HW Problems for Assignment 5 - Part II
Due 6:00 PM Tuesday, November 29, 2016

Note : this is the second half of HW 5. It covers material from Lecture 11 on Tuesday, November 22, 2016. The first half of HW 5 was posted on Tuesday, November 25, 2016, and covered material from Lecture 10 on Tuesday, November 15, 2016. Both halves are to be submitted by 6 PM on Tuesday, November 29, 2016. There are 50 points possible for the questions covering Lecture 10, and 50 points possible for the questions covering Lecture 11.

1. (50 Points) VaR with Unknown Parameters. The file

"SP500_Log_Returns_20150101_20160101.xlsx"

contains daily log return data for the S&P 500 index for 2015. In this exercise, you will estimate the VaR associated to a $\$1,000,000$ portfolio in the S&P 500 index for the one (business) day period 12/31/15 – 1/4/2016 (i.e. we assume $t = 12/31/15$ and Δ is one business day. In addition to estimating the VaR you will output confidence intervals for your estimate based off the methodologies described in class.

To perform these calculations, note first that from the data file we can obtain the number of days of data n, along with the standard estimators for the mean and variance of the log returns. Denote by $\bar{X}_n$ the mean, and S_n^2 the variance. Next, assume at t we hold $V = \$1,000,000$ in the S&P 500 index, and that given $\mathcal{F}_t$ we have $X_{t+\Delta} \overset{\mathcal{F}_t}{\sim} N\left(\mu, \sigma^2\right)$ where μ, σ^2 are the (unknown) true values of the mean and variance. Using the full loss operator

$$L_{t+\Delta} = -V\left(e^{X_{t+\Delta}} - 1\right),$$

we know that the theoretical value at risk given $\mathcal{F}_t$ takes the form

$$\mathrm{VaR}_\alpha\left(L_{t+\Delta}\right) = V\left(1 - e^{\mu + \sigma N^{-1}(1-\alpha)}\right). \tag{0.1}$$

Write a simulation to estimate $\mathrm{VaR}_\alpha\left(L_{t+\Delta}\right)$ as well as the confidence intervals in two ways:

(a) (**25 Points**) Assuming μ is known, and given by the standard estimator $\bar{X}_n$. Here, estimate $\mathrm{VaR}_\alpha\left(L_{t+\Delta}\right)$ using the empirical distribution of log returns as well as using the theoretical formula in (0.1) with the sample mean and variance. Furthermore, produce a $100(1-\beta)\%$ confidence interval for your estimate following the methodology on slides 25-28 of Lecture 11.

(b) (**25 Points**) Assume now that both μ and σ are unknown. Here, write a simulation to estimate $\mathrm{VaR}_\alpha\left(L_{t+\Delta}\right)$ using the methodology described on slide 31 of class. Note that the simulation with produce samples $Y_m = \mathrm{VaR}_\alpha\left(L_{t+\Delta}\right)^m$ for $m = 1, ..., M$. With these simulated values, output the average

$$\bar{Y}_M = \frac{1}{M} \sum_{m=1}^{m} Y_m,$$

as well as the confidence interval given by (A, B) where A is the $\beta/2$ quantile of the empirical distribution of the $\{Y_m\}$ and B is the $1 - \beta/2$ quantile of the empirical distribution of the $\{Y_m\}$.

For parameter values use $\alpha = .95$, $\beta = .025$, and $M = 100,000$ trials for your simulation in part (b). How do your VaR estimates and confidence intervals compare?