Code `OllinSphere-BiB`
User's manual

Miguel Alcubierre

Instituto de Ciencias Nucleares, UNAM

malcubi@nucleares.unam.mx

June, 2018

Contents

1   Introduction

The program `OllinSphere-BiB` solves the Einstein evolution equations in spherical symmetry, using a curvilinear version of the BSSN formulation for the 3+1 evolution equations, with different types of matter and different gauge conditions.

The main difference of this version of the code with respect to previous ones is the fact that it uses box-in-box mesh refinement (hence the BiB part of the name), and is parallelized with MPI.

Note: This manual is perpetually incomplete since I keep adding new things to the code. Still, it should give you a good idea of the basics. If you find something wrong please let me know!

2   Downloading the code

If you are reading this it means you probably already downloaded the code. But if for some reason you need to download it here is how.

2.1   From CVS

The easiest way to obtain the code is to download it from the CVS server of the Ollin group (unfortunately, since CVS is rather old, now this only works from within my own Institute's network). In order to obtain a username and password for this sever you need to contact Miguel Alcubierre (malcubi@nucleares.unam.mx). Once you have a username and password you first need to log into the repository by typing at the terminal:

```
cvs -d :pserver:username@dulcinea.nucleares.unam.mx:/usr/local/ollincvs login
```

where you should change the word "username" for your personal username! You then need to type your password. After this you must download the code by typing:

```
cvs -d :pserver:username@dulcinea.nucleares.unam.mx:/usr/local/ollincvs co Codes/OllinSphere-BiB
```

This will create a directory `Codes` and inside it directory `OllinSphere-BiB`. This is the main directory for the code.

2.2 From GitHub

Another way to dowload the code is from the public GitHub repository. This is in fact easier than CVS:

```
git clone https://github.com/malcubi/OllinSphere-BiB
```

Once you have the code, you can get updated versions by just doing "git pull" inside the code main directory.

3 Directory structure

The main directory for the code is `OllinSphere-BiB`. There are several sub-directories inside this main directory:

CVS	Contains information about the CVS root and server (see Sec. 21).
doc	Contains the tex and pdf files for this user's manual.
exe	This directory is created at compile time and contains the executable file. It also contains a copy of the parameter files found in directory `par` (see below).
fakempi	Contains fake MPI routines so that the compiler won't complain if MPI is not installed.
gnuplot	Contains a couple of simple gnuplot macros for visualization.
objs	This directory is created at compile time and contains all the object and module files.
ollingraph	Contains the visualization package "ollingraph" for convenient "quick and dirty" visualization (see Section 22 below).
par	Contains examples of parameter files (see Section 5 below).
prl	Contains perl scripts used at compile time to create the subroutines that manage parameters and arrays.
src	Contains the source files for all the code routines.

tools   Contains some useful analysis tools (for example an FFT code).

The directory `src` is itself divided into a series of sub-directories in order to better classify the different routines. These sub-directories are:

auto   Contains `FORTRAN` files that are automatically generated at compile time by the perl scripts. These files should not be edited manually!

base   Contains the routines that control the basic execution of the code, including the parameter and array declarations, the parameter parser, the output routines, the main evolution controllers, and generic routines for calculating derivatives, dissipation, etc. The code in fact starts execution at the routine `src/base/main.f90` contained in this directory.

geometry   Contains routines related to initial data, evolution and analysis of the spacetime geometric variables, including sources, gauge conditions, constraints, horizon finders, etc.

matter   Contains routines related to the initial data, evolution and analysis of the different matter models, including a generic routine for calculating the basic matter variables, and routines for evolving scalar fields, electric fields, fluids, etc.

4   Compiling and running the code

The code is written in `FORTRAN 90` and is parallelized with `MPI` (Message Passing Interface). All subroutines are in separate files inside the directory `src` and its sub-directories.

4.1   Compiling

To compile just move inside the `OllinSphere-BiB` directory and type:

```
make
```

This will first run some perl scripts that create a series of automatically generated `FORTRAN` files that will be placed inside the directory `src/auto`. It will then compile all the `FORTRAN`

routines that it can find inside any of the sub-directories of `src` (it will attempt to compile any file with the extension `.f90`).

The resulting object files and `FORTRAN` module files will be placed inside the sub-directory `objs`. The Makefile will then create a directory `exe` and will place in it the final executable file called `ollinsphere`. It will also copy to this directory all the sample parameter files inside the sub-directory `par`, and the visualization package `ollingraph`.

Notice that at this time the Makefile can use the compilers `g95`, `gfortran`, or the Intel compilers `ifc` (and the new version `ifort`), and it will automatically check if they are installed. If you have a different compiler then the Makefile will have to be modified (hopefully it won't be very difficult). The code will also attempt to find an `MPI` installation (it looks for the command `mpif90`), and if it does not find it it will use the fake routines inside the directory `fakempi`.

The Makefile has several other useful targets that can be listed by typing: `make help`.

4.2   Running

To run the code move into the directory `exe` and type:

`ollinsphere name.par`

Where `name.par` is the name of your parameter file (more on parameter files below). The code will then read data from the parameter file silently and hopefully start producing some output to the screen. The code will also create an output directory and will write the data files to that directory.

For parallel runs using `MPI` one must use instead the command:

`mpirun -np N ollinsphere name.par`

where `N` should be an integer number that specifies the number of processors to be used.

5   Parameter files

At run time the code reads the parameter values from a parameter file (parfile), with a name of the form `name.par`, that must be specified in the command line after the executable:

`ollinsphere name.par`

The data in this parameter file can be given in any order, using the format:

`parameter = value`

Comments (anything after a `#`) and blank lines are ignored. Only one parameter is allowed per line, and only one value is allowed per parameter, with the exception of the parameters `outvars0D` and `outvars1D` that control which arrays get output and take lists of arrays as values, for example:

`outvars0D = alpha,A,B`

There is in fact one other parameter that can also take multiple values as input, it is the parameter `mattertype` that can accept several types of matter at once (see Section 11 below).

Parameters that do not appear in the parfile get the default values given in the file `src/base/param.f90`. Examples of parameter files can be found in the subdirectory `par`.

IMPORTANT: Even though `FORTRAN` does not distinguish between upper and lower case in variable names, the names of parameters are handled as strings by the parameter parser, so lower and upper case are in fact different. The name of parameters in the parameter file should then be identical to the way in which they appear in the file `param.f90`.

6   Output files

At run time, the codes creates an output directory whose name should be given in the parameter file. It then produces a series of output files with the data from the run. There are so called `0D` files (with extension `*.tl`) and `1D` files (with extension `*.rl`).

The `0D` files refer to scalar quantities obtained from the spatial arrays as functions of time.

8

These scalar quantities include the maximum (`max`), the minimum (`min`), and three different norms of the spatial arrays: maximum absolute value (`nm1`), root mean square (`nm2`), and total variation (`var`).

The `1D` files contain the complete spatial arrays at different times. Even though the code is only one-dimensional, these files can become quite big if you are not careful, so beware.

Since we can have several grid refinement levels, the file names are appended with a number corresponding to the specific level (all grid levels have output). For example:

```
alpha0.rl:     Level 0 (coarsest grid)
alpha1.rl:     Level 1
...
```

All files are written in ASCII, and using a format adapted to XGRAPH (but other graphic packages should be able to read them).

Output is controlled by the following parameters:

`directory`	Name of directory for output.
`Ninfo`	How often do we output information to screen?
`Noutput0D`	How often do we do 0D output?
`Noutput1D`	How often do we do 1D output?
`outvars0D`	Arrays that need 0D output (a list separated by commas).
`outvars1D`	Arrays that need 1D output (a list separated by commas).

7 Checkpointing

For very long runs, or in cases when we want to save the final state of the simulation so we can restart from that point, it is useful to have the ability to save the current state of the whole simulation at a given time. The code therefore is capable of doing a "checkpoint" every so often.

In order for the code to do checkpointing, we must add the following to the parameter file:

```
checkpoint = .true.
Ncheckpoint = N
```

where N is an integer number that specifies how often we want to do a checkpoint (it shouldn't be too often, checkpointing is slow and produces a lot of data). This will create a series of directories named `checkpoint_t=T` inside the output directory, with T a real number that indicates the time corresponding to the given checkpoint. Each of these directories will contain all the data needed to restart the simulation from that point.

