NUS
National University of Singapore

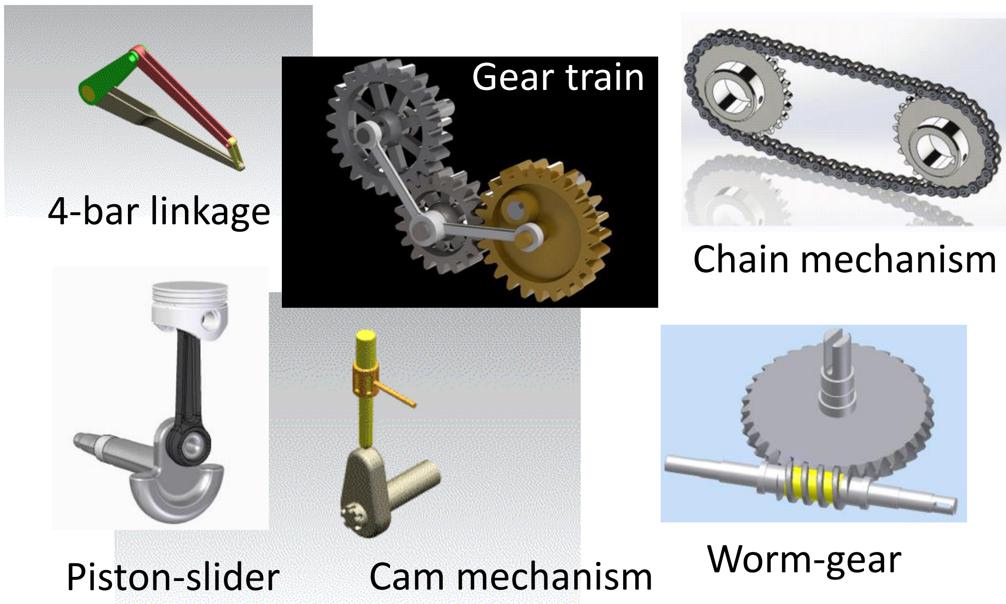
ME2115/TME2115

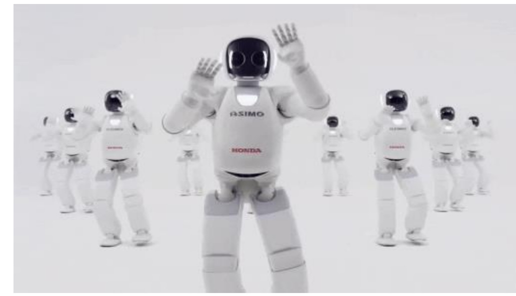
Mechanics of Machines

REVIEW ON BASIC MECHANICS

Study of Mechanism

- A mechanical system integrates *power and control systems* with *basic mechanisms* to perform specific tasks.

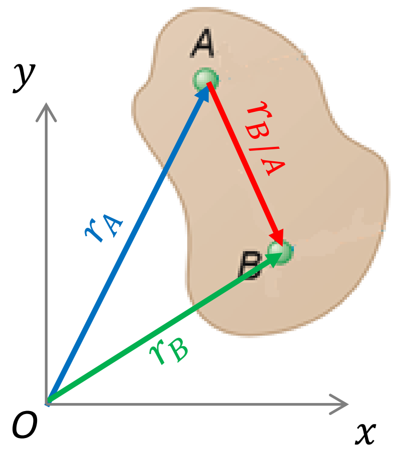




- It involves either *analyses* and/or *synthesis* of the system via *Kinematic* and *Kinetic* studies. In either cases, they involves *vector equations*.

Kinematics of Rigid Body – Position

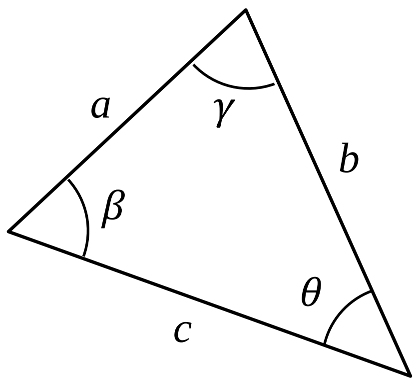
- *Position vector* represents the position of a point in space in relation to a given reference frame, e.g. $\vec{r}_A$ and $\vec{r}_B$. For rigid body, the important points of interest are the center of mass and joints at which it is connected to other bodies.



- It is also common to determine the *relative position* vector

$$\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A = -\vec{r}_{A/B}$$

- For 2D problem, the following trigonometry relations can be used to determine the geometry information, namely:



  - Sine rule:

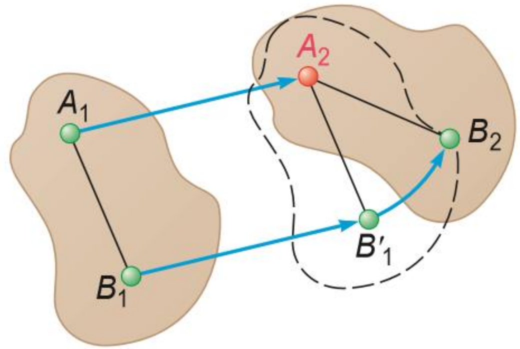
$$\frac{sin\theta}{a} = \frac{sin\beta}{b} = \frac{sin\gamma}{c}$$

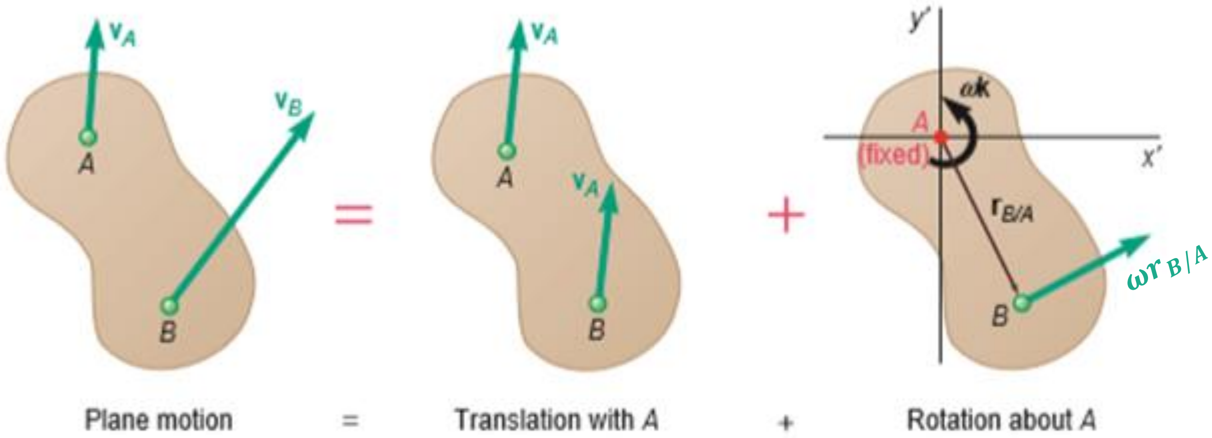
  - Cosine rule:

$$a^2 = b^2 + c^2 - 2bccos(\theta)$$

Kinematics of Rigid Body – Velocity

- A general 2D plane motion of a rigid body comprises a translation and a rotation components.





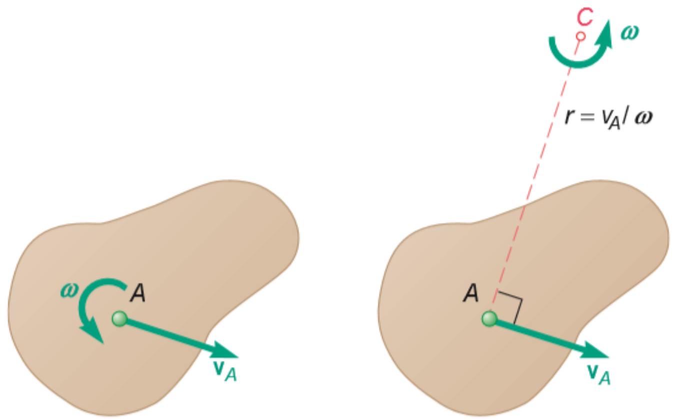
- At any given instant, the velocities of any two points *A* and *B* on the rigid body are related by the following relation:

$$\boxed{\vec{v}_B = \vec{v}_A + \underbrace{\vec{\omega} \times \vec{r}_{B/A}}_{\text{Tangential to } \vec{r}_{B/A}}}$$

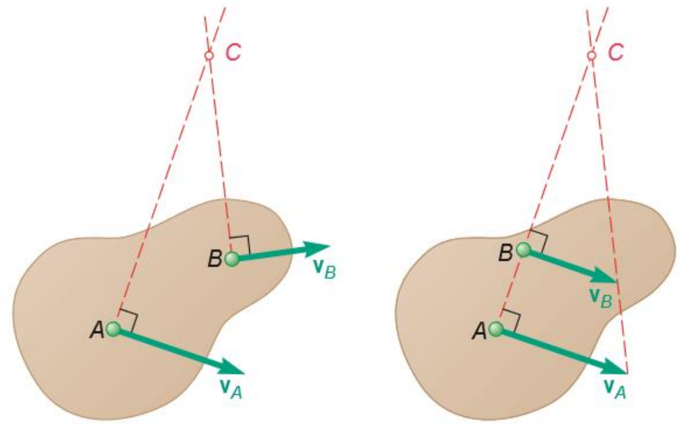
Kinematics of Rigid Body – Velocity…

- At any given instant, there exist a point at which the rigid body appears to be rotating about. This point is called the *instantaneous center of rotation* (*I.C.R*) in plane motion.

- It can be determined if $\vec{v}_A$ and ω are given.



- It can also be determined if two velocities on the rigid body were defined.

