

## LEARNING ASSESSMENTS

**E8.10** Compute the impedance  $Z_T$  in the network in Fig. E8.10.

**ANSWER:**

$$Z_T = 3.38 + j1.08 \Omega.$$

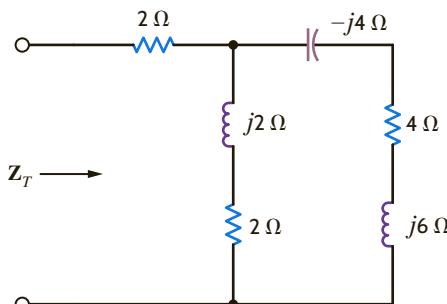


Figure E8.10

**E8.11** Find  $Z$  in Fig. E8.11.

**ANSWER:**

$$Z = 1.95 + j0.29 \Omega.$$

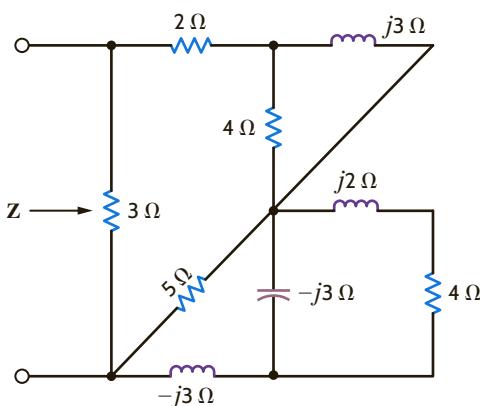


Figure E8.11

Impedance and admittance are functions of frequency, and therefore their values change as the frequency changes. These changes in  $Z$  and  $Y$  have a resultant effect on the current–voltage relationships in a network. This impact of changes in frequency on circuit parameters can be easily seen via a phasor diagram. The following examples will serve to illustrate these points.

8.6

## Phasor Diagrams

Let us sketch the phasor diagram for the network shown in Fig. 8.13.

## EXAMPLE 8.12

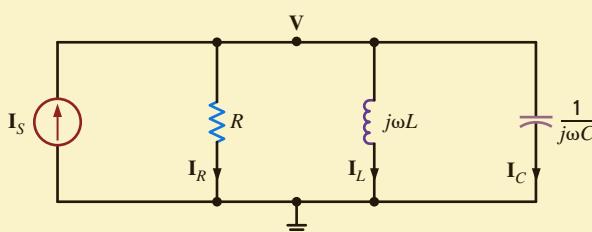


Figure 8.13

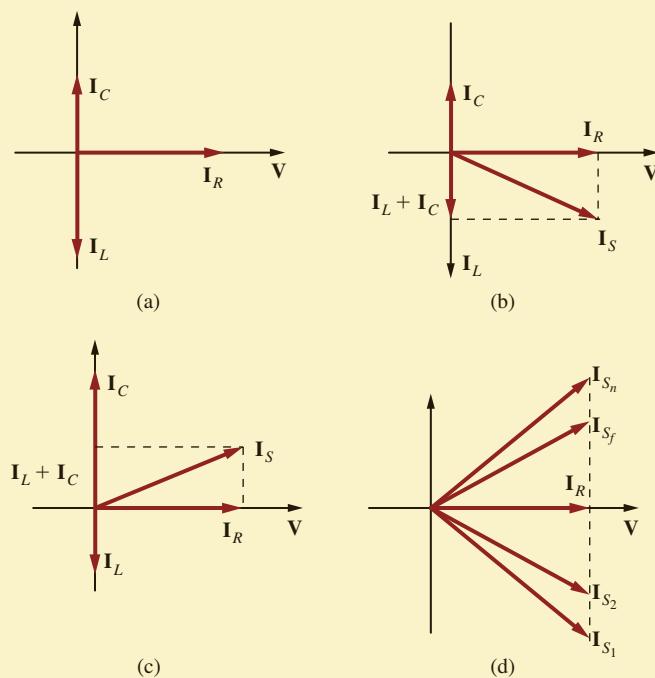
Example parallel circuit.

**SOLUTION**

The pertinent variables are labeled on the figure. For convenience in forming a phasor diagram, we select  $\mathbf{V}$  as a reference phasor and arbitrarily assign it a  $0^\circ$  phase angle. We will, therefore, measure all currents with respect to this phasor. We suffer no loss of generality by assigning  $\mathbf{V}$  a  $0^\circ$  phase angle, since if it is actually  $30^\circ$ , for example, we will simply rotate the entire phasor diagram by  $30^\circ$  because all the currents are measured with respect to this phasor.

**Figure 8.14**

Phasor diagrams for the circuit in Fig. 8.13.



