LEARNING ASSESSMENTS

E3.11 Use nodal analysis to find I_o in the circuit in Fig. E3.11.

ANSWER: $I_o = \frac{4}{3}$ mA.

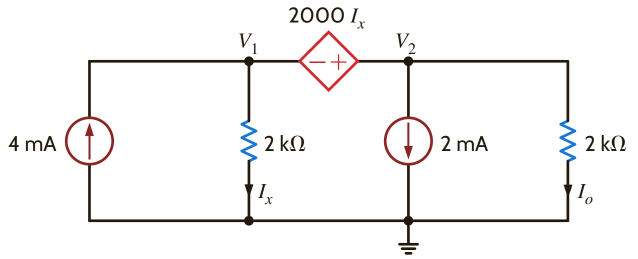


Figure E3.11

E3.12 Find V_o in Fig. E3.12 using nodal analysis.

ANSWER: $V_o = 6.29$ V.

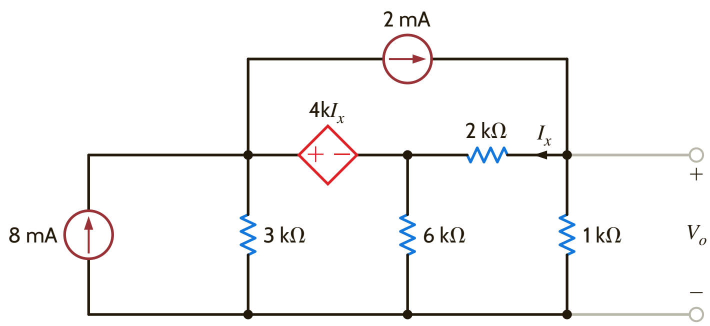


Figure E3.12

3.2 Loop Analysis

We found that in a nodal analysis the unknown parameters are the node voltages and KCL was employed to determine them. Once these node voltages have been calculated, all the branch currents in the network can easily be determined using Ohm's law. In contrast to this approach, a loop analysis uses KVL to determine a set of loop currents in the circuit. Once these loop currents are known, Ohm's law can be used to calculate any voltages in the network. Via network topology we can show that, in general, there are exactly $B - N + 1$ linearly independent KVL equations for any network, where B is the number of branches in the circuit and N is the number of nodes. For example, if we once again examine the circuit in Fig. 2.5, we find that there are eight branches and five nodes. Thus, the number of linearly independent KVL equations necessary to determine all currents in the network is $B - N + 1 = 8 - 5 + 1 = 4$. The network in Fig. 2.5 is redrawn as shown in Fig. 3.18 with four loop currents labeled as shown. The branch currents are then determined as

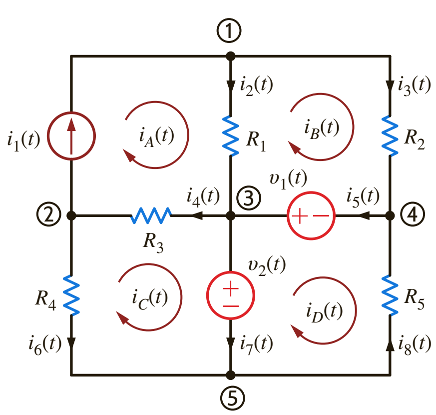


Figure 3.18
Figure 2.5 redrawn with loop currents.

$$i_1(t) = i_A(t)$$
$$i_2(t) = i_A(t) - i_B(t)$$
$$i_3(t) = i_B(t)$$
$$i_4(t) = i_A(t) - i_C(t)$$
$$i_5(t) = i_B(t) - i_D(t)$$
$$i_6(t) = -i_C(t)$$
$$i_7(t) = i_C(t) - i_D(t)$$
$$i_8(t) = -i_D(t)$$

All the circuits we will examine in this text will be *planar*, which simply means that we can draw the circuit on a sheet of paper in such a way that no conductor crosses another conductor. If a circuit is planar, the loops are more easily identified. For example, recall in Chapter 2 that we found that a single equation was sufficient to determine the current in a circuit containing a single loop. If the circuit contains N independent loops, we will show (and the general topological formula $B - N + 1$ can be used for verification) that N independent simultaneous equations will be required to describe the network.

Our approach to loop analysis will mirror the approach used in nodal analysis (i.e., we will begin with simple cases and systematically proceed to those that are more difficult). Then at the end of this section we will outline a general strategy for employing loop analysis.

CIRCUITS CONTAINING ONLY INDEPENDENT VOLTAGE SOURCES To begin our analysis, consider the circuit shown in Fig. 3.19. We note that this network has seven branches and six nodes, and thus the number of linearly independent KVL equations necessary to determine all currents in the circuit is $B - N + 1 = 7 - 6 + 1 = 2$. Since two linearly independent KVL equations are required, we identify two independent loops, *A-B-E-F-A* and *B-C-D-E-B*. We now define a new set of current variables called *loop currents*, which can be used to find the physical currents in the circuit. Let us assume that current i_1 flows in the first loop and that current i_2 flows in the second loop. Then the branch current flowing from B to E through R_3 is $i_1 - i_2$. The directions of the currents have been assumed. As was the case in the nodal analysis, if the actual currents are not in the direction indicated, the values calculated will be negative.

Applying KVL to the first loop yields

HINT

The equations employ the passive sign convention.

$$+v_1 + v_3 + v_2 - v_{S1} = 0$$

KVL applied to loop 2 yields

$$+v_{S2} + v_4 + v_5 - v_3 = 0$$

where $v_1 = i_1R_1$, $v_2 = i_1R_2$, $v_3 = (i_1 - i_2)R_3$, $v_4 = i_2R_4$, and $v_5 = i_2R_5$.

