E9.13 The voltage across a 2-W resistor is given by the waveform in Fig. E9.13. Find the average power absorbed by the resistor.

ANSWER:
$P = 38.22$ W.

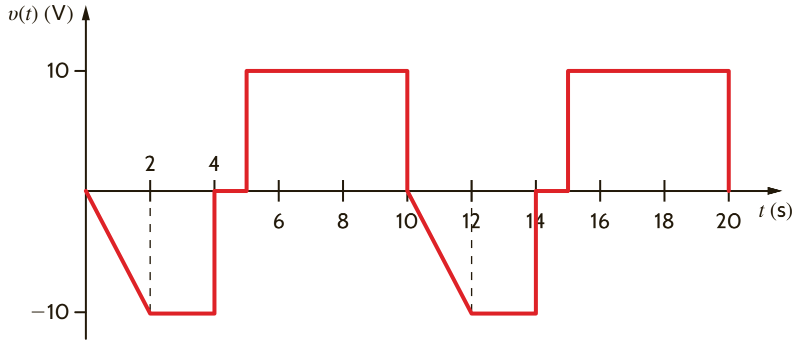

Figure E9.13

E9.14 Compute the rms value of the voltage waveform shown in Fig. E9.14.

ANSWER:
$V_{\text{rms}} = 1.633$ V.

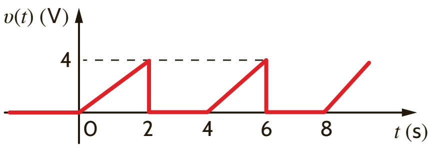

Figure E9.14

9.5 The Power Factor

The power factor is a very important quantity. Its importance stems in part from the economic impact it has on industrial users of large amounts of power. In this section we carefully define this term and then illustrate its significance via some practical examples.

In Section 9.4 we showed that a load operating in the ac steady state is delivered an average power of

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

We will now further define the terms in this important equation. The product $V_{\text{rms}} I_{\text{rms}}$ is referred to as the *apparent power*. Although the term $\cos(\theta_v - \theta_i)$ is a dimensionless quantity, and the units of P are watts, apparent power is normally stated in volt-amperes (VA) or kilovolt-amperes (kVA) to distinguish it from average power.

We now define the *power factor* (pf) as the ratio of the average power to the apparent power; that is,

$$\text{pf} = \frac{P}{V_{\text{rms}} I_{\text{rms}}} = \cos(\theta_v - \theta_i) \qquad \textbf{9.27}$$

where

$$\cos(\theta_v - \theta_i) = \cos\theta_{\mathbf{Z}_L} \qquad \textbf{9.28}$$

The angle $\theta_v - \theta_i = \cos\theta_{\mathbf{Z}_L}$ is the phase angle of the load impedance and is often referred to as the *power factor angle*. The two extreme positions for this angle correspond to a purely resistive load where $\theta_{\mathbf{Z}_L} = 0$ and the pf is 1, and the purely reactive load where $\theta_{\mathbf{Z}_L} = \pm 90°$ and the pf is 0. It is, of course, possible to have a unity pf for a load containing R, L, and C elements if the values of the circuit elements are such that a zero phase angle is obtained at the particular operating frequency.

There is, of course, a whole range of power factor angles between $\pm 90°$ and $0°$. If the load is an equivalent RC combination, then the pf angle lies between the limits $-90° < \theta_{\mathbf{Z}_L} < 0°$. On the other hand, if the load is an equivalent RL combination, then the pf angle lies between the limits $0 < \theta_{\mathbf{Z}_L} < 90°$. Obviously, confusion in identifying the type of load could result, due to the fact that $\cos\theta_{\mathbf{Z}_L} = \cos(-\theta_{\mathbf{Z}_L})$. To circumvent this problem, the pf is said to be either *leading* or *lagging,* where these two terms *refer to the phase of the current with respect to the voltage*. Since the current leads the voltage in an *RC* load, the load has a leading pf. In a similar manner, an *RL* load has a lagging pf; therefore, load impedances of $\mathbf{Z}_L = 1 - j1\ \Omega$ and $\mathbf{Z}_L = 2 + j1\ \Omega$ have power factors of $\cos(-45°) = 0.707$ leading and $\cos(26.57°) = 0.894$ lagging, respectively.

EXAMPLE 9.10

An industrial load consumes 88 kW at a pf of 0.707 lagging from a 480-V rms line. The transmission line resistance from the power company's transformer to the plant is 0.08 Ω. Let us determine the power that must be supplied by the power company (a) under present conditions and (b) if the pf is somehow changed to 0.90 lagging. (It is economically advantageous to have a power factor as close to one as possible.)

HINT

Technique

1. Given P_L, pf, and V_{rms}, determine I_{rms}.
2. Then $P_S = P_L + I^2_{\text{rms}}R_{\text{line}}$, where R_{line} is the line resistance.

SOLUTION

a. The equivalent circuit for these conditions is shown in Fig. 9.11. Using Eq. (9.27), we obtain the magnitude of the rms current into the plant:

$$\begin{aligned} I_{\text{rms}} &= \frac{P_L}{(\text{pf})(V_{\text{rms}})} \\ &= \frac{(88)(10^3)}{(0.707)(480)} \\ &= 259.3 \text{ A rms} \end{aligned}$$

The power that must be supplied by the power company is

$$\begin{aligned} P_S &= P_L + (0.08)I^2_{\text{rms}} \\ &= 88{,}000 + (0.08)(259.3)^2 \\ &= 93.38 \text{ kW} \end{aligned}$$

b. Suppose now that the pf is somehow changed to 0.90 lagging but the voltage remains constant at 480 V. The rms load current for this condition is

$$\begin{aligned} I_{\text{rms}} &= \frac{P_L}{(\text{pf})(V_{\text{rms}})} \\ &= \frac{(88)(10^3)}{(0.90)(480)} \\ &= 203.7 \text{ A rms} \end{aligned}$$

Under these conditions, the power company must generate

$$\begin{aligned} P_S &= P_L + (0.08)I^2_{\text{rms}} \\ &= 88{,}000 + (0.08)(203.7)^2 \\ &= 91.32 \text{ kW} \end{aligned}$$

Note carefully the difference between the two cases. A simple change in the pf of the load from 0.707 lagging to 0.90 lagging has had an interesting effect. Note that in the first case the power company must generate 93.38 kW in order to supply the plant with 88 kW of power because the low power factor means that the line losses will be high—5.38 kW. However, in the second case the power company need only generate 91.32 kW in order to supply the plant with its required power, and the corresponding line losses are only 3.32 kW.

Figure 9.11

Example circuit for examining changes in power factor.