In order to later restart a simulation from a given checkpoint file, one must first move the corresponding checkpoint directory to the main directory containing the executable and parameter files. One then deletes the line containing the parameter "idata" in the parameter file (or simply comments the line with #), and adds the lines:

```
idata = checkpoint
checkpointfile = checkpointfile
```

where `checkpointfile` should correspond to the full name of the checkpoint directory from which we want to restart. For example, adding the line:

```
checkpointfile = checkpoint_t=0.00000
```

causes the code to restart from $t = 0$ (notice that in that case the code will not calculate the initial data again, it will just take the data from the checkpoint file).

WARNING: When restarting from a checkpoint always remember to change the name of the output directory, as otherwise the code will just rewrite it and you will loose all your data until that point!

8   Numerical grid

8.1   Staggering the origin

In order to avoid having divisions by zero in some terms, we stagger the origin. This means that there is no grid point at the origin. Instead, the first grid point (grid point 0) is located
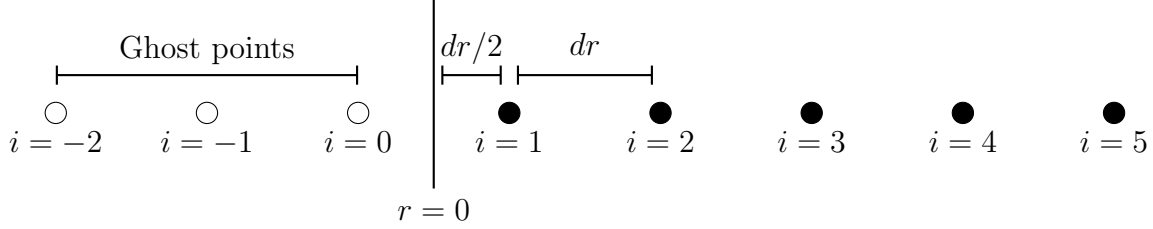
Figure 1: Basic grid structure close to $r = 0$ showing how the origin is staggered. We show the case of 3 ghost points to the left of the origin (for fourth order spatial differencing).

at $r = -dr/2$ (with dr the grid spacing), the second one (grid point 1) at $r = +dr/2$, and so on. This also makes it easy to apply symmetry boundary conditions, since for an even grid function f_i we can simply say $f_0 = +f_1$ and for an odd function $f_0 = -f_1$.

Grid points to the left of the origin are known as "ghost points", and the code usually adds more than one in order to be able to use higher order differencing stencils. The positions of the grid points then correspond to $r_i = (i - 1/2) * dr$, where i runs from $(1 - g)$ to some maximum number Nr, and where g is the number of ghost points given by the parameter ghost. In fact, for second order spatial differencing the code takes ghost=2, while for fourth order differencing it takes ghost=3, see Figure 1 (the reason for this has to do with dissipation, which typically needs one extra ghost point that would normally be required for a given differencing order, see below). Notice that the parameter ghost is determined by the code at run time and should NOT be fixed in the parameter file.

8.2 Box-in-box grid refinement

The also code uses "box-in-box" type grid refinement. That is, it has refinement levels with higher resolution close to the origin set up at the beginning from the parameter file. The base resolution for the coarsest grid is set in the parameter file using the parameter dr0, and the total number of grids is set by the parameter Nl (Nl=1 means a uni-grid run). Higher resolution grids are set up with refinement factors of two for each level (see Figure 2). It is important to notice that the grid point positions at different refinement levels do not coincide because of the staggering of the origin.

When using grid refinement the time step dt is also refined in the same way, so that a the first
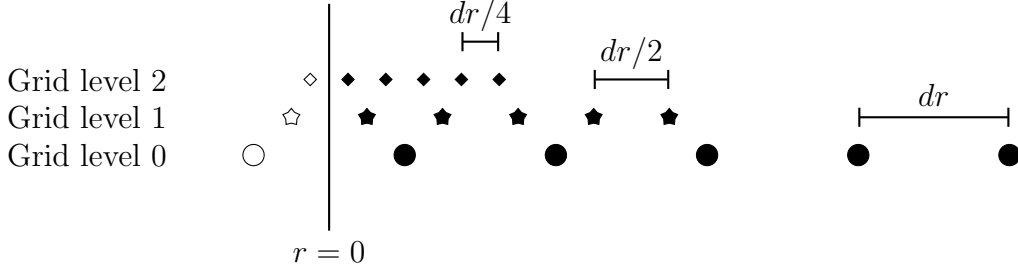
Figure 2: Box-in-box grid refinement close to the origin. Notice that the positions of the grid points at different refinement levels do not coincide because of the staggering of the origin.

refinement level takes twice as many time steps as the coarsest grid, the second refinement level four times as many time steps, and so on. There are two important concepts that should be mentioned here, the operations known as "restriction" and "prolongation". Restriction refers to the injection of fine grid information into a coarser grid once they both reach the same time level. Prolongation refers, on the other hand, to the use of coarse grid information to create new fine grids, which we don't do here since the grid structure is defined from the beginning. However, coarse grid information is still needed in order to give boundary conditions to the fine grids.

In order to understand how the integration in time proceeds let us consider the case of only two grid levels (if there are more levels the same procedure is applied recursively). Assuming we already have initial data for both grid levels, first the coarse grid is advanced one full time step. After this, the fine grid is advanced two half time steps until it coincides again with the coarse grid (see Figure 3). Notice that in order to do this we need to give boundary data to the fine grid, which is obtained from the coarse grid via interpolation in both time and space. At the moment the code uses cubic interpolation in space, and quadratic interpolation in time by keeping three coarse time steps in memory (we only need to keep the data close to the fine grid boundary). This does not work for the very first steps since we don't have enough coarse time steps yet, so for the first two fine grid time steps we use linear interpolation in time.

Once both grids coincide in time again we inject (restrict) data from the fine grid back into the coarse grid at those locations that are covered by both grids. Notice that if the coarse grid points coincided with fine grid points one could simply just copy the information from the fine grid into the coarse grid (though in practice some kind of average is often used to eliminate high frequency noise). However, due to the staggering of the origin, our fine and coarse grids do not coincide, so in practice we interpolate from the fine grid into the coarse grid (using cubic
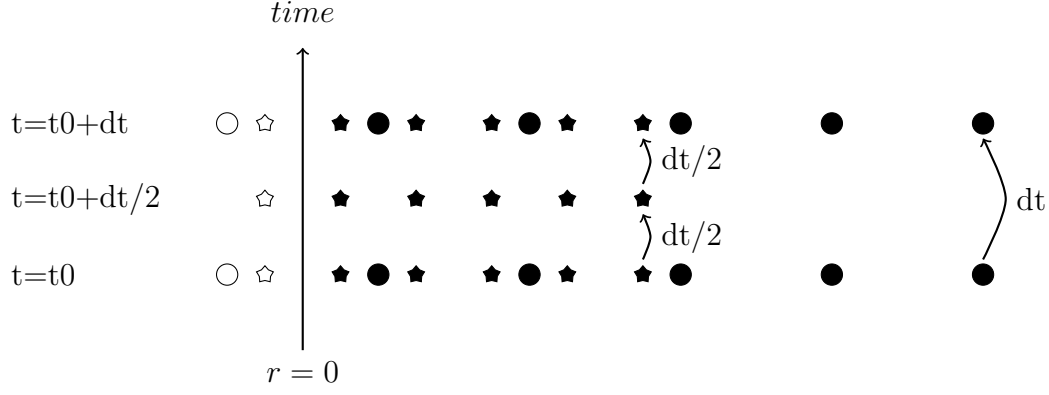
Figure 3: Time stepping for two refinement levels (for the case of 1 ghost point). The coarse grid is first advanced one full time step, and after that the fine grid is advanced two half time steps, using information from the coarse grid to set up boundary data.

interpolation).

8.3   Domain decomposition paralellization

The code uses a "domain decomposition" model for paralellization, that is, it distributes the full grid among the different processors. In order to minimize communications between processors, ghost points are also added to the sides of each grid so that there is enough information to calculate spatial differences. Later, at the end of each time step, these ghost points are "synchronized", that is, information is copied to the ghost points of a given processor from the corresponding interior points of neighboring processors, see Figure 4.

