Tutorial 12

Question 1.
Show that

(a) $\begin{vmatrix} \lambda a & \lambda b \\ c & d \end{vmatrix} = \lambda \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ \lambda c & \lambda d \end{vmatrix}$

(b) $\begin{vmatrix} a+e & b+f \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} e & f \\ c & d \end{vmatrix}$

(c) $\begin{vmatrix} a & \lambda a \\ c & \lambda c \end{vmatrix} = \begin{vmatrix} a & b \\ \lambda a & \lambda b \end{vmatrix} = 0$

(d) $\begin{vmatrix} a & c \\ b & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

The properties of determinants you have just verified for 2×2 determinants hold for all determinants, as proved in PMTH213. You may use them in Assignment 12.

Question 2.

Take $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 1 & 1 & 5 \end{bmatrix}$.

(i) Use the properties of determinants in Question 1 to evaluate $\det(A)$.

(ii) Calculate the co-factors of A, and hence determine $\operatorname{adj}(A)$.

(iii) Verify that $A \operatorname{adj}(A) = \operatorname{adj}(A)\, A = \det(A)\underline{\mathbf{1}}_3$

Question 3.
Using elementary row operations, transform the augmented matrix

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 4 & 5 & 0 & 1 & 0 \\ 1 & 1 & 5 & 0 & 0 & 1 \end{array}\right]$$

to reduced row echelon form (Gauß-Jordan algorithm) and determine whether the matrix A in Question 2 is invertible, and, if so, find is inverse.

Question 4. Take three points P, Q, R in $\mathbb{R}^3$, with co-ordinates $(r, s, t), (u, v, w)$ and (x, y, z) respectively. Let S be the parallelepiped spanned by the three line segments OP, OQ, OR. show that the volume of S is the absolute value of the determinant

$$\begin{vmatrix} r & s & t \\ u & v & w \\ x & y & z \end{vmatrix}$$