

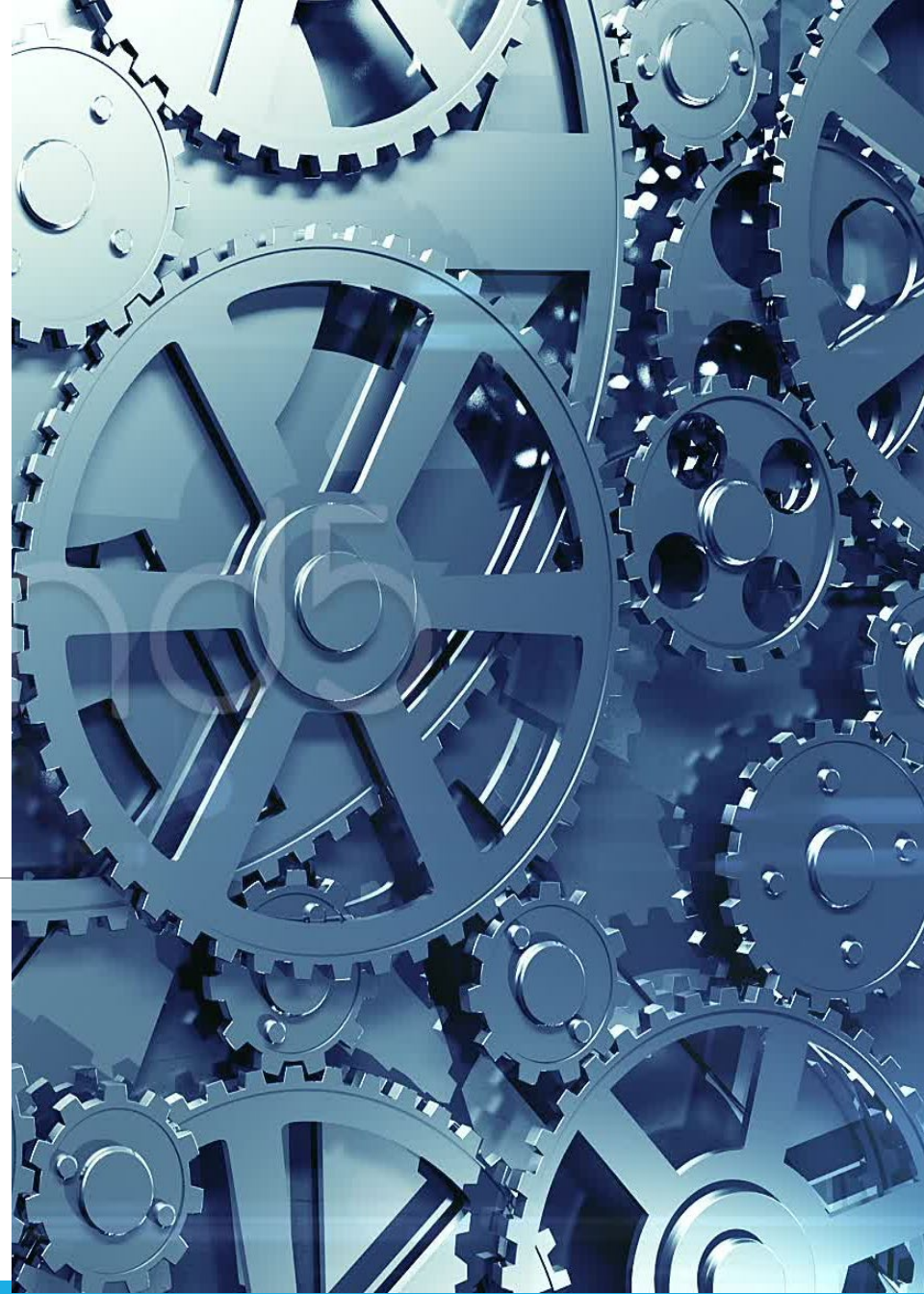
# ME2115/ME2115E/TME2115 - **Mechanics of Machines**

## **Review of free-body diagram**

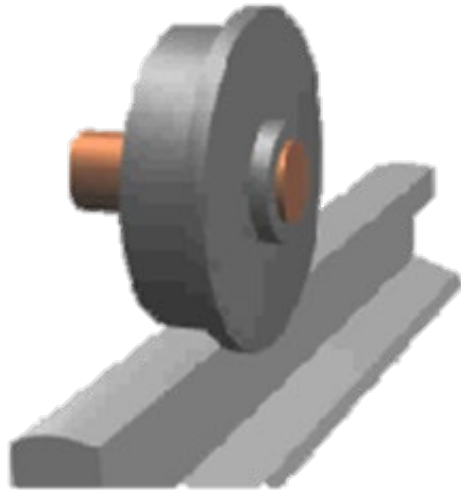
Dr. SHEN Lei (Part I)

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Tel: 66013813; Office: EA-05-09



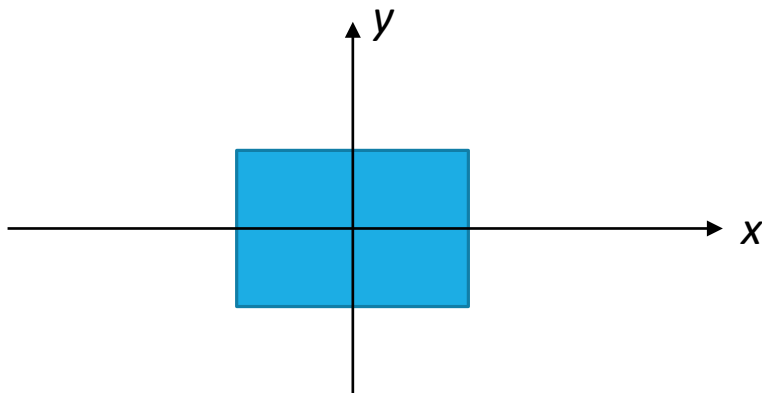
## Free body vs. constrained body



Constrained body



Free body  
No constraint



In a 3D space, a free body has 6 degrees of freedom: translation along  $x$ ,  $y$ ,  $z$ ; rotation about  $x$ ,  $y$ ,  $z$ .

However, in a 2D plane, a free body only has 3 degrees of freedom: translation along  $x$ ,  $y$ ; rotation about  $z$ . **WHY?**

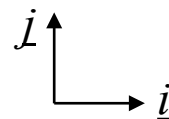
Will move out of the plane if rotates about  $x$  or  $y$ .

