruler is best.
7. If you need the slope of a line or other information, then you may print it on the graph. **NOTE**: a slope usually has **UNITS**.

The graph below illustrates the above points

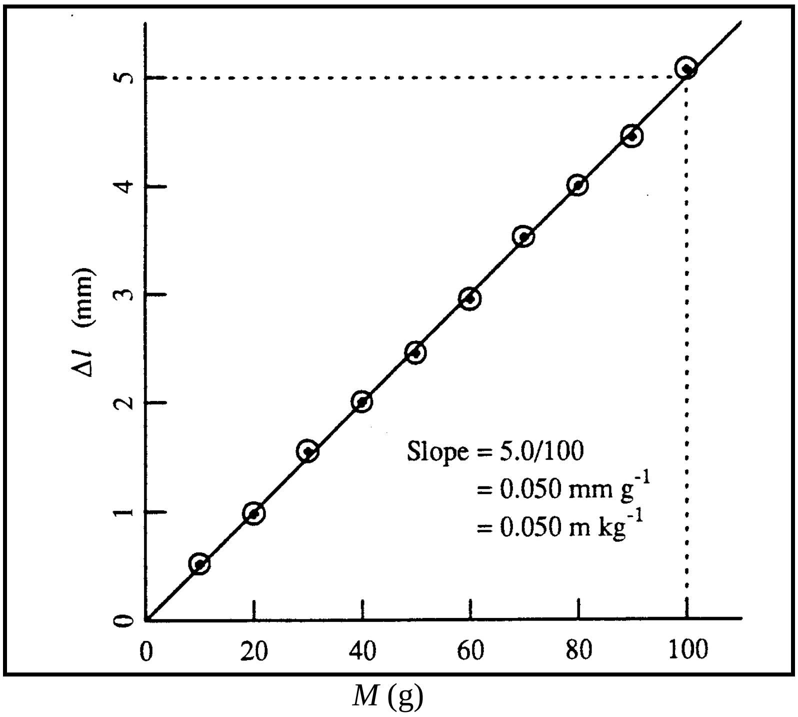


Fig. 1. Cumulative extension Δl of a thin nichrome wire as a function of the load from a mass M applied to one end.

GOOD EXPERIMENTAL PROCEDURE

- Record all measurements directly into your log book. Recording observations on scraps of paper and subsequently copying into the log book is very bad practice.

- As far as practicable, make several observations of each quantity. This minimises the effects of mistakes and it assists with the determination of the uncertainty in the measurement.

- ALWAYS record exactly what you read from an instrument. DO NOT perform unit conversions or any other calculations before writing the result down. This is simply good experimental practice.

- It is usually best to record observations in the form of a table. Each column of the table must have a heading, INCLUDING UNITS. A tabular form can shorten the time taken in doing calculations, since they can then be performed in a systematic way.

Example

We measure height h as a function of time t and need to plot a graph of h^2 against $1/t$. We expect the graph to be a straight line.

The results should be recorded as follows:

H (cm)	h^2 (m^2)	t (min)	$\frac{1}{t}$ (sec^{-1})	Comments

The units in columns 1 and 3 are the units in which the respective instruments are calibrated. The final comment column is useful for recording other necessary experimental conditions or other comments on the reading. Note anything that you think might affects the accuracy of a measurement.

- Handle apparatus carefully. You may be required to pay for the repair of apparatus damaged.

- At the end of the afternoon, the apparatus must be left tidily in the condition in which you found it.

HOW TO PRESENT A TECHNICAL REPORT

A report must contain the following sections**:**

- Title of the experiment and date that it was conducted.

- Aims of the experiment**.**

- Theoretical background.

  Do not just copy the theory from the experiment description. Paraphrase it. We are looking for evidence that you understand what you are reporting on.

- Method, experimental observations, and comments.

  The presentation of your work should be clearly and logically arranged so that the reader can easily follow what you have done. You should write in the past tense (for example "the object was weighed on electronic scales"). Do not reproduce large quantities of the experimental notes, merely state, in your own words what was done. The comments on your work should include the following:

  - A brief explanation of what was done at each stage of the experiment.

    Relevant detailed information not included in the experimental notes.

    A description of mistakes you may have made and how they were corrected, or their effect if it was not possible to correct them.

- Results and calculations. The following should be included:

  - Your results should be presented in tabular format, with uncertainties quoted where appropriate. Tables can also be used for calculations where appropriate. Graphs and calculations should be included in this section.
  - An uncertainty analysis where asked. All direct measurements must be stated with an uncertainty.
  - The final, result should be quoted and compared with tabulated data, giving reasons for any discrepancy.

- A conclusion.

  Irrespective of whether an experiment was successful or not, you MUST write a conclusion. As well as providing a final statement of the results obtained, it affords an opportunity for a critical appraisal of the experiment. In general, you should address the following points:

  - Summarise the experiment in terms of the fundamental principles of physics demonstrated.

- State the final result(s) and the estimated uncertainty or major source of uncertainty, including the appropriate units.
- Compare your result with tabulated data, citing the reference.
- Discuss any discrepancy between the experimental and tabulated results, highlighting the major sources of uncertainty.
- If possible, suggest ways of improving the experiment by reducing the uncertainty factors and removing or reducing difficulties.

LAB PRACTICAL MARKING SCHEME

The following is a general marking scheme and may be modified for certain practicals:

Practical:

Effort put in and ability *demonstrated* during practical:	50%

NOTE: a mark for the effort put in and ability demonstrated during a practical is assigned once a lab report is submitted. Where a student completes a lab but fails to submit a report, a mark of zero for the practical is recorded.

Report:

Overall presentation:	3%
Title, date:	2%
Aims:	3%
Theory:	5%
Method:	15%
Results, calculations:	15%
Conclusions:	7%

FREQUENTLY USED SYMBOLS

There are many symbols used in physics. Here is a list of some that you should become familiar with.

Greek alphabet

Learn to recognise the Greek letters and practice writing them legibly.

	α	alpha		ν	nu
	β	beta	Ξ	ξ	xi
	γ	gamma		ο	omicron
Δ	δ	delta		π	pi
	ε	epsilon		ρ	rho
	ζ	zeta	Σ	σ	sigma
	η	eta		τ	tau
	θ	theta		υ	upsilon
	ι	iota	Φ	ϕ or φ	phi
	κ	kappa		χ	chi
Λ	λ	lambda	Ψ	ψ	psi
	μ	mu	Ω	ω	omega

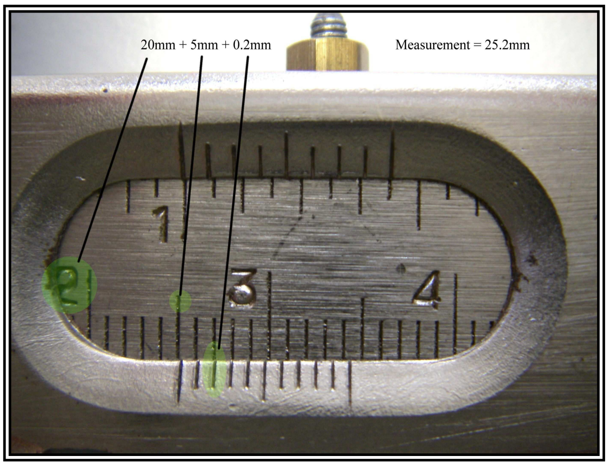
SI prefixes

Use these as often as possible. You should learn the prefixes down to tera and femto.

kilo	k	$x10^{3}$	milli	m	$x10^{-3}$
mega	M	$x10^{6}$	micro	μ	$x10^{-6}$
giga	G	$x10^{9}$	nano	n	$x10^{-9}$
tera	T	$x10^{12}$	pico	p	$x10^{-12}$
peta	P	$x10^{15}$	femto	f	$x10^{-15}$
exa	E	$x10^{18}$	atto	a	$x10^{-18}$
zetta	Z	$x10^{21}$	zepto	z	$x10^{-21}$
yotta	Y	$x10^{24}$	yocto	y	$x10^{-24}$

DIMENSIONAL MEASUREMENTS

Vernier Callipers



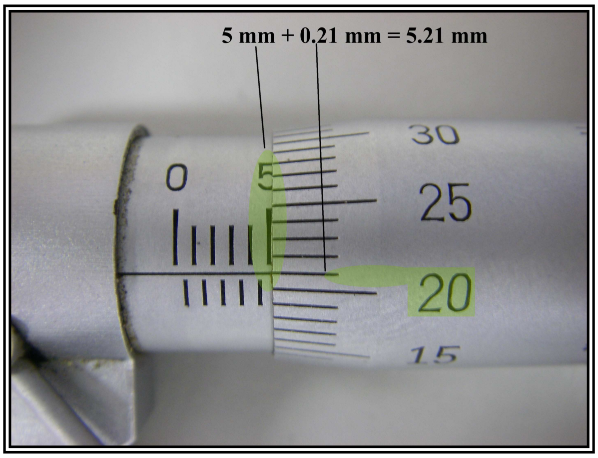
The reading on the vernier scale is 25.2 mm.

On this vernier, the lower scale reads in mm and the upper scale in inches. We will always use the lower scale.

The 2, 3 and 4 visible represent cm or 20, 30 and 40 mm. You can work in either units as you prefer.

Take the position of the left of the lower sliding scale to start; it is past 2cm, counting along another 5 mm your initial reading is 2.5cm (or 25mm).
The next value is determined by the graduation mark on the lower scale that best aligns with a graduation mark on the upper fixed scale.
Here the best alignment is at the second graduation so the reading will be 0.2 mm.
Your final reading is 2.52 cm (or 25.2 mm).

Micrometer



The reading on this micrometer is 5.21 mm.

On the shaft the upper graduations are 1mm and the lower indicate 0.5mm i.e. the second of the lower graduations will represent 1.5mm.

One complete rotation of the barrel is 0.5mm therefore each division equals 0.01 mm.

The reading shown is determined in this manner, the 5mm is visible on the upper scale of the shaft but no 0.5 graduation is visible, so, then taking the reading from the barrel that aligns with the centre mark on the shaft we get 0.21 added to the value from the shaft this gives a reading of 5.21mm.

INTRODUCTORY EXERCISES
Graphing

Usually, one measures one variable as a function of another variable (e.g. velocity as a function of time) and then deduces some property (e.g. the acceleration) from the data. Since each measurement has an uncertainty associated with it, the data will not exactly follow a simple relation. Drawing an appropriate graph of the data is a good way of averaging out random errors and so determining how closely a given relation is obeyed. It also provides an accurate way of deducing other properties from the data. This exercise gives you the opportunity to practice using graphs.

1. A ball is thrown vertically upward on the moon and its velocity is measured as a function of time, giving the following data.

Time (s)	Velocity (ms^{-1})
1.0	14.6
2.0	12.9
3.0	10.8
4.0	9.4
5.0	8.0
6.0	5.7
7.0	4.5
8.0	2.9

Draw a graph showing the variation of velocity with time and determine

- the acceleration,
- the initial velocity,
- the time for the ball to reach its highest point.

2. Straight-line graphs are very useful because they are straightforward to interpret and simple to analyse. Often, the relationship between the two measured variables is not linear, but it is sometimes possible to rearrange the relationship algebraically to produce a linear relationship between two different variables. This then tells you how to manipulate your measured data in order to plot a linear graph.

Example

Suppose that measured quantities n and t are related by

$$n = n_o \exp(-t / \tau) \tag{1}$$

where n_o and τ are constants. The goal of the experiments might be to determine τ by measuring many n, t pairs. Clearly, n and t are not related linearly. But some algebra gives:

$$\ln n = \ln n_o - t / \tau \tag{2}$$

If we compare this to the standard form of a straight line:

$$Y = c + mX, \tag{3}$$

we see that a linear relationship results by taking:

$$Y = \ln n, \;\; X = t, \;\; m = -1 / \tau \tag{4}$$

That is, a plot of ln n against t is linear with slope -1 / τ.

Another Example (*more complicated*)

Measured quantities x and y are related by

$$x^2 = \frac{a}{b + 1/y} \tag{5}$$

Some algebra gives

$$\frac{1}{y} = \frac{a}{x^2} - b \tag{6}$$

Comparing this with the equation (3), we see that a plot of $Y = 1 / y$ against $X = 1 / x^2$ will give a straight line of slope a and intercept $-b$.

Exercise

The elevation s above sea level of an object sliding down a curved cliff face varies with time t according to the formula

$$s = \frac{a}{b + t^2}$$

Measured data:

t (s) (+/- 1.0s)	s (m) (+/- 1.0m)
5	29.4
10	22.4
15	14.7
20	9.8
25	7.2

It is required to determine a and b from these data.

- Determine a suitable way of plotting a graph so that the results will be a straight line and the constants a and b can be obtained.
- Draw up a table which will help with the calculations. Assume that the uncertainty on values for x and y on graph = 10% for simplicity.
- Plot the graph and determine a and b from the graph.
- Estimate the uncertainty in the slope of the graph (and hence in a).

A simple measurement

5. Select one of the metal rods and make measurements to determine its density.

    - Measure the length with vernier callipers, the diameter with a micrometer and the mass with an electronic balance.
    - Estimate the uncertainty in each measurement and state it.
        - Do not simply assume that the digital display on the balance is accurate to the last figure. You can check this by checking what value is given for zero mass.
    - Using your measured values, calculate the density of the rod.
    - Calculate the uncertainty in your measurement of the density by applying the rules for uncertainty combinations.
    - Guess the material of your rod, and look up the accepted value of the density in tables.
    - Does your measured result agree with the tabulated result to within the uncertainty? If not, explain why.

Name:______________________________

EXPERIMENT NO. 1 **FORCE TABLE**

Note: In this experiment it is only necessary for you to complete the results of your work in the following pages of this Laboratory Manual. On completion of the experiment you may carefully detach and submit these completed pages as your Experiment Report (ie no other written report is required).

Aim:
To determine the resultant of several concurrent forces using three methods: graphical method (tail-to-head scale drawings), vector component method (addition of components) and a force table.

INTRODUCTION

Measurable quantities may be classified as either:
1. scalar quantities or
2. vector quantities.

A scalar quantity has magnitude only but a vector quantity has both magnitude and direction. For example, since to specify completely the velocity of a body it is necessary to state not only how fast it is travelling but also in what direction it is going, velocity is a vector quantity. We need to state the size of a vector (its magnitude) and its direction. However, the mass of a body is completely specified by a magnitude and mass is a scalar quantity. Since the weight of a body is the force with which it is attracted by the earth, weight is a vector quantity, having units of Newtons (N). In this experiment we will be working with forces, discovering how to add them up, given their vector nature.

A vector can be represented as an arrow, its length corresponding to the size of the vector, and the direction of the arrow representing the direction of the vector. For instance, force is a vector and so can be drawn as an arrow. This is illustrated in Figure 1 for a force of 10 N acting to the right.

Figure 1. A line representing a force of about 10 N acting to the right.

The length of the arrow is drawn to scale to indicate the magnitude of the force. For example, 1cm might be taken to represent 1 N of force. The direction of the arrow indicates the direction of the force and the arrowhead indicates the sense, in this case from left to right.

If two or more forces are exerted at a common point they may be added together and replaced by a single force, called the resultant. For instance, a number of forces act on the shoulder and they may be represented by a single resultant force (see Figure 2).

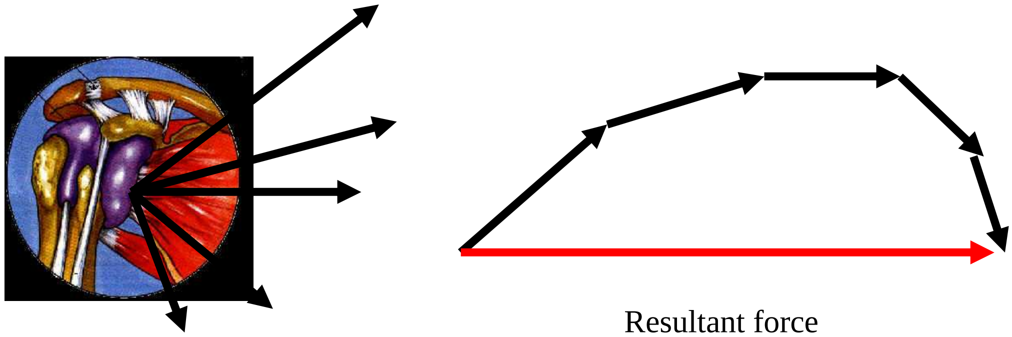


Figure 2. The forces acting on the shoulder can be represented by a single resultant force.