Substituting these values into the two KVL equations produces the two simultaneous equations required to determine the two loop currents; that is,

$$i_1(R_1 + R_2 + R_3) - i_2(R_3) = v_{S1}$$
$$-i_1(R_3) + i_2(R_3 + R_4 + R_5) = -v_{S2}$$

or in matrix form

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_{S1} \\ -v_{S2} \end{bmatrix}$$

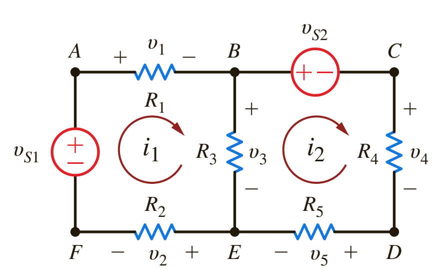


Figure 3.19

A two-loop circuit.

At this point, it is important to define what is called a *mesh*. A mesh is a special kind of loop that does not contain any loops within it. Therefore, as we traverse the path of a mesh, we do not encircle any circuit elements. For example, the network in Fig. 3.19 contains two meshes defined by the paths *A-B-E-F-A* and *B-C-D-E-B*. The path *A-B-C-D-E-F-A* is a loop, but it is not a mesh. Since the majority of our analysis in this section will involve writing KVL equations for meshes, we will refer to the currents as mesh currents and the analysis as a *mesh analysis*.

EXAMPLE 3.12

Consider the network in **Fig. 3.20a**. We wish to find the current I_o.

SOLUTION

We will begin the analysis by writing mesh equations. Note that there are no + and − signs on the resistors. However, they are not needed, since we will apply Ohm's law to each resistive element as we write the KVL equations. The equation for the first mesh is

$$-12 + 6\text{k}I_1 + 6\text{k}(I_1 - I_2) = 0$$

The KVL equation for the second mesh is

$$6\text{k}(I_2 - I_1) + 3\text{k}I_2 + 3 = 0$$

where $I_o = I_1 - I_2$.

Solving the two simultaneous equations yields $I_1 = 5/4$ mA and $I_2 = 1/2$ mA. Therefore, $I_o = 3/4$ mA. All the voltages and currents in the network are shown in **Fig. 3.20b**. Recall from nodal analysis that once the node voltages were determined, we could check our analysis using KCL at the nodes. In this case, we know the branch currents and can use KVL around any closed path to check our results. For example, applying KVL to the outer loop yields

$$-12 + \frac{15}{2} + \frac{3}{2} + 3 = 0$$
$$0 = 0$$

Since we want to calculate the current I_o, we could use loop analysis, as shown in **Fig. 3.20c**. Note that the loop current I_1 passes through the center leg of the network and, therefore, $I_1 = I_o$. The two loop equations in this case are

$$-12 + 6\text{k}(I_1 + I_2) + 6\text{k}I_1 = 0$$

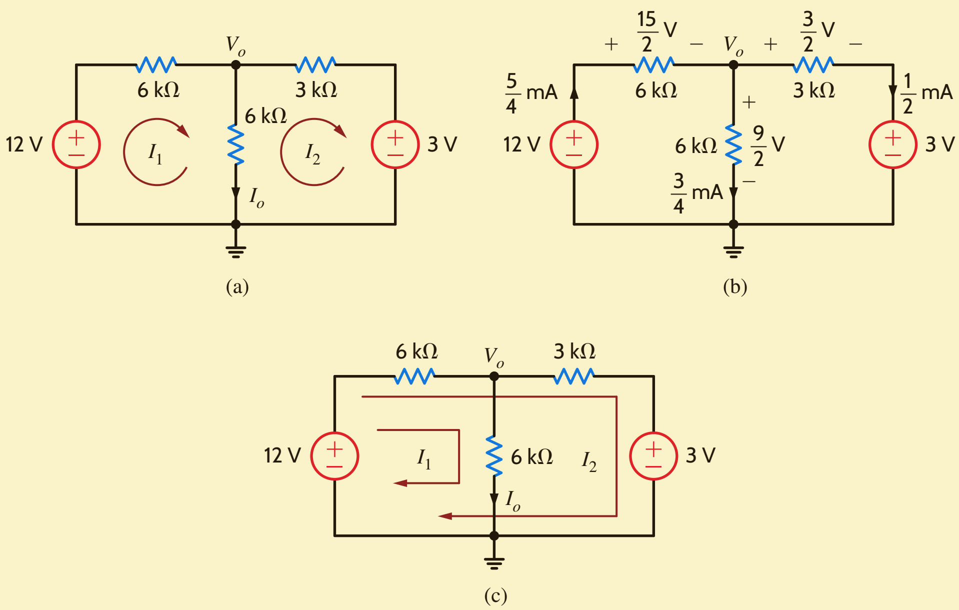


Figure 3.20 Circuits used in Example 3.12.

and

$$-12 + 6\text{k}(I_1 + I_2) + 3\text{k}I_2 + 3 = 0$$

Solving these equations yields $I_1 = 3/4$ mA and $I_2 = 1/2$ mA. Since the current in the 12-V source is $I_1 + I_2 = 5/4$ mA, these results agree with the mesh analysis.

Finally, for purposes of comparison, let us find I_o using nodal analysis. The presence of the two voltage sources would indicate that this is a viable approach. Applying KCL at the top center node, we obtain

$$\frac{V_o - 12}{6\text{k}} + \frac{V_o}{6\text{k}} + \frac{V_o - 3}{3\text{k}} = 0$$

and, hence,

$$V_o = \frac{9}{2}\,\text{V}$$

and then

$$I_o = \frac{V_o}{6\text{k}} = \frac{3}{4}\,\text{mA}$$

Note that in this case we had to solve only one equation instead of two.

Once again, we are compelled to note the symmetrical form of the mesh equations that describe the circuit in Fig. 3.19. Note that the coefficient matrix for this circuit is symmetrical.

