ME2142 Formulae
Please help

Vector Mechanics

$$\overrightarrow{OB}-\overrightarrow{OA}=\overrightarrow{AB}$$

$$u=|\mathbf{u}|=\sqrt{u_x^2+u_y^2+u_z^2}$$

$$\mathbf{u}\cdot\mathbf{v}=\begin{pmatrix}u_x\\u_y\\u_z\end{pmatrix}\cdot\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix}=u_xv_x+u_yv_y+u_zv_z$$

$$\mathbf{u}\times\mathbf{v}=\begin{pmatrix}u_x\\u_y\\u_z\end{pmatrix}\times\begin{pmatrix}v_x\\v_y\\v_z\end{pmatrix}=\begin{pmatrix}u_yv_z-u_zv_y\\u_zv_x-u_xv_z\\u_xv_y-u_yv_x\end{pmatrix}$$

$$\mathbf{u}\times\mathbf{u}=0$$

$$\mathbf{u}\cdot\mathbf{v}=0\text{ (if }\mathbf{u}\perp\mathbf{v})$$

$$\mathbf{u}\cdot\mathbf{v}=uv\cos\theta$$

$$\mathbf{u}\times\mathbf{v}=uv\sin\theta\cdot\mathbf{n}$$

Particle Kinematics
Rectilinear Motion

$$dx=v\,dt\parallel\int_{x_0}^{x}dx=\int_{t_0}^{t}v(t)\,dt$$

$$dv=a\,dt\parallel\int_{v}^{v_0}dv=\int_{t_0}^{t}a(t)\,dt$$

$$v\,dv=a\,dx\parallel\int_{v_0}^{v}v\,dv=\int_{x_0}^{x}a(x)\,dx$$

Given $a=a(t)$:

$$\int_{v_0}^{v}dv=\int_{t_0}^{t}a(t)\,dt\parallel\int_{x_0}^{x}dx=\int_{t_0}^{t}v(t)\,dt$$

Given $a=a(x)$:

$$\int_{v_0}^{v}v\,dv=\int_{x_0}^{x}a(x)\,dx\parallel\int_{t_0}^{t}dt=\int_{x_0}^{x}\frac{1}{v(x)}\,dx$$

Given $a=a(v)$:

$$\int_{t_0}^{t}dt=\int_{v_0}^{v}\frac{1}{a(v)}\,dv\parallel\int_{x_0}^{x}dx=\int_{v_0}^{v}\frac{v}{a(v)}\,dv$$

If v is constant:

$$x=x_0+v(t-t_0)$$

If a is constant:

