ME2115/ME2115E/TME2115 -

Mechanics of Machines

Review of vectors

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Why vectors?

Because of complicated 3D space and motion

It is hard to solve 3D problems using trigonometric and graphing methods.

Just like it is convenient and easy to solve a complicated math word problem using algebra instead of the model method.

Mechanical quantities

Basic quantities: **SI unit**

r	*space*	Meter (m)
m	*mass*	Kilogram (kg)
t	*time*	Second (s)
F	*force*	Newton (N)

→ In Newtonian mechanics, these are absolute quantities, independent of each other. But force is dependent on the three.

$1\ N = 1\ Kg \cdot m/s^2$

Other derived quantities:

Velocity, speed, acceleration, moment, momentum, work, kinetic energy, angular velocity, angular acceleration, …..

→ These quantities can be derived from basic quantities.

Some of physical quantities possess direction, which are vectors, such as **velocity**.
If a quantity is independent of direction, it is a scalar, such as speed.

? Question: Which physical quantity above is a vector?

Vectors of physical quantities

Take "force" as an example

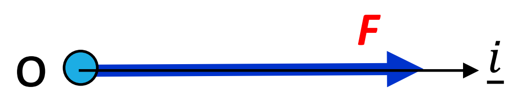
- Force is a **vector.** It possesses both magnitude and direction.

- The symbol of force vector can be ***F*** or $\underline{F}$ or $\vec{F}$
- Four factors of a force vector: magnitude, unit, direction and the point of action
- The direction of a vector can be represented by its **unit vector**, $\underline{\lambda}_F = \underline{F}/F$
  Dimensionless and magnitude is 1. Recall, what are vectors $\underline{i}$ $\underline{j}$ *and* $\underline{k}$?

<u>Answer</u>: they are unit vectors of x, y and z, respectively, representing the direction of x, y, z

- Alternatively, the above force vector can be represented by

F = $1\underline{i}$ N



*Direction of vectors

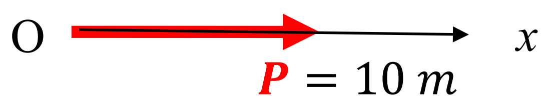
It is worth noting that in the text book and lecture notes,

i) signs are often used to present the direction of vectors in 1D structures

ii) arrows are commonly used in 2D planar structures

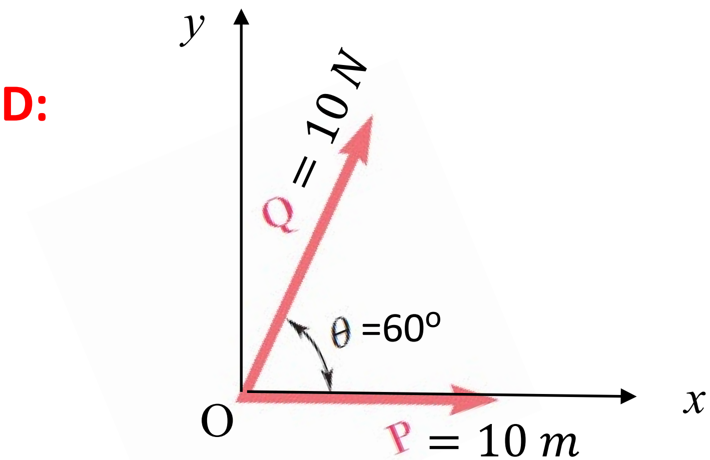
For example:

1D:

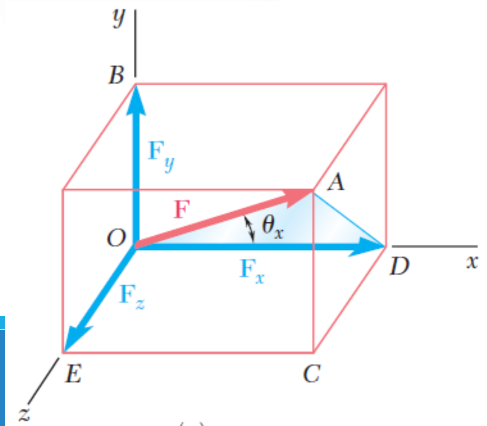


Vector format	Sign/arrow format
In general, $\underline{P} = 10\underline{i}\ m$;	Simply, $P = +10\ m$
In general, $\underline{Q} = 5\underline{i} + 8.66\underline{j}\ N$; $\underline{V} = \underline{P} \times \underline{Q} = 86.6\underline{k}\ Nm$	Simply, $Q = 10\ N \measuredangle 60^{o}$; $\underline{V} = \underline{P} \times \underline{Q} = 86.6\ Nm$