Since this symmetry is generally exhibited by networks containing resistors and independent voltage sources, we can learn to write the mesh equations by inspection. In the first equation, the coefficient of i_1 is the sum of the resistances through which mesh current 1 flows, and the coefficient of i_2 is the negative of the sum of the resistances common to mesh current 1 and mesh current 2. The right-hand side of the equation is the algebraic sum of the voltage sources in mesh 1. The sign of the voltage source is positive if it aids the assumed direction of the current flow and negative if it opposes the assumed flow. The first equation is KVL for mesh 1. In the second equation, the coefficient of i_2 is the sum of all the resistances in mesh 2, the coefficient of i_1 is the negative of the sum of the resistances common to mesh 1 and mesh 2, and the right-hand side of the equation is the algebraic sum of the voltage sources in mesh 2. In general, if we assume all of the mesh currents to be in the same direction (clockwise or counterclockwise), then if KVL is applied to mesh j with mesh current i_j, the coefficient of i_j is the sum of the resistances in mesh j and the coefficients of the other mesh currents (e.g., i_{j-1}, i_{j+1}) are the negatives of the resistances common to these meshes and mesh j. The right-hand side of the equation is equal to the algebraic sum of the voltage sources in mesh j. These voltage sources have a positive sign if they aid the current flow i_j and a negative sign if they oppose it.

EXAMPLE 3.13

Let us write the mesh equations by inspection for the network in Fig. 3.21. Then we will use MATLAB to solve for the mesh currents.

SOLUTION

The three linearly independent simultaneous equations are

$$\begin{aligned}
(4\text{k} + 6\text{k})I_1 - (0)I_2 - (6\text{k})I_3 &= -6 \\
-(0)I_1 + (9\text{k} + 3\text{k})I_2 - (3\text{k})I_3 &= 6 \\
-(6\text{k})I_1 - (3\text{k})I_2 + (3\text{k} + 6\text{k} + 12\text{k})I_3 &= 0
\end{aligned}$$

or in matrix form

$$\begin{bmatrix} 10\text{k} & 0 & -6\text{k} \\ 0 & 12\text{k} & -3\text{k} \\ -6\text{k} & -3\text{k} & 21\text{k} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$$

Note the symmetrical form of the equations.

Dividing the equation by 1000 yields the matrix

$$\begin{bmatrix} 10 & 0 & -6 \\ 0 & 12 & -3 \\ 0 & -3 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -0.006 \\ 0.006 \\ 0 \end{bmatrix}$$

The MATLAB solution is then

```
>> G = [10 0 -6;0 12 -3;-6 -3 21]
   G =
        10     0    -6
         0    12    -3
        -6    -3    21

>> I = [-0.006;0.006;0]
   I =
        -0.0060
         0.0060
              0

>> V = inv(G)*I
   V =
        1.0e-003*

          -0.6757
           0.4685
          -0.1261
```

or

$I_1 = -0.6757$ mA
$I_2 = 0.4685$ mA
$I_3 = -0.1261$ mA

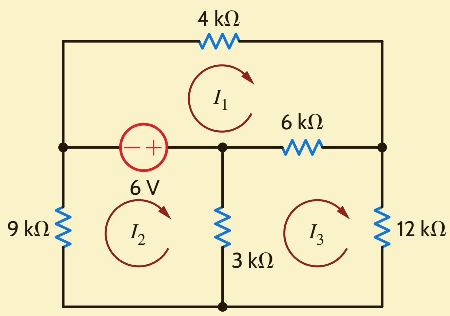


Figure 3.21
Circuit used in Example 3.13.

CIRCUITS CONTAINING INDEPENDENT CURRENT SOURCES Just as the presence of a voltage source in a network simplified the nodal analysis, the presence of a current source simplifies a loop analysis. The following examples illustrate the point.

LEARNING ASSESSMENTS

E3.13 Use mesh equations to find V_o in the circuit in Fig. E3.13.

ANSWER: $V_o = \frac{33}{5}$ V.

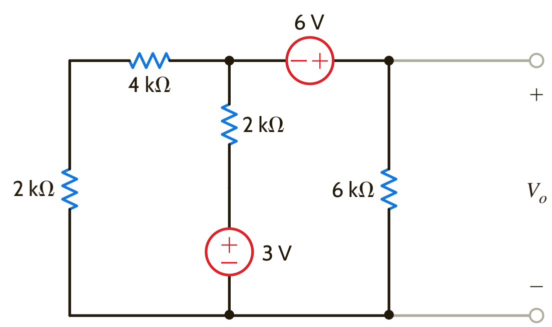


Figure E3.13

E3.14 Find V_o in Fig. E3.14 using mesh analysis.

ANSWER: $V_o = 8.96$ V.

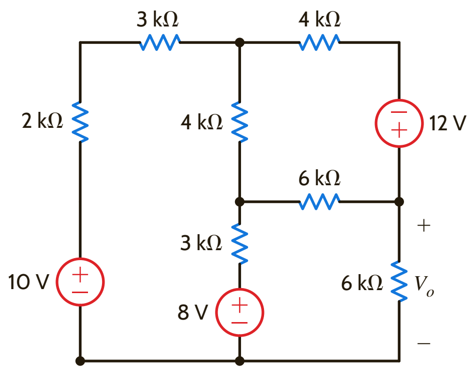


Figure E3.14

EXAMPLE 3.14

Let us find both V_o and V_1 in the circuit in Fig. 3.22.

