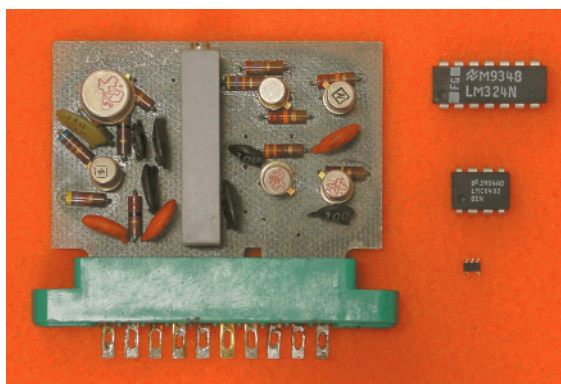


## 4.1

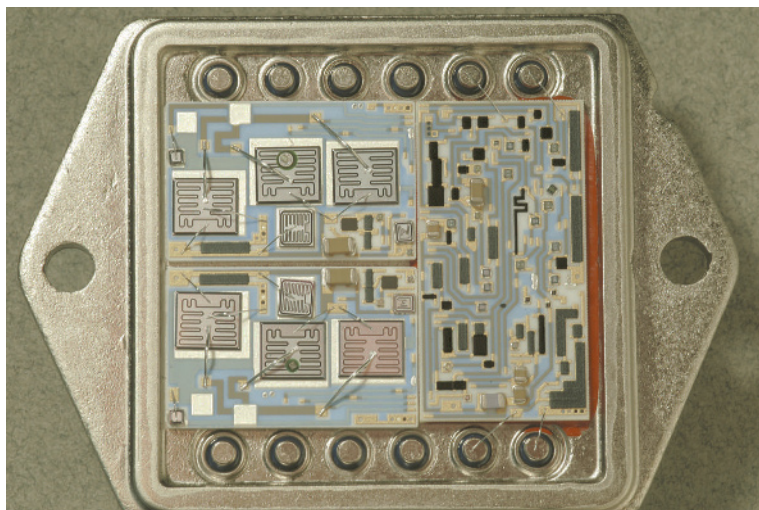
## Introduction

It can be argued that the operational amplifier, or op-amp as it is commonly known, is the single most important integrated circuit for analog circuit design. It is a versatile interconnection of transistors and resistors that vastly expands our capabilities in circuit design, from engine control systems to cellular phones. Early op-amps were built of vacuum tubes, making them bulky and power hungry. The invention of the transistor at Bell Labs in 1947 allowed engineers to create op-amps that were much smaller and more efficient. Still, the op-amp itself consisted of individual transistors and resistors interconnected on a printed circuit board (PCB). When the manufacturing process for integrated circuits (ICs) was developed around 1970, engineers could finally put all of the op-amp transistors and resistors onto a single IC chip. Today, it is common to find as many as four high-quality op-amps on a single IC for as little as \$0.40. A sample of commercial op-amps is shown in **Fig. 4.1**.

Why are they called operational amplifiers? Originally, the op-amp was designed to perform mathematical operations such as addition, subtraction, differentiation, and integration. By adding simple networks to the op-amp, we can create these “building blocks” as well as voltage scaling, current-to-voltage conversion, and myriad more complex applications.



(a)



(b)

**Figure 4.1**

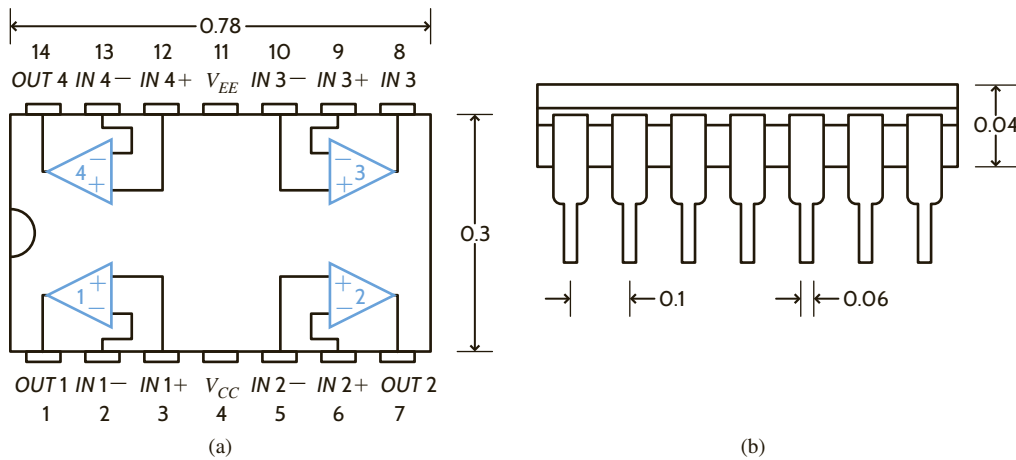
A selection of op-amps. On the left (a) is a discrete op-amp assembled on a printed circuit board (PCB). On the right, top-down, a LM324 DIP, LMC6492 DIP, and MAX4240 in a SO-5 package (small outline/5 pins). The Apex Microtechnology PA03 with its lid removed (b) showing individual transistors and resistors. (Left, Courtesy of Mark Nelms and Jo Ann Loden; right, Courtesy of Apex Microtechnology Corp.)

