

TABLE 8.1 Phasor representation

TIME DOMAIN	FREQUENCY DOMAIN
$A \cos (\omega t \pm \theta)$	$A / \pm \theta$
$A \sin (\omega t \pm \theta)$	$A / \pm \theta - 90^\circ$

PROBLEM-SOLVING STRATEGY

PHASOR ANALYSIS

- STEP 1. Using phasors, transform a set of differential equations in the time domain into a set of algebraic equations in the frequency domain.
- STEP 2. Solve the algebraic equations for the unknown phasors.
- STEP 3. Transform the now-known phasors back to the time domain.

However, if a network contains only sine sources, there is no need to perform the 90° shift. We simply perform the normal phasor analysis, and then the *imaginary* part of the time-varying complex solution is the desired response. Simply put, cosine sources generate a cosine response, and sine sources generate a sine response.

LEARNING ASSESSMENTS

E8.3 Convert the following voltage functions to phasors.

$v_1(t) = 12 \cos (377t - 425^\circ) \text{ V}$
 $v_2(t) = 18 \sin (2513t + 4.2^\circ) \text{ V}$

ANSWER:
 $V_1 = 12 \angle -425^\circ \text{ V};$
 $V_2 = 18 \angle -85.8^\circ \text{ V}.$

E8.4 Convert the following phasors to the time domain if the frequency is 400 Hz.

$V_1 = 10 \angle 20^\circ \text{ V}$
 $V_2 = 12 \angle -60^\circ \text{ V}$

ANSWER:
 $v_1(t) = 10 \cos (800\pi t + 20^\circ) \text{ V};$
 $v_2(t) = 12 \cos (800\pi t - 60^\circ) \text{ V}.$

8.4
Phasor Relationships for Circuit Elements

As we proceed in our development of the techniques required to analyze circuits in the sinusoidal steady state, we are now in a position to establish the phasor relationships between voltage and current for the three passive elements *R*, *L*, and *C*.

In the case of a resistor as shown in Fig. 8.6a, the voltage–current relationship is known to be

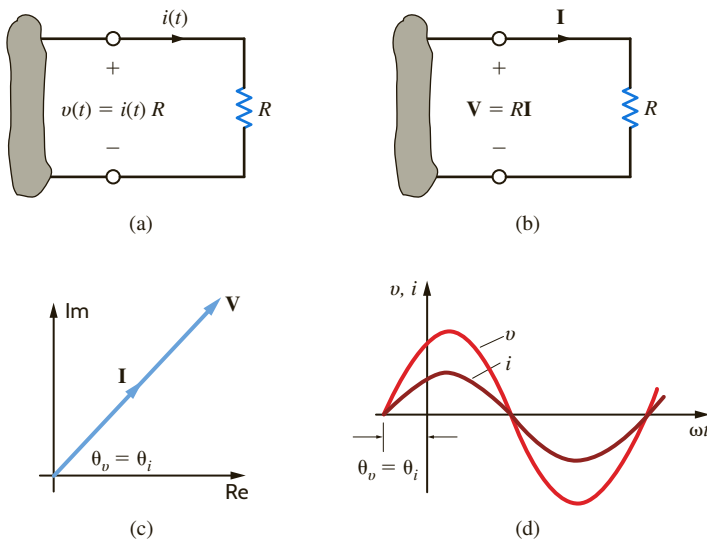
$v(t) = Ri(t)$ 8.20

Applying the complex voltage $V_M e^{j(\omega t + \theta_v)}$ results in the complex current $I_M e^{j(\omega t + \theta_i)}$, and therefore Eq. (8.20) becomes

$V_M e^{j(\omega t + \theta_v)} = RI_M e^{j(\omega t + \theta_i)}$

which reduces to

$V_M e^{j\theta_v} = RI_M e^{j\theta_i}$ 8.21

**Figure 8.6**

Voltage–current relationships for a resistor.

