E9.18 The source in Fig. E9.18 supplies 40 kW at a power factor of 0.9 lagging. The real and reactive losses of the transmission-line feeder are 1.6 kW and 2.1 kvar, respectively. Find the load voltage and the real and reactive power absorbed by the load.

ANSWER:
$\mathbf{V}_L = 416.83\underline{/-162°}$ V;
$P_L = 38.4$ kW;
$Q_L = 17.27$ kvar.

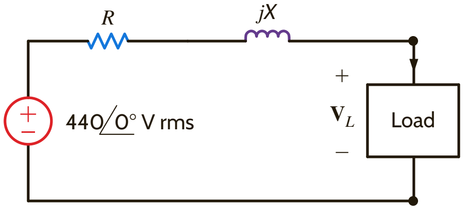

Figure E9.18

E9.19 Find the power factor of the source and $v_S(t)$ in Fig. E9.19 if $f = 60$ Hz.

ANSWER:
$\text{pf}_{\text{tn}} = 0.9457$ lagging; $v_S(t) = 765.94 \cos(377t - 7.77°)$ V.

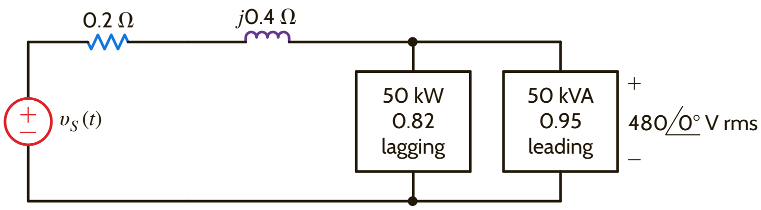

Figure E9.19

9.7 Power Factor Correction

Industrial plants that require large amounts of power have a wide variety of loads. However, by nature the loads normally have a lagging power factor. In view of the results obtained in Example 9.10, we are naturally led to ask whether there is any convenient technique for raising the power factor of a load. Since a typical load may be a bank of induction motors or other expensive machinery, the technique for raising the pf should be an economical one to be feasible.

To answer the question we pose, consider the diagram in **Fig. 9.16**. A typical industrial load with a lagging pf is supplied by an electrical source. Also shown is the power triangle for the load. The load pf is $\cos(\theta_{\text{old}})$. If we want to improve the power factor, we need to reduce the angle shown on the power triangle in Fig. 9.16. From Eq. (9.38) we know that the tangent of this angle is equal to the ratio of Q to P. We could decrease the angle by increasing P. This is not an economically attractive solution because our increased power consumption would increase the monthly bill from the electric utility.

The other option we have to reduce this angle is to decrease Q. How can we decrease Q? Recall from a previous section that a capacitor is a source of reactive power and does not absorb real power. Suppose we connect a capacitor in parallel with our industrial load as shown in **Fig. 9.17**. The corresponding power triangles for this diagram are also shown in Fig. 9.17. Let's define

$$\mathbf{S}_{\text{old}} = P_{\text{old}} + jQ_{\text{old}} = |\mathbf{S}_{\text{old}}|\underline{/\theta_{\text{old}}} \quad \text{and} \quad \mathbf{S}_{\text{new}} = P_{\text{old}} + jQ_{\text{new}} = |\mathbf{S}_{\text{new}}|\underline{/\theta_{\text{new}}}$$

Then with the addition of the capacitor,

$$\mathbf{S}_{\text{new}} = \mathbf{S}_{\text{old}} + \mathbf{S}_{\text{cap}}$$

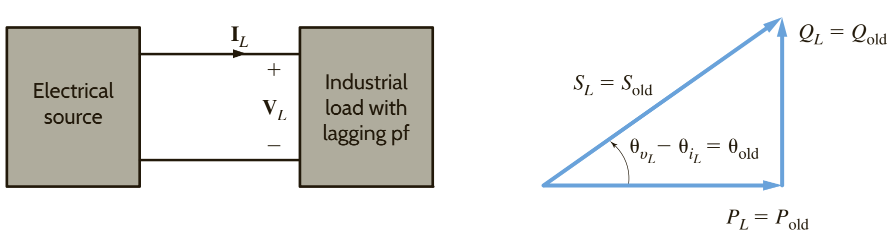

Figure 9.16 Diagram for power factor correction.

