5.3 Thévenin's and Norton's Theorems

Thus far we have presented a number of techniques for circuit analysis. At this point we will add two theorems to our collection of tools that will prove to be extremely useful. The theorems are named after their authors, M. L. Thévenin, a French engineer, and E. L. Norton, a scientist formerly with Bell Telephone Laboratories.

Suppose that we are given a circuit and that we wish to find the current, voltage, or power that is delivered to some resistor of the network, which we will call the load. *Thévenin's theorem* tells us that we can replace the entire network, exclusive of the load, by an equivalent circuit that contains only an independent voltage source in series with a resistor in such a way that the current–voltage relationship at the load is unchanged. *Norton's theorem* is identical to the preceding statement except that the equivalent circuit is an independent current source in parallel with a resistor.

Note that this is a very important result. It tells us that if we examine any network from a pair of terminals, we know that with respect to those terminals, the entire network is equivalent to a simple circuit consisting of an independent voltage source in series with a resistor or an independent current source in parallel with a resistor.

In developing the theorems, we will assume that the circuit shown in **Fig. 5.5a** can be split into two parts, as shown in **Fig. 5.5b**. In general, circuit B is the load and may be linear or nonlinear. Circuit A is the balance of the original network exclusive of the load and must be linear. As such, circuit A may contain independent sources, dependent sources and resistors, or any other linear element. We require, however, that a dependent source and its control variable appear in the same circuit.

Circuit A delivers a current i to circuit B and produces a voltage v_o across the input terminals of circuit B. From the standpoint of the terminal relations of circuit A, we can replace circuit B by a voltage source of v_o volts (with the proper polarity), as shown in **Fig. 5.5c**. Since the terminal voltage is unchanged and circuit A is unchanged, the terminal current i is unchanged.

Now, applying the principle of superposition to the network shown in Fig. 5.5c, the total current i shown in the figure is the sum of the currents caused by all the sources in circuit A and the source v_o that we have just added. Therefore, via superposition the current i can be written

$$i = i_o + i_{sc} \qquad \textbf{5.1}$$

where i_o is the current due to v_o with all independent sources in circuit A made zero (i.e., voltage sources replaced by short circuits and current sources replaced by open circuits), and i_{sc} is the short-circuit current due to all sources in circuit A with v_o replaced by a short circuit.

The terms i_o and v_o are related by the equation

$$i_o = \frac{-v_o}{R_{Th}} \qquad \textbf{5.2}$$

where R_{Th} is the equivalent resistance looking back into circuit A from terminals A-B with all independent sources in circuit A made zero.

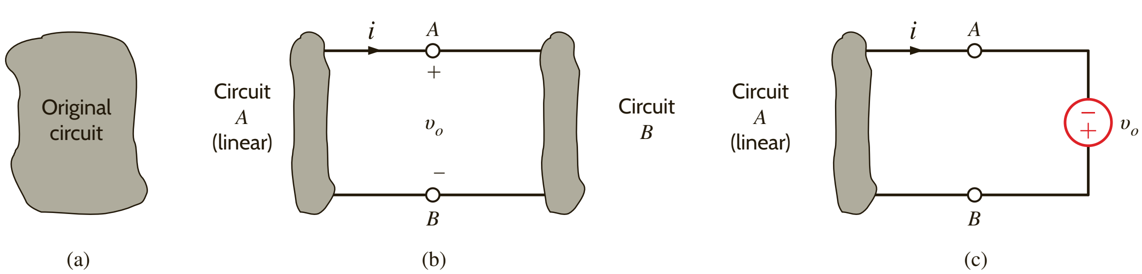

Figure 5.5

Concepts used to develop Thévenin's theorem.

Substituting Eq. (5.2) into Eq. (5.1) yields

$$i = -\frac{v_o}{R_{\text{Th}}} + i_{\text{sc}} \qquad 5.3$$

This is a general relationship and, therefore, must hold for any specific condition at terminals A-B. As a specific case, suppose that the terminals are open-circuited. For this condition, $i = 0$ and v_o is equal to the open-circuit voltage v_{oc}. Thus, Eq. (5.3) becomes

$$i = 0 = \frac{-v_{\text{oc}}}{R_{\text{Th}}} + i_{\text{sc}} \qquad 5.4$$

Hence,

$$v_{\text{oc}} = R_{\text{Th}} i_{\text{sc}} \qquad 5.5$$

This equation states that the open-circuit voltage is equal to the short-circuit current times the equivalent resistance looking back into circuit A with all independent sources made zero. We refer to R_{Th} as the Thévenin equivalent resistance.

Substituting Eq. (5.5) into Eq. (5.3) yields

$$i = \frac{-v_o}{R_{\text{Th}}} + \frac{v_{\text{oc}}}{R_{\text{Th}}}$$

or

$$v_o = v_{\text{oc}} - R_{\text{Th}} i \qquad 5.6$$

Let us now examine the circuits that are described by these equations. The circuit represented by Eq. (5.6) is shown in **Fig. 5.6a**. The fact that this circuit is equivalent at terminals A-B to circuit A in Fig. 5.5 is a statement of *Thévenin's theorem*. The circuit represented by Eq. (5.3) is shown in Fig. 5.6b. The fact that this circuit is equivalent at terminals A-B to circuit A in Fig. 5.5 is a statement of *Norton's theorem*.

Having demonstrated that there is an inherent relationship between the Thévenin equivalent circuit and the Norton equivalent circuit, we now proceed to apply these two important and useful theorems. The manner in which these theorems are applied depends on the structure of the original network under investigation. For example, if only independent sources are present, we can calculate the open-circuit voltage or short-circuit current and the Thévenin equivalent resistance. However, if dependent sources are also present, the Thévenin equivalent will be determined by calculating v_{oc} and i_{sc}, since this is normally the best approach for determining R_{Th} in a network containing dependent sources. Finally, if circuit A contains no *independent* sources, then both v_{oc} and i_{sc} will necessarily be zero. (Why?) Thus, we cannot determine R_{Th} by $v_{\text{oc}}/i_{\text{sc}}$, since the ratio is indeterminate. We must look for another approach. Notice that if $v_{\text{oc}} = 0$, then the equivalent circuit is merely the unknown resistance R_{Th}. If we apply an external source to circuit A—a test source v_t—and determine the current, i_t, which flows into circuit A from v_t, then R_{Th} can be determined from $R_{\text{Th}} = v_t/i_t$. Although the numerical value of v_t need not be specified, we could let $v_t = 1$ V and then $R_{\text{Th}} = 1/i_t$. Alternatively, we could use a current source as a test source and let $i_t = 1$ A; then $v_t = (1)R_{\text{Th}}$.

Before we begin our analysis of several examples that will demonstrate the utility of these theorems, remember that these theorems, in addition to being another approach, often

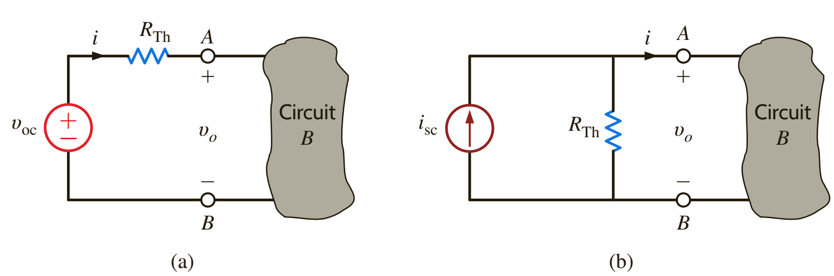

Figure 5.6
(a) Thévenin and (b) Norton equivalent circuits.

permit us to solve several small problems rather than one large one. They allow us to replace a network, no matter how large, *at a pair of terminals* with a Thévenin or Norton equivalent circuit. In fact, we could represent the entire U.S. power grid at a pair of terminals with one of the equivalent circuits. Once this is done, we can quickly analyze the effect of different loads on a network. Thus, these theorems provide us with additional insight into the operation of a specific network.