SOLUTION

Although it appears that there are two unknown mesh currents, the current I_1 goes directly through the current source and, therefore, I_1 is constrained to be 2 mA. Hence, only the current I_2 is unknown. KVL for the rightmost mesh is

$$2\text{k}(I_2 - I_1) - 2 + 6\text{k}I_2 = 0$$

And, of course,

$$I_1 = 2 \times 10^{-3}$$

These equations can be written as

$$-2\text{k}I_1 + 8\text{k}I_2 = 2$$
$$I_1 = 2/\text{k}$$

Solving these equations for I_2 yields $I_2 = 3/4\text{kA}$ and thus

$$V_o = 6\text{k}I_2 = \frac{9}{2}\,\text{V}$$

To obtain V_1, we apply KVL around any closed path. If we use the outer loop, the KVL equation is

$$-V_1 + 4kI_1 - 2 + 6kI_2 = 0$$

And, therefore,

$$V_1 = \frac{21}{2}\,\text{V}$$

Note that since the current I_1 is known, the 4-kΩ resistor did not enter the equation in finding V_o. However, it appears in every loop containing the current source and, thus, is used in finding V_1.

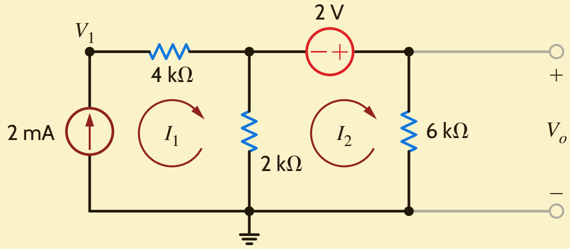


Figure 3.22

Circuit used in Example 3.14.

EXAMPLE 3.15

We wish to find V_o in the network in Fig. 3.23.

SOLUTION

Since the currents I_1 and I_2 pass directly through a current source, two of the three required equations are

$$I_1 = 4 \times 10^{-3}$$
$$I_2 = -2 \times 10^{-3}$$

The third equation is KVL for the mesh containing the voltage source; that is,

$$4k(I_3 - I_2) + 2k(I_3 - I_1) + 6kI_3 - 3 = 0$$

These equations yield

$$I_3 = \frac{1}{4}\,\text{mA}$$

and, hence,

$$V_o = 6kI_3 - 3 = \frac{-3}{2}\,\text{V}$$

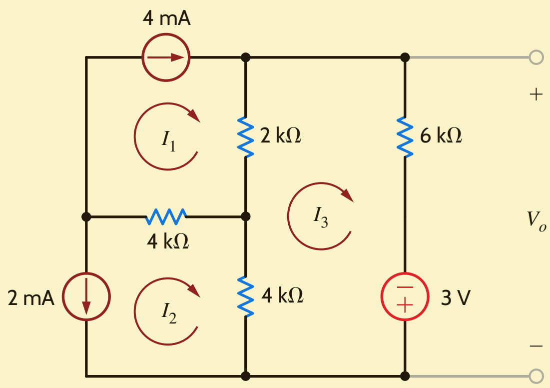


Figure 3.23

Circuit used in Example 3.15.

What we have demonstrated in the previous example is the general approach for dealing with independent current sources when writing KVL equations; that is, use one loop through each current source. The number of "window panes" in the network tells us how many equations we need. Additional KVL equations are written to cover the remaining circuit elements in the network. The following example illustrates this approach.

EXAMPLE 3.16

Let us find I_o in the network in Fig. 3.24a.

SOLUTION

First, we select two loop currents I_1 and I_2 such that I_1 passes directly through the 2-mA source, and I_2 passes directly through the 4-mA source, as shown in Fig. 3.24b. Therefore, two of our three linearly independent equations are

$$I_1 = 2 \times 10^{-3}$$
$$I_2 = 4 \times 10^{-3}$$

HINT

In this case, the 4-mA current source is located on the boundary between two meshes. Thus, we will demonstrate two techniques for dealing with this type of situation. One is a special loop technique, and the other is known as the supermesh approach.

The remaining loop current I_3 must pass through the circuit elements not covered by the two previous equations and cannot, of course, pass through the current sources. The path for this remaining loop current can be obtained by open-circuiting the current sources, as shown in Fig. 3.24c. When all currents are labeled on the original circuit, the KVL equation for this last loop, as shown in Fig. 3.24d, is

$$-6 + 1\text{k}I_3 + 2\text{k}(I_2 + I_3) + 2\text{k}(I_3 + I_2 - I_1) + 1\text{k}(I_3 - I_1) = 0$$

Solving the equations yields

$$I_3 = \frac{-2}{3}\text{ mA}$$

and, therefore,

$$I_o = I_1 - I_2 - I_3 = \frac{-4}{3}\text{ mA}$$

Next, consider the supermesh technique. In this case, the three mesh currents are specified as shown in Fig. 3.24e, and since the voltage across the 4-mA current source is unknown, it is assumed to be V_x. The mesh currents constrained by the current sources are

$$I_1 = 2 \times 10^{-3}$$
$$I_2 - I_3 = 4 \times 10^{-3}$$

The KVL equations for meshes 2 and 3, respectively, are

$$2\text{k}I_2 + 2\text{k}(I_2 - I_1) - V_x = 0$$
$$-6 + 1\text{k}I_3 + V_x + 1\text{k}(I_3 - I_1) = 0$$

Adding the last two equations yields

$$-6 + 1\text{k}I_3 + 2\text{k}I_2 + 2\text{k}(I_2 - I_1) + 1\text{k}(I_3 - I_1) = 0$$

Note, that the unknown voltage V_x has been eliminated. The two constraint equations, together with this latter equation, yield the desired result.

The purpose of the supermesh approach is to avoid introducing the unknown voltage V_x. The supermesh is created by mentally removing the 4-mA current source, as shown in Fig. 3.24f. Then writing the KVL equation around the dotted path, which defines the supermesh, using the original mesh currents as shown in Fig. 3.24e, yields

$$-6 + 1\text{k}I_3 + 2\text{k}I_2 + 2\text{k}(I_2 - I_1) + 1\text{k}(I_3 - I_1) = 0$$

Note that this supermesh equation is the same as that obtained earlier by introducing the voltage V_x.

