

## LEARNING ASSESSMENTS

**E11.9** An  $abc$ -sequence three-phase voltage source connected in a balanced wye supplies power to a balanced delta-connected load. The line current for the  $a$  phase is  $I_{aA} = 12/40^\circ$  A rms. Find the phase currents in the delta-connected load.

**ANSWER:**

$$I_{AB} = 6.93/70^\circ \text{ A rms};$$

$$I_{BC} = 6.93/-50^\circ \text{ A rms};$$

$$I_{CA} = 6.93/-170^\circ \text{ A rms}.$$

**E11.10** Find the line currents and the power absorbed by the delta-connected load in Fig. E11.10.

**ANSWER:**

$$I_{aA} = 35.76/-34.74^\circ \text{ A rms};$$

$$I_{bB} = 35.76/-154.74^\circ \text{ A rms};$$

$$I_{cC} = 35.76/85.26^\circ \text{ A rms};$$

$$17.29 - j6.92 \text{ kVA}.$$

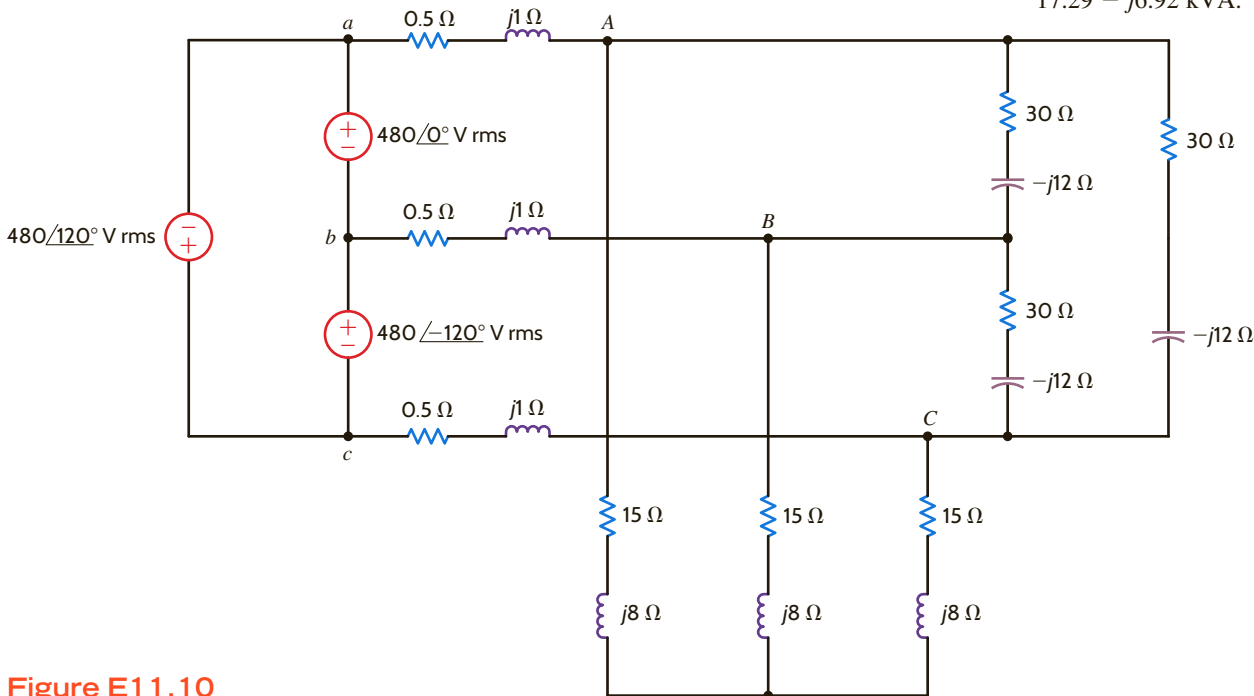


Figure E11.10

### 11.4 Power Relationships

Whether the load is connected in a wye or a delta, the real and reactive power per phase is

$$P_p = V_p I_p \cos \theta \quad 11.25$$

$$Q_p = V_p I_p \sin \theta$$

where  $\theta$  is the angle between the phase voltage and the line current. For a Y-connected system,  $I_p = I_L$  and  $V_p = V_L/\sqrt{3}$ , and for a  $\Delta$ -connected system,  $I_p = I_L/\sqrt{3}$  and  $V_p = V_L$ . Therefore,

