11.1 Three-Phase Circuits

In this chapter we add a new dimension to our study of ac steady-state circuits. Up to this point, we have dealt with what we refer to as single-phase circuits. Now we extend our analysis techniques to polyphase circuits or, more specifically, three-phase circuits (that is, circuits containing three voltage sources that are one-third of a cycle apart in time).

We study three-phase circuits for a number of important reasons. It is more advantageous and economical to generate and transmit electric power in the polyphase mode than with single-phase systems. As a result, most electric power is transmitted in polyphase circuits. In the United States the power system frequency is 60 Hz, whereas in other parts of the world 50 Hz is common.

The generation of electric power in the polyphase mode is accomplished with an electric generator, which converts mechanical energy to electrical energy. This mechanical energy can be produced at a dam or hydroelectric facility, as shown in **Fig. 11.1**. As illustrated in **Fig. 11.2**, water stored in a reservoir falls through a turbine to the river below. The turbine drives the electric generator to produce three-phase voltages. In the fossil-fuel generating facility in **Fig. 11.3**, the turbine is driven by steam. In the diagram of **Fig. 11.4**, fuel and air are combusted in the boiler, turning water into steam to drive the turbine. Cooling water is circulated through the condenser to change the steam exhaust from the turbine back to water to complete the cycle. A nuclear generating facility, shown in **Fig. 11.5**, also utilizes steam to drive the turbine. The heat from fission in the reactor core produces the steam.

Note that all three types of generating facilities are located close to a body of water such as a river and are not often close to the loads that consume the electrical energy. Power transmission lines, such as those shown in **Fig. 11.6**, are constructed to transport electrical energy from the generating facilities to the loads. The transmission of electrical energy is most efficiently accomplished at very high voltages. Because this voltage can be extremely high in comparison to the level at which it is normally used (e.g., in the household), there is a need to raise and lower the voltage. This can be easily accomplished in ac systems using transformers, which we studied in Chapter 10. An example of a three-phase power transformer is shown in **Fig. 11.7**.

Figure 11.1

Hydroelectric generating facility (Courtesy of Mark Nelms).

