

Homework 5 Question 3

Question 1:

Write explicitly the losses $L_{t+\Delta}$ in terms of quantities known at time t and the random variables $X_{t+\Delta}, S_{t+\Delta}$

$$L_{t+\Delta} = -\Delta_t (S_{t+\Delta}^b - S_t)$$

where

$$\frac{1}{2} S_{t+\Delta} = \frac{S_{t+\Delta} - S_{t+\Delta}^b}{S_{t+\Delta}}$$

$$S_{t+\Delta}^b = -\frac{1}{2} S_{t+\Delta} * S_{t+\Delta} + S_{t+\Delta}$$

$$S_{t+\Delta}^b = -S_{t+\Delta} \left(\frac{1}{2} S_{t+\Delta} + 1 \right)$$

and

$$S_{t+\Delta} = S_t * e^{X_{t+\Delta}}$$

so

$$S_{t+\Delta}^b = -S_t * e^{X_{t+\Delta}} * \left(\frac{1}{2} S_{t+\Delta} + 1 \right)$$

The Losses can then be calculated using::

$$L_{t+\Delta} = -\Delta_t [S_t * e^{X_{t+\Delta}} * \left(\frac{1}{2} S_{t+\Delta} + 1 \right) - S_t * e^{X_{t+\Delta}}]$$

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}} \left[\left(\frac{1}{2} S_{t+\Delta} + 1 \right) - 1 \right]$$

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}} * \frac{1}{2} S_{t+\Delta}$$

Question 2

$$LVAR_{\alpha}^{ind} = VaR_{\alpha} + LC$$

$$LC = 1/2 \Delta_t S_t (\mu_{s,t+\Delta} + k \sigma_{s,t+\Delta})$$

The LC cost can be computed using $k = 3$, $\mu_{s,t+\Delta} = 0.2$, $\sigma_{s,t+\Delta} = 0.08$, $S_t = 59$, $\Delta_t = 100$

```
In [1]: k = 3
mu_s = 0.002
sigma_s = 0.0008
St = 59
delta_t = 100

LC = 0.5*delta_t*St*(mu_s+k*sigma_s)
LC
```

Out[1]: 12.98

$$VaR_{\alpha} = \Delta_t S_t (1 - e^{\mu_{t+\Delta} + \sigma_{t+\Delta} N^{-1}(1-\alpha)})$$

$$Z_{1-\alpha} = Z_{0.01} = -2.326348$$

```
In [2]: import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[2]: 337.202669461597

```
In [3]: LVar_ind_alpha = VaR+LC
LVar_ind_alpha
```

Out[3]: 350.182669461597

Question 3:

Simulation

Start my importing packages and setting plot parameters

```
In [4]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

# Specify Parameters for graphs
fig = plt.figure(figsize=(18,6), dpi=1600)
alpha=alpha_scatterplot = 0.2
alpha_bar_chart = 0.55
```

<matplotlib.figure.Figure at 0x24046191208>

Create function to sample s and X from normal distributions with the respective mean and standard deviation Next we calculate the loss using the formula from part 1

We draw X and s from their respective normal distributions and then use the equation

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}} \left[\frac{1}{2} s_{t+\Delta} \right]$$

```
In [5]: def Lossfunction(mu_s, mu_t, sigma_s, sigma_t, St, delta_t):
        s_sample = np.random.normal(loc = mu_s, scale = sigma_s)
        X_sample = np.random.normal(loc = mu_t, scale= sigma_t)
        Loss = delta_t * St * np.exp(X_sample)*(0.5*s_sample)
        return(Loss)
```

```
In [6]: s_sample = np.random.normal(loc = mu_s, scale = sigma_s)
        X_sample = np.random.normal(loc = mu_t, scale= sigma_t)
        Loss = delta_t * St * np.exp(X_sample)*(0.5*s_sample)
        print(s_sample, X_sample)
        Loss
```

```
0.0018411905963245103 0.018215565172591704
```

```
Out[6]: 5.5313569274847314
```

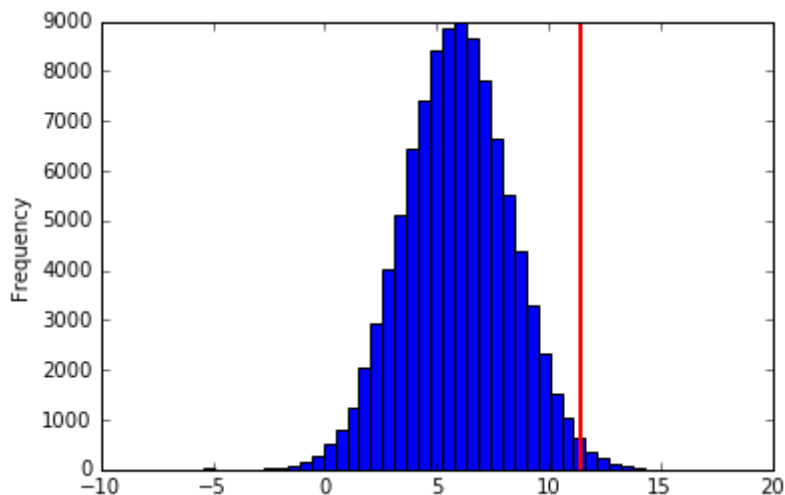
Pre allocate space and then simulate returns 100000 (numb_sim) times

```
In [7]: numb_sim = 100000 # M
        LVAR_sim = pd.Series(data = np.repeat(0,numb_sim))
        for sim in range(numb_sim):
            LVAR_sim.loc[sim] = Lossfunction(mu_s, mu_t, sigma_s, sigma_t, St, delta_t)
        LVAR_sim.describe()
```

```
Out[7]: count    100000.000000
        mean       5.900997
        std        2.367700
        min       -5.387702
        25%        4.301821
        50%        5.895013
        75%        7.483499
        max       15.922148
        dtype: float64
```

```
In [8]: LVAR_sim.plot(kind = 'hist', bins = 40)
quantile99 = LVAR_sim.quantile(q=0.99)
plt.axvline(x = quantile99, color = 'r', linewidth = 2.0)
```

Out[8]: <matplotlib.lines.Line2D at 0x240491cdb70>



Part 1

The confidence α

Not really sure what this means....

Part 2

The estimate of $LVAR_{\alpha}^{sim}$ found via simulation

```
In [9]: LVAR_99 = LVAR_sim.quantile(q=0.99)
LVAR_99
```

Out[9]: 11.469697751928559

Part 3

The theoretical VaR_{α} from question 2

```
In [10]: import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[10]: 337.202669461597

Part 4

The estimated liquidity cost $LC^{sim} = LVAR_{\alpha}^{sim} - VaR_{\alpha}$

In [11]: `LC_sim = LVAR_99 - VaR`
`LC_sim`

Out[11]: -325.73297170966845

Part 5

The estimated percentage increase in the risk measure:

$$100 * \left(\frac{LVAR_{\alpha}^{sim}}{VaR_{\alpha}} - 1 \right)$$

In [12]: `100*(LVAR_99/VaR -1)`

Out[12]: -96.598574450717749

Part 6

The industry approximate $LVAR_{\alpha}^{ind}$

In [13]: `LVar_ind_alpha`

Out[13]: 350.182669461597

Part 7

The industry liquidity cost LC

In [14]: `LC`

Out[14]: 12.98

Part 8

The industry percentage increase in the risk measure: $100 \frac{LC}{VaR_{\alpha}}$

In [15]: `100*(LC/VaR)`

Out[15]: 3.8493170948868345

How do the risk measures and liquidity costs compare?

Not sure...