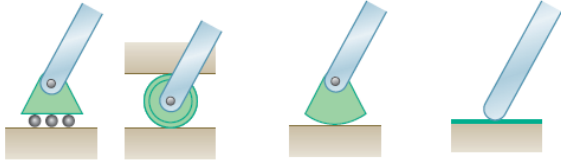

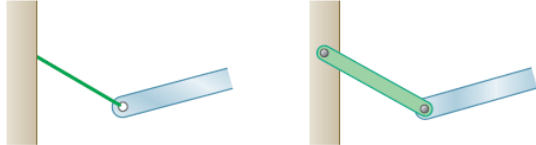
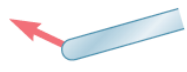
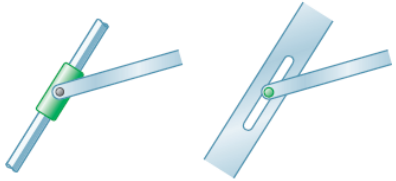
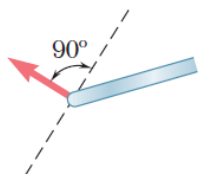

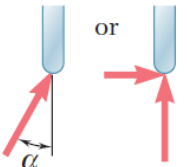
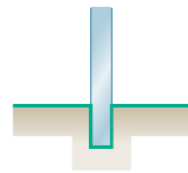
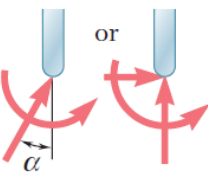
# Reactions at Supports and Connections for a two-dimensional structure

**Must remember!**

The first step in the solution of any mechanical problem concerning the equilibrium of a rigid body is to construct an appropriate free-body diagram of the body.

Without a proper F.B.D, one will construct wrong equations accordingly and then wrong answers.



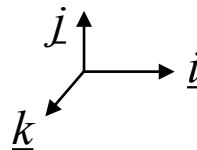
Support or Connection	Reaction	Number of Unknowns
 Rollers      Rocker      Frictionless surface	 Force with known line of action	1
 Short cable      Short link	 Force with known line of action	1
 Collar on frictionless rod      Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge      Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

# Reactions at Supports and Connections for a **three-dimensional** structure

**Must remember!**

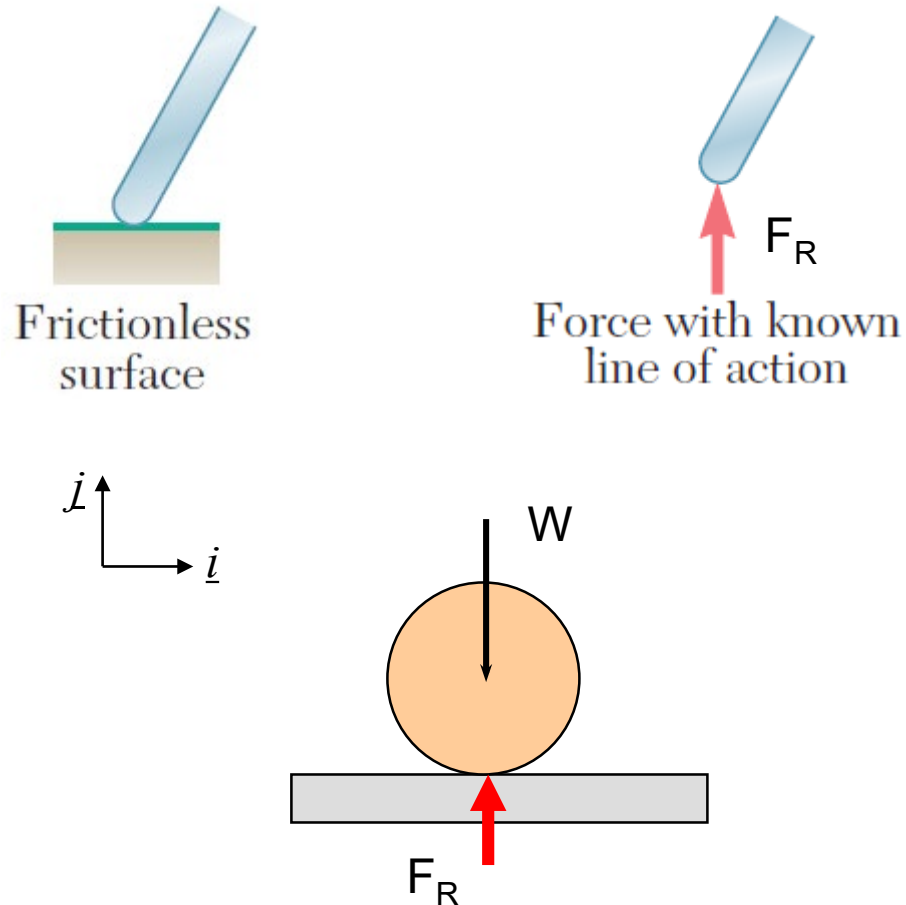
It is necessary to show on the diagram the reactions through which the ground and other bodies oppose a possible motion of the body.

In summary, if a support prevents translation of a body in a particular direction, then the support exerts a constrained force on the body in that direction. If a rotation is prevented, then the support exerts a moment on the body.

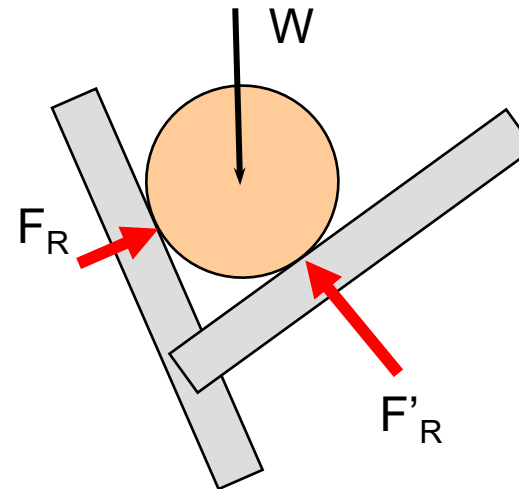


<p>Ball</p>	<p>Frictionless surface</p>	<p>Force with known line of action (one unknown)</p>	<p>Cable</p>	<p>Force with known line of action (one unknown)</p>
<p>Roller on rough surface</p>	<p>Wheel on rail</p>	<p>Two force components</p>		
<p>Rough surface</p>	<p>Ball and socket</p>	<p>Three force components</p>		
<p>Universal joint</p>	<p>Three force components and one couple</p>	<p>Fixed support</p>	<p>Three force components and three couples</p>	
<p>Hinge and bearing supporting radial load only</p>	<p>Hinge and bearing supporting axial thrust and radial load</p>	<p>Two force components (and two couples; see page 191)</p>		
<p>Pin and bracket</p>	<p>Hinge and bearing supporting axial thrust and radial load</p>	<p>Three force components (and two couples; see page 191)</p>		

## Surface support



Constrained force,  $F_R$   
Direction normal to the surface  
at the contact point





# Review of F. B. D

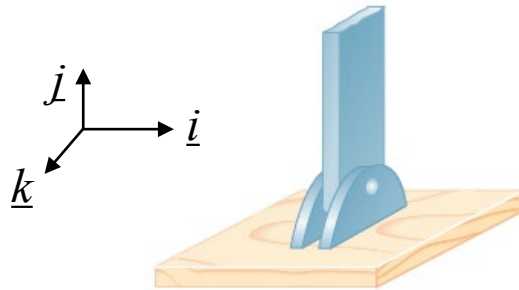
## Pin/pin and bracket/roller supports

2D structure

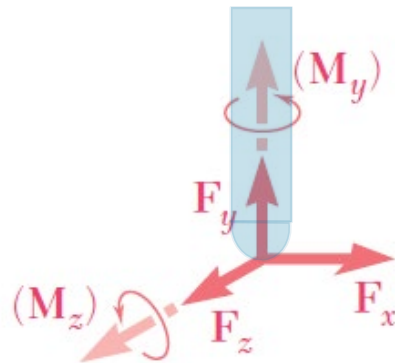
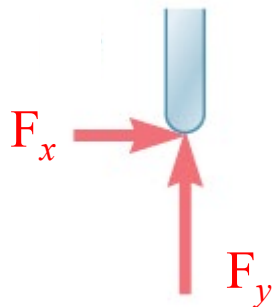