CIRCUITS CONTAINING ONLY INDEPENDENT SOURCES Consider for a moment some salient features of this example. Note that in applying the theorems there is no point in breaking the network to the left of the 3-V source, since the resistors in parallel with the current source are already a Norton equivalent. Furthermore, once the network has been simplified using a Thévenin or Norton equivalent, we simply have a new network with which we can apply the theorems again. The following example illustrates this approach.

EXAMPLE 5.5

Let us use Thévenin's and Norton's theorems to find V_o in the network in Example 5.3.

SOLUTION

The circuit is redrawn in Fig. 5.7a. To determine the Thévenin equivalent, we break the network at the 6-kΩ load as shown in Fig. 5.7b. KVL indicates that the open-circuit voltage, V_{oc}, is equal to 3 V plus the voltage V_1, which is the voltage across the current source. The 2 mA from the current source flows through the two resistors (where else could it possibly go!) and, therefore, $V_1 = (2 \times 10^{-3})(1\text{k} + 2\text{k}) = 6$ V. Therefore, $V_{oc} = 9$ V. By making both sources zero, we can find the Thévenin equivalent resistance, R_{Th}, using the circuit in Fig. 5.7c. Obviously, $R_{Th} = 3$ kΩ. Now our Thévenin equivalent circuit, consisting of V_{oc} and R_{Th}, is connected back to the original terminals of the load, as shown in Fig. 5.7d. Using a simple voltage divider, we find that $V_o = 6$ V.

To determine the Norton equivalent circuit at the terminals of the load, we must find the short-circuit current as shown in Fig. 5.7e. Note that the short circuit causes the 3-V source to be directly across (i.e., in parallel with) the resistors and the current source. Therefore, $I_1 = 3/(1\text{k} + 2\text{k}) = 1$ mA. Then, using KCL, $I_{sc} = 3$ mA. We have already determined R_{Th} and, therefore, connecting the Norton equivalent to the load results in the circuit in Fig. 5.7f. Hence, V_o is equal to the source current multiplied by the parallel resistor combination, which is 6 V.

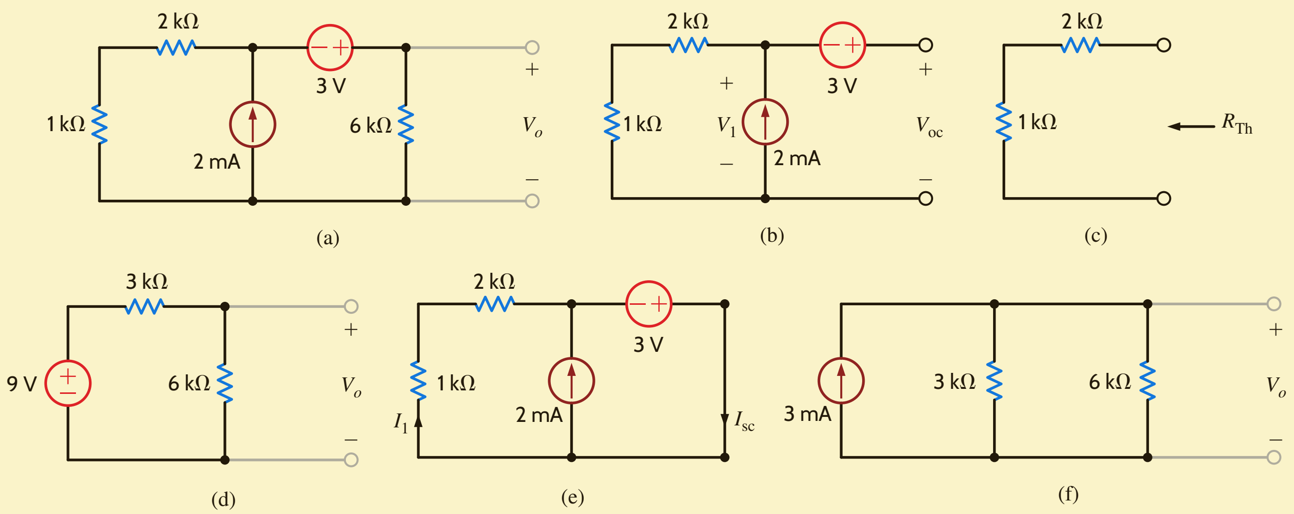

Figure 5.7
Circuits used in Example 5.5.

EXAMPLE 5.6

Let us use Thévenin's theorem to find V_o in the network in Fig. 5.8a.

SOLUTION

If we break the network to the left of the current source, the open-circuit voltage V_{oc_1} is as shown in Fig. 5.8b. Since there is no current in the 2-kΩ resistor and therefore no voltage across it, V_{oc_1} is equal to the voltage across the 6-kΩ resistor, which can be determined by voltage division as

$$V_{oc_1} = 12\left(\frac{6k}{6k + 3k}\right) = 8 \text{ V}$$

The Thévenin equivalent resistance, R_{Th_1}, is found from Fig. 5.8c as

$$R_{Th_1} = 2k + \frac{(3k)(6k)}{3k + 6k} = 4 \text{ k}\Omega$$

Connecting this Thévenin equivalent back to the original network produces the circuit shown in Fig. 5.8d. We can now apply Thévenin's theorem again, and this time we break the network to the right of the current source as shown in Fig. 5.8e. In this case, V_{oc_2} is

$$V_{oc_2} = (2 \times 10^{-3})(4k) + 8 = 16 \text{ V}$$

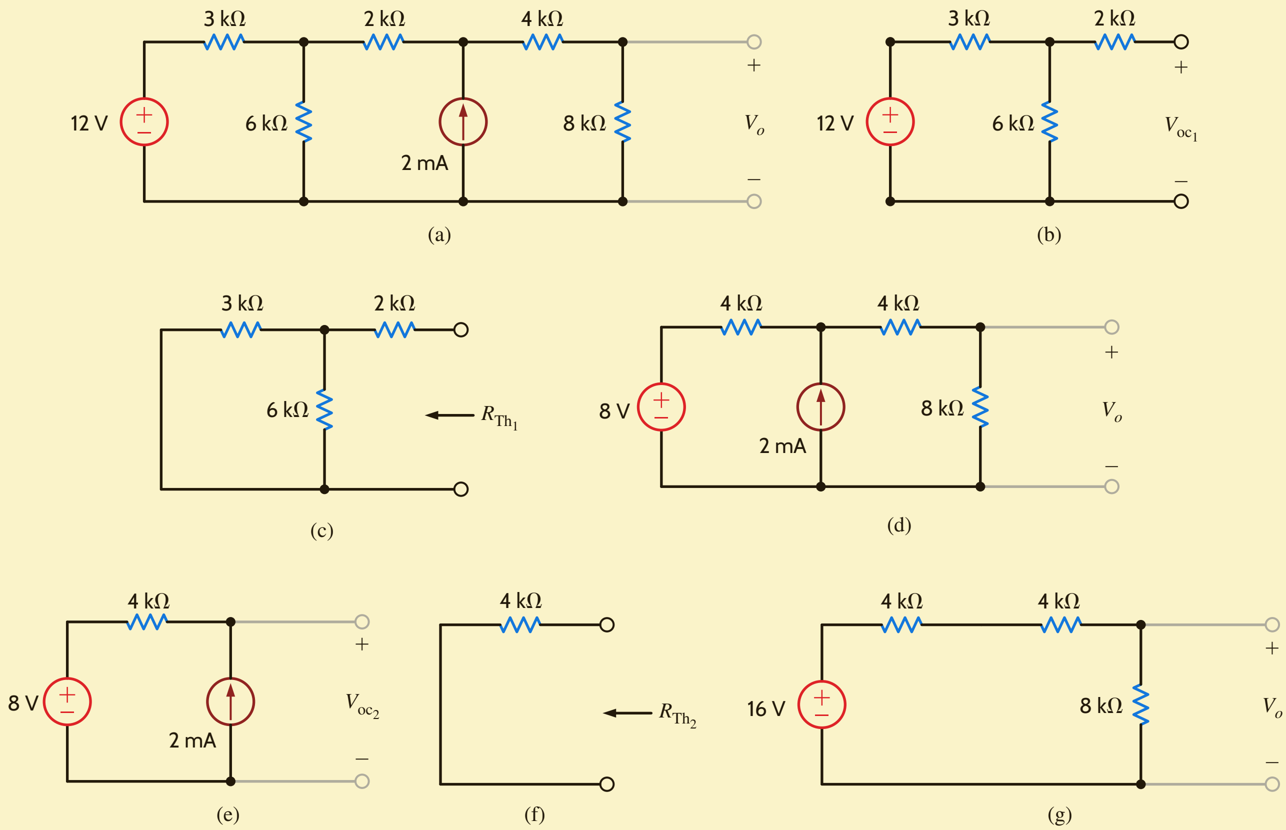

Figure 5.8
Circuits used in Example 5.6.

and R_{Th_2} obtained from Fig. 5.8f is 4 kΩ. Connecting this Thévenin equivalent to the remainder of the network produces the circuit shown in Fig. 5.8g. Simple voltage division applied to this final network yields $V_o = 8$ V. Norton's theorem can be applied in a similar manner to solve this network; however, we save that solution as an exercise.

