Finally, it is important to note that our electrical networks satisfy the principle of conservation of energy. Because of the relationship between energy and power, it can be implied that power is also conserved in an electrical network. This result was formally stated in 1952 by B. D. H. Tellegen and is known as Tellegen's theorem—the sum of the powers absorbed by all elements in an electrical network is zero. Another statement of this theorem is that the power supplied in a network is exactly equal to the power absorbed. Checking to verify that Tellegen's theorem is satisfied for a particular network is one way to check our calculations when analyzing electrical networks.

1.3 Circuit Elements

Thus far, we have defined voltage, current, and power. In the remainder of this chapter we will define both independent and dependent current and voltage sources. Although we will assume ideal elements, we will try to indicate the shortcomings of these assumptions as we proceed with the discussion.

In general, the elements we will define are terminal devices that are completely characterized by the current through the element and/or the voltage across it. These elements, which we will employ in constructing electric circuits, will be broadly classified as being either active or passive. The distinction between these two classifications depends essentially on one thing—whether they supply or absorb energy. As the words themselves imply, an *active* element is capable of generating energy and a *passive* element cannot generate energy.

However, later we will show that some passive elements are capable of storing energy. Typical active elements are batteries and generators. The three common passive elements are resistors, capacitors, and inductors.

In Chapter 2 we will launch an examination of passive elements by discussing the resistor in detail. Before proceeding with that element, we first present some very important active elements.

1. Independent voltage source
2. Independent current source
3. Two dependent voltage sources
4. Two dependent current sources

INDEPENDENT SOURCES An *independent voltage source* is a two-terminal element that maintains a specified voltage between its terminals *regardless of the current through it* as shown by the v-i plot in Fig. 1.14a. The general symbol for an independent source, a circle, is also shown in Fig. 1.14a. As the figure indicates, terminal A is $v(t)$ volts positive with respect to terminal B.

In contrast to the independent voltage source, the *independent current source* is a two-terminal element that maintains a specified current *regardless of the voltage across its terminals*, as illustrated by the v-i plot in Fig. 1.14b. The general symbol for an independent

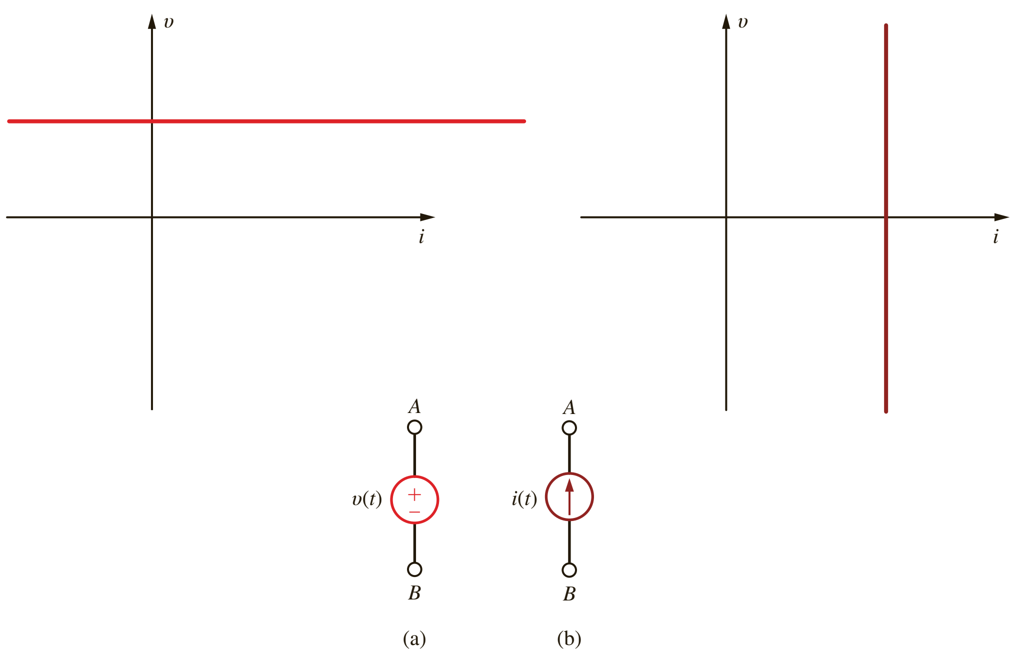

Figure 1.14

Symbols for (a) independent voltage source and (b) independent current source.

current source is also shown in Fig. 1.14b, where $i(t)$ is the specified current and the arrow indicates the positive direction of current flow.

