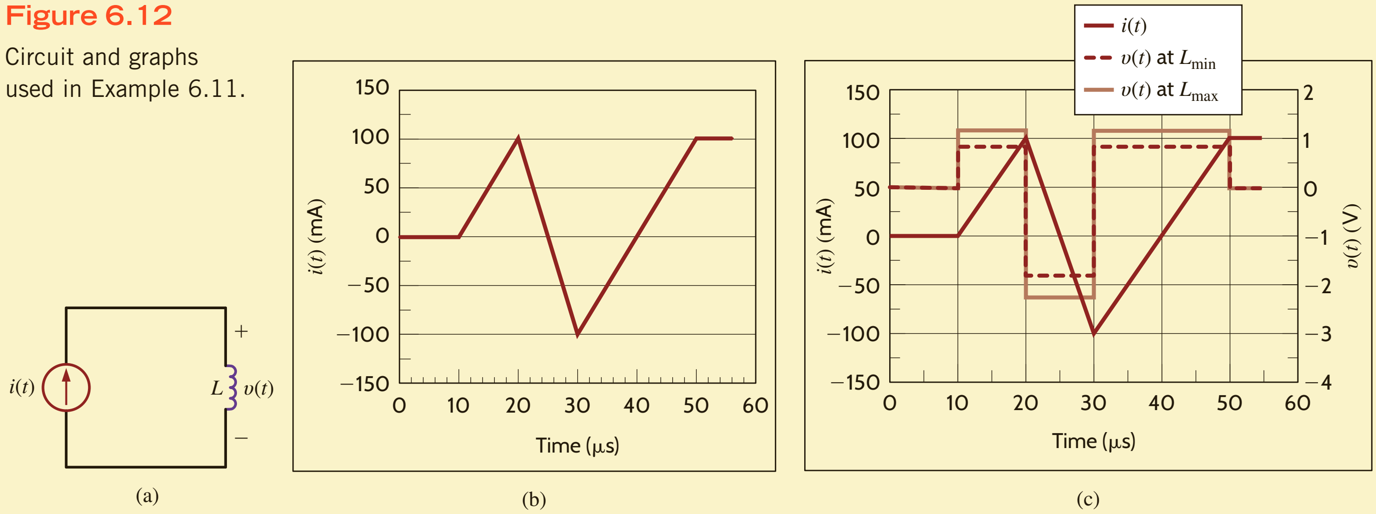
Figure 6.12

Circuit and graphs used in Example 6.11.



6.3 Capacitor and Inductor Combinations

SERIES CAPACITORS If a number of capacitors are connected in series, their equivalent capacitance can be calculated using KVL. Consider the circuit shown in **Fig. 6.13a**. For this circuit

$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t) \tag{6.13}$$

but

$$v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)dt + v_i(t_0) \tag{6.14}$$

Therefore, Eq. (6.13) can be written as follows using Eq. (6.14):

$$v(t) = \left(\sum_{i=1}^{N} \frac{1}{C_i}\right)\int_{t_0}^{t} i(t)dt + \sum_{i=1}^{N} v_i(t_0) \tag{6.15}$$

$$= \frac{1}{C_S}\int_{t_0}^{t} i(t)\,dt + v(t_0) \tag{6.16}$$

where

$$v(t_0) = \sum_{i=1}^{N} v_i(t_0)$$

and

$$\frac{1}{C_S} = \sum_{i=1}^{N} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \tag{6.17}$$

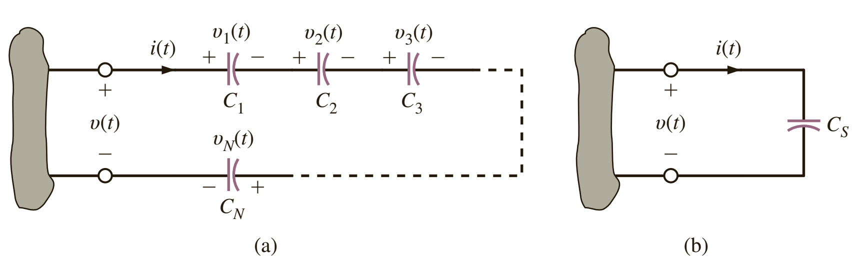
HINT

Capacitors in series combine like resistors in parallel.