Adding vectors to find the resultant is easy if they happen to line up (ie. be in the same direction, we simply add up the magnitude of the forces), but when they don't, as in the case of the shoulder, addition is a little harder. There are two methods of calculating the resultant theoretically that we will attempt in this experiment: the graphical method (tail-to-head scale drawings), and the vector component method (addition of components).

The graphical method of vector addition

The starting point for this method is to draw a diagram of what's going on. We simply draw the vectors to scale and in the correct direction. We draw the forces as arrows with a head and a tail. The head shows the direction of the force. We draw the arrow so that its length is the magnitude of the force. For instance, a 10 N force in the positive x direction would be drawn pointing in the positive x direction with a size indicating the 10 N (eg it may be drawn 10 cm long). Then we draw the next force, the one we're going to add to it, with its tail at the first one's head and at the correct angle. Again draw it to scale (ie 10 cm for 10 N). We can then measure the magnitude of the resultant force with a ruler and its direction with a protractor.

Take the example in figure 3, where two forces are applied to a body at right angles to one another. We have 40 N along the x-axis and 30 N along the y-axis ((a)). What is the resultant of these two forces? What does the sum of these two forces look like? What would be the one force that could represent both these forces? Well to find out we draw a diagram as shown, to scale ((b)). The 40 N force would be say 4 cm long and the 30 N one 3 cm. We draw them head to tail. The resultant can be found by joining the tail of the first force to the head of the last one ((c)). We can measure this resultant force with a ruler using our scale (ie if it measures 5 cm then it will be a 50 N force) and the angle at which it acts with a protractor. This can be done for any number of forces acting at the same point.

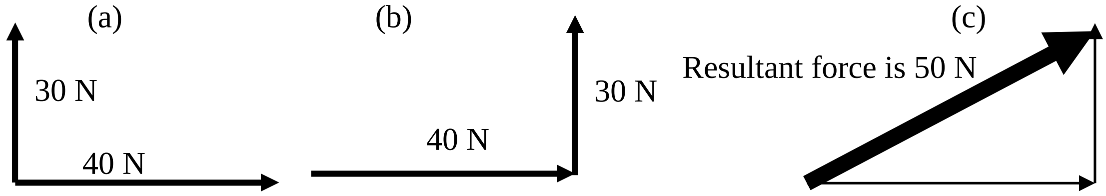


Figure 3. The graphical method for vector addition - (a) two forces acting on a body are represented as arrows whose length represents their magnitude and their direction is that of the force. (b) Drawing them tail-to- head helps work out the resultant of the two forces, shown in (c).

The vector component method of vector addition

A second method of vector addition is called the vector component method of vector addition. We'll start simply using the same example shown in figure 3, one force of 40 N acting along the *x*-axis and another of 30 N acting along the *y*-axis. In this method, instead of drawing the forces to scale and measuring the resultant force, we can use trigonometric calculations to work out the magnitude and direction of the resultant force. These two forces form a right-angled triangle (see figure 4). In these triangles, if we know the size of the two sides (the two forces A and B here) we can calculate the size of the third side (the resultant, C) and the angle at which the resultant acts. The relationship is $A^2 + B^2 = C^2$, where A, B and C are the three sides of the triangle (see figure 4). So to get C we need to calculate $\sqrt{A^2 + B^2} = C$.

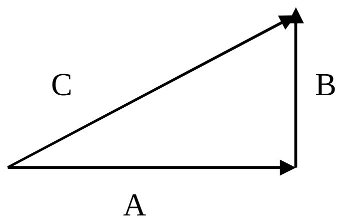


Figure 4. A right-angled triangle

So the resultant of the two forces acting at right angles to one another can be found by putting in the values for the magnitude of A and B. In our case it is $\sqrt{40^2 + 30^2} = 50$. We can calculate the angle between A and C by using the relationship $\theta = \tan^{-1}\frac{B}{A}$. In this case it is $\theta = \tan^{-1}\frac{30}{40} = 37^0$.

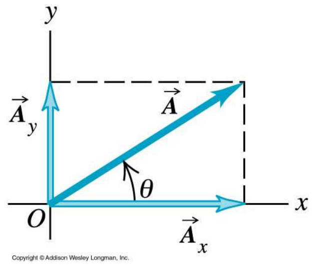
When the forces do not act at right angles to each other we can still add vectors numerically using the following steps:

1. **Sketch** the vectors to see the size and direction

2. Split the vectors into their **components** (horizontal and vertical)
3. Add the **horizontal** components together
4. Add the **vertical** components together
5. **Combine** the components for the final vector

What does all this mean? The horizontal components are how much of the force acts in the *x*-direction, the vertical components are how much of the force acts in the *y*-direction. Figure 5 demonstrates this. The horizontal component of force A is $A_x = A\cos\theta$, the vertical component is $A_y = A\sin\theta$

Figure 5. Splitting vectors into horizontal and vertical components.



So take each force and calculate each one's horizontal component (using $A_x = A\cos\theta$) and vertical component (using $A_y = A\sin\theta$). Then add all the horizontal components from all the forces together, to get the total horizontal component. This is the horizontal component of the resultant. Say we had two forces A and B, then $R_x = A_x + B_x$. Now do the same for the vertical components, so $R_y = A_y + B_y$. Now we have a resultant in the *x*-direction and a resultant in the *y*-direction. This is just like the simple case where we had a right-angled triangle, but instead of the forces A and B we have R_x and R_y. The components R_x and R_y can be combined to form the final vector R, as shown in figure 6.

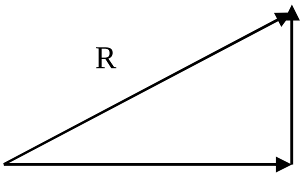


Figure 6. Combining R_x and R_y to get the final resultant.

We can also calculate the magnitude and direction of this resultant using the equation we had before for right-angled triangles, where $\sqrt{R_x^2 + R_y^2} = R$ and $\theta = \tan^{-1}\frac{R_y}{R_x}$.

The following diagram shows two vectors A and B being added. Each vector is separated into its horizontal and vertical components and the components are added:

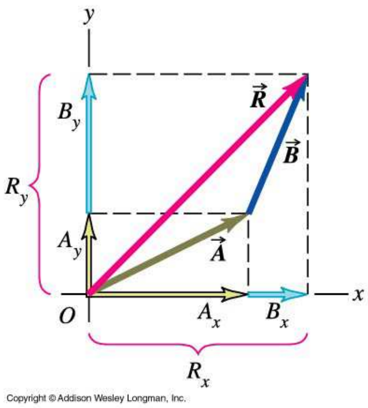


Figure 7. Adding two vectors A and B to find the resultant R using horizontal and vertical components of A and B.

We can see that in the horizontal direction: $R_x = A_x + B_x$ and in the vertical direction: $R_y = A_y + B_y$. These are the resultant in the *x*-direction and a resultant in the *y*-direction. The components R_x and R_y can be combined to calculate the magnitude and direction of the resultant R using $\sqrt{R_x^{\ 2} + R_y^{\ 2}} = R$ and $\theta = \tan^{-1}\frac{R_y}{R_x}$. Please note that the function $\tan^{-1}(x)$ (or arctan(x)) is a periodical function with the period of $\pi = 180^\circ$. Therefore, you should check whether $\theta = \tan^{-1}\frac{R_y}{R_x}$ or $\theta = \tan^{-1}\frac{R_y}{R_x} \pm \pi$ is the correct answer. The appropriate verification is based on the knowledge in which quadrant (e.g., either in $0 \rightarrow 90^\circ$ or in $180^\circ \rightarrow 270^\circ$) the vector $\vec{R}$ lies.

PART A

In this part of the prac you will be required to add forces together using both the graphical tail-to-head and vector component techniques for vector addition.

On graph paper provided, use the graphical tail-to-head technique to find the vector sums of the indicated forces. <u>Compare this with results using the vector component method.</u>

A1. Force **A** = 20 N at 0^0 (so along the *x*-axis)
Force **B** = 30 N at 90^0 (so along the *y*-axis)

A2. Force **A** = 20 N at 45^0 to the *x*-axis
Force **B** = 30 N at 130^0 to the *x*-axis

Problem A1 Graphical 'tail-to-head'

Problem A1 Vector component

Problem A2 Graphical 'tail-to-head'

Problem A2 Vector component

PART B

In this part of the experiment you will use the force table to see how well practical methods for vector addition compare with theoretical ones. You will be given several force combinations for which you will find the resultant using the force table first, and then you will use the graphical method and the vector component method. The experimental and theoretical resultant will then be compared. Before we use the force table we need to understand how it works and what it measures. Figure 8 shows what the force table looks like.

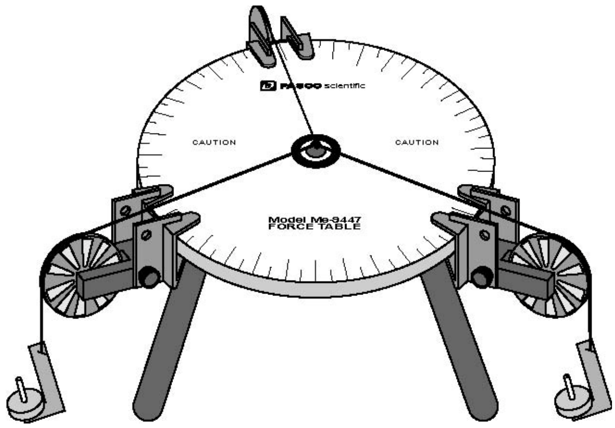


Figure 8. The assembled force table

Two forces are applied on the force table by hanging masses over pulleys positioned at certain angles. Then the angle and mass hung over a third pulley are adjusted until it balances the other two forces. This third force is called the equilibrant ($\vec{F}_E$) since it is the force which establishes equilibrium. The equilibrant is not the same as the resultant ($\vec{F}_R$). The resultant is the addition of the two forces. While the equilibrant is equal in magnitude to the resultant, it is in the opposite direction because it balances the resultant (see Figure 9). So the equilibrant is the negative of the resultant:

$$-\vec{F}_E = \vec{F}_R = \vec{F}_A + \vec{F}_B$$

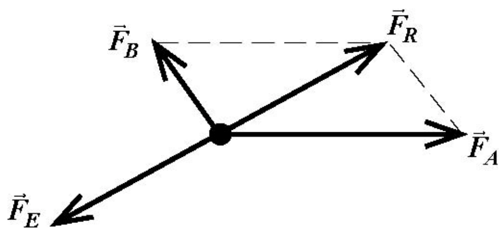


Figure 9: The Equilibrant balances the resultant

So when we use the force table we will measure the equilibrant, the resultant being easily determined using this by remembering that the resultant is equal in magnitude but in the opposite direction to the equilibrant.

Apparatus

The Force Table apparatus includes the following:
Force Table Assembly with Centre Post and three Detachable Legs
Three Super Pulley Clamps
Three Mass Hangers
Plastic Ring
Spool of Thread

You will also require masses and Sellotape (supplied), a protractor, ruler and pencil for theoretical calculations.

Assembly

Remove the three legs from the clips on the bottom of the plastic force table disk. Screw the legs into the holes on the bottom of the disk. The protractors printed on the pages at the end of this experiment are smaller versions of the top surface of the Force Table. These can be trimmed and used as overlays on the Force Table for drawing and tracing of the string positions. If you wish to use these then attach them to the Force table with Sellotape. Next attach three pulleys and clamps to the rim of the disk. There are two ways to attach the strings to the table: the first way uses the conventional ring in the centre of the table and the second way uses an anchor string through the hole in the centre of the table. The advantage of the anchor string is that a higher precision can be achieved because a single knot is being centred instead of the massive ring. The anchor string keeps the masses from falling to one side when the system is not in equilibrium.
NOTE: In both methods it is important to adjust the pulleys so that the strings are parallel to the top surface of the Force Table, and as close to the top surface as possible. When adjusting the pulleys, don't let the ring rest on the top surface.

Ring Method

The correct set up for this method is shown in Figure 10. To use this method, screw the centre post up until it stops so that it sticks up above the table. Place the ring over the post and tie one 30 cm long string to the ring for each pulley. The strings must be long enough to reach over the pulleys. Place each string over a pulley and tie a mass hanger on the end of each string.

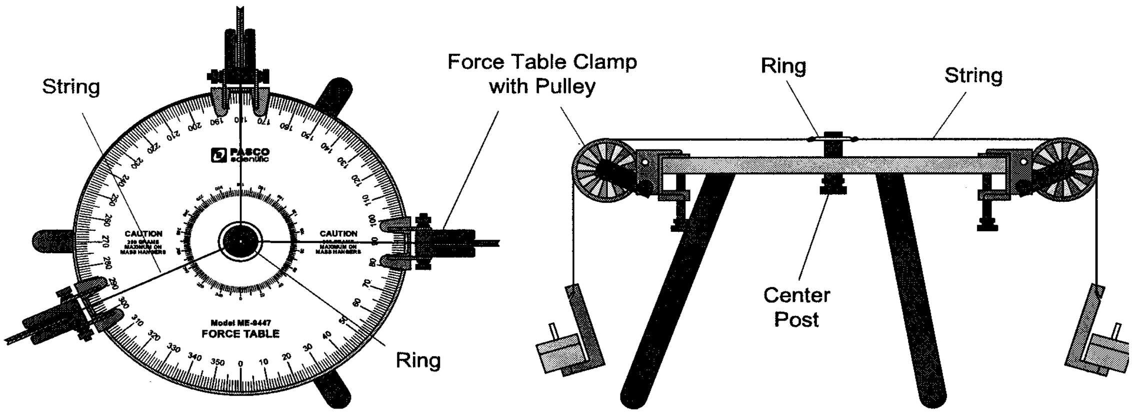


Figure 10: The table set up for the Ring method

Anchor String Method

The correct set up for this method is shown in Figure 11. Cut two 60 cm lengths of string and tie them together at their centres (to form an “X”). Three of the ends will reach from the centre of the table over a pulley; the fourth will be threaded down through the hole in the centre post to act as the anchor string. Screw the centre post down so it is flush with the top surface of the table. Thread the anchor string down through the hole in the centre post and tie that end to one of the legs. Put each of the other strings over a pulley and tie a mass hanger on the end of each string.

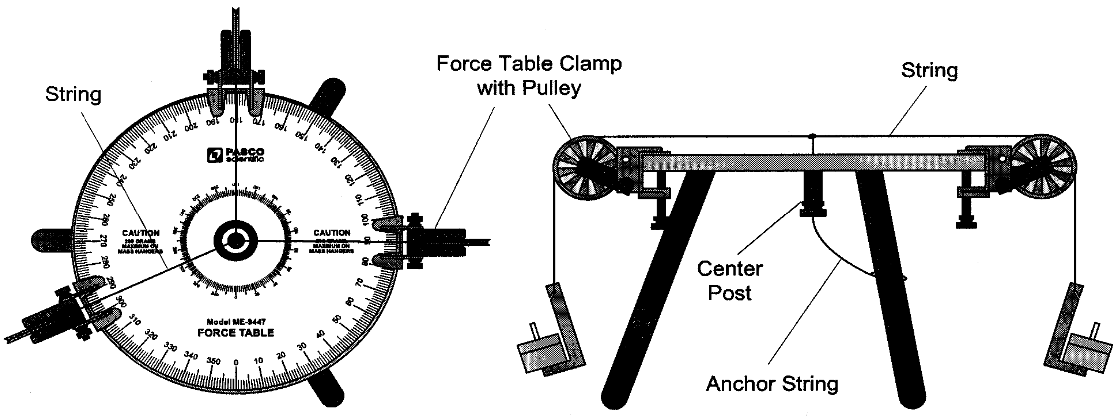


Figure 11: The table set up for the Anchor String method

NOTE: In all cases, the force caused by the mass hanging over the pulley is found by multiplying the mass by the acceleration due to gravity (*mg*).

EXPERIMENTAL METHOD

1. Assemble the force table as described in the Assembly section. Use three pulleys (two for the forces that will be added and one for the force that balances the sum of the two forces).
2. Decide whether to use the Ring Method or the Anchor String Method. If you are using the Ring Method, screw the centre post up so that it will hold the ring in place when the masses are suspended from the two pulleys. If you are using the Anchor String Method, leave the centre post so that it is flush with the top surface of the force table. Make sure the anchor string is tied to one of the legs of the force table so that the anchor string will hold the strings that are attached to the masses that will be suspended from the two pulleys.
3. Hang the following masses on two of the pulleys and clamp the pulleys at the given angles:

   Pulley A = 50g at 60°
   Pulley B = 100g at 160°

   This is problem 1.
4. By trial and error, find the angle for the third pulley and the mass which must be suspended from it that will balance the forces exerted on the strings by the other two masses. To determine whether the system is in equilibrium, use either of the following criteria:

Ring Method of Finding Equilibrium

The ring should be centred over the post when the system is in equilibrium. Screw the centre post down so that it is flush with the top surface of the force table and no longer able to hold the ring in position. Pull the ring slightly to one side and let it go. Check to see that the ring returns to the centre. If not, adjust the mass and/or angle of the pulley until the ring always returns to the centre when pulled slightly to one side

Anchor String Method of Finding Equilibrium

The knot should be centred over the hole in the middle of the centre post when the system is in equilibrium. The anchor string should be slack. Adjust the pulleys downward until the strings are close to the top surface of the force table. Pull the knot slightly to one side and let it go. Check to see that the knot returns to the centre. If not, adjust the mass and/or angle of the third pulley until the knot always returns to the centre when pulled slightly to one side.