2D:

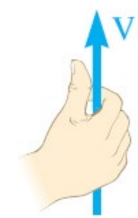


3D:



$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}\ \ N$$

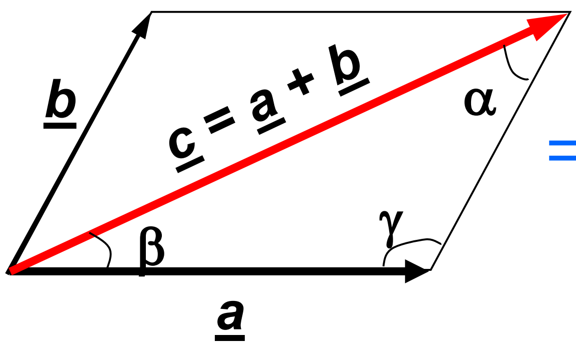
Vector format only



Right-hand rule

*Addition or Resultant of vectors

The addition of forces obeys the **parallelogram law** of vector addition.



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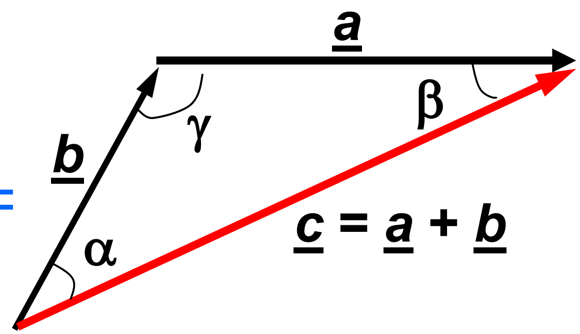


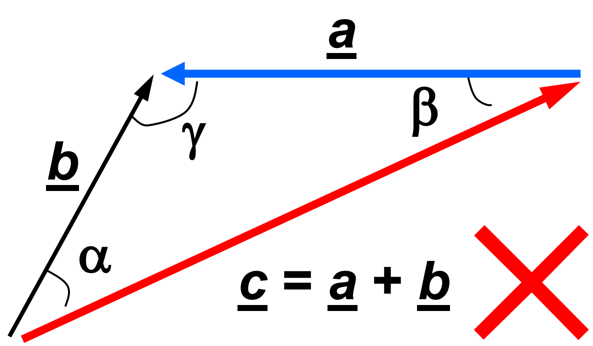
Vector triangle

Magnitude and direction of $\underline{c}$ may be obtained by the law of cosines

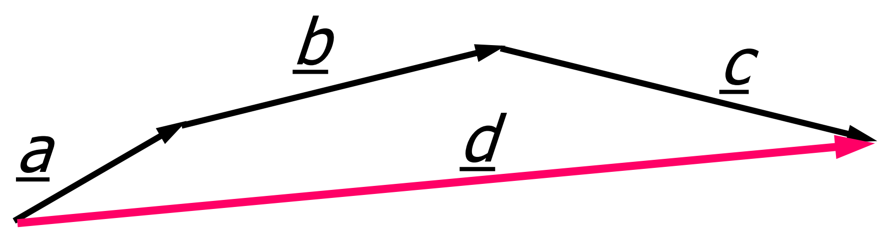
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

or the law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



The addition of 3 or more forces is carried out by arranging the given vectors in tail-to-tip fashion and connecting the tail of the first vector with the tip of the last one - **polygon rule** for addition of vectors



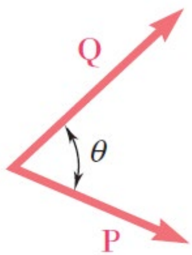
$$\underline{d} = \underline{a} + \underline{b} + \underline{c}$$

*Product of vectors

The two important vector operations in this module are i) dot product or scale product and ii) cross product or vector project of two vectors.

