at node D yields

$$I_3 + I_7 = I_8$$

Solving for I_7 we obtain

$$I_7 = -1 \text{ A}$$

Then since

$$\begin{aligned} V_{CE} &= V_{CD} + V_{DE} \\ &= 8 + 4 \\ &= 12 \text{ V} \end{aligned}$$

I_6 can be obtained from Ohm's law as 4 A. Kirchhoff's current law at node E yields

$$I_4 + I_6 + I_8 = 0$$

and hence $I_4 = -6$ A. Then since

$$\begin{aligned} V_{CB} &= V_{CE} + V_{EB} \\ &= 12 + (6)(1) \\ &= 18 \text{ V} \end{aligned}$$

Ohm's law yields $I_5 = -3$ A. At node C,

$$I_2 + I_5 = I_6 + I_7$$

Solving for the only unknown I_2 yields $I_2 = 6$ A. Then

$$\begin{aligned} V_{AC} &= (6)(2) \\ &= 12 \text{ V} \end{aligned}$$

The only remaining unknown current is I_1. At node A

$$\begin{aligned} I_1 &= I_2 + I_3 \\ &= 9 \text{ A} \end{aligned}$$

Now Kirchhoff's voltage law around the upper left-hand loop yields

$$V_S - V_{AC} - V_{CB} = 0$$

or

$$V_S = 30 \text{ V}$$

Kirchhoff's voltage law around the upper right-hand loop yields

$$V_{AC} - V_{AD} + 8 = 0$$

or

$$V_{AD} = 20 \text{ V}$$

The circuit with all voltage and currents labeled is shown in **Fig. 2.32b**. Note carefully that Kirchhoff's current law is satisfied at every node and Kirchhoff's voltage law is satisfied around every loop.

2.6 Wye $\rightleftharpoons$ Delta Transformations

To provide motivation for this topic, consider the circuit in **Fig. 2.33**. Note that this network has essentially the same number of elements as contained in our recent examples. However, when we attempt to reduce the circuit to an equivalent network containing the source V_1 and an equivalent resistor R, we find that nowhere is a resistor in series or parallel with another. Therefore, we cannot attack the problem directly using the techniques that we have learned thus far. We can, however, replace one portion of the network with an equivalent circuit, and this

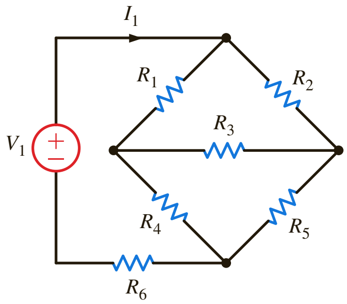


Figure 2.33

Network used to illustrate the need for the wye $\rightleftharpoons$ delta transformation.

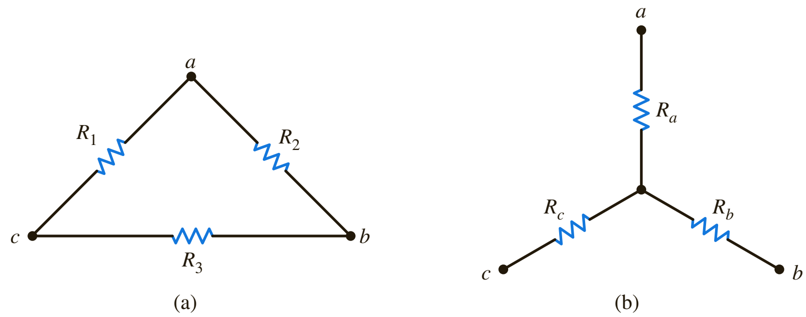


Figure 2.34

Delta and wye resistance networks.

conversion will permit us, with ease, to reduce the combination of resistors to a single equivalent resistance. This conversion is called the wye-to-delta or delta-to-wye transformation.

Consider the networks shown in Fig. 2.34. Note that the resistors in Fig. 2.34a form a Δ (delta) and the resistors in Fig. 2.34b form a Y (wye). If both of these configurations are connected at only three terminals a, b, and c, it would be very advantageous if an equivalence could be established between them. It is, in fact, possible to relate the resistances of one network to those of the other such that their terminal characteristics are the same. This relationship between the two network configurations is called the Y-Δ transformation.

The transformation that relates the resistances R_1, R_2, and R_3 to the resistances R_a, R_b, and R_c is derived as follows. For the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal (e.g., the resistance at terminals a and b with c open-circuited must be the same for both networks).

Therefore, if we equate the resistances for each corresponding set of terminals, we obtain the following equations:

$$
\begin{aligned}
R_{ab} &= R_a + R_b = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3} \\
R_{bc} &= R_b + R_c = \frac{R_3(R_1 + R_2)}{R_3 + R_1 + R_2} \\
R_{ca} &= R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}
\end{aligned} \tag{2.27}
$$

Solving this set of equations for R_a, R_b, and R_c yields

$$
\begin{aligned}
R_a &= \frac{R_1 R_2}{R_1 + R_2 + R_3} \\
R_b &= \frac{R_2 R_3}{R_1 + R_2 + R_3} \\
R_c &= \frac{R_1 R_3}{R_1 + R_2 + R_3}
\end{aligned} \tag{2.28}
$$

Similarly, if we solve Eq. (2.27) for R_1, R_2, and R_3, we obtain

$$
\begin{aligned}
R_1 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_b} \\
R_2 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} \\
R_3 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_a}
\end{aligned} \tag{2.29}
$$

