

**E2.9** Find  $V_{bd}$  in the circuit in Fig. E2.9.

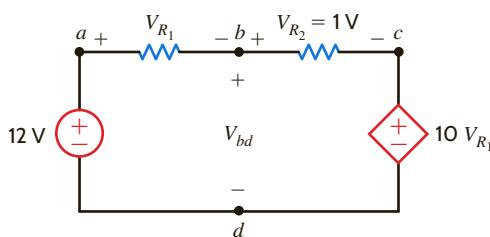


Figure E2.9

**ANSWER:**

$$V_{bd} = 11 \text{ V.}$$

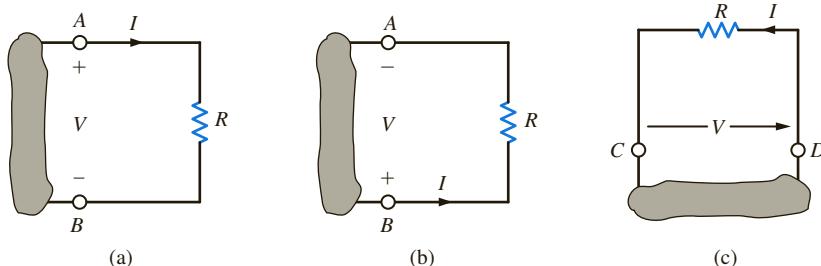


**HINT**  
The subtleties associated with Ohm's law, as described here, are important and must be adhered to in order to ensure that the variables have the proper sign.

Before proceeding with the analysis of simple circuits, it is extremely important that we emphasize a subtle but very critical point. Ohm's law as defined by the equation  $V = IR$  refers to the relationship between the voltage and current as defined in **Fig. 2.14a**. If the direction of either the current or the voltage, but not both, is reversed, the relationship between the current and the voltage would be  $V = -IR$ . In a similar manner, given the circuit in **Fig. 2.14b**, if the polarity of the voltage between the terminals *A* and *B* is specified as shown, then the direction of the current *I* is from point *B* through *R* to point *A*. Likewise, in **Fig. 2.14c**, if the direction of the current is specified as shown, then the polarity of the voltage must be such that point *D* is at a higher potential than point *C* and, therefore, the arrow representing the voltage *V* is from point *C* to point *D*.

Figure 2.14

Circuits used to explain Ohm's law.



## 2.3

### Single-Loop Circuits

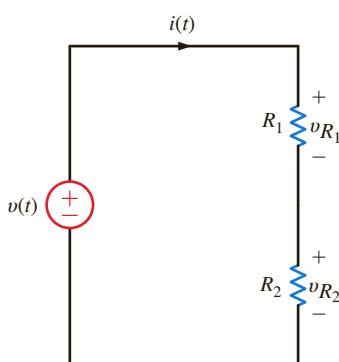


Figure 2.15

Single-loop circuit.

**VOLTAGE DIVISION** At this point we can begin to apply the laws presented earlier to the analysis of simple circuits. To begin, we examine what is perhaps the simplest circuit—a single closed path, or loop, of elements.

Applying KCL to every node in a single-loop circuit reveals that the same current flows through all elements. We say that these elements are connected in series because they carry the same current. We will apply Kirchhoff's voltage law and Ohm's law to the circuit to determine various quantities in the circuit.

Our approach will be to begin with a simple circuit and then generalize the analysis to more complicated ones. The circuit shown in **Fig. 2.15** will serve as a basis for discussion. This circuit consists of an independent voltage source that is in series with two resistors. We have assumed that the current flows in a clockwise direction. If this assumption is correct, the solution of the equations that yields the current will produce a positive value. If the current is actually flowing in the opposite direction, the value of the current variable will simply be negative, indicating that the current is flowing in a direction opposite to that assumed. We have also made voltage polarity assignments for  $v_{R_1}$  and  $v_{R_2}$ . These assignments have been made using the convention employed in our discussion of Ohm's law and our choice for the direction of  $i(t)$ —that is, the convention shown in **Fig. 2.14a**.