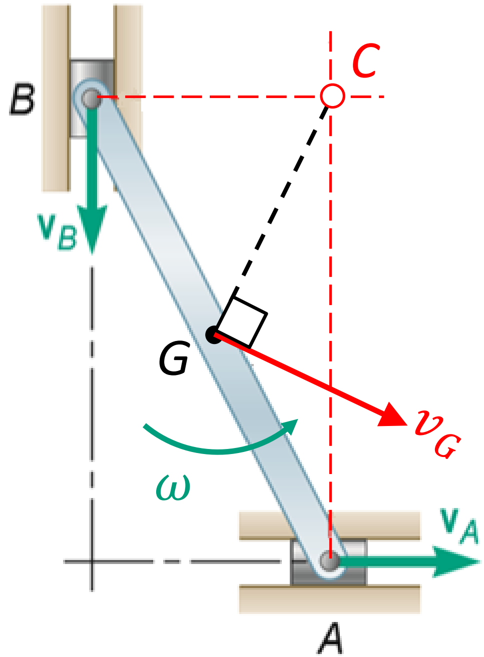


- Once it is found, velocity at any other point can be found as

$$\vec{v}_B = \vec{\omega} \times \vec{r}_{B/C}$$

Kinematics of Rigid Body – Velocity…

➢ This graphical approach provides an alternate way to solve for velocity, instead of using the vector equation. For example,



Using velocity equation,

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

And then

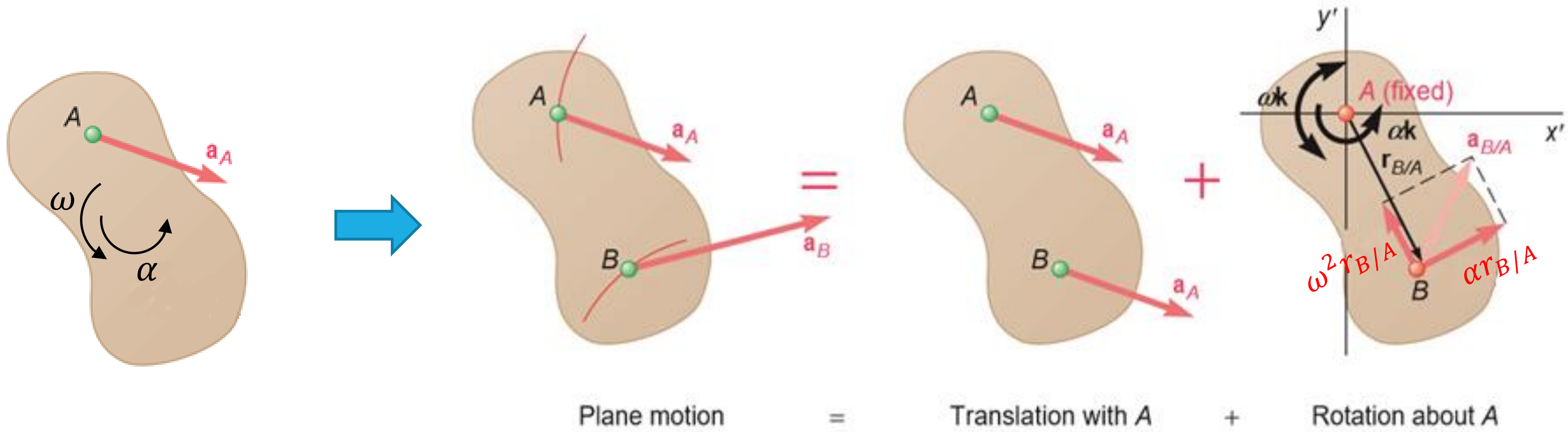
$$\vec{v}_G = \vec{v}_B + \vec{\omega} \times \vec{r}_{G/B}, \ \ or$$

$$\vec{v}_G = \vec{v}_A + \vec{\omega} \times \vec{r}_{G/A}$$

Based on instantaneous center of rotation

$$\vec{v}_G = \vec{\omega} \times \vec{r}_{G/C}$$

Kinematics of Rigid Body – Acceleration



- At the given instant, the accelerations of any two points *A* and *B* on the rigid body are related by the following relation:

$$\boxed{\vec{a}_B = \vec{a}_A + \underbrace{\vec{\alpha} \times \vec{r}_{B/A}}_{\text{Tangential to } \vec{r}_{B/A}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})}_{\text{Centripetal}\left(-\omega^2 \vec{r}_{B/A} \;\; or \;\; \omega^2 \vec{r}_{A/B}\right)}}$$

Note: The concept of instantaneous center of rotation *CANNOT* be applied for acceleration.

Vector Calculus

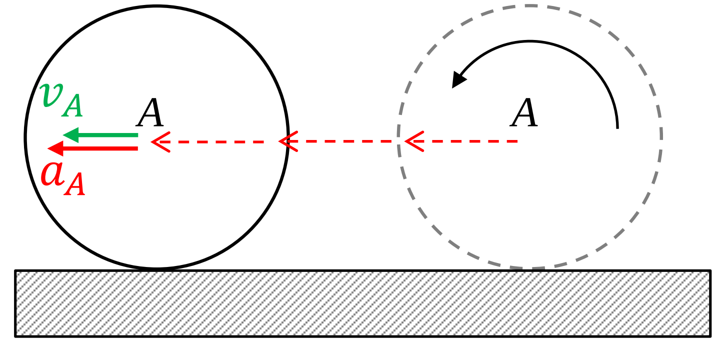
- Given three vectors, $\vec{p} = p_x\vec{\imath} + p_y\vec{\jmath}$; $\vec{q} = q_x\vec{\imath} + q_y\vec{\jmath}$; and $\vec{r} = r\vec{k}$.

Summary of vector operations for dynamics in *2D planar motion*

Vector addition	$\vec{p} + \vec{q} = \vec{q} + \vec{p} = (p_x + q_x)\vec{\imath} + (p_y + q_y)\vec{\jmath}$
Scalar (Dot) product	$\vec{p}.\vec{q} = \vec{q}.\vec{p} = p_x q_x + p_y q_y$
	$\vec{p}.\vec{r} = \vec{q}.\vec{r} = 0$
Vector (Cross) product	$\vec{k} \times \vec{\imath} = \vec{\jmath}$; $\vec{k} \times \vec{\jmath} = -\vec{\imath}$
	$\vec{p} \times \vec{q} = -\vec{q} \times \vec{p} = (p_x q_y - p_y q_x)\vec{k}$
	$\vec{r} \times \vec{p} = -rp_y\vec{\imath} + rp_x\vec{\jmath}$
	$\vec{p} \times \vec{p} = \vec{q} \times \vec{q} = \vec{r} \times \vec{r} = 0$

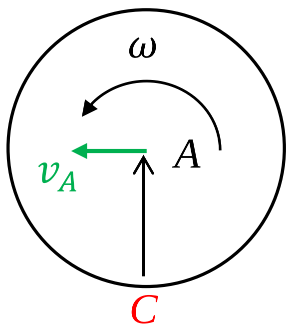
Kinematics of Rigid Body – Rolling on ground

- For a *circular disc* or *sphere* rolling (with or without slipping) on a flat ground, its *center of geometric* is always moving parallel to the ground.



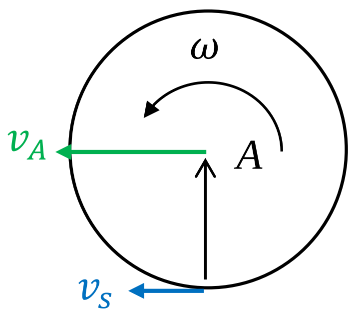
- This means that the velocity and acceleration at its geometric centre A must also be parallel to the ground.

- Suppose it is *rolling without sliding* (slipping), the point in contact with the ground is the *instantaneous centre of rotation*.



In this case,

$$\vec{v}_A = \vec{\omega} \times \vec{r}_{A/C}$$

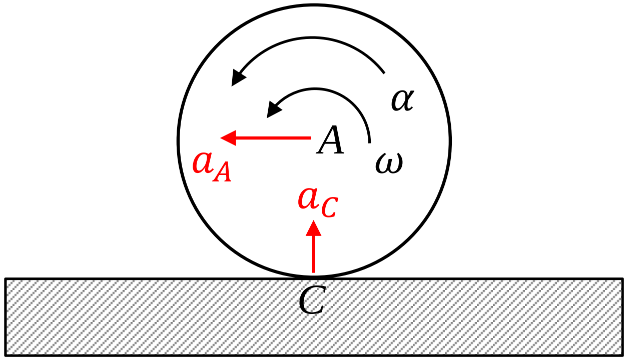


But if the body *slips* at $\vec{v}_s$, then

$$\vec{v}_A = \vec{v}_s + \vec{\omega} \times \vec{r}_{A/C}$$

which means that *$\vec{v}_s$ and $\vec{\omega}$ are independent* quantities.

Kinematics of Rigid Body – Rolling on ground…



- Likewise, in terms of acceleration, we can imposed the relation for *non-slip rolling*

$$\vec{a}_A = \vec{\alpha} \times \vec{r}_{A/C}$$

- For any other point, it is given by

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

- At the point of contact, it will experience centripetal acceleration, as given by

$$\vec{a}_C = \vec{a}_A + \vec{\alpha} \times \vec{r}_{C/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{C/A}\right)$$

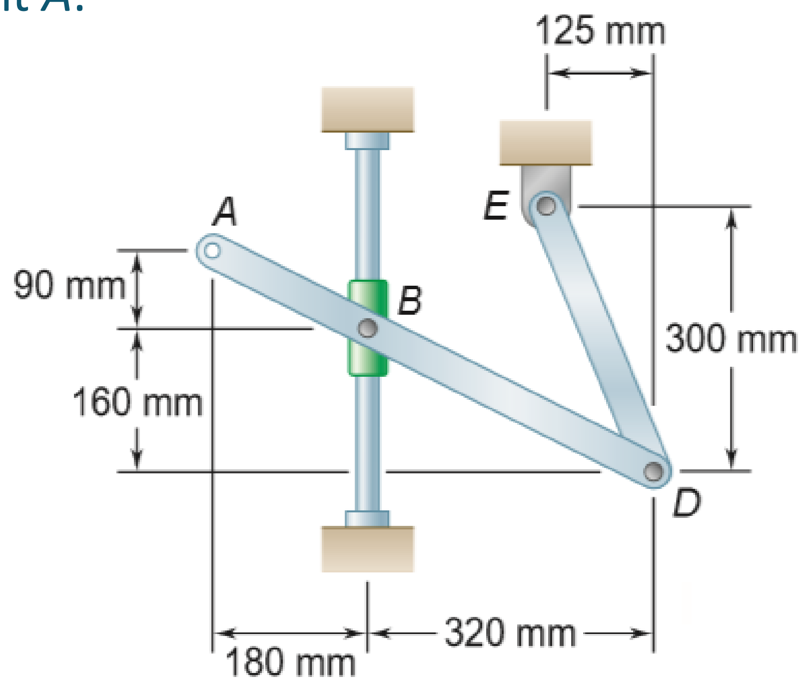
$$\vec{a}_C = \left(\vec{a}_A - \vec{\alpha} \times \vec{r}_{A/C}\right) - \omega^2 \vec{r}_{C/A} = \omega^2 \vec{r}_{A/C}$$