5. The third force is called the equilibrant $(\vec{F}_E)$ since it is the force which establishes equilibrium. Record the mass and angle required for the third pulley to put the system into equilibrium in Table 1 (in Results section) in the column titled “Equilibrant”.
6. Calculate the force for each pulley (mass times acceleration due to gravity, *mg*, remembering that mass must be in **kg** for this calculation) and fill in this part of the table.
7. The equilibrant is equal in magnitude to the resultant, but acts in the opposite direction because it balances the resultant (see Figure 9). In order to work out

the direction of the resultant, either subtract 180^0 from the angle found for the equilibrant or draw a line on the protractor on top of the Force Table along the equilibrant, but back in the opposite direction and measure the angle of this on the Table top. Now fill in Table 2 in the Results section for this problem in the line for experimental measurements.

8. Calculate the resultant theoretically using the component method and the graphical method. Space is given for this in the Results section. Complete Table 2 in the Results section.
9. Repeat for the following pulley configurations:

   Problem 2. **Pulley A = 150g at 10°**
   Pulley B = 110g at 70°

   Problem 3. **Pulley A = 200g at 0°**
   Pulley B = 100g at 55°

In your own words, write down the experimental method. Comment on what you did and anything that went wrong or could be improved.

Experimental Method: *(write what you did, in your own words)*

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RESULTS

This section contains the results from the experiment. Fill in the gaps as instructed in the Experimental Method section. **REMEMBER TO USE SI UNITS, SO KG FOR MASS IN THE FORCE CALCULATION.**

Problem No.	**Pulley A**			**Pulley B**			**Equilibrant (F_E)**		
	Angle (°)	Mass A (g)	Force (N)	Angle (°)	Mass B (g)	Force (N)	Angle (°)	Mass F_E (g)	Force (N)
B1	60	50		160	100				
B2	10	150		70	110				
B3	0	200		55	100				

Table 1 Forces and Equilibrant found for each problem

Problem B1 Graphical 'tail-to-head'

Problem B1 Vector component

Problem B2 Graphical 'tail-to-head'

Problem B2 Vector component

Problem B3 Graphical 'tail-to-head'

Problem B3 Vector component

Method	Resultant for Problem B1		Resultant for Problem B2		Resultant for Problem B3	
	Magnitude (N)	Direction (θ)	Magnitude (N)	Direction (θ)	Magnitude (N)	Direction (θ)
Experiment:						
Graphical ('tail-to-head')						
Vector component:						

Table 2 Results of the Three Methods of Vector Addition

How do the theoretical values for the magnitude and direction of the resultant compare to the experimental magnitude and direction?

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CONCLUSION

Comment on the success of the various methods and any likely source(s) of uncertainty in the experimental apparatus.

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EXPERIMENT NO. 2 **YOUNG'S MODULUS**

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

Aim:
To determine Young's modulus of nichrome.

Write up this experiment as you go. In this experiment you are required to give a value for the uncertainty in direct measurements only.

Theory:

This experiment is designed to investigate elastic behaviour and the relationship between forces and deformations. This is seen at its simplest when stretching a wire, applying a force to it by attaching a weight to one end. The wire is said to be in "tension". Let us start with a wire of length l_0 and cross-sectional area *A* and hang a weight with mass *m* on one end. Assuming that the force is perpendicular to the cross-sectional area of the wire and distributed evenly over the section we define tensile stress (tensile as to do with tension) as

$$\text{stress} = \frac{\text{applied force}}{\text{cross - sectional area of wire}} = \frac{mg}{A}$$

If the wire of original length ℓ_o extends by an amount $\Delta\ell$ under tension produced by the weight mg we define tensile strain (related to the amount of deformation produced by the applied force) as

$$\text{strain} = \frac{\Delta\ell}{\ell_o}$$

For small tensile stress the tensile stress is proportional to the tensile strain (ie the more force applied, the more deformation). They are related by Young's modulus Y which is a measure of how hard it is to stretch the wire. It is defined as

$$Y = \frac{\text{stress}}{\text{strain}}$$

Hence $$Y = \frac{mgl_0}{A\Delta l}$$

Hooke's law states that Y is constant for a given material.

To measure *Y* it is therefore necessary to measure ℓ_o, *A* and the ratio $m / \Delta\ell$. You may assume g = 9.800 + 0.010 m s^{-2}.

Question 1
What are the units of Young's modulus?

Apparatus:

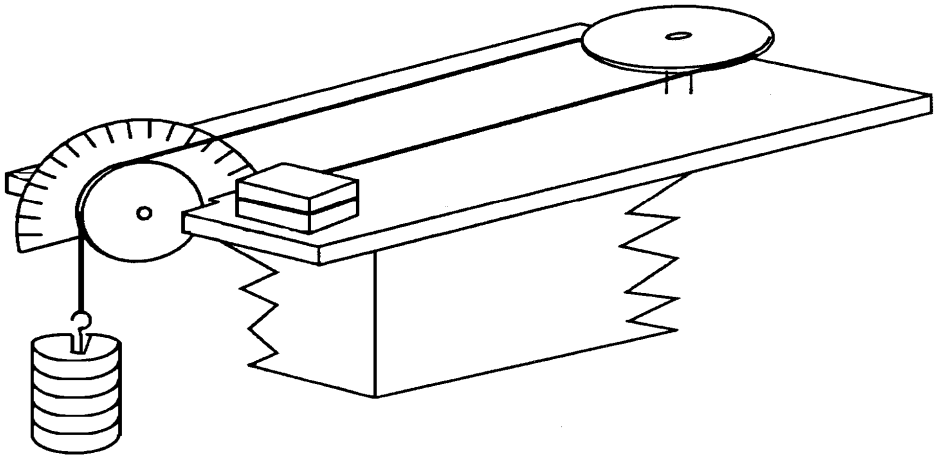


Figure 1.

As shown in figure 1, the nichrome wire is clamped in a block at one end of the apparatus. It is stretched round the pulley at the other end and back over the pointer pulley at the first end**.** The weight of the hooked mass stand is sufficient to hold the wire in place. The masses are each approximately, but not exactly, 50 g. Weigh each one as you use it.

PRECAUTION: *Ensure that there are no kinks in the wire.*

As the wire is stretched, the pulley turns through a small angle which is read from the protractor placed behind the pulley and pointer. The extension $\Delta\ell$ produced in the wire is therefore related to the angle through which the pointer turns.

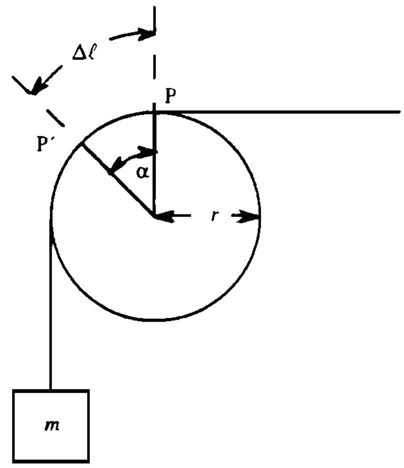
Method:

Place the masses on the stand one at a time, noting and recording the total mass and total pulley rotation after each addition. Tabulate the data in the following form.

mass (g)	Protractor reading (degrees)	cumulative angular deviation (degrees)

Analysis of results:

If a circular arc of length *s* subtends an angle α radians, and the radius of the circle is *r*, then $s = r\alpha$. If a mass *m* is added to the pulley, the point P on the wire will move to P' and hence $\Delta\ell$ = length of the arc PP'.



- The protractor is calibrated in degrees, whereas radians is needed. It is convenient to avoid conversion of all measurements by determining a relationship between $\Delta\ell$ and the angular deviation α (measured in degrees):

$$\Delta\ell = r\alpha = \frac{D}{2} \text{ x } \frac{2\pi}{360}\alpha$$

(the $\frac{2\pi}{360}$ is converting the degrees to radians and D is the diameter of the pulley)

- Substitute this into the expression for Y

- We can now rearrange this formula so that it is in the form $\alpha = km$ where k is the gradient, and hence produce a graph of angular deviation (α) against load (*m*) from which Y can be calculated using the gradient. See if you can rearrange the equation to get this.

- Plot a graph of angular deviation against load and determine the constant *k* from the slope of the graph.

- Rearrange the expression for *k* to make *Y* the subject.

- Measure the diameter of the wire with a micrometer, the diameter of the pulley wheel with vernier callipers and the length of the wire. (Think carefully about what length ℓ_o you should measure.)

- Calculate Y.

Conclusion:

As well as stating the result, you should make a short critical appraisal of the experiment, the apparatus and the method.

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

PART A- Centripetal Force

Aim

To study the effect of varying the radius of the circle, on an object rotating in a circular path.

Theory

When an object of mass m, attached to a string of length r, is rotated in a horizontal circle, the centripetal force F_c on the mass is given by

$$F_c = \frac{mv^2}{r} = mrw^2 \qquad 1.$$

where v is the tangential velocity and ω is the angular speed ($v = r\omega$).

Rearranging equation 1 gives $w^2 = \left(\frac{F_c}{m}\right)\frac{1}{r}$. 2.

Apparatus

Consult the assembly diagram in Figure 1 below.

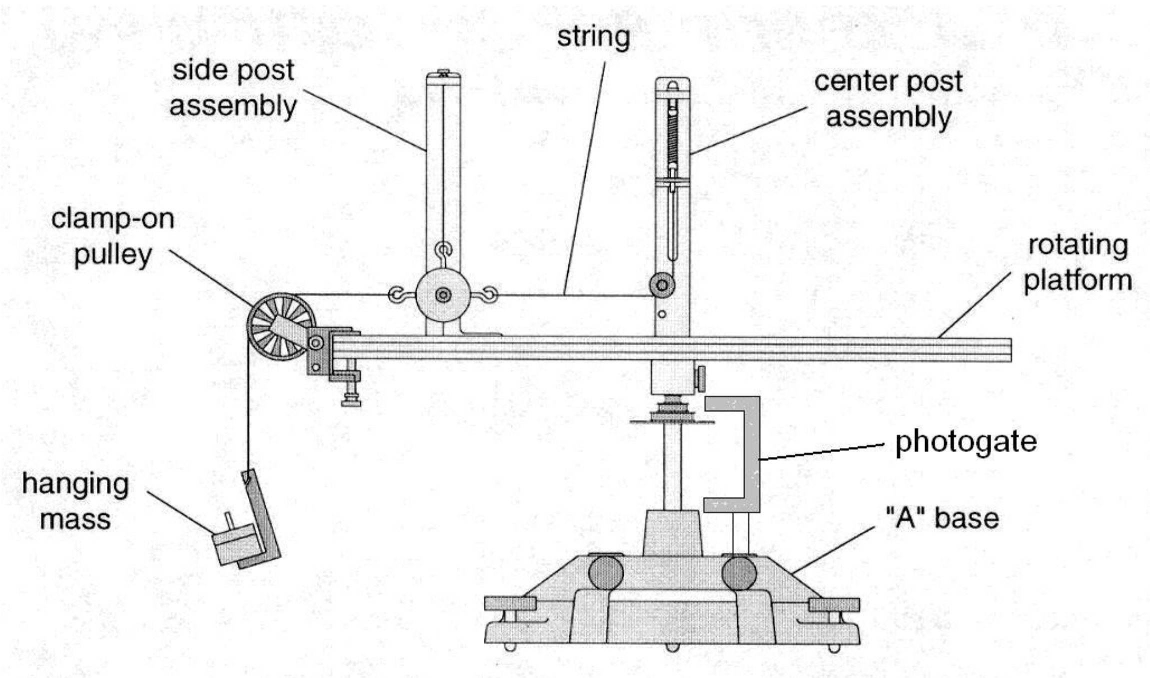


Figure 1 Apparatus.

Method

Level the rotating beam by placing a spirit level on the rotating beam, and adjusting the thumb screws on the “A” base. Check two directions, at right angles to each other.

1. The centripetal force F_c and the mass of the hanging object will be held constant for this experiment. Weigh the object (the brass disc with three hooks), and record its mass *m*. Hang the object from the side post, and connect the string from the spring to the object.
2. Attach the clamp-on-pulley to the end of the track nearer to the hanging object. Attach a string to the hanging object and hang a 50 g (plus hook stand) mass over the clamp on pulley. Record this weight (*Weight = mg*). This establishes the constant centripetal force.
3. Select a radius by aligning the line on the side post with any desired position on the measuring tape. Record this position.
4. The object on the side bracket must hang vertically. On the centre post, adjust the spring bracket vertically until the string from which the object hangs on the side post is aligned with the vertical line on the side post.
5. Align the indicator on the centre post with the orange indicator.
6. Remove the weight and attached string that is hanging over the pulley.
7. Rotate the apparatus, increasing the speed until the orange indictor is centred in the indicator bracket on the centre post. This indicates that the string supporting the hanging object is once again vertical and thus the hanging object is at the desired radius.

8. **Use of timing program**.
   a. Clip the cord from the interface into the photogate and plug the other end of the interface into the USB port of the computer.
   b. Use the software “*Datastudio*” →*Create experiment*→ *Smart Pulley (rotational)* → *table* (lower left) →*velocity*
   c. When you have the beam spinning at the desired speed, press the start button at the top of the screen page. Data will fill the column for time between gates and time in gates. When you have about 10 data point press the stop button. Use of the summation button “Σ” to find the mean of the column.
9. Move the side post to a new radius and repeat the procedure, finding an angular speed for each radius. Do this for a total of five radii and place the results in a table.

Results

Plot the angular speed **squared** (ie ω^2) versus the inverse radius (ie $\frac{1}{r}$).

According to equation 2. (above), and given the mass, *m*¸ is constant, this should give a straight line of slope $\left(\frac{F_c}{m}\right)$.

1. Calculate the centripetal force from the line of best fit of the graph.
   a. Estimate the uncertainty in the slope of this line.

b. What are the major causes of uncertainty in your value of F_c ?
c. How does the value of F_c compare with the weight force of the mass itself.
d. Has this method allowed for friction of the bearings of the A frame ? Discuss.

PART B- Conservation of Angular Momentum

Aim

A non-rotating ring is dropped onto a rotating disk and the final angular speed of the system is compared with the value predicted using conservation of angular momentum.

Theory

When the ring is dropped onto the rotating disk, there is no net torque on the system since the torque on the ring is equal and opposite to the torque on the disk. Therefore there is no change in angular momentum. Angular momentum is conserved.

$$L = I_i \omega_i = I_f \omega_f$$

Where $\boldsymbol{I_i}$ is the initial rotational inertia and ω_i is the initial angular speed. The initial rotational inertia is that of a disk

$$I_i = I_{disk} = \frac{1}{2} M_d R_d^2$$ where R is the radius and M is the mass of disk

The final rotational inertia adds the ring, which has a rotational inertia of:

$$I_{ring} = M_r R_r^2$$ where r is the average radius and m is the mass of the ring

So the final rotational inertia of the combined disk and ring is

$$I_f = \frac{1}{2} M_d R_d^2 + M_r R_r^2$$

The final rotational speed ω_f is given by

$$\omega_f = \frac{I_i \omega_i}{I_f}$$

Thus $$\omega_f = \left(\frac{\frac{1}{2} M_d R_d^2}{\frac{1}{2} M_d R_d^2 + M_r R_r^2} \right) \omega_i \qquad 3.$$

Setup

Replace the beam on the A stand with the plastic disk as per Figure 2 below.