Frictionless pin

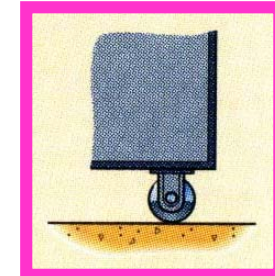
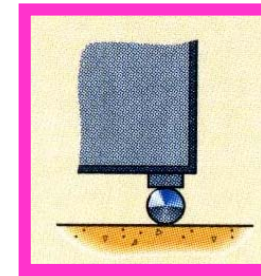
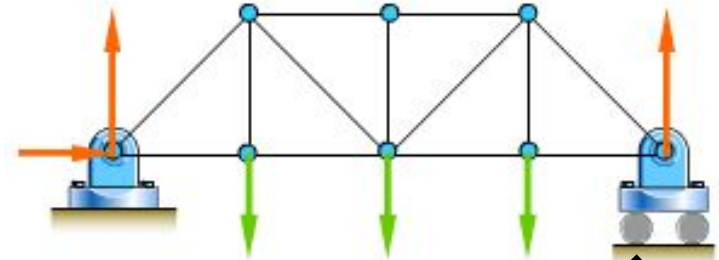
3D structure



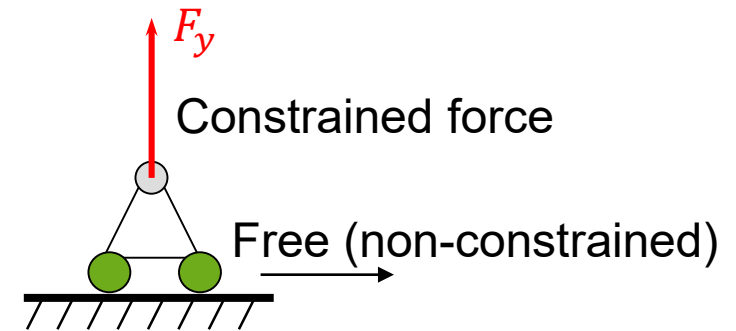
Pin and bracket



2D structure



Roller support

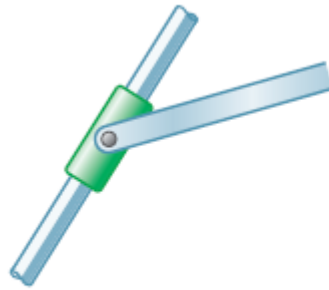


**Sign convention** of pin (bracket) support: positive along  $\underline{i}, \underline{j}, \underline{k}$

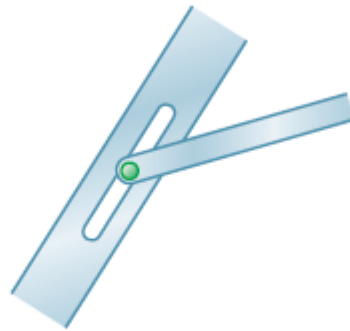
# Review of F. B. D

## Collar-rod and pin-slot supports

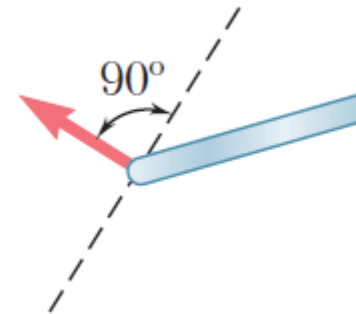
2D structure  $\begin{matrix} \uparrow j \\ \rightarrow i \end{matrix}$



Collar on  
frictionless rod



Frictionless pin in slot

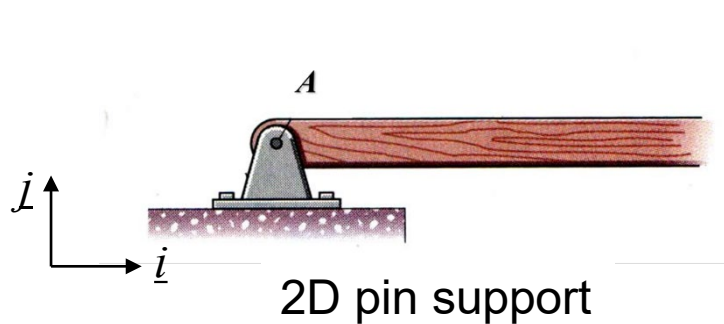


Force with known  
line of action

Only one reaction (unknown force) perpendicular to the rod or slot

# How to find all constrained forces/couples (named reactions) of a support ?

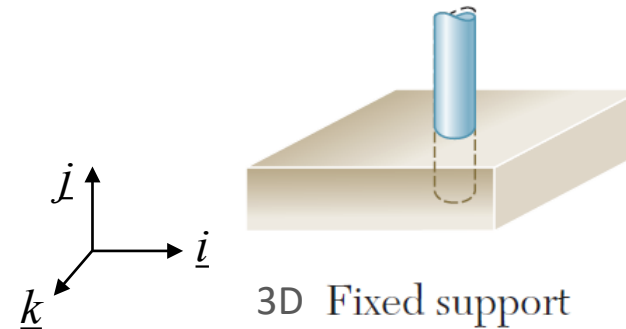
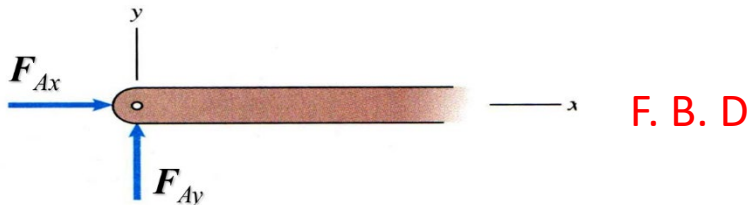
**Tip:** find the degree of freedom of the body!



2D free body has 3 degrees of freedom

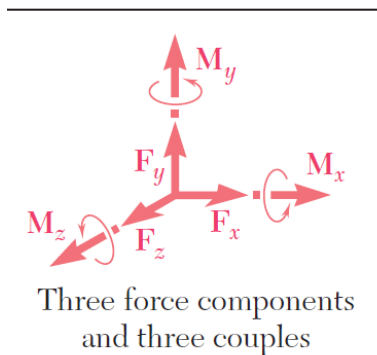
- 1) Can translate along  $\underline{i}$ ?
- 2) Can translate along  $\underline{j}$ ?
- 3) Can rotate in the 2D plane?

