ing a system that was struck by lightning, we might consider modeling the lightning stroke as a unit impulse function. Another example is the process of sampling where an analog-to-digital converter (ADC) is utilized to convert a time signal into values that can be used in a computer. The ADC captures the value of the time signal at certain instants of time. The sampling property of the unit impulse function described above is very useful in modeling the sampling process.

EXAMPLE 13.2

Let us determine the Laplace transform of an impulse function.

SOLUTION

The Laplace transform of the impulse function is

$$\mathbf{F}(s) = \int_0^\infty \delta(t - t_0)\, e^{-st}\, dt$$

Using the sampling property of the delta function, we obtain

$$\mathcal{L}[\delta(t - t_0)] = e^{-t_0 s}$$

In the limit as $t_0 \rightarrow 0$, $e^{-t_0 s} \rightarrow 1$, and therefore

$$\mathcal{L}[\delta(t)] = \mathbf{F}(s) = 1$$

13.3 Transform Pairs

We will now illustrate the development of a number of basic transform pairs that are very useful in circuit analysis.

EXAMPLE 13.3

Let us find the Laplace transform of $f(t) = t$.

SOLUTION

The Laplace transform of the function $f(t) = t$ is

$$\mathbf{F}(s) = \int_0^\infty t e^{-st}\, dt$$

Integrating the function by parts, we let

$$u = t \qquad \text{and} \qquad dv = e^{-st}\, dt$$

Then

$$du = dt \qquad \text{and} \qquad v = \int e^{-st}\, dt = -\frac{1}{s} e^{-st}$$

Therefore,

$$\begin{aligned}\mathbf{F}(s) &= \frac{-t}{s} e^{-st} \bigg|_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\, dt \\ &= \frac{1}{s^2} \qquad \sigma > 0\end{aligned}$$

HINT

$t \leftrightarrow \dfrac{1}{s^2}$

EXAMPLE 13.4

Let us determine the Laplace transform of the cosine function.

SOLUTION

The Laplace transform for the cosine function is

$$\begin{aligned}\mathbf{F}(s) &= \int_0^\infty \cos \omega t\, e^{-st}\, dt \\ &= \int_0^\infty \frac{e^{+j\omega t} + e^{-j\omega t}}{2} e^{-st}\, dt \\ &= \int_0^\infty \frac{e^{-(s-j\omega)t} + e^{-(s+j\omega)t}}{2}\, dt \\ &= \frac{1}{2}\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) \qquad \sigma > 0 \\ &= \frac{s}{s^2+\omega^2}\end{aligned}$$

HINT

$\cos \omega t \leftrightarrow \dfrac{s}{s^2+\omega^2}$

A short table of useful Laplace transform pairs is shown in Table 13.1.

Once the transform pairs are known, we can easily move back and forth between the time domain and the complex frequency domain without having to use Eqs. (13.1) and (13.4).

LEARNING ASSESSMENTS

E13.1 If $f(t) = e^{-at}$, show that $\mathbf{F}(s) = 1/(s + a)$.

E13.2 If $f(t) = \sin \omega t$, show that $\mathbf{F}(s) = \omega/(s^2 + \omega^2)$.

TABLE 13.1 Short table of Laplace transform pairs

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\dfrac{1}{s}$
e^{-at}	$\dfrac{1}{s+a}$
t	$\dfrac{1}{s^2}$
$\dfrac{t^n}{n!}$	$\dfrac{1}{s^{n+1}}$
te^{-at}	$\dfrac{1}{(s+a)^2}$
$\dfrac{t^n e^{-at}}{n!}$	$\dfrac{1}{(s+a)^{n+1}}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$
$e^{-at}\sin bt$	$\dfrac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\dfrac{s+a}{(s+a)^2+b^2}$