## 4.2

## Op-Amp Models

How can we, understanding only sources and resistors, hope to comprehend the performance of the op-amp? The answer lies in modeling. When the bells and whistles are removed, an op-amp is just a really good voltage amplifier. In other words, the output voltage is a scaled replica of the input voltage. Modern op-amps are such good amplifiers that it is easy to create an accurate, first-order model. As mentioned earlier, the op-amp is very popular and is used extensively in circuit design at all levels. We should not be surprised to find that op-amps are available for every application—low voltage, high voltage, micro-power, high speed, high current, and so forth. Fortunately, the topology of our model is independent of these issues.

We start with the general-purpose LM324 quad (four in a pack) op-amp from National Semiconductor, shown in the upper right corner of **Fig. 4.1a**. The pinout for the LM324 is shown in **Fig. 4.2** for a DIP (Dual Inline Pack) style package with dimensions in inches. Recognizing there are four identical op-amps in the package, we will focus on amplifier 1. Pins 3 and 2 are the

**Figure 4.2**

The pinout (a) and dimensional diagram (b) of the LM324 quad op-amp. Note the pin pitch (distance pin-to-pin) is 0.1 inches—standard for DIP packages.

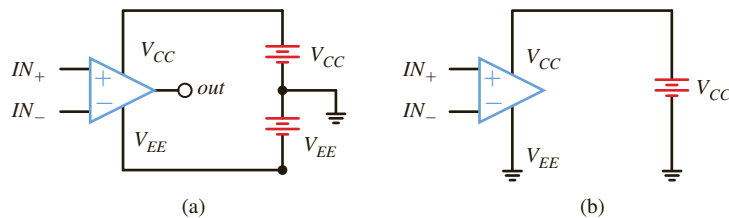
input pins,  $IN\ 1+$  and  $IN\ 1-$ , and are called the noninverting and inverting inputs, respectively. The output is at pin 1. A relationship exists between the output and input voltages,

$$V_o = A_o (IN_+ - IN_-) \quad 4.1$$

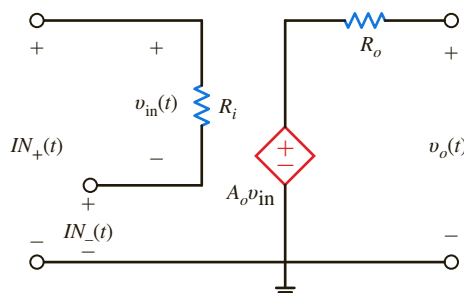
where all voltages are measured with respect to ground and  $A_o$  is the gain of the op-amp. (The location of the ground terminal will be discussed shortly.) From Eq. (4.1), we see that when  $IN_+$  increases, so will  $V_o$ . However, if  $IN_-$  increases, then  $V_o$  will decrease—hence, the names noninverting and inverting inputs. We mentioned earlier that op-amps are very good voltage amplifiers. How good? Typical values for  $A_o$  are between 10,000 and 1,000,000!

Amplification requires power that is provided by the dc voltage sources connected to pins 4 and 11, called  $V_{CC}$  and  $V_{EE}$ , respectively. **Fig. 4.3** shows how the power supplies, or rails, are connected for both dual- and single-supply applications and defines the ground node to which all input and output voltages are referenced. Traditionally,  $V_{CC}$  is a positive dc voltage with respect to ground, and  $V_{EE}$  is either a negative voltage or ground itself. Actual values for these power supplies can vary widely depending on the application, from as little as one volt up to several hundred.