Equation (8.21) can be written in phasor form as

$$\mathbf{V} = R\mathbf{I}$$

8.22

where

$$\mathbf{V} = V_M e^{j\theta_v} = V_M \angle \theta_v \quad \text{and} \quad \mathbf{I} = I_M e^{j\theta_i} = I_M \angle \theta_i$$

This relationship is illustrated in **Fig. 8.6b**. From Eq. (8.21) we see that $\theta_v = \theta_i$ and thus the current and voltage for this circuit are *in phase*.

Historically, complex numbers have been represented as points on a graph in which the x -axis represents the real axis and the y -axis the imaginary axis. The line segment connecting the origin with the point provides a convenient representation of the magnitude and angle when the complex number is written in a polar form. A review of the Appendix will indicate how these complex numbers or line segments can be added, subtracted, and so on. Since phasors are complex numbers, it is convenient to represent the phasor voltage and current graphically as line segments. A plot of the line segments representing the phasors is called a *phasor diagram*. This pictorial representation of phasors provides immediate information on the relative magnitude of one phasor with another, the angle between two phasors, and the relative position of one phasor with respect to another (i.e., leading or lagging). A phasor diagram and the sinusoidal waveforms for the resistor are shown in **Figs. 8.6c** and **d**, respectively. A phasor diagram will be drawn for each of the other circuit elements in the remainder of this section.



Current and voltage are in phase.

If the voltage $v(t) = 24 \cos(377t + 75^\circ)$ V is applied to a $6\text{-}\Omega$ resistor as shown in Fig. 8.6a, we wish to determine the resultant current.

Since the phasor voltage is

$$\mathbf{V} = 24 \angle 75^\circ \text{ V}$$

the phasor current from Eq. (8.22) is

$$\mathbf{I} = \frac{24 \angle 75^\circ}{6} = 4 \angle 75^\circ \text{ A}$$

which in the time domain is

$$i(t) = 4 \cos(377t + 75^\circ) \text{ A}$$

EXAMPLE 8.6

SOLUTION

LEARNING ASSESSMENT

E8.5 The current in a $4\text{-}\Omega$ resistor is known to be $\mathbf{I} = 12\angle 60^\circ$ A. Express the voltage across the resistor as a time function if the frequency of the current is 4 kHz.

ANSWER:

$$v_1(t) = 48 \cos(8000\pi t + 60^\circ) \text{ V.}$$

The voltage–current relationship for an inductor, as shown in Fig. 8.7a, is

$$v(t) = L \frac{di(t)}{dt} \quad 8.23$$

Substituting the complex voltage and current into this equation yields

$$V_M e^{j(\omega t + \theta_v)} = L \frac{d}{dt} I_M e^{j(\omega t + \theta_i)}$$

which reduces to

$$V_M e^{j\theta_v} = j\omega L I_M e^{j\theta_i} \quad 8.24$$

Equation (8.24) in phasor notation is

$$\mathbf{V} = j\omega L \mathbf{I} \quad 8.25$$

Note that the differential equation in the time domain (8.23) has been converted to an algebraic equation with complex coefficients in the frequency domain. This relationship is shown in Fig. 8.7b. Since the imaginary operator $j = 1\angle 90^\circ = 1/90^\circ = \sqrt{-1}$, Eq. (8.24) can be written as

$$V_M e^{j\theta_v} = \omega L I_M e^{j(\theta_i + 90^\circ)} \quad 8.26$$

Therefore, the voltage and current are 90° out of phase, and in particular the voltage leads the current by 90° or the current lags the voltage by 90° . The phasor diagram and the sinusoidal waveforms for the inductor circuit are shown in Figs. 8.7c and d, respectively.