For simplicity, the number of ghost points used here is the same as the number used to the left of the origin (see above), and is controlled by the same parameter `ghost` (determined at evolution time). Notice that when we use grid refinement, each grid level is decomposed in exactly the same way among the same processors.
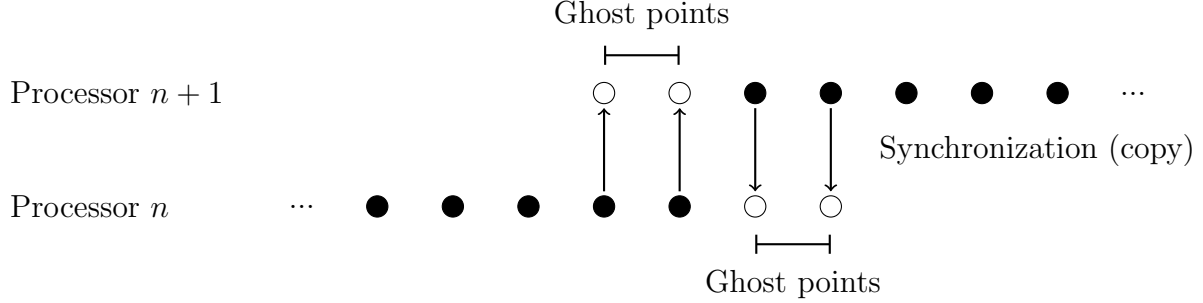
Figure 4: Overlapping grids for two processors showing the domain decomposition model with the ghost points (in this case two) for each processor, and the synchronization operation.

9   Spacetime and evolution equations

9.1   Spacetime metric

In the code, the spatial metric is written in the following way:

$$
\begin{aligned}
dl^2 &= \psi(r,t)^4 \left(A(r,t)dr^2 + r^2 B(r,t)\, d\Omega^2 \right) \ , \\
&= e^{4\phi(r,t)} \left(A(r,t)dr^2 + r^2 B(r,t)\, d\Omega^2 \right) \ ,
\end{aligned}
\tag{9.1}
$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ the solid angle element, and with $\psi = e^\phi$ the conformal factor. The full spacetime metric takes the form:

$$
ds^2 = \left(-\alpha^2 + \beta_r \beta^r \right) dt^2 + 2\beta_r dt dr + dl^2 \ ,
\tag{9.2}
$$

where α is the lapse function and β^r the shift vector (which only has a radial component), and where $\beta_r = A\beta^r$.

It is also useful to define an inverse to the conformal factor as:

$$
\chi := 1/\psi^n = \exp(-n\phi) \ ,
\tag{9.3}
$$

with n a power controlled by the parameter `chipower`. For black hole spacetimes the conformal factor ϕ is singular at the origin, while χ remains regular, so it is best to evolve χ instead of ϕ. This can be controlled with the logical parameter `chimethod` (which by default is set to `.true.`).

14

9.2 Evolution equations

For the evolution equations, the code uses a Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation adapted to spherical symmetry. The specific form of the evolution equations used here can be found in [3].

There are two BSSN variants or "flavors" controlled by the parameter `bssnflavor`, which can take the values `lagrangian` or `eulerian` (the default is `bssnflavor=lagrangian`). They refer to the way in which the shift terms are added to make determinant of the conformal metric constant along time-lines (lagrangian), or constant along the normal-lines (eulerian).

There is also a parameter that allows one to switch off the evolution of the spacetime, which is useful in case one wants to evolve some matter field in a fixed background spacetime. The parameter is called `spacetime` and can have the values `dynamic` or `static` (default is `spacetime=dynamic`).

The main evolution variables (arrays) are:

`alpha`	The lapse function α.
`beta`	The contravariant radial shift component β^r.
`A`	The conformal radial metric component $\mathtt{A} := \tilde{g}_{rr}$.
`B`	The conformal radial angular component $\mathtt{B} := \tilde{g}_{\theta\theta}/r^2$.
`phi`	The conformal factor ϕ (see equation 9.1).
`KTA`	The mixed radial component of the trace-free extrinsic curvature K_{ij}^{TF}, $\mathtt{KTA} := (K^{TF})^r{}_r \equiv K_A$.
`KTB`	The mixed angular component of the trace-free extrinsic curvature K_{ij}^{TF}, $\mathtt{KTB} := (K^{TF})^\theta{}_\theta \equiv K_B$.
`trK`	The trace of the extrinsic curvature $\mathtt{trK} := K^m{}_m$.
`Deltar`	The auxiliary BSSN variable $\mathtt{Deltar} = \Delta^r$, which is defined as:

$$
\begin{aligned}
\Delta^r \;\; &:= \;\; \frac{1}{A}\left[\frac{\partial_r A}{2A} - \frac{\partial_r B}{B} - \frac{2}{r}\left(1 - \frac{A}{B}\right)\right] \, , \\
&= \;\; \frac{1}{A}\left[\frac{\partial_r A}{2A} - \frac{\partial_r B}{B} - 2r\lambda\right] \, ,
\end{aligned}
\tag{9.4}
$$

15

with λ one of the regularization variables (see Section 10).

dtbeta This is the time derivative of the shift: $\partial_t\beta = \texttt{dtbeta}$. When we use advection terms for the evolution of the shift (see Section 14), dtbeta is not quite the time derivative, and we have instead: $\partial_t\beta = \texttt{dtbeta} + \beta\partial_r\beta$.

Notice that for the trace free extrinsic curvature one must have $K_A + 2K_B = 0$. Also, even though the auxiliary BSSN variable Δ^r is initially defined in terms of metric derivatives, it is later evolved independently with its own evolution equation (modified using the momentum constraint), as required in the BSSN formulation.

The general BSSN formulation allows for a free parameter η that controls how much of the momentum constraint has been added to the evolution equation for Δ^r. Standard BSSN takes $\eta = 2$, and it can be shown that one needs to take $\eta > 1/2$ in order to have a strongly hyperbolic system. The choice $\eta = 2$ guarantees that the travelling modes associated with the constraints travel at the speed of light. In the code this is controlled by the parameter eta with default value equal to 2 (it is not a good idea to change this).

The code also calculates a series of auxiliary geometric quantities defined as:

psi The conformal factor $\texttt{psi} := \psi = e^\phi$. The code also calculates $\texttt{psi2} := \psi^2$ and $\texttt{psi4} := \psi^4$.

chi Another form of the conformal factor $\texttt{chi} = \chi = 1/\psi^2$. This function is in fact evolved instead of ϕ if one sets the logical parameter chimethod=.true. (by default it is false). This is useful for black hole evolutions as it improves the treatment of the central puncture.

AB2 Determinant of the conformal spatial metric $\texttt{AB2} := AB^2$. This should be time independent for "lagrangian" evolutions, while for "eulerian" evolutions it should change with time.

K2 Square of the extrinsic curvature:¡ $\texttt{K2} = K_{ij}{}^{ij} = K_A^2 + 2K_B^2 + (\mathrm{tr}K)^2/3$.

RICA Radial mixed component of the spatial Ricci tensor $\texttt{RICA} = R^r{}_r$.

RSCAL Spatial Ricci scalar $\texttt{RSCAL} = R^i{}_i$.

10 Regularization

For regular spacetimes one has to make sure the equations are regular near $r = 0$. There are several important issues about regularization of the evolution equations at the origin:

10.1 Symmetry conditions at the origin

The first thing that needs to be taken into account is the symmetry properties of the different dynamical variables at the origin. Since variables must be differentiable at the origin, this implies that metric variables must behave as:

$$\alpha \sim \alpha^0 + \mathcal{O}(r^2) \tag{10.1}$$
$$\beta^r \sim \mathcal{O}(r) \tag{10.2}$$
$$\phi \sim \phi^0 + \mathcal{O}(r^2) \tag{10.3}$$
$$A \sim A^0 + \mathcal{O}(r^2) \tag{10.4}$$
$$B \sim B^0 + \mathcal{O}(r^2) \tag{10.5}$$

And since this must hold for all time we must also have:

$$K_A \sim K_A^0 + \mathcal{O}(r^2) \tag{10.6}$$
$$K_B \sim K_B^0 + \mathcal{O}(r^2) \tag{10.7}$$

Imposing these conditions at the origin is then reduced to asking for some variables to be odd and others to be even. To do this in practice we stagger the origin. This means that there is no grid point at the origin, instead the first grid point is at $r = -dr/2$, the second one at $r = dr/2$ and so on. This makes it very easy to apply symmetry boundary conditions, since for an even function f we can simply say $f_0 = +f_1$ and for an odd function $f_0 = -f_1$.