EXAMPLE 5.7

It is instructive to examine the use of Thévenin's and Norton's theorems in the solution of the network in Fig. 5.4a, which is redrawn in Fig. 5.9a.

SOLUTION

If we break the network at the 6-kΩ load, the open-circuit voltage is found from Fig. 5.9b. The equations for the mesh currents are

$$-6 + 4\text{k}I_1 + 2\text{k}(I_1 - I_2) = 0$$

and

$$I_2 = 2 \times 10^{-3}$$

from which we easily obtain $I_1 = 5/3$ mA. Then, using KVL, V_{oc} is

$$\begin{aligned} V_{oc} &= 4\text{k}I_1 + 2\text{k}I_2 \\ &= 4\text{k}\left(\frac{5}{3} \times 10^{-3}\right) + 2\text{k}(2 \times 10^{-3}) \\ &= \frac{32}{3}\text{ V} \end{aligned}$$

R_{Th} is derived from Fig. 5.9c and is

$$R_{Th} = (2\text{k}//4\text{k}) + 2\text{k} = \frac{10}{3}\text{ k}\Omega$$

Attaching the Thévenin equivalent to the load produces the network in Fig. 5.9d. Then using voltage division, we obtain

$$\begin{aligned} V_o &= \frac{32}{3}\left(\frac{6\text{k}}{6\text{k} + \frac{10}{3}\text{k}}\right) \\ &= \frac{48}{7}\text{ V} \end{aligned}$$

In applying Norton's theorem to this problem, we must find the short-circuit current shown in Fig. 5.9e. At this point, the quick-thinking reader stops immediately! Three mesh equations applied to the original circuit will immediately lead to the solution, but the three mesh equations in the circuit in Fig. 5.9e will provide only part of the answer, specifically the short-circuit current. Sometimes the use of the theorems is more complicated than a straightforward attack using node or loop analysis. This would appear to be one of those situations. Interestingly, it is not. We can find I_{sc} from the network in Fig. 5.9e without using the mesh equations. The technique is simple, but a little tricky, and so we ignore it at this time. Having said all these things, let us now finish what we have started. The mesh equations for the network in Fig. 5.9e are

$$\begin{aligned} -6 + 4\text{k}(I_1 - I_{sc}) + 2\text{k}(I_1 - 2 \times 10^{-3}) &= 0 \\ 2\text{k}(I_{sc} - 2 \times 10^{-3}) + 4\text{k}(I_{sc} - I_1) &= 0 \end{aligned}$$

where we have incorporated the fact that $I_2 = 2 \times 10^{-3}$ A. Solving these equations yields $I_{sc} = 16/5$ mA. R_{Th} has already been determined in the Thévenin analysis. Connecting the Norton equivalent to the load results in the circuit in Fig. 5.9f. Solving this circuit yields $V_o = 48/7$ V.

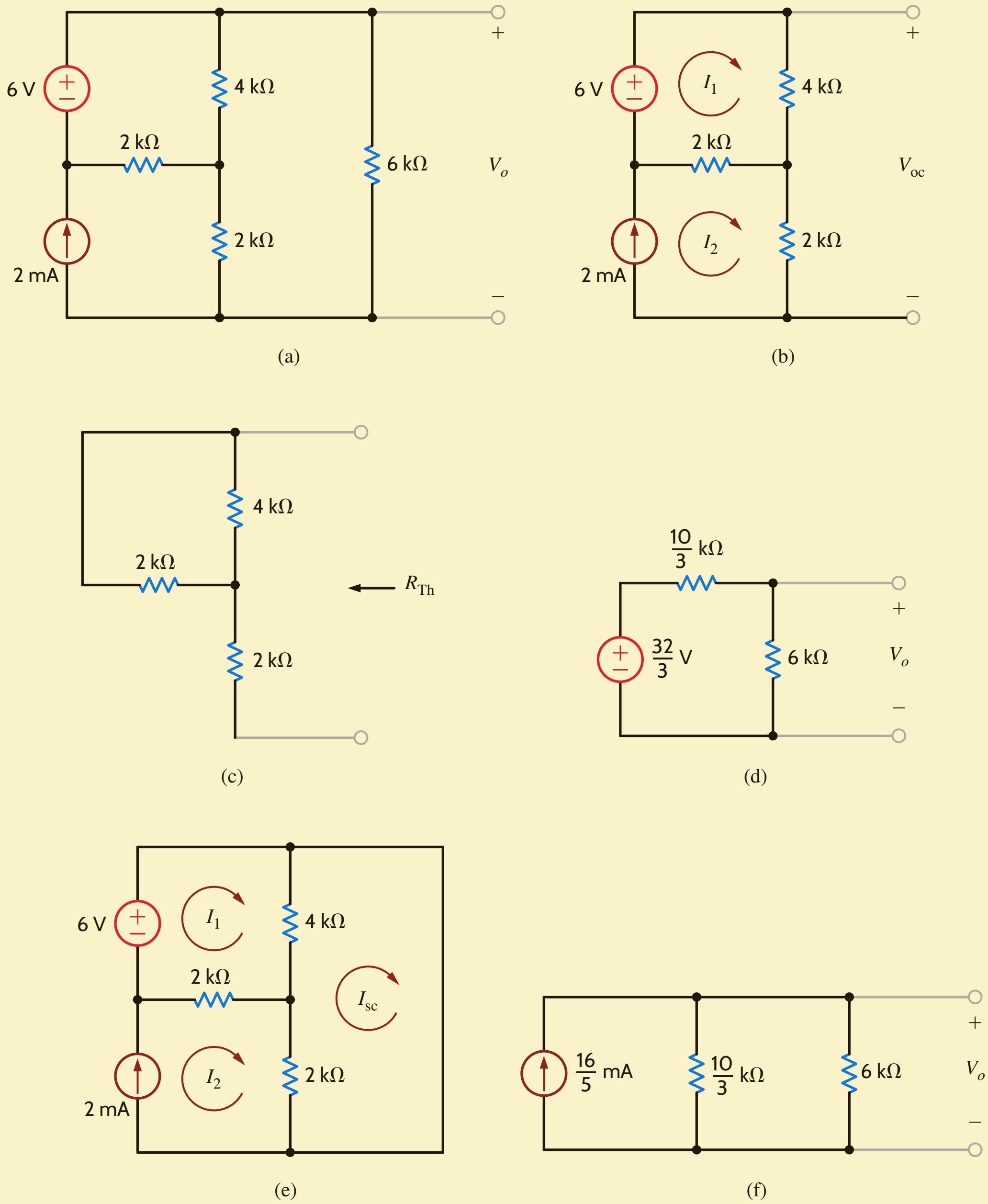

Figure 5.9
Circuits used in Example 5.7.

LEARNING ASSESSMENTS

E5.6 Use Thévenin's theorem to find V_o in the network in Fig. E5.6.