If answer is no, there is a constrained force



3D free body has 6 degrees of freedom

- 1) Can translate along  $\underline{i}$ ?
- 2) Can translate along  $\underline{j}$ ?
- 3) Can translate along  $\underline{k}$ ?
- 4) Can rotate about  $\underline{i}$ ?
- 5) Can rotate about  $\underline{j}$ ?
- 6) Can rotate about  $\underline{k}$ ?

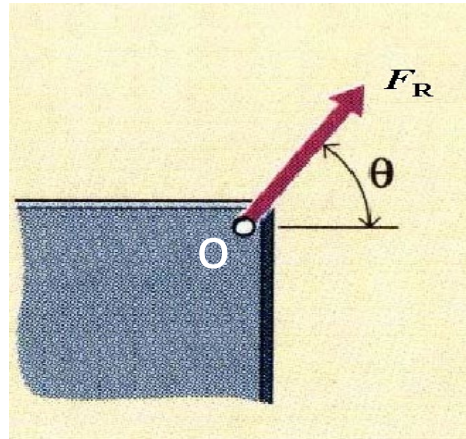
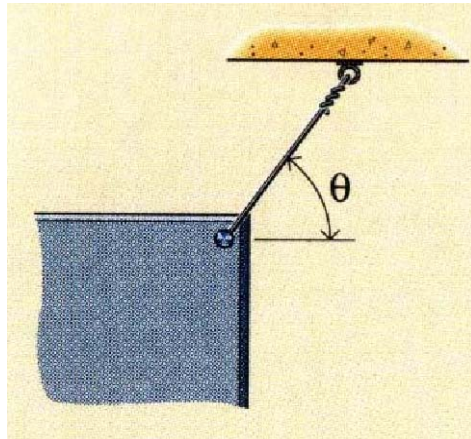


F. B. D



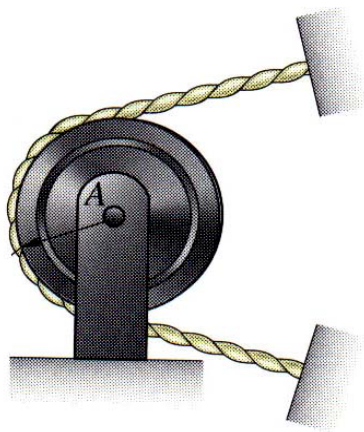
# Review of F. B. D

## Constraint of rope, cable, chain, link...

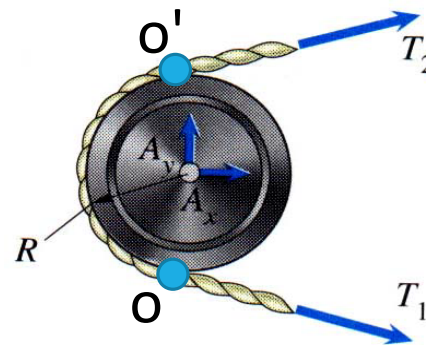


**The point of action** is the connection point between the studied object and the rope/cable/chain.

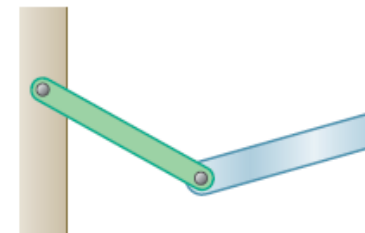
**Direction** is always along the rope/cable/chain away the point of action.



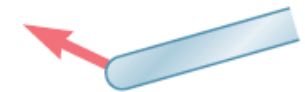
(a)



(b)



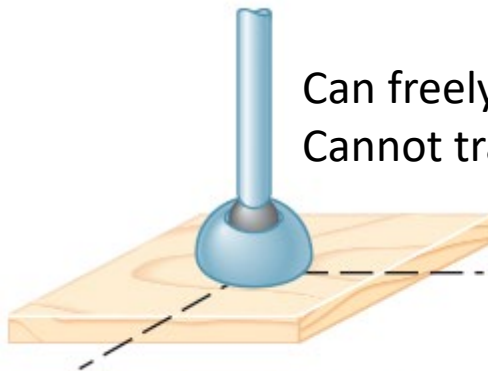
Short link



Force with known  
line of action

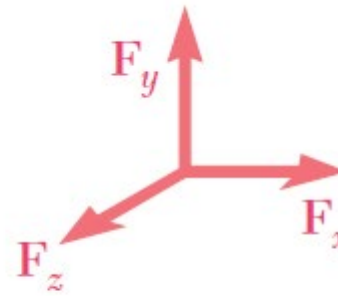
# Review of F. B. D

## Three dimensional (3D) constrains



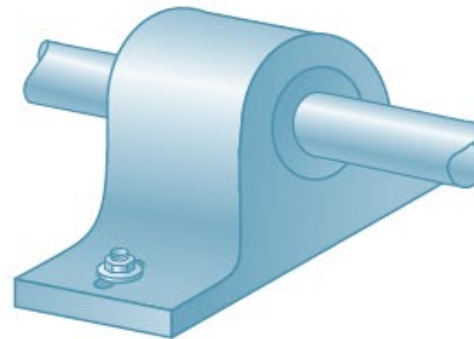
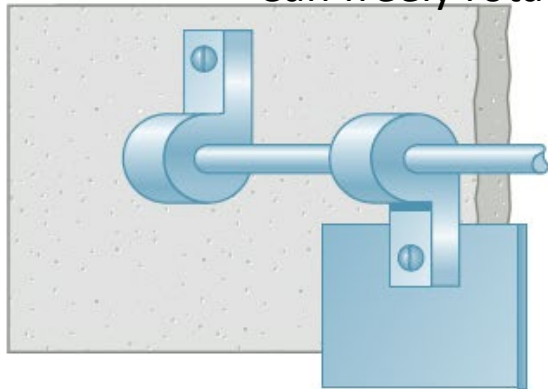
Can freely rotate about  $x, y, z$   
Cannot translate along  $x, y, z$

Ball and socket

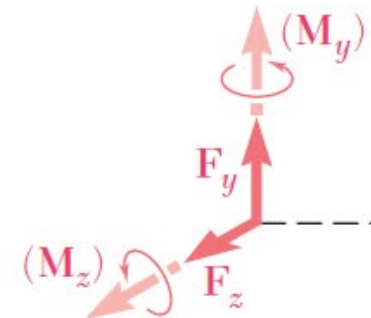


Three force components

Can freely rotate about and translate along axial  $x$



Hinge and bearing supporting radial load only



Two force components  
(and two couples; see page 191)

## \*Review of F. B. D

### Free-body diagram (very important !!!)

A **free-body diagram** is a sketch of an object or a connected group of objects, modeled as a single particle/rigid body that is completely isolated from its environment or surrounding bodies and represents the interactions of its environment by appropriate external forces.