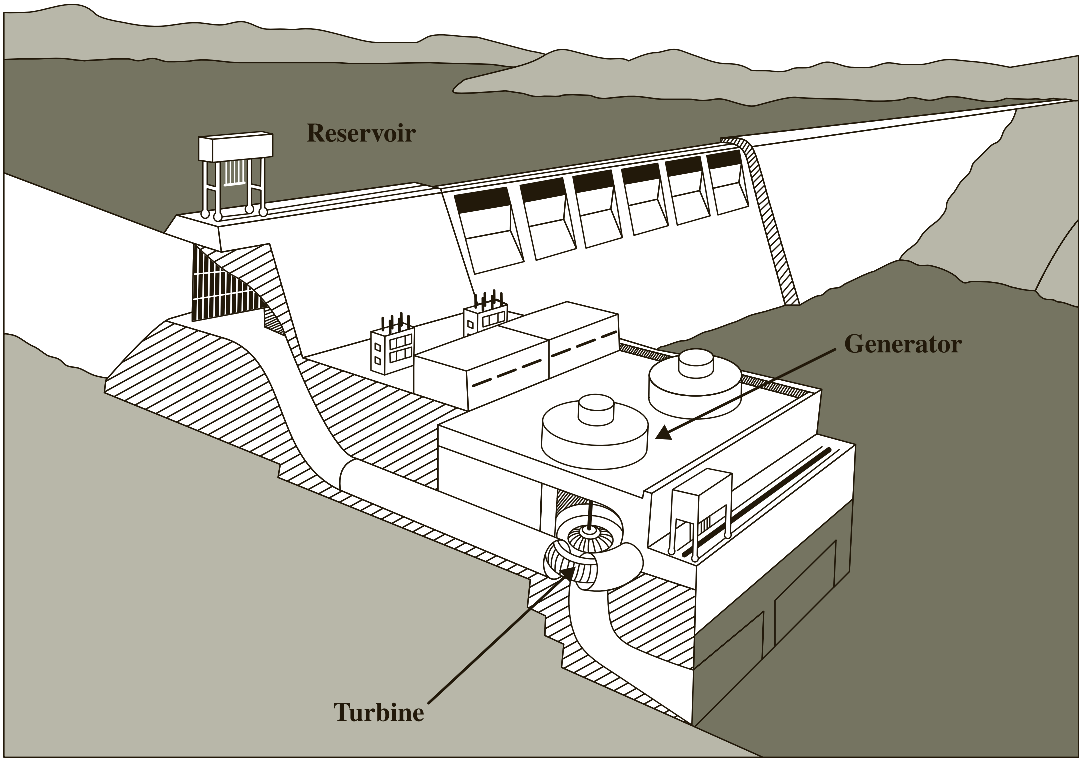

Figure 11.2

Diagram of a hydroelectric generating facility (Diagram courtesy of Southern Company).

Figure 11.3

A fossil-fuel generating facility (Courtesy of Mark Nelms).

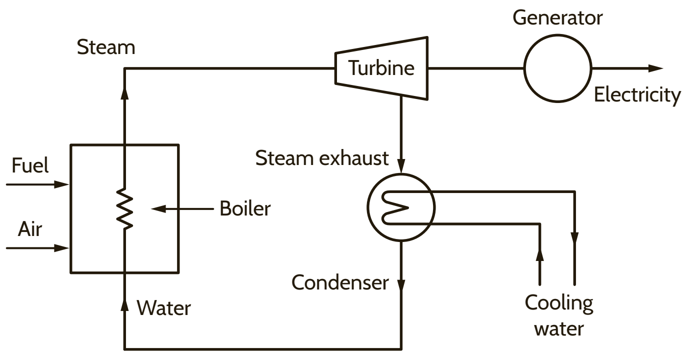

Figure 11.4
Conceptual diagram for a fossil-fuel generating facility (Diagram courtesy of Southern Company).

Figure 11.5
A nuclear generating facility (Stockbyte/SUPERSTOCK).

Figure 11.6
Power transmission lines (Courtesy of Mark Nelms).

Figure 11.7

A three-phase power transformer (Courtesy of Jeremy Nelms, Talquin Electric Cooperative, Inc.).

As the name implies, three-phase circuits are those in which the forcing function is a three-phase system of voltages. If the three sinusoidal voltages have the same magnitude and frequency and each voltage is 120° out of phase with the other two, the voltages are said to be *balanced*. If the loads are such that the currents produced by the voltages are also balanced, the entire circuit is referred to as a *balanced three-phase circuit*.

A balanced set of three-phase voltages can be represented in the frequency domain as shown in **Fig. 11.8a**, where we have assumed that their magnitudes are 120 V rms. From the figure we note that

$$\begin{aligned} \mathbf{V}_{an} &= 120\angle 0^\circ \text{ V rms} \\ \mathbf{V}_{bn} &= 120\angle -120^\circ \text{ V rms} \\ \mathbf{V}_{cn} &= 120\angle -240^\circ \text{ V rms} \\ &= 120\angle 120^\circ \text{ V rms} \end{aligned} \tag{11.1}$$