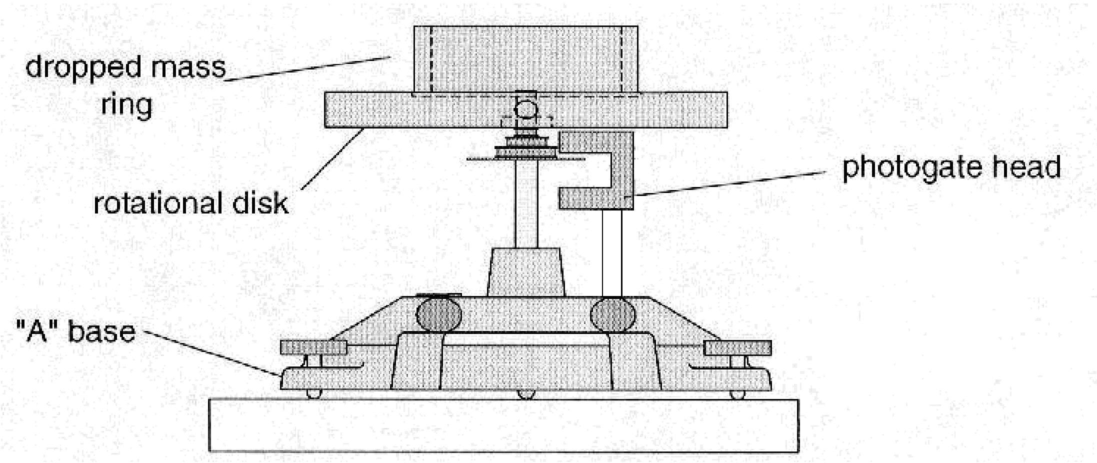


Figure 2 Setup to measure conservation of angular momentum .

Method

1. Spin the disc and measure the rotational velocity ω_i. by averaging about 10 data points, as in PART A. Then hold the metal cylinder above the disk, and drop it onto the disk. Measure the new angular velocity ω_f. in the same way.
2. Weigh the disk and ring and measure the radius.

Results

1. What are the major causes of uncertainty in the experimental value of angular velocity $\boldsymbol{\omega_f}$.
2. The theoretical final angular velocity . ω_f. can be calculated using**Thus**

$$\omega_f = \left(\frac{\frac{1}{2} M_d R_d^2}{\frac{1}{2} M_d R_d^2 + M_r R_r^2} \right) \omega_i$$

   What are the major causes of uncertainty in the experimental value? Does the experimental value agree with the theoretical value?
3. Is Kinetic energy = $\frac{1}{2} I\omega^2$ conserved in this collision? Explain.

EXPERIMENT No. 4 ROTATIONAL INERTIA

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

Aim

To find the rotational inertia of various masses experimentally and to verify they agree with the theoretical values.

Theory

Theoretically, the rotational inertia, I of a point mass is given by $I = Mr^2$, where M is the mass and r is the distance the mass is from the axis of rotation.

To find the rotational inertia experimentally, a known torque $\boldsymbol{\tau}$ is applied to the object and the resulting angular acceleration is measured.

Since $\tau = I\alpha$ then $I = \dfrac{\tau}{\alpha}$ where α is the angular acceleration which is equal to a/R, and τ is the torque caused by the weight handing from the thread which is wrapped around the base of the apparatus.

Now $\tau = RT$ where R is the radius of the cylinder about which the thread is wound and T is the tension in the thread when the apparatus is rotating.

Applying Newton's Second law for the mass weight, m_w, gives

$$\sum F = m_w g - T = m_w a$$

Solving for the tension in the thread gives

$$T = m_w(g-a)$$

$$\text{So } \tau = RT = Rm_w(g-a)$$

$$\therefore \quad I = \frac{\tau}{\alpha} = \frac{m_w(g-a)R^2}{a} = m_w\left(\frac{g}{a}-1\right)R^2$$

$$If \quad \frac{g}{a} >> 1 \quad then \quad I = m_w \frac{g\,R^2}{a}$$

This can be rearranged to $$a = \left(\frac{g\,R^2}{I}\right) m_w \qquad \textbf{1.}$$

If a graph is plotted of a (y axis) versus m_w (x axis), the gradient can be used to calculate the moment of inertia I if R and g are known.

Setup

1. Consult the assembly diagram in Figure 1. Level the rotating beam by placing a spirit level on the rotating beam, and adjusting the thumb screws on the "A" base. Check two directions, at right angles to each other.

2. Attach the square mass (point mass) to the track on the rotating platform at a fairly large radius.
3. Mount the *Smart Pulley* and *Photogate* as shown in Figure 1. and connect it through the Power link interface to the USB port of a computer
4. Run the program "*Data studio*"→ *Create experiment*→ *Smart pulley (linear)*.

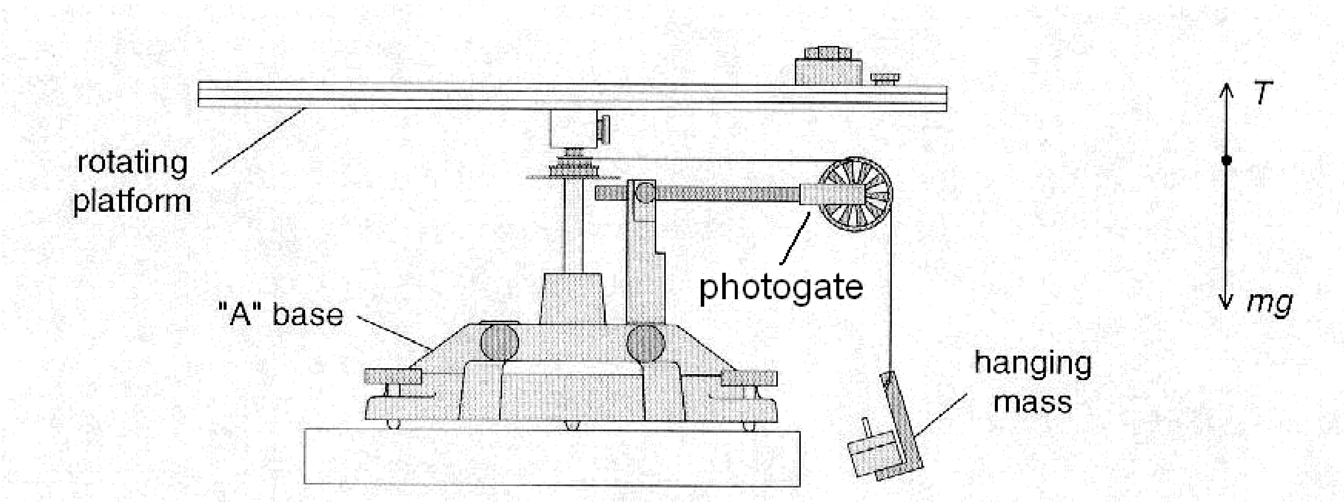


Figure 1 Apparatus

Procedure

<u>PART A. Point mass</u>

Theoretical calculation of Rotational Inertia of the Point Mass

1. Weigh the square mass to find the mass *M* and record.
2. Measure the distance from the axis of rotation to the centre of the square mass and record this radius. Calculate the theoretical *I* assuming it is a point mass.

Experimental measurement of Rotational Inertia
First, find the Angular Acceleration of the Point Mass and Apparatus

3. To find the acceleration, start with enough mass m_w to overcome friction and start accelerating the apparatus, probably 30 g, as shown in Figure 1. Wind the thread up and press *start* as the m_w falls from the table level to the floor, hitting *stop* just before the mass hits the floor.
4. The slope of the velocity graph will be the linear acceleration of the mass before it hits the floor. Press the fit button to fit a linear relationship. Note this down and increase the m_w by 10 g each time for 5 weights.

Using calipers, measure the diameter of the cylinder about which the thread is wrapped and calculate the radius.

So far in this experiment, the apparatus is rotating as well as the point mass. It is necessary to determine the acceleration and the rotational inertia of the apparatus by itself so this rotational inertia can be subtracted from the total, leaving only the rotational inertia of the point mass.

5. Take the point mass off the rotational apparatus and repeat 3 and 4, but for the apparatus alone.

Calculations

6. On the same graph plot acceleration against m_w for the apparatus and point mass, and also acceleration against m_w for the apparatus alone. Calculate the gradient of each line, and using Equation 1., the rotational inertia I for each.

(The rotational inertia of the point mass will be the difference between the two.)

7. Discuss major causes of uncertainty in your value of rotational inertia of the point mass.
    a. Compare this value with the theoretical value.
    b. Have you allowed for friction in this experiment? Discuss.

PART B. Ring mass

Take off the rotational bar and place the solid disk on the axis. Repeat all steps in **Section A. Point mass**, with the circular ring now used in place of the point mass.

(Note: The rotational inertia of a ring is the addition of an array of point masses, all of equal radius. Thus $I = MR^2$ is still the way to calculate the rotational inertia.)

Name:______________________________

EXPERIMENT No. 5 **VOLTAGE SOURCES**

Note: In this experiment it is only necessary for you to complete the results of your work in the following pages of this Laboratory Manual. On completion of the experiment you may carefully detach and submit these completed pages as your Experiment Report (ie no other written report is required).

Introduction

The following diagrams show an analogy between fluid flow and flow of current in an electric circuit. This can be helpful in gaining an understanding of current, voltage and resistance in electric circuits.

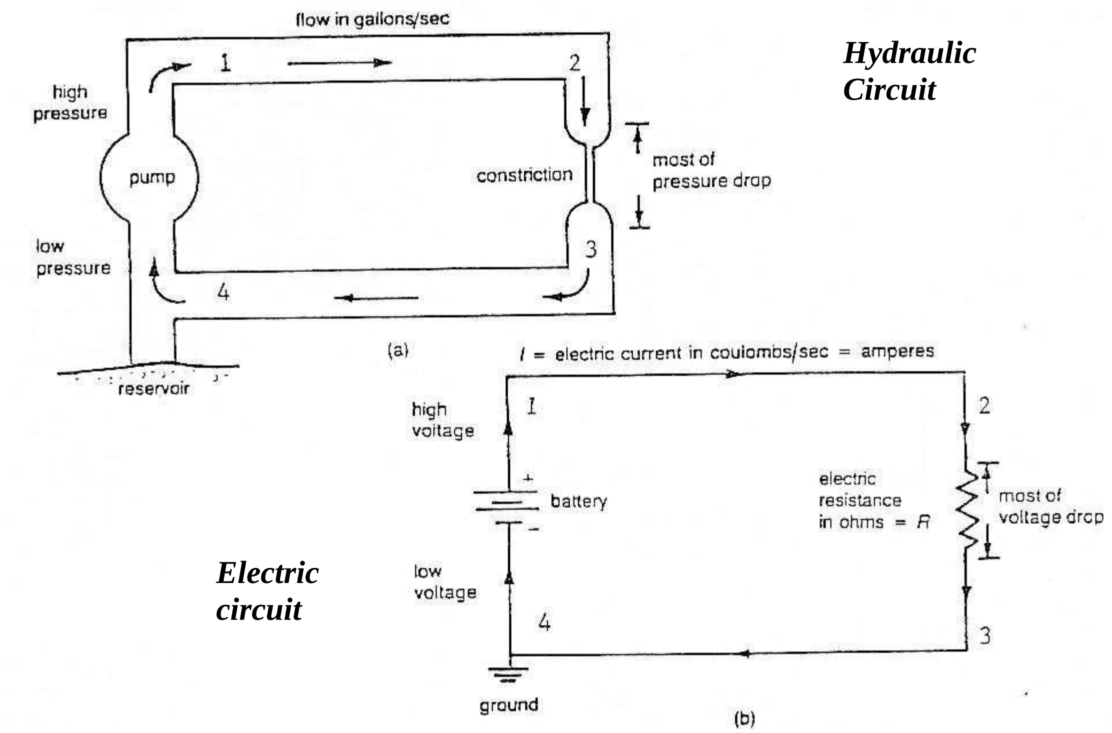


Figure 1. Hydraulic and Electric Circuits

In the hydraulic circuit the fluid flows in pipes from high pressure to low pressure. For example, the pressure P_1 at point 1 is slightly larger than the pressure P_2 at point 2 ($P_1 > P_2$). Similarly $P_2 > P_3$ and $P_3 > P_4$; the pressure decreases continually around the circuit, with most of the pressure drop occurring across the constriction. This can only happen if a source of energy (the pump) pushes the fluid from 4 to 1.

In electric circuits, current flows in conductors from high voltage to low voltage. For example, the voltage (or potential) at point 1 is higher than at point 2 and the voltage decreases from points 1 to 4. The role of the battery is to supply the moving charges with energy to get from 4 to 1; it thus increases the voltage back to that at the point 4. The battery does this by converting chemical energy into electrical energy.

A battery is an example of a voltage source, which is a circuit element that maintains a voltage difference between its two terminals. The voltage difference between the two terminals of an "ideal" voltage source is independent of the current flowing through it. Most practical voltage sources are not like this; they are equivalent to an ideal voltage source V_0 in series with a resistance R_i, as shown in Fig. 2. The voltage V_0 is called the open circuit voltage and R_i is called the *internal resistance* of the battery.

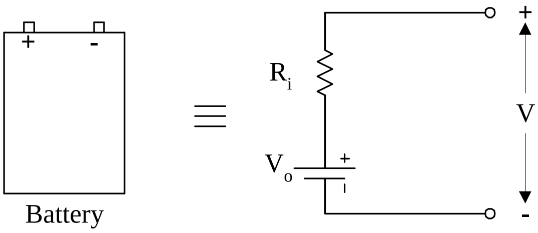


Figure 2

When the circuit is open and the current is zero, the voltage drop across R_i is 0 and the voltage difference between the terminals of the battery is $V = V_0$. When a load resistance R_L is connected across the battery (Fig. 3) and a current I flows, the voltage drop across R_i is IR_i and the voltage difference between the terminals is

$$V = V_0 - IR_i. \qquad (1)$$

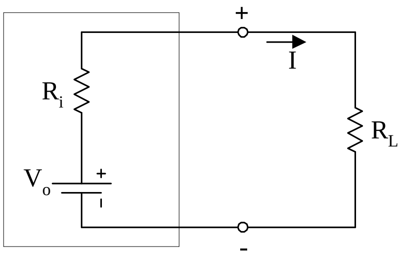


Figure 3

Note that V is the voltage drop across R_L and hence

$$V = IR_L. \qquad (2)$$

The relationship between V and I as given by (1) can represented graphically as shown in Fig. 4.

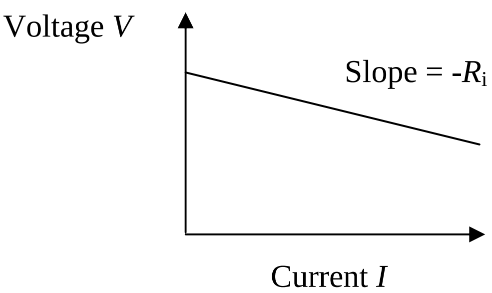


Figure 4

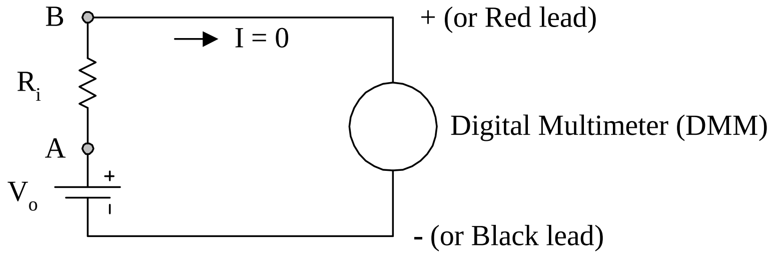


Figure 5

The open circuit voltage of a battery can be measured by connecting a digital multimeter (properly set to voltage) across the battery without connecting any load resistance, as shown in Fig. 5. The digital multimeter (DMM) is equivalent to a very high resistance and hence the current flowing in the circuit is 0. From eqn. (1) the DMM therefore reads $V = V_0$.

It is not possible to measure R_i directly by using the DMM to measure the resistance between A and B in Fig. 5. This is because this resistance is a result of the internal workings of the battery and the point A in the circuit is inaccessible. It can, however, be measured by plotting a graph like Fig. 4.

In this experiment you will measure V_0 and R_i for (i) a type AA battery, (ii) a type D battery and (iii) a solar cell.

Apparatus

The circuit used in the experiment is shown in Figure 6.

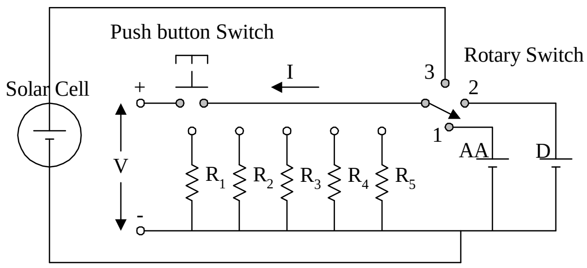


Figure 6

A rotary switch is used to select (i) the type AA battery, (ii) the type D battery or (iii) the solar cell. The circuit will not be complete until you (a) connect one of the resistors R_1 to R_5 to the positive terminal (+) **and** (b) push the push-button switch.

