Homework 5 Question 3 Malcolm Taylor, Karen Pardo, Kulin Chheda

Question 1:

Write explicitly the losses $L_{t+\Delta}$ in terms of quantities known at time t and the random variables $X_{t+\Delta}, s_{t+\Delta}$

$$L_{t+\Delta} = -\Delta_t (S^b_{t+\Delta} - S_t)$$

where

$$\frac{1}{2} s_{t+\Delta} = \frac{S_{t+\Delta} - S^b_{t+\Delta}}{S_{t+\Delta}}$$

$$S^b_{t+\Delta} = -\frac{1}{2} s_{t+\Delta} * S_{t+\Delta} + S_{t+\Delta}$$

$$S^b_{t+\Delta} = -S_{t+\Delta} (\frac{1}{2} s_{t+\Delta} + 1)$$

and

$$S_{t+\Delta} = S_t * e^{X_{t+\Delta}}$$

so

$$S^b_{t+\Delta} = -S_t * e^{X_{t+\Delta}} * (\frac{1}{2} s_{t+\Delta} + 1)$$

The Losses can then be calculated using::

$$L_{t+\Delta} = -\Delta_t [S_t * e^{X_{t+\Delta}} * (\frac{1}{2} s_{t+\Delta} + 1) - S_t * e^{X_{t+\Delta}}]$$

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}} [(\frac{1}{2} s_{t+\Delta} + 1) - 1]$$

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}} * \frac{1}{2} s_{t+\Delta}$$

Question 2

$$LVAR^{ind}_{\alpha} = VaR_{\alpha} + LC$$

$$LC = 1/2 \Delta_t S_t (\mu_{s,t+\Delta} + k\sigma_{s,t+\Delta})$$

The LC cost can be computed using k = 3, $\mu_{s,t+\Delta} = 0.2$, $\sigma_{s,t+\Delta} = 0.08, S_t = 59, \Delta_t = 100$

In [1]:
```
k = 3
mu_s = 0.002
sigma_s = 0.0008
St = 59
delta_t = 100

LC = 0.5*delta_t*St*(mu_s+k*sigma_s)
LC
```

Out[1]: 12.98

$$VaR_\alpha = \Delta_t S_t (1 - e^{\mu_{t+\Delta} + \sigma_{t+\Delta} N^{-1}(1-\alpha)})$$
$$Z_{1-\alpha} = Z_{0.01} = -2.326348$$

In [2]:
```
import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[2]: 337.202669461597

In [3]:
```
LVar_ind_alpha = VaR+LC
LVar_ind_alpha
```

Out[3]: 350.182669461597

Question 3:

Simulation

Start my importing packages and setting plot parameters

In [4]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline

# Specify Parameters for graphs
fig = plt.figure(figsize=(18,6), dpi=1600)
alpha=alpha_scatterplot = 0.2
alpha_bar_chart = 0.55
```

```
<matplotlib.figure.Figure at 0x24046191208>
```

Create function to sample s and X from normal distributions with the respective mean and standard deviation Next we calculate the loss using the formula from part 1

We draw X and s from their respective normal distributions and then use the equation

$$L_{t+\Delta} = -\Delta_t * S_t * e^{X_{t+\Delta}}[\frac{1}{2}s_{t+\Delta}]$$

In [5]:
```
def Lossfunction(mu_s, mu_t, sigma_s, sigma_t, St, delta_t):
    s_sample = np.random.normal(loc = mu_s, scale = sigma_s)
    X_sample = np.random.normal(loc = mu_t, scale= sigma_t)
    Loss = delta_t * St * np.exp(X_sample)*(0.5*s_sample)
    return(Loss)
```

In [6]:
```
s_sample = np.random.normal(loc = mu_s, scale = sigma_s)
X_sample = np.random.normal(loc = mu_t, scale= sigma_t)
Loss = delta_t * St * np.exp(X_sample)*(0.5*s_sample)
print(s_sample, X_sample)
Loss
```

```
0.0018411905963245103 0.018215565172591704
```