$$P_p = \frac{V_L I_L}{\sqrt{3}} \cos \theta \quad 11.26$$

$$Q_p = \frac{V_L I_L}{\sqrt{3}} \sin \theta$$

The total real and reactive power for all three phases is then

$$P_T = 3 P_p = \sqrt{3} V_L I_L \cos \theta \quad 11.27$$

$$Q_T = 3 Q_p = \sqrt{3} V_L I_L \sin \theta$$

and, therefore, the magnitude of the complex power (apparent power) is

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

$$= \sqrt{3} V_L I_L$$

and

$$\angle \mathbf{S}_T = \theta$$

A three-phase balanced wye–delta system has a line voltage of 208 V rms. The total real power absorbed by the load is 1200 W. If the power factor angle of the load is  $20^\circ$  lagging, we wish to determine the magnitude of the line current and the value of the load impedance per phase in the delta.

The line current can be obtained from Eq. (11.26). Since the real power per phase is 400 W,

$$400 = \frac{208 I_L}{\sqrt{3}} \cos 20^\circ$$

$$I_L = 3.54 \text{ A rms}$$

The magnitude of the current in each leg of the delta-connected load is

$$I_\Delta = \frac{I_L}{\sqrt{3}}$$

$$= 2.05 \text{ A rms}$$

Therefore, the magnitude of the delta impedance in each phase of the load is

$$|Z_\Delta| = \frac{V_L}{I_\Delta}$$

$$= \frac{208}{2.05}$$

$$= 101.46 \Omega$$

Since the power factor angle is  $20^\circ$  lagging, the load impedance is

$$Z_\Delta = 101.46/20^\circ$$

$$= 95.34 + j34.70 \Omega$$

## EXAMPLE 11.5

### SOLUTION

For the circuit in Example 11.2 we wish to determine the real and reactive power per phase at the load and the total real power, reactive power, and complex power at the source.

From the data in Example 11.2 the complex power per phase at the load is

$$\mathbf{S}_{\text{load}} = \mathbf{V}\mathbf{I}^*$$

$$= (113.15/-1.08^\circ)(5.06/27.65^\circ)$$

$$= 572.54/26.57^\circ$$

$$= 512.07 + j256.09 \text{ VA}$$

Therefore, the real and reactive power per phase at the load are 512.07 W and 256.09 var, respectively.

The complex power per phase at the source is

$$\mathbf{S}_{\text{source}} = \mathbf{V}\mathbf{I}^*$$

$$= (120/0^\circ)(5.06/27.65^\circ)$$

$$= 607.2/27.65^\circ$$

$$= 537.86 + j281.78 \text{ VA}$$

Therefore, total real power, reactive power, and apparent power at the source are 1613.6 W, 845.2 var, and 1821.6 VA, respectively.

## EXAMPLE 11.6

### SOLUTION

**EXAMPLE 11.7**

A balanced three-phase source serves three loads, as follows:

Load 1: 24 kW at 0.6 lagging power factor

Load 2: 10 kW at unity power factor

Load 3: 12 kVA at 0.8 leading power factor

If the line voltage at the loads is 208 V rms at 60 Hz, we wish to determine the line current and the combined power factor of the loads.

**SOLUTION**

From the data we find that

$$\mathbf{S}_1 = 24,000 + j32,000$$

$$\mathbf{S}_2 = 10,000 + j0$$

$$\mathbf{S}_3 = 12,000 / -36.9^\circ = 9600 - j7200$$

Therefore,

$$\mathbf{S}_{\text{load}} = 43,600 + j24,800$$

$$= 50,160 / 29.63^\circ \text{ VA}$$

$$I_L = \frac{|\mathbf{S}_{\text{load}}|}{\sqrt{3} V_L}$$

$$= \frac{50,160}{208\sqrt{3}}$$

$$I_L = 139.23 \text{ A rms}$$

and the combined power factor is

$$\begin{aligned} \text{pf}_{\text{load}} &= \cos 29.63^\circ \\ &= 0.869 \text{ lagging} \end{aligned}$$



The sum of three complex powers:

$$\mathbf{S}_{\text{load}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$

**EXAMPLE 11.8**

Given the three-phase system in Example 11.7, let us determine the line voltage and power factor at the source if the line impedance is  $\mathbf{Z}_{\text{line}} = 0.05 + j0.02 \Omega$ .