Procedure

Part A Resistance Measurement

If the push-button switch is **not** pushed, the circuit is not complete and no current flows. It is therefore possible to measure the resistance of each of the 5 resistors directly. Set the DMM to measure resistance and connect the *common* (black) lead to the negative (-) terminal. Connect the other lead to the other terminal of R_1 and measure its resistance. Repeat for the other resistors and record the results in the table below.

Nominal value (Ω)	Measured value (Ω)
R_1 = 2.2 Ω	
R_2 = 4.7 Ω	
R_3 = 10 Ω	
R_4 = 47 Ω	
R_5 = 100 Ω	

Comment on the comparison.

Part B Measurements on a type-AA battery

(a) Set the DMM to measure voltage and connect it between the + and - points. Do not connect any of the resistors to +. Set the rotary switch to Position 1.

(b) Now push the button and measure and record the voltage.

Question: Is the reading equal to the open-circuit voltage V_0 of the type AA battery?

Explain.

(c) Connect R_1 to +, push the button and read the value of V. Note that $I = V/R_1$. Repeat for the other resistors and record the results in the table below. Use your measured values of R rather than the nominal values.

Record the value in the table below, as soon as the button is pressed. The large current drawn when the load resistance is small causes the voltage to drop steadily.)

	V	$I = V/R$
$R_1 =$		
$R_2 =$		
$R_3 =$		
$R_4 =$		
$R_5 =$		

Draw the V versus I graph on the following graph paper, similar to Figure 4. Include the measured open circuit voltage on the graph.

Obtain the value of R_i from the graph.

Part C Measurements on type-D battery

Repeat the above measurements for the type D battery (rotary switch at position 2) and note your answer in the table below.

	V	$I = V/R$
R_1 =		
R_2 =		
R_3 =		
R_4 =		
R_5 =		

Obtain the value of R_i from the graph.

Part D Maximum power available from a battery

The power dissipated in the load resistance R_L (figure 7) is $P = I^2 R_L$.

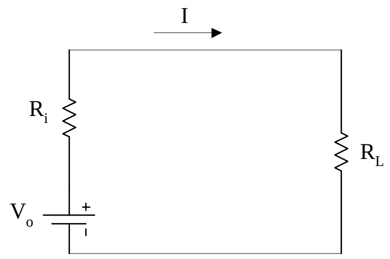


Figure 7

It can be shown that this power is a maximum P_{max} when $R_L = R_i$. Show this yourself !

Hint:

1. Write the equation for the Power dissipated in the load resistance (R_L) as a function of V, R_i and R_L.
2. The maximum power P_{max} dissipated in R_L occurs when the derivative of $P_{max}(R_L)$ with respect to R_L is zero

Your attempt:

Now, from your equation written as part of Hint 1 above, we can derive P_{max} in terms of R_i

$$P_{max} = \frac{1}{4}\frac{V_0^2}{R_i}.$$

Show this yourself :

Calculate the maximum power P_{max} for the type AA and type D batteries and record in the table below.

	type AA	type D
V_0 (V)		
R_i (Ω)		
P_{max} (W)		

Part E Measurements using the solar cell

The solar cell acts as a battery by converting light falling on it to electrical energy. The voltage V_0 produced and the internal resistance R_i depend on the light intensity.

In this experiment you will use a floodlight as a source and you can vary its intensity by changing the distance between the lamp and the solar cell. Use the rotary switch in the circuit to choose the solar cell (Position 3).

(i) Locate the lamp 30 cm above the solar cell and repeat the procedure of Part B to measure V_0 and R_i of the solar cell. Record values as quickly as possible after turning on the lamp and the immediately turn the lamp off (to prevent heating the cell) before preparing the next resistance value. Tabulate your data in the table given below

	V	$I = V/R$
R_1 =		
R_2 =		
R_3 =		
R_4 =		
R_5 =		

(ii) Locate the lamp 60 cm above the solar cell and repeat the experiment, again recording your data in the table below.

	V	$I = V/R$
R_1 =		
R_2 =		
R_3 =		
R_4 =		
R_5 =		

Plot both sets of data ((i) and (ii) above) on a single graph on the following graph page.

Conclusion

How did the internal resistance of the solar cell vary with intensity of illumination ?

EXPERIMENT No. 6 POTENTIOMETER

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

Aim:

The measurement of potential difference is important for several reasons. For example, it is necessary to determine the emf of a battery, it can be used to measure an unknown resistance or compare resistances and it can be used to measure current if the resistance through which the current is flowing is known.

The aim of this experiment is to measure potential difference with a potentiometer. The potentiometer is an important device because it employs a null method and is capable of great accuracy.

PART A. THE POTENTIOMETER

The potential divider circuit reproduced here as Figure 1, was studied in Experiment No. 9. Review Part B of that experiment to ensure that you understand the principles of operation of a potential divider. In particular, write down the potential difference V_{AB} between A and B in terms of E, R_1 and R_2.

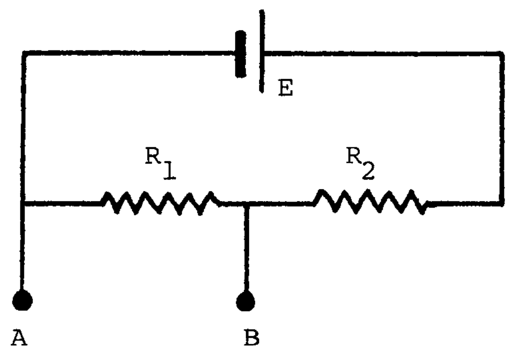


Figure 1.

This circuit is a means of measuring the emf of the dry cell (or *any* potential difference) if E, R_1 and R_2 are known and is the principle of operation of the potentiometer.

In practice, the value of E is not used and is eliminated by comparing the unknown potential difference with a known potential difference as follows:

1. A standard cell with emf E_S is connected in series with the galvanometer and its protective resistance between A and B such that the emf is *opposite* to that of V_{AB}.

Draw a complete circuit diagram of this arrangement. When no current flows in the galvanometer, what is the relationship between R_1, R_2, E_S, and E.

PRECAUTION: *Ensure that you have the protective resistance on its highest value before connecting the circuit. The highest resistance corresponds to minimum sensitivity.*

2. The standard cell is replaced by the source of the unknown potential difference whose value is V, and the values of the decade resistance boxes are R_1' and R_2' when no current flows in the galvanometer. What is the relationship between R_1', R_2', V and E?

3. Eliminate E from these two expressions and show the simplification that occurs if $R_1' + R_2' = R_1 + R_2$.

4. Explain in your practical book the procedure that could be used to measure an unknown potential difference using the potentiometer.

Comments on the Apparatus:

Note that the value of the emf E is not required. The only requirements of it are that it is large enough to produce a large enough potential difference across AB and that it has sufficient capacity to not run down while the experiment is being performed.

The galvanometer used is a sensitive moving coil microammeter. Its suitability for the purpose is determined by its voltage sensitivity.

A simple pointer type moving coil instrument capable of detecting a potential difference of about 10^{-3} volts is adequate for many purposes. It is important to note that the voltage indicating meter need not be calibrated; it needs only to have adequate sensitivity for the purpose.

Note that at balance, there is no current through the galvanometer. It is this feature that is of outstanding importance. It is much easier to detect whether or not a current is flowing than to measure a current.

Procedure:

1. Set $R_1 + R_2$ = 5,000 ohms in the potentiometer circuit, insert the standard cell and adjust R_1, keeping $R_1 + R_2$ constant, such that no current flows in the galvanometer.
   The uncertainty in R_1 is *not* half the limit of reading of the resistance box. It is determined by the maximum amount R_1 can be varied (keeping $R_1 + R_2$ constant) without detecting any current flow.

2. Replace the standard cell by the dry cell, balance the potentiometer and calculate the emf of the dry cell, and its uncertainty.

   NOTE: *Use the same dry cell as used in* Part B *of* Experiment No. 9.

PART B. DIRECT-READING POTENTIOMETER

The procedure for measuring a potential difference on the potentiometer can be simplified by adjusting the instrument such that the value of R_1 at balance is numerically equal to the voltage of the unknown potential difference. The instrument is then said to be direct reading and this can be achieved as follows.

The total resistance $R_1 + R_2$ is arbitrary (although once a value is chosen it is kept constant). The potentiometer is made direct reading by choosing a suitable value of $R_1 + R_2$. If R_1 is set equal to the value of the standard cell with the standard cell across AB and R_2 is adjusted to balance the instrument, then when the standard cell is replaced by an unknown source the new potential at balance will be numerically equal to the value of R_1' (because as before $R_1 + R_2 = R_1' + R_2'$ = constant).

Prove this statement.

Procedure:

1. Put the standard cell across AB and make the instrument direct reading ($R_1 = 1019\ \Omega$ since the standard cell is 1.019 V).

2. Measure the emf of the dry cell keeping $R_1' + R_2' = R_1 + R_2$ = constant.

PART C. THE STUDENT'S POTENTIOMETER

The Student's potentiometer is basically the same as that used in Part B, but is more versatile and compact. The arrangement of components and connections is shown in Figure 2.

The external source is connected between the terminals marked ACCUMULATOR.

With the source selector switch set at TEST and the Galvanometer Sensitivity set at INFINITY, connect the standard cell to the terminals marked STD.CELL. To the terminals marked UNKNOWN connect the dry cell.

The potentiometer is first standardised by turning the source selector switch to STD., setting the voltage switches to the voltage of the standard cell (marked on it) and obtaining a balance by adjusting the rheostat knobs marked COARSE and FINE for a null point on the galvanometer. When balance has almost been reached, turn the GALV.SENS switch to zero for final balance.

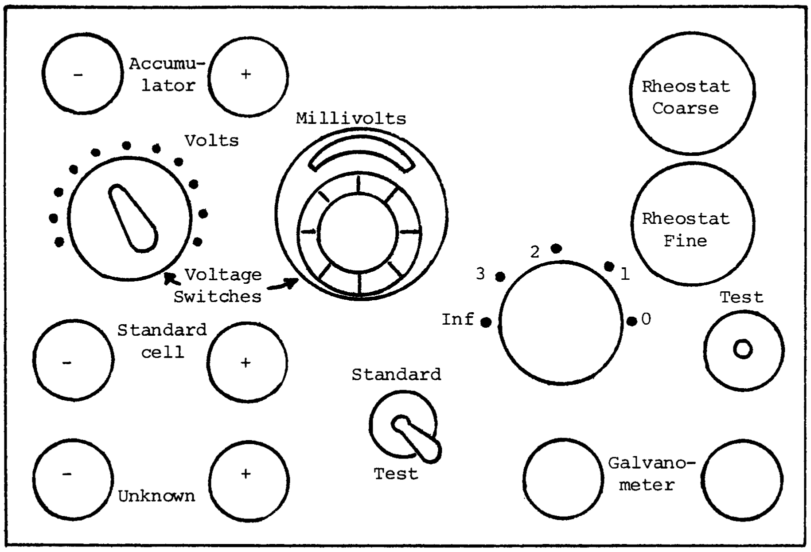


Figure 2.

The voltage reading is now the *actual* voltage of the standard cell. If an unknown source of potential difference is now balanced by turning the voltage switches (VOLTS and MILLIVOLTS) the unknown potential difference is the actual reading of the voltage switches.

This standardisation procedure is called making the instrument *direct reading*, because the unknown potential difference is the actual value read directly from the potentiometer. The equivalent procedure in the potentiometer used in Part B was to set R_1 numerically equal to the standard cell voltage and then vary R_2 until balance was achieved. Then an unknown potential difference was balanced as before keeping $R_1 + R_2$ constant.

Having made the potentiometer direct reading, switch to TEST, balance the potentiometer and read the emf of the dry cell.

PART D. CALIBRATION OF A VOLTMETER

With the student's potentiometer still direct reading, connect AB in the circuit below (Figure 3) to the UNKNOWN terminals.

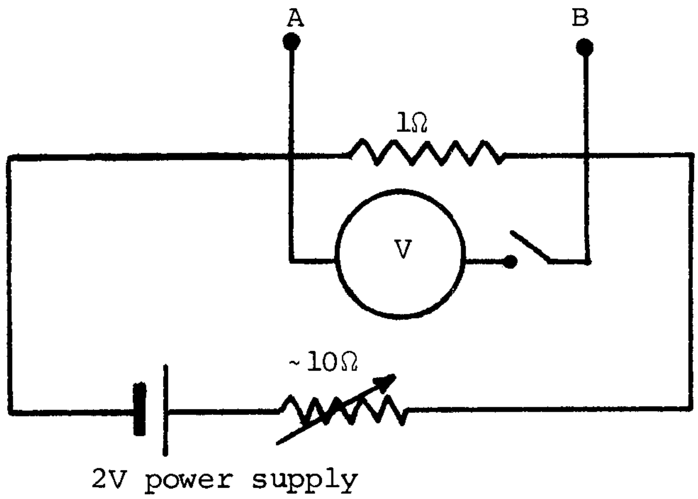


Figure 3.

The voltage between A and B can be read directly from the potentiometer at balance.

Procedure:

1. Explain how the above circuit can be used to calibrate the voltmeter.

2. Choose the 1.0 V range and calibrate the voltmeter throughout that range. Choose points spaced such that there are about 10 readings for the range. Plot a calibration curve for the readings.

Questions:

1. Compare the value of the emf of the dry cell obtained by the various measurements. Which of the methods, voltmeter and potentiometer, is the more accurate and why?

2. Should you show the "line of best fit" or should you draw a smooth line through all the points on the graph to represent the *actual calibration*? In answering this question consider:

   (a) physical phenomena are usually continuous and a smooth curve is perhaps the best representation of the variables concerned because it is able to take account of experimental error;

   (b) an accurate calibration is able to account for non-linearities in the construction and scale accuracy of instruments used in the measurements.

Name:______________________________

EXPERIMENT No. 7 ELECTRICITY AND INSTRUMENTS

Note: In this experiment it is only necessary for you to complete the results of your work in the following pages of this Laboratory Manual. On completion of the experiment you may carefully detach and submit these completed pages as your Experiment Report (ie no other written report is required).

Introduction

The basic measurements in electric circuits are:

(i) measurement of the potential (voltage *V*) drop *across* a circuit element.

(ii) measurement of the current *I through* a circuit element.

The practical unit for current is the ampere *A* and for voltage is the volt *V*.

The voltage drop across any circuit element depends on the current passing through it. The relationship between the voltage and current is usually represented by a graph. For example, Figure 1 shows the '*V-I* characteristic' of a resistor and Figure 2 shows the *V-I* characteristic of a Zener diode.

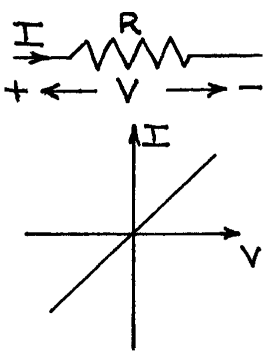


Figure 1.

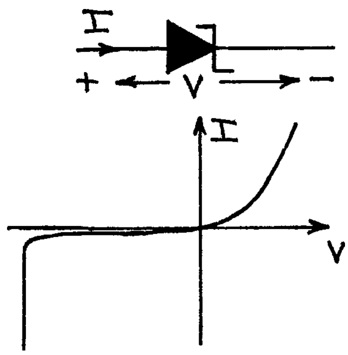


Figure 2.

Note that a *V-I* curve displays the properties of the circuit element under consideration (not the circuit).

Resistors have a simple 'linear' relationship between *V* and *I*. This is expressed in terms of Ohm's Law:

$$V = RI,$$

where *R* is the resistance.

If the current does not change with time it is called a direct current (DC). It is common practice to refer to a constant voltage as a 'DC voltage'.

If the current varies with time as $I(t) = I_O \cos \omega t$ as shown in Figure 3, it is called alternating current (AC).

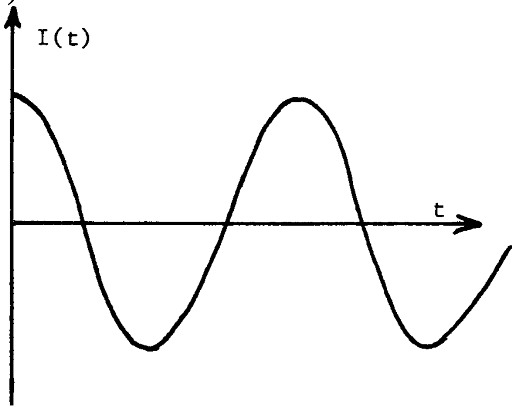


Figure 3. Alternating Current

A car battery is an example of a DC voltage source and household power mains is an example of an AC voltage source.

In this experiment we will study DC circuits only.
The instrument used to measure the voltage difference between two points is called a *voltmeter* and the instrument used to measure the current through an element is called an *ammeter* (refer to Figures 4 and 5).

