Homework 5 Question 3

Question 1:

Write explicitly the losses $L_{t+\Delta}$ in terms of quantities known at time t and the random variables $X_{t+\Delta}$, $s_{t+\Delta}$

$$L_{t+\Delta} = -\Delta_t(S^b_{t+\Delta} - S_t)$$

where

$$\frac{1}{2}s_{t+\Delta} = \frac{S_{t+\Delta} - S^b_{t+\Delta}}{S_{t+\Delta}}$$

$$S^b_{t+\Delta} = -\frac{1}{2}s_{t+\Delta} * S_{t+\Delta} + S_{t+\Delta}$$

$$S^b_{t+\Delta} = -S_{t+\Delta}(\frac{1}{2}s_{t+\Delta} - 1)$$

and

$$S_{t+\Delta} = S_t * e^{X_{t+\Delta}}$$

so

$$S^b_{t+\Delta} = -S_t * e^{X_{t+\Delta}} * (\frac{1}{2}s_{t+\Delta} - 1)$$

The Losses can then be calculated using::

$$L_{t+\Delta} = -\Delta_t[-S_t * e^{X_{t+\Delta}} * (\frac{1}{2}s_{t+\Delta} - 1) - S_t * e^{X_{t+\Delta}}]$$

$$L_{t+\Delta} = -\Delta_t * [S_t * e^{X_{t+\Delta}} * (1 - \frac{1}{2}s_{t+\Delta}) + S_t]$$

Question 2

$$LVAR^{ind}_\alpha = VaR_\alpha + LC$$

$$LC = 1/2\Delta_t S_t(\mu_{s,t+\Delta} + k\sigma_{s,t+\Delta})$$

The LC cost can be computed using k = 3, $\mu_{s,t+\Delta} = 0.2$, $\sigma_{s,t+\Delta} = 0.08, S_t = 59, \Delta_t = 100$

In [1]:
```
k = 3
mu_s = 0.002
sigma_s = 0.0008
St = 59
delta_t = 100

LC = 0.5*delta_t*St*(mu_s+k*sigma_s)
LC
```

Out[1]: 12.98

$$VaR_\alpha = \Delta_t S_t(1 - e^{\mu_{t+\Delta} + \sigma_{t+\Delta} N^{-1}(1-\alpha)})$$
$$Z_{1-\alpha} = Z_{0.01} = -2.326348$$

In [2]:
```
import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[2]: 337.202669461597

In [3]:
```
LVar_ind_alpha = VaR+LC
LVar_ind_alpha
```

Out[3]: 350.182669461597

Question 3:

Simulation

Start my importing packages and setting plot parameters

In [4]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

# Specify Parameters for graphs
fig = plt.figure(figsize=(18,6), dpi=1600)
alpha=alpha_scatterplot = 0.2
alpha_bar_chart = 0.55
```

```
<matplotlib.figure.Figure at 0x15d8db18b00>
```

Create function to sample s and X from normal distributions with the respective mean and standard deviation Next we calculate the loss using the formula from part 1

We draw X and s from their respective normal distributions and then use the equation

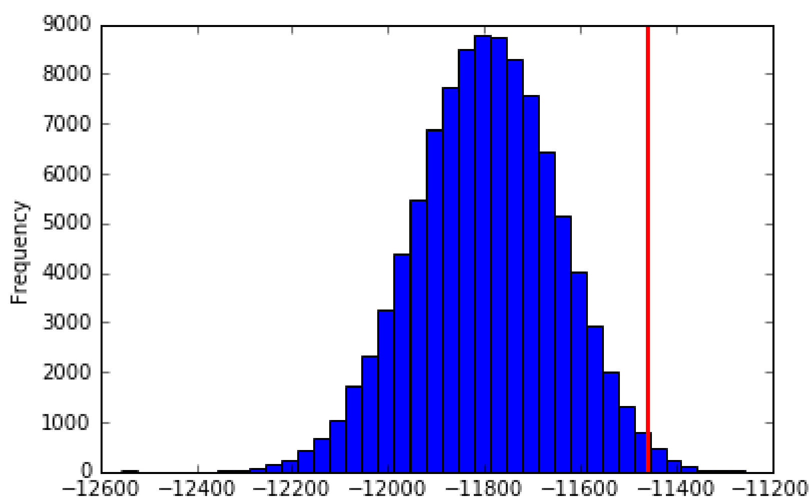
$$L_{t+\Delta} = -\Delta_t * [S_t * e^{X_{t+\Delta}} * (1 - \frac{1}{2}s_{t+\Delta}) + S_t]$$

In [5]:
```
numb_sim = 100000 # M
LVAR_sim = pd.Series(data = np.repeat(0,numb_sim))
X_sample = np.random.normal(loc = mu_t, scale= sigma_t, size = numb_sim)
s_sample = np.random.normal(loc = mu_s, scale = sigma_s, size = numb_sim)
for sim in range(numb_sim):
    LVAR_sim.loc[sim] = -delta_t *( St * np.exp(X_sample[sim])*( 1-0.5*s_sample[s
LVAR_sim.describe()
```

Out[5]:
```
count     100000.000000
mean      -11796.311207
std          149.010395
min       -12554.489931
25%       -11896.048703
50%       -11794.921249
75%       -11694.334072
max       -11221.573224
dtype: float64
```

In [6]:
```
LVAR_sim.plot(kind = 'hist', bins = 40)
quantile99 = LVAR_sim.quantile(q=0.99)
plt.axvline(x = quantile99, color = 'r', linewidth = 2.0)
```

Out[6]:
```
<matplotlib.lines.Line2D at 0x15d8b47ae80>
```



Part 1

The confidence α = 0.99

Part 2

The estimate of $LVAR_{\alpha}^{sim}$ found via simulation

In [7]:
```
LVAR_99 = LVAR_sim.quantile(q=0.99)
LVAR_99
```

Out[7]: -11458.088557942423

Part 3

The theoretical VaR_α from question 2

In [8]:
```
import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[8]: 337.202669461597

Part 4

The estimated liquidity cost $LC^{sim} = LVAR_\alpha^{sim} - VaR_\alpha$

In [9]:
```
LC_sim = LVAR_99 - VaR
LC_sim
```

Out[9]: -11795.291227404021

Part 5

The estimated percentage increase in the risk measure:

$$100 * (\frac{LVAR_\alpha^{sim}}{VaR_\alpha} - 1)$$

In [10]:
```
100*(LVAR_99/VaR -1)
```

Out[10]: -3497.9827550704931

Part 6

The industry approximate $LVAR_\alpha^{ind}$

In [11]:
```
LVar_ind_alpha
```

Out[11]: 350.182669461597

Part 7

The industry liquidity cost LC

In [12]:
```
LC
```

Out[12]: 12.98

Part 8

The industry percentage increase in the risk measure: $100\frac{LC}{VaR_\alpha}$

In [13]:
```
100*(LC/VaR)
```

Out[13]: 3.8493170948868345

How do the risk measures and liquidity costs compare?

Both estimates are lower than the analytical solution. The liquidity cost in the simulated case is much lower than in the analytical case. This accounts for most of the different in the VaR calculations.