Applying Kirchhoff's voltage law to this circuit yields

$$-v(t) + v_{R_1} + v_{R_2} = 0$$

or

$$v(t) = v_{R_1} + v_{R_2}$$

However, from Ohm's law we know that

$$\begin{aligned} v_{R_1} &= R_1 i(t) \\ v_{R_2} &= R_2 i(t) \end{aligned}$$

Therefore,

$$v(t) = R_1 i(t) + R_2 i(t)$$

Solving the equation for  $i(t)$  yields

$$i(t) = \frac{v(t)}{R_1 + R_2} \quad 2.9$$

Knowing the current, we can now apply Ohm's law to determine the voltage across each resistor:

$$\begin{aligned} v_{R_1} &= R_1 i(t) \\ &= R_1 \left[ \frac{v(t)}{R_1 + R_2} \right] \\ &= \frac{R_1}{R_1 + R_2} v(t) \end{aligned} \quad 2.10$$

Similarly,

$$v_{R_2} = \frac{R_2}{R_1 + R_2} v(t) \quad 2.11$$

Though simple, Eqs. (2.10) and (2.11) are very important because they describe the operation of what is called a *voltage divider*. In other words, the source voltage  $v(t)$  is divided between the resistors  $R_1$  and  $R_2$  in direct proportion to their resistances.

In essence, if we are interested in the voltage across the resistor  $R_1$ , we bypass the calculation of the current  $i(t)$  and simply multiply the input voltage  $v(t)$  by the ratio

$$\frac{R_1}{R_1 + R_2}$$

As illustrated in Eq. (2.10), we are using the current in the calculation, but not explicitly.

Note that the equations satisfy Kirchhoff's voltage law, since

$$-v(t) + \frac{R_1}{R_1 + R_2} v(t) + \frac{R_2}{R_1 + R_2} v(t) = 0$$

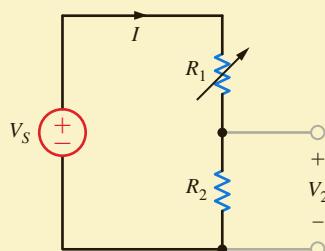


**HINT**  
The manner in which voltage divides between two series resistors.

Consider the circuit shown in **Fig. 2.16**. The circuit is identical to Fig. 2.15 except that  $R_1$  is a variable resistor such as the volume control for a radio or television set. Suppose that  $V_S = 9$  V,  $R_1 = 90$  k $\Omega$ , and  $R_2 = 30$  k $\Omega$ .

Let us examine the change in both the voltage across  $R_2$  and the power absorbed in this resistor as  $R_1$  is changed from 90 k $\Omega$  to 15 k $\Omega$ .

## EXAMPLE 2.13



**Figure 2.16**  
Voltage-divider circuit.

**SOLUTION** Since this is a voltage-divider circuit, the voltage  $V_2$  can be obtained directly as

$$\begin{aligned} V_2 &= \left[ \frac{R_2}{R_1 + R_2} \right] V_s \\ &= \left[ \frac{30\text{k}}{90\text{k} + 30\text{k}} \right] (9) \\ &= 2.25 \text{ V} \end{aligned}$$

Now suppose that the variable resistor is changed from  $90 \text{ k}\Omega$  to  $15 \text{ k}\Omega$ . Then

$$\begin{aligned} V_2 &= \left[ \frac{30\text{k}}{30\text{k} + 15\text{k}} \right] (9) \\ &= 6 \text{ V} \end{aligned}$$

The direct voltage-divider calculation is equivalent to determining the current  $I$  and then using Ohm's law to find  $V_2$ . Note that the larger voltage is across the larger resistance. This voltage-divider concept and the simple circuit we have employed to describe it are very useful because, as will be shown later, more complicated circuits can be reduced to this form.

Finally, let us determine the instantaneous power absorbed by the resistor  $R_2$  under the two conditions  $R_1 = 90 \text{ k}\Omega$  and  $R_1 = 15 \text{ k}\Omega$ . For the case  $R_1 = 90 \text{ k}\Omega$ , the power absorbed by  $R_2$  is

$$\begin{aligned} P_2 &= I^2 R_2 = \left( \frac{9}{120\text{k}} \right)^2 (30\text{k}) \\ &= 0.169 \text{ mW} \end{aligned}$$

In the second case

$$\begin{aligned} P_2 &= \left( \frac{9}{45\text{k}} \right)^2 (30\text{k}) \\ &= 1.2 \text{ mW} \end{aligned}$$

The current in the first case is  $75 \mu\text{A}$ , and in the second case it is  $200 \mu\text{A}$ . Since the power absorbed is a function of the square of the current, the power absorbed in the two cases is quite different.

Let us now demonstrate the practical utility of this simple voltage-divider network.