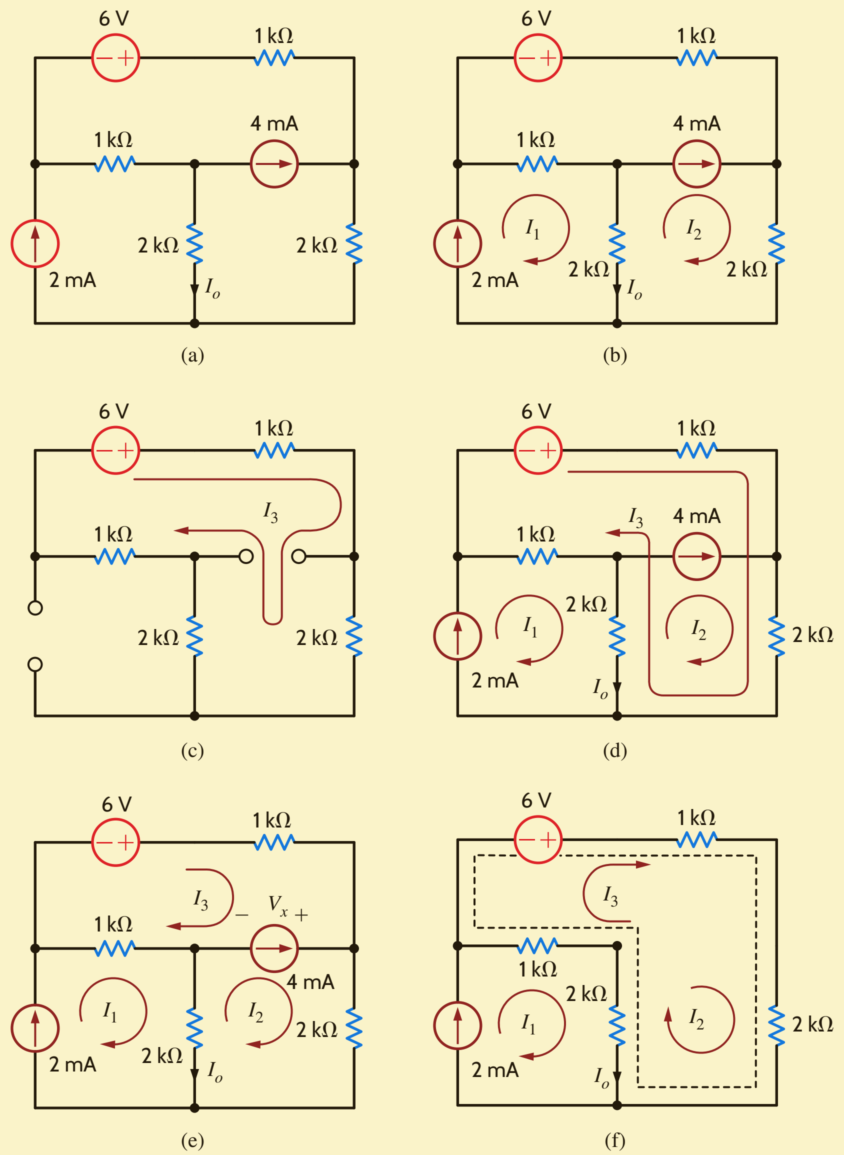


Figure 3.24
Circuits used in Example 3.16.

LEARNING ASSESSMENTS

E3.15 Find V_o in the network in Fig. E3.15.

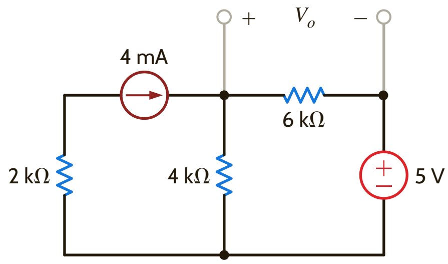


Figure E3.15

ANSWER:
$V_o = \frac{33}{5}$ V.

E3.16 Find V_o in the network in Fig. E3.16.

ANSWER: $V_o = \frac{32}{5}$ V.

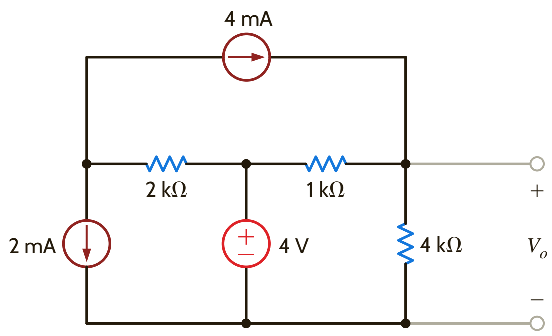


Figure E3.16

E3.17 Find V_o in Fig. E3.17 using loop analysis.

ANSWER: $V_o = 9.71$ V.

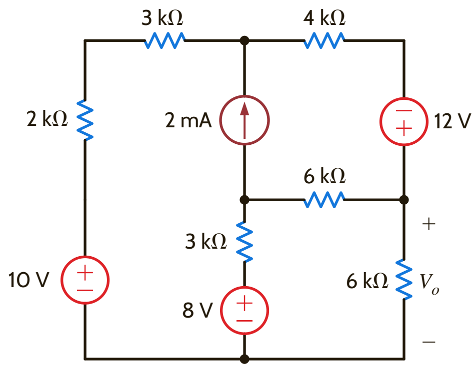


Figure E3.17

E3.18 Find V_o in Fig. E3.17 using mesh analysis.

ANSWER: $V_o = 9.71$ V.

CIRCUITS CONTAINING DEPENDENT SOURCES We deal with circuits containing dependent sources just as we have in the past. First, we treat the dependent source as though it were an independent source when writing the KVL equations. Then we write the controlling equation for the dependent source. The following examples illustrate the point.