NOTE: *Most moving coil meters have a definite polarity, ie current must flow into the meter at the positive (red) terminal and out at the negative (black) terminal. When used as a voltmeter, such an instrument must be wired so that the positive terminal is connected to the point at higher voltage. Most meters with digital displays do have positive and negative terminals but can be connected in either direction and the display indicates whether the current or voltage is positive or negative.*

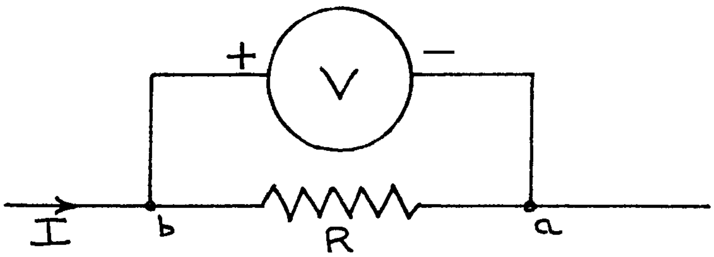


Figure 4. A voltmeter connected to measure the voltage difference between point *b* and point 1. The reading of this voltmeter is equal to $V_{ba} = V_b - V_a$ which will be positive in this case.

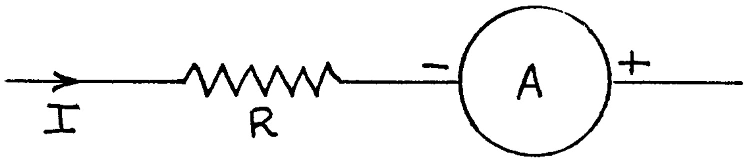


Figure 5. An ammeter connected to measure the current through the resistor *R*. The reading of this ammeter is *-I*.

Question 1:

Redraw Figures 4 and 5 so that the voltmeter reading is V_{ab} and the ammeter reading is *+I*. Will the reading on the voltmeter be positive or negative?

Drawing/Answer:

The instrument used to measure resistance is the *ohmmeter*. The measurement will be meaningful if the instrument is connected directly across the resistor but not if other components intervene (refer to Figure 6).

NOTE: In most instances the resistor must be disconnected from the circuit before a correct measurement can be made.

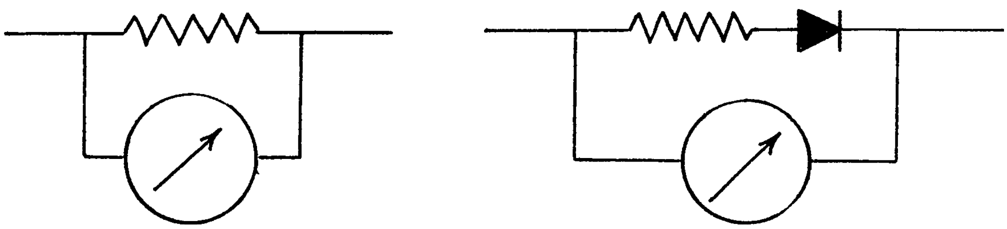


Figure 6. (a) Correct (b) Incorrect

Multimeters:

Very often the three instruments are combined together in one instrument usually called a *multimeter*. The reading of an analog multimeter is indicated by the motion of a dial against a scale. A digital multimeter shows the readings in a numeric display which often includes the sign and the magnitude of the measured quantity as well as the unit (*V*, *A*, or Ω (ohm) for resistance).

NOTE: *The terminals of digital and analog bipolar instruments (which can read both negative and positive voltage or current) may be red and black, may consist of red and black leads, and the black terminal or lead is often marked COMMON.*

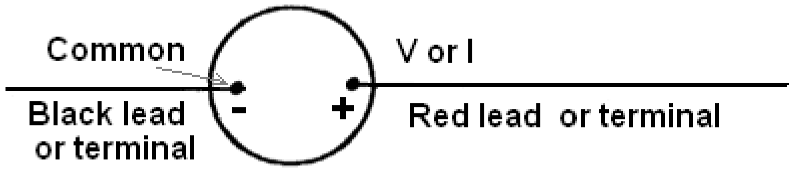


Figure 7. Digital Multimeter

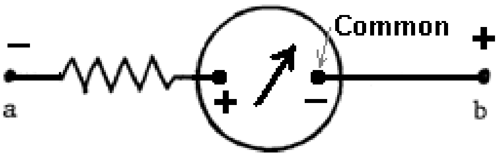
For the meter to give a positive reading:

(i) when connected as an ammeter current must flow in at the red terminal and out from the common terminal.

(ii) when connected as a voltmeter the voltage at the red terminal must be higher than the voltage at the common terminal.

Question 2:

For the values of current and voltage given, write down the reading on the digital multimeter in each of the circuits shown in Figure 8 below.



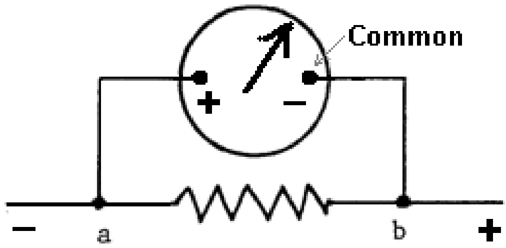
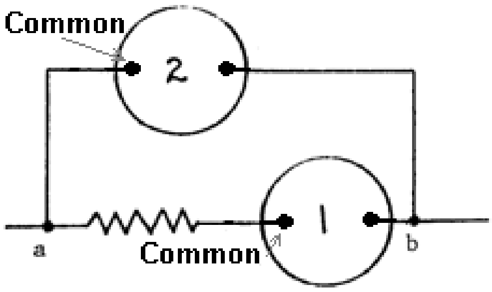


Figure 8. (a) I = 5 mA, V_{ba} = 5 V (b) I = 5 mA, V_{ba} = 5 V

Type of meter = _________ Type of meter = _________

Meter reading = _________ Meter reading = _________



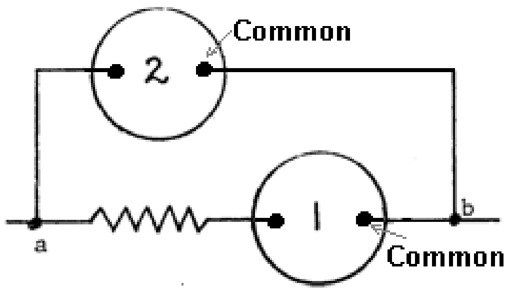


Figure 8. (c) I = 5 mA, V_{ba} = 5 V (d) I = 5 mA, V_{ba} = 5 V

Meter 1 reading = _________ Meter 1 reading = _________

Meter 2 reading = _________ Meter 2 reading = _________

Voltage and Current Measurements:

Apparatus:

Figure 9 shows a simple DC circuit which consists of:

(i) DC voltage source (9 V battery)

(ii) Switch S_1

(iii) Resistance R_1

(iv) Light-emitting-diode (LED)

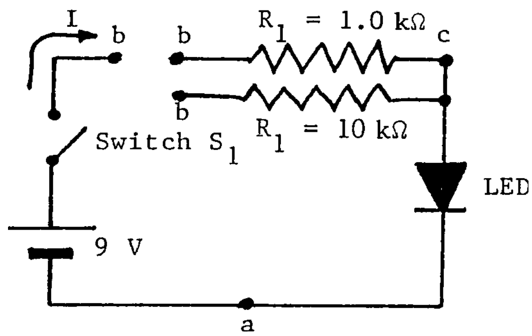


Figure 9.

An analog ammeter will be used to measure the current I and a digital multimeter to measure the voltage differences V_{ba}, V_{cb} and V_{ac}.

Question 3:

Redraw the circuit diagram to include both the ammeter and the digital multimeter connected to measure V_{cb}. (You may well decide that completing the b - b connection via the ammeter is convenient.)

Drawing:

Procedure:

Complete the connection to select R_1 = 1.0 kΩ.

Connect the multimeter to measure V_{ba}.

Pushing switch S_1 will complete the circuit and enable measurements to be recorded.

WARNING: *Use adequate thought and care - excessive current easily damages the diode. Before pressing S_1 ensure that R_1 has not been short-circuited.*
Hence complete Table 1.

Table 1.

	R_1 = 1.0 kΩ	R_1 = 10 kΩ
I		
V_{ba}		
V_{cb}		
V_{ac}		

Question 4:

Calculate the sum of the three voltage drops $V_{ba} + V_{cb} + V_{ac}$ for both values of R_1 using Table 1 and comment on the result.

Answer:

Question 5:

Compare the intensity of the light emitted from the LED for both values of R_1.

The power P dissipated in a circuit element is given by:

$$P = VI$$

where I is the current through that element and V is the voltage drop across it. Calculate the power dissipated in the LED for both values of R_1 and comment on the result.

Comparison/Calculation:

Measurement of Resistance

Figure 10 shows an arrangement of three resistors. The 'nominal' values of the resistance are indicated in the figure, but actual values can be different from the nominal value by up to ± 5%.

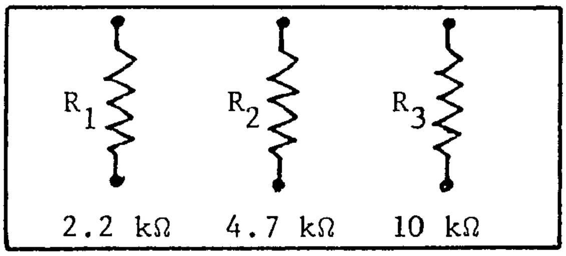
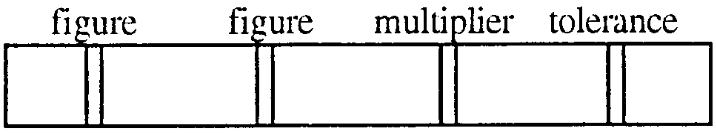


Figure 10.

Colour coding scheme

colour	Bl	Br	R	O	Y	G	B	P	Gr	W
figure	0	1	2	3	4	5	6	7	8	9
multiplier	10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7		

Tolerances Silver 10%
Gold 5%



Procedure:

Measure the value of resistance for each of these resistors using the digital multimeter and comment on the results.

Results/Comments:

R_1 = _____ R_2 = _____ R_3 = _____

Theory of resistance combinations:

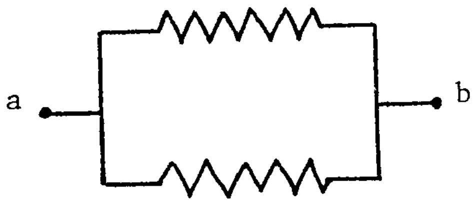
A number of resistors is said to be connected:

(i) in *parallel* if they share the *same voltage difference,*

(ii) in *series* if they have the *same current* passing through them.

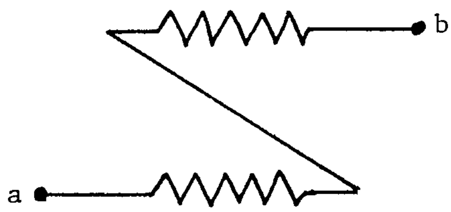
Question 6:

Indicate *in terms of the above criteria* whether the resistor combinations in Figure 11 below are connected between points a and b in series or in parallel or a combination of these.



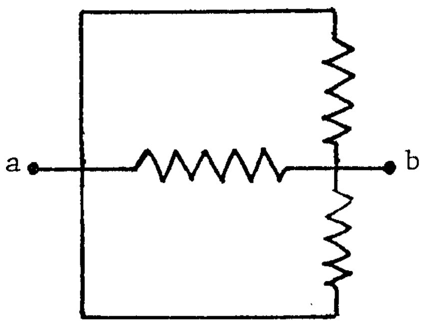
Answer and why:

(i)



Answer and why:

(ii)



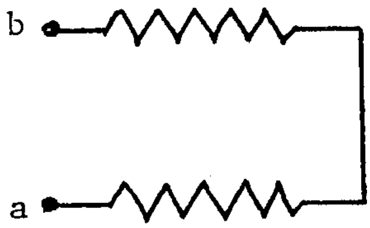
Answer and why:

(iii)

Answer and why:

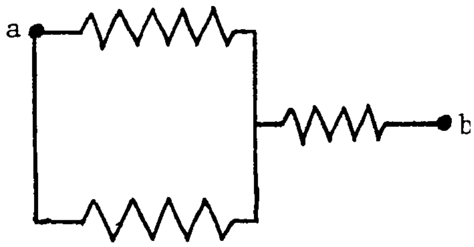
(iv)

Answer and why:



(v)

Answer and why:



(vi)

Figure 11.

If a number of resistors of resistance R_1, R_2, R_3 . . . is connected in *series*, the total resistance R of the combination is given by:

$$R = R_1 + R_2 + R_3 + \ldots$$

If a number of resistors of resistance R_1, R_2, R_3 . . . is connected in *parallel*, the total resistance R of the combination is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ \ldots$$

Using the resistors for which you have already measured the value of resistance, and for each of the arrangements specified below:

(i) Make the appropriate connections.

(ii) In the table below, draw a circuit diagram for the arrangement, and

(iii) calculate the expected value of the equivalent resistance, **giving details of the calculations**.

(iv) Measure the equivalent resistance of the combinations.

Combination	**Circuit diagram**	**Calculated value of *R***	**Measured *R***
R_1, R_2, R_3 in *series.*			
R_1, R_2, R_3 in *parallel.*			
R_1 and R_2 in *parallel* and the combination in *series* with R_3.			
R_1 and R_2 in *series* and the combination in *parallel* with R_3.			

Question 7: In calculating expected values, should the nominal or measured values of each resistance be used?

_V-I_ Characteristics:

(a) Connect a circuit so that the voltage across a resistor *R* can be varied and measured and the current through it can also be measured. To do this you have a 2 V power supply and a 30 Ω rheostat. The resistor under investigation has a nominal value of 10 Ω.

Draw your proposed circuit below. Wire up the circuit and **check with the demonstrator before proceeding**.

Circuit diagram:

Take 5 readings of *V* and *I* and plot the *V-I* characteristic (V on y-axis, I on x-axis) on the graph paper provided on the following pages. Space the values of *V* approximately uniformly throughout the 2 V range. Calculate *R* from the slope of the graph.

V	*I*

(b) Replace the 10 Ω resistor with a lamp bulb and repeat the experiment, using values of *V* of 0.1, 0.4, 0.8, 1.2, and 1.8 volts. Also plot the values on the SAME graph paper

V	*I*

From the graph, what can you say about the resistance of the light globe as a function of applied voltage (or resulting current flowing through it)? Why ?

Which of the resistor or light globe exhibits “Ohmic” behaviour ?

EXPERIMENT No. 8 **OSCILLOSCOPE AND CIRCUITS**

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

PART A: CATHODE RAY OSCILLOSCOPE (CRO)

Aim:
The aim of this first part of the practical is to learn the operation of a cathode ray oscilloscope (CRO) and how to use it to measure properties of electrical signals.
The oscilloscope you will use for this practical is termed a DSO, digital storage oscilloscope. The main variation being the colour display using LCD technology rather than a cathode ray tube.
The cathode ray principle is still of interest in Physics.

Introduction to the Cathode-Ray Oscilloscope
The oscilloscope is an extremely important laboratory test instrument, it is a device for observing and measuring electrical signals. The oscilloscope is used to observe and measure signals produced by electronic circuits and to observe the electrical output from measuring devices such as microphones, photocells, antennas, etc.

After completing this experiment you will be able to use the oscilloscope to:
1. observe an unknown signal
2. measure the amplitude of the signal
3. measure the period of a periodic signal and determine the frequency.

The game of the name:
a) Oscilloscope: An instrument that displays electrical signals (oscillating voltage) in a graphical format.
b) Cathode-Ray: A cathode ray is an electron beam. The name (cathode ray) goes back to early experiments with cathodes (negatively charged electrodes). The electron beams looked like rays emitted from the cathode and hence were called cathode rays. These experiments were done before the discovery of the electron. It was later recognised that the cathode ray is actually a stream of electrons (an electron beam).

How does it work?

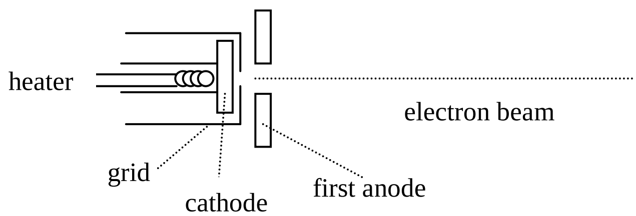


Figure 1. The beam-forming region of the electron gun

The cathode ray tube is evacuated as electrons cannot travel far in air. The heater filament heats the cathode composed of barium, calcium and strontium oxides and electrons are released by thermionic emission. The cathode current is controlled by the potentials of the grid and the first anode, and the electron beam is focussed by the focus anode and accelerated to the phosphor screen with a positive potential of typically 10KV to 25KV. When the energetic electrons strike the phosphor screen, light is emitted permitting a signal trace to be observed.