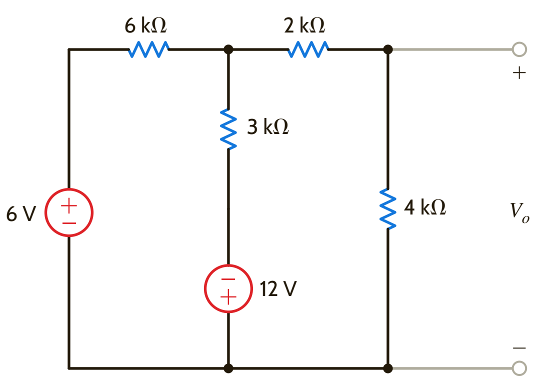

Figure E5.6

ANSWER:

$V_o = -3$ V.

E5.7 Find V_o in the circuit in Fig. E5.3 using both Thévenin's and Norton's theorems. When deriving the Norton equivalent circuit, break the network to the left of the 4-kΩ resistor. Why?

ANSWER:
$V_o = \frac{4}{3}$ V.

E5.8 Find V_o in Fig. E5.8 using Thévenin's theorem.

ANSWER:
$V_o = 3.88$ V.

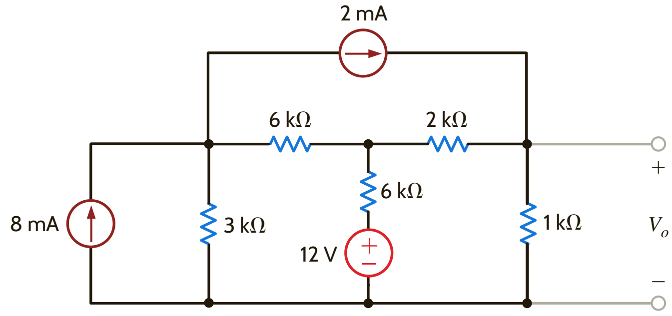

Figure E5.8

E5.9 Find I_0 in Fig. E5.9 using Norton's theorem.

ANSWER:
$I_0 = -0.857$ mA.

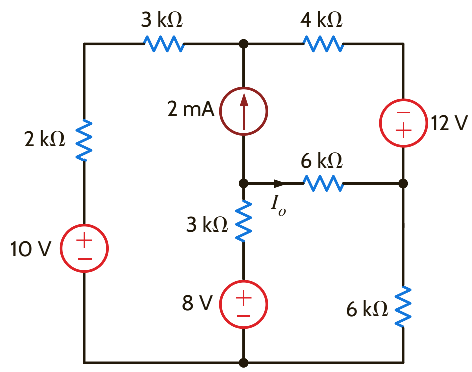

Figure E5.9

CIRCUITS CONTAINING ONLY DEPENDENT SOURCES As we have stated earlier, the Thévenin or Norton equivalent of a network containing only dependent sources is R_{Th}. The following examples will serve to illustrate how to determine this Thévenin equivalent resistance.

EXAMPLE 5.8

We wish to determine the Thévenin equivalent of the network in Fig. 5.10a at the terminals *A-B*.

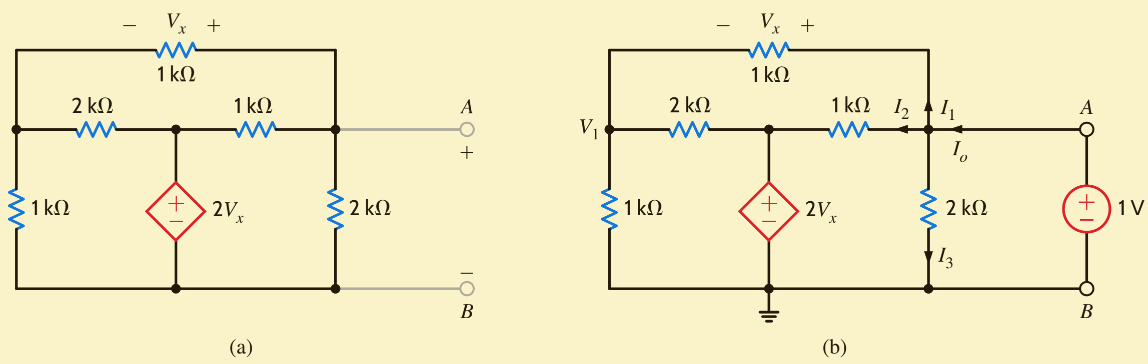

Figure 5.10 Networks employed in Example 5.8.

SOLUTION

Our approach to this problem will be to apply a 1-V source at the terminals as shown in Fig. 5.10b and then compute the current I_o and $R_{Th} = 1/I_o$.

The equations for the network in Fig. 5.10b are as follows. KVL around the outer loop specifies that

$$V_1 + V_x = 1$$

The KCL equation at the node labeled V_1 is

$$\frac{V_1}{1k} + \frac{V_1 - 2V_x}{2k} + \frac{V_1 - 1}{1k} = 0$$

Solving the equations for V_x yields $V_x = 3/7$ V. Knowing V_x, we can compute the currents I_1, I_2, and I_3. Their values are

$$I_1 = \frac{V_x}{1k} = \frac{3}{7}\text{ mA}$$

$$I_2 = \frac{1 - 2V_x}{1k} = \frac{1}{7}\text{ mA}$$

$$I_3 = \frac{1}{2k} = \frac{1}{2}\text{ mA}$$

Therefore,

$$\begin{aligned} I_o &= I_1 + I_2 + I_3 \\ &= \frac{15}{14}\text{ mA} \end{aligned}$$

and

$$\begin{aligned} R_{Th} &= \frac{1}{I_o} \\ &= \frac{14}{15}\text{ k}\Omega \end{aligned}$$

EXAMPLE 5.9

Let us determine R_{Th} at the terminals A-B for the network in Fig. 5.11a.

SOLUTION

Our approach to this problem will be to apply a 1-mA current source at the terminals A-B and compute the terminal voltage V_2 as shown in Fig. 5.11b. Then $R_{Th} = V_2/0.001$.

The node equations for the network are

$$\frac{V_1 - 2000I_x}{2k} + \frac{V_1}{1k} + \frac{V_1 - V_2}{3k} = 0$$

$$\frac{V_2 - V_1}{3k} + \frac{V_2}{2k} = 1 \times 10^{-3}$$

and

$$I_x = \frac{V_1}{1k}$$

Solving these equations yields

$$V_2 = \frac{10}{7}\text{ V}$$

and hence,

$$\begin{aligned} R_{Th} &= \frac{V_2}{1 \times 10^{-3}} \\ &= \frac{10}{7}\text{ k}\Omega \end{aligned}$$

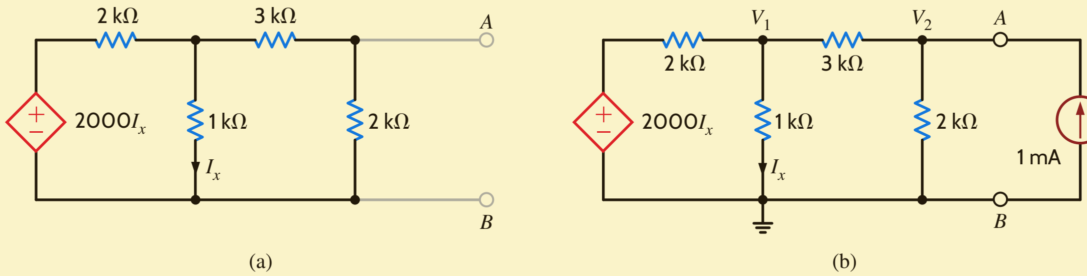

Figure 5.11 Networks used in Example 5.9.

CIRCUITS CONTAINING BOTH INDEPENDENT AND DEPENDENT SOURCES

In these types of circuits we must calculate both the open-circuit voltage and short-circuit current to calculate the Thévenin equivalent resistance. Furthermore, we must remember that we cannot split the dependent source and its controlling variable when we break the network to find the Thévenin or Norton equivalent.

We now illustrate this technique with a circuit containing a current-controlled voltage source.