EXAMPLE 3.17

Let us find V_o in the circuit in **Fig. 3.25**, which contains a voltage-controlled voltage source.

SOLUTION

The equations for the loop currents shown in the figure are

$$-2V_x + 2k(I_1 + I_2) + 4kI_1 = 0$$
$$-2V_x + 2k(I_1 + I_2) - 3 + 6kI_2 = 0$$

where

$$V_x = 4kI_1$$

These equations can be combined to produce

$$-2kI_1 + 2kI_2 = 0$$
$$-6kI_1 + 8kI_2 = 3$$

In matrix form, the equations are

$$\begin{bmatrix} -2000 & 2000 \\ -6000 & 8000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

The MATLAB solution is then

```
>> R = [-2000 2000;-6000 8000]
   R =
       -2000   2000
       -6000   8000

>> V = [0;3]
   V =
       0
       3

>> I = inv(R)*V
   I =
       0.0015
       0.0015
```

Or

$I_1 = 1.5$ mA
$I_2 = 1.5$ mA

and, therefore,

$$V_o = 6\text{k}I_2 = 9 \text{ V}$$

For comparison, we will also solve the problem using nodal analysis. The presence of the voltage sources indicates that this method could be simpler. Treating the 3-V source and its connecting nodes as a supernode and writing the KCL equation for this supernode yields

$$\frac{V_x - 2V_x}{2\text{k}} + \frac{V_x}{4\text{k}} + \frac{V_x + 3}{6\text{k}} = 0$$

where

$$V_o = V_x + 3$$

These equations also yield $V_o = 9$ V.

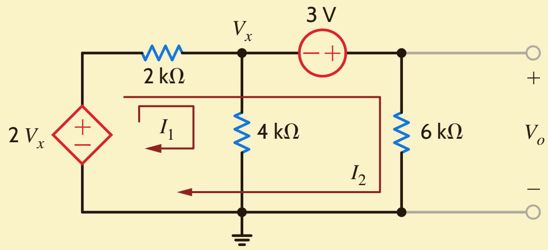


Figure 3.25
Circuit used in Example 3.17.

EXAMPLE 3.18

Let us find V_o in the circuit in Fig. 3.26, which contains a voltage-controlled current source.

SOLUTION

The currents I_1 and I_2 are drawn through the current sources. Therefore, two of the equations needed are

$$I_1 = \frac{V_x}{2000}$$
$$I_2 = 2 \times 10^{-3}$$

The KVL equation for the third mesh is

$$-3 + 2k(I_3 - I_1) + 6kI_3 = 0$$

where

$$V_x = 4k\,(I_1 - I_2)$$

Combining these equations yields

$$\begin{aligned} -I_1 + 2I_2 &= 0 \\ I_2 &= 2/k \\ -2kI_2 + 8kI_3 &= 3 \end{aligned}$$

In matrix form, the equations are

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 0 \\ -2000 & 0 & 8000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.002 \\ 3 \end{bmatrix}$$

The MATLAB solution is then

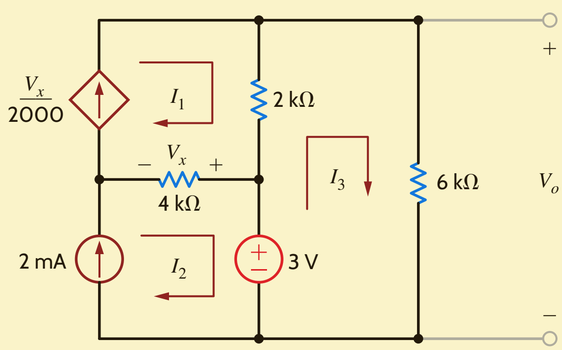
```
>> R = [-1 2 0;0 1 0;-2000 0 8000]
   R =
         -1    2      0
          0    1      0
      -2000    0   8000

>> V = [0;0.002;3]
   V =
             0
        0.0020
        3.0000

>> I = inv(R)*V
   I =
        0.0040
        0.0020
        0.0014
```

The actual numbers are $I_1 = 4.0$ mA, $I_2 = 2.0$ mA, and $I_3 = 1.375$ mA, where MATLAB has rounded off the value of I_3.

And, hence, $V_o = 8.25$ V.

Figure 3.26

Circuit used in Example 3.18.



EXAMPLE 3.19

The network in Fig. 3.27 contains both a current-controlled voltage source and a voltage-controlled current source. Let us use MATLAB to determine the loop currents.

SOLUTION

The equations for the loop currents shown in the figure are

$$I_1 = \frac{4}{\text{k}}$$

$$I_2 = \frac{V_x}{2\text{k}}$$

$$-1\text{k}I_x + 2\text{k}(I_3 - I_1) + 1\text{k}(I_3 - I_4) = 0$$

$$1\text{k}(I_4 - I_3) + 1\text{k}(I_4 - I_2) + 12 = 0$$

where

$$V_x = 2\text{k}(I_3 - I_1)$$

$$I_x = I_4 - I_2$$

Combining these equations yields

$$I_1 = \frac{4}{\text{k}}$$

$$I_1 + I_2 - I_3 = 0$$

$$1\text{k}I_2 + 3\text{k}I_3 - 2\text{k}I_4 = 8$$

$$1\text{k}I_2 + 1\text{k}I_3 - 2\text{k}I_4 = 12$$

In matrix form, the equations are

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1000 & 3000 & -2000 \\ 0 & 1000 & 1000 & -2000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0.004 \\ 0 \\ 8 \\ 12 \end{bmatrix}$$

The MATLAB solution is then

```
>> R = [1 0 0 0;1 1 -1 0;0 1000 3000 -2000;0 1000 1000 -2000]
   R =
        1       0       0       0
        1       1      -1       0
        0    1000    3000   -2000
        0    1000    1000   -2000

>> V = [0.004;0;8;12]
   V =
         0.0040
              0
         8.0000
        12.0000
```

