4.3 Fundamental Op-Amp Circuits

As a general rule, when analyzing op-amp circuits we write nodal equations at the op-amp input terminals, using the ideal op-amp model conditions. Thus, the technique is straightforward and simple to implement.

EXAMPLE 4.2

Let us determine the gain of the basic inverting op-amp configuration shown in Fig. 4.13a using both the nonideal and ideal op-amp models.

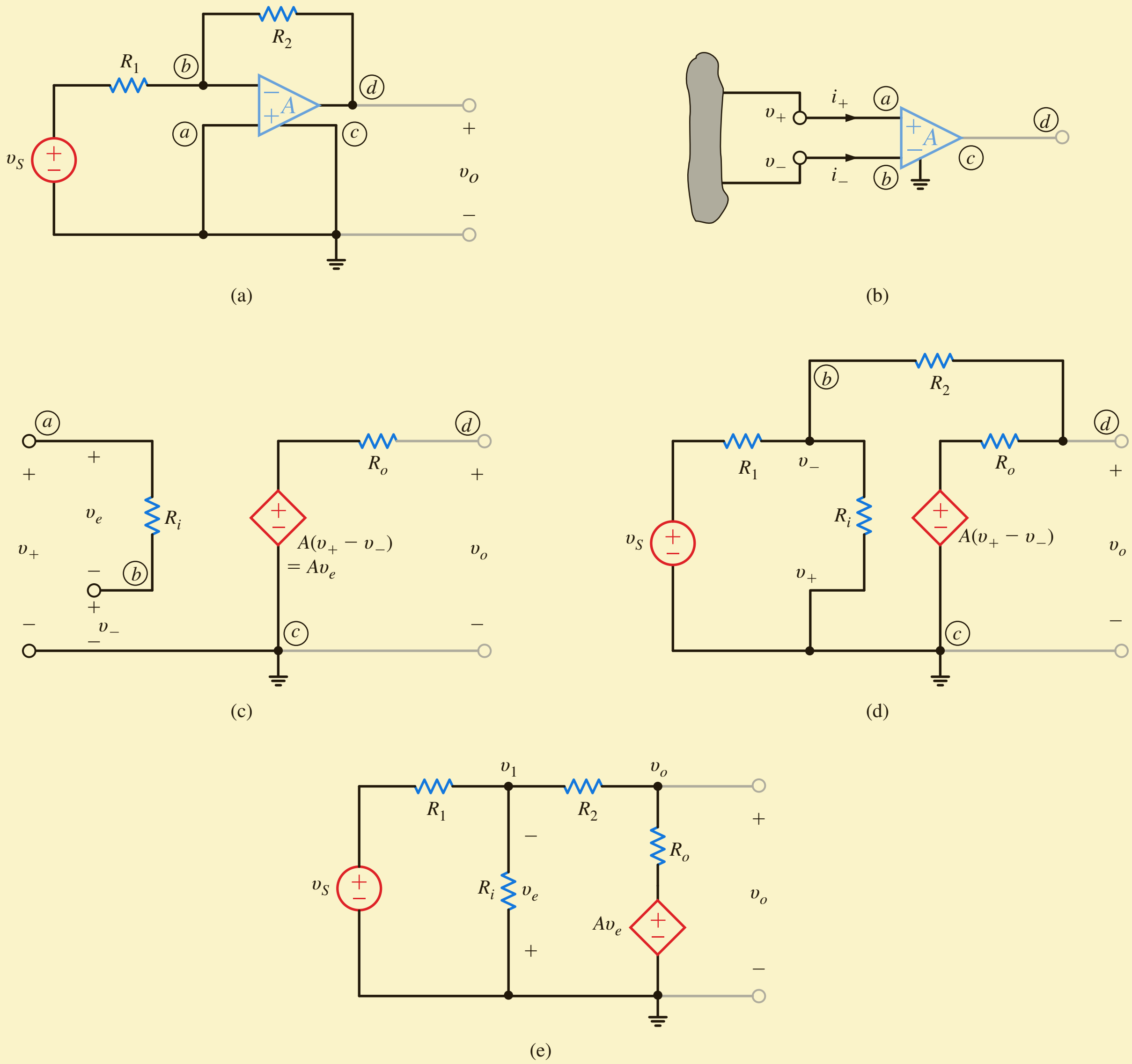


Figure 4.13
Op-amp circuit.