EXAMPLE 5.10

Let us use Thévenin's theorem to find V_o in the network in Fig. 5.12a.

SOLUTION

To begin, we break the network at points A-B. Could we break it just to the right of the 12-V source? No! Why? The open-circuit voltage is calculated from the network in Fig. 5.12b. Note that we now use the source $2000I_x'$ because this circuit is different from that in Fig. 5.12a. KCL for the supernode around the 12-V source is

$$\frac{(V_{oc}+12)-(-2000I_x')}{1\text{k}}+\frac{V_{oc}+12}{2\text{k}}+\frac{V_{oc}}{2\text{k}}=0$$

where

$$I_x'=\frac{V_{oc}}{2\text{k}}$$

yielding $V_{oc} = -6\text{V}$.

I_{sc} can be calculated from the circuit in Fig. 5.12c. Note that the presence of the short circuit forces I_x'' to zero and, therefore, the network is reduced to that shown in Fig. 5.12d.

Therefore,

$$I_{sc}=\frac{-12}{\frac{2}{3}\text{k}}=-18\text{ mA}$$

Then

$$R_{Th}=\frac{V_{oc}}{I_{sc}}=\frac{1}{3}\text{ k}\Omega$$

Connecting the Thévenin equivalent circuit to the remainder of the network at terminals A-B produces the circuit in Fig. 5.12e. At this point, simple voltage division yields

$$V_o=(-6)\left(\frac{1\text{k}}{1\text{k}+1\text{k}+\frac{1}{3}\text{k}}\right)=\frac{-18}{7}\text{ V}$$

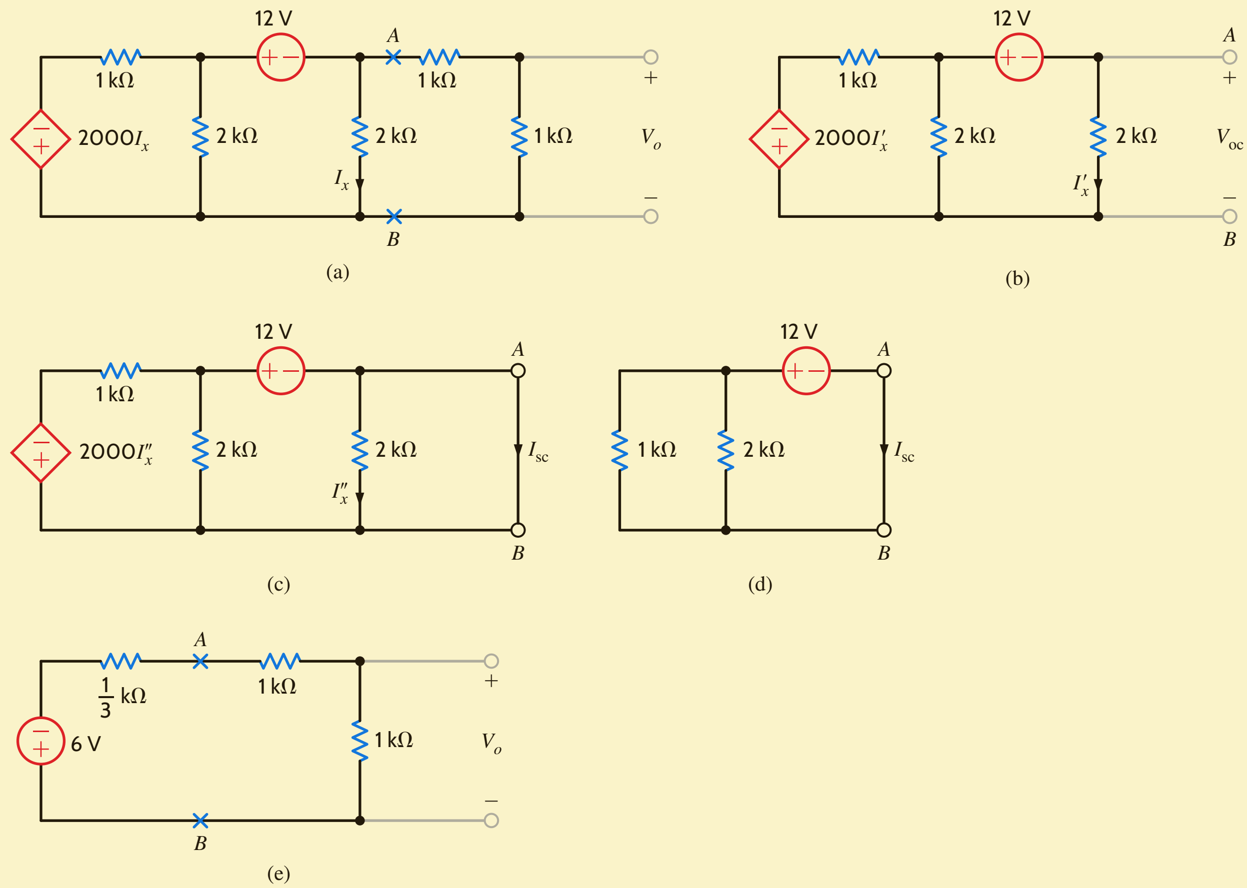

Figure 5.12

Circuits used in Example 5.10.

EXAMPLE 5.11

Let us find the current I_o in the network in **Fig. 5.13a** using Thévenin's theorem.

SOLUTION

V_{oc} is determined from the network shown in **Fig. 5.13b**. The presence of the three loops indicates three equations are required, together with the constraint equation for the dependent source. Since two of the currents are shown to go directly through the current sources, two of the necessary equations are

$$I_1 = \frac{4}{\text{k}}$$

$$I_3 = 2I_x'$$

The third equation is obtained by applying KVL around the two upper loops; that is,

$$1\text{k}(I_1 + I_2) + 12 + 1\text{k}(I_2 - I_3) + 1\text{k}(I_1 + I_2 - I_3) = 0$$

And the constraint equation for the dependent source is

$$I_x' = I_2 - 2I_x'$$

Solving these equations yields

$$I_2 = -\frac{12}{k}\,\text{A}$$

$$I_3 = -\frac{8}{k}\,\text{A}$$

Then V_{oc} can be determined from the KVL equation

$$-6 + 1\text{k}(I_3 - I_2 - I_1) + V_{oc} = 0$$

which yields $V_{oc} = 6$ V.

Because of the presence of the dependent source, R_{Th} must be determined from the equation

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

I_{sc} is derived from the circuit in **Fig. 5.13c**. The node equation for the supernode is

$$\frac{(V_1 - 6)}{1\text{k}} + \frac{4}{\text{k}} + \frac{(V_1 - 12)}{1\text{k}} + \frac{2(V_1 - 12)}{1\text{k}} = 0$$

which yields $V_1 = 19/2$ V, and since $V_2 = V_1 - 12$, $V_2 = -5/2$ V. Then KCL at the center node is

$$\frac{6}{1\text{k}} + \frac{4}{\text{k}} = \frac{5}{2\text{k}} + I_{sc}$$

Yielding $I_{sc} = 15/2\text{k}$ A, and since $R_{Th} = V_{oc}/I_{sc}$, $R_{Th} = 12\text{k}/15$ ohms. Finally, forming the Thévenin equivalent circuit and connecting up the 1k ohm resistor produces the network in **Fig. 5.13d**. The current I_0 is then

$$I_0 = \frac{6}{\left(\frac{12}{15} + 1\right)\text{k}} = \frac{10}{3\text{k}}\,\text{A}$$