In their normal mode of operation, independent sources supply power to the remainder of the circuit. However, they may also be connected into a circuit in such a way that they absorb power. A simple example of this latter case is a battery-charging circuit such as that shown in Example 1.1.

It is important that we pause here to interject a comment concerning a shortcoming of the models. In general, mathematical models approximate actual physical systems only under a certain range of conditions. Rarely does a model accurately represent a physical system under every set of conditions. To illustrate this point, consider the model for the voltage source in Fig. 1.14a. We assume that the voltage source delivers v volts regardless of what is connected to its terminals. Theoretically, we could adjust the external circuit so that an infinite amount of current would flow, and therefore the voltage source would deliver an infinite amount of power. This is, of course, physically impossible. A similar argument could be made for the independent current source. Hence, the reader is cautioned to keep in mind that models have limitations and thus are valid representations of physical systems only under certain conditions.

For example, can the independent voltage source be utilized to model the battery in an automobile under all operating conditions? With the headlights on, turn on the radio. Do the headlights dim with the radio on? They probably won't if the sound system in your automobile was installed at the factory. If you try to crank your car with the headlights on, you will notice that the lights dim. The starter in your car draws considerable current, thus causing the voltage at the battery terminals to drop and dimming the headlights. The independent voltage source is a good model for the battery with the radio turned on; however, an improved model is needed for your battery to predict its performance under cranking conditions.

EXAMPLE 1.4

Determine the power absorbed or supplied by the elements in the network in **Fig. 1.15**.

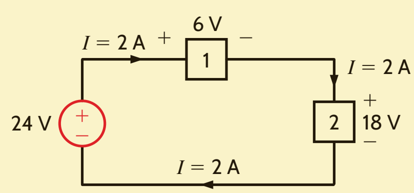

Figure 1.15
Network for Example 1.4.

HINT
Elements that are connected in series have the same current.

SOLUTION

The current flow is out of the positive terminal of the 24-V source, and therefore this element is supplying (2)(24) = 48 W of power. The current is into the positive terminals of elements 1 and 2, and therefore elements 1 and 2 are absorbing (2)(6) = 12 W and (2)(18) = 36 W, respectively. Note that the power supplied is equal to the power absorbed.

LEARNING ASSESSMENT

E1.3 Find the power that is absorbed or supplied by the elements in Fig. E1.3.

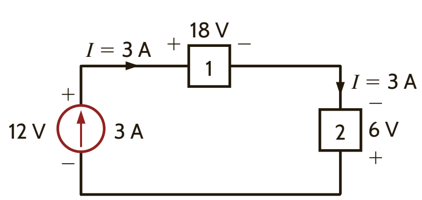

Figure E1.3

ANSWER:
Current source supplies 36 W, element 1 absorbs 54 W, and element 2 supplies 18 W.