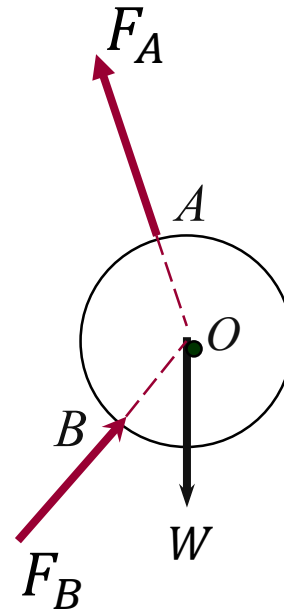
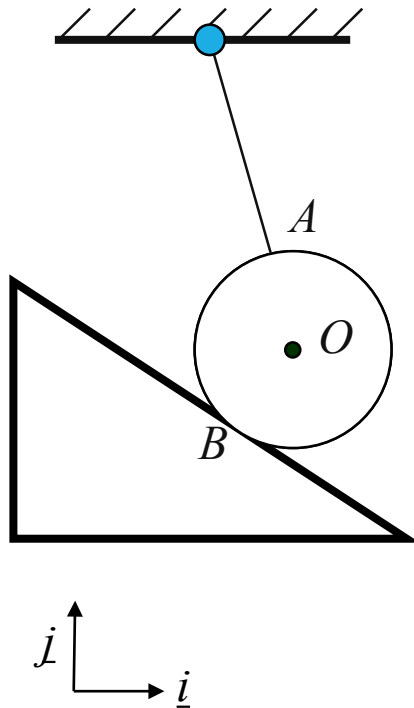
Drawing a free-body diagram is an art, and can be learned **only by practice**. If a correct free-body diagram is constructed, then the balance of the forces can be carried out in a very systematic manner.

### Steps of drawing F. B. D:

- 1) **Isolate the body of interest** (to be free body).
- 2) Draw your axis system (2D or 3D? Cartesian, polar...).
- 3) Draw all non-constrained applied forces first, such as weight, friction, wind forces.
- 4) **Identify all supports/connections, then draw reactions for each one based on the F.B.D. Tables.**
- 5) Draw appropriate dimensions (angles and distances).
- 6) Remove the internal forces of two free bodies if have.