10.2 Local flatness at the origin

This problem is more subtle. Local flatness can be shown to imply that, in the above expansions for small r, one must have:

$$A^0 = B^0 \,, \tag{10.8}$$
$$K_A^0 = K_B^0 \,. \tag{10.9}$$

The problem with this is that one now has an over-determined boundary condition: The derivatives of both A and B must vanish at $r = 0$, plus both functions must be equal, that is, three boundary conditions for two quantities (and the same happens for K_A and K_B).

We deal with this problem by introducing the auxiliary variables:

$$\lambda = \frac{1}{r^2}\left(1 - \frac{A}{B}\right), \tag{10.10}$$

$$K_\lambda = \frac{1}{r^2}(K_A - K_B), \tag{10.11}$$

which are used to rewrite the problematic terms, and imposing the extra boundary conditions on λ and K_λ (basically, local flatness implies that both are even at $r = 0$). The details of this procedure can be found in [1, 2].

11 Matter

When we write the Einstein equations in 3+1 form, the components of the stress-energy tensor that will be important are the energy density, momentum density and spatial stress tensor as measured by the so-called Eulerian observers, *i.e.* those that move along the normal direction to the spatial hypersurfaces:

$$\rho = n^\mu n^\nu T_{\mu\nu}, \tag{11.1}$$
$$J_i = -n^\mu P_i^\nu T_{\mu\nu}, \tag{11.2}$$
$$S_{ij} = P_i^\mu P_i^\nu T_{\mu\nu}, \tag{11.3}$$

with n^μ the unit normal vector to the spatial hypersurfaces of constant coordinate time, and P_ν^μ the projector operator onto these hypersurfaces.

Notice that since this code is for spherically symmetric spacetimes, we in fact only have one component for the momentum density, which the code takes to be the covariant one (index down): $J_A := J_r$. Also, we only have two independent components of the spatial stress tensor which the code takes with mixed indices (one up, one down): $S_A := S_r^r$ and $S_B := S_\theta^\theta$.

In the code, these quantities are defined even for vacuum (they are set to zero), since they are required both in the evolution equations and for the calculation of the constraints.

If you want to add a new matter model, you need to be sure that you add the appropriate expressions for these quantities in the file `src/matter/matter.f90`. Just have a look at what is already there to guide you, and try to follow the same conventions.

The type of matter used by the code is controlled by the character-type parameter `mattertype`. At the moment it allows the following values:

`vacuum`	There is no matter (this is the default).
`cosmo`	Cosmological constant.
`scalar`	Real scalar field. The scalar field is assumed to have a potential whose form is controlled by the parameter `scalarpotential`, which can take the values (`none`,`phi2`,`phi4`), corresponding to a free scalar field (`none`), a massive scalar field with a potential of the form $V = m^2 \Phi^2/2$ (`phi2`), where the mass m is give by the parameter `scalar_mass`, or a potential of the form $V = m^2 \Phi^2/2 + \lambda \Phi^4/4$ (`phi4`), with the coefficient of the self-interaction term given by the parameter `scalar_lambda`.
`ghost`	Ghost scalar field. Similar to the scalar field above, but the sign of the stress-energy tensor is reversed.
`complex`	Complex scalar field. The complex scalar field is assumed to have a potential whose form is controlled by the parameter `complexpotential`, which can take the values (`none`,`phi2`,`phi4`), corresponding to a free scalar field (`none`), a massive scalar field with a potential of the form $V = m^2 \vert\Phi\vert^2/2$ (`phi2`), where the mass m is give by the parameter `complex_mass`, or a potential of the form $V(\phi) = m^2 \vert\Phi\vert^2/2 + \lambda\vert\Phi\vert^4/4$ (`phi4`), with the coefficient of the self-interaction term given by the parameter `complex_lambda`.
`nonmin`	Non-minimally coupled scalar field (Note: I need to explain this).
`proca`	Real Proca field. This is essentially a massive electromagnetic field, with mass given by the parameter `proca_mass`.
`complexproca`	Complex Proca field. This is a complex Proca field, with mass given by the parameter `cproca_mass`.
`dirac`	Dirac spinor field.
`electric`	Electric field.

`fluid`	Perfect fluid.
`dust`	Dust (perfect fluid with zero pressure).

Any new type of matter model should be added to the list of allowed values for this parameter in the file `param.f90`. Notice that the code allows one to have several different types of matter at the same time, the corresponding stress-energy tensors are just added together. For example, if you want to have a real scalar field and a complex scalar field evolving together you can add the following line to the parameter file:

```
mattertype = scalar,complex
```

Similarly, for a charged complex scalar field you must have:

```
mattertype = complex,electric
```

12   Constraints

12.1   Hamiltonian and momentum constraints

When required for output, the code calculates the Hamiltonian and momentum constraints and saves them in the arrays `ham` and `mom`. Notice that since we are in spherical symmetry, the momentum constraint only has a non-zero radial component.

The expression used for the Hamiltonian constraint is:

$$\texttt{ham} := H = R - \left(K_A^2 + 2K_B^2 \right) + \frac{2}{3}\,\mathrm{tr}K - 16\pi\rho \, , \tag{12.1}$$

with R the spatial Ricci scalar, and ρ the energy density of matter.

For the radial (covariant) momentum constraint we have:

$$\texttt{mom} := M_r = \partial_r K_A - \frac{2}{3}\,\partial_r \mathrm{tr}K + 6K_A\,\partial_r\phi + r^2 \left(\frac{\partial_r B}{B} + \frac{2}{r} \right) - 8\pi J_A \, , \tag{12.2}$$

with ϕ the conformal factor and J_A the covariant radial component of the momentum density.

Notice that for an exact solution both these constraints should be zero, so monitoring their behaviour, and particularly how they approach zero as we increase the resolution, gives us information about the accuracy of the code.

12.2   BSSN extra constraint

The BSSN formalism requires the introduction of an auxiliary variable Δ^r defined in terms of derivatives of the conformal metric components (see [3]). This is then promoted to an independent variable, and its evolution equation is modified using the momentum constraint.

We therefore end up with a new constraint related to the definition of Δ^r and stored for output int he array `CDeltar`:

$$\texttt{CDelatr} := A\Delta^r - \left(\frac{\partial_r A}{2A} - \frac{\partial_r B}{B} - 2r\lambda \right) \ , \tag{12.3}$$

with λ a regularization variable (see below). Again, this quantity should be zero for an exact solution.

12.3   Regularization constraints

Finally, for the regularization procedure we need to introduce two more auxiliary variables that again are evolved independently so we have two more constraints coming from their definitions that are saved for output in the arrays `Clambda` and `CKlambda`:

$$\texttt{Clambda} \ := \ r^2\lambda - \left(1 - \frac{A}{B} \right) \ , \tag{12.4}$$

$$\texttt{CKlambda} \ := \ r^2 K_\lambda - (K_A - K_B) \ . \tag{12.5}$$

13   Slicing conditions

The slicing condition is controlled by the parameter `slicing`, which at the moment can take one of the following values (default is `slicing=harmonic`):

> `static`   The lapse remains static at its initial value and does not evolve.

harmonic   This is a harmonic-type slicing condition of the form:

$$\partial_t \alpha = -\alpha^2 f \, \text{tr}K + \beta^r \partial_r \alpha \ , \tag{13.1}$$

where f is a real positive number controlled by the parameter `gauge_f` (default is `gauge_f=1`). True harmonic slicing really corresponds to `gauge_f=1`, but here we allow any positive real number. This is a hyperbolic slicing condition corresponding to a speed of propagation of the gauge given by $\sqrt{f}$.