Out[6]:
```
5.5313569274847314
```

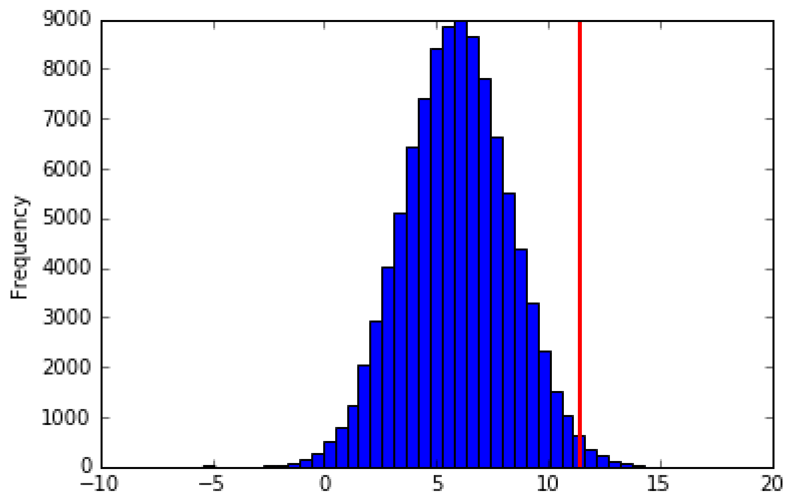
Pre allocate space and then simulate returns 100000 (numb_sim) times

In [7]:
```
numb_sim = 100000 # M
LVAR_sim = pd.Series(data = np.repeat(0,numb_sim))
for sim in range(numb_sim):
    LVAR_sim.loc[sim] = Lossfunction(mu_s, mu_t, sigma_s, sigma_t, St, delta_t)
LVAR_sim.describe()
```

Out[7]:
```
count    100000.000000
mean          5.900997
std           2.367700
min          -5.387702
25%           4.301821
50%           5.895013
75%           7.483499
max          15.922148
dtype: float64
```

In [8]:
```
LVAR_sim.plot(kind = 'hist', bins = 40)
quantile99 = LVAR_sim.quantile(q=0.99)
plt.axvline(x = quantile99, color = 'r', linewidth = 2.0)
```

Out[8]: <matplotlib.lines.Line2D at 0x240491cdb70>



Part 1

The confidence α

Not really sure what this means....

Part 2

The estimate of $LVAR_{\alpha}^{sim}$ found via simulation

In [9]:
```
LVAR_99 = LVAR_sim.quantile(q=0.99)
LVAR_99
```

Out[9]: 11.469697751928559

Part 3

The theoretical VaR_{α} from question 2

In [10]:
```
import math
mu_t = 0
sigma_t = 0.4/math.sqrt(250)
Z_alpha = -2.3263
VaR = St*delta_t*(1-math.exp(mu_t + sigma_t*Z_alpha))
VaR
```

Out[10]: 337.202669461597

Part 4

The estimated liquidity cost $LC^{sim} = LVAR_{\alpha}^{sim} - VaR_{\alpha}$

In [11]:
```
LC_sim = LVAR_99 - VaR
LC_sim
```

Out[11]: -325.73297170966845

Part 5

The estimated percentage increase in the risk measure:

$$100 * (\frac{LVAR_{\alpha}^{sim}}{VaR_{\alpha}} - 1)$$

In [12]:
```
100*(LVAR_99/VaR -1)
```

Out[12]: -96.598574450717749

Part 6

The industry approximate $LVAR_{\alpha}^{ind}$

In [13]:
```
LVar_ind_alpha
```

Out[13]: 350.182669461597

Part 7

The industry liquidity cost LC

In [14]:
```
LC
```

Out[14]: 12.98

Part 8

The industry percentage increase in the risk measure: $100 \frac{LC}{VaR_{\alpha}}$

In [15]:
```
100*(LC/VaR)
```

Out[15]: 3.8493170948868345

How do the risk measures and liquidity costs compare?

Not sure...