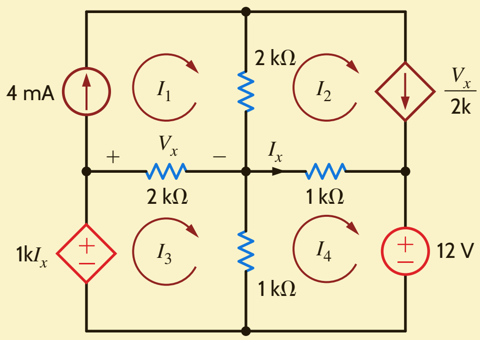


Figure 3.27
Circuit used in Example 3.19.

```
>> I = inv(R)*V
   I =
        0.0040
       -0.0060
       -0.0020
       -0.0100
```

or

$I_1 = 4.0$ mA
$I_2 = 6.0$ mA
$I_3 = -2.0$ mA
$I_4 = -1.0$ mA

EXAMPLE 3.20

At this point, we will again examine the circuit in Example 3.10 and analyze it using loop equations. Recall that because the network has two voltage sources, the nodal analysis was somewhat simplified. In a similar manner, the presence of the current sources should simplify a loop analysis.

Clearly, the network has four loops, and thus four linearly independent equations are required to determine the loop currents. The network is redrawn in Fig. 3.28 where the loop currents are specified. Note that we have drawn one current through each of the independent current sources. This choice of currents simplifies the analysis since two of the four equations are

$$I_1 = 2/\text{k}$$
$$I_3 = -2/\text{k}$$

The two remaining KVL equations for loop currents I_2 and I_4 are

$$-2V_x + 1\text{k}I_2 + (I_2 - I_3)1\text{k} = 0$$
$$(I_4 + I_3 - I_1)1\text{k} - 2V_x + 1\text{k}I_4 + 4 = 0$$

where

$$V_x = 1\text{k}(I_1 - I_3 - I_4)$$

Substituting the equations for I_1 and I_3 into the two KVL equations yields

$$2\text{k}I_2 + 2\text{k}I_4 = 6$$
$$4\text{k}I_4 = 8$$

Solving these equations for I_2 and I_4, we obtain

$$I_4 = 2 \text{ mA}$$
$$I_2 = 1 \text{ mA}$$

and thus

$$V_o = 1\text{V}$$

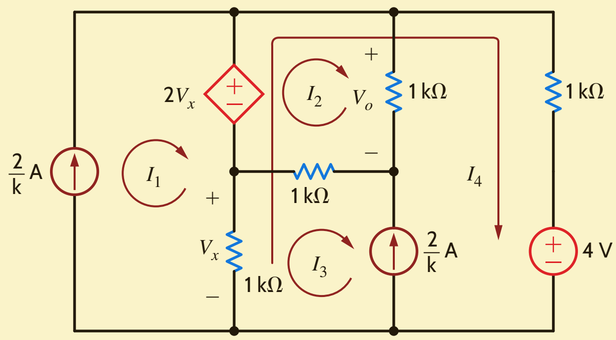


Figure 3.28 Circuit used in Example 3.20.

EXAMPLE 3.21

Let us once again consider Example 3.11. In this case, we will examine the network using loop analysis. Although there are four sources, two of which are dependent, only one of them is a current source. Thus, from the outset we expect that a loop analysis will be more difficult than a nodal analysis. Clearly, the circuit contains six loops. Thus, six linearly independent equations are needed to solve for all the unknown currents.

The network is redrawn in Fig. 3.29 where the loops are specified. The six KVL equations that describe the network are

$$\begin{aligned} 1\text{k}I_1 + 1\text{k}(I_1 - I_2) + 1\text{k}(I_1 - I_4) &= 0 \\ 1\text{k}(I_2 - I_1) - 6 + 1\text{k}(I_2 - I_5) &= 0 \\ I_3 &= 2I_x \\ -12 + 1\text{k}(I_4 - I_1) + 2V_x &= 0 \\ -2V_x + 1\text{k}(I_5 - I_2) + 1\text{k}(I_5 - I_o) &= 0 \\ 1\text{k}(I_o - I_5) + 1\text{k}(I_o - I_3) + 1\text{k}I_o &= 0 \end{aligned}$$

The control variables for the two dependent sources are

$$\begin{aligned} V_x &= -1\text{k}I_1 \\ I_x &= I_5 - I_o \end{aligned}$$

Substituting the control parameters into the six KVL equations yields

$$\begin{array}{cccccccc} 3I_1 & -I_2 & 0 & -I_4 & 0 & 0 & = & 0 \\ -I_1 & +2I_2 & 0 & 0 & -I_5 & 0 & = & 6/\text{k} \\ 0 & 0 & I_3 & 0 & -2I_5 & +2I_o & = & 0 \\ -3I_1 & 0 & 0 & +I_4 & 0 & 0 & = & 12/\text{k} \\ 2I_1 & -I_2 & 0 & 0 & +2I_5 & -I_o & = & 0 \\ 0 & 0 & 0 & 0 & -3I_5 & +5I_o & = & 0 \end{array}$$

In matrix form, the equations are

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -3 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -3 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.006 \\ 0 \\ 0.012 \\ 0 \\ 0 \end{bmatrix}$$

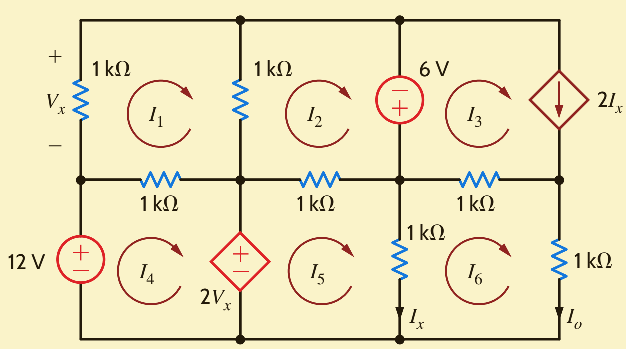


Figure 3.29
Circuit used in Example 3.21.

