ME2115 Formulae

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Vector Mechanics

$$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB}$$

$$u = |\mathbf{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

$$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = u_x v_x + u_y v_y + u_z v_z$$

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \times \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{pmatrix}$$

$$\mathbf{u} \times \mathbf{u} = 0$$

$$\mathbf{u} \cdot \mathbf{v} = 0 \text{ (if } \mathbf{u} \perp \mathbf{v})$$

$$\mathbf{u} \cdot \mathbf{v} = uv \cos\theta$$

$$\mathbf{u} \times \mathbf{v} = uv \sin\theta \cdot \mathbf{n}$$

Particle Kinematics

Rectilinear Motion

$$dx = v\,dt \,\|\, \int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt$$

$$dv = a\,dt \,\|\, \int_{v}^{v_0} dv = \int_{t_0}^{t} a(t)\,dt$$

$$v\,dv = a\,dx \,\|\, \int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$$

Given $a = a(t)$:

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)\,dt \,\|\, \int_{x_0}^{x} dx = \int_{t_0}^{t} v(t)\,dt$$

Given $a = a(x)$:

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx \,\|\, \int_{t_0}^{t} dt = \int_{x_0}^{x} \frac{1}{v(x)}\,dx$$

Given $a = a(v)$:

$$\int_{t_0}^{t} dt = \int_{v_0}^{v} \frac{1}{a(v)}\,dv \,\|\, \int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v}{a(v)}\,dv$$

If v is constant:

$$x = x_0 + v(t - t_0)$$

If a is constant:

$$v = v_0 + a(t - t_0)$$

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Curvilinear Motion

The previous section can apply to angular motion by replacing x with θ, v with ω, and a with α.

$$s = r\theta$$

$$v_t = r\omega$$

$$a_t = r\alpha$$

$$a_n = \frac{v_t^2}{\rho}$$

Rigid Body Mechanics

General Plane Motion

$$v_B = v_A + v_{B/A}$$

$$\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A} = v_t$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$a_{B/A} = \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) + \vec{\alpha} \times \vec{r}_{B/A} = \vec{a}_n + \vec{a}_t$$

Rolling without Sliding

* velocity at contact point always 0

$$v_o = r\omega$$

$$a_t = r\alpha$$

$$\vec{a_c} = \vec{\alpha} \times \vec{r}_{c/o}$$

$$\vec{v_c} = \vec{\omega} \times \vec{r}_{c/o}$$

Mass Properties

$$dW = \gamma t\,dA \equiv W = \gamma t A$$

$$\bar{x}A = \int x\,dA = Q_y$$

$$\bar{y}A = \int y\,dA = Q_x$$

Compound Shapes

$$Q_y = \bar{X} \sum A = \sum \bar{x} A$$

$$Q_y = \bar{Y} \sum A = \sum \bar{y} A$$

Mass Moment of Inertia

$$I_O = \int r^2\,dm = mk_o^2$$

$$I = I_O + md^2$$

Rigid Body Kinetics

$$\Sigma F = m\bar{a} = \Sigma F_{eff.}$$

$$\Sigma M_G = \bar{I}\alpha = \Sigma M_{eff.}$$

More generally.

$$\Sigma M_i + \Sigma(\vec{r}_{i/A} \times \vec{F}_i) = M_A = I_G\vec{\alpha} + \vec{r}_{G/A} \times m\vec{a}_G$$

$$M_A = (I_G + mr_{G/A}^2)\vec{\alpha} = I_A\vec{\alpha}$$

$$\bar{a} = a_{ref} + r\omega^2 e_n + r\alpha e_t$$

Principle of Work and Energy

$$U_{1\to2} = \int \vec{F}\,d\vec{r} + \int M\,d\theta = T_2 - T_1$$

$$V_{grav.} = mg$$

$$V_{elas.} = \frac{1}{2}k\delta x$$

$$T_i = \frac{1}{2}m|\vec{v_i}|^2 + \frac{1}{2}I_G\omega^2$$

Work of Conservative Forces

$$U_{1\to2} = U_2 - U_1 = V_1 - V_2$$

$$V_1 + T_1 = V_2 + T_2$$

Free Vibration Without Damping

$$\ddot{u} + \omega_n^2 u = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Natural Parameters

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$f_n = \frac{\omega_n}{2\pi} \text{ (in Hz)}$$

General Solution

$$u = A\sin(\omega_n t + \phi)$$

$$A = \sqrt{x_o^2 + \left(\frac{v_o}{\omega_n}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{x_o\omega_n}{v_o}\right)$$