1+log   This is a slicing condition of the 1+log family, which is very similar to the harmonic family above:

$$\partial_t \alpha = -\alpha f \, \text{tr}K + \beta^r \partial_r \alpha \ , \tag{13.2}$$

where again f is a positive real number controlled by the same parameter `gauge_f`. Standard 1+log slicing corresponds to `gauge_f=2` (though as mentioned above the default value is 1).

shockavoid   This corresponds to the family of shock avoiding slicing conditions, which is similar to the 1+log condition above but with $f(\alpha)$ a function given by (see comments in file `src/geometry/auxiliary_geometry.f90`):

$$f(\alpha) = 1 + k/\alpha^2 \ . \tag{13.3}$$

maximal   This is maximal slicing which corresponds to the condition that the trace of the extrinsic curvature remains equal to zero during evolution (which guarantees that the spatial volume elements remain constant), and which implies that the lapse must satisfy the following elliptic equation:

$$\nabla^2 \alpha = \alpha K_{ij} K^{ij} + 4\pi \alpha \left(\rho + \text{tr}S \right) \ . \tag{13.4}$$

In spherical symmetry the Laplacian operator reduces to:

$$\nabla^2 = \partial_r^2 + \left(\frac{2}{r} - \frac{\partial_r A}{2A} + \frac{\partial_r B}{B} + 2 \, \partial_r \phi \right) \partial_r \ . \tag{13.5}$$

Notice that the choices `harmonic`, `1+log` and `shockavoid` are special cases of the Bona-Masso family of slicing conditions that has the general form:

$$\partial_t \alpha = -\alpha^2 f(\alpha) \text{tr}K + \beta^r \partial_r \alpha \ . \tag{13.6}$$

with $f(\alpha)$ an arbitrary (positive) function of the lapse.

The initial value of the lapse can also be controlled by the parameter `ilapse` which can take the values (default `ilapse=none`):

none	The initial lapse it first set to 1, but can later be modified by initial data routines and will not be touched after this.
one	The initial lapse is set to 1 again AFTER all initial data routines have been called.
isotropic	The initial lapse it set to $\alpha = 2/\psi - 1$ after all initial data routines have been called, with ψ the conformal factor. Notice that this is the correct value of the lapse for a static Schwarzschild spacetime in isotropic coordinates, but we can use it for any type of initial data and it should still result in a lapse that keeps the spacetime approximately static far away.
psiminus2	The initial lapse is set to $\alpha = 1/\psi^2$ after all initial data routines have been called, with ψ the conformal factor. This is useful for black hole evolutions where ψ becomes infinite at the origin, and results in a '"pre-collapsed" initial lapse that is already zero and smooth at $r = 0$.
psiminus4	Similar to the case above, but sets the initial lapse to $\alpha = 1/\psi^4$.

There is also the possibility of adding a Gaussian to the initial value of the lapse. This is useful for perturbing the initial data and causing interesting gauge dynamics. This is controlled by the parameters:

lapsegauss	Logical parameter that if set to `.true.` adds a Gaussian to the initial lapse (default is `.false.`).
lapse_a0	Amplitude of Gaussian.
lapse_r0	Center of Gaussian.
lapse_s0	Width of Gaussian.

14 Shift conditions

Notice that since we are in spherical symmetry the shift vector only has a radial component. The shift condition is controlled by the parameter `shift`, which at the moment can take one of the following values (default is `shift=none`):

none   There is no shift, and all shift terms in the evolution equations are ignored. This is the default.

zero   The shift is set to zero, but all the shift terms in the evolution equations are calculated. This is useful for testing.

static   The shift is non-zero but it does not evolve in time. Again, useful for testing.

Gammadriver1   Parabolic Gamma-driver shift. The shift condition is given by:

$$\partial_t \beta^r = \xi \partial_t \Delta^r \ . \tag{14.1}$$

Gammadriver2   Standard second order hyperbolic Gamma-driver shift. The shift condition is given by:
$$\partial_t^2 \beta^r = \xi \partial_t \Delta^r - \eta \partial_t \beta^r \ . \tag{14.2}$$

In the code this is written in second order form as:

$$\partial_t \beta^r = \mathcal{B} \ , \tag{14.3}$$
$$\partial_t \mathcal{B} = \xi \partial_t \Delta^r - \eta \mathcal{B} \ . \tag{14.4}$$

This is in fact the standard Gamma-driver used in most BSSN codes. Notice that in the code $\mathcal{B}$ is in fact called `dtbeta`.

The Gamma-driver shift conditions above relate the evolution of the shift to that of the BSSN auxiliary variable Δ^r. The values of the coefficients ξ and η are controlled with the real parameters `drivercsi` and `drivereta`. There is also the logical parameter `driveradv` which if set to `.true.` modifies the Gamma driver conditions by adding advection terms on the shift (by default it is `.false.`).

Just as in the case of the lapse, there is the possibility of adding a Gaussian to the initial value of the shift. This is useful for perturbing the initial data and causing interesting gauge dynamics. This is controlled by the parameters:

shiftgauss	Logical parameter that if set to `.true.` adds a Gaussian to the initial shift (default is `.false.`).
shift_a0	Amplitude of Gaussian.
shift_r0	Center of Gaussian.
shift_s0	Width of Gaussian.

15 Boundary conditions

The outer boundary conditions for the evolution are controlled by the character-type parameter `boundary`, which at the moment can take one of the following values (the default is `flat`):

none	No boundary condition is applied. The sources are applied all the way to the boundary using one-sided differences.
static	The sources are set to zero at the boundary. This is done only for evolving arrays that are not declared with the keyword NOBOUND.
flat	The sources at the boundary are copied from their value one grid point in. This is done only for evolving arrays that are not declared with the keyword NOBOUND.
radiative	Outgoing-wave boundary condition. This is somewhat more involved as it uses some information from eigenfields and eigenspeeds, and works quite well in practice.
constraint	Constraint preserving boundary condition. This condition uses information from the constraints in order to avoid introducing constraint violations at the boundary.

Notice that all the boundary conditions are always applied at the level of the source terms, and are applied only for some of the evolving arrays and not all of them. In general, evolving arrays that are declared with the keyword NOBOUND have no need for a boundary condition since their source terms can be calculated all the way to the boundary.

Static and flat boundary conditions are applied in the automatically generated file `src/auto/simpleboundary.f90`. The radiative and constraint preserving boundary conditions are explained with more detail below.

15.1 Radiative boundaries

In this case we apply outgoing wave boundary conditions taking into account information about the characteristic structure of the evolution system. For the case of the geometric boundaries we apply the radiative boundary condition only to the evolving arrays (`trK,KTA,Klambda`), and when we are using a shift condition of the Gammadriver family also to `dtbeta`. For matter variables (for example scalar fields), radiative boundaries must be coded in their respective source routines. Here we will explain in some detail how the radiative boundaries are applied to the geometric variables, since they are somewhat involved.

The characteristic structure of the BSSN system in spherical symmetry is analysed in detail in [4]. For the case of zero or static shift, one finds that there are two families of travelling modes: One family associated with the slicing condition that travels with the gauge speed $\alpha\sqrt{f/A}$, with $f(\alpha)$ the Bona-Masso function that in the code is controlled by the parameter `gauge_f`, and a second family whose speed depends on the multiple of the constraints that has been added to the evolution equation for Δ^r, which in the code is controlled by the parameter `eta`. For standard BSSN (`eta=2`) the second family in fact travels at the speed of light, but these modes are NOT associated with gravitational waves (there are no gravitational waves in spherical symmetry), but rather with the violation of the constraints.

From the characteristic analysis one can show that the trace of the extrinsic curvature $\mathrm{tr}K$ is a pure gauge quantity associated with the slicing condition. For the boundary condition we then assume that far away $\mathrm{tr}K$ behaves as an outgoing spherical wave with speed $v_f \sim \sqrt{f}$:

$$\mathrm{tr}K \sim g(r - v_f t)/r \ , \tag{15.1}$$

with g an arbitrary wave profile. This can easily be shown to imply that:

$$\partial_t \mathrm{tr}K \sim -v_f \left(\partial_r \mathrm{tr}K + \mathrm{tr}K/r \right) \ . \tag{15.2}$$

This is the radiative boundary condition used in the code for $\mathrm{tr}K$ (except for maximal slicing when we just set $\mathrm{tr}K = 0$ everywhere). It works remarkably well and results in very small reflections from the boundary. [1]

In order to obtain a boundary condition for K_A (`KTA` in the the code) we use a similar idea, but somewhat more involved. From the characteristic analysis one also finds that the eigenfields

---

[1]Notice that a similar boundary condition can be used for some matter fields, for example real and complex scalar fields, but assuming that the outgoing spherical waves travel at the speed of light.

that move at the speed of light for $\eta = 2$ mentioned above (associated with the constraints) are related to $K_A - 2/3\,\mathrm{tr}K$ (this combination is in fact the sum of the outgoing and incoming eigenfields). It is clear that the outgoing eigenfield will behave as an outgoing spherical wave moving at the speed of light, while the incoming field should be small (it will not be zero since they are coupled through source terms). So, if we assume to first approximation that this combination behaves as:

$$K_A - 2/3\,\mathrm{tr}K \sim h(r - v_l t)/r \ , \tag{15.3}$$

with $v_l \sim 1$ the speed of light far away and h some wave profile, we find that

$$\partial_t \left(K_A - 2/3\,\mathrm{tr}K \right) \sim - \left[\partial_r \left(K_A - 2/3\,\mathrm{tr}K \right) + \left(K_A - 2/3\,\mathrm{tr}K \right)/r \right] \ . \tag{15.4}$$

And combining this with the boundary condition for $\mathrm{tr}K$ above, we finally find:

$$\partial_t K_A \sim - \left(\partial_r K_A + K_A/r \right) - 2/3 \left(v_f - 1 \right) \left(\partial_r \mathrm{tr}K + \mathrm{tr}K/r \right) \ . \tag{15.5}$$

This boundary condition again turns out to be very well-behaved, and results in very small reflections at the boundary, but it will violate the constraints (see the following Section). Notice also that K_A and the regularization variable K_λ are closely related to each other, so having a boundary condition for K_A immediately gives us a boundary condition for K_λ.