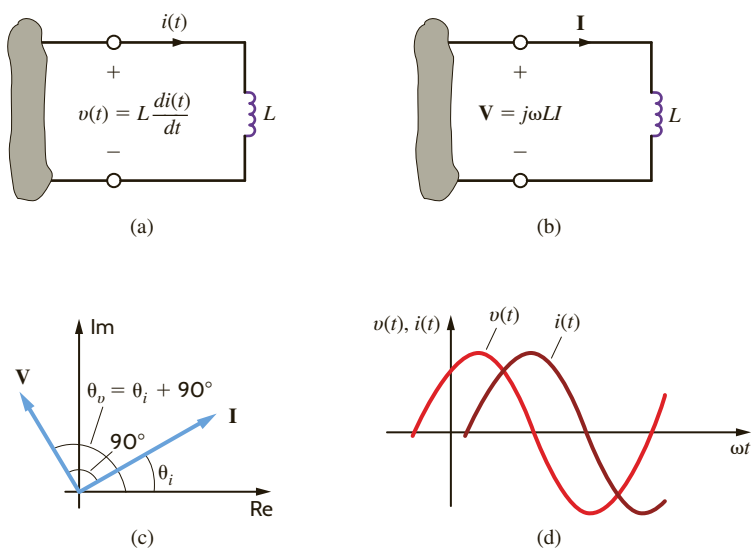
HINT

The derivative process yields a frequency-dependent function.

HINT

The voltage leads the current or the current lags the voltage.

Figure 8.7
Voltage–current relationships for an inductor.



The voltage $v(t) = 12 \cos(377t + 20^\circ)$ V is applied to a 20-mH inductor, as shown in Fig. 8.7a. Find the resultant current.

The phasor current is

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}}{j\omega L} = \frac{12 \angle 20^\circ}{\omega L \angle 90^\circ} \\ &= \frac{12 \angle 20^\circ}{(377)(20 \times 10^{-3}) \angle 90^\circ} \\ &= 1.59 \angle -70^\circ \text{ A}\end{aligned}$$

or

$$i(t) = 1.59 \cos(377t - 70^\circ) \text{ A}$$

EXAMPLE 8.7

SOLUTION



Applying $\mathbf{V} = j\omega L \mathbf{I}$

$$\frac{x_1 \angle \theta_1}{x_2 \angle \theta_2} = \frac{x_1}{x_2} \angle \theta_1 - \theta_2$$

LEARNING ASSESSMENT

E8.6 The current in a 0.05-H inductor is $\mathbf{I} = 4 \angle -30^\circ$ A. If the frequency of the current is 60 Hz, determine the voltage across the inductor.

ANSWER:

$$v_L(t) = 75.4 \cos(377t + 60^\circ) \text{ V.}$$

The voltage–current relationship for our last passive element, the capacitor, as shown in Fig. 8.8a, is

$$i(t) = C \frac{dv(t)}{dt} \quad 8.27$$

Once again employing the complex voltage and current, we obtain

$$I_M e^{j(\omega t + \theta_i)} = C \frac{d}{dt} V_M e^{j(\omega t + \theta_v)}$$

which reduces to

$$I_M e^{j\theta_i} = j\omega C V_M e^{j\theta_v} \quad 8.28$$

In phasor notation this equation becomes

$$\mathbf{I} = j\omega C \mathbf{V} \quad 8.29$$

Eq. (8.27), a differential equation in the time domain, has been transformed into Eq. (8.29), an algebraic equation with complex coefficients in the frequency domain. The phasor relationship is shown in Fig. 8.8b. Substituting $j = 1e^{j90^\circ}$ into Eq. (8.28) yields

$$I_M e^{j\theta_i} = \omega C V_M e^{j(\theta_v + 90^\circ)} \quad 8.30$$

Note that the voltage and current are 90° out of phase. Eq. (8.30) states that the current leads the voltage by 90° or the voltage lags the current by 90° . The phasor diagram and the sinusoidal waveforms for the capacitor circuit are shown in Figs. 8.8c and d, respectively.



The current leads the voltage or the voltage lags the current.