Pendulum

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

$$\omega_n = \sqrt{\frac{g}{l}}$$

Generic Rigid Body

$$\omega_n = \sqrt{\frac{mgd}{I_O}}$$

Free Vibration with Damping

Spring-Mass-Damper System

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

Overdamped $\left(\left(\frac{c}{2m}\right)^2 - \frac{k}{m} > 0\right)$

$$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

Critically Damped $\left(\left(\frac{c}{2m}\right)^2 - \frac{k}{m} = 0\right)$

$$c_{cr} = 2\sqrt{mk} = 2m\omega_n$$

$$x = (A_1 + A_2 t)e^{-\omega_n t}$$

Underdamped $\left(\left(\frac{c}{2m}\right)^2 - \frac{k}{m} < 0\right)$

Equation of Motion

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$x = Xe^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

Exponential Decay Coefficient

$$\alpha = \frac{c}{2m} = \zeta\omega_n$$

Damped Oscillation Frequency

$$\omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} = \omega_n\sqrt{1 - \zeta^2}$$

Damping Ratio

$$\zeta = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}}$$

Initial Conditions

$$X = \sqrt{C_1^2 + C_2^2} = \sqrt{\left(\frac{v_o + \zeta\omega_n x_o}{\omega_d}\right)^2 + x_o^2}$$

$$\phi = \tan^{-1}\left(\frac{C_1}{C_2}\right) = \tan^{-1}\left(\frac{\omega_d x_o}{v_o + \zeta\omega_n x_o}\right)$$

Logarithmic Decrement

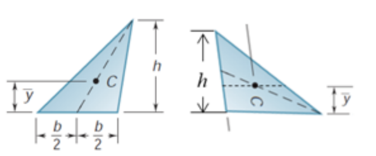
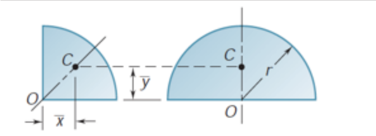
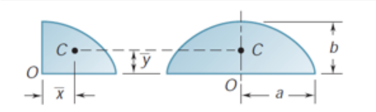
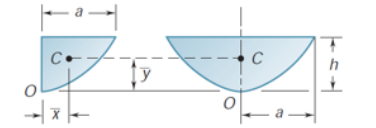
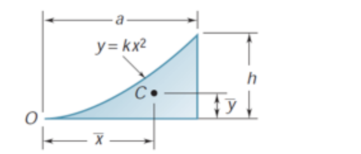
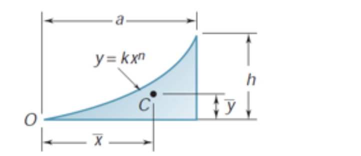
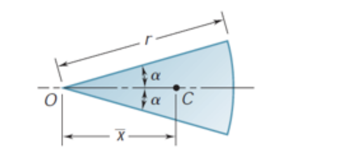
$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \zeta\omega_n\tau_d$$

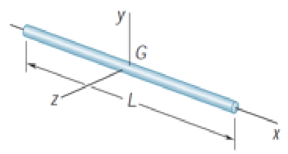
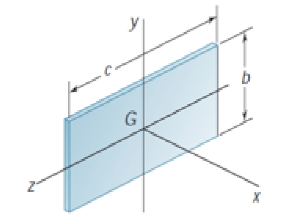
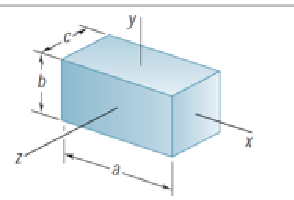
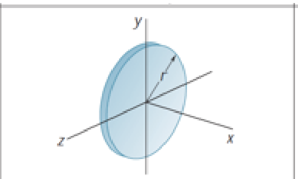
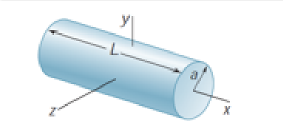
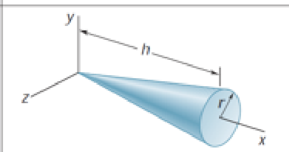
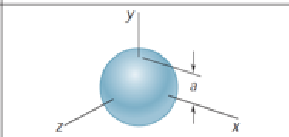
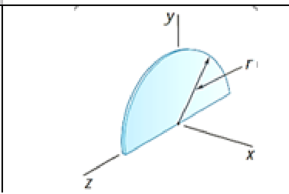
$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

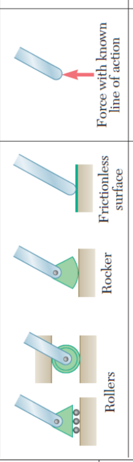
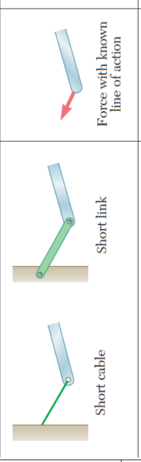
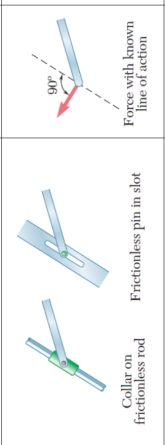
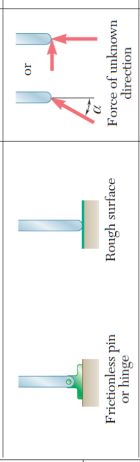
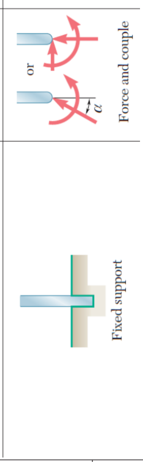
$$\delta = \frac{1}{N}\ln\left(\frac{x_1}{x_{1+N}}\right)$$