The last variable to consider is the BSSN function Δ^r. In practice we have found that when the shift is zero, or non-zero but time-independent, or if we are using the parabolic shift condition `shift=Gammadriver1`, we can just calculate the source for Δ^r all the way to the boundary using one-sided differences and this results in a stable and well-behaved evolution. For the `shift=Gammadriver2` shift condition the sources for Δ^r can still be calculated all the way to the boundary, but we now need to impose a boundary condition on the time derivative of the shift $\mathcal{B}$ (which in the code is called `dtbeta`).

The characteristic analysis for the lagrangian case when we take `shift=Gammadriver2` is not very difficult, but for the eurlerian case it becomes much more complicates, so in the code the boundary conditions assume that we always use the lagrangian formulation. In that case it turns out that the shift eigenfields are related to the quantity $\mathcal{B} + Q\partial_r\alpha$, with Q given far away as $Q = v_\xi^2/(v_\xi^2 - v_f^2)$, and where $v_\xi \sim \sqrt{4\xi/3}$ is the shift gauge speed and $v_f \sim \sqrt{f}$ the lapse gauge speed defined above. Notice that Q diverges when $v_f = v_\xi$, which happens for example for pure harmonic slicing and the standard Gamma driver shift, in which case we have $v_f = v_\xi = 1$. This is a flaw of the standard Gamma driver shift condition, the characteristic structure becomes ill-defined in this case. However, the boundary condition below should still

work in this case.

We now assume that far away we have:

$$\mathcal{B} + Q\partial_r\alpha \sim u(r - v_\xi t)/r \ , \tag{15.6}$$

One can show that the above assumption then implies:

$$\partial_t\mathcal{B} \sim -v_\xi \left(\partial_r\mathcal{B} + \mathcal{B}/r\right) - \frac{v_\xi^2}{v_\xi + v_f}\left(\partial_r^2\alpha + \partial_r\alpha/r\right) \ . \tag{15.7}$$

We can now apply this condition at the boundary, using one-sided derivatives for the lapse, and it works quite well. But we still need to make one final transformation. The above expression has two drawbacks: First, it involves second derivatives of the lapse, which at the boundary are less accurate and should be avoided if possible, but worse, even though it works well for travelling waves, for stationary solutions with a non-trivial lapse it introduces a drift at the boundary. Both these drawbacks can be addressed if we notice that, for an outgoing travelling wave the incoming slicing eigenfield should be very small. From the characteristic analysis we find that this implies $\partial_r\alpha \sim v_f\,\mathrm{tr}K$. We can then rewrite the above expression as:

$$\partial_t\mathcal{B} \sim -v_\xi \left(\partial_r\mathcal{B} + \mathcal{B}/r\right) - \frac{v_f v_\xi^2}{v_\xi + v_f}\left(\partial_r\mathrm{tr}K + \mathrm{tr}K/r\right) \ . \tag{15.8}$$

This expression does not involve second derivatives of the lapse, and has the advantage that for static solutions $\mathrm{tr}K$ vanishes, so it should reduce the drift at the boundary. This boundary condition works surprisingly well and is the one used in the code.

15.2   Constraint preserving boundaries

The radiative boundaries explained above introduce constraint violations at the boundaries, which arise directly from the condition applied to K_A. Here we use a method based on [4], but modified in a way that makes it both simpler and more robust.

We start from the fact, mentioned above, that the incoming and outgoing modes associated with the constraints, ω_l^+ and ω_l^-, are such that

$$\omega_l^+ + \omega_l^- = K_A - 2/3\,\mathrm{tr}K \ , \tag{15.9}$$

which clearly implies:

$$\partial_t K_A = 2/3\,\partial_t\mathrm{tr}K + \partial_t\left(\omega_l^+ + \omega_l^-\right) \ . \tag{15.10}$$

28

Assume now that the time derivative of $\mathrm{tr}K$ at the boundary has already been obtained from the radiative condition above. This will not in itself violate the constraints as $\mathrm{tr}K$ is pure gauge.

It turns out that the time derivative of $\left(\omega_l^+ + \omega_l^-\right)$ can be written in terms of the constraints in such a way that if one then asks for the constraints to vanish this quantity evolves only through source terms (that is, no derivatives of the extrinsic curvature or second derivatives of the metric). The resulting expression is not terribly complicated but it is not particularly illuminating either, so I will not write it here.

We can the simply use the expression above as a boundary condition for K_A that will not violate the constraints. In practice, we find that as a pulse of constraint violation reaches the boundary, it gets reflected with an amplitude of roughly the same size, and this converges away with increased resolution.

16   Initial data

The type of initial data is controlled by the character-type parameter `idata`. If you add a new type of initial data it should be appended to the list of allowed values for this parameter in the file `src/base/param.f90`. You should also add a corresponding call to your initial data routine in the file `src/base/initial.f90`.

There are in fact many more initial data types than the ones discussed here that have been added through the years (like charged boson stars, Proca stars, Dirac stars, etc.). Have a look at the file `src/base/param.f90` to see the values allowed by `idata`, and the corresponding routines that calculate them (routines with names that typically start with `idata_` and can be found in the directories `src/geometry` and `src/matter`).

Simple types of vacuum initial data already there are:

`idata=minkowski`	The initial metric is simply set to Minkowski and the extrinsic curvature is set to zero. Notice that one can still have non-trivial gauge evolution if one adds a Gaussian perturbation to the initial lapse (see Section 13).
`idata=schwarzschild`	The initial metric is set to that corresponding to a Schwarzschild black hole in isotropic coordinates: $A = B = 1$, $\psi = 1 + M/(2r^2)$. Here M is the mass of the black hole controlled by the parameter

	BHmass (default is 1). The initial extrinsic curvature is set to zero.
idata=trumpetBH	The initial data corresponds to a Schwarzschild black hole in the so-called "trumpet" gauge. This data is not analytic and must be solved numerically.
idata=reissnernordstrom	The initial data corresponds to a Reissner-Nordstrom charged black hole.

There is also already some initial data with matter for the case of real or complex scalar fields:

idata=scalarpulse	The initial data is a simple Gaussian pulse of a real scalar field. The amplitude, center and width of the Gaussian are controlled by the parameters scalar_a0, scalar_r0, scalar_s0. The initial data is assumed to be time-symmetric, and the Hamiltonian constraint is solved for the conformal factor.
idata=ghostpulse	Similar as the case above, but for a ghost scalar field.
idata=complexpulse	The initial data is a simple Gaussian pulse of a complex scalar field. The amplitude, center and width of the Gaussian, for the real and imaginary parts, are controlled by the parameters complexR_a0, complexR_r0, complexR_s0 and complexI_a0, complexI_r0, complexI_s0, respectively. The initial data is assumed to be time-symmetric, and the Hamiltonian constraint is solved for the conformal factor.
idata=bosonstar	The initial data corresponds to a Boson Star. This requires complex scalar field matter with non-zero mass. For this initial data one must set the central value of the real part of the scalar field with the parameter complex_phi0, and one must also give two bracketing values of the frequency with the parameters omega_left and omega_right. Notice that these values need to be given with high precision or the solution will fail.