At the upper node in the circuit KCL is

$$\mathbf{I}_S = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C = \frac{\mathbf{V}}{R} + \frac{\mathbf{V}}{j\omega L} + \frac{\mathbf{V}}{1/j\omega C}$$

Since  $\mathbf{V} = V_M \angle 0^\circ$ , then

$$\mathbf{I}_S = \frac{V_M \angle 0^\circ}{R} + \frac{V_M \angle -90^\circ}{\omega L} + V_M \omega C \angle 90^\circ$$

The phasor diagram that illustrates the phase relationship between  $\mathbf{V}$ ,  $\mathbf{I}_R$ ,  $\mathbf{I}_L$ , and  $\mathbf{I}_C$  is shown in [Fig. 8.14a](#). For small values of  $\omega$  such that the magnitude of  $\mathbf{I}_L$  is greater than that of  $\mathbf{I}_C$ , the phasor diagram for the currents is shown in [Fig. 8.14b](#). In the case of large values of  $\omega$ —that is, those for which  $\mathbf{I}_C$  is greater than  $\mathbf{I}_L$ —the phasor diagram for the currents is shown in [Fig. 8.14c](#). Note that as  $\omega$  increases, the phasor  $\mathbf{I}_S$  moves from  $\mathbf{I}_{S_1}$  to  $\mathbf{I}_{S_n}$  along a locus of points specified by the dashed line shown in [Fig. 8.14d](#).

Note that  $\mathbf{I}_S$  is in phase with  $\mathbf{V}$  when  $\mathbf{I}_C = \mathbf{I}_L$  or, in other words, when  $\omega L = 1/\omega C$ . Hence, the node voltage  $\mathbf{V}$  is in phase with the current source  $\mathbf{I}_S$  when

$$\omega = \frac{1}{\sqrt{LC}}$$

This can also be seen from the KCL equation

$$\mathbf{I} = \left[ \frac{1}{R} + j \left( \omega C - \frac{1}{\omega L} \right) \right] \mathbf{V}$$



**HINT**  
From a graphical standpoint, phasors can be manipulated like vectors.

Let us determine the phasor diagram for the series circuit shown in Fig. 8.15a.

KVL for this circuit is of the form

$$\begin{aligned}\mathbf{V}_S &= \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C \\ &= \mathbf{I}R + \omega L \mathbf{I}/90^\circ + \frac{\mathbf{I}}{\omega C} / -90^\circ\end{aligned}$$

If we select  $\mathbf{I}$  as a reference phasor so that  $\mathbf{I} = I_M/0^\circ$ , then if  $\omega L I_M > I_M/\omega C$ , the phasor diagram will be of the form shown in Fig. 8.15b. Specifically, if  $\omega = 377$  rad/s (i.e.,  $f = 60$  Hz), then  $\omega L = 6$  and  $1/\omega C = 2$ . Under these conditions, the phasor diagram is as shown in Fig. 8.15c. If, however, we select  $\mathbf{V}_S$  as reference with, for example,

$$v_S(t) = 12\sqrt{2} \cos(377t + 90^\circ) \text{ V}$$

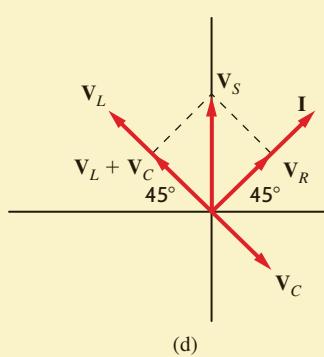
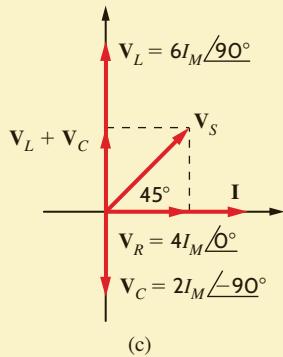
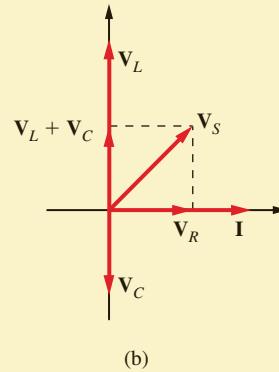
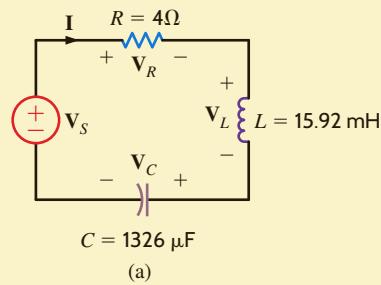
then

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}}{\mathbf{Z}} = \frac{12\sqrt{2}/90^\circ}{4 + j6 - j2} \\ &= \frac{12\sqrt{2}/90^\circ}{4\sqrt{2}/45^\circ} \\ &= 3/45^\circ \text{ A}\end{aligned}$$

and the entire phasor diagram, as shown in Figs. 8.15b and c, is rotated  $45^\circ$ , as shown in Fig. 8.15d.

## EXAMPLE 8.13

### SOLUTION



**Figure 8.15**

Series circuit and certain specific phasor diagrams (plots are not drawn to scale).