Example 1

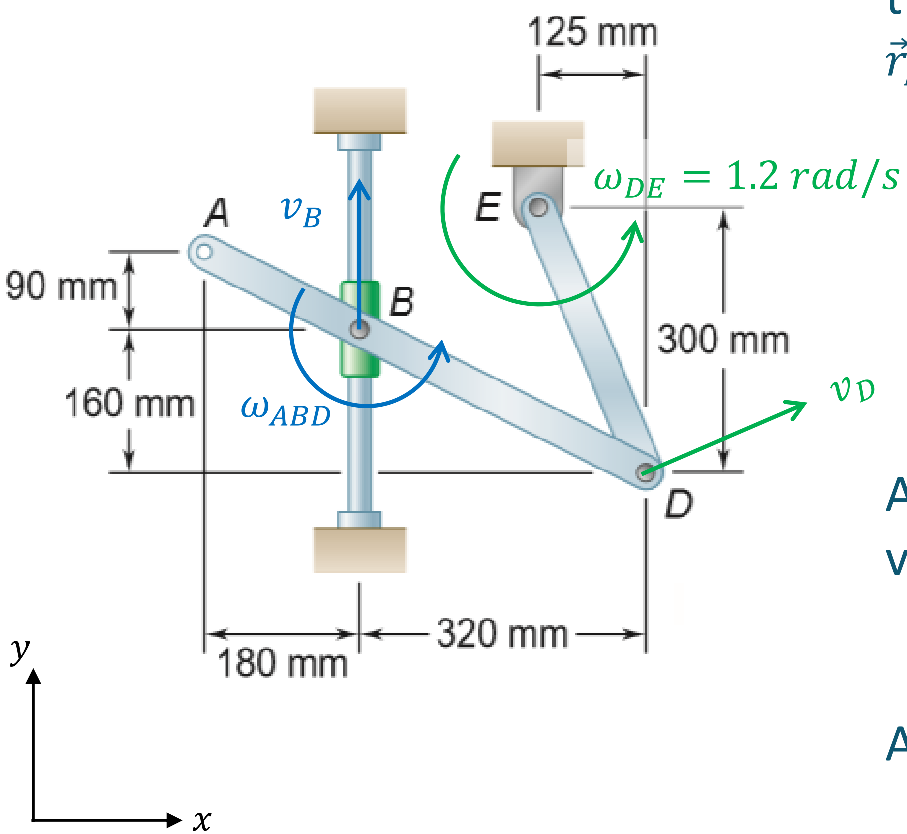
Arm *ABD* is connected by pins to a collar at *B* and to crank *DE*. Knowing that the angular velocity of crank *DE* is 1.2 *rad/s* counter-clockwise, determine at this given instant:

(a) the angular velocity of arm *ABD*, and

(b) the velocity of point *A*.



Example 1…



Since the crank *DE* rotates about a fixed point *E*, the velocity at *D* is expected to be tangential to $\vec{r}_{D/E}$, which is

$$\vec{v}_D = \vec{\omega}_{DE} \times \vec{r}_{D/E}$$

$$= 1.2\vec{k} \times (0.125\vec{\imath} - 0.3\vec{\jmath})$$

$$= 0.36\vec{\imath} + 0.15\vec{\jmath}$$

And the velocity at *B* is constrained to move vertically along the rod, i.e.

$$\vec{v}_B = v_B\vec{\jmath}$$

And angular velocity of rod *ABD* is given by

$$\vec{\omega}_{ABD} = \omega_{ABD}\vec{k}$$

Example 1...

(a) Applying the velocity equation at point *D* and *B*, we have

$$\vec{v}_D = \vec{v}_B + \vec{\omega}_{ABD} \times \vec{r}_{D/B}$$

$$\Rightarrow \vec{v}_D = v_B \vec{j} + \omega_{ABD}\vec{k} \times (0.32\vec{i} - 0.16\vec{j})$$

$$\Rightarrow 0.36\vec{i} + 0.15\vec{j} = 0.16\omega_{ABD}\vec{i} + (v_B + 0.32\omega_{ABD})\vec{j}$$

Now, consider the *i*- and *j*- components separately, gives

$$0.36 = 0.16\omega_{ABD} \;\Rightarrow \omega_{ABD} = \mathbf{2.25}\, rad/s \qquad (\vec{i} - component)$$

$$0.15 = v_B + 0.32\omega_{ABD} \;\Rightarrow v_B = -0.57\, m/s \quad (\vec{j} - component)$$

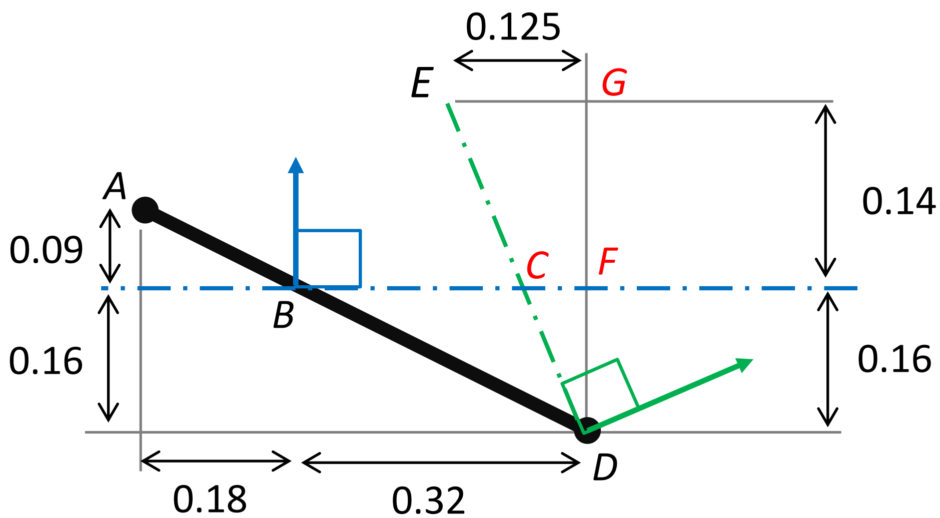
(b) Finally, the velocity of *A* can be determined as:

$$\vec{v}_A = \vec{v}_B + \vec{\omega}_{ABD} \times \vec{r}_{A/B} = -0.57\vec{j} + \left(2.25\vec{k}\right) \times (-0.18\vec{i} + 0.09\vec{j})$$

$$\Rightarrow \vec{v}_A = \mathbf{-0.2025\vec{i} - 0.975\vec{j}}$$

Example 1…

Alternatively: Based on these velocity information, we can determine the *I.C.R* for rod *ABD*, which reveals the following geometrical information.



Using similar triangles of *EGD* and *CFD*, we have

$$\frac{CF}{EG} = \frac{DF}{DG}$$

$$\Rightarrow CF = \frac{0.16}{0.30}(0.125) = 0.06667$$

Hence, position vector of point *D* w.r.t. *C* is

$$\vec{r}_{D/C} = 0.06667\,\vec{\imath} - 0.16\,\vec{\jmath}$$

Example 1…

(a) Now, consider the velocity of *D* from *ABD*, which is

$$\vec{v}_D = \vec{\omega}_{ABD} \times \vec{r}_{D/C}$$

$$\vec{v}_D = \omega_{ABD}\vec{k} \times (0.06667\,\vec{\imath} - 0.16\,\vec{\jmath})$$

$$\Rightarrow 0.36\vec{\imath} + 0.15\vec{\jmath} = 0.16\omega_{ABD}\vec{\imath} + 0.06667\omega_{ABD}\vec{\jmath}$$

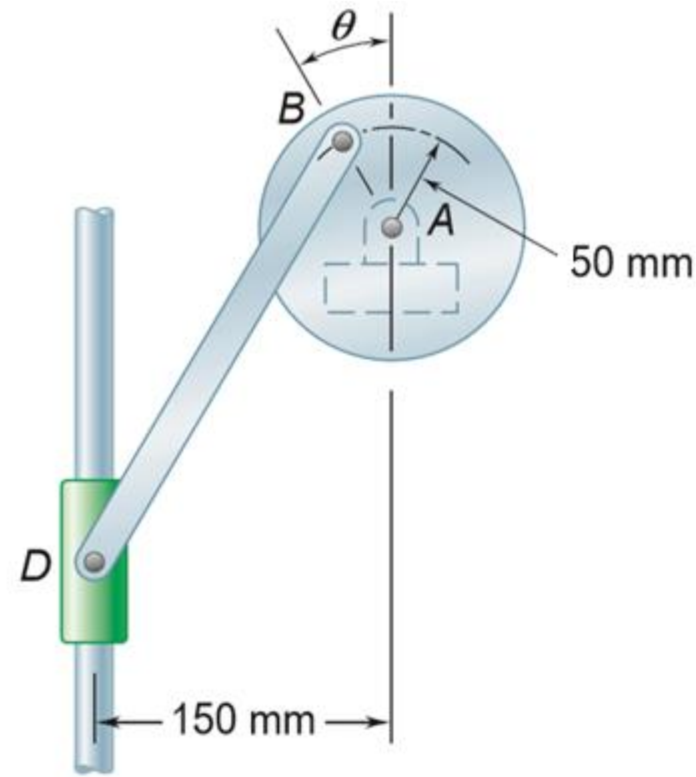
$$\Rightarrow \begin{array}{l} \omega_{ABD} = \dfrac{0.36}{0.16} = \mathbf{2.25}\, rad/s \quad (\vec{\imath} - component), or \\ \omega_{ABD} = \dfrac{0.15}{0.06667} = \mathbf{2.25}\, rad/s, \ \ (\vec{\jmath} - component) \end{array}$$

(b) The velocity of *A* can be determined as:

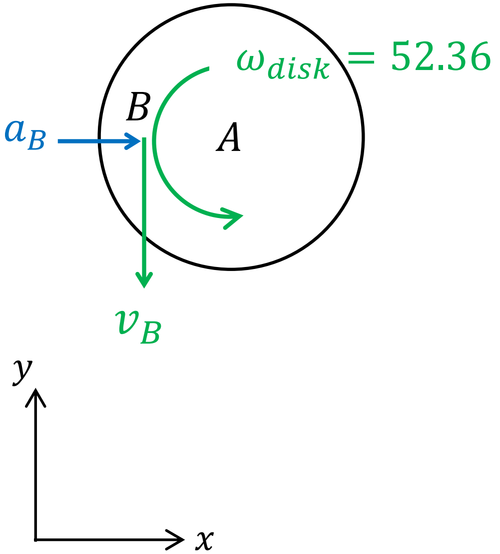
$$\begin{aligned} \vec{v}_A &= \vec{\omega}_{ABD} \times \vec{r}_{A/C} \\ &= \left(2.25\vec{k}\right) \times (-(0.5 - 0.06667)\vec{\imath} + 0.09\vec{\jmath}) \\ &= \mathbf{-0.2025\vec{\imath} - 0.975\vec{\jmath}} \end{aligned}$$

Example 2

The disk shown has a constant angular velocity of 500 *rpm* counter-clockwise. Knowing that rod *BD* is 250 *mm* long, determine the acceleration of collar *D* when θ = 90°.