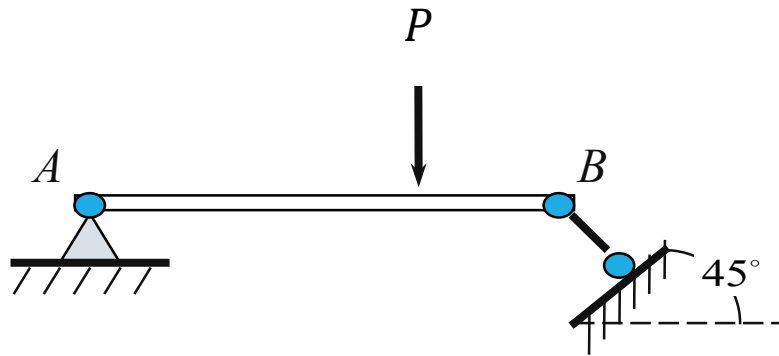
# Example 1

## FBD for the ball:



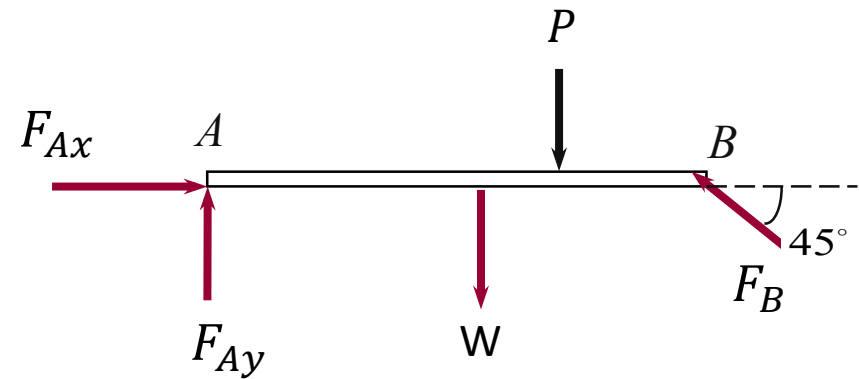
## Example 2

### FBD for the rod:



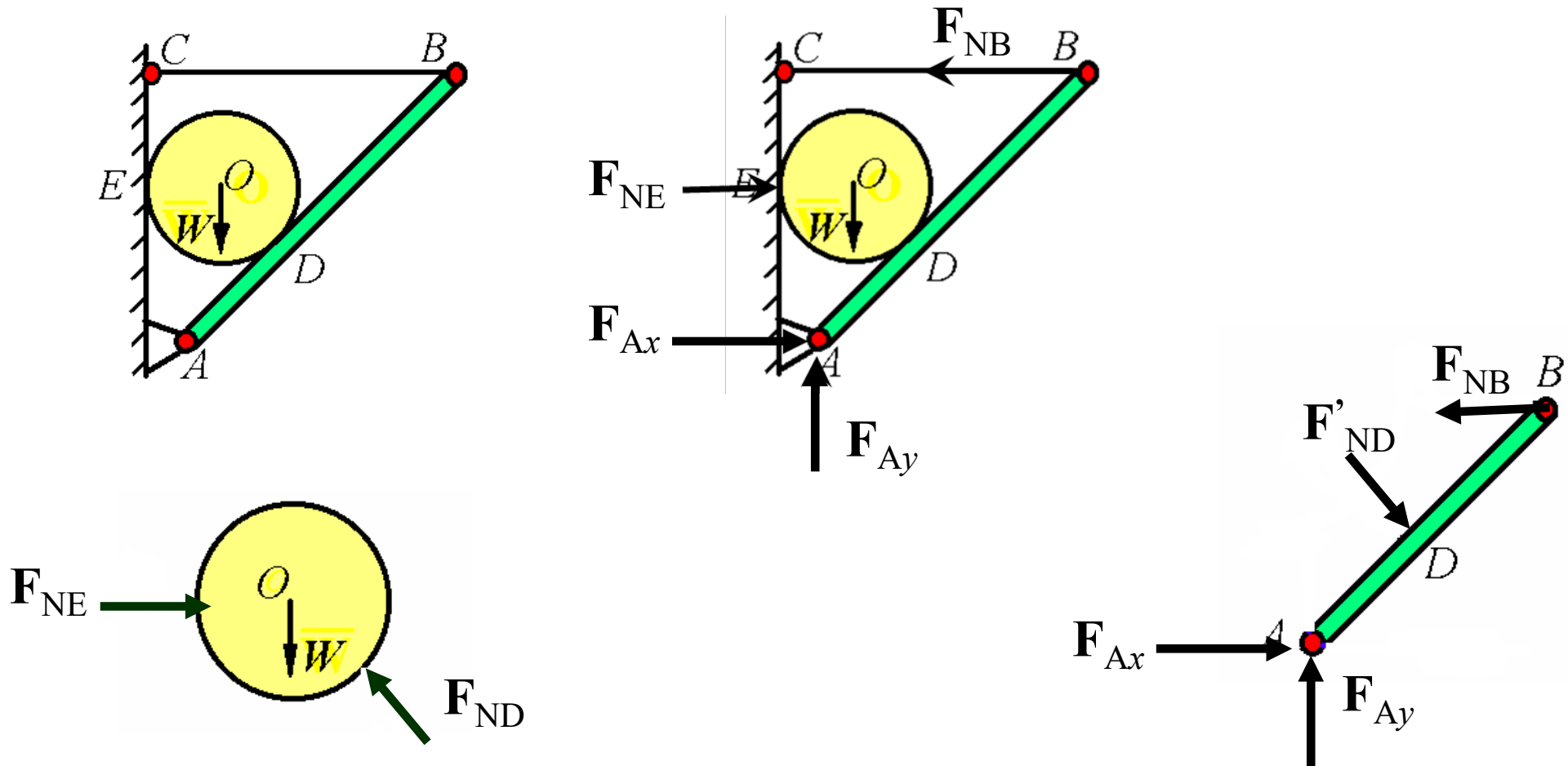
Pin support

Roller support  
on frictionless surface



## Example 3

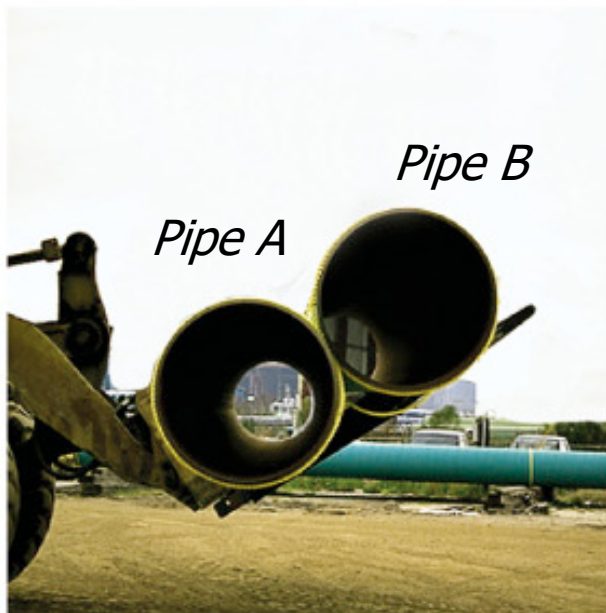
FBD for the ball, slim rod (massless), and whole structure:



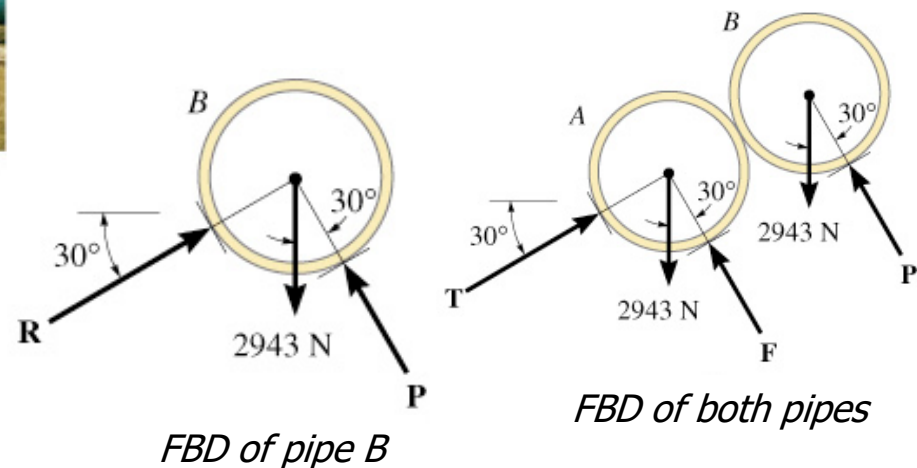
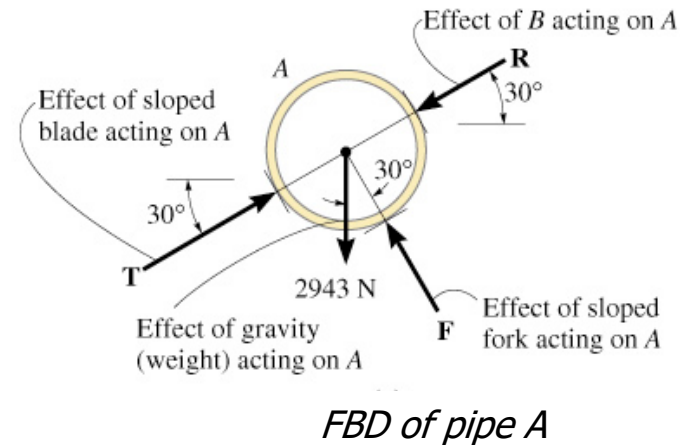


## Example 4

Two smooth pipes, each weighing  $W$ , are supported by the forks of a tractor as shown in the figure. Draw free body diagrams for each pipe and for both pipes.