The cathode ray tube
The picture tube in older TV sets or computer monitors is a cathode ray tube. A schematic diagram of a cathode ray tube is shown below in Figure 2.

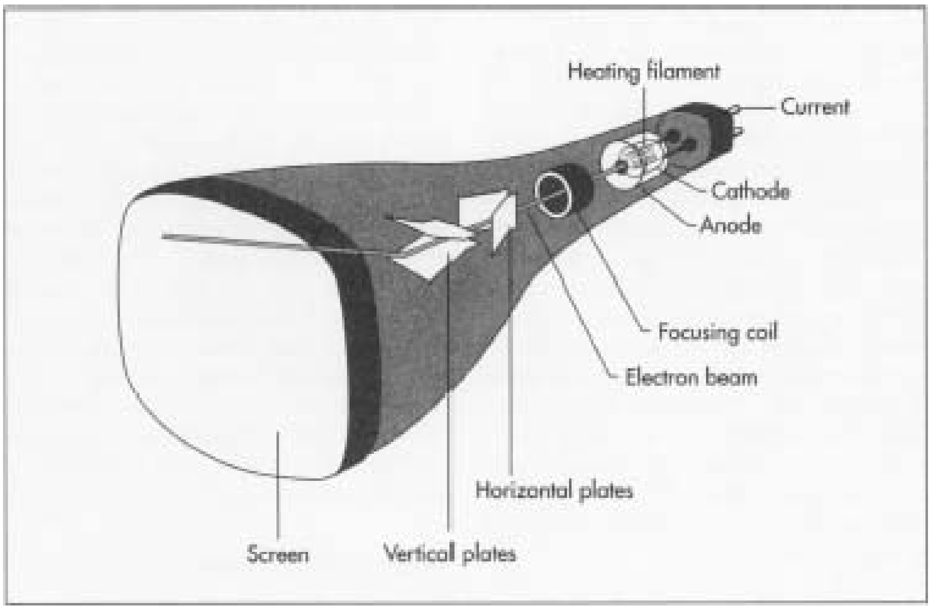


Figure 2. Cathode Ray Tube with Electrostatic Deflection

The vertical position of the bright spot on the screen can be moved by varying the voltage difference between the vertical deflection plates. The input signal is fed to the vertical deflection plates. Hence, the CRO responds to a voltage signal and is therefore essentially a display version of a voltmeter. The CRO is connected into a circuit just as any other voltmeter. The timebase signal is connected to the horizontal deflection plates to deflect the beam in the horizontal direction. The steady sweep across the screen can be used to time variations in the voltage signal (and hence to determine the frequency of regular variations), and so in that sense the instrument operates as a clock.

Instrument familiarisation:
In this part, you will learn how to use the oscilloscope. This instrument is a two-channel oscilloscope – it can display two signals simultaneously. You will come across many different brands of oscilloscope in your future careers but they do have more or less the same operational features and controls; here we will keep this section

as generic as possible. You will need to look closely at the oscilloscope in front of you to identify the key controls.

Connect a signal generator (white box) to the CH1 (channel 1) input of the DSO. The signal generator produces several types of electrical signals. A particular signal type can be selected on the signal generator by pushing the red MODE button. Push the MODE button as many times as necessary to select the sine MODE. Choose the frequency range 0.1 K – 1 K (which means 0.1 kHz to 1 kHz) and set the central dial to 20. This means that the frequency of the signal being produced is $0.20 \times 1.0 = 0.2$ kHz.

Function of Controls:
A brief description of the key controls follows.
Operate each control in turn and observe the effect it has on the display.

a) Vertical controls; Vertical

Channel 1 and Channel 2: Vertical - Volts/Div.
These select the vertical sensitivity for CH1 and CH2. For example, if CH1 is set to 5.00V, the vertical scale for CH1 is 5.0 volts per division. This calibration is accurate to within 3%.
The voltage/division is displayed at the lower left of the screen.
Position: This provides for vertical positioning of the display.

Input connectors for CH1 and CH2 signals are coaxial (BNC)-type.

Coupling DC – Ground - AC
To select the input coupling for the vertical amplifier.

DC: The input signal is displayed without modification.
AC: The DC component of the signal is blocked.
Ground: The display corresponds to zero signal (ground level).

Pressing the CH1 button will display a menu where the coupling mode may be selected. (the Menu on/off will also do this)

Vertical display selection;

CH1 Button - only Channel 1 signal is displayed.
CH2 Button - only Channel 2 signal is displayed.
(Note that both can be depressed to give a dual display)
Using the MATH button allows signals applied to CH1 and CH2 to be added, subtracted and other functions applied and displayed as a single trace.

b) Time base controls; Horizontal

TIME/DIV: Selects the horizontal scale. The time/division is displayed in the lower centre of the screen.
Position: This provides for horizontal positioning of the display.

c) Trigger controls;
Pressing the Trig Menu displays the triggering options.

Type - Selects the level of the triggering waveform that will initiate the time base. Sets the type of triggering signal the DSO will expect. Set to AC position for normal trigger.
Slope - Selects slope of the triggering waveform at the trigger level.
AC Line - Selects trigger from the 50 Hz power line.
Source - Sets the source of the trigger signal. Usually this is set to CH1 or CH2 for internal triggering.

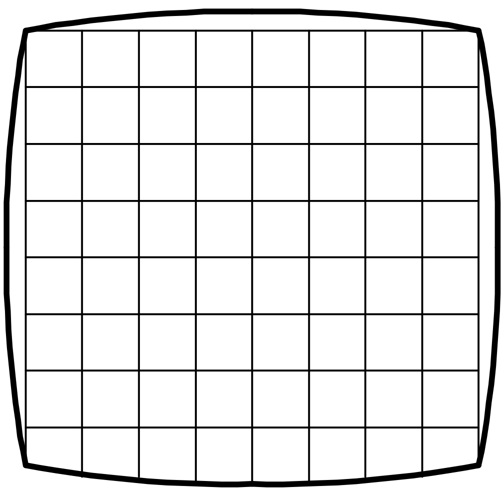
EXT Trigger - Input socket (again a BNC-type) for application of signals to provide an external trigger.

d) Run/Stop Button;

This can be used to freeze a displayed waveform. It will be useful in part B below.

Voltage and frequency measurements

A) Turn the Gain control of the signal generator fully clockwise and leave the range and dial setting as before so that the nominal frequency of the output signal is 0.2 kHz. Sketch the DSO display on a section of graph paper (eg like diagram below) and show clearly: the vertical scale and the horizontal scale and the trigger level.



B) Press the Math button and select CH1 Invert (On) and sketch the CRO Display

- Use the DSO display to measure the magnitude of the signal, V_o. In practice, the peak-to-peak signal is measured and, if amplitude is required, the peak-to-peak signal is divided by 2.
- Use the DSO to measure the period of the signal
- Calculate the frequency of the signal and compare to the nominal frequency (as selected by the signal generator controls).

- Determine the absolute and relative errors in the nominal frequency (by comparing to your measured value)

PART B: OBSERVING OTHER WAVEFORMS.

The other signal generator (small grey box) has pulse, ramp, square and sine outputs. Use the DSO to observe each of these outputs.
Sketch the observed waveforms and indicate clearly on the graph

- the vertical and horizontal scales
- the ground level and the trigger level (ramp and sine only).

PART C: RECTIFYING AND SMOOTHING CIRCUITS

There are occasions when mains alternating voltage is supplied but a constant voltage is needed. The process which achieves this is called rectification and smoothing. The alternating voltage oscillates from positive to negative and back again. Rectification removes the negative part leaving a voltage which varies between zero and the maximum value. That oscillation is then smoothed out to a steady value. The steady voltage supply on the bench in this laboratory is obtained by rectifying and smoothing the mains voltage.

Aim:

To investigate the effects of various rectifying and smoothing circuits on a sinusoidally varying voltage.

Apparatus:

One of the most important applications of the diode is in the rectification of alternating voltage. Rectification and smoothing may be examined using the circuit shown in Figure 3, and as assembled in the circuit box provided.

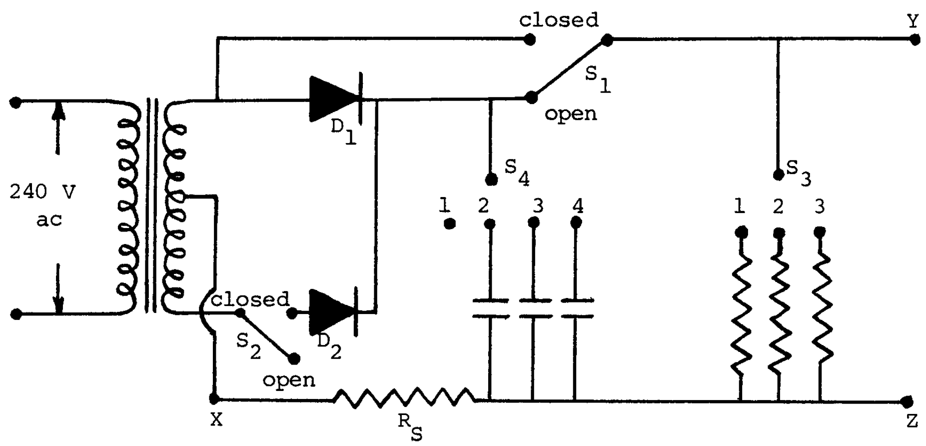


Figure 3.

The box contains a step-down transformer T, with centre tap (240 V $\rightarrow$ 6.3 V); two semi-conductor diodes, D_1 and D_2; two on-off switches S_1 and S_2; three capacitors, any one of which may be included in the circuit using the four-way switch S_4; three load resistors, any one of which may be included in the circuit using the three-way switch S_3; and a series resistor R_S (= 27 Ω).
The transformer is supplied by the 240 V AC mains voltage, and output voltages across the load may be examined between terminals Y and Z. The current flowing in the diodes may be examined by observing the potential drop across R_S, i.e. using terminals X and Z.

Different rectifying and smoothing circuits are obtained using different switching combinations.

Setting-up Procedure:

The following procedure is designed to provide voltage and current wave-forms which can readily be studied and directly compared with each other.

. Connect Y and Z output terminals of the circuit to the oscilloscope. On the oscilloscope, use CH1 and select DC coupling.

. To produce a sinusoidally varying voltage from the circuit of Figure 1, close S_1, open S_2 and set each of S_3 and S_4 at position 1.

Exercise A:

The wave form now being displayed represents an unrectified, unsmoothed voltage signal.

Draw a clear diagram of the basic circuit (without switches) which is now in operation. Include only those components in which current flows, which in this case are just two resistors and half of the secondary windings of the transformer.

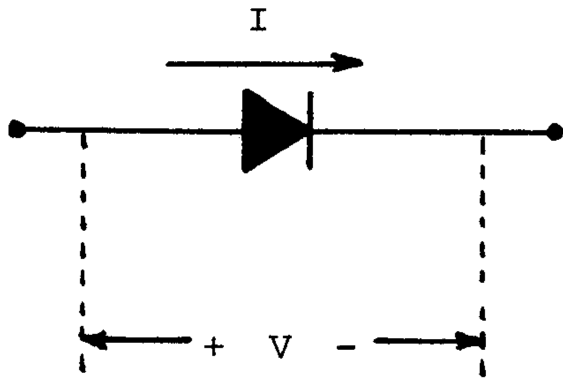
Draw a clear diagram of the output voltage waveform on graph paper. (Draw it at the top of a page as there are seven more to be drawn during the experiment, one below the other, for purposes of comparison. Choose a scale so that this diagram covers about 4 cm of graph paper in the vertical direction).

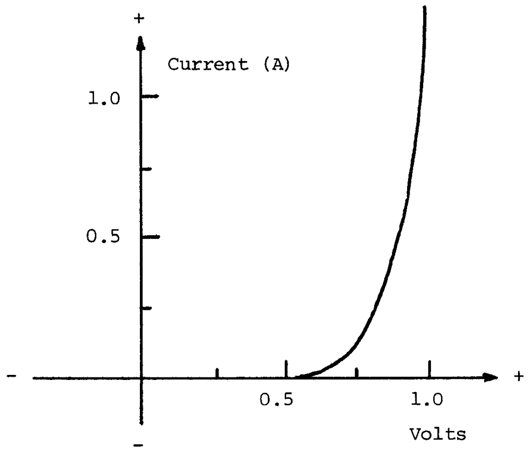
Measure the *peak-to-peak* voltage of the wave.

Measure the average output voltage (between Y and Z) using a DC voltmeter, and comment on the result obtained.

Exercise B:

This exercise involves introducing one of the two semiconductor diodes into the circuit to provide half-wave rectification. A semiconductor diode is conventionally represented in a circuit by the symbol shown in Figure 4(a).





(a) (b)

Figure 4 Semiconductor diode (a) symbol (b) I-V characteristic

The important property of a diode is that substantial currents can flow through it only in one direction (in the direction shown, when the polarity of the applied voltage difference is as shown - the diode is then said to be forward biased). If the polarity of the applied potential difference is the reverse of that shown in Figure 4(a) (a reverse bias), only a negligibly small current can flow through the diode in the opposite direction. The current-voltage characteristic for the semiconductor diodes used in the present experiment appears in Figure 4(b).

Half-wave rectification involving the use of a single diode may be examined with the circuit of Figure 3 by opening both S_1 and S_2 and maintaining each of S_3 and S_4 at position 1.

Draw a clear diagram of the basic circuit which is now in operation. As before include only those components in which current flows. In this case they are one diode, two resistors and half of the secondary windings of the transformer.

Draw a clear diagram of this output voltage waveform. (Draw it below the trace from Exercise A in such a way that the positions of the peaks on the two traces can be compared directly. By using the same scale as before it will cover about 2 cm in the vertical direction).

Measure the voltage amplitude V_O of the wave-form. In view of the Note in Part A, it might be expected that this value would be exactly half of the peak-to-peak value measured in Exercise A. However, it is a little less because the circuit now contains a diode. Use Figure 4(b) to explain the difference quantitatively.

Measure the average output voltage for half-wave rectification using a DC voltmeter. Compare this measured value with the calculated average voltage for such a wave-form, which is V_O/π, where V_O is the voltage amplitude

Exercise C:

Full-wave rectification involving the use of two diodes may be examined with the circuit of Figure 3 by opening S_1, closing S_2, and maintaining each of S_3 and S_4 at position 1.

Draw a clear diagram of the basic circuit which is now in operation.

Draw a clear diagram of this full-wave rectified wave-form, again in such a way that the positions of the peaks can be compared directly with those of the traces drawn in Exercises A and B.

Measure the voltage amplitude of the wave-form.

Measure the average output voltage for full-wave rectification using a DC voltmeter. Compare this measured value with the calculated average voltage for such a wave-form, which is $2V_O/\pi$, where V_O is the voltage amplitude.

Exercise D:

Examine the current through the diodes for this full-wave rectifying circuit by connecting the circuit output terminals X and Z to the oscilloscope. (Z remains connected to the same oscilloscope input terminal while X replaced Y).

To obtain a readily observable trace it is necessary to increase the sensitivity of the oscilloscope to 500mV/cm. Draw a clear diagram of the current trace, relating to the other traces as before. (Note the similarity of shape of the full-wave rectified current and voltage traces. However the current trace is inverted compared with the voltage trace because the voltage difference between X and Z has opposite polarity to the voltage difference between Y and Z).

Exercise E:

Return the voltage sensitivity of the oscilloscope to 2 volts/cm, and reconnect the circuit output terminals Y and Z to the oscilloscope so that output voltage wave-forms can again be examined.

Full-wave rectification with smoothing is obtained using the circuit of Figure 3 by maintaining S_1 open, S_2 closed and S_3 at position 1, but with S_4 at position 2, 3 or 4. This introduces a capacitor across the load resistor.

Draw a clear diagram of the basic circuit which is now in operation.

Draw one of the smooth wave-forms thus obtained, again so that it can be compared directly with previously drawn traces.

Note qualitatively how the degree of smoothing depends on capacitor value, and calculate the % ripple for the largest capacitor, (when across the load obtained with S_3 in position 1). The % ripple is defined as shown in Figure 5.