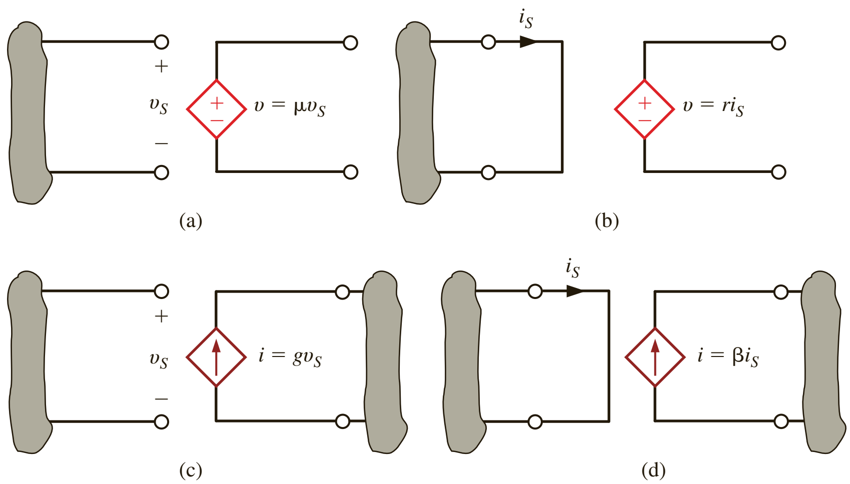

Figure 1.16

Four different types of dependent sources.

DEPENDENT SOURCES In contrast to the independent sources, which produce a particular voltage or current completely unaffected by what is happening in the remainder of the circuit, dependent sources generate a voltage or current that is determined by a voltage or current at a specified location in the circuit. These sources are very important because they are an integral part of the mathematical models used to describe the behavior of many electronic circuit elements.

For example, metal-oxide-semiconductor field-effect transistors (MOSFETs) and bipolar transistors, both of which are commonly found in a host of electronic equipment, are modeled with dependent sources, and therefore the analysis of electronic circuits involves the use of these controlled elements.

In contrast to the circle used to represent independent sources, a diamond is used to represent a dependent or controlled source. **Fig. 1.16** illustrates the four types of dependent sources. The input terminals on the left represent the voltage or current that controls the dependent source, and the output terminals on the right represent the output current or voltage of the controlled source. Note that in **Figs. 1.16a** and **d**, the quantities μ and β are dimensionless constants because we are transforming voltage to voltage and current to current. This is not the case in **Figs. 1.16b** and **c**; hence, when we employ these elements a short time later, we must describe the units of the factors r and g.

EXAMPLE 1.5

Given the two networks shown in **Fig. 1.17**, we wish to determine the outputs.

SOLUTION

In **Fig. 1.17a**, the output voltage is $V_o = \mu V_S$ or $V_o = 20\ V_S = (20)(2\text{ V}) = 40$ V. Note that the output voltage has been amplified from 2 V at the input terminals to 40 V at the output terminals; that is, the circuit is a voltage amplifier with an amplification factor of 20.

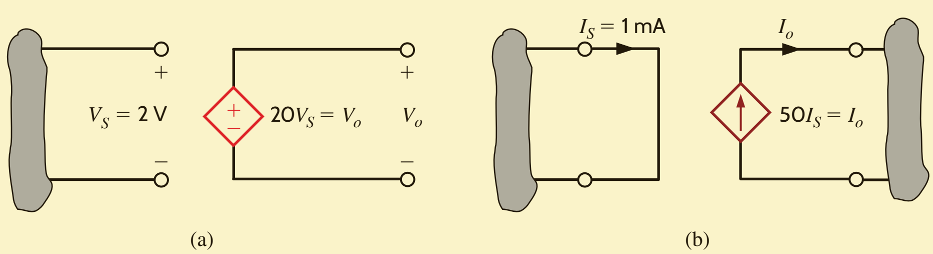

Figure 1.17

Circuits for Example 1.5.

In **Fig. 1.17b**, the output current is $I_o = \beta I_S = (50)(1\text{ mA}) = 50$ mA; that is, the circuit has a current gain of 50, meaning that the output current is 50 times greater than the input current.

LEARNING ASSESSMENT

E1.4 Determine the power supplied by the dependent sources in Fig. E1.4.

ANSWER:
(a) Power supplied = 80 W;
(b) power supplied = 160 W.

