14.1 Laplace Circuit Solutions

To introduce the utility of the Laplace transform in circuit analysis, let us consider the *RL* series circuit shown in Fig. 14.1. In particular, let us find the current, $i(t)$.

Using Kirchhoff's voltage law, we can write the time-domain differential equation,

$$v_S(t) = L\left(\frac{di(t)}{dt}\right) + Ri(t)$$

The complementary differential equation is

$$L\left(\frac{di(t)}{dt}\right) + Ri(t) = 0 \qquad \textbf{14.1}$$

and has the solution

$$i_C(t) = K_C e^{-\alpha t}$$

Substituting $i_C(t)$ into the complementary equation yields the relationship

$$R - \alpha L = 0$$

or

$$\alpha = \frac{R}{L} = 1000$$

The particular solution is of the same form as the forcing function, $v_S(t)$:

$$i_p(t) = K_p$$

Substituting $i_p(t)$ into the original differential equation yields the expression

$$1 = RK_p$$

or

$$K_p = 1/R = 1/100$$

The final solution is the sum of $i_p(t)$ and $i_C(t)$,

$$i(t) = K_p + K_C e^{-\alpha t} = \frac{1}{100} + K_C e^{-1000t}$$

To find K_C, we must use the value of the current at some particular instant of time. For $t < 0$, the unit step function is zero and so is the current. At $t = 0$, the unit step goes to one; however, the inductor forces the current to instantaneously remain at zero. Therefore, at $t = 0$, we can write

$$i(0) = 0 = K_p + K_C$$

or

$$K_C = -K_p = -\frac{1}{100}$$

Thus, the current is

$$i(t) = 10(1 - e^{-1000t})u(t) \text{ mA}$$

Let us now try a different approach to the same problem. Making use of Table 13.2, let us take the Laplace transform of both sides of Eq. (14.1):

$$\mathscr{L}[v_S(t)] = \mathbf{V}_S(s) = L[s\mathbf{I}(s) - i(0)] + R\mathbf{I}(s)$$

Figure 14.1

RL series network.