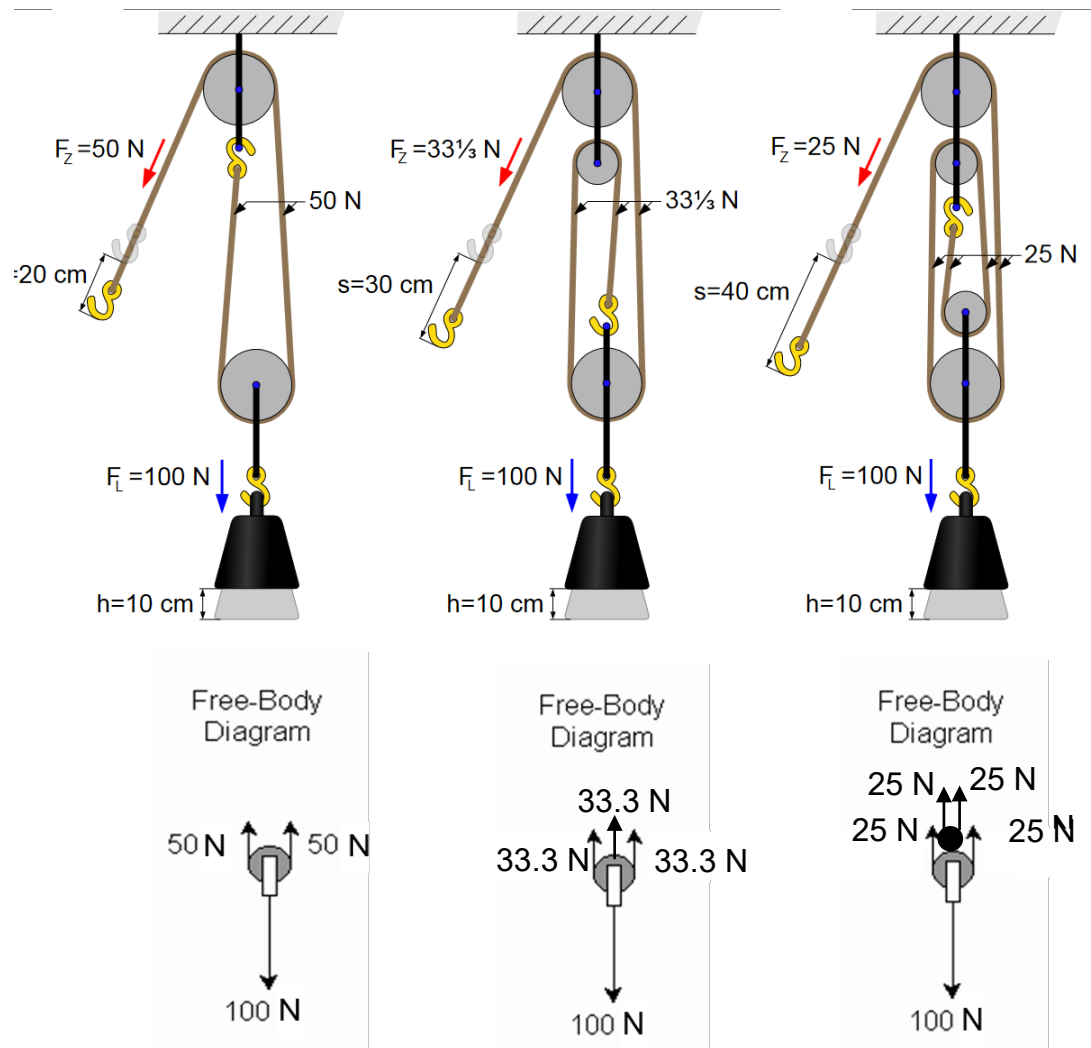


*The weight of a body is an external force and its effect is shown as a single resultant force acting vertically down through the body's centre of gravity.*



# Example 5

## FBD for movable pulleys:



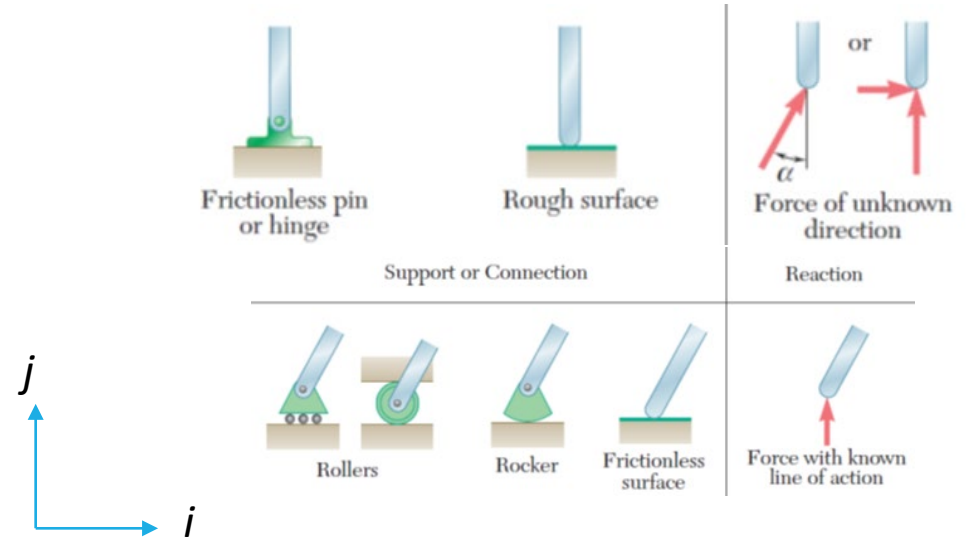
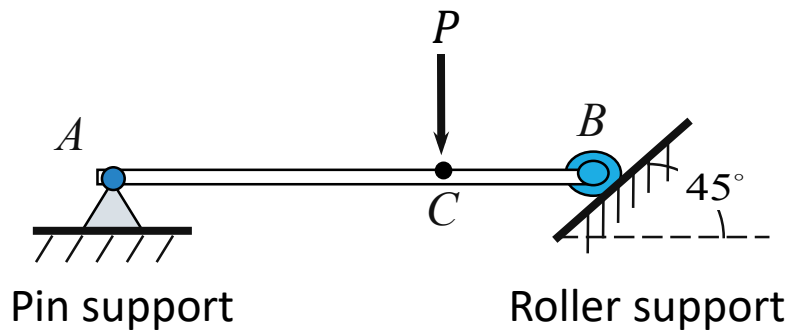
### Tips:

1. Isolate movable pulleys as one rigid body.
2. Each cable carries equal load.

- ❑ Drawing F.B.D is the **first** step for solving almost all mechanical problems.
- ❑ Drawing a free-body diagram is an art, and can be learned **only by practice**.
- ❑ Do the F.B.D test on Quiz/LumiNUS for self-evaluation. To improve your learning of this important concept in this module, personalized practice problems will be provided to you later based on your test results.

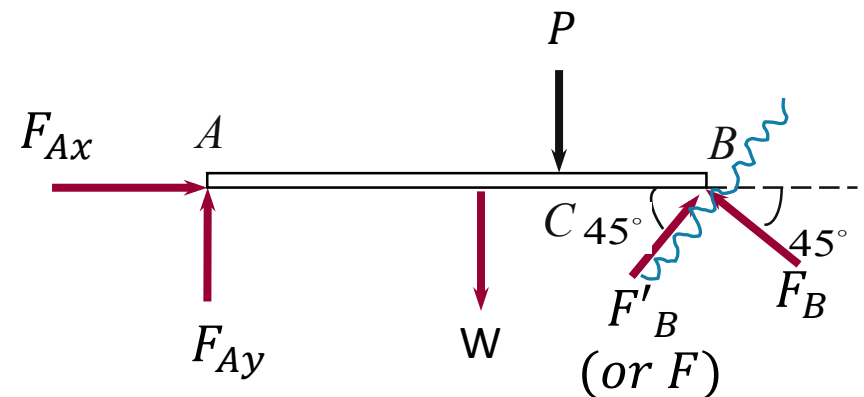
# Practice problem 1

A rod  $AB$  has mass  $m$ . It is the pin support at the end  $A$ . An external force  $P$  is applied on point  $C$ . Draw the F.B.D of the rod (a) if the roller support of  $B$  is on a frictionless surface or (b) on a rough surface.



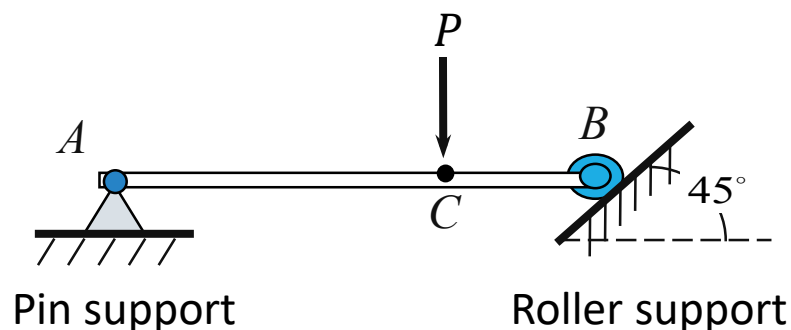
## Steps of drawing F. B. D:

- 1) Isolate the body of interest (to be free body).
- 2) Draw your axis system (2D or 3D? Cartesian, polar...).
- 3) Draw all non-constrained applied forces first, such as weight, friction, wind forces.
- 4) Identify all supports/connections, then draw reactions for each one based on the F.B.D. Tables.
- 5) Draw appropriate dimensions (angles and distances).
- 6) Remove the internal forces of two free bodies if have.