$Given\ two\ vectors\ \underline{P}\ and\ \underline{Q}$

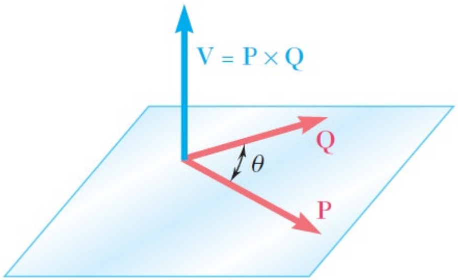
i) Dot or scalar product of them:$V = \underline{P} \cdot \underline{Q}$

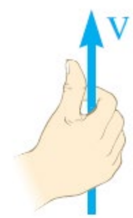


Magnitude of V :

$V = PQ \cos\theta$

ii) Cross or vector product of them:$\underline{V} = \underline{P} \times \underline{Q}$





Magnitude of $\underline{V}$:

$V = PQ \sin\theta$

Right-hand screw rule

Direction of $\underline{V}$:

Perpendicular to the plane containing $\underline{P}$ and $\underline{Q}$

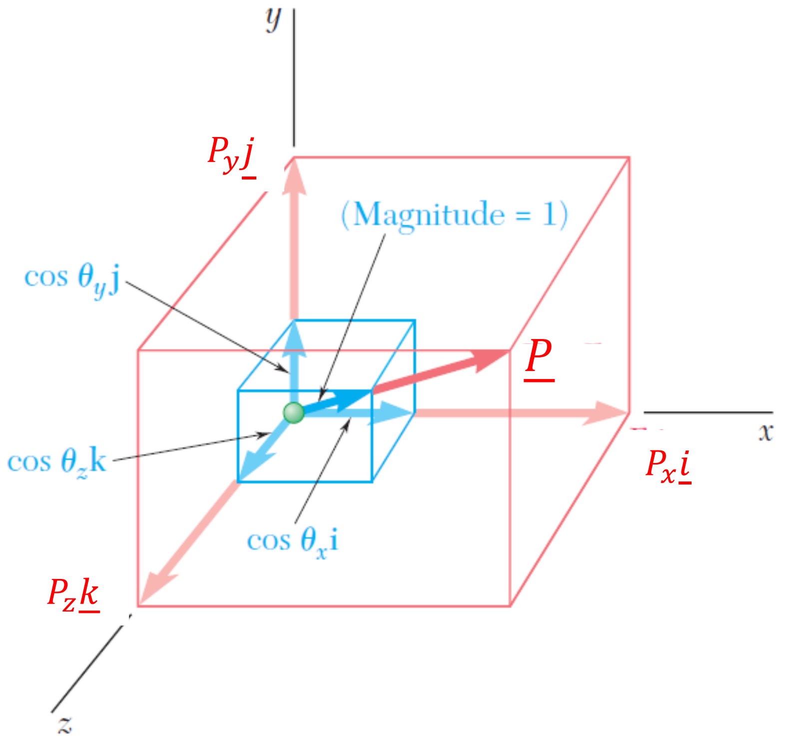
Vector operations (general)

If we don't know θ between two vectors or in more general way, we can use the rectangular components of vectors so that they can be analyzed mathematically.

Notice that we can resolve any vector into components, defined w.r.t. a rectangular coordinate system, for example $\underline{P} = P_x\underline{i} + P_y\underline{j} + P_z\underline{k}$

The magnitude of $\underline{P}$

$$P = \sqrt{P_x^2 + P_y^2 + P_z^2}$$



Vector operations (general)

Given two vectors, $\underline{P} = P_x\underline{i} + P_y\underline{j} + P_z\underline{k}$ and $\underline{Q} = Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k}$