Example 2…



Consider the disk rotating anti-clockwise at a *constant* angular velocity of 500 *rpm*, which means

$$\vec{\omega}_{disk} = 500\left(\frac{2\pi}{60}\right)\vec{k} = 52.36\,\vec{k}\,rad/s\,,$$

and

$$\vec{\alpha}_{disk} = 0\,\vec{k}$$

Since the disk is fixed to rotate about *A*, this gives

$$\vec{v}_B = \vec{\omega}_{disk} \times \vec{r}_{B/A} = -\omega_{disk} r_{B/A}\,\vec{j}$$

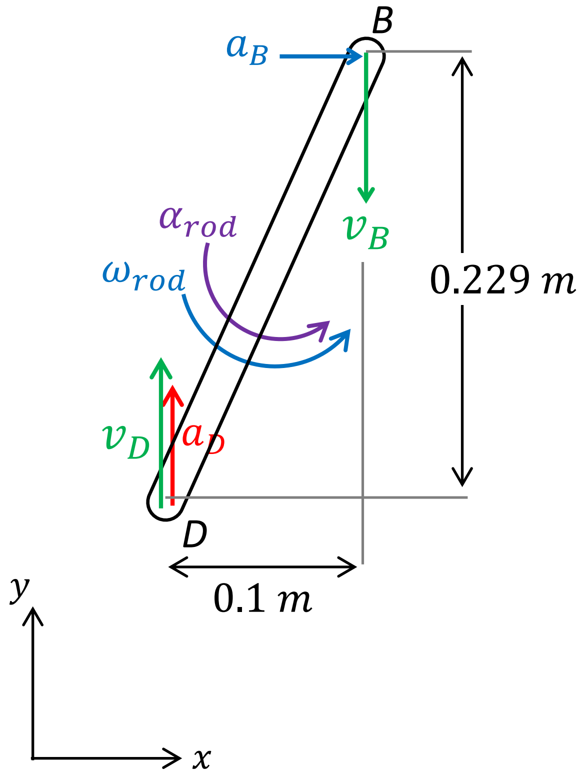
$$= -(52.36)(0.05)\vec{j} = -2.618\vec{j}$$

And the acceleration at *B* is only due to the centripetal component, i.e.

$$\vec{a}_B = \vec{\alpha}_{disk} \times \vec{r}_{B/A} + \vec{\omega}_{disk} \times \left(\vec{\omega}_{disk} \times \vec{r}_{B/A}\right) = \omega_{disk}{}^2 r_{B/A}\,\vec{i}$$

$$= (52.36)^2(0.05) = 137.08\,\vec{i}$$

Example 2...



Now, consider the kinematics of rod *BD*.

Since collar *D* is guided to slide along the vertical rod, hence its velocity $\vec{v}_D$ and acceleration $\vec{a}_D$ can be expressed as $v_D\,\vec{j}$ and $a_D\,\vec{j}$, respectively.

Based on the velocity analysis of point *B* and *D*, we have

$$\vec{v}_D = \vec{v}_B + \vec{\omega}_{rod} \times \vec{r}_{D/B}$$

$$\Rightarrow v_D\,\vec{j} = -2.618\vec{j} + \omega_{rod}\vec{k} \times (-0.1\vec{i} - 0.229\vec{j})$$

$$= 0.229\omega_{rod}\vec{i} - (2.618 + 0.229\omega_{rod})\vec{j}$$

From the *i*-component equation, we see that $\omega_{rod} = 0$.

Note that you can also infer this by looking at the ICR, which does not intercept and hence implies $\omega_{rod} = 0$.

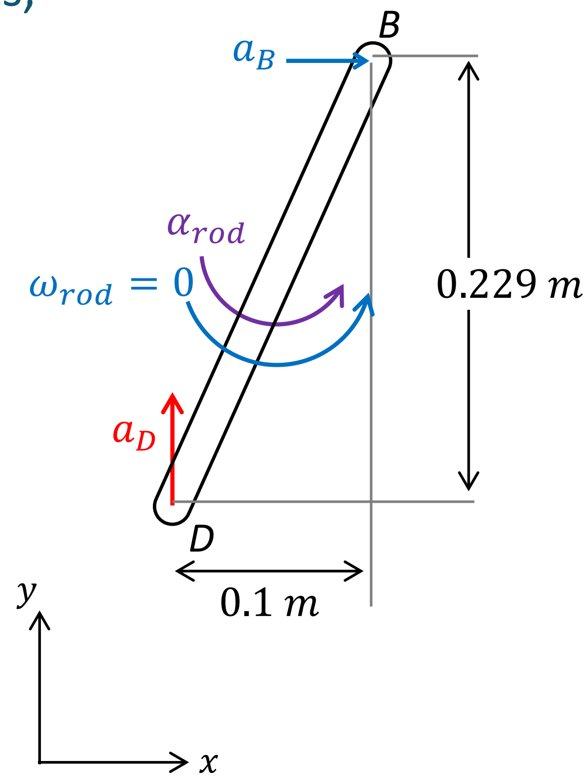
Example 2...

Now, looking at the acceleration equation at the two points, we have

$$\vec{a}_D = \vec{a}_B + \vec{\alpha}_{rod} \times \vec{r}_{D/B} + \vec{\omega}_{rod} \times \left(\vec{\omega}_{rod} \times \vec{r}_{D/B}\right)$$

$$\Rightarrow a_D\,\vec{j} = 137.08\,\vec{\imath} + \alpha_{rod}\vec{k} \times (-0.1\vec{\imath} - 0.229\vec{j})$$