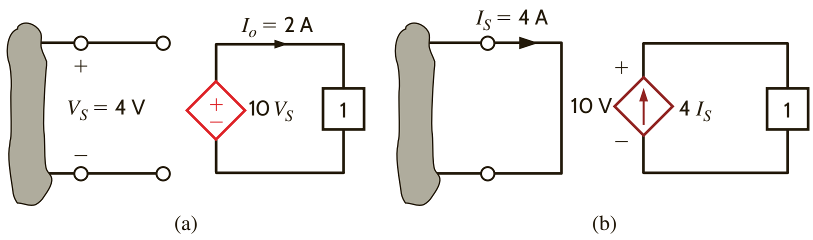

Figure E1.4

EXAMPLE 1.6

Calculate the power absorbed by each element in the network of Fig. 1.18. Also verify that Tellegen's theorem is satisfied by this network.

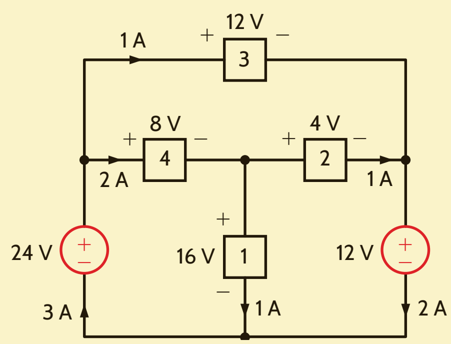

Figure 1.18
Circuit used in Example 1.6.

SOLUTION

Let's calculate the power absorbed by each element using the sign convention for power.

$$
\begin{aligned}
P_1 &= (16)(1) = 16 \text{ W} \\
P_2 &= (4)(1) = 4 \text{ W} \\
P_3 &= (12)(1) = 12 \text{ W} \\
P_4 &= (8)(2) = 16 \text{ W} \\
P_{12\text{ V}} &= (12)(2) = 24 \text{ W} \\
P_{24\text{ V}} &= (24)(-3) = -72 \text{ W}
\end{aligned}
$$

Note that to calculate the power absorbed by the 24-V source, the current of 3 A flowing up through the source was changed to a current −3 A flowing down through the 24-V source.

Let's sum up the power absorbed by all elements: 16 + 4 + 12 + 16 + 24 − 72 = 0

This sum is zero, which verifies that Tellegen's theorem is satisfied.

EXAMPLE 1.7

Use Tellegen's theorem to find the current I_o in the network in Fig. 1.19.

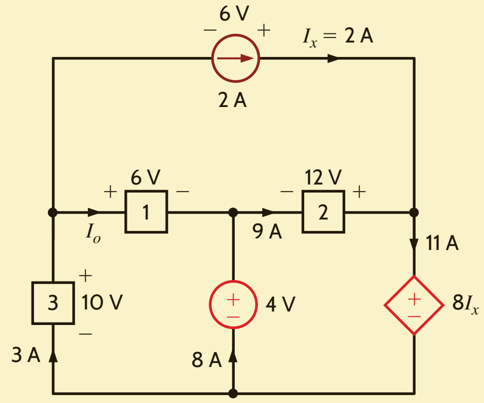

Figure 1.19
Circuit used in Example 1.7.

SOLUTION

First, we must determine the power absorbed by each element in the network. Using the sign convention for power, we find

$$P_{2\text{A}} = (6)(-2) = -12 \text{ W}$$
$$P_1 = (6)(I_o) = 6I_o \text{ W}$$
$$P_2 = (12)(-9) = -108 \text{ W}$$
$$P_3 = (10)(-3) = -30 \text{ W}$$
$$P_{4\text{V}} = (4)(-8) = -32 \text{ W}$$
$$P_{DS} = (8I_x)(11) = (16)(11) = 176 \text{ W}$$

Applying Tellegen's theorem yields

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

or

$$6I_o + 176 = 12 + 108 + 30 + 32$$

Hence,

$$I_o = 1\text{A}$$

LEARNING ASSESSMENTS

E1.5 Find the power that is absorbed or supplied by the circuit elements in the network in Fig. E1.5.