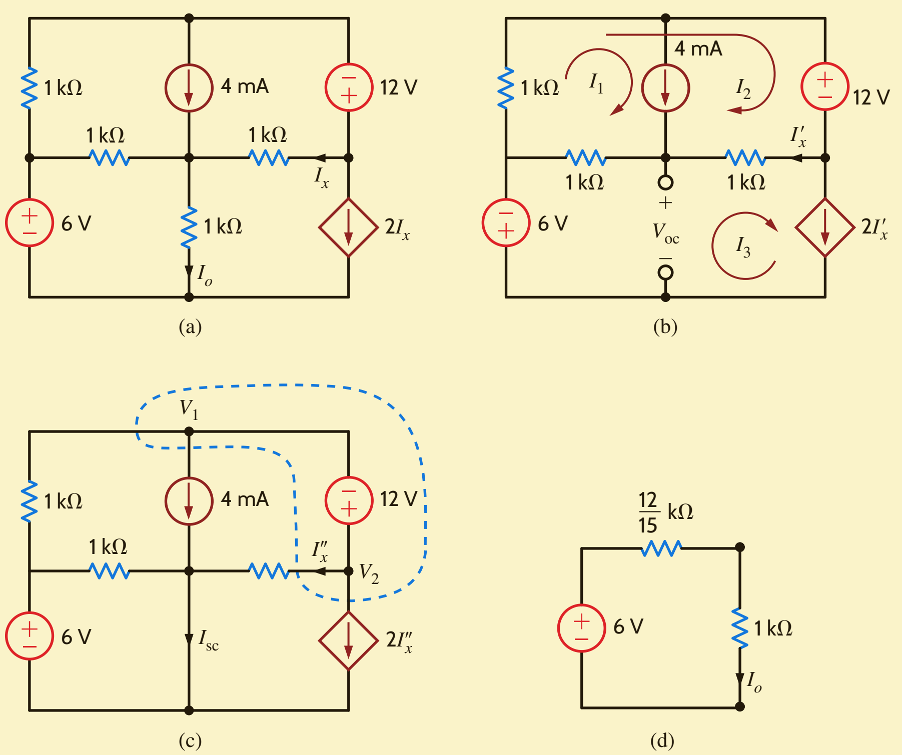

Figure 5.13 Circuits used in Example 5.11.

EXAMPLE 5.12

We will now reexamine a problem that was solved earlier using both nodal and loop analyses. The circuit used in Examples 3.10 and 3.20 is redrawn in Fig. 5.14a. Since a dependent source is present, we will have to find the open-circuit voltage and the short-circuit current in order to employ Thévenin's theorem to determine the output voltage V_o.

SOLUTION

As we begin the analysis, we note that the circuit can be somewhat simplified by first forming a Thévenin equivalent for the leftmost and rightmost branches. Note that these two branches are in parallel and neither branch contains the control variable. Thus, we can simplify the network by reducing these two branches to one via a Thévenin equivalent. For the circuit shown in Fig. 5.14b, the open-circuit voltage is

$$V_{oc_1} = \frac{2}{k}(1k) + 4 = 6 \text{ V}$$

And the Thévenin equivalent resistance at the terminals, obtained by looking into the terminals with the sources made zero, is

$$R_{Th_1} = 1 \text{ k}\Omega$$

The resultant Thévenin equivalent circuit is now connected to the remaining portion of the circuit producing the network in Fig. 5.14c.

Now we break the network shown in Fig. 5.14c at the output terminals to determine the open-circuit voltage V_{oc_2} as shown in Fig. 5.14d. Because of the presence of the voltage sources, we will use a nodal analysis to find the open-circuit voltage with the help of a supernode. The node equations for this network are

$$V_1 = 3V_x'$$

$$\frac{V_1 - 6}{1k} + \frac{V_1 - 2V_x'}{1k} = \frac{2}{k}$$

and thus $V_x' = 2$ V and $V_1 = 6$ V. Then, the open-circuit voltage, obtained using the KVL equation

$$-2V_x' + V_{oc_2} + \frac{2}{k}(1k) = 0$$

is

$$V_{oc_2} = 2 \text{ V}$$

The short-circuit current is derived from the network shown in Fig. 5.14e. Once again, we employ the supernode, and the network equations are

$$V_2 = 3V_x''$$

$$\frac{V_2 - 6}{1k} + \frac{V_2 - 2V_x''}{1k} = \frac{2}{k}$$

The node voltages obtained from these equations are $V_x'' = 2$ V and $V_2 = 6$ V. The line diagram shown in Fig. 5.14f displays the node voltages and the resultant branch currents. (Node voltages are shown in the circles, and branch currents are identified with arrows.) The node voltages and resistors are used to compute the resistor currents, while the remaining currents are derived by KCL. As indicated, the short-circuit current is

$$I_{sc_2} = 2 \text{ mA}$$

Then, the Thévenin equivalent resistance is

$$R_{Th_2} = \frac{V_{oc_2}}{I_{sc_2}} = 1 \text{ k}\Omega$$

The Thévenin equivalent circuit now consists of a 2-V source in series with a 1-kΩ resistor. Connecting this Thévenin equivalent circuit to the load resistor yields the network shown in Fig. 5.14g. A simple voltage divider indicates that $V_o = 1$ V.

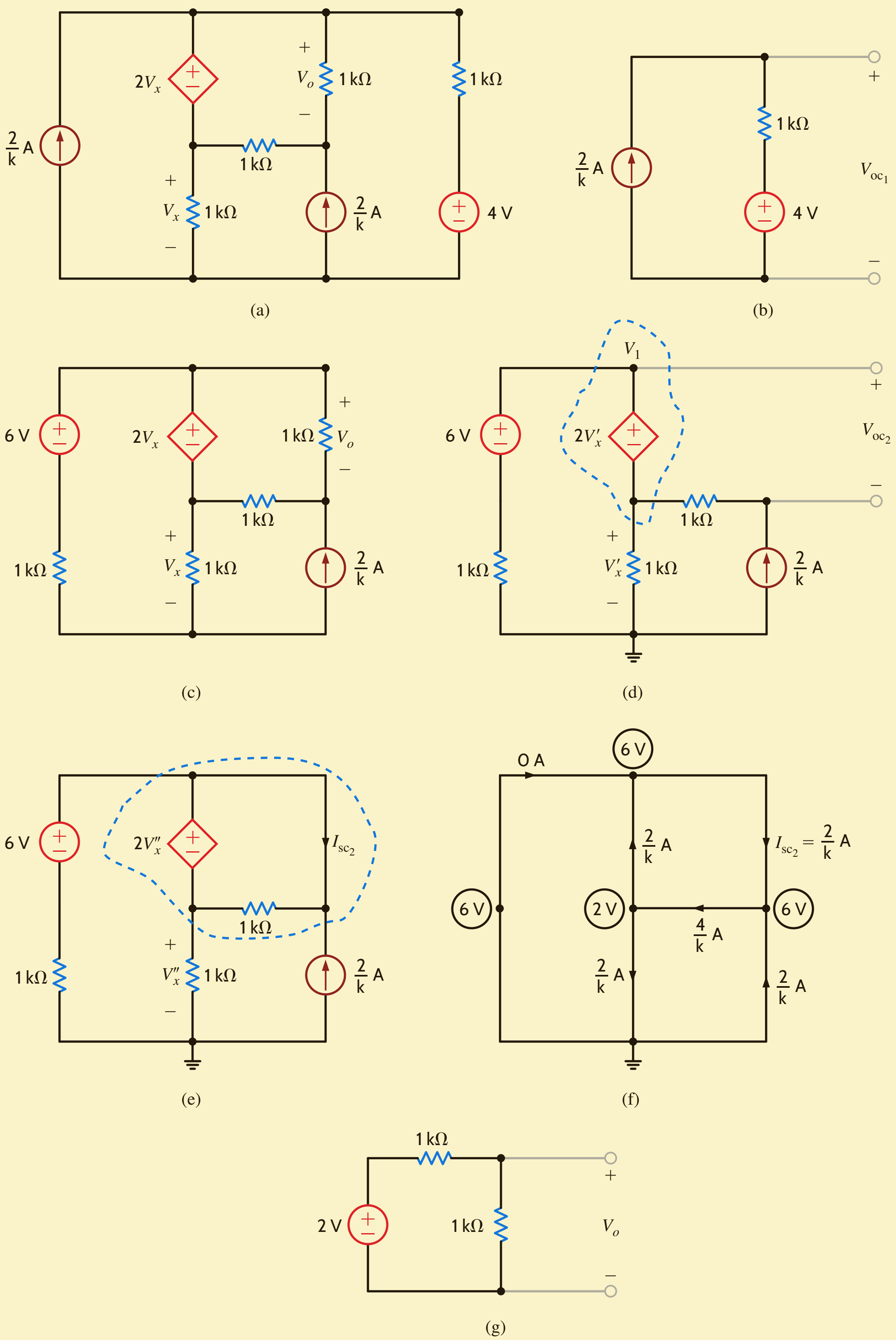

Figure 5.14
Circuits used in Example 5.12.