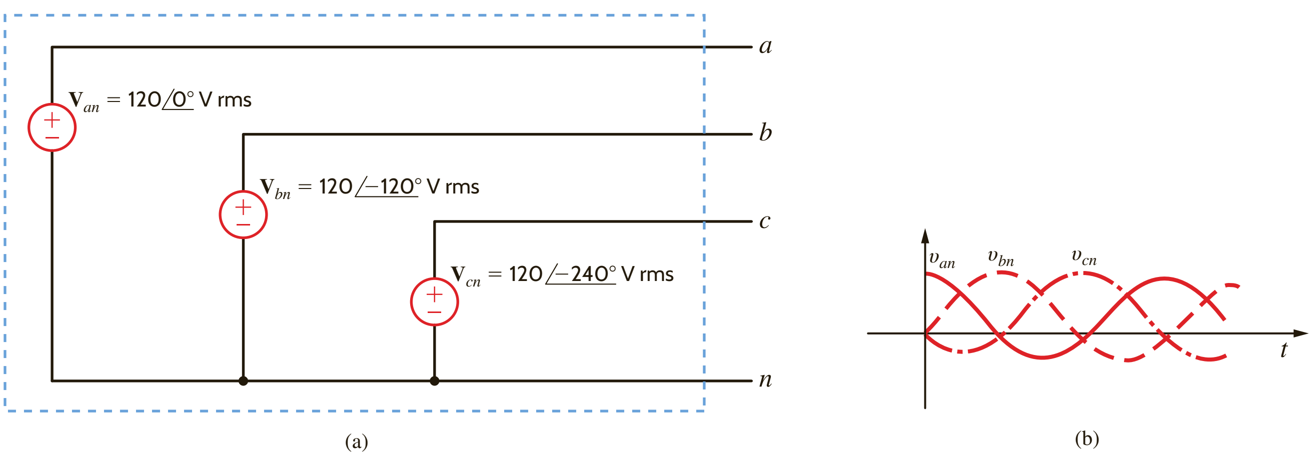

Figure 11.8

Balanced three-phase voltages.

Our double-subscript notation is exactly the same as that employed in the earlier chapters; that is, $\mathbf{V}_{an}$ means the voltage at point a with respect to point n. We will also employ the double-subscript notation for currents; that is, $\mathbf{I}_{an}$ is used to represent the current from a to n. However, we must be very careful in this case to describe the precise path, since in a circuit there will be more than one path between the two points. For example, in the case of a single loop the two possible currents in the two paths will be 180° out of phase with each other.

The preceding phasor voltages can be expressed in the time domain as

$$\begin{aligned} v_{an}(t) &= 120\sqrt{2}\cos\omega t \text{ V} \\ v_{bn}(t) &= 120\sqrt{2}\cos(\omega t - 120^\circ) \text{ V} \\ v_{cn}(t) &= 120\sqrt{2}\cos(\omega t - 240^\circ) \text{ V} \end{aligned} \tag{11.2}$$

These time functions are shown in Fig. 11.8b.

Finally, let us examine the instantaneous power generated by a three-phase system. Assume that the voltages in Fig. 11.8 are

$$\begin{aligned} v_{an}(t) &= V_m\cos\omega t \text{ V} \\ v_{bn}(t) &= V_m\cos(\omega t - 120^\circ) \text{ V} \\ v_{cn}(t) &= V_m\cos(\omega t - 240^\circ) \text{ V} \end{aligned} \tag{11.3}$$

If the load is balanced, the currents produced by the sources are

$$\begin{aligned} i_a(t) &= I_m\cos(\omega t - \theta) \text{ A} \\ i_b(t) &= I_m\cos(\omega t - \theta - 120^\circ) \text{ A} \\ i_c(t) &= I_m\cos(\omega t - \theta - 240^\circ) \text{ A} \end{aligned} \tag{11.4}$$

The instantaneous power produced by the system is

$$\begin{aligned} p(t) &= p_a(t) + p_b(t) + p_c(t) \\ &= V_m I_m[\cos\omega t\cos(\omega t - \theta) + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t - 240^\circ)\cos(\omega t - \theta - 240^\circ)] \end{aligned} \tag{11.5}$$

Using the trigonometric identity,

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \tag{11.6}$$

Eq. (11.5) becomes

$$\begin{aligned} p(t) = \frac{V_m I_m}{2}&[\cos\theta + \cos(2\omega t - \theta) + \cos\theta \\ &+ \cos(2\omega t - \theta - 240^\circ) + \cos\theta + \cos(2\omega t - \theta - 480^\circ)] \end{aligned} \tag{11.7}$$

which can be written as

$$\begin{aligned} p(t) = \frac{V_m I_m}{2}&[3\cos\theta + \cos(2\omega t - \theta) \\ &+ \cos(2\omega t - \theta - 120^\circ) + \cos(2\omega t - \theta + 120^\circ)] \end{aligned} \tag{11.8}$$

There exists a trigonometric identity that allows us to simplify the preceding expression. The identity, which we will prove later using phasors, is

$$\cos\phi + \cos(\phi - 120^\circ) + \cos(\phi + 120^\circ) = 0 \tag{11.9}$$

If we employ this identity, the expression for the power becomes

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta \text{ W} \tag{11.10}$$