ME2142 Formulae

Please help

Vector Mechanics

$$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$$

$$u = |\mathbf{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = u_x v_x + u_y v_y + u_z v_z$$

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \times \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{pmatrix}$$

$$\mathbf{u} \times \mathbf{u} = 0$$

$$\mathbf{u} \cdot \mathbf{v} = 0 \text{ (if } \mathbf{u} \perp \mathbf{v})$$

$$\mathbf{u} \cdot \mathbf{v} = uv\cos\theta$$

$$\mathbf{u} \times \mathbf{v} = uv\sin\theta \cdot \mathbf{n}$$

Particle Kinematics

Rectilinear Motion

$$dx = v\,dt \,\|\, \int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt$$

$$dv = a\,dt \,\|\, \int_{v}^{v_0} dv = \int_{t_0}^{t} a(t)\,dt$$

$$v\,dv = a\,dx \,\|\, \int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$$

Given $a = a(t)$:

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)\,dt \,\|\, \int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt$$

Given $a = a(x)$:

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx \,\|\, \int_{t_0}^{t} dt = \int_{x_0}^{x} \frac{1}{v(x)}\,dx$$

Given $a = a(v)$:

$$\int_{t_0}^{t} dt = \int_{v_0}^{v} \frac{1}{a(v)}\,dv \,\|\, \int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v}{a(v)}\,dv$$

If v is constant:

$$x = x_0 + v(t - t_0)$$

If a is constant:

$$v = v_0 + a(t - t_0)$$

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Curvilinear Motion

$$a_n = \frac{v_t^2}{\rho}$$

Rigid Body Mechanics

General Plane Motion

$$v_{B/A} = v_B - v_A$$

$$v_{B/A} = \omega k \times r_{B/A} = r\omega$$

Rolling without Sliding

* velocity at contact point always 0

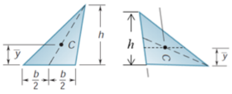
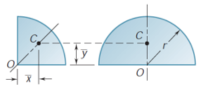
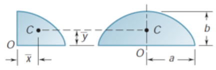
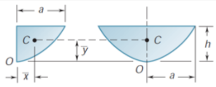
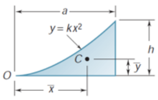
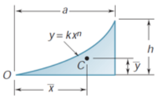
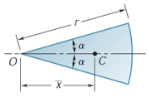
$$v_O = r\omega$$

Mass Properties

$$dW = \gamma t\,dA \equiv W = \gamma t A$$

$$\bar{x}A = \int x\,dA = Q_y$$

$$\bar{y}A = \int y\,dA = Q_x$$

	$\bar{x}$	$\bar{y}$	Area
		$\frac{h}{3}$	$\frac{bh}{2}$
	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
	0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
	$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
	0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
	$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
	0	$\frac{3h}{5}$	$\frac{4ah}{3}$
	$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
	$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
	$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Compound Shapes

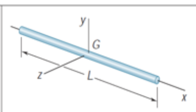
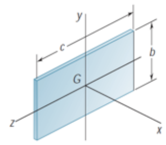
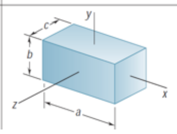
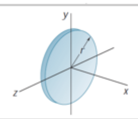
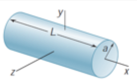
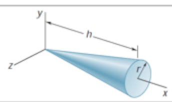
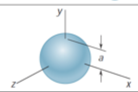
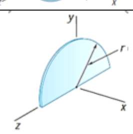
$$Q_y = \bar{X}\sum A = \sum \bar{x}A$$

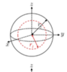
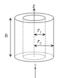
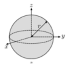
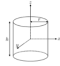
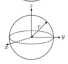
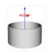
$$Q_y = \bar{Y}\sum A = \sum \bar{y}A$$

Mass Moment of Inertia

$$I_O = \int r^2\,dm = k_o^2 m$$

$$I = I_O + md^2$$

Slender rod		$I_y = I_z = \frac{1}{12}mL^2$
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$
Rectangular prism		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}m(c^2 + a^2)$ $I_z = \frac{1}{12}m(a^2 + b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$
Circular cone		$I_x = \frac{3}{10}ma^2$ $I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$
Sphere		$I_x = I_y = I_z = \frac{2}{5}ma^2$
Semicircular disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$

Thick-walled hollow sphere		$I = \frac{2m}{5}\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$	Thick-walled hollow cylinder		$I_z = \frac{1}{2}m(r_1^2 + r_2^2)$ $I_x = I_y = \frac{1}{12}m[3(r_1^2 + r_2^2) + h^2]$
Solid sphere		$I = \frac{2}{5}mr^2$	Solid cylinder		$I_z = \frac{1}{2}mr^2$
Hollow sphere		$I = \frac{2}{3}mr^2$	Thin-walled hollow cylinder		$I = mr^2$

Rigid Body Kinetics

$$\Sigma F = m\bar{a}$$

$$\bar{a} = a_{ref} + r\omega^2 e_n + r\alpha e_t$$

$$\Sigma M_G = \bar{I}\alpha$$