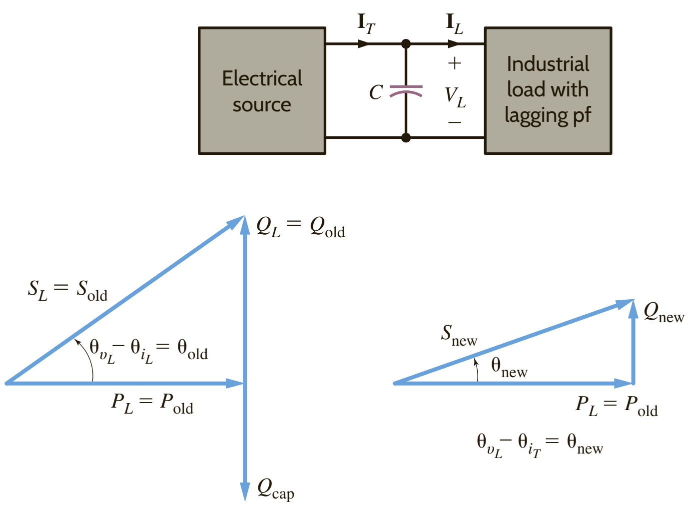

Figure 9.17
Power factor correction diagram including capacitor.

Therefore,

$$\begin{aligned}\mathbf{S}_{cap} &= \mathbf{S}_{new} - \mathbf{S}_{old}\\ &= (P_{new} - jQ_{new}) - (P_{old} + jQ_{old})\\ &= j(Q_{new} - Q_{old})\\ &= j(Q_{cap})\end{aligned}$$

Recall from Eqs. (9.36) and (9.37) that in general

$$\mathbf{S} = \mathrm{V}_{rms}^2/\mathbf{Z}^*$$

and for a capacitor

$$\mathbf{Z}^* = -1/j\omega C$$

so that

$$\mathbf{S}_{cap} = Q_{cap} = -j\omega C \mathrm{V}_{rms}^2$$

This equation can be used to find the required value of C in order to achieve the new specified power factor defined by the new power factor angle illustrated in Fig. 9.17.

Hence, we can obtain a particular power factor for the total load (industrial load and capacitor) simply by judiciously selecting a capacitor and placing it in parallel with the original load. In general, we want the power factor to be large, and therefore the power factor angle must be small [i.e., the larger the desired power factor, the smaller the angle $(\theta_{v_L} - \theta_{i_T})$].

EXAMPLE 9.13

Every month our electrical energy provider sends us a bill for the amount of electrical energy that we have consumed. The rate is often expressed in cents per kWh and consists of at least two components: (1) the demand charge, which covers the cost of lines, poles, transformers, and so on, and (2) the energy charge, which covers the cost to produce electric energy at power plants. The energy charge is the subject of the deregulation of the electric utility industry where you, as a customer, choose your energy provider.

It is common for an industrial facility operating at a poor power factor to be charged more by the electric utility providing electrical service. Let's suppose that our industrial facility operates at 277 V rms and requires 500 kW at a power factor of 0.75 lagging. Assume an energy charge of 2¢ per kWh and a demand charge of \$3.50 per kW per month if the power factor is between 0.9 lagging and unity and \$5 per kVA per month if the power factor is less than 0.9 lagging.

The monthly energy charge is 500 × 24 × 30 × \$0.02 = \$7200. Let's calculate the monthly demand charge with the 0.75 lagging power factor. The complex power absorbed by the industrial facility is

$$\mathbf{S}_{\text{old}} = \frac{500}{0.75}\underline{/\cos^{-1}(0.75)} = 666.67\underline{/41.4^\circ} = 500 + j441 \text{ kVA}$$

The monthly demand charge is 666.67 × \$5 = \$3333.35. The total bill from the energy provider is \$7200 + \$3333.35 = \$10,533.35 per month.

Let's consider installing a capacitor bank, as shown in Fig. 9.18, to correct the power factor and reduce our demand charge. The demand charge is such that we only need to correct the power factor to 0.9 lagging. The monthly demand charge will be the same whether the power factor is corrected to 0.9 or unity. The complex power absorbed by the industrial facility and capacitor bank will be

$$\mathbf{S}_{\text{new}} = \frac{500}{0.9}\underline{/\cos^{-1}(0.9)} = 555.6\underline{/25.84^\circ} = 500 + j242.2 \text{ kVA}$$

The monthly demand charge for our industrial facility with the capacitor bank is 500 × \$3.50 = \$1750 per month. The average power absorbed by our capacitor bank is negligible compared to the average power absorbed by the industrial facility, so our monthly energy charge remains \$7200 per month. With the capacitor bank installed, the total bill from the energy provider is \$7200 + \$1750 = \$8950 per month.