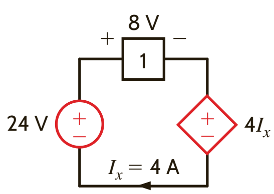

Figure E1.5

ANSWER:
$P_{24\text{ V}} = 96$ W supplied;
$P_1 = 32$ W absorbed;
$P_{4I_x} = 64$ W absorbed.

E1.6 Find the power that is absorbed or supplied by the network elements in Fig. E1.6.

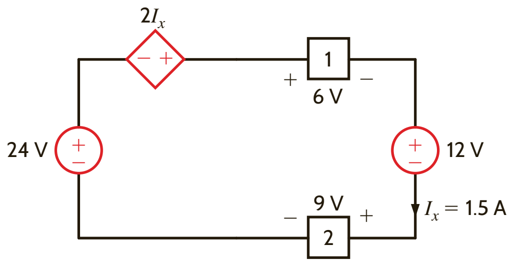

Figure E1.6

ANSWER:
$P_{24\,V} = 36$ W supplied;
$P_{12\,V} = 18$ W absorbed;
$P_{2I_x} = 4.5$ W supplied;
$P_1 = 9$ W absorbed;
$P_2 = 13.5$ W absorbed.

E1.7 Find I_x in Fig. E1.7 using Tellegen's theorem.

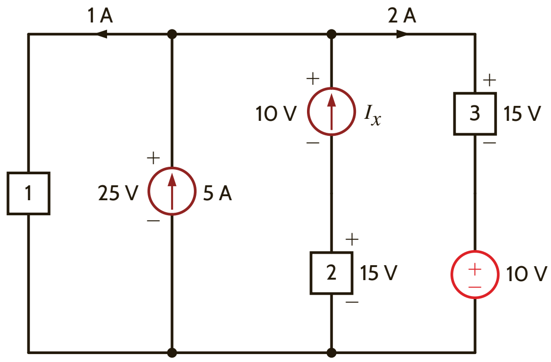

Figure E1.7

ANSWER:
$I_x = -2$ A.

EXAMPLE 1.8

The charge that enters the BOX is shown in **Fig. 1.20**. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.