The MATLAB solution is then

```
>> R = [3 -1 0 -1 0 0;-1 2 0 0 -1 0;0 0 1 0 -2 2;-3 0 0 1
0 0;2 -1 0 0 2 -1;0 0 0 0 -3 5]
    R =
          3   -1    0   -1    0    0
         -1    2    0    0   -1    0
          0    0    1    0   -2    2
         -3    0    0    1    0    0
          2   -1    0    0    2   -1
          0    0    0    0   -3    5

>> V = [0;0.006;0;0.012;0;0]
    V =
              0
         0.0060
              0
         0.0120
              0
              0

>> I = inv(R)*V
    I =
          0.0500
         -0.0120
         -0.0640
          0.1620
         -0.0800
         -0.0480
```

or

$I_1 = 50.0$ mA
$I_2 = -12.0$ mA
$I_3 = -64.0$ mA
$I_4 = 162.0$ mA
$I_5 = -80.0$ mA
$I_6 = -48.0$ mA

As a final point, it is very important to examine the circuit carefully before selecting an analysis approach. One method could be much simpler than another, and a little time invested up front may save a lot of time in the long run. For an N-node circuit, $N - 1$ linearly independent equations must be formulated to solve for $N - 1$ node voltages. An N-loop circuit requires the formulation of N linearly independent equations. One consideration in the selection of a method should be the number of linearly independent equations that must be formulated. The same circuit was solved in Example 3.10 using nodal analysis and in Example 3.20 using loop analysis. The circuit in Fig. 3.16 has four unknown node voltages. As a result, four linearly independent equations are required. Because there are two voltage sources, two constraint equations are needed. It was pointed out in Example 3.20 that this same circuit has four loops, which requires four linearly independent equations. The two current sources produce two constraint equations.

The effort required to solve this circuit using either nodal or loop analysis is similar. However, this is not true for many circuits. Consider the circuit in **Fig. 3.30**. This circuit has eight loops. Selection of the loop currents such that only one loop current flows through the independent current source leaves us with seven unknown loop currents. Since this circuit has seven nodes, there are six node voltages, and we must formulate six linearly independent equations. By judicious selection of the bottom node as the reference node, four of the node

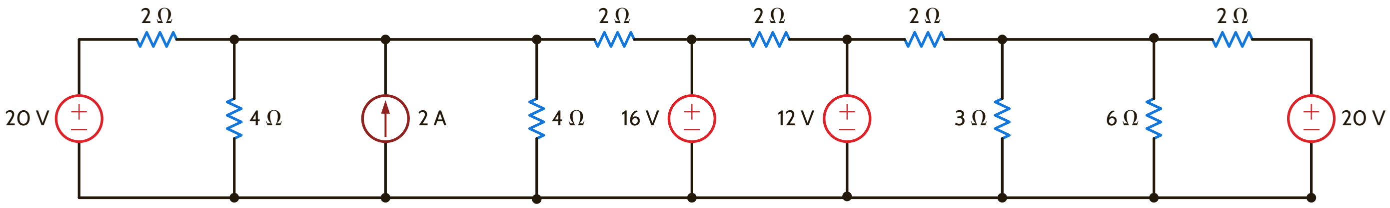


Figure 3.30

A circuit utilized in a discussion of the selection of an analysis technique.

voltages are known, leaving just two unknown node voltages—the node voltage across the current source and the node voltage across the 3-Ω and 6-Ω resistors. Applying KCL at these two nodes yields two equations that can be solved for the two unknown node voltages. Even with the use of a modern calculator or a computer program such as MATLAB, the solution of two simultaneous equations requires less effort than the solution of the seven simultaneous equations that the loop analysis would require.

PROBLEM-SOLVING STRATEGY

LOOP ANALYSIS

STEP 1. Determine the number of independent loops in the circuit. Assign a loop current to each independent loop. For an N-loop circuit, there are N-loop currents. As a result, N linearly independent equations must be written to solve for the loop currents.

If current sources are present in the circuit, either of two techniques can be employed. In the first case, one loop current is selected to pass through one of the current sources. The remaining loop currents are determined by open-circuiting the current sources in the circuit and using this modified circuit to select them. In the second case, a current is assigned to each mesh in the circuit.

STEP 2. Write a constraint equation for each current source—independent or dependent—in the circuit in terms of the assigned loop current using KCL. Each constraint equation represents one of the necessary linearly independent equations, and N_I current sources yield N_I linearly independent equations. For each dependent current source, express the controlling variable for that source in terms of the loop currents.

STEP 3. Use KVL to formulate the remaining $N - N_I$ linearly independent equations. Treat dependent voltage sources like independent voltage sources when formulating the KVL equations. For each dependent voltage source, express the controlling variable in terms of the loop currents.

LEARNING ASSESSMENTS

E3.19 Use mesh analysis to find V_o in the circuit in Fig. E3.19.

ANSWER:
$V_o = 12$ V.

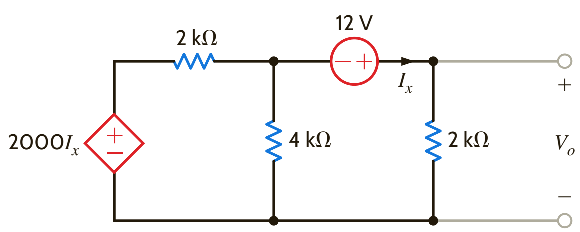


Figure E3.19