Thus, the circuit in **Fig. 6.13b** is equivalent to that in Fig. 6.13a under the conditions stated previously.

Figure 6.13

Equivalent circuit for *N* series-connected capacitors.



It is also important to note that since the same current flows in each of the series capacitors, each capacitor gains the same charge in the same time period. The voltage across each capacitor will depend on this charge and the capacitance of the element.

EXAMPLE 6.12

Determine the equivalent capacitance and the initial voltage for the circuit shown in Fig. 6.14.

SOLUTION

Note that these capacitors must have been charged before they were connected in series or else the charge of each would be equal and the voltages would be in the same direction.

The equivalent capacitance is

$$\frac{1}{C_S} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

where all capacitance values are in microfarads.

Therefore, $C_S = 1\ \mu\text{F}$ and, as seen from the figure, $v(t_0) = -3$ V. Note that the total energy stored in the circuit is

$$\begin{aligned} w(t_0) &= \frac{1}{2}[2 \times 10^{-6}(2)^2 + 3 \times 10^{-6}(-4)^2 + 6 \times 10^{-6}(-1)^2] \\ &= 31\ \mu\text{J} \end{aligned}$$

However, the energy recoverable at the terminals is

$$\begin{aligned} w_C(t_0) &= \frac{1}{2} C_S v^2(t) \\ &= \frac{1}{2}[1 \times 10^{-6}(-3)^2] \\ &= 4.5\ \mu\text{J} \end{aligned}$$

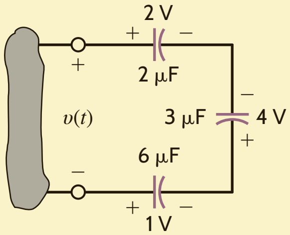


Figure 6.14
Circuit containing multiple capacitors with initial voltages.

EXAMPLE 6.13

Two previously uncharged capacitors are connected in series and then charged with a 12-V source. One capacitor is 30 μF and the other is unknown. If the voltage across the 30-μF capacitor is 8 V, find the capacitance of the unknown capacitor.

SOLUTION

The charge on the 30-μF capacitor is

$$Q = CV = (30\ \mu\text{F})(8\ \text{V}) = 240\ \mu\text{C}$$

Since the same current flows in each of the series capacitors, each capacitor gains the same charge in the same time period:

$$C = \frac{Q}{V} = \frac{240\ \mu\text{C}}{4\ \text{V}} = 60\ \mu\text{F}$$

PARALLEL CAPACITORS To determine the equivalent capacitance of N capacitors connected in parallel, we employ KCL. As can be seen from Fig. 6.15a,

$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t) \qquad \textbf{6.18}$$

$$= C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt} + \cdots + C_N \frac{dv(t)}{dt}$$

$$= \left(\sum_{i=1}^{N} C_i\right) \frac{dv(t)}{dt}$$

$$= C_p \frac{dv(t)}{dt} \qquad \textbf{6.19}$$

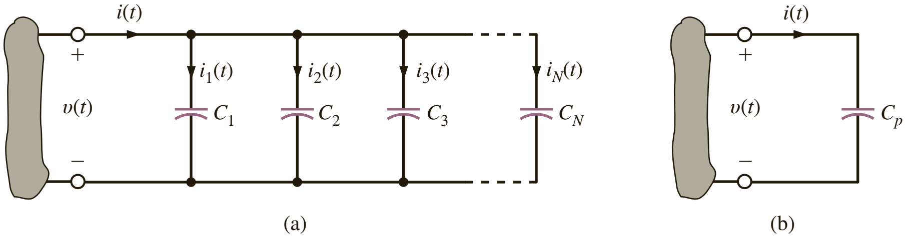


Figure 6.15 Equivalent circuit for N capacitors connected in parallel.