How many kvars of capacitance do we need to correct the power factor to 0.9 lagging?

$$\mathbf{S}_{\text{new}} - \mathbf{S}_{\text{old}} = \mathbf{S}_{\text{cap}} = (500 + j242.2) - (500 + \text{j}441) = -j198.8 \text{ kvar}$$

Let's assume that it costs \$100 per kvar to install the capacitor bank at the industrial facility for an installation cost of \$19,880. How long will it take to recover the cost of installing the capacitor bank? The difference in the monthly demand charge without the bank and with the bank is \$3333.35 − \$1750 = \$1583.35. Dividing this value into the cost of installing the bank yields \$19,880/\$1583.35 = 12.56 months.

Figure 9.18
A bank of capacitors (Courtesy of Jeremy Nelms, Talquin Electric Cooperative, Inc.).

EXAMPLE 9.14

Plastic kayaks are manufactured using a process called rotomolding, which is diagrammed in Fig. 9.19. Molten plastic is injected into a mold, which is then spun on the long axis of the kayak until the plastic cools, resulting in a hollow one-piece craft. Suppose that the induction motors used to spin the molds consume 50 kW at a pf of 0.8 lagging from a $220\underline{/0^\circ}$-V rms, 60-Hz line. We wish to raise the pf to 0.95 lagging by placing a bank of capacitors in parallel with the load.

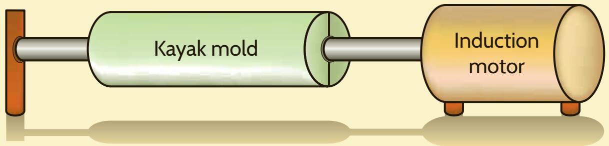

Figure 9.19 Rotomolding manufacturing process.

SOLUTION

The circuit diagram for this problem is shown in Fig. 9.20. $P_L = 50$ kW and since $\cos^{-1} 0.8 = 36.87°$, $\theta_{old} = 36.87°$. Therefore,

$$Q_{old} = P_{old} \tan \theta_{old} = (50)(10^3)(0.75) = 37.5 \text{ kvar}$$

Hence,

$$\mathbf{S}_{old} = P_{old} + jQ_{old} = 50{,}000 + j37{,}500$$

and

$$\mathbf{S}_{cap} = 0 + jQ_{cap}$$

Since the required power factor is 0.95,

$$\begin{aligned}\theta_{new} &= \cos^{-1}(\text{pf}_{new}) = \cos^{-1}(0.95)\\ &= 18.19°\end{aligned}$$

Then

$$\begin{aligned}Q_{new} &= P_{old} \tan \theta_{new}\\ &= 50{,}000 \tan (18.19°)\\ &= 16{,}430 \text{ var}\end{aligned}$$

Hence

$$\begin{aligned}Q_{new} - Q_{old} = Q_{cap} &= -\omega C V^2_{\text{rms}}\\ 16{,}430 - 37{,}500 &= -\omega C V^2_{\text{rms}}\end{aligned}$$

Solving the equation for C yields

$$\begin{aligned}C &= \frac{21{,}070}{(377)(220)^2}\\ &= 1155 \ \mu\text{F}\end{aligned}$$

By using a capacitor of this magnitude in parallel with the industrial load, we create, from the utility's perspective, a load pf of 0.95 lagging. However, the parameters of the actual load remain unchanged. Under these conditions, the current supplied by the utility to the kayak manufacturer is less, and therefore they can use smaller conductors for the same amount of power. Or, if the conductor size is fixed, the line losses will be less since these losses are a function of the square of the current.

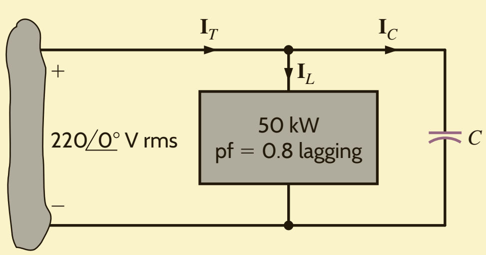

Figure 9.20 Example circuit for power factor correction.