PROBLEM-SOLVING STRATEGY

APPLYING THÉVENIN'S THEOREM

STEP 1. Remove the load and find the voltage across the open-circuit terminals, V_{oc}. All the circuit analysis techniques presented here can be used to compute this voltage.

STEP 2. Determine the Thévenin equivalent resistance of the network at the open terminals with the load removed. Three different types of circuits may be encountered in determining the resistance, R_{Th}.

(a) If the circuit contains only independent sources, they are made zero by replacing the voltage sources with short circuits and the current sources with open circuits. R_{Th} is then found by computing the resistance of the purely resistive network at the open terminals.

(b) If the circuit contains only dependent sources, an independent voltage or current source is applied at the open terminals and the corresponding current or voltage at these terminals is measured. The voltage/current ratio at the terminals is the Thévenin equivalent resistance. Since there is no energy source, the open-circuit voltage is zero in this case.

(c) If the circuit contains both independent and dependent sources, the open-circuit terminals are shorted and the short-circuit current between these terminals is determined. The ratio of the open-circuit voltage to the short-circuit current is the resistance R_{Th}.

STEP 3. If the load is now connected to the Thévenin equivalent circuit, consisting of V_{oc} in series with R_{Th}, the desired solution can be obtained.

The problem-solving strategy for Norton's theorem is essentially the same as that for Thévenin's theorem, with the exception that we are dealing with the short-circuit current instead of the open-circuit voltage.

LEARNING ASSESSMENTS

E5.10 Find V_o in the circuit in Fig. E5.10 using Thévenin's theorem.

ANSWER: $V_o = \frac{36}{13}$ V.

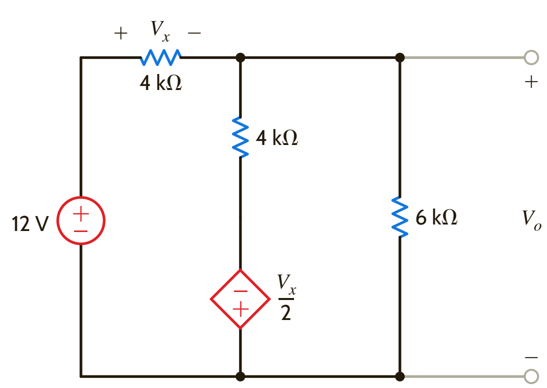

Figure E5.10

E5.11 Find V_o in Fig. E5.11 using Thévenin's theorem.

ANSWER:
$V_o = 6.29$ V.

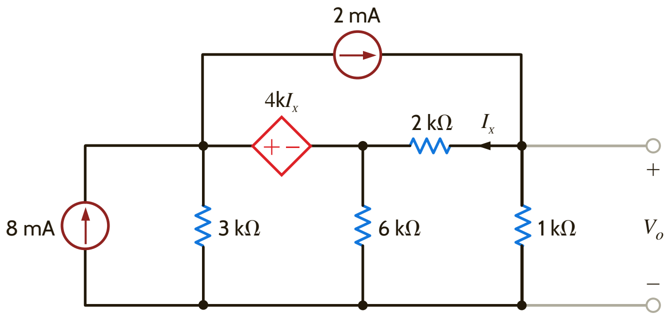

Figure E5.11

E5.12 Use Thévenin's theorem to find the power supplied by the 12-V source in Fig. E5.12.

ANSWER:
8.73 mW.

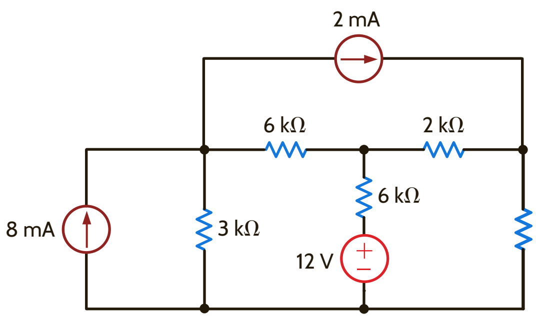

Figure E5.12

E5.13 Find the Thévenin equivalent of the network at terminals $A - B$ in Fig. E5.13.

ANSWER:
$R_{Th} = 1619\ \Omega$.

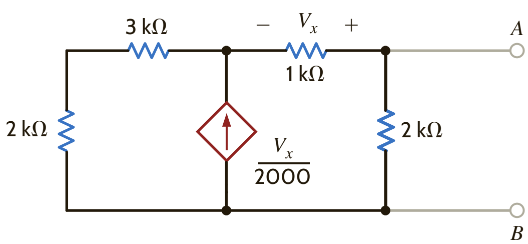

Figure E5.13

Having examined the use of Thévenin's and Norton's theorems in a variety of different types of circuits, it is instructive to look at yet one other aspect of these theorems that we find useful in circuit analysis and design. This additional aspect can be gleaned from the Thévenin equivalent and Norton equivalent circuits.

The relationships specified in Fig. 5.6 and Eq. (5.5) have special significance because they represent what is called a *source transformation* or *source exchange*. What these relationships tell us is that if we have embedded within a network a current source i in parallel with a resistor R, we can replace this combination with a voltage source of value $v = iR$ in series with the resistor R. The reverse is also true; that is, a voltage source v in series with a resistor R can be replaced with a current source of value $i = v/R$ in parallel with the resistor R. Parameters within the circuit (e.g., an output voltage) are unchanged under these transformations.

We must emphasize that the two equivalent circuits in Fig. 5.6 *are equivalent only at the two external nodes*. For example, if we disconnect circuit B from both networks in Fig. 5.6, the equivalent circuit in Fig. 5.6b dissipates power, but the one in Fig. 5.6a does not.

EXAMPLE 5.13

We will now demonstrate how to find V_o in the circuit in **Fig. 5.15a** using the repeated application of source transformation.

SOLUTION

If we begin at the left end of the network in Fig. 5.15a, the series combination of the 12-V source and 3-kΩ resistor is converted to a 4-mA current source in parallel with the 3-kΩ resistor. If we combine this 3-kΩ resistor with the 6-kΩ resistor, we obtain the circuit in **Fig. 5.15b**. Note that at this point we have eliminated one circuit element. Continuing the reduction, we convert the 4-mA source and 2-kΩ resistor into an 8-V source in series with this same 2-kΩ resistor. The two 2-kΩ resistors that are in series are now combined to produce the network in **Fig. 5.15c**. If we now convert the combination of the 8-V source and 4-kΩ resistor into a 2-mA source in parallel with the 4-kΩ resistor and combine the resulting current source with the other 2-mA source, we arrive at the circuit shown in **Fig. 5.15d**. At this point, we can simply apply current division to the two parallel resistance paths and obtain

$$I_o = (4 \times 10^{-3})\left(\frac{4\text{k}}{4\text{k} + 4\text{k} + 8\text{k}}\right) = 1 \text{ mA}$$

and hence,

$$V_o = (1 \times 10^{-3})(8\text{k}) = 8 \text{ V}$$

The reader is encouraged to consider the ramifications of working this problem using any of the other techniques we have presented.