Equations (2.28) and (2.29) are general relationships and apply to any set of resistances connected in a Y or Δ. For the balanced case where $R_a = R_b = R_c$ and $R_1 = R_2 = R_3$, the equations above reduce to

$$R_Y = \frac{1}{3}R_\Delta \tag{2.30}$$

and

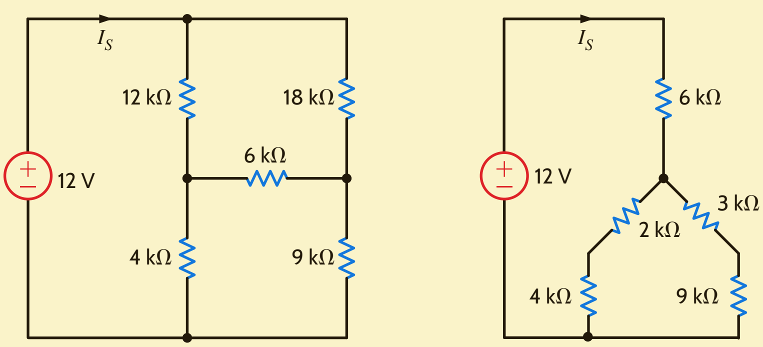
$$R_\Delta = 3R_Y \tag{2.31}$$

It is important to note that it is not necessary to memorize the formulas in Eqs. (2.28) and (2.29). Close inspection of these equations and **Fig. 2.34** illustrates a definite pattern to the relationships between the two configurations. For example, the resistance connected to point a in the wye (i.e., R_a) is equal to the product of the two resistors in the Δ that are connected to point a divided by the sum of all the resistances in the delta. R_b and R_c are determined in a similar manner. Similarly, there are geometrical patterns associated with the equations for calculating the resistors in the delta as a function of those in the wye.

Let us now examine the use of the delta ⇌ wye transformation in the solution of a network problem.

EXAMPLE 2.25

Given the network in **Fig. 2.35a**, let us find the source current I_S.



(a) (b)

Figure 2.35 Circuits used in Example 2.25.

SOLUTION

Note that none of the resistors in the circuit are in series or parallel. However, careful examination of the network indicates that the 12k-, 6k-, and 18k-ohm resistors, as well as the 4k-, 6k-, and 9k-ohm resistors each form a delta that can be converted to a wye. Furthermore, the 12k-, 6k-, and 4k-ohm resistors, as well as the 18k-, 6k-, and 9k-ohm resistors, each form a wye that can be converted to a delta. Any one of these conversions will lead to a solution. We will perform a delta-to-wye transformation on the 12k-, 6k-, and 18k-ohm resistors, which leads to the circuit in **Fig. 2.35b**. The 2k- and 4k-ohm resistors, like the 3k- and 9k-ohm resistors, are in series and their parallel combination yields a 4k-ohm resistor. Thus, the source current is

$$\begin{aligned} I_S &= 12/(6\text{k} + 4\text{k}) \\ &= 1.2 \text{ mA} \end{aligned}$$

EXAMPLE 2.26

A Wheatstone bridge circuit is an accurate device for measuring resistance. This circuit, shown in **Fig. 2.36**, is used to measure the unknown resistor R_x. The center leg of the circuit contains a galvanometer, which is a very sensitive device that can be used to measure current in the microamp range When the unknown resistor is connected to the bridge, R_3 is adjusted until the current in the galvanometer is zero, at which point the bridge is balanced. In this balanced condition

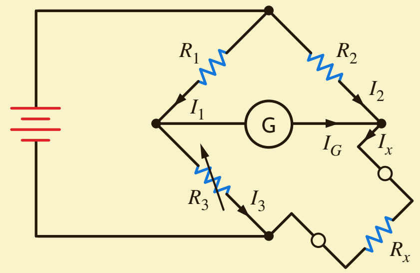
$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

so that

$$R_x = \left(\frac{R_2}{R_1}\right)R_3$$

Figure 2.36

The Wheatstone bridge circuit.



Engineers also use this bridge circuit to measure strain in solid material. For example, a system used to determine the weight of a truck is shown in **Fig. 2.37a**. The platform is supported by cylinders on which strain gauges are mounted. The strain gauges, which measure strain when the cylinder deflects under load, are connected to a Wheatstone bridge as shown in **Fig. 2.37b**. The strain gauge has a resistance of 120 Ω under no-load conditions and changes value under load. The variable resistor in the bridge is a calibrated precision device.

Weight is determined in the following manner. The ΔR_3 required to balance the bridge represents the Δ strain, which when multiplied by the modulus of elasticity yields the Δ stress. The Δ stress multiplied by the cross-sectional area of the cylinder produces the Δ load, which is used to determine weight.

Let us determine the value of R_3 under no load when the bridge is balanced and its value when the resistance of the strain gauge changes to 120.24 Ω under load.

SOLUTION Using the balance equation for the bridge, the value of R_3 at no load is

$$\begin{aligned} R_3 &= \left(\frac{R_1}{R_2}\right)R_x \\ &= \left(\frac{100}{110}\right)(120) \\ &= 109.0909\ \Omega \end{aligned}$$

Under load, the value of R_3 is

$$\begin{aligned} R_3 &= \left(\frac{100}{110}\right)(120.24) \\ &= 109.3091\ \Omega \end{aligned}$$

Therefore, the ΔR_3 is

$$\begin{aligned} \Delta R_3 &= 109.3091 - 109.0909 \\ &= 0.2182\ \Omega \end{aligned}$$

Figure 2.37

Diagrams used in Example 2.26.