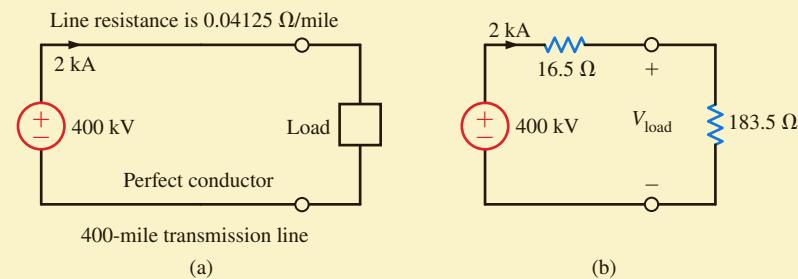
## EXAMPLE 2.14



Consider the circuit in **Fig. 2.17a**, which is an approximation of a high-voltage dc transmission facility. We have assumed that the bottom portion of the transmission line is a perfect conductor and will justify this assumption in the next chapter. The load can be represented by a resistor of value  $183.5 \Omega$ . Therefore, the equivalent circuit of this network is shown in **Fig. 2.17b**.

**Figure 2.17**

A high-voltage dc transmission facility.



Let us determine both the power delivered to the load and the power losses in the line.

Using voltage division, the load voltage is

### SOLUTION

$$V_{\text{load}} = \left[ \frac{183.5}{183.5 + 16.5} \right] (400\text{k}) \\ = 367 \text{kV}$$

The input power is 800 MW and the power transmitted to the load is

$$P_{\text{load}} = I^2 R_{\text{load}} \\ = 734 \text{ MW}$$

Therefore, the power loss in the transmission line is

$$P_{\text{line}} = P_{\text{in}} - P_{\text{load}} = I^2 R_{\text{line}} \\ = 66 \text{ MW}$$

Since  $P = VI$ , suppose now that the utility company supplied power at 200 kV and 4 kA. What effect would this have on our transmission network? Without making a single calculation, we know that because power is proportional to the square of the current, there would be a large increase in the power loss in the line and, therefore, the efficiency of the facility would decrease substantially. That is why, in general, we transmit power at high voltage and low current.

**MULTIPLE-SOURCE/RESISTOR NETWORKS** At this point we wish to extend our analysis to include a multiplicity of voltage sources and resistors. For example, consider the circuit shown in **Fig. 2.18a**. Here we have assumed that the current flows in a clockwise direction, and we have defined the variable  $i(t)$  accordingly. This may or may not be the case, depending on the value of the various voltage sources. Kirchhoff's voltage law for this circuit is

$$+v_{R_1} + v_2(t) - v_3(t) + v_{R_2} + v_4(t) + v_5(t) - v_l(t) = 0$$

or, using Ohm's law,

$$(R_1 + R_2)i(t) = v_l(t) - v_2(t) + v_3(t) - v_4(t) - v_5(t)$$

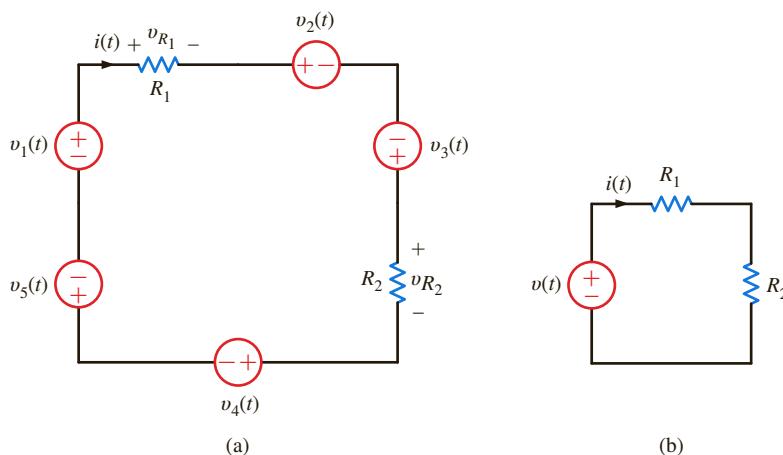
which can be written as

$$(R_1 + R_2)i(t) = v(t)$$

where

$$v(t) = v_l(t) + v_3(t) - [v_2(t) + v_4(t) + v_5(t)]$$

so that under the preceding definitions, **Fig. 2.18a** is equivalent to **Fig. 2.18b**. In other words, the sum of several voltage sources in series can be replaced by one source whose value is the algebraic sum of the individual sources. This analysis can, of course, be generalized to a circuit with  $N$  series sources.