How can we model the op-amp? A dependent voltage source can produce  $V_o$ ! What about the currents into and out of the op-amp terminals (pins 3, 2, and 1)? Fortunately for us, the currents are fairly proportional to the pin voltages. That sounds like Ohm's law. So, we model the  $I$ - $V$  performance with two resistors, one at the input terminals ( $R_i$ ) and another at the output ( $R_o$ ). The circuit in **Fig. 4.4** brings everything together.

**Figure 4.3**

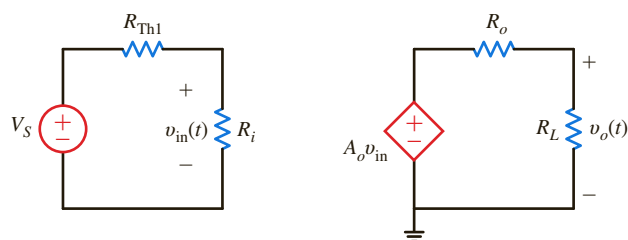
Schematics showing the power supply connections and ground location for (a) dual-supply and (b) single-supply implementations.

**Figure 4.4**

A simple model for the gain characteristics of an op-amp.

**Figure 4.5**

A network that depicts an op-amp circuit.  $V_S$  and  $R_{Th1}$  model the driving circuit, while the load is modeled by  $R_L$ . The circuit in Fig. 4.4 is the op-amp model.



What values can we expect for  $A_o$ ,  $R_i$ , and  $R_o$ ? We can reason through this issue with the help of Fig. 4.5 where we have drawn an equivalent for the circuitry that drives the input nodes and we have modeled the circuitry connected to the output with a single resistor,  $R_L$ . Since the op-amp is supposed to be a great voltage amplifier, let's write an equation for the overall gain of the circuit  $V_o/V_S$ . Using voltage division at the input and again at the output, we quickly produce the expression

$$\frac{V_o}{V_S} = \left[ \frac{R_i}{R_i + R_{Th1}} \right] A_o \left[ \frac{R_L}{R_o + R_L} \right]$$

To maximize the gain regardless of the values of  $R_{Th1}$  and  $R_L$ , we make the voltage division ratios as close to unity as possible. The ideal scenario requires that  $A_o$  be infinity,  $R_i$  be infinity, and  $R_o$  be zero, yielding a large overall gain of  $A_o$ . Table 4.1 shows the actual values of  $A_o$ ,  $R_i$ , and  $R_o$  for a sampling of commercial op-amps intended for very different applications. While  $A_o$ ,  $R_i$ , and  $R_o$  are not ideal, they do have the correct tendencies.

The power supplies affect performance in two ways. First, each op-amp has minimum and maximum supply ranges over which the op-amp is guaranteed to function. Second, for proper operation, the input and output voltages are limited to no more than the supply voltages.\* If the inputs/output can reach within a few dozen millivolts of the supplies, then the inputs/output are called rail-to-rail. Otherwise, the inputs/output voltage limits are more severe—usually a volt or so away from the supply values. Combining the model in Fig. 4.4, the values in Table 4.1, and these I/O limitations, we can produce the graph in Fig. 4.6 showing the output–input relation for each op-amp in Table 4.1. From the graph we see that LMC6492 and MAX4240 have rail-to-rail outputs, while the LM324 and PA03 do not.

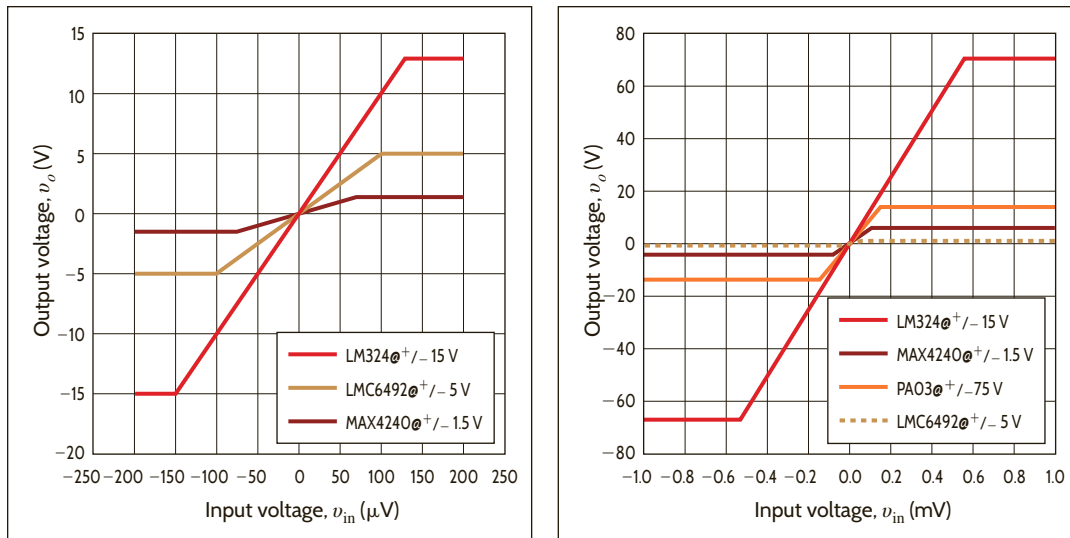
Even though the op-amp can function within the minimum and maximum supply voltages, because of the circuit configuration, an increase in the input voltage may not yield a corresponding increase in the output voltage. In this case, the op-amp is said to be in saturation. The following example addresses this issue.