$$= \ (137.08 + 0.229\alpha_{rod})\vec{\imath} - 0.1\alpha_{rod}\vec{j}$$



Again, by looking at the *i*-component equation, we get

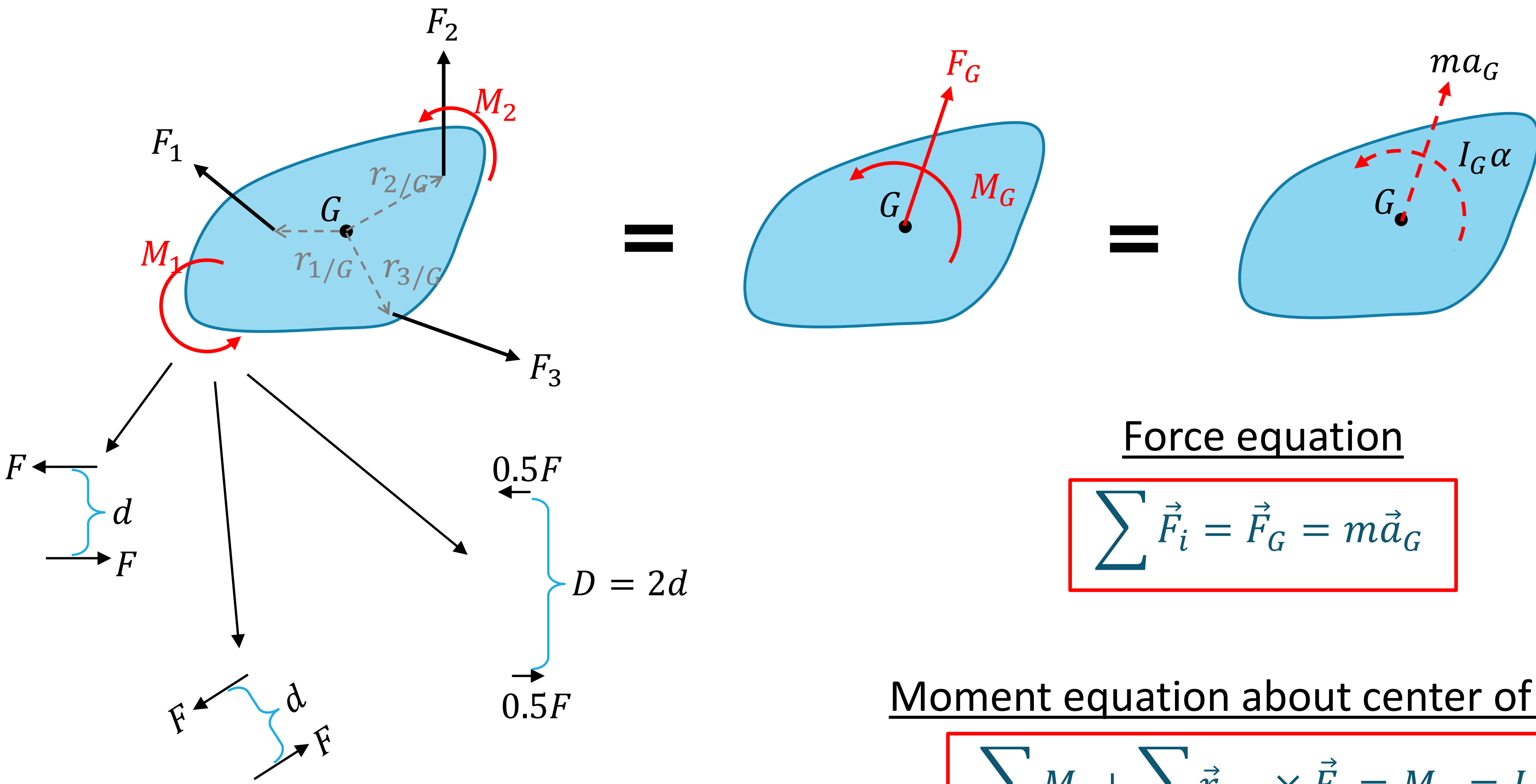
$$137.08 + 0.229\alpha_{rod} = 0$$

$$\Rightarrow \alpha_{rod} = -598.6\, rad/s^2$$

And finally substituting α into the *j*- component equation, gives

$$a_D = -0.1\alpha_{rod} = -0.1(-598.6) = \mathbf{59.9}\ m/s^2$$

Kinetics of Rigid Body - Newton's 2nd Law



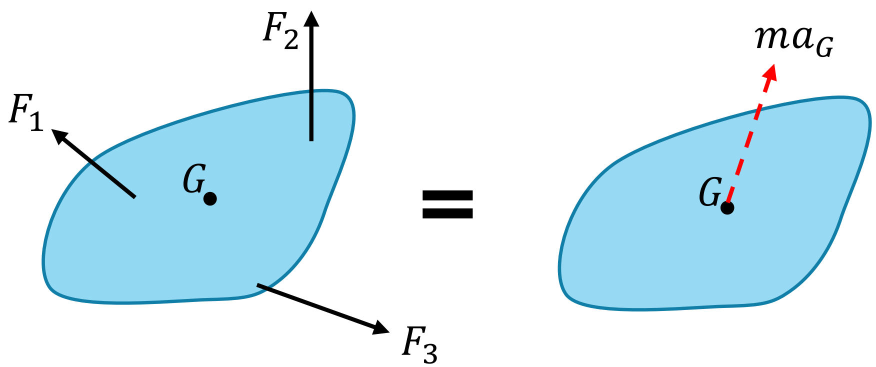
Force equation

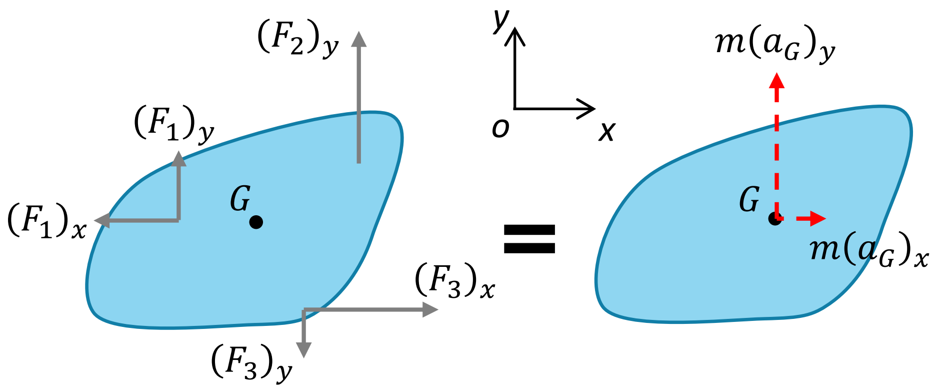
$$\boxed{\sum \vec{F}_i = \vec{F}_G = m\vec{a}_G}$$

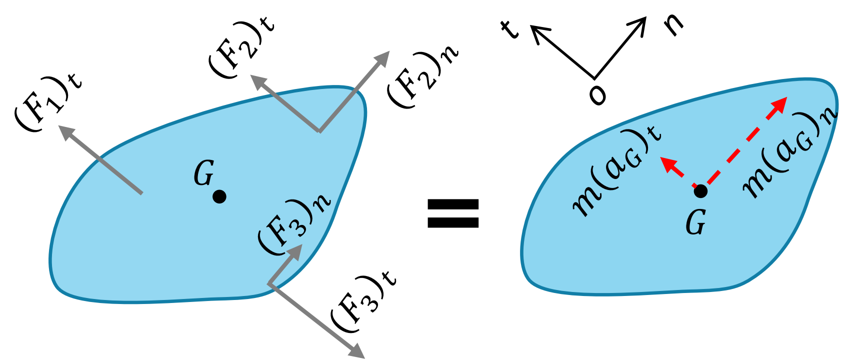
Moment equation about center of mass

$$\boxed{\sum M_i + \sum \vec{r}_{i/G} \times \vec{F}_i = M_G = I_G\alpha}$$

Newton's 2nd Law – Force equation





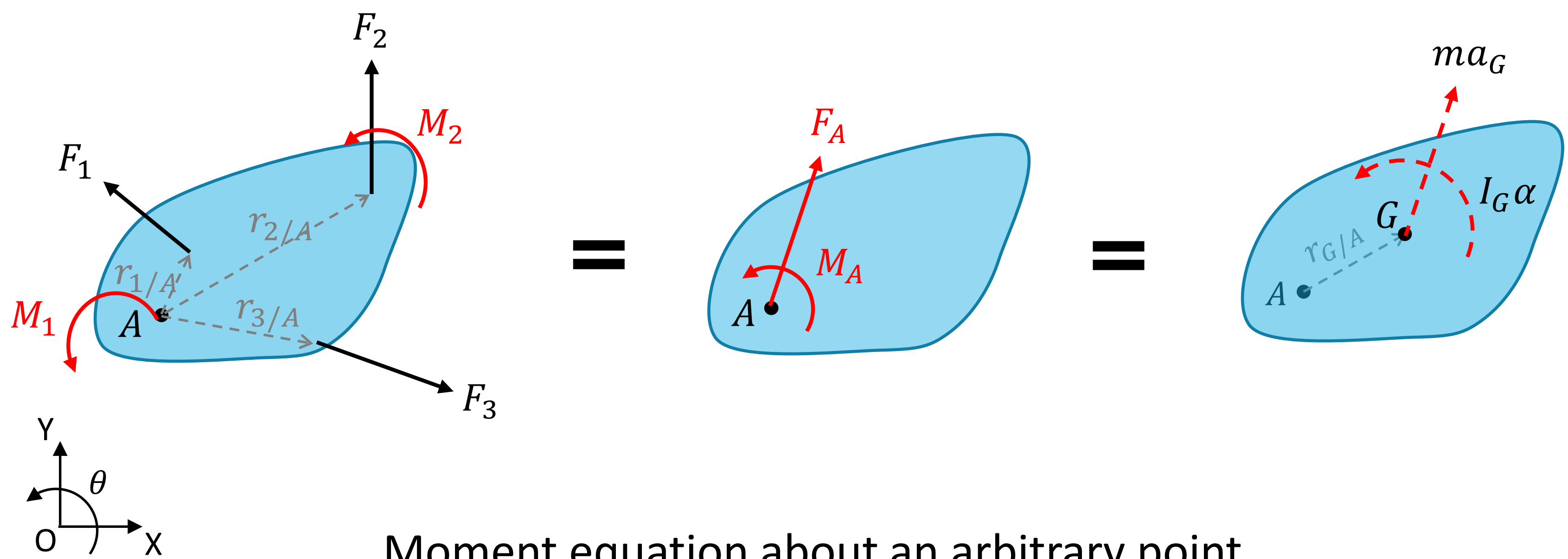


$$\sum(\vec{F}_i)_x = m(\vec{a}_G)_x, and \sum(\vec{F}_i)_y = m(\vec{a}_G)_y$$

$$\sum(\vec{F}_i)_n = m(\vec{a}_G)_n, and \sum(\vec{F}_i)_t = m(\vec{a}_G)_t$$

Newton's 2nd Law – Moment equation

Taking moment about *A*,



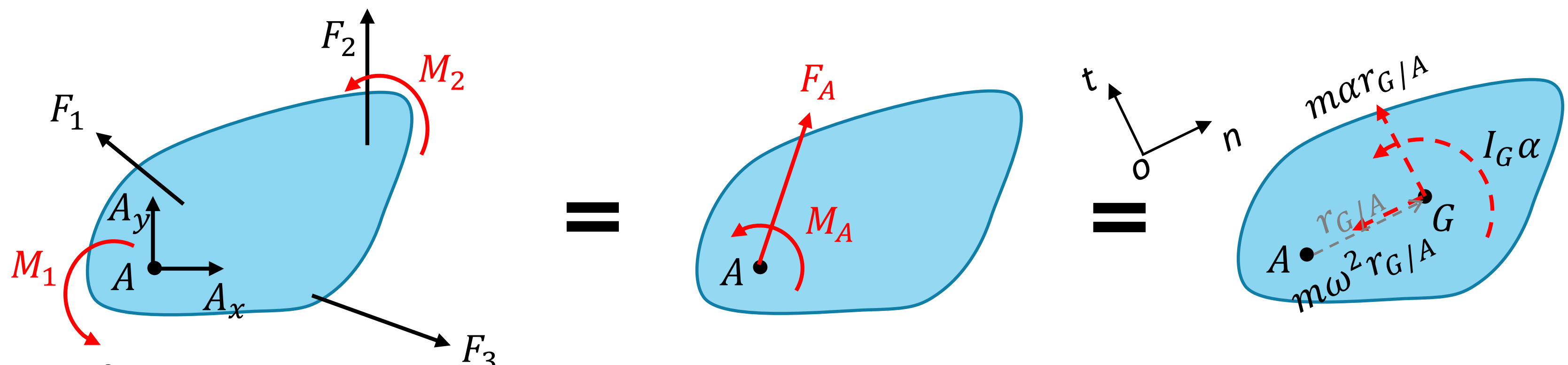
<u>Moment equation about an arbitrary point</u>

$$\boxed{\sum M_i + \sum \vec{r}_{i/A} \times \vec{F}_i = M_A = I_G\vec{\alpha} \ + \ \vec{r}_{G/A} \times m\vec{a}_G}$$

Note that $I_G\vec{\alpha}$ is always presented on the RHS, which is independent on where the moment is taken about.

Newton's 2nd Law – Moment equation…

Suppose the body rotates about a *fixed axis* at A, then the acceleration at the mass centre $\vec{a}_G$ can be expressed in the normal and tangent coordinate.