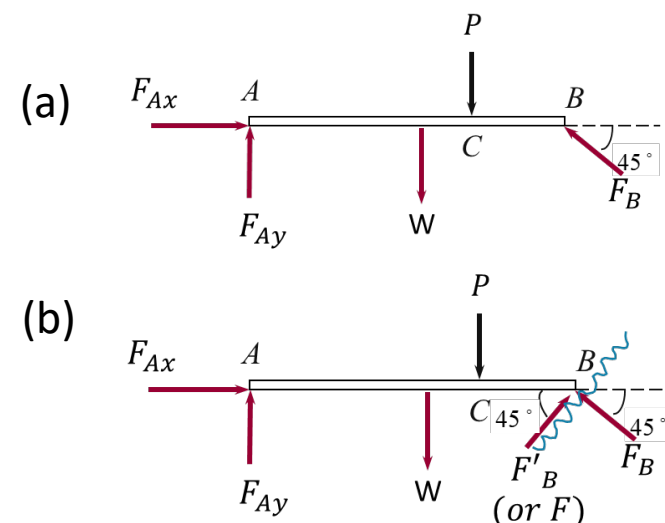
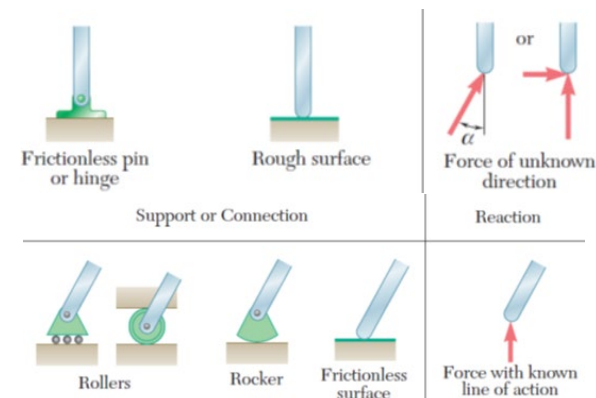
# Practice problem 1

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## Steps of drawing F. B. D:

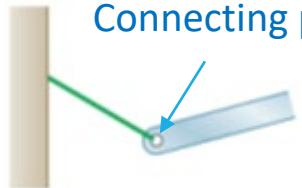

- 1) Isolate the body of interest (to be free body).
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- 3) Draw all non-constrained applied forces first, such as weight, friction, wind forces.
- 4) Identify all supports/connections, then draw reactions for each one based on the F.B.D. Tables.
- 5) Draw appropriate dimensions (angles and distances).
- 6) Remove the internal forces of two free bodies if have.



# Practice problem 2

To demonstrate the importance of the step - Isolate the body of interest (to be a free body) from its supports

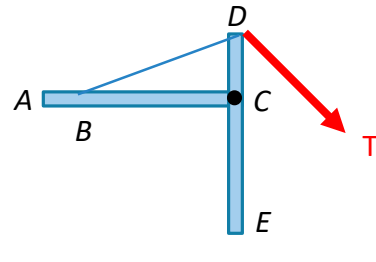
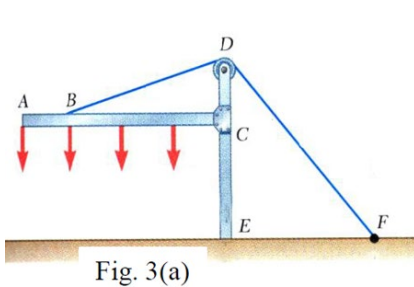
Recall the reaction of a soft non-stretchable rope/string/cable/chain... connection.

 <p>Connecting point</p> <p>Short cable</p>	<ol style="list-style-type: none"><li>1. <b>Point of action of the reaction (tension):</b> connecting point</li><li>2. <b>Line of action of tension:</b> the line of cable.</li><li>3. <b>Sense of tension:</b> away the point of action</li></ol>	 <p>Force with known line of action</p>
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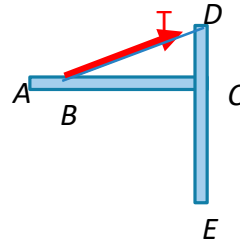


# Practice problem 3

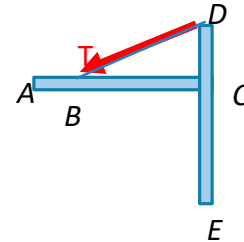
Draw the tension force on the frame.  $C$  is a pin and  $D$  is a pulley. Mass of the frame and cable is negligible.



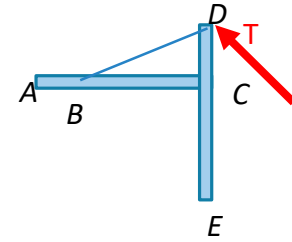
(a)



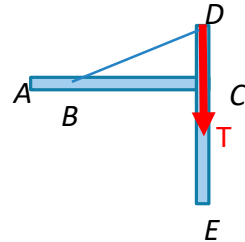
(b)



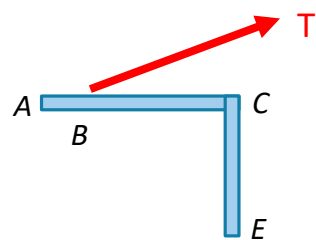
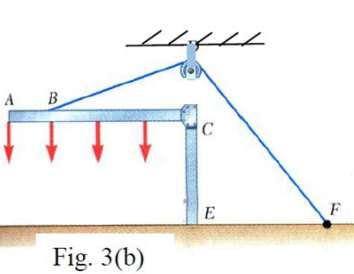
(c)



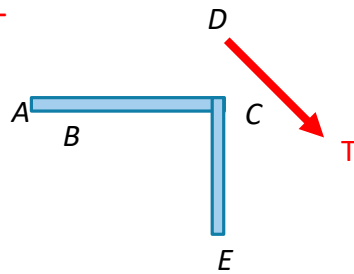
(d)



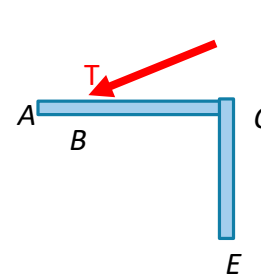
(e)



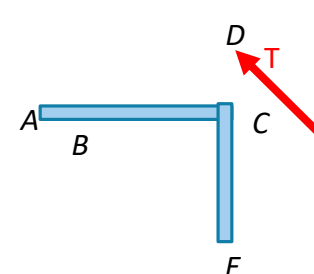
(a)



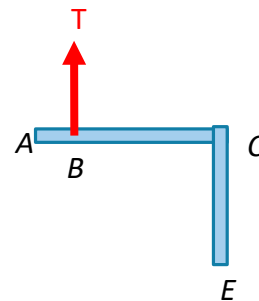
(b)



(c)



(d)



(e)