Dot (or scalar) Product of Vectors $\underline{P}$ and $\underline{Q}$

$$V = \underline{P}\cdot\underline{Q} = \left(P_x\underline{i} + P_y\underline{j} + P_z\underline{k}\right)\cdot\left(Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k}\right)$$

$$= P_xQ_x\underline{i}\cdot\underline{i} + \cancel{P_xQ_y\underline{i}\cdot\underline{j}} + \cancel{P_xQ_z\underline{i}\cdot\underline{k}}$$

$$+ \cancel{P_yQ_x\underline{j}\cdot\underline{i}} + P_yQ_y\underline{j}\cdot\underline{j} + \cancel{P_yQ_z\underline{j}\cdot\underline{k}}$$

$$+ \cancel{P_zQ_x\underline{k}\cdot\underline{i}} + \cancel{P_zQ_y\underline{k}\cdot\underline{j}} + P_zQ_z\underline{k}\cdot\underline{k}$$

$$V = P_xQ_x + P_yQ_y + P_zQ_z$$

a scalar

Cross (or vector) Product of Vectors $\underline{P}$ and $\underline{Q}$

$$\underline{V} = \underline{P}\times\underline{Q} = \left(P_x\underline{i} + P_y\underline{j} + P_z\underline{k}\right)\times\left(Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k}\right)$$

$$= \cancel{P_xQ_x\underline{i}\times\underline{i}} + P_xQ_y\underline{i}\times\underline{j} + P_xQ_z\underline{i}\times\underline{k}$$

$$+ P_yQ_x\underline{j}\times\underline{i} + \cancel{P_yQ_y\underline{j}\times\underline{j}} + P_yQ_z\underline{j}\times\underline{k}$$

$$+ P_zQ_x\underline{k}\times\underline{i} + P_zQ_y\underline{k}\times\underline{j} + \cancel{P_zQ_z\underline{k}\times\underline{k}}$$

$$= P_xQ_y\underline{k} - P_xQ_z\underline{j} - P_yQ_x\underline{k} + P_yQ_z\underline{i} + P_zQ_x\underline{j} - P_zQ_y\underline{i}$$

$$\underline{V} = \left(P_yQ_z - P_zQ_y\right)\underline{i} \boxed{-} \left(P_xQ_z - P_zQ_x\right)\underline{j} + \left(P_xQ_y - P_yQ_x\right)\underline{k}$$

a vector

Tips:

$\underline{i}\cdot\underline{i} = ii\cos(0^{\circ}) = 1$

Similarly, $\underline{j}\cdot\underline{j} = \underline{k}\cdot\underline{k} = 1$

Others = 0

The direction of $\underline{i}$, $\underline{j}$, $\underline{k}$ follows the right hand screw rule

Tips:

$\underline{i}\times\underline{i} = ii\sin(0^{\circ}) = 0$

$\underline{i}\times\underline{j} = \underline{k}$ $\quad$ $\underline{j}\times\underline{k} = \underline{i}$

$\underline{k}\times\underline{i} = \underline{j}$

$\underline{i}\times\underline{k} = -\ \underline{k}\times\underline{i} = -\ \underline{j}$

……

*Summary of equations

Given two vectors, $\underline{P} = P_x\underline{i} + P_y\underline{j} + P_z\underline{k}$ and $\underline{Q} = Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k}$

Vector addition	$\underline{P} + \underline{Q} = (P_x + Q_x)\underline{i} + (P_y + Q_y)\,\underline{j} + (P_z + Q_z)\,\underline{k}$
Scalar (Dot) product	$\underline{P}.\underline{Q} = \underline{Q}.\underline{P} = P_xQ_x + P_yQ_y + P_zQ_z$
	$\underline{P}.\underline{P} = P^2 = P_x^2 + P_y^2 + P_z^2$
Vector (Cross) product	$\underline{P} \times \underline{Q} = -\underline{Q} \times \underline{P}$ $= \left(P_yQ_z - P_zQ_y\right)\underline{i} - (P_xQ_z - P_zQ_x)\underline{j}$ $+ \left(P_xQ_y - P_yQ_x\right)\underline{k}$
	$\underline{P} \times \underline{P} = \underline{Q} \times \underline{Q} = 0$