SOLUTION Our model for the op-amp is shown generically in Fig. 4.13b and specifically in terms of the parameters R_i, A, and R_o in Fig. 4.13c. If the model is inserted in the network in Fig. 4.13a, we obtain the circuit shown in Fig. 4.13d, which can be redrawn as shown in Fig. 4.13e.

The node equations for the network are

$$\frac{v_1 - v_S}{R_1} + \frac{v_1}{R_i} + \frac{v_1 - v_o}{R_2} = 0$$

$$\frac{v_o - v_1}{R_2} + \frac{v_o - Av_e}{R_o} = 0$$

where $v_e = -v_1$. The equations can be written in matrix form as

$$\begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2} & -\left(\dfrac{1}{R_2}\right) \\ -\left(\dfrac{1}{R_2} - \dfrac{A}{R_o}\right) & \dfrac{1}{R_2} + \dfrac{1}{R_o} \end{bmatrix} \begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \begin{bmatrix} \dfrac{v_s}{R_1} \\ 0 \end{bmatrix}$$

Solving for the node voltages, we obtain

$$\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \dfrac{1}{R_2} + \dfrac{1}{R_o} & \dfrac{1}{R_2} \\ \dfrac{1}{R_2} - \dfrac{A}{R_o} & \dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_o} \end{bmatrix} \begin{bmatrix} \dfrac{v_s}{R_1} \\ 0 \end{bmatrix}$$

where

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_o}\right) - \left(\frac{1}{R_2}\right)\left(\frac{1}{R_2} - \frac{A}{R_o}\right)$$

Hence,

$$v_o = \frac{\left(\dfrac{1}{R_2} - \dfrac{A}{R_o}\right)\left(\dfrac{v_s}{R_1}\right)}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}\right)\left(\dfrac{1}{R_2} + \dfrac{1}{R_o}\right) - \left(\dfrac{1}{R_2}\right)\left(\dfrac{1}{R_2} - \dfrac{A}{R_o}\right)}$$

which can be written as

$$\frac{v_o}{v_s} = \frac{-(R_2/R_1)}{1 - \left[\left(\dfrac{1}{R_1} + \dfrac{1}{R_i} + \dfrac{1}{R_2}\right)\left(\dfrac{1}{R_2} + \dfrac{1}{R_o}\right) \Big/ \left(\dfrac{1}{R_2}\right)\left(\dfrac{1}{R_2} - \dfrac{A}{R_o}\right)\right]}$$

If we now employ typical values for the circuit parameters (e.g., $A = 10^5$, $R_i = 10^8\ \Omega$, $R_o = 10\ \Omega$, $R_1 = 1\ \text{k}\Omega$, and $R_2 = 5\ \text{k}\Omega$), the voltage gain of the network is

$$\frac{v_o}{v_s} = -4.9996994 \approx -5.000$$

However, the ideal op-amp has infinite gain. Therefore, if we take the limit of the gain equation as $A \to \infty$, we obtain

$$\lim_{A \to \infty} \left(\frac{v_o}{v_S}\right) = -\frac{R_2}{R_1} = -5.000$$

Note that the ideal op-amp yielded a result accurate to within four significant digits of that obtained from an exact solution of a typical op-amp model. These results are easily repeated for the vast array of useful op-amp circuits.

We now analyze the network in Fig. 4.13a using the ideal op-amp model. In this model,

$$i_+ = i_- = 0$$

$$v_+ = v_-$$

As shown in Fig. 4.13a, $v_+ = 0$ and, therefore, $v_- = 0$. If we now write a node equation at the negative terminal of the op-amp, we obtain

$$\frac{v_S - 0}{R_1} + \frac{v_o - 0}{R_2} = 0$$

or

$$\frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

and we have immediately obtained the results derived previously.

Notice that the gain is a simple resistor ratio. This fact makes the amplifier very versatile in that we can control the gain accurately and alter its value by changing only one resistor. Also, the gain is essentially independent of op-amp parameters. Since the precise values of A_o, R_i, and R_o are sensitive to such factors as temperature, radiation, and age, their elimination results in a gain that is stable regardless of the immediate environment. Since it is much easier to employ the ideal op-amp model rather than the nonideal model, unless otherwise stated we will use the ideal op-amp assumptions to analyze circuits that contain operational amplifiers.

