9.1 Instantaneous Power

By employing the sign convention adopted in the earlier chapters, we can compute the instantaneous power supplied or absorbed by any device as the product of the instantaneous voltage across the device and the instantaneous current through it.

Consider the circuit shown in Fig. 9.1. In general, the steady-state voltage and current for the network can be written as

$$v(t) = V_M \cos(\omega t + \theta_v) \tag{9.1}$$

$$i(t) = I_M \cos(\omega t + \theta_i) \tag{9.2}$$

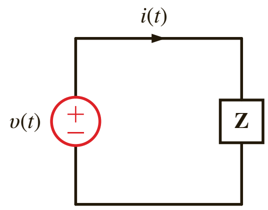

Figure 9.1

Simple ac network.

The instantaneous power is then

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i) \end{aligned} \tag{9.3}$$

Employing the following trigonometric identity,

$$\cos\phi_1 \cos\phi_2 = \frac{1}{2}[\cos(\phi_1 - \phi_2) + \cos(\phi_1 + \phi_2)] \tag{9.4}$$

we find that the instantaneous power can be written as

$$p(t) = \frac{V_M I_M}{2}[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \tag{9.5}$$

Note that the instantaneous power consists of two terms. The first term is a constant (i.e., it is time independent), and the second term is a cosine wave of twice the excitation frequency. We will examine this equation in more detail in Section 9.2.

EXAMPLE 9.1

The circuit in Fig. 9.1 has the following parameters: $v(t) = 4\cos(\omega t + 60°)$ V and $\mathbf{Z} = 2\underline{/30°}\ \Omega$. We wish to determine equations for the current and the instantaneous power as a function of time and plot these functions with the voltage on a single graph for comparison.

SOLUTION

Since

$$\begin{aligned} \mathbf{I} &= \frac{4\underline{/60°}}{2\underline{/30°}} \\ &= 2\underline{/30°}\ \text{A} \end{aligned}$$

then

$$i(t) = 2\cos(\omega t + 30°)\ \text{A}$$

From Eq. (9.5),

$$\begin{aligned} p(t) &= 4[\cos(30°) + \cos(2\omega t + 90°)] \\ &= 3.46 + 4\cos(2\omega t + 90°)\ \text{W} \end{aligned}$$

HINT

Note that $p(t)$ contains a dc term and a cosine wave with twice the frequency of $v(t)$ and $i(t)$.

A plot of this function, together with plots of the voltage and current, is shown in Fig. 9.2. As can be seen in this figure, the instantaneous power has a dc or constant term and a second term whose frequency is twice that of the voltage or current.

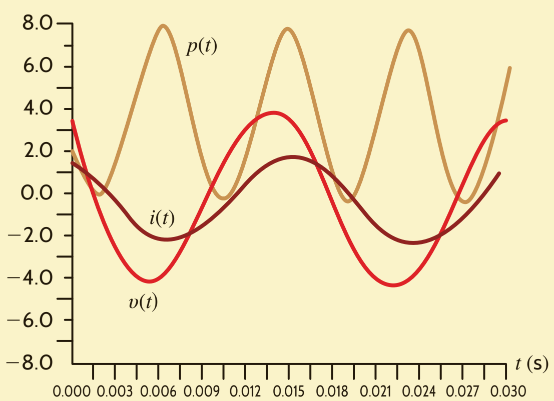

Figure 9.2

Plots of $v(t)$, $i(t)$, and $p(t)$ for the circuit in Example 9.1 using $f = 60$ Hz.