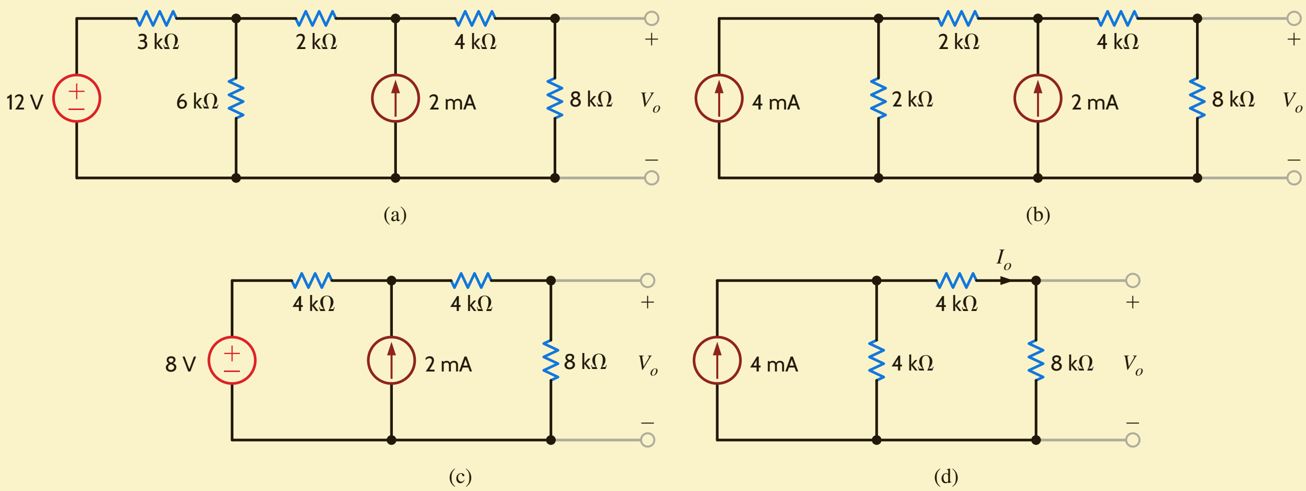

Figure 5.15

Circuits used in Example 5.13.

Note that this systematic, sometimes tedious, transformation allows us to reduce the network methodically to a simpler equivalent form with respect to some other circuit element. However, we should also realize that this technique is worthless for circuits of the form shown in Fig. 5.4. Furthermore, although applicable to networks containing dependent sources, it is not as useful as other techniques, and care must be taken not to transform the part of the circuit that contains the control variable.

LEARNING ASSESSMENTS

E5.14 Find V_o in the circuit in Fig. E5.3 using source exchange.

ANSWER:
$V_o = \frac{4}{3}$ V.

E5.15 Find the I_o in Fig. E5.15 using source transformations.

ANSWER:
$I_o = -1.94$ mA.

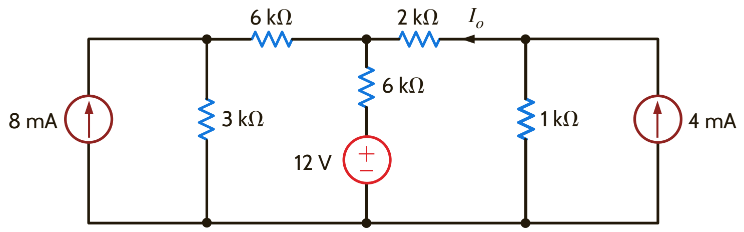

Figure E5.15

At this point, let us pause for a moment and reflect on what we have learned; that is, let us compare the use of node or loop analysis with that of the theorems discussed in this chapter. When we examine a network for analysis, one of the first things we should do is count the number of nodes and loops. Next we consider the number of sources. For example, are there a number of voltage sources or current sources present in the network? All these data, together with the information that we expect to glean from the network, give a basis for selecting the simplest approach. With the current level of computational power available to us, we can solve the node or loop equations that define the network in a flash.

With regard to the theorems, we have found that in some cases the theorems do not necessarily simplify the problem and a straightforward attack using node or loop analysis is as good an approach as any. This is a valid point, provided that we are simply looking for some particular voltage or current. However, the real value of the theorems is the insight and understanding that they provide about the physical nature of the network. For example, superposition tells us what each source contributes to the quantity under investigation. However, a computer solution of the node or loop equations does not tell us the effect of changing certain parameter values in the circuit. It does not help us understand the concept of loading a network or the ramifications of interconnecting networks or the idea of matching a network for maximum power transfer. The theorems help us to understand the effect of using a transducer at the input of an amplifier with a given input resistance. They help us explain the effect of a load, such as a speaker, at the output of an amplifier. We derive none of this information from a node or loop analysis. In fact, as a simple example, suppose that a network at a specific pair of terminals has a Thévenin equivalent circuit consisting of a voltage source in series with a 2-kΩ resistor. If we connect a 2-Ω resistor to the network at these terminals, the voltage across the 2-Ω resistor will be essentially nothing. This result is fairly obvious using the Thévenin theorem approach; however, a node or loop analysis gives us no clue as to why we have obtained this result.

We have studied networks containing only dependent sources. This is a very important topic because all electronic devices, such as transistors, are modeled in this fashion. Motors in power systems are also modeled in this way. We use these amplification devices for many different purposes, such as speed control for automobiles.

In addition, it is interesting to note that when we employ source transformation as we did in Example 5.13, we are simply converting back and forth between a Thévenin equivalent circuit and a Norton equivalent circuit.

Finally, we have a powerful tool at our disposal that can be used to provide additional insight and understanding for both circuit analysis and design. That tool is Microsoft Excel, and it permits us to study the effects, on a network, of varying specific parameters. The following example will illustrate the simplicity of this approach.

EXAMPLE 5.14

We wish to use Microsoft Excel to plot the Thévenin equivalent parameters V_{oc} and R_{Th} for the circuit in Fig. 5.16 over the R_x range 0 to 10 kΩ.

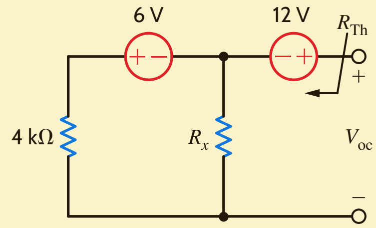

Figure 5.16
Circuit used in Example 5.14.

SOLUTION

The Thévenin resistance is easily found by replacing the voltage sources with short circuits. The result is

$$R_{Th} = 4//R_x = \frac{4R_x}{4 + R_x} \qquad \textbf{5.7}$$

where R_x and R_{Th} are in kΩ. Superposition can be used effectively to find V_{oc}. If the 12-V source is replaced by a short circuit

$$V_{oc_1} = -6\left[\frac{R_x}{R_x + 4}\right]$$

Applying this same procedure for the 6-V source yields

$$V_{oc_2} = 12$$

and the total open-circuit voltage is

$$V_{oc} = 12 - 6\left[\frac{R_x}{R_x + 4}\right] \qquad \textbf{5.8}$$

In Excel we wish to (1) vary R_x between 0 and 10 kΩ, (2) calculate R_{Th} and V_{oc} at each R_x value, and (3) plot V_{oc} and R_{Th} versus R_x. We begin by opening Excel and entering column headings as shown in Fig. 5.17a. Next, we enter a zero in the first cell of the R_x column at column-row location A4. To automatically fill the column with values, go to the Edit menu and select Fill/Series to open the window shown in Fig. 5.17b, which has already been edited appropriately for 101 data points. The result is a series of R_x values from 0 to 10 kΩ in 100 Ω steps. To enter Eq. (5.8), go to location B4 (right under the V_{oc} heading). Enter the following text and do not forget the equal sign:

```
=12-6*A4/(A4+4)
```

This is Eq. (5.8) with R_x replaced by the first value for R_x, which is at column-row location A4. Similarly for R_{Th}, enter the following expression at C4.

```
=4*A4/(A4+4)
```

To replicate the expression in cell B4 for all R_x values, select cell B4, grab the lower right corner of the cell, hold and drag down to cell B104, and release. Repeat for R_{Th} by replicating cell C4.

To plot the data, first drag the cursor across all cells between A4 and C104. Next, from the Insert menu, select Chart. We recommend strongly that you choose the XY (Scatter) chart type. Excel will take you step by step through the basic formatting of your chart, which, after some manipulations, might look similar to the chart in Fig. 5.17c.