PROBLEM-SOLVING STRATEGY

OP-AMP CIRCUITS

STEP 1. Use the ideal op-amp model: $A_o = \infty$, $R_i = \infty$, $R_o = 0$.

- $i_+ = i_- = 0$
- $v_+ = v_-$

STEP 2. Apply nodal analysis to the resulting circuit.

STEP 3. Solve nodal equations to express the output voltage in terms of the op-amp input signals.

EXAMPLE 4.3

Let us now determine the gain of the basic noninverting op-amp configuration shown in Fig. 4.14.

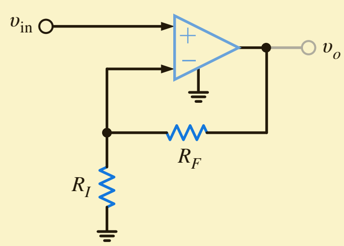


Figure 4.14 The noninverting op-amp configuration.

SOLUTION

Once again, we employ the ideal op-amp model conditions; that is, $v_- = v_+$ and $i_- = i_+$. Using the fact that $i_- = 0$ and $v_- = v_{in}$, the KCL equation at the negative terminal of the op-amp is

$$\frac{v_{in}}{R_I} = \frac{v_o - v_{in}}{R_F}$$

or

$$v_{in}\left(\frac{1}{R_I} + \frac{1}{R_F}\right) = \frac{v_o}{R_F}$$

Thus,

$$\frac{v_o}{v_{in}} = 1 + \frac{R_F}{R_I}$$

Note the similarity of this case to the inverting op-amp configuration in the previous example. We find that the gain in this configuration is also controlled by a simple resistor ratio but is not inverted; that is, the gain ratio is positive.

The remaining examples, though slightly more complicated, are analyzed in exactly the same manner as those already outlined.

EXAMPLE 4.4

Gain error in an amplifier is defined as

$$GE = \left[\frac{\text{actual gain} - \text{ideal gain}}{\text{ideal gain}}\right] \times 100\%$$

We wish to show that for a standard noninverting configuration with finite gain A_o, the gain error is

$$GE = \frac{-100\%}{1 + A_o\beta}$$

where $\beta = R_1/(R_1 + R_2)$.

SOLUTION

The standard noninverting configuration and its equivalent circuit are shown in **Figs. 4.15a** and **b**, respectively. The circuit equations for the network in Fig. 4.15b are

$$v_S = v_{in} + v_1, \qquad v_{in} = \frac{v_o}{A_o}, \qquad \text{and} \qquad v_1 = \frac{R_1}{R_1 + R_2} v_o = \beta v_o$$

The expression that relates the input and output is

$$v_S = v_o\left[\frac{1}{A_o} + \beta\right] = v_o\left[\frac{1 + A_o\beta}{A_o}\right]$$

and thus the actual gain is

$$\frac{v_o}{v_S} = \frac{A_o}{1 + A_o\beta}$$

Recall that the ideal gain for this circuit is $(R_1 + R_2)/R_1 = 1/\beta$. Therefore, the gain error is

$$GE = \left[\frac{\dfrac{A_o}{1 + A_o\beta} - \dfrac{1}{\beta}}{1/\beta}\right]100\%$$

which, when simplified, yields

$$GE = \frac{-100\%}{1 + A_o\beta}$$

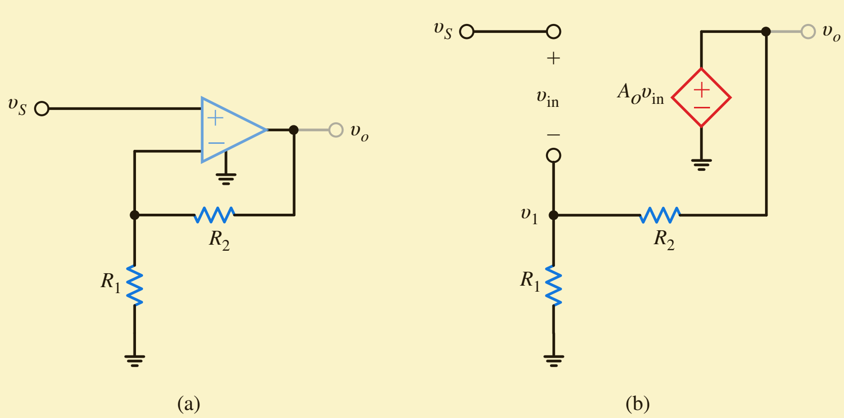


Figure 4.15
Circuits used in Example 4.4.