$$v=v_0+a(t-t_0)$$

$$x=x_0+v_0(t-t_0)+\frac{1}{2}a(t-t_0)^2$$

$$v^2-v_0^2=2a(x-x_0)$$

Curvilinear Motion

$$a_n=\frac{v_t^2}{\rho}$$

Rigid Body Mechanics
General Plane Motion

$$v_{B/A}=v_B-v_A$$

$$v_{B/A}=\omega k\times r_{B/A}=r\omega$$

Rolling without Sliding
* velocity at contact point always 0

$$v_O=r\omega$$

Mass Properties

$$dW=\gamma t\,dA\equiv W=\gamma tA$$

$$\bar{x}A=\int x\,dA=Q_y$$

$$\bar{y}A=\int y\,dA=Q_x$$

Compound Shapes

$$Q_y=\bar{X}\sum A=\sum\bar{x}A$$

$$Q_y=\bar{Y}\sum A=\sum\bar{y}A$$

Mass Moment of Inertia

$$I_O=\int r^2\,dm=k_o^2m$$

$$I=I_O+md^2$$

Rigid Body Kinetics

$$\Sigma F=m\bar{a}$$

$$\bar{a}=a_{ref}+r\omega^2e_n+r\alpha e_t$$

$$\Sigma M_G=\bar{I}\alpha$$

Principle of Work and Energy

$$U_{1\to2}=\int\vec{F}\,d\vec{r}+\int M\,d\theta=T_2-T_1$$

$$T_i=\frac{1}{2}m|\vec{v_i}|^2+\frac{1}{2}I_G\omega^2$$

Work of Conservative Forces

$$U_{1\to2}=U_2-U_1=V_1-V_2$$

$$V_1+T_1=V_2+T_2$$

Free Vibration Without Damping

$$\ddot{u}+\omega_n^2u=0$$

$$\omega_n=\sqrt{\frac{k}{m}}$$

Natural Parameters

$$\tau_n=\frac{2\pi}{\omega_n}$$

$$f_n=\frac{\omega_n}{2\pi}\text{ (in Hz)}$$

General Solution

$$u=A\sin(\omega_nt+\phi)$$

$$A=\sqrt{x_o^2+\left(\frac{v_o}{\omega_n}\right)^2}$$

$$\phi=\tan^{-1}(\frac{x_o\omega_n}{v_o})$$

Pendulum

$$\ddot{\theta}+\frac{g}{l}\theta=0$$

$$\omega_n=\sqrt{\frac{g}{l}}$$

Generic Rigid Body

$$\omega_n=\sqrt{\frac{mgd}{I_O}}$$

Free Vibration with Damping

Spring-Mass-Damper System

$$m\ddot{x}+c\dot{x}+kx=0$$

$$\lambda=-\frac{c}{2m}\pm\sqrt{\left(\frac{c}{2m}\right)^2-\frac{k}{m}}$$

Overdamped $\left(\left(\frac{c}{2m}\right)^2-\frac{k}{m}>0\right)$

$$x=A_1e^{\lambda_1t}+A_2e^{\lambda_2t}$$

Overdamped $\left(\left(\frac{c}{2m}\right)^2-\frac{k}{m}=0\right)$

$$c_{cr}=2\sqrt{mk}=2m\omega_n$$

$$x=(A_1+A_2t)e^{-\omega_nt}$$

Underdamped $\left(\left(\frac{c}{2m}\right)^2-\frac{k}{m}<0\right)$

Equation of Motion

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^2x=0$$

$$x=Xe^{-\zeta\omega_nt}\sin(\omega_dt+\phi)$$

Exponential Decay Coefficient

$$\alpha=\frac{c}{2m}=\zeta\omega_n$$

Damped Oscillation Frequency

$$\omega_d=\sqrt{\frac{k}{m}-\left(\frac{c}{2m}\right)^2}=\omega_n\sqrt{1-\zeta^2}$$

Damping Ratio

$$\zeta=\frac{c}{c_{cr}}=\frac{c}{2\sqrt{km}}$$

Initial Conditions

$$X=\sqrt{C_1^2+C_2^2}=\sqrt{\left(\frac{v_o+\zeta\omega_nx_o}{\omega_d}\right)^2+x_o^2}$$

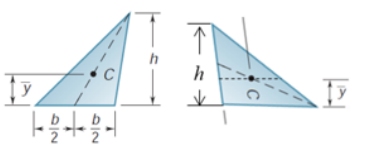
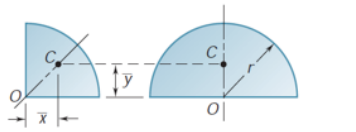
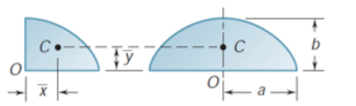
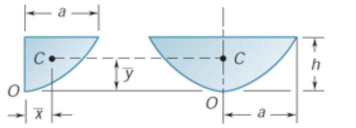
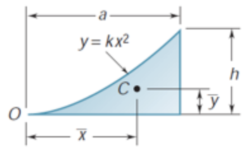
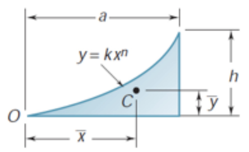
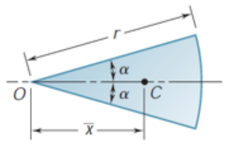
$$\phi=\tan^{-1}\left(\frac{C_1}{C_2}\right)=\tan^{-1}\left(\frac{\omega_dx_o}{v_o+\zeta\omega_nx_o}\right)$$

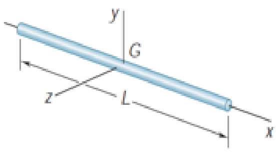
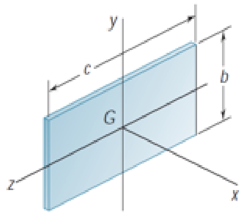
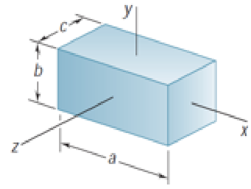
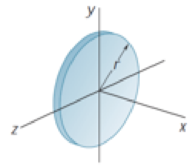
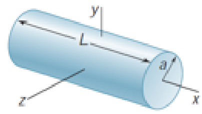
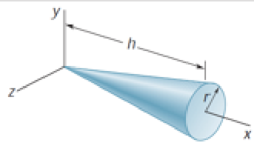
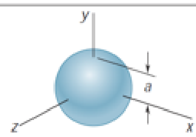
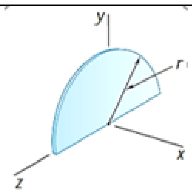
Logarithmic Decrement

$$\delta=\ln\left(\frac{x_1}{x_2}\right)=\zeta\omega_n\tau_d$$

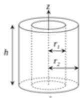
$$\zeta=\frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$$

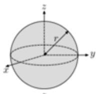
$$\delta=\frac{1}{N}\ln\left(\frac{x_1}{x_{1+N}}\right)$$

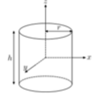
	$\bar{x}$	$\bar{y}$	Area
		$\frac{h}{3}$	$\frac{bh}{2}$
	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
	0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
	$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
	0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
	$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
	0	$\frac{3h}{5}$	$\frac{4ah}{3}$
	$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
	$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
	$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

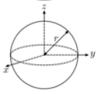
Slender rod		$I_y = I_z = \frac{1}{12}mL^2$
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$
Rectangular prism		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}m(c^2 + a^2)$ $I_z = \frac{1}{12}m(a^2 + b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$
Circular cone		$I_x = \frac{3}{10}ma^2$ $I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$
Sphere		$I_x = I_y = I_z = \frac{2}{5}ma^2$
Semicircular disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$

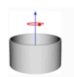
Thick-walled hollow sphere  $I = \frac{2m}{5}\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$

Thick-walled hollow cylinder  $I_z = \frac{1}{2}m(r_1^2 + r_2^2)$, $I_x = I_y = \frac{1}{12}m[3(r_1^2 + r_2^2) + h^2]$

Solid sphere  $I = \frac{2}{5}mr^2$

Solid cylinder  $I_z = \frac{1}{2}mr^2$

Hollow sphere  $I = \frac{2}{3}mr^2$

Thin-walled hollow cylinder  $I = mr^2$