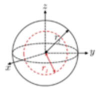
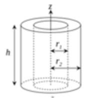
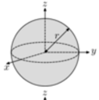
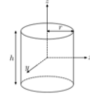
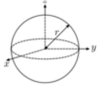
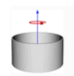
Tips

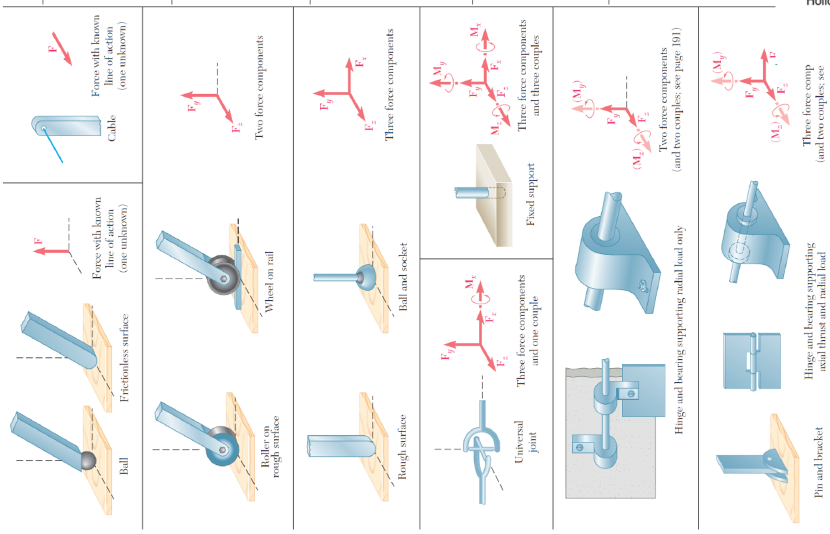
- Only tangential movement with fixed rotation (hinges, links)
- For **Constrained Motion** - do kinematic analysis (relative velocity/acceleration) then kinetic analysis (equate FBD to EFD)
- Use moments about points that eliminate as many forces as possible
- **Law of Sines**: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ for any triangle

	$\bar{x}$	$\bar{y}$	Area
		$\frac{h}{3}$	$\frac{bh}{2}$
	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
	0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
	$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
	0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
	$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
	0	$\frac{3h}{5}$	$\frac{4ah}{3}$
	$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
	$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
	$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Slender rod		$I_y = I_z = \frac{1}{12}mL^2$
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2+c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$
Rectangular prism		$I_x = \frac{1}{12}m(b^2+c^2)$ $I_y = \frac{1}{12}m(c^2+a^2)$ $I_z = \frac{1}{12}m(a^2+b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2+L^2)$
Circular cone		$I_x = \frac{3}{10}ma^2$ $I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2+h^2)$
Sphere		$I_x = I_y = I_z = \frac{2}{5}ma^2$
Semicircular disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$

Support or Connection	Reaction	Number of Unknowns
 Frictionless surface; Rocker; Rollers	Force with known line of action	1
 Short link; Short cable	Force with known line of action	1
 Frictionless pin in slot; Collar on frictionless rod	$90°$ Force with known line of action	1
 Rough surface; Frictionless pin or hinge	or α Force of unknown direction	2
 Fixed support	or α Force and couple	3

Thick-walled hollow sphere 	$I = \frac{2m}{5}\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$	Thick-walled hollow cylinder  $I_z = \frac{1}{2}m(r_1^2+r_2^2)$; $I_x = I_y = \frac{1}{12}m[3(r_1^2+r_2^2)+h^2]$
Solid sphere 	$I = \frac{2}{5}mr^2$	Solid cylinder  $I_z = \frac{1}{2}mr^2$
Hollow sphere 	$I = \frac{2}{3}mr^2$	Thin-walled hollow cylinder  $I = mr^2$



Ball; Frictionless surface	Force with known line of action (one unknown)
Cable	Force with known line of action (one unknown)
Roller on rough surface; Wheel on rail	Two force components
Rough surface; Ball and socket	Three force components
Universal joint	Three force components and one couple
Fixed support	Three force components and three couples
Hinge and bearing supporting radial load only	Two force components (and two couples; see page 191)
Pin and bracket; Hinge and bearing supporting axial thrust and radial load	Three force comp (and two couples; see