Reaction force at *A* arises
to constraint its motion

<u>Moment equation about a fixed point</u>

$$M_A = I_G\vec{\alpha} + \vec{r}_{G/A} \times m\vec{a}_t = I_G\vec{\alpha} + \left(r_{G/A}\vec{i'}\right) \times \left(m\alpha r_{G/A}\vec{j'}\right)$$

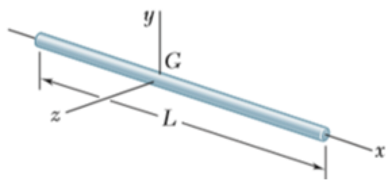
$$\boxed{M_A = \underbrace{\left(I_G + m{r_{G/A}}^2\right)}_{\text{Parallel-axis theorem}}\vec{\alpha} = I_A\vec{\alpha}}$$

Moment of Inertia

- *Centriodal Moment of inertia* I_G measures the extent to which an object resists rotational acceleration about its *centre of mass (C.M).*

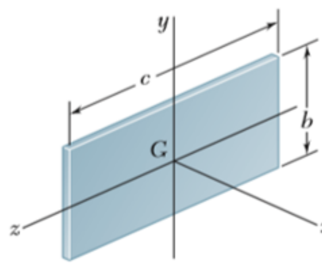
Slender rod,

$$I_y = I_z = \frac{1}{12}mL^2$$



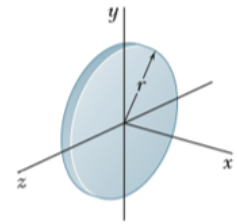
Thin rectangular plate,

$$I_x = \frac{1}{12}m(b^2 + c^2)$$
$$I_y = \frac{1}{12}mc^2$$
$$I_z = \frac{1}{12}mb^2$$



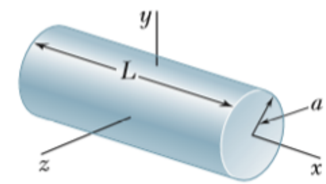
Thin disk,

$$I_x = \frac{1}{2}mr^2$$
$$I_y = I_z = \frac{1}{4}mr^2$$



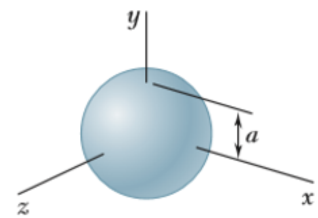
Circular cylinder,

$$I_x = \frac{1}{2}ma^2$$
$$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$$



Sphere,

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

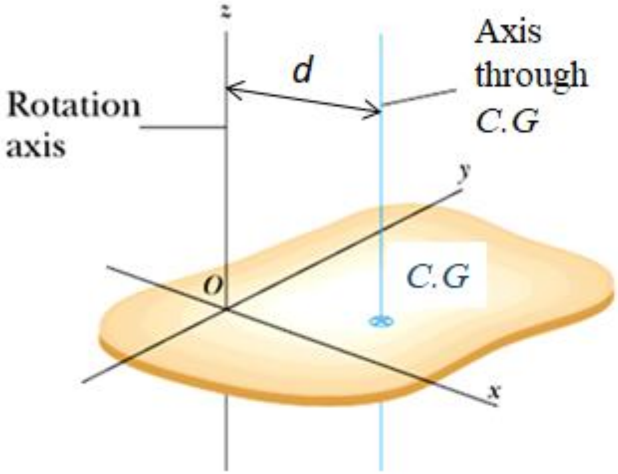


- In general, it can be expressed as,

$$I_G = mk^2$$

where k is the *radius of gyration*.

Moment of inertia – Parallel-axis theorem

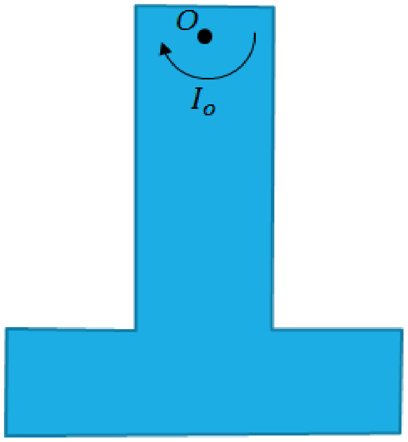


➢ *Parallel-axis theorem* is given by

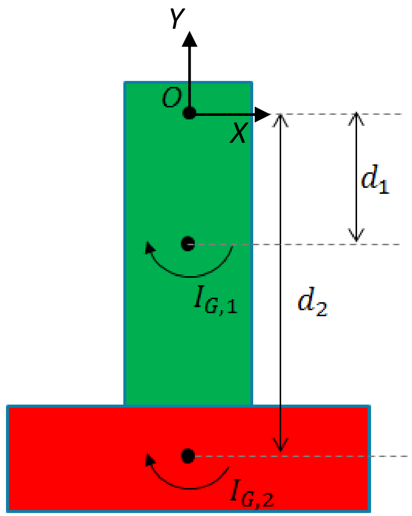
$$I_O = I_G + md^2$$

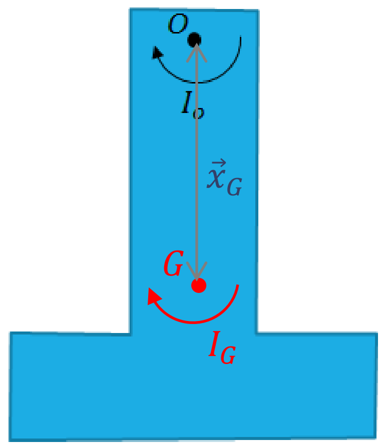
where d is the perpendicular distance between the two axes.

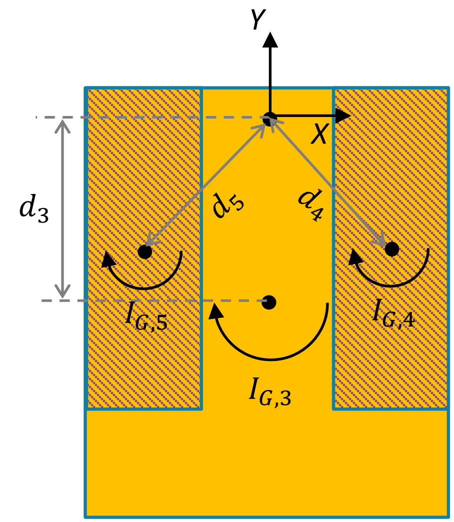
➢ *Parallel-axis theorem* can be used to calculate the moment of inertia of more complex bodies that are made up of simple geometries with known *C.M.s* and their respective centroidal moment of inertia.



Moment of inertia – Parallel-axis theorem…







$$\vec{x}_G = \frac{M_1\vec{x}_{G,1} + M_2\vec{x}_{G,2}}{M_1 + M_2}$$

$$I_O = \left(I_{G,1} + m_1 d_1^2\right) + \left(I_{G,2} + m_2 d_2^2\right)$$

$$I_G = I_O - m|\vec{x}_G|^2$$

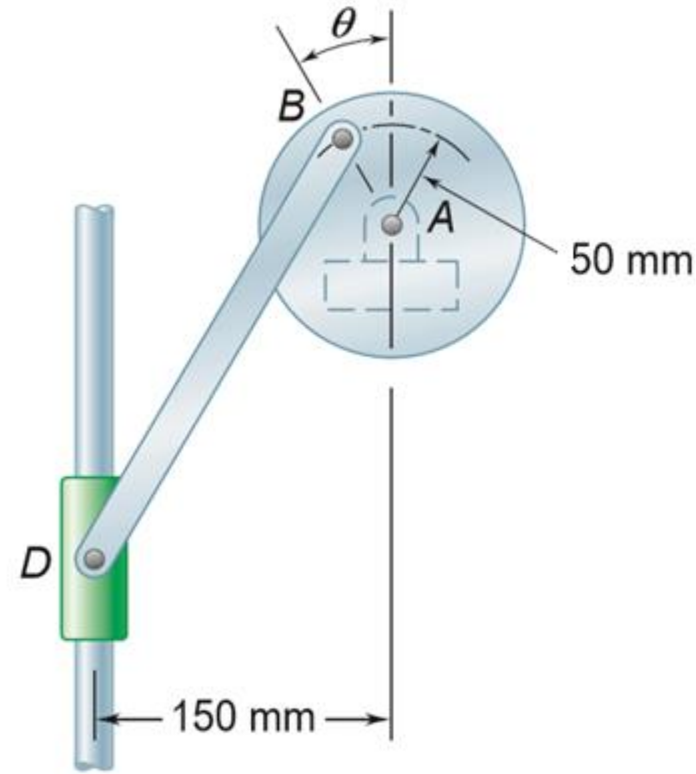
$$\vec{x}_G = \frac{M_3\vec{x}_{G,3} - M_4\vec{x}_{G,4} - M_5\vec{x}_{G,5}}{M_3 - M_4 - M_5}$$

$$I_O = \left(I_{G,3} + m_3 d_3^2\right) - \left(I_{G,4} + m_4 d_4^2\right)$$

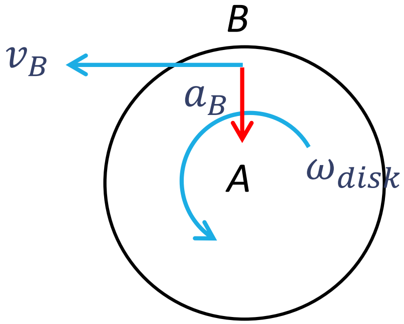
$$-\left(I_{G,5} + m_5 d_5^2\right)$$

Example 3

The 250-mm uniform rod *BD*, of mass 5 *kg*, is connected as shown to disk *A* and to a collar of negligible mass, that may slide freely along a vertical rod. Knowing that disk *A* rotates counter-clockwise at a constant rate of 500 *rpm*, determine the reactions at *D* when $\theta = 0$.



Example 3...

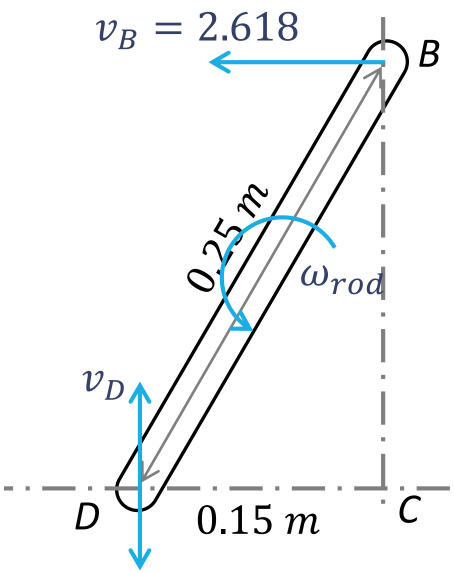


Kinematics analysis of the disk.