HINT Capacitors in parallel combine like resistors in series.

where

$$C_p = C_1 + C_2 + C_3 + \cdots + C_N \tag{6.20}$$

EXAMPLE 6.14

Determine the equivalent capacitance at terminals A-B of the circuit shown in Fig. 6.16.

SOLUTION

$$C_p = 15\ \mu\text{F}$$

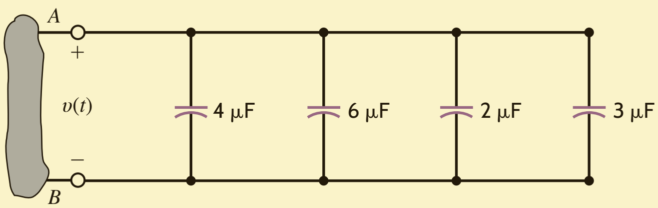


Figure 6.16 Circuit containing multiple capacitors in parallel.

LEARNING ASSESSMENTS

E6.11 Two initially uncharged capacitors are connected as shown in Fig. E6.11. After a period of time, the voltage reaches the value shown. Determine the value of C_1.

ANSWER: 4 μF.

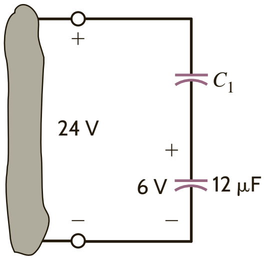


Figure E6.11

E6.12 Compute the equivalent capacitance of the network in Fig. E6.12.

ANSWER: 1.5 μF.

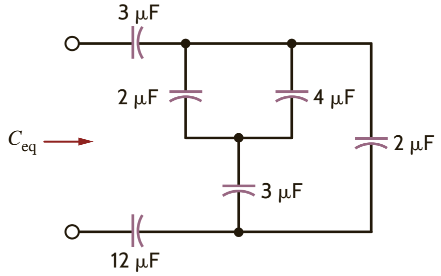


Figure E6.12

E6.13 Determine C_T in Fig. E6.13.

ANSWER: 1.667 μF.

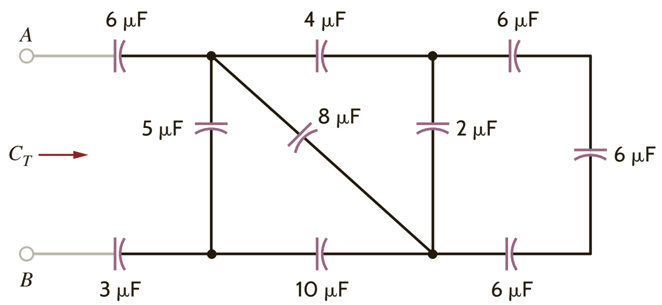


Figure E6.13

SERIES INDUCTORS If N inductors are connected in series, the equivalent inductance of the combination can be determined as follows. Referring to Fig. 6.17a and using KVL, we see that

$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t) \tag{6.21}$$

and therefore,

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \cdots + L_N \frac{di(t)}{dt} \tag{6.22}$$

$$= \left(\sum_{i=1}^{N} L_i \right) \frac{di(t)}{dt}$$

$$= L_S \frac{di(t)}{dt} \tag{6.23}$$

where

$$L_S = \sum_{i=1}^{N} L_i = L_1 + L_2 + \cdots + L_N \tag{6.24}$$

HINT

Inductors in series combine like resistors in series.

Therefore, under this condition the network in Fig. 6.17b is equivalent to that in Fig. 6.17a.