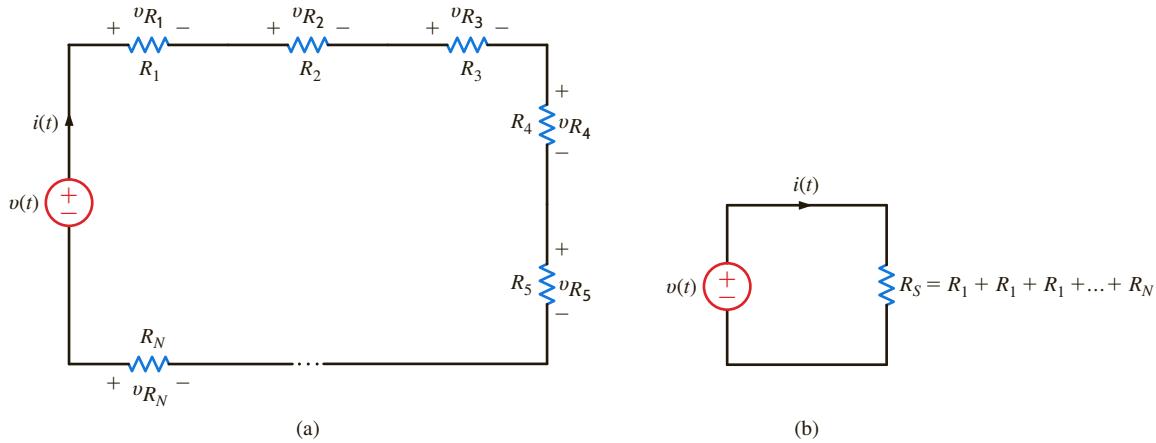


**Figure 2.18**

Equivalent circuits with multiple sources.

**Figure 2.19**

Equivalent circuits.



Now consider the circuit with  $N$  resistors in series, as shown in **Fig. 2.19a**. Applying Kirchhoff's voltage law to this circuit yields

$$\begin{aligned} v(t) &= v_{R_1} + v_{R_2} + \cdots + v_{R_N} \\ &= R_1 i(t) + R_2 i(t) + \cdots + R_N i(t) \end{aligned}$$

and therefore,

$$v(t) = R_S i(t) \quad 2.12$$

where

$$R_S = R_1 + R_2 + \cdots + R_N \quad 2.13$$

and hence,

$$i(t) = \frac{v(t)}{R_S} \quad 2.14$$

Note also that for any resistor  $R_i$  in the circuit, the voltage across  $R_i$  is given by the expression

$$v_{R_i} = \frac{R_i}{R_S} v(t) \quad 2.15$$

which is the voltage-division property for multiple resistors in series.

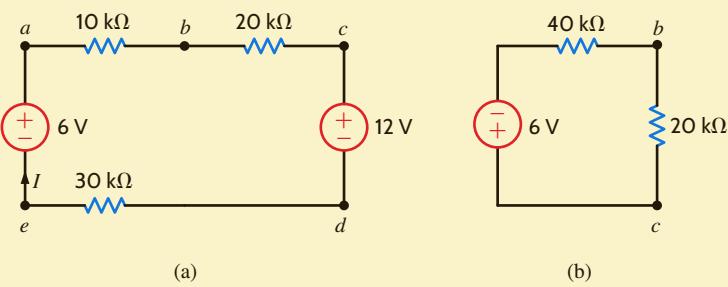
Equation (2.13) illustrates that *the equivalent resistance of  $N$  resistors in series is simply the sum of the individual resistances*. Thus, using Eq. (2.13), we can draw the circuit in **Fig. 2.19b** as an equivalent circuit for the one in **Fig. 2.19a**.

## EXAMPLE 2.15

Given the circuit in **Fig. 2.20a**, let us find  $I$ ,  $V_{bd}$ , and the power absorbed by the  $30\text{-k}\Omega$  resistor. Finally, let us use voltage division to find  $V_{bc}$ .

**Figure 2.20**

Circuit used in Example 2.15.



KVL for the network yields the equation

$$\begin{aligned} 10kI + 20kI + 12 + 30kI - 6 &= 0 \\ 60kI &= -6 \\ I &= -0.1 \text{ mA} \end{aligned}$$

Therefore, the magnitude of the current is 0.1 mA, but its direction is opposite to that assumed.