Given that $\omega_{disk} = 500\,rpm = 52.36\,rad/s,$ and $\alpha_{disk} = 0,$ we get

$$\vec{v}_B = \vec{\omega}_{disk} \times \vec{r}_{B/A} = 52.36\vec{k} \times 0.05\vec{j} = -2.618\vec{i}$$

$$\vec{a}_B = -(\omega_{disk})^2\vec{r}_{B/A} = -52.36^2(0.05\vec{j}) = -137.08\vec{j}$$



Velocity analysis of the rod, using *I.C.R*, gives

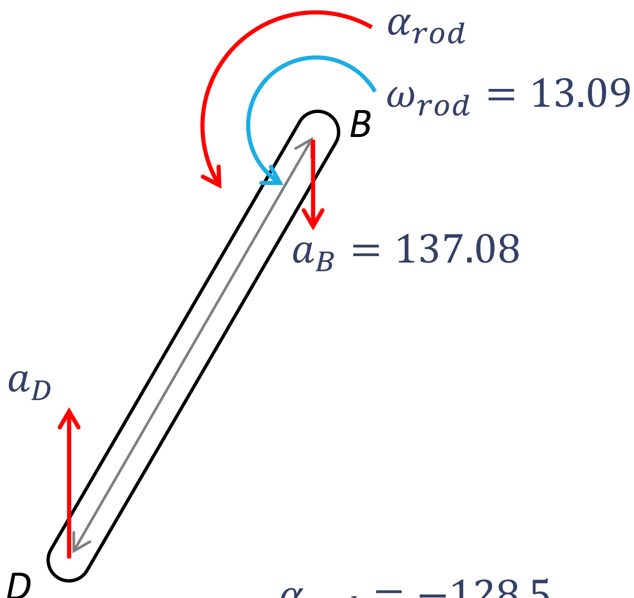
$$BC = \sqrt{0.25^2 - 0.15^2} = 0.20\,m$$

$$\vec{v}_B = \vec{\omega}_{rod} \times \vec{r}_{B/C}$$

$$\Rightarrow -2.618\vec{i} = \omega_{rod}\vec{k} \times 0.20\vec{j}$$

$$\Rightarrow \omega_{rod} = 13.09\,rad/s$$

Example 3…



Acceleration analysis of the rod ,

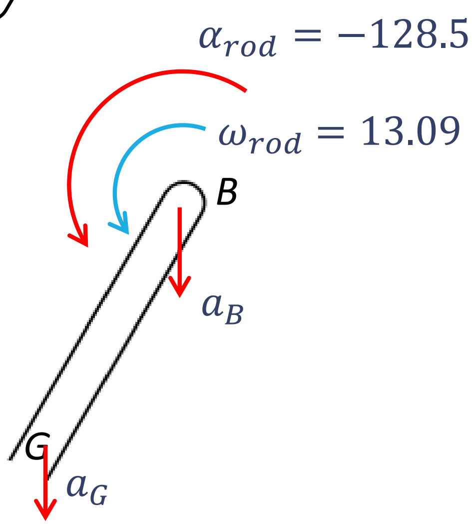
$$\vec{a}_D = \vec{a}_B + \left(\vec{\alpha}_{rod} \times \vec{r}_{D/B}\right) - \omega_{rod}{}^2 \vec{r}_{D/B}$$

$$\Rightarrow a_D\vec{j} = -137.08\vec{j} + \alpha_{rod}\vec{k} \times (-0.15\vec{i} - 0.20\vec{j})$$

$$-(13.09)^2(-0.15\vec{i} - 0.20\vec{j})$$

$$\Rightarrow a_D\vec{j} = \underbrace{(0.2\alpha_{rod} + 25.7)}\vec{i} + (-102.8 - 0.15\alpha_{rod})\vec{j}$$

$$0.2\alpha_{rod} + 25.7 = 0 \Rightarrow \alpha_{rod} = -128.5\, rad/s^2$$



The acceleration at the mass centre of the rod *G* is

$$\vec{a}_G = \vec{a}_B + \left(\vec{\alpha}_{rod} \times \vec{r}_{G/B}\right) - \omega_{rod}{}^2 \vec{r}_{G/B}$$

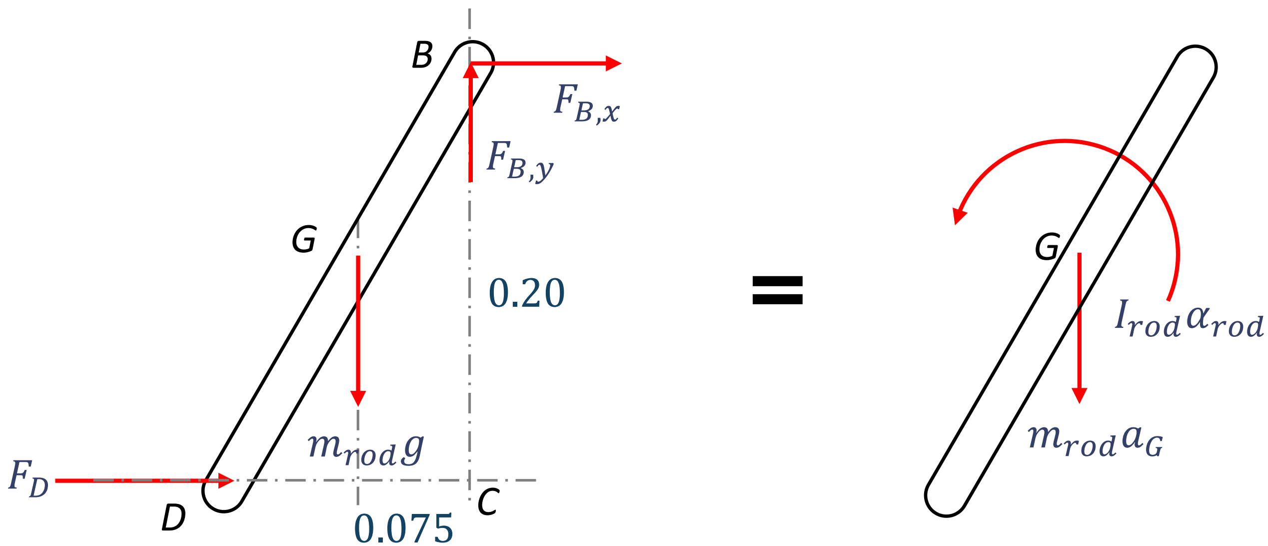
$$= -137.08\vec{j} - 128.5\vec{k} \times (-0.075\vec{i} - 0.10\vec{j})$$

$$-(13.09)^2(-0.075\vec{i} - 0.10\vec{j})$$

$$\Rightarrow \vec{a}_G = -110.3\vec{j}$$

Example 3…

Kinetic analysis of the rod,



From the *FBD*, and taking moment about *B*, we get

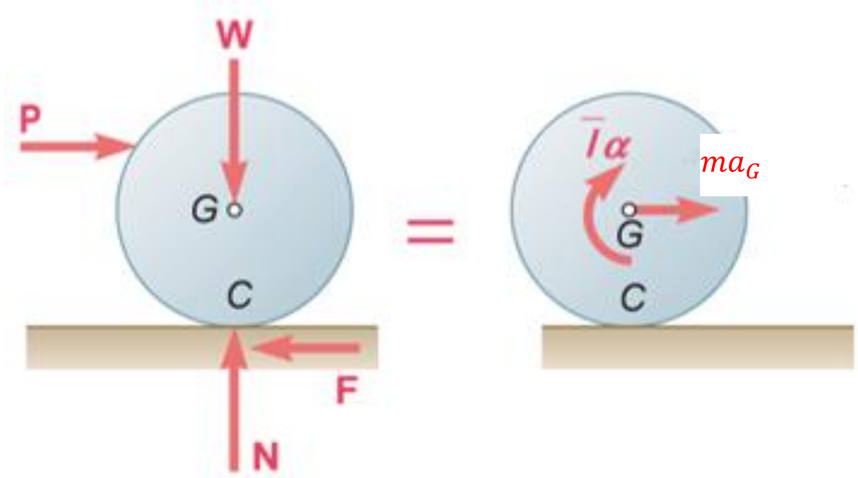
$$F_D(BC) + m_{rod}g(0.075) = I_{rod}\alpha_{rod} + m_{rod}a_G(0.075)$$

$$\Rightarrow 0.2F_D + 5(9.81)(0.075) = \left(\frac{1}{12}(5)(0.25)^2\right)(-128.5) + (5)(110.3)(0.075)$$

$$\Rightarrow F_D = \mathbf{172}\ N$$

Kinetics of Rolling Body

- For a circular body in rolling motion,



Rolling without sliding:

$$F \leq \mu_s N, \text{ and } \vec{a}_G = \vec{\alpha} \times \vec{r}_{G/C}$$

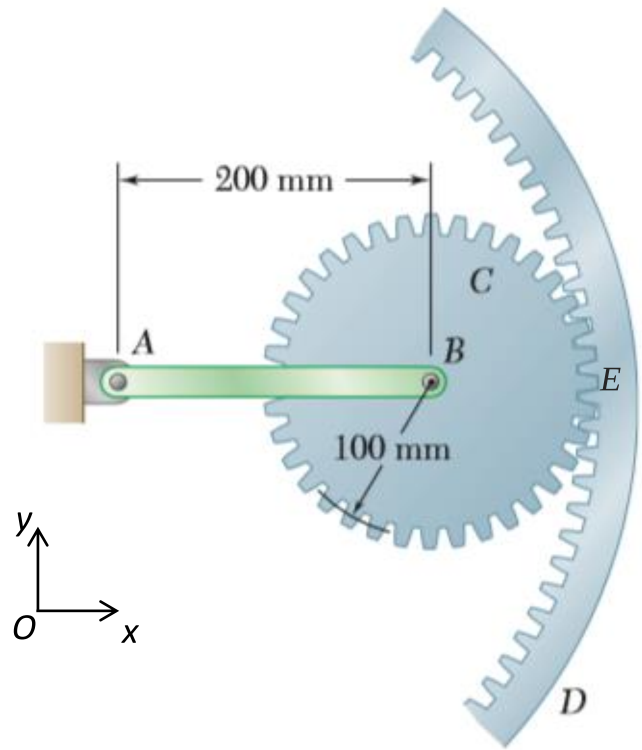
Rotating and sliding:

$F = \mu_k N$, but $\vec{a}_G$ and $\vec{\alpha}$ are ***independent***.

where μ_s and μ_k are static and kinetic friction coefficients, respectively.

Example 4

Gear C has a mass of 5 kg and a centroidal radius of gyration of 75 mm. The uniform bar AB has a mass of 3 kg and gear D is stationary. If the system is released from rest in the position shown, determine the angular acceleration of gear C.

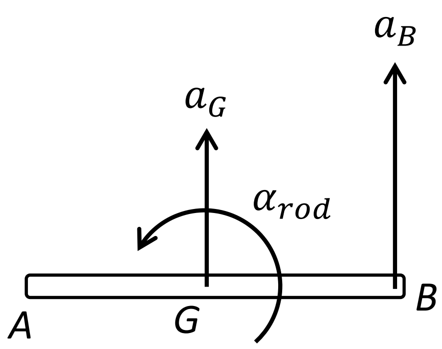


$g = 9.8\ m/s^2$

Example 4…

Consider the kinematics of the system.