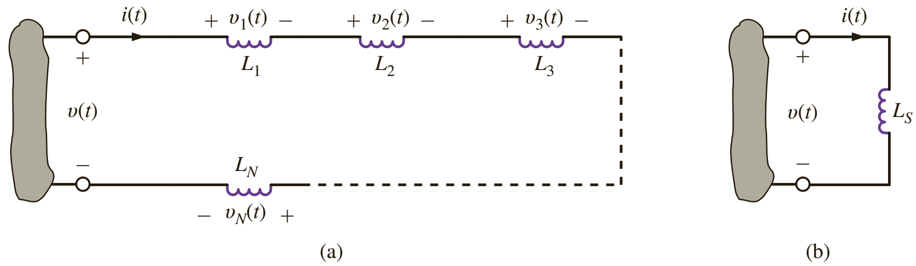


Figure 6.17

Equivalent circuit for N series-connected inductors.

EXAMPLE 6.15

Find the equivalent inductance of the circuit shown in Fig. 6.18.

SOLUTION

The equivalent inductance of the circuit shown in Fig. 6.18 is

$$L_S = 1\text{ H} + 2\text{ H} + 4\text{ H}$$
$$= 7\text{ H}$$

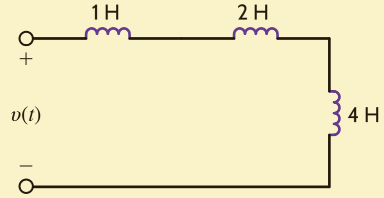


Figure 6.18

Circuit containing multiple inductors.

PARALLEL INDUCTORS Consider the circuit shown in Fig. 6.19a, which contains N parallel inductors. Using KCL, we can write

$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t) \tag{6.25}$$

However,

$$i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)dx + i_j(t_0) \tag{6.26}$$

Substituting this expression into Eq. (6.25) yields

$$i(t) = \left(\sum_{j=1}^{N}\frac{1}{L_j}\right)\int_{t_0}^{t} v(x)dx + \sum_{j=1}^{N} i_j(t_0) \tag{6.27}$$

$$= \frac{1}{L_p}\int_{t_0}^{t} v(x)dx + i(t_0) \tag{6.28}$$

where

HINT
Inductors in parallel combine like resistors in parallel.

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N} \tag{6.29}$$

and $i(t_0)$ is equal to the current in L_p at $t = t_0$. Thus, the circuit in Fig. 6.19b is equivalent to that in Fig. 6.19a under the conditions stated previously.

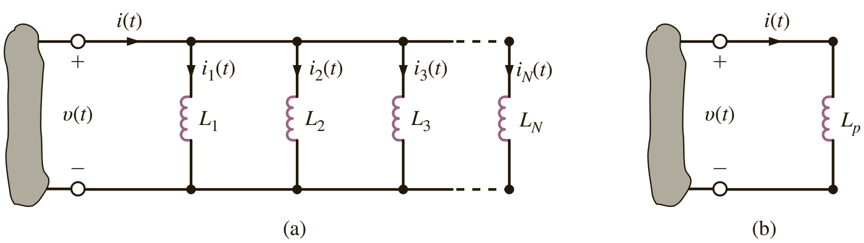


Figure 6.19
Equivalent circuit for N inductors connected in parallel.

EXAMPLE 6.16

Determine the equivalent inductance and the initial current for the circuit shown in Fig. 6.20.

SOLUTION

The equivalent inductance is

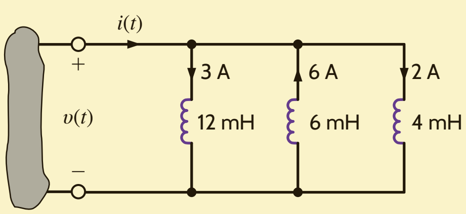
$$\frac{1}{L_p} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

where all inductance values are in millihenrys:

$$L_p = 2 \text{ mH}$$

and the initial current is $i(t_0) = -1$ A.

Figure 6.20
Circuit containing multiple inductors with initial currents.



The previous material indicates that capacitors combine like conductances, whereas inductances combine like resistances.