The voltage  $V_{bd}$  can be calculated using either of the closed paths *abdea* or *bcd*. The equations for both cases are

$$10kI + V_{bd} + 30k - 6 = 0$$

and

$$20kI + 12 - V_{bd} = 0$$

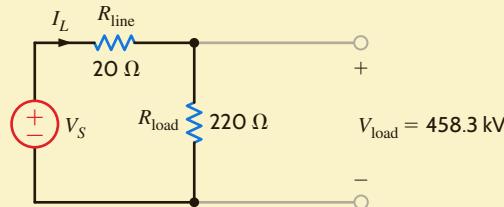
Using  $I = -0.1$  mA in either equation yields  $V_{bd} = 10$  V. Finally, the power absorbed by the 30-k $\Omega$  resistor is

$$P = I^2R = 0.3 \text{ mW}$$

Now from the standpoint of determining the voltage  $V_{bc}$ , we can simply add the sources since they are in series, add the remaining resistors since they are in series, and reduce the network to that shown in **Fig. 2.20b**. Then

$$\begin{aligned} V_{bc} &= \frac{20k}{20k + 40k} (-6) \\ &= -2 \text{ V} \end{aligned}$$

A dc transmission facility is modeled by the approximate circuit shown in **Fig. 2.21**. If the load voltage is known to be  $V_{\text{load}} = 458.3$  kV, we wish to find the voltage at the sending end of the line and the power loss in the line.



**Figure 2.21**

Circuit used in Example 2.16.

## EXAMPLE 2.16

Knowing the load voltage and load resistance, we can obtain the line current using Ohm's law:

$$\begin{aligned} I_L &= 458.3 \text{ kV} / 220 \Omega \\ &= 2.083 \text{ kA} \end{aligned}$$

The voltage drop across the line is

$$\begin{aligned} V_{\text{line}} &= (I_L)(R_{\text{line}}) \\ &= 41.66 \text{ kV} \end{aligned}$$

Now, using KVL,

$$\begin{aligned} V_S &= V_{\text{line}} + V_{\text{load}} \\ &= 500 \text{ kV} \end{aligned}$$

## SOLUTION

## SOLUTION

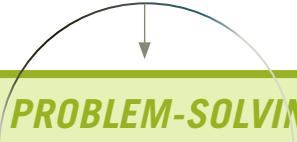
Note that since the network is simply a voltage-divider circuit, we could obtain  $V_S$  immediately from our knowledge of  $R_{\text{line}}$ ,  $R_{\text{load}}$ , and  $V_{\text{load}}$ . That is,

$$V_{\text{load}} = \left[ \frac{R_{\text{load}}}{R_{\text{load}} + R_{\text{line}}} \right] V_S$$

and  $V_S$  is the only unknown in this equation.

The power absorbed by the line is

$$\begin{aligned} P_{\text{line}} &= I_L^2 R_{\text{line}} \\ &= 86.79 \text{ MW} \end{aligned}$$



## PROBLEM-SOLVING STRATEGY

### SINGLE-LOOP CIRCUITS

- STEP 1.** Define a current  $i(t)$ . We know from KCL that there is only one current for a single-loop circuit. This current is assumed to be flowing either clockwise or counterclockwise around the loop.
- STEP 2.** Using Ohm's law, define a voltage across each resistor in terms of the defined current.
- STEP 3.** Apply KVL to the single-loop circuit.
- STEP 4.** Solve the single KVL equation for the current  $i(t)$ . If  $i(t)$  is positive, the current is flowing in the direction assumed; if not, then the current is actually flowing in the opposite direction.

## LEARNING ASSESSMENTS

**E2.10** Find  $I$  and  $V_{bd}$  in the circuit in Fig. E2.10.

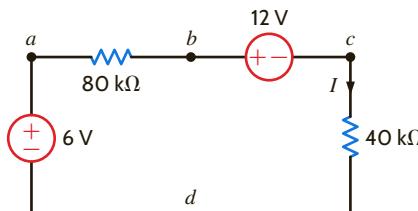


Figure E2.10

**ANSWER:**

$$\begin{aligned} I &= -0.05 \text{ mA;} \\ V_{bd} &= 10 \text{ V.} \end{aligned}$$

**E2.11** In the network in Fig. E2.11, if  $V_{ad}$  is 3 V, find  $V_S$ .

**ANSWER:**

$$V_S = 9 \text{ V.}$$

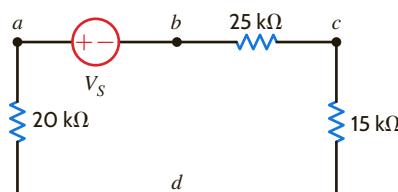


Figure E2.11