EXAMPLE 4.5

Consider the op-amp circuit shown in Fig. 4.16. Let us determine an expression for the output voltage.

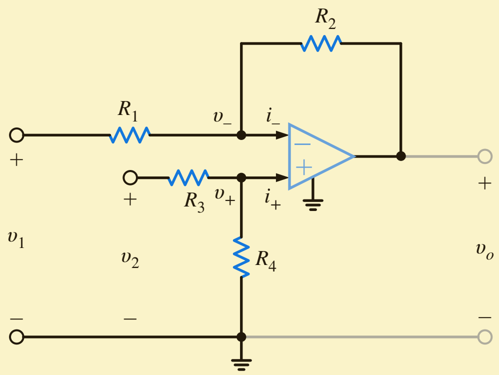


Figure 4.16 Differential amplifier operational amplifier circuit.

SOLUTION

The node equation at the inverting terminal is

$$\frac{v_1 - v_-}{R_1} + \frac{v_o - v_-}{R_2} = i_-$$

At the noninverting terminal, KCL yields

$$\frac{v_2 - v_+}{R_3} = \frac{v_+}{R_4} + i_+$$

However, $i_+ = i_- = 0$ and $v_+ = v_-$. Substituting these values into the two preceding equations yields

$$\frac{v_1 - v_-}{R_1} + \frac{v_o - v_-}{R_2} = 0$$

and

$$\frac{v_2 - v_-}{R_3} = \frac{v_-}{R_4}$$

Solving these two equations for v_o results in the expression

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{R_1}{R_2}\right)\frac{R_4}{R_3 + R_4}v_2 - \frac{R_2}{R_1}v_1$$

Note that if $R_4 = R_2$ and $R_3 = R_1$, the expression reduces to

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

Therefore, this op-amp can be employed to subtract two input voltages.

EXAMPLE 4.6

The circuit shown in Fig. 4.17a is a precision differential voltage-gain device. It is used to provide a single-ended input for an analog-to-digital converter. We wish to derive an expression for the output of the circuit in terms of the two inputs.

SOLUTION

To accomplish this, we draw the equivalent circuit shown in Fig. 4.17b. Recall that the voltage across the input terminals of the op-amp is approximately zero and the currents into the op-amp input terminals are approximately zero. Note that we can write node equations for node voltages v_1 and v_2 in terms of v_o and v_a. Since we are interested in an expression for v_o

in terms of the voltages v_1 and v_2, we simply eliminate the v_a terms from the two node equations. The node equations are

$$\frac{v_1 - v_o}{R_2} + \frac{v_1 - v_a}{R_1} + \frac{v_1 - v_2}{R_G} = 0$$

$$\frac{v_2 - v_a}{R_1} + \frac{v_2 - v_1}{R_G} + \frac{v_2}{R_2} = 0$$

Combining the two equations to eliminate v_a, and then writing v_o in terms of v_1 and v_2, yields

$$v_o = (v_1 - v_2)\left(1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}\right)$$