**SOLUTION**

The complex power absorbed by the line impedances is

$$\begin{aligned} \mathbf{S}_{\text{line}} &= 3(R_{\text{line}} I_L^2 + jX_{\text{line}} I_L^2) \\ &= 2908 + j1163 \text{ VA} \end{aligned}$$

The complex power delivered by the source is then

$$\begin{aligned} \mathbf{S}_S &= \mathbf{S}_{\text{load}} + \mathbf{S}_{\text{line}} \\ &= 43,600 + j24,800 + 2908 + j1163 \\ &= 53,264 / 29.17^\circ \text{ VA} \end{aligned}$$

The line voltage at the source is then

$$\begin{aligned} V_{L_S} &= \frac{S_S}{\sqrt{3} I_L} \\ &= 220.87 \text{ V rms} \end{aligned}$$

and the power factor at the source is

$$\begin{aligned} \text{pf}_S &= \cos 29.17^\circ \\ &= 0.873 \text{ lagging} \end{aligned}$$



Recall that the complex power for all three lines is

$$\mathbf{S}_{\text{line}} = 3 I_L^2 \mathbf{Z}_{\text{line}}$$

Let's consider the three-phase system shown in **Fig. 11.20**. Calculate the real power loss in the line resistance for  $V_L = 500$  kV rms and 50 kV rms.

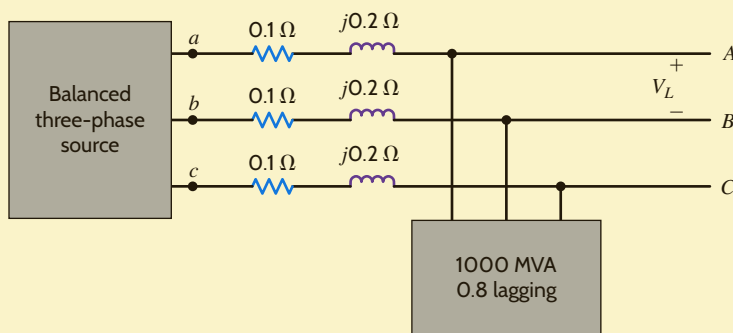
For  $V_L = 500$  kV rms,  $I_L = \frac{S_{\text{load}}}{\sqrt{3}V_L} = \frac{1000}{\sqrt{3}(500)} = 1.155$  kA rms, and the real power

losses in the line are  $P_{\text{line}} = 3I_L^2 R_{\text{line}} = 3(1.155)^2(0.1) = 0.4$  MW.

For  $V_L = 50$  kV rms,  $I_L = \frac{1000}{\sqrt{3}(50)} = 11.55$  kA rms and

$P_{\text{line}} = 3I_L^2 R_{\text{line}} = 3(11.55)^2(0.1) = 40$  MW

The line losses at 50 kV rms are 100 times larger than those at 500 kV rms. This example illustrates that power transmission at higher voltages is more efficient because of the reduced losses. The transformer discussed in Chapter 10 allows voltage levels in ac systems to be changed easily. Electric generators at power plants generate line voltages up to 25 kV. Transformers are utilized to step up this voltage for transmission from the plants to the load centers.



**Figure 11.20**

Three-phase system for calculation of line losses for different load voltages.

## EXAMPLE 11.9

### SOLUTION

## LEARNING ASSESSMENTS

**E11.11** A three-phase balanced wye–wye system has a line voltage of 208 V rms. The total real power absorbed by the load is 12 kW at 0.8 pf lagging. Determine the per-phase impedance of the load.

**ANSWER:**  
 $Z = 2.88/36.87^\circ \Omega$ .

**E11.12** For the balanced wye–wye system described in Learning Assessment E11.3, determine the real and reactive power and the complex power at both the source and the load.