**TABLE 4.1** A list of commercial op-amps and their model values

MANUFACTURER	PART NO.	$A_o$ (V/V)	$R_i$ (M $\Omega$ )	$R_o$ ( $\Omega$ )	COMMENTS
National	LM324	100,000	1.0	20	General purpose, up to $\pm 16$ V supplies, very inexpensive
National	LMC6492	50,000	$10^7$	150	Low-voltage, rail-to-rail inputs and outputs <sup>†</sup>
Maxim	MAX4240	20,000	45	160	Micro-power (1.8 V supply @ 10 $\mu$ A), rail-to-rail inputs and outputs
Apex Microtechnology	PA03	125,000	$10^5$	2	High-voltage, $\pm 75$ V and high-output current capability, 30 A. That's 2 kW!

<sup>†</sup>Rail-to-rail is a trademark of Motorola Corporation. This feature is discussed further in the following paragraphs.

\*Op-amps are available that have input and/or output voltage ranges beyond the supply rails. However, these devices constitute a very small percentage of the op-amp market and will not be discussed here.

**Figure 4.6**

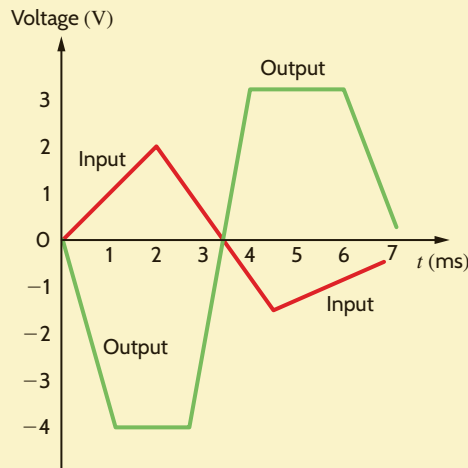
Transfer plots for the op-amps listed in Table 4.1. The supply voltages are listed in the plot legends. Note that the LMC6492 and MAX4240 have rail-to-rail output voltages (output voltage range extends to power supply values), while the LM324 and PA03 do not.

The input and output signals for an op-amp circuit are shown in Fig. 4.7. We wish to determine (a) if the op-amp circuit is linear and (b) the circuit's gain.

- We know that if the circuit is linear, the output must be linearly related, that is, proportional, to the input. An examination of the input and output waveforms in Fig. 4.7 clearly indicates that in the region  $t = 1.25$  to  $2.5$  and  $4$  to  $6$  ms the output is constant while the input is changing. In this case, the op-amp circuit is in saturation and therefore not linear.
- In the region where the output is proportional to the input, that is,  $t = 0$  to  $1$  ms, the input changes by  $1$  V and the output changes by  $3.3$  V. Therefore, the circuit's gain is  $3.3$ .

## EXAMPLE 4.1

### SOLUTION

**Figure 4.7**

An op-amp input-output characteristic.

To introduce the performance of the op-amp in a practical circuit, consider the network in Fig. 4.8a called a unity gain buffer. Notice that the op-amp schematic symbol includes the power supplies. Substituting the model in Fig. 4.4 yields the circuit in Fig. 4.8b, containing just resistors and sources, which we can easily analyze. Writing loop equations, we have

$$\begin{aligned} V_S &= IR_i + IR_o + A_o V_{in} \\ V_{out} &= IR_o + A_o V_{in} \\ V_{in} &= IR_i \end{aligned}$$

Figure 4.8

Circuit (a) and model (b) for the unity gain buffer.