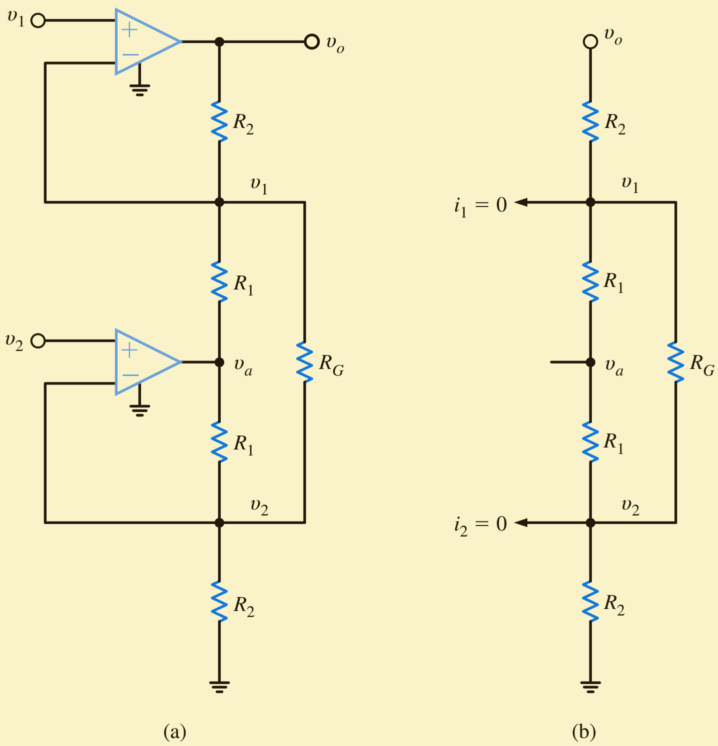


Figure 4.17 Instrumentation amplifier circuit.

LEARNING ASSESSMENTS

E4.1 Find I_o in the network in Fig. E4.1.

ANSWER: $I_o = 8.4$ mA.

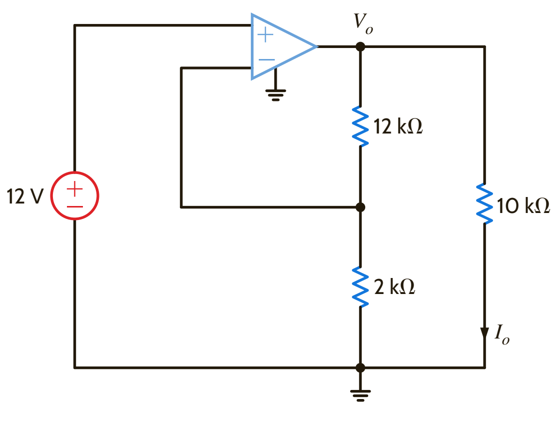


Figure E4.1

E4.2 Determine the gain of the op-amp circuit in Fig. E4.2.

ANSWER:
$$\frac{V_o}{V_S} = 1 + \frac{R_2}{R_1}.$$

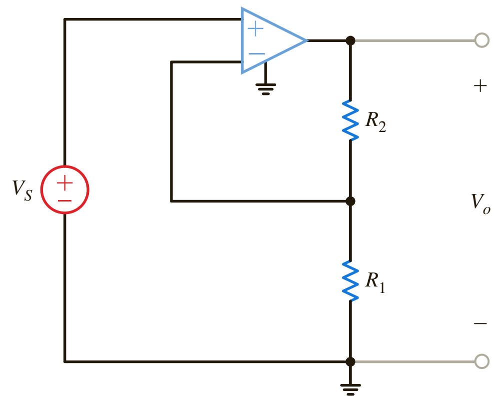


Figure E4.2

E4.3 Determine both the gain and the output voltage of the op-amp configuration shown in Fig. E4.3.

ANSWER:
$V_o = 0.101$ V;
gain = 101.

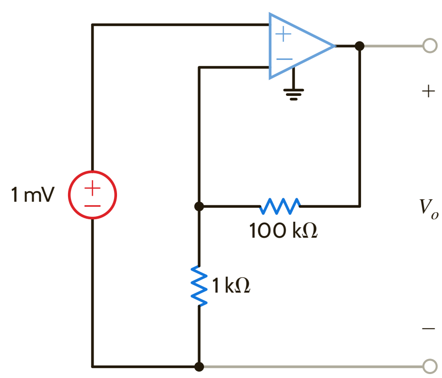


Figure E4.3

E4.4 Find I_1, I_2, I_3, and I_4 in Fig. E4.4.

ANSWER:
$I_1 = 0$, $I_2 = 1.25$ mA,
$I_3 = -0.5$ mA, and $I_4 = 0.75$ mA.

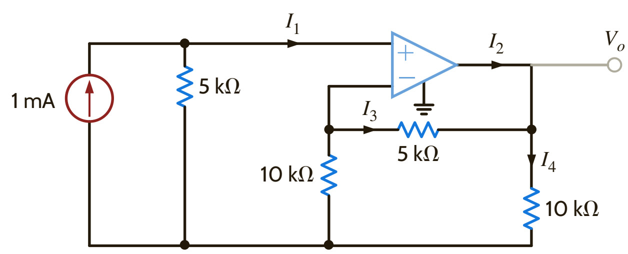


Figure E4.4

E4.5 Find V_o in terms of V_1 and V_2 in Fig. E4.5. If $V_1 = V_2 = 4$ V, find V_o. If the op-amp power supplies are ± 15 V and $V_2 = 2$ V, what is the allowable range of V_1?