**ANSWER:**  
 $S_{\text{load}} = 1186.77 + j444.66 \text{ VA};$   
 $S_{\text{source}} = 1335.65 + j593.55 \text{ VA}.$

**E11.13** A 480-V rms line feeds two balanced three-phase loads. If the two loads are rated as follows,

Load 1: 5 kVA at 0.8 pf lagging

Load 2: 10 kVA at 0.9 pf lagging

determine the magnitude of the line current from the 480-V rms source.

**ANSWER:**  
 $I_L = 17.97 \text{ A rms}.$

**E11.14** If the line voltage at the load is 480 V rms in Fig. E11.14, find the line voltage and power factor at the source.

**ANSWER:**

$V_L = 501.7$  V rms; pf = 0.9568 lagging.

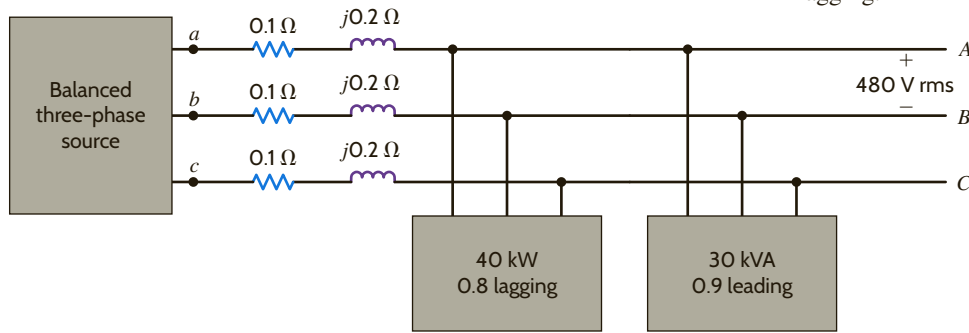


Figure E11.14

## EXAMPLE 11.10

Two balanced three-phase systems, X and Y, are interconnected with lines having impedance  $\mathbf{Z}_{\text{line}} = 1 + j2 \Omega$ . The line voltages are  $\mathbf{V}_{ab} = 12\angle 0^\circ$  kV rms and  $\mathbf{V}_{AB} = 12\angle 5^\circ$  kV rms, as shown in Fig. 11.21a. We wish to determine which system is the source, which is the load, and the average power supplied by the source and absorbed by the load.

### SOLUTION

When we draw the per-phase circuit for the system as shown in Fig. 11.21b, the analysis will be essentially the same as that of Example 9.12.

The network in Fig. 11.21b indicates that

$$\begin{aligned} \mathbf{I}_{aA} &= \frac{\mathbf{V}_{an} - \mathbf{V}_{AN}}{\mathbf{Z}_{\text{line}}} \\ &= \frac{\frac{12,000}{\sqrt{3}} \angle -30^\circ - \frac{12,000}{\sqrt{3}} \angle -25^\circ}{\sqrt{5} \angle 63.43^\circ} \\ &= 270.30 \angle -180.93^\circ \text{ A rms} \end{aligned}$$

The average power absorbed by system Y is

$$\begin{aligned} P_Y &= \sqrt{3} V_{AB} I_{aA} \cos(\theta_{V_{AB}} - \theta_{I_{aA}}) \\ &= \sqrt{3} (12,000)(270.30) \cos(-25^\circ + 180.93^\circ) \\ &= -5.130 \text{ MW} \end{aligned}$$

Note that system Y is not the load, but rather the source and supplies 5.130 MW.

System X absorbs the following average power:

$$P_X = \sqrt{3} V_{ab} I_{Aa} \cos(\theta_{V_{ab}} - \theta_{I_{Aa}})$$

where

$$\mathbf{I}_{Aa} = -\mathbf{I}_{aA} = 270.30 \angle -0.93^\circ \text{ A rms}$$

Therefore,

$$\begin{aligned} P_X &= \sqrt{3} (12,000)(270.30) \cos(-30^\circ + 0.93^\circ) \\ &= 4.910 \text{ MW} \end{aligned}$$

and hence system X is the load.

The difference in the power supplied by system Y and that absorbed by system X is, of course, the power absorbed by the resistance of the three lines.