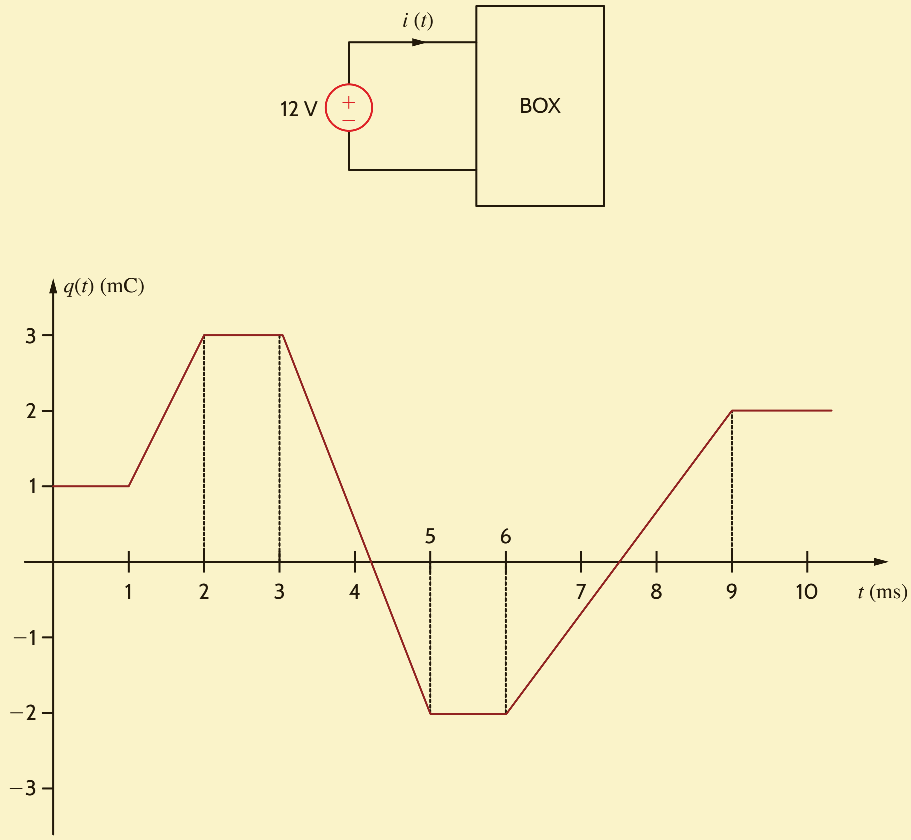

Figure 1.20
Diagrams for Example 1.8.

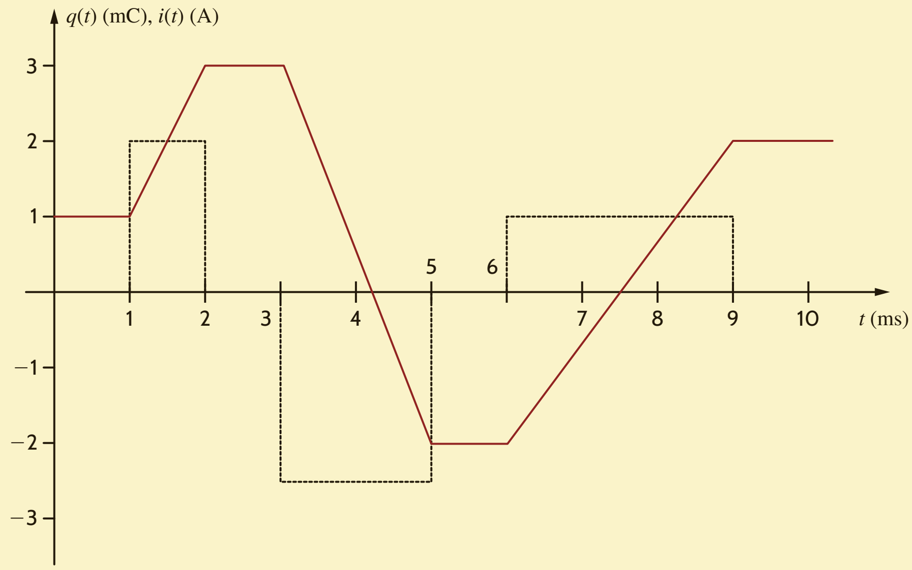

Figure 1.21
Charge and current waveforms for Example 1.8.

SOLUTION

Recall that current is related to charge by $i(t) = \dfrac{dq(t)}{dt}$. The current is equal to the slope of the charge waveform.

$$i(t) = 0 \qquad 0 \leq t \leq 1 \text{ ms}$$

$$i(t) = \frac{3 \times 10^{-3} - 1 \times 10^{-3}}{2 \times 10^{-3} - 1 \times 10^{-3}} = 2 \text{ A} \qquad 1 \leq t \leq 2 \text{ ms}$$

$$i(t) = 0 \qquad 2 \leq t \leq 3 \text{ ms}$$

$$i(t) = \frac{-2 \times 10^{-3} - 3 \times 10^{-3}}{5 \times 10^{-3} - 3 \times 10^{-3}} = -2.5 \text{ A} \qquad 3 \leq t \leq 5 \text{ ms}$$

$$i(t) = 0 \qquad 5 \leq t \leq 6 \text{ ms}$$

$$i(t) = \frac{2 \times 10^{-3} - (-2 \times 10^{-3})}{9 \times 10^{-3} - 6 \times 10^{-3}} = 1.33 \text{ A} \qquad 6 \leq t \leq 9 \text{ ms}$$

$$i(t) = 0 \qquad t \geq 9 \text{ ms}$$

The current is plotted with the charge waveform in **Fig. 1.21**. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

The power absorbed by the BOX is $12 \times i(t)$.