ANSWER:
$V_o = -2\,V_1 + 3.5\,V_2$;
6 V; $-4\text{ V} \le V_1 \le 11$ V.

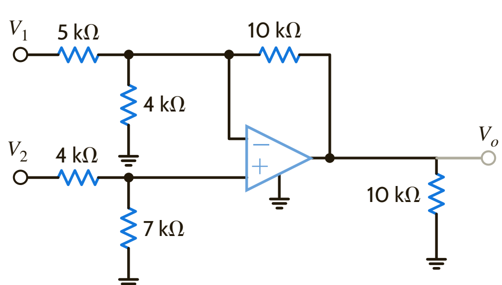


Figure E4.5

E4.6 Find V_o and V_3 in Fig. E4.6.

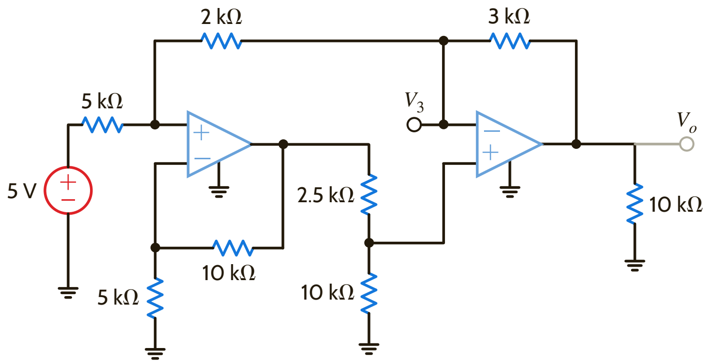


Figure E4.6

ANSWER:
$V_o = -9$ V;
$V_3 = -4.8$ V.

E4.7 Find V_o in Fig. E4.7.

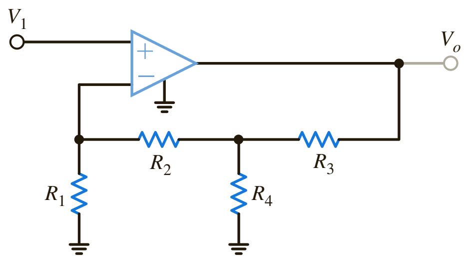


Figure E4.7

ANSWER:

$$V_o = \left[\left(\frac{R_3}{R_2} + \frac{R_3}{R_4} + 1\right)\left(1 + \frac{R_2}{R_1}\right) - \frac{R_3}{R_2}\right]V_1$$

EXAMPLE 4.7

The circuit in Fig. 4.18 is an electronic ammeter. It operates as follows: the unknown current, I, through R_I produces a voltage, V_I. V_I is amplified by the op-amp to produce a voltage, V_o, which is proportional to I. The output voltage is measured with a simple voltmeter. We want to find the value of R_2 such that 10 V appears at V_o for each milliamp of unknown current.

SOLUTION

Since the current into the op-amp + terminal is zero, the relationship between V_I and I is

$$V_I = IR_I$$

The relationship between the input and output voltages is

$$V_o = V_I\left(1 + \frac{R_2}{R_1}\right)$$

or, solving the equation for V_o/I, we obtain

$$\frac{V_o}{I} = R_I\left(1 + \frac{R_2}{R_1}\right)$$

Using the required ratio V_o/I of 10^4 and resistor values from Fig. 4.18, we can find that

$$R_2 = 9\ \text{k}\Omega$$

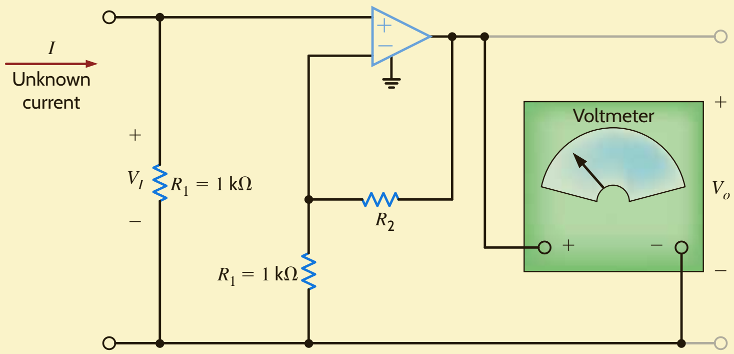


Figure 4.18
Electronic ammeter.