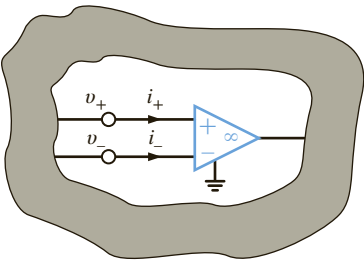
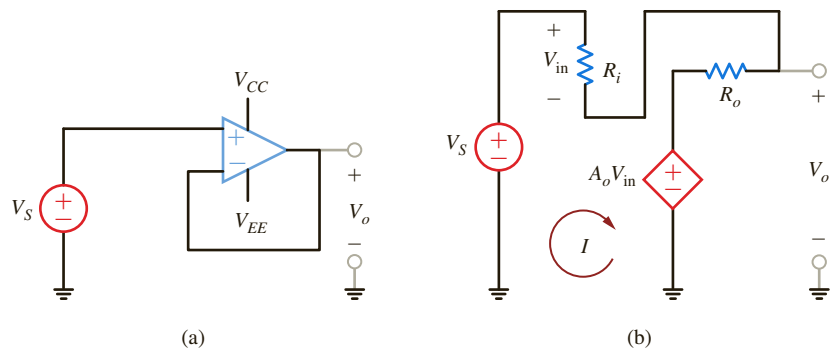


Figure 4.9

Ideal model for an operational amplifier. Model parameters:  $i_+ = i_- = 0$ ,  $v_+ = v_-$ .

Solving for the gain,  $V_o/V_S$ , we find

$$\frac{V_o}{V_S} = \frac{1}{1 + \frac{R_i}{R_o + A_o R_i}}$$

For  $R_o \ll R_i$ , we have

$$\frac{V_o}{V_S} \approx \frac{1}{1 + \frac{1}{A_o}}$$

And, if  $A_o$  is indeed  $\gg 1$ ,

$$\frac{V_o}{V_S} \approx 1$$

The origin of the name *unity gain buffer* should be apparent. Table 4.2 shows the actual gain values for  $V_S = 1$  V using the op-amps listed in Table 4.1. Notice how close the gain is to unity and how small the input voltage and current are. These results lead us to simplify the op-amp in Fig. 4.4 significantly. We introduce the *ideal op-amp model*, where  $A_o$  and  $R_i$  are infinite and  $R_o$  is zero. This produces two important results for analyzing op-amp circuitry, listed in Table 4.3.

From Table 4.3 we find that the ideal model for the op-amp is reduced to that shown in Fig. 4.9. The important characteristics of the model are as follows: (1) since  $R_i$  is extremely large, the input currents to the op-amp are approximately zero (i.e.,  $i_+ \approx i_- \approx 0$ ); and (2) if the output voltage is to remain bounded, then as the gain becomes very large and approaches infinity, the voltage across the input terminals must simultaneously become infinitesimally small so that as  $A_o \rightarrow \infty$ ,  $v_+ - v_- \rightarrow 0$  (i.e.,  $v_+ - v_- = 0$  or  $v_+ = v_-$ ). The difference between these input voltages is often called the *error signal* for the op-amp (i.e.,  $v_+ - v_- = v_e$ ).

The ground terminal  $\perp$  shown on the op-amp is necessary for signal current return, and it guarantees that Kirchhoff's current law is satisfied at both the op-amp and the ground node in the circuit.

In summary, then, our ideal model for the op-amp is simply stated by the following conditions:

$$\begin{aligned} i_+ &= i_- = 0 \\ v_+ &= v_- \end{aligned} \tag{4.2}$$

TABLE 4.2 Unity gain buffer performance for the op-amps listed in Table 4.1

OP-AMP	BUFFER GAIN	$V_{in}$ (mV)	$I$ (pA)
LM324	0.999990	9.9999	9.9998
LMC6492	0.999980	19.999	$1.9999 \times 10^{-6}$
MAX4240	0.999950	49.998	1.1111
PA03	0.999992	7.9999	$7.9999 \times 10^{-5}$

TABLE 4.3 Consequences of the ideal op-amp model on input terminal I/V values

MODEL ASSUMPTION	TERMINAL RESULT
$A_o \rightarrow \infty$	input voltage $\rightarrow 0$ V
$R_i \rightarrow \infty$	input current $\rightarrow 0$ A

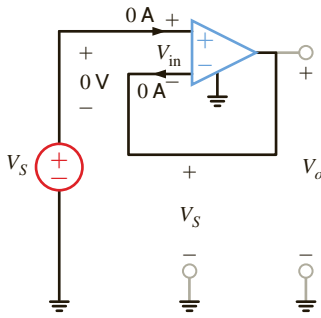


Figure 4.10

An ideal op-amp configured as a unity gain buffer.