In order to restart from a checkpoint file, we must have:

idata=checkpoint

17 Apparent horizons

For spacetimes that contain black holes, either from the initial data or through gravitational collapse during the evolution, the code can look for apparent horizons. Since we are in spherical symmetry, this in fact reduces to finding a zero of the expansion for null light rays given by:

$$\Theta := \frac{1}{\psi^2 A^{1/2}} \left(\frac{2}{r} + \frac{\partial_r B}{B} + 4\,\partial_r \phi \right) - 2K_B \; , \tag{17.1}$$

where ψ is the conformal factor. An apparent horizon will correspond to the outermost value of r for which the expansion Θ is equal to zero.

The code will look for apparent horizons whenever the logical parameter `ahfind` is set to `.true.` (by default it is `.false.`). It will look for horizons every certain number of time steps, controlled by the value of the integer parameter `ahfind_every` (by default 1).

Once it locates a horizon, it will output its position as a function of time in the file `ah_radius.tl` (if no horizon is found this value is 0). It will also output its physical area, its Schwarzschild radius, and its mass to the files `ah_area.tl`, `ah_schw.tl` and `ah_mass.tl`, respectively. Notice that the mass of the apparent horizon is defined in terms of its area A as: $M_{AH} := \sqrt{A/16\pi}$.

It is sometimes useful to know the value of the lapse at the position of the horizon, so the code also outputs this value in the file `ah_alpha.tl`.

18 Other analysis tools

The code can also calculate many other quantities for analysis that are not listed here, such as for example the total Schwarzschild-like mass (`mass_sch`), the total integrated mass (`mass_int`), the areal radial coordinate (`r_area`), the expansion of null geodesics, the Weyl tensor, the 4D Ricci scalar, the curvature invariants I and J, total conserved charges for different types of fields, etc.

I will not enlist all those quantities here as they continue to grow all the time, but have a look at the files `src/geometry/analysis_geometry.f90` and `src/matter/matter_geometry.f90`. In order to calculate any such quantities you MUST ask for output of them in the parameter file.

19 Numerical methods

19.1 Time integration

For the time integration the code uses a method of lines, where the time integration and spatial differentiation are considered independent of each other.

For the evolution of the field variables we can use one of two methods:

1. Standard fourth order Runge-Kutta.

2. 3-step iterative Crank-Nicholson scheme (ICN). The ICN scheme for N iterations is defined as follows (with $S(f)$ the source term of the evolution equation):

$$
\begin{aligned}
f_0 &= f(t) \,, & (19.1) \\
f_{i+1} &= f_0 + dt/2 \; S(f_i) \quad \text{i from 1 to N-1} \,, & (19.2) \\
f_N &= f_0 + dt \; S(f_{N-1}) \,, & (19.3) \\
f_t + dt &= f_N & (19.4)
\end{aligned}
$$

That is, we do $N-1$ half steps starting always from $f(t)$ but evaluating the source term using the results from the previous step. Finally, we do one last full step. Doing only 2 ($N = 2$) steps is enough for second order accuracy, but we need to do at least 3 ($N = 3$) for stability. ICN is a rather robust method that can be applied to non-linear systems of equations, but it is somewhat dissipative.

The choice of the integration method is done through the parameter `integrator` which can take the values {`rk4,icn`}.

19.2 Spatial differencing

For spatial differentiation we can use either second or fourth order differencing (I have also added sixth and eight order, but this is ussually not needed as the time integration is at most fourth order). Close to the outer boundaries we in fact reduce the differencing always to second order. The choice of the differencing order is controlled by the parameter `order` which can take the values {`two,four`}.

19.3 Dissipation

The code uses Kreiss-Oliger type dissipation to improve stability. Often one can turn this off and things will still work, but is seems to be important for scalar field evolutions and also to keep a black hole spacetime stable for a long time. The dissipation of the geometric variables is controlled by the parameter `geodiss`, and the dissipation for the scalar field (both real and complex) is controlled by the parameter `scalardiss`. Both are set by default to a value of 0.01. Other type of matter fields have similar dissipation parameters (`nonmindiss`, `procadiss`, etcetera).

20   List of main code parameters

Here is a list of the main code parameters with their default values and ranges when applicable. For a list of all declared parameters see the file `src/base/param.f90`. But do notice that some of the "parameters" declared in `param.f90` should not be fixed in the parameter file since they are in fact defined at run time.

There are many more parameters that are not mentioned here, particularly related to new matter models or initial data routines. Have a look at the file `src/base/param.f90` where they are defined, they are usually explained in the comments).

- Grid parameters:

`dr0 (real(8))`	Spatial interval for base (coarsest) (default 1.0).
`Nrtotal (integer)`	Total number of grid points for base grid (default 10).
`Nl (integer)`	Number of refinement levels (default 1).
`ghost (integer)`	Number of ghost zones (default 0). Should NOT be fixed in the parameter file.
`intorder (integer)`	Order of interpolation (default 3). Don't change this unless you know what you are doing!
`boundary (character)`	Type of outer boundary condition. Can take the following values: `none`, `static`, `flat`, `radiative` (default `flat`).

- Time stepping parameters:

dt0 (real(8))	Time step for base (coarsest) grid (default 0.0). This should NOT be set in the parameter file, as it is calculated at run time based on the value of dt0 and dtfac.
dtfac (real(8))	Courant parameter dt/dr (default 0.5).
adjuststep (logical)	Dynamically adjust the time step in order to satisfy the Courant stability condition (default .false.).
Nt (integer)	Total number of time steps (default 10).

- Output parameters:

directory (character)	Name of output directory (default output).
Ninfo (integer)	Frequency of output to screen (default 1).
Noutput0D (integer)	Frequency of 0D output (default 1).
Noutput1D (integer)	Frequency of 1D output (default 1).
outvars0D (character)	Comma separated lists of variables that need 0D output (default alpha).
outvars1D (character)	Comma separated lists of variables that need 1D output (default alpha).
commentype (character)	Type of comment lines used on the output files (range = {xgraph, gnuplot}, default xgraph).

- Checkpoint parameters:

checkpoint (logical)	Do we want to output checkpoint files? (default .false.)
checkpointinitial (logical)	Do we want to output checkpoint the initial data? (default .true.)
Ncheckpoint (integer)	Frequency of checkpoint output (default 1000).
checkpointfile (character)	Name of checkpoint directory for restarting (default checkpoint).

- Evolution parameters:

spacetime (character)	Can take the values dynamic or background to choose if we want a dynamic spacetime (evolve the Einstein field equations) or a static background spacetime (default dynamic).

formulation (character)	Defines the formulation of the 3+1 equations to use, and can take the values `bssn` or `z4c` (default `bssn`).
bssnflavor (character)	Controls the evolution of the volume element in BSSN, and can take the values `lagrangian` or `eulerian` (default `lagrangian`).
integrator (character)	Method for time integration. Can take the values `icn` for 3-step iterative Crank-Nicholson, or `rk4` for fourth order Runge-Kutta (default `icn`).
order (character)	Order of spatial finite-differencing. Can take the values `two` o `four` (default `two`).
nolambda (logical)	If true do not use regularization variables (default `.false.`).
noDeltar (logical)	If true do not use the BSSN `Deltar` variable, and calculate it in terms of metric derivatives (default `.false.`).
noKTA (logical)	If true do not evolve `KTA` as an independent variable and calculate it fdrom `Klambda` (default `.true.`).
chimethod (logical)	If true evolve $\chi = \exp(-n\phi)$ instead of ϕ. This is useful for black hole evolutions (default `.false.`).
chipower (integer)	Value of n in the definition $\chi = \exp(-n\phi)$ (default 2).
regular2 (logical)	If true evolve $K_{\lambda 2} = K_\lambda / \psi^4$ instead of K_λ. This is useful for black hole evolutions (default `.false.`).
eta (real)	Parameter that controls the multiple of the momentum constraints that is added to the evolution equation for Δ^r in the BSSN formulation (default `2.0`).

21 Editing the code

If you are planning to edit the code and add your own new parameters, arrays or routines, here are some details you should know.

21.1 Adding parameters

All parameters for the code should be declared in the file `src/base/param.f90`, using a very specific format:

```
type ::  name = value
```

Here `type` can be any of the standard `FORTRAN` variable types (`logical`, `integer`, `real`, `character`), `name` is the name of the parameter, and `value` is a default initial value that makes some sense for the code. Only one parameter can be declared per line, since this file will be read at compilation time by a perl script that expects that structure.