Acceleration analysis of the bar *AB*.



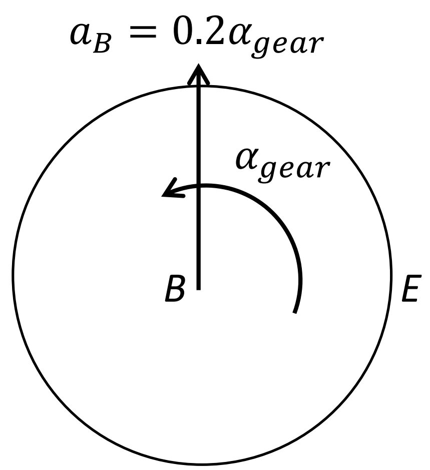
$$\vec{a}_B = \vec{\alpha}_{rod} \times \vec{r}_{B/A}$$

$$\Rightarrow \vec{a}_B = \left(\alpha_{rod}\vec{k}\right) \times (0.2\vec{\imath}\,) = 0.2\alpha_{rod}\,\vec{\jmath}$$

and

$$\vec{a}_G = \left(\alpha_{rod}\vec{k}\right) \times (0.1\vec{\imath}) = 0.1\alpha_{rod}\vec{\jmath}$$

Acceleration analysis of the gear *C*.



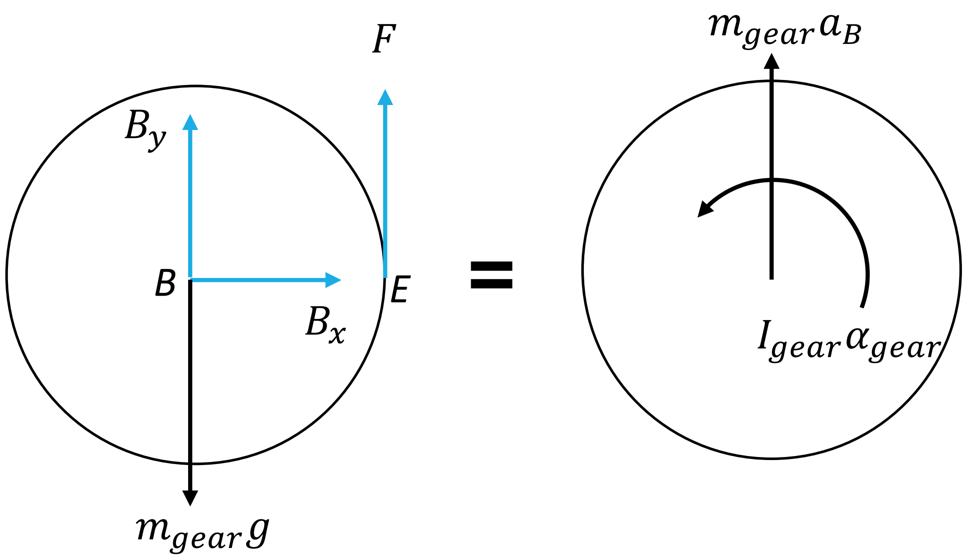
$$\vec{a}_E = \vec{a}_B + \left(\vec{\alpha}_{gear} \times \vec{r}_{E/B}\right)$$

$$\Rightarrow 0 = 0.2\alpha_{rod}\,\vec{\jmath} + \left(\alpha_{gear}\,\vec{k}\right) \times (0.1\,\vec{\imath})$$

$$\Rightarrow \alpha_{rod} = -\frac{1}{2}\alpha_{gear}$$

Example 4…

Kinetic analysis of gear C.



Force in x-direction,

$$B_x = 0$$

Force in y-direction,

$$B_y + F - m_{gear}g = m_{gear}a_B$$

$$\Rightarrow B_y + F = m_{gear}(g + 0.2\alpha_{rod})$$

Taking moment about B, gives

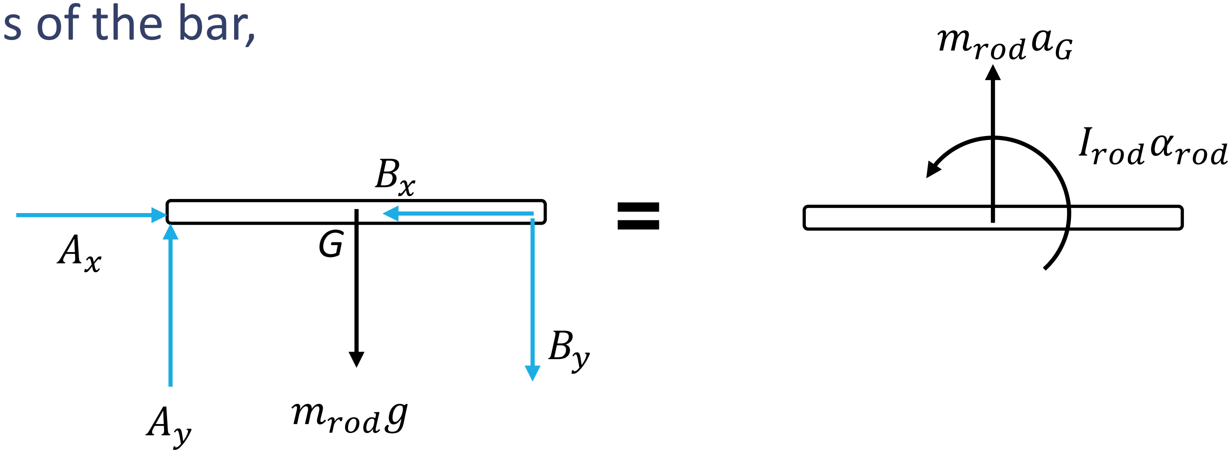
$m_c k^2$

$$F(BE) = I_{gear}\alpha_{gear} \Rightarrow F = \frac{(5)(0.075)^2}{0.1}\alpha_{gear}$$

$$\Rightarrow F = 0.28125\alpha_{gear}$$

Example 4...

Kinetics analysis of the bar,



Force in *x*-direction,

$$A_x - B_x = 0 \Rightarrow A_x = 0$$

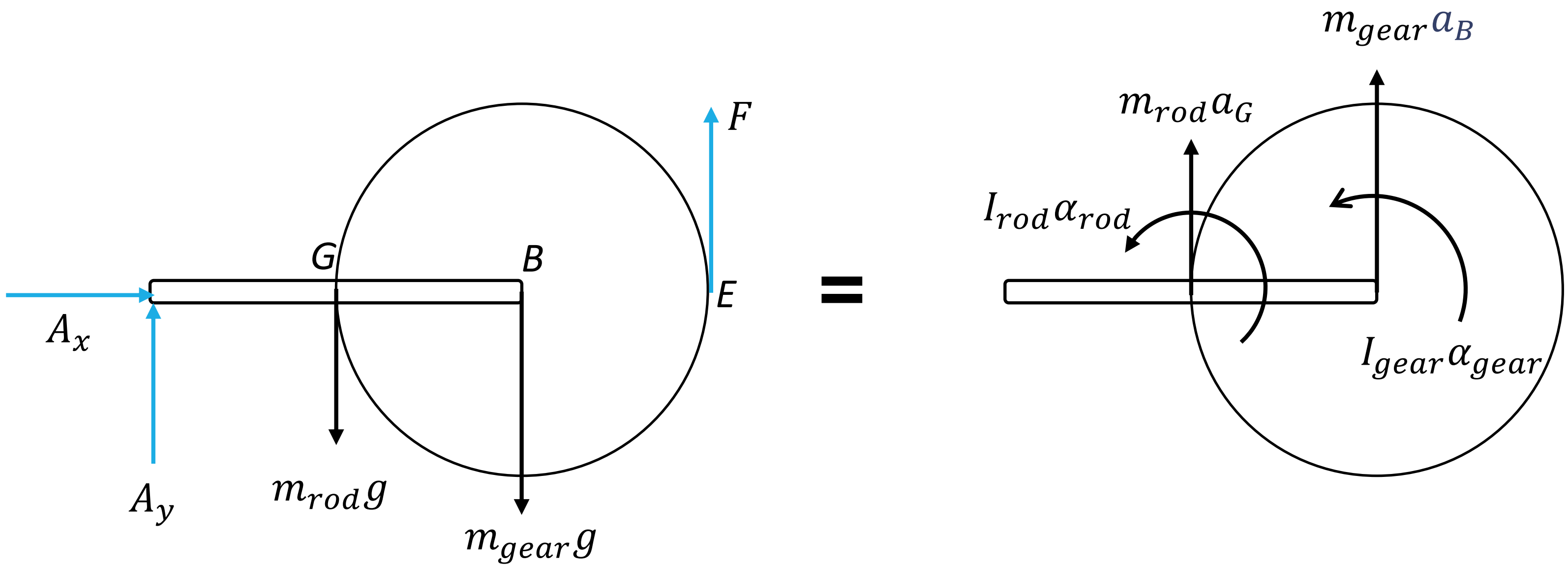
Force in *y*-direction,

$$A_y - B_y - m_{rob}g = m_{rod}a_G$$
$$\Rightarrow A_y - B_y = m_{rod}(g + 0.1\alpha_{rod})$$

Moment equation would involve reaction force at either *A* and/or *B*.

Example 4…

Kinetics analysis of the bar and gear sub-system,



Taking the moment about *A*,

$$F(AE) - m_{rod}g(AG) - m_{gear}g(AB) = (m_{rod}a_G)(AG) + I_{rod}\alpha_{rod}$$

$$+(m_{gear}a_B)(AB) + I_{gear}\alpha_{gear}$$

Example 4...

Using the numeric and further evaluating it, gives

$$0.3F - (3)(9.81)(0.1) - (5)(9.81)(0.2) = 3(0.1\alpha_{rod})(0.1) + \left(\frac{1}{12}(3)(0.2)^2\right)\alpha_{rod}$$

$$+ 5(0.2\alpha_{rod})(0.2) + \left((5)(0.075)^2\right)\alpha_{gear}$$

$$\Rightarrow 0.3F - 12.753 = 0.24\left(-\frac{1}{2}\alpha_{gear}\right) + 0.028125\alpha_{gear}$$

$$\Rightarrow 0.3F - 12.753 = -0.09188\alpha_{gear}$$

Finally, substituting $F = 0.28125\alpha_{gear}$ into the above equation, gives

$$0.3(0.28125\alpha_{gear}) - 12.753 = -0.09188\alpha_{gear}$$

$$\Rightarrow \alpha_{gear} = \mathbf{72.36}\ rad/s^2$$