EXAMPLE 4.8

The two op-amp circuits shown in **Fig. 4.19** produce an output given by the equation

$$V_o = 8\,V_1 - 4\,V_2$$

where

$$1\text{ V} \leq V_1 \leq 2\text{ V} \qquad \text{and} \qquad 2\text{ V} \leq V_2 \leq 3\text{ V}$$

We wish to determine (a) the range of V_o and (b) if both of the circuits will produce the full range of V_o given that the dc supplies are ± 10 V.

SOLUTION

a. Given that $V_o = 8\,V_1 - 4\,V_2$ and the range for both V_1 and V_2 is $1\text{ V} \leq V_1 \leq 2\text{ V}$ and $2\text{ V} \leq V_2 \leq 3\text{ V}$, we find that

$$V_{o\,\max} = 8(2) - 4(2) = 8\text{ V} \qquad \text{and} \qquad V_{o\,\min} = 8(1) - 4(3) = -4\text{ V}$$

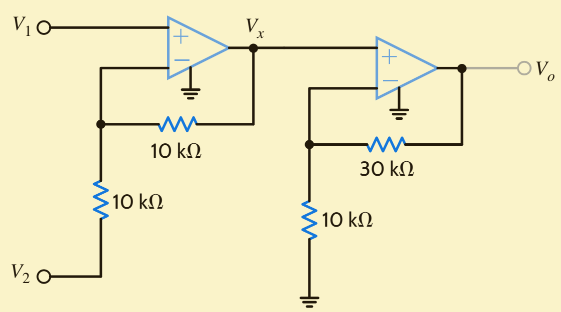
and thus the range of V_o is -4 V to $+8$ V.

b. Consider first the network in **Fig. 4.19a**. The signal at V_x, which can be derived using the network in Example 4.5, is given by the equation $V_x = 2\,V_1 - V_2$. V_x is a maximum when $V_1 = 2$ V and $V_2 = 2$ V; that is, $V_{x\,\max} = 2(2) - 2 = 2$ V. The minimum value for V_x occurs when $V_1 = 1$ V and $V_2 = 3$ V; that is, $V_{x\,\min} = 2(1) - 3 = -1$ V. Since both the max and min values are within the supply range of ± 10 V, the first op-amp in Fig. 4.19a will not saturate. The output of the second op-amp in this circuit is given by the expression $V_o = 4\,V_x$. Therefore, the range of V_o is $-4\text{ V} \leq V_o \leq 8$ V. Since this range is also within the power supply voltages, the second op-amp will not saturate, and this circuit will produce the full range of V_o.

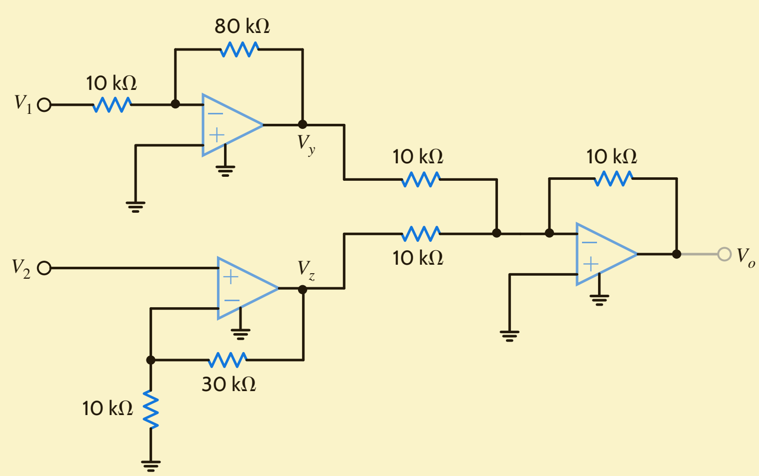
Next, consider the network in **Fig. 4.19b**. The signal $V_y = -8\,V_1$ and so the range of V_y is $-16\text{ V} \leq V_y \leq -8$ V and the range of V_y is outside the power supply limits. This circuit will saturate and fail to produce the full range of V_o.

Figure 4.19

Circuits used in Example 4.7.



(a)



(b)