The following variations are permitted: Character-type parameters that are allowed to receive multiple values at the same time (separated by commas) should be declared as:

```
character ::  name = value !  multiple
```

Also, a range can be defined for character-type parameters as:

```
character ::  name = value !  range = (value1,value2,...,valuen)
```

The range is not compulsory. If it is not there, any value is allowed (for example, directory names).

REMEMBER: All parameters must have a default value, as otherwise the code will not compile. The initial value should basically correspond to the code NOT doing anything weird. That is, initialize to Minkowski, static slicing, no shift, vacuum, and all special features turned off.

21.2 Adding arrays

Since this is a 1D code, it works with a series of 1D arrays that have all the dynamical and analysis variables. All arrays must be declared in the file `src/base/arrays.config`, using a very specific format.

Arrays must be declared to be either `REAL` or `COMPLEX`, and only one array can be declared per line. Control, statements are indicated in a comment and must include the following three

keywords:

```
SYMMETRY, INTENT, STORAGE
```

Notice that they must always come in this order, and they must be separated by commas.

The keyword SYMMETRY can only have three values:

- `SYMMETRY = +1`. The variable is even at origin (no space is allowed between `+` and 1).

- `SYMMETRY = -1`. The variable is odd at origin (no space is allowed between `-` and 1).

- `SYMMETRY = 0`. The variable will NOT be touched when updating the symmetries at the origin.

NOTE: In fact it is possible to set `SYMMETRY` equal to a regular expression that evaluates to plus or minus 1, but there can be NO white spaces or the perl script will complain, for example:

```
SYMMETRY = +(1.d0-2.d0*mod(var,2)) or SYMMETRY = -(1.d0-2.d0*mod(var,2))
```

with `var` some code variable.

The keyword `INTENT` can have one of the following values:

- `INTENT = EVOLVE`. The array is a main dynamical variable and will be updated during the evolution loop. For each array of this type named `var`, three extra arrays will be created: `var_p` = previous time level, `svar` = source term for evolution, `var_a` = auxiliary array.

- `INTENT = AUXILIARY`. The array is not one of the main evolved variables, but can contain other important variables (radius, derivatives, etc.).

- `INTENT = OUTPUT`. The array will be calculated for output, so memory will only be allocated if we want output for this array.

- `INTENT = POINTER`. This array is actually a pointer. They are declared with STORAGE always, but they are not actually allocated.

The keyword `STORAGE` can have one of the following values:

- `STORAGE = ALWAYS`. Storage for this variable is always on. If the variable is declared with `INTENT=OUTPUT` this means it will be on when we need output.

- `STORAGE = CONDITIONAL(*)`. In this case storage will be conditional on the condition inside the parenthesis. Notice that the condition must be a correctly formatted `FORTRAN` logical statement involving one of the parameters of the code (see e.g. the arrays associated with the shift).

The are other optional keywords that can also be added to an array definition. They are:

- The keyword `ONELEVEL`, when present, indicates that the array has only one grid level. This is useful for example for the synchronization pointer.

- The keyword `NOBOUND`, when present, indicates that an evolving array has no need for a boundary condition since its source can be calculated all the way to the boundary.

- The keyword `ZEROD`, when present, indicates that the "array" is not really an array but rather just a global real variable. This is useful, for example, when integrating the background in a cosmological spacetime.

21.3   Adding routines

Routines should be added directly in the sub-directories `src/geometry` or `src/matter` with the extension `.f90`. There is no need to modify the Makefile, as it will automatically compile and link all files it finds with that extension.

Since all the parameters and arrays are declared respectively in the files `src/base/param.f90` and `src/base/arrays.config`, if you want your routine to see them you need to add the following two lines immediately after the name of the routine (before declaring any new variables):

```
use param
use arrays
```

Since the code can run in parallel, if you need information about what processo your are on, of you want to use MPI commands, you should also add:

```
use mpi
use procinfo
```

Finally, if you need to use the standard finite-differencing routines that are already there you should also add:

```
use derivatives
use derivadvect
```

21.4   Adding new initial data

If you want to add a new initial data the first thing you need to do is to add a new option to the allowed values of the parameter `idata` in the file `src/base/param.f90`.

Once you have done this you need to edit the file `src/base/initial.f90`. If you new initial data is analytic and simple, you can code it directly in that file, otherwise, you need to create a new initial data routine and add a call to it there. Your initial data routine can be added to the adequate directory (`src/geometry` if it is for vacuum, or `src/matter` if it is for matter) with a name of the form `idata_name` (see the examples already in the code).

21.5   Adding new matter models

The code currently has the following matter models: `scalar, complex ,ghost, proca, complexproca, dirac, fluid, dust`. If you want to add a new type of matter you need to edit a few things.

First, you must name you matter model and add it to the list of allowed values of the parameter `mattertype` in the file `src/base/param.f90`. You should also add a new section to that file with whichever extra parameters you need to control the evolution and initial data for you matter model.

Next you need to add initial data routines and evolution routines for your matter model in the directory `src/matter`. You need to make sure this routines are called by modifying the files `src/base/initial.f90` and `src/matter/sources_matter.f90`.

Finally, don't forget to add the expressions for the BSSN matter terms (energy density, momentum density, stress tensor, charge density, etc.) for your new matter model to the file `src/matter/stresstensor.f90` (see the examples already there).

21.6   CVS

CVS stands for "Concurrent Versions System", and chances are you got this code through CVS so you already know what it is. If you don't, CVS is a system for keeping well organized versions of a code developed by several people, by having the "official version" on a central repository and keeping track of all new changes. It is amazing how rapidly one looses track of what is the

"most up-to-date version" of a multi-author code (or any multi-author files for that matter) without using something like CVS. [2]

If you add new routines to the code that you think might be useful for other users, it would be nice to add them to the CVS repository. Just ask Miguel Alcubierre if you need access to the repository (malcubi@nucleares.unam.mx).

22 Ollingraph

The code includes a simple visualization package called `ollingraph`. It is written in Python and uses Matplotlib to plot simple line plots and animations. At the moment it should work fine under python 3.11. It is supposed to have similar functionality to the old xgraph and ygraph packages, and expects the data files in the same format (see below).

The plots are quite simple, and meant for quick/dirty interactive visualization. These plots are not supposed to be used for figures intended for publication (use gnuplot or something similar for that). To use the package type:

```
ollingraph file1 file2 ...  filen
```

When plotting several data files at once, they are all assumed to be of the same type. One can also add a custom name for the plot using the option –title:

```
ollingraph --title="Plot name" file1 file2 ...  filen
```

If this option is not there the plot is just named after the name of the data file being plotted, assuming there is only one, or just says "Multiple data files" if there is more than one file.

Before using it make sure that `ollingraph` has execution permission:

```
chmod +x ollingraph
```

The data files are expected to have the following format:

---

[2]I know CVS is old fashioned, and you might prefer to use SVN for your own codes, or even something like GitHub. But I am used to CVS, so for this code you are stuck with it. Come to think of it, you might also think `FORTRAN` it old fashioned, but tough luck!

- 0D files (those ending in .tl):

  1. One comment line starting with `#` or `"` with the file name (this line could be missing and it should still work).

  2. A series of lines with the data points, with the x and y values separated by blank spaces.

- 1D evolution-type files (those ending in .rl):

  1. Each time step begins with a comment line starting with `#` or `"` that contains the time in the format:

     ```
     #Time = (real number)
     ```

  2. A series of lines with the data points, with the x and y values separated by blank spaces.

  3. One or more blank lines to separate the next time step.

References

[1] "Regularization of spherically symmetric evolution codes in numerical relativity", M. Alcubierre, J.A. Gonzalez, Comput.Phys.Commun.**167**, 76-84 (2005); gr-qc/0401113.

[2] "Regularization of spherical and axisymmetric evolution codes in numerical relativity", M. Ruiz, M. Alcubierre, D. Nuñez, Gen.Rel.Grav. **40**, 159-182 (2008); arXiv:0706.0923 [gr-qc].

[3] "Formulations of the 3+1 evolution equations in curvilinear coordinates", M. Alcubierre and M. D. Mendez, Gen.Rel.Grav. **43**, 2769 (2011); arXiv:1010.4013 [gr-qc].

[4] "Constraint preserving boundary consitions for the BSSN formulation in spherical symmetry", M. Alcubierre and J.M. Torres, Class.Quant.Grav. **32**, 035006 (2015); arXiv:1407.8529 [gr-qc].