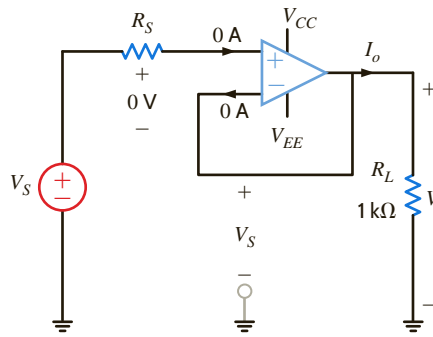


Figure 4.11

A unity gain buffer with a load resistor.

These simple conditions are extremely important because they form the basis of our analysis of op-amp circuits.

Let's use the ideal model to reexamine the unity gain buffer, drawn again in **Fig. 4.10**, where the input voltage and currents are shown as zero. Given that  $V_{in}$  is zero, the voltage at both op-amp inputs is  $V_S$ . Since the inverting input is physically connected to the output,  $V_o$  is also  $V_S$ —unity gain!

Armed with the ideal op-amp model, let's change the circuit in **Fig. 4.10** slightly as shown in **Fig. 4.11** where  $V_S$  and  $R_S$  are an equivalent for the circuit driving the buffer and  $R_L$  models the circuitry connected to the output. There are three main points here. First, the gain is still unity. Second, the op-amp requires no current from the driving circuit. Third, the output current ( $I_o = V_o/R_L$ ) comes from the power supplies, through the op-amp and out of the output pin. In other words, the load current comes from the power supplies, which have plenty of current output capacity, rather than the driving circuit, which may have very little. This isolation of current is called *buffering*.

An obvious question at this point is this: if  $V_o = V_S$ , why not just connect  $V_S$  to  $V_o$  via two parallel connection wires; why do we need to place an op-amp between them? The answer to this question is fundamental and provides us with some insight that will aid us in circuit analysis and design.

Consider the circuit shown in **Fig. 4.12a**. In this case  $V_o$  is not equal to  $V_S$  because of the voltage drop across  $R_S$ :

$$V_o = V_S - IR_S$$

However, in **Fig. 4.12b**, the input current to the op-amp is zero and, therefore,  $V_S$  appears at the op-amp input. Since the gain of the op-amp configuration is 1,  $V_o = V_S$ . In **Fig. 4.12a** the resistive network's interaction with the source caused the voltage  $V_o$  to be less than  $V_S$ . In other words, the resistive network loads the source voltage. However, in **Fig. 4.12b** the op-amp isolates the source from the resistive network; therefore, the voltage follower is referred to as a *buffer amplifier* because it can be used to isolate one circuit from another. The energy supplied to the resistive network in the first case must come from the source  $V_S$ , whereas in the second case it comes from the power supplies that supply the amplifier, and little or no energy is drawn from  $V_S$ .

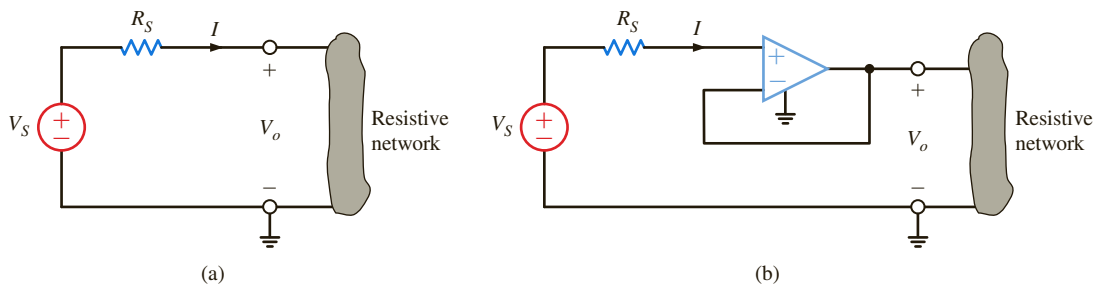


Figure 4.12

Illustration of the isolation capability of a voltage follower.