$$p(t) = 12(0) = 0 \qquad 0 \leq t \leq 1 \text{ ms}$$

$$p(t) = 12(2) = 24 \text{ W} \qquad 1 \leq t \leq 2 \text{ ms}$$

$$p(t) = 12(0) = 0 \qquad 2 \leq t \leq 3 \text{ ms}$$

$$p(t) = 12(-2.5) = -30 \text{ W} \qquad 3 \leq t \leq 5 \text{ ms}$$

$$p(t) = 12(0) = 0 \qquad 5 \leq t \leq 6 \text{ ms}$$

$$p(t) = 12(1.33) = 16 \text{ W} \qquad 6 \leq t \leq 9 \text{ ms}$$

$$p(t) = 12(0) = 0 \qquad t \geq 9 \text{ ms}$$

The power absorbed by the BOX is plotted in **Fig. 1.22**. For the time intervals, $1 \leq t \leq 2$ ms and $6 \leq t \leq 9$ ms, the BOX is absorbing power. During the time interval $3 \leq t \leq 5$ ms, the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.

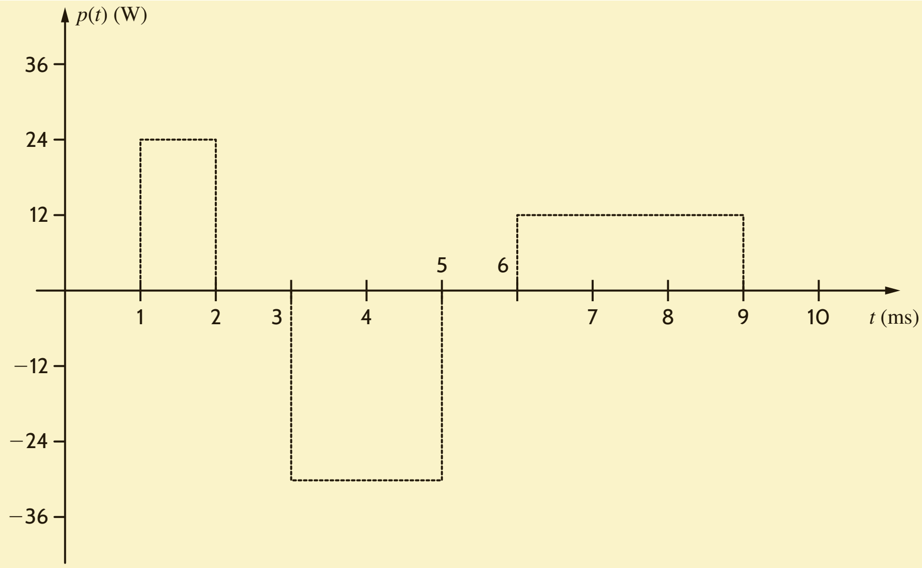

Figure 1.22
Power waveform for Example 1.8.

LEARNING ASSESSMENTS

E1.8 The power absorbed by the BOX in Fig. E1.8 is $p(t) = 2.5e^{-4t}$ W. Compute the energy and charge delivered to the BOX in the time interval $0 < t < 250$ ms.

ANSWER:
395.1 mJ; 8.8 mC.

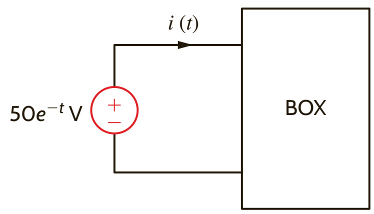

Figure E1.8

E1.9 The energy absorbed by the BOX in Fig. E1.9 is given below. Calculate and sketch the current flowing into the BOX. Also calculate the charge that enters the BOX between 0 and 12 seconds.

ANSWER:
$Q = 0$.

