Numerical Methods Assignment 2

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February 20, 2018

Chaper 3 Exercises

Question 3.3

The linear least squares system is given as follows:

$$Ax = \begin{bmatrix} 1 & e \\ 2 & e^2 \\ 3 & e^3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \approx \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = b$$

Question 3.18

Consider the vector

$$a = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

.

(a)

We need to annihilate the third component of a, using an elementary elimination matrix. The easiest is to subtract twict the first column from the last column. Therefore the elementary row elimination matrix is given as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(b)

Now we need to do the same thing but with a Householder transformation. We want to apply two Householder transformations, the first H_1 eliminating all but the first element, and the second H_2 eliminating all but the second element. Then it stand that the Householder matrix H which eliminates only the third element is simply the matrix multiplication of $H = H_1 H_2$.

Let's find H_1

$$v = a - \alpha(e_1) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \alpha \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where $\alpha = \pm||a||_2 = \pm\sqrt{29}$. Since a_1 is positive, we should use the negative α to avoid cancellation.

Following this we have

$$v = a - \sqrt{29}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2-\sqrt{29}\\3\\4\end{bmatrix}$$

and Householder matrix

$$H_1 = I - 2\frac{vv^T}{v^Tv} = \begin{bmatrix}0.6885828 & 0.37139068 & -0.62283441\\0.37139068 & 0.55708601 & 0.74278135\\-0.62283441 & 0.74278135 & -0.24566881\end{bmatrix}$$

by a similar process we find

$$H_2 = \begin{bmatrix}1.45796221 & 0.64642473 & -1.19938586\\0.64642473 & 1.22908525 & 1.43036648\\-1.19938586 & 1.43036648 & -0.68704746\end{bmatrix}$$

and the desired

$$H = H_1 + H_2 = \begin{bmatrix}1.37716559 & 0.74278135 & -1.24566881\\0.74278135 & 1.11417203 & 1.48556271\\-1.24566881 & 1.48556271 & -0.49133762\end{bmatrix}$$

(c)

We pick the Given's Rotation matrix such that G is of the form

$$\begin{bmatrix}1 & 0 & 0\\0 & c & s\\0 & -s & c\end{bmatrix}$$

Since $|a_3| > |a_2|$ we use the cotangent formulation to find c and s. We have $\tau = c/s = \frac{a_3}{a_2} = 0.75$ and therefore

$$s = \frac{1}{\sqrt{1+\tau^2}} = \frac{4}{5}$$

and

$$c = \frac{3}{5}.$$

so the givens rotation matrix is

$$G = \begin{bmatrix}1 & 0 & 0\\0 & \frac{3}{5} & \frac{4}{5}\\0 & \frac{-4}{5} & \frac{3}{5}\end{bmatrix}$$

Question 3.24

Question 3.25