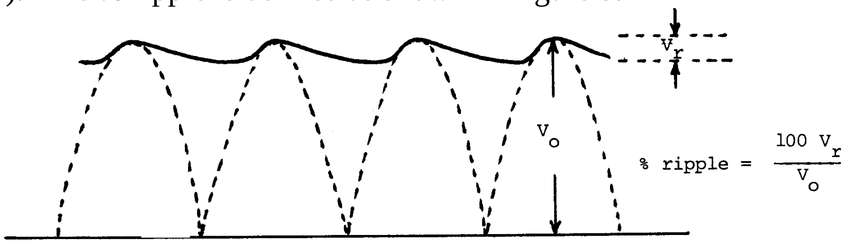


Figure 5. Percentage Ripple in Rectified Circuit

Exercise F:

With S_4 in position 4, switch S_3 to position 2, thus decreasing the load on the circuit and hence the current. Why is improved smoothing obtained with a higher load resistance (decreased load on the circuit)? The ripple voltage is now so small that it is difficult to measure using the existing oscilloscope settings. Note that if the voltage sensitivity is increased to, say, 1 volt/cm, in an effort to expand the ripple voltage (V_r in Figure 5), the trace goes off the screen, since the steady DC component V_o is also amplified.

The DC component can however be removed from the oscilloscope display and the AC ripple voltage examined separately. This is done by selecting AC coupling. The voltage signal is now fed to the oscilloscope through a capacitor which cuts out any DC component.

Increase the voltage sensitivity to 50mV/cm and measure the ripple voltage V_r. Calculate the % ripple for the present combination of capacitor and load resistor.

EXPERIMENT No. 9 **AC CIRCUITS I**

Note: In this experiment you will be required to write up the entire Experiment Report yourself (ie. no proforma sections are provided) and submit it in loose-leaf form.

PART A: AC CURRENTS AND VOLTAGES

An alternating sinusoidal current varies with time according to

$$I(t) = I_O \sin \omega t$$

where ω is the angular frequency.

The peak-to-peak current is the difference between the maximum and minimum values of the current.

The rms (root-mean-square) current is the square root of the mean of the current squared and it is straightforward to show that

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

Similar relations hold for AC voltages.

The rms current and voltage are of interest because these are the quantities measured by AC ammeters and voltmeters and because the power dissipated in a resistance by an AC current of rms value I_{rms} is the same as that of a DC current of this value.

Procedure:

1. Write down the relation between the rms and the peak-to-peak values of a sinusoidal AC voltage.

2. Connect the oscillator to oscilloscope and satisfy yourself that you understand the operation of the oscillator controls.
   Choose a frequency of 1000 Hz and a sinusoidal wave form and measure the peak-to-peak voltage of the oscillator for a given amplitude setting.

3. Now measure the rms voltage with the AC range on the multimeter and verify your answer to 1.

PART B: IMPEDANCE OF RESISTOR, CAPACITOR AND INDUCTOR

If an AC voltage is applied to any combination of resistors, capacitors and inductors the resulting current is related to the voltage by

$$V = IZ$$

where V and I are rms or peak-to-peak values. The quantity Z is called the impedance of the circuit and is independent of V, although it may depend on the angular frequency ω.

This is the AC analogue of Ohm's law for DC circuits.

1. *Resistance*

Connect the following circuit (Figure 1).

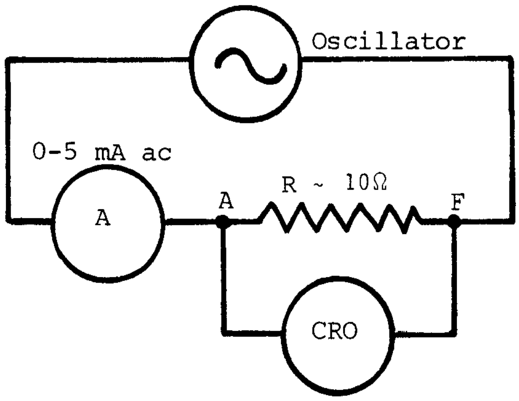


Figure 1.

PRECAUTION: *Take care! The oscillator gain control is very sensitive and must be increased very slowly.*

Choose a frequency of 1000 Hz and plot a graph of V_{pp} (peak-to-peak) versus I_{rms}. Take five readings of the voltage for current values spread between 0 and 5 mA, and plot the data directly on the graph. Deduce a value for Z and show that it is equal to the resistance of the resistor within experimental uncertainty.

Repeat for a frequency of 10 000 Hz and show that Z is independent of the frequency.

2. *Capacitance*

Replace the resistor by the 5 μF capacitor (connection points A and F become B and E) and repeat the above procedure and deduce a value of Z for frequencies of 1000, 3000 and 10 000 Hz. Before taking accurate readings, investigate the range of voltage values for currents of up to 5 mA and choose a suitable scale so that all three sets of data can be plotted directly on to the one graph. The value of the frequency is important for this and subsequent parts.

By using the oscilloscope, check the oscillator scale calibration and hence use precisely 1000, 3000 and 10 000 Hz. Show that Z is inversely proportional to frequency and that $Z = \dfrac{1}{\omega C}$ within the experimental uncertainty.

3. *Inductance*

Replace the capacitor by the inductor (coil 3) (use the two connection points E) and repeat the above procedure for the same three frequencies. Show that Z is proportional to frequency and that $Z = \omega L$ within experimental uncertainty.

Note: For a frequency of 10 000 Hz the current may not reach 5 mA, in which case readings of 0.5 mA intervals should be recorded. (In this case it has been possible to ignore the resistance of the coil. Part C, Section 2, illustrates a case when it is not valid to ignore the coil resistance).

EXPERIMENT No. 10 (Advanced) **AC CIRCUITS II**

PART A: COMBINATIONS OF L, R AND C

It has been shown experimentally, and can be proved theoretically, that the impedance in the above cases is

$$
\begin{aligned}
Z &= R && \text{resistor}\\
&= 1/\omega C && \text{capacitor}\\
&= \omega L && \text{inductor}
\end{aligned}
$$

The peak-to-peak (or rms) values of V and I are proportional for any combination of L, R and C.

The resultant impedance of a combination of resistors can be calculated in the same way as for DC circuits but this is not the case for capacitors or inductors. This is because the voltage and current are not in phase across a capacitor or inductor.

It can be shown that

(i) voltage and current are in phase across a resistance,
(ii) the voltage across a capacitor lags the current by 90°,
(iii) the voltage across an inductor leads the current by 90°.

These relations can be represented by showing the quantities as vectors (Figure 1).

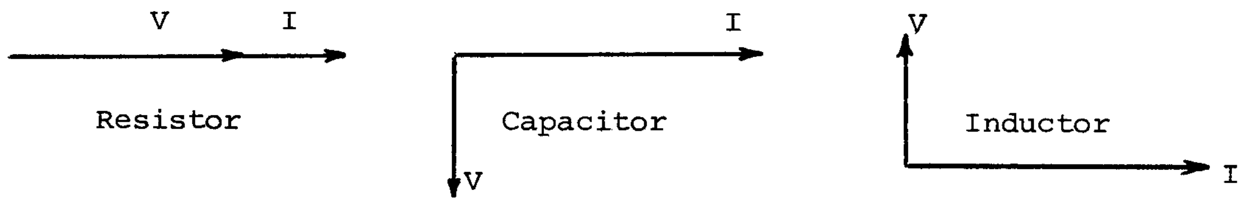


Figure 1.

The magnitude of each voltage is IZ where Z is the relevant impedance.

The resultant voltage across components in series is therefore the *vector* sum of the above voltages.

Procedure:

1. Connect the following circuit using the circuit board provided and using a lead between F and B to bypass the 1 μF capacitor.

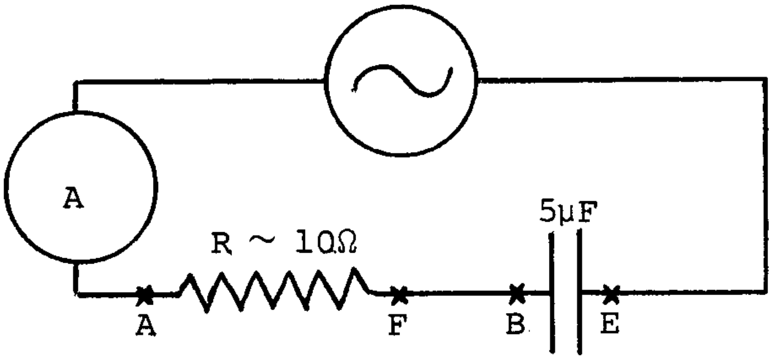


Figure 2.

For a frequency of 1000 Hz and a current of 3 mA, use the CRO to measure the voltage between A and F, B and E, A and E.

Note that $V_{AE} \neq V_{AF} + V_{BE}$ since the voltages must be added vectorially.

Construct a vector diagram for the sum of the voltages.

Verify that the angle between V_{AF} and V_{BE} is 90˚ and explain why this is so.

Calculate the product RC from V_{AF}/V_{BE} and compare it with the product calculated using values stated on the components.

2. Connect the following circuit (Figure 3) using the circuit board provided.

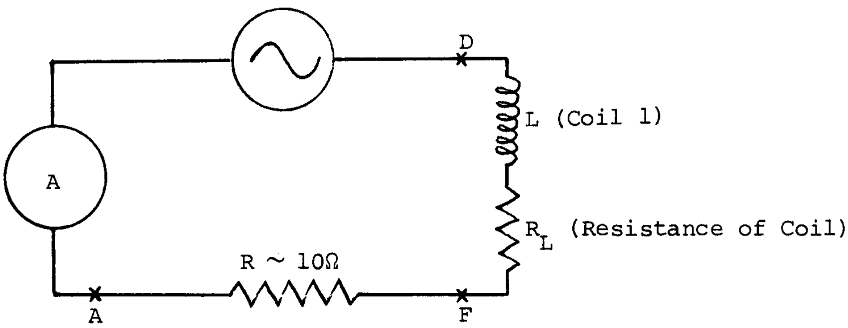


Figure 3.

For a frequency of 1000 Hz and a current of 3 mA, use the CRO to measure the voltages across AF, FD and AD.

Draw the vector diagram and determine L and R_L from the diagram.

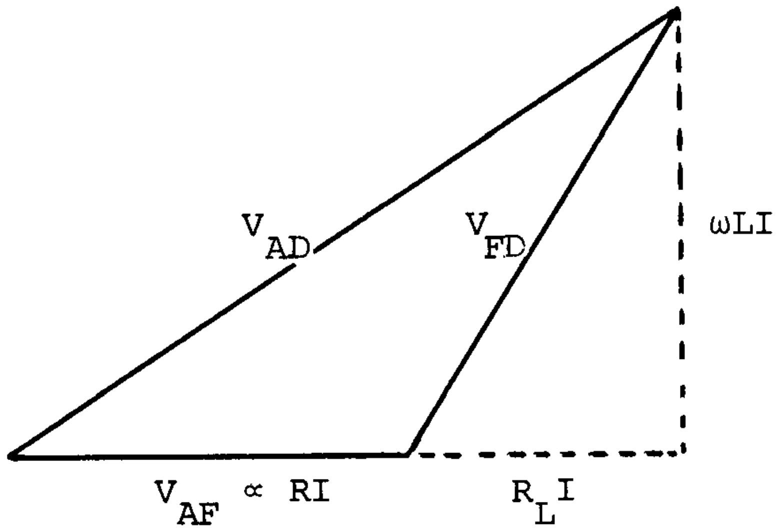


Figure 4.

3. With the same circuit as above, measure the voltage across the coil as a function of the current.

   Plot V_{pp} versus I_{rms} and calculate the inductance L and its uncertainty from the slope of the graph. Take the value of R_L written on the coil.

PART D: RESONANCE CURVE

The theory of a series L, R, C resonant circuit is given in the Appendix.

Write down an expression for the resonant frequency of an *LRC* circuit.

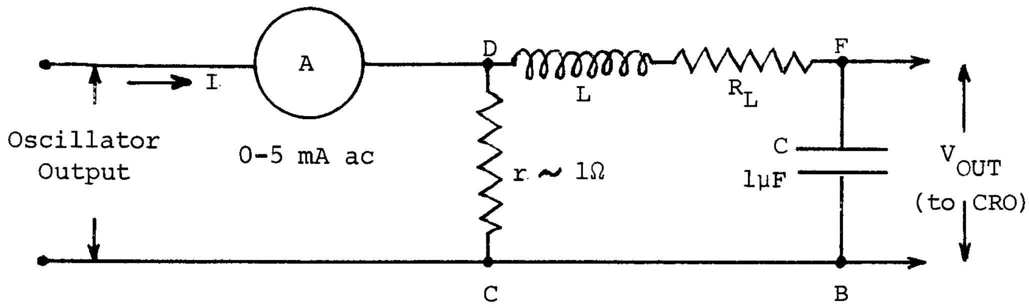


Figure 5.

Set up the above circuit using C = 1μF and Coil 3 (800 turns). The resistance R_L in the circuit is the resistance of the coil.

The left-hand part of the circuit is an arrangement to inject a known voltage into the resonant circuit. If $r \ll R_L$ almost all the current flows through r and the input voltage is $V_{IN} = Ir$.

Keeping I constant at 4 mA, plot V_{OUT} as a function of frequency. Cover the range 200 Hz each side of resonance and choose the spacing of points to clearly define the curve, especially close to resonance.

Calculations:

Calculate Q by each of the three methods specified below and compare them. Comment on any discrepancy.

1. From the resonance curve, note the frequencies f_1 and f_2 at which V_{OUT} drops to $\frac{1}{\sqrt{2}}$ of its value at the resonant frequency f_o.

$$Q = \frac{\omega_o}{\omega_2 - \omega_1} = \frac{f_o}{f_2 - f_1}$$

2. At Resonance; $Q = \dfrac{\omega_o L}{R}$

3. At Resonance; $Q = \dfrac{V_{OUT}}{V_{IN}}$ where $V_{IN} \approx Ir$

PART E: VARIATION OF RESONANT FREQUENCY WITH CAPACITANCE

Retain Coil 3, but replace the 1.0 μF capacitor by the variable capacitance box. Determine the resonant frequency of the circuit for each of the capacitors, plot a suitable graph and determine the value of *L* from the graph. Compare the value of *L* with that written on the coil.

APPENDIX

A.C. Resonance:

A series circuit is shown in figure A1 below.

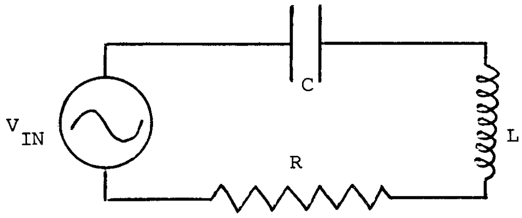


Figure A1.

The impedance of the circuit is given by

$$Z = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}} \tag{1}$$

As ω increases, the reactance term $X = \omega L - \frac{1}{\omega C}$ changes from negative to positive.

When

$$\omega = \omega_o = \frac{1}{\sqrt{LC}} \tag{2}$$

the reactance is zero *Z* has its minimum value *R* and the current is at a maximum.

1. The quality factor *Q* of the circuit is defined as

$$Q = \frac{\omega_o}{\omega_2 - \omega_1} \tag{3}$$

where ω_1, ω_2 are the angular frequencies on either side of ω_O at which the current falls to $\frac{1}{\sqrt{2}}$ of its maximum value.

2. Using (1), ω_1 and ω_2 satisfy

$$\left(\sqrt{2}\,R\right)^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

This is a quartic which has two positive roots whose difference is

$$\omega_2 - \omega_1 = \frac{R}{L}$$

Therefore from (3)

$$Q = \frac{\omega_o L}{R} \qquad (4)$$

3. The voltage across the capacitor V_{OUT} is given at any frequency by

$$\frac{V_{OUT}}{V_{IN}} = \frac{1/\omega C}{Z}$$

At resonance $Z = R$, $\dfrac{1}{\omega_o C} = \omega_o L$, and so

$$\frac{V_{OUT}}{V_{IN}} = \frac{\omega_O L}{R} = Q \qquad (5)$$

ie. Q is the voltage magnification of the tuned circuit.

If the resonance is sharp (Q large), the impedance of the capacitor ($1/\omega C$) is practically constant near the resonance peak, and so we can assume that the current through the capacitor is proportional to V_{OUT}. Therefore a plot of V_{OUT} as a function of ω for constant V_{IN} will produce the resonance curve.

NOTE: *R* in equation (4) is strictly an effective resistance which includes all dissipative effects in the circuit, and so *Q* is a parameter of the circuit as a whole. However in a well-designed circuit the main losses will be in the coil and we can interpret *R* as the resistance of the coil at the resonant frequency. In that case *Q* can be regarded as a parameter of the coil, but the measured *Q* will always be lower than the actual *Q* of the coil because of other sources of power dissipation, eg. voltmeter resistance, losses in dielectrics and insulators, radiation, etc.