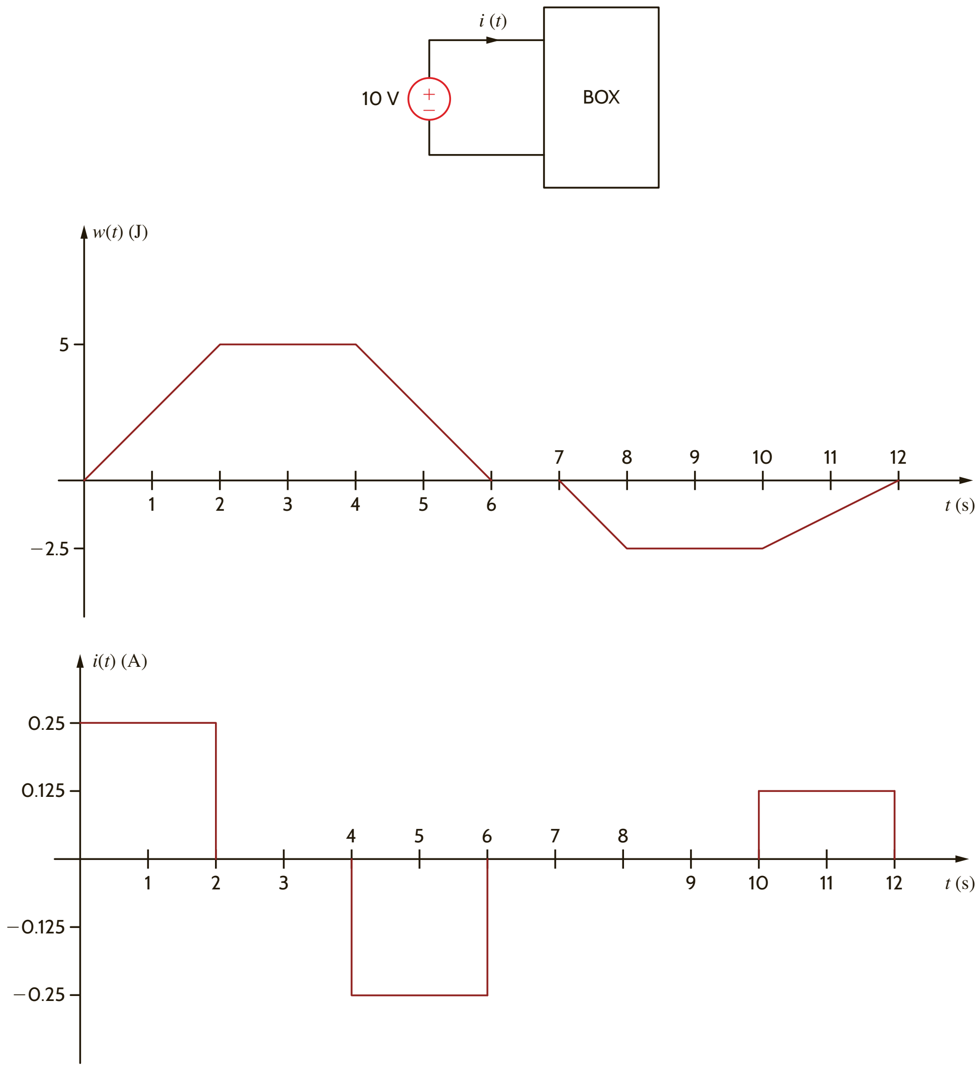

Figure E1.9

EXAMPLE 1.9

The ubiquitous universal serial bus (USB) port is commonly utilized to charge smartphones, as shown in Fig. 1.23. Technical details for USB specifications can be found at www.usb.org. The amount of current that can be provided over a USB port is defined in the USB specifications.

According to the USB 2.0 standard, a device is classified as low power if it draws 100 mA or less and high power if it draws between 100 and 500 mA.

1. A 1000 mAh lithium-ion battery has been fully discharged (i.e., 0 mAh). How long will it take to recharge it from a USB port supplying a constant current of 250 mA? How much charge is stored in the battery when it is fully charged?
2. A fully charged 1000 mAh lithium-ion battery supplies a load, which draws a constant current of 200 mA for 4 hours. How much charge is left in the battery at the end of the 4 hours? Assuming that the load remains constant at 3.6 V, how much energy is absorbed by the load in joules?