E5.2 Find V_o in Fig. E5.2 using linearity and the assumption that $V_o = 1$ V.

ANSWER:
$V_o = 5/3$ V.

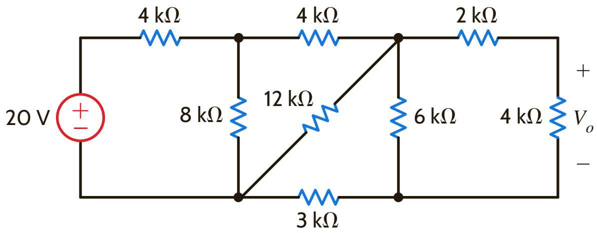


Figure E5.2

5.2 Superposition

EXAMPLE 5.2

To provide motivation for this subject, let us examine the simple circuit of Fig. 5.2a, in which two sources contribute to the current in the network. The actual values of the sources are left unspecified so that we can examine the concept of superposition.

SOLUTION

The mesh equations for this network are

$$6ki_1(t) - 3ki_2(t) = v_1(t)$$
$$-3ki_1(t) + 9ki_2(t) = -v_2(t)$$

Solving these equations for $i_1(t)$ yields

$$i_1(t) = \frac{v_1(t)}{5k} - \frac{v_2(t)}{15k}$$

In other words, the current $i_1(t)$ has a component due to $v_1(t)$ and a component due to $v_2(t)$. In view of the fact that $i_1(t)$ has two components, one due to each independent source, it would be interesting to examine what each source acting alone would contribute to $i_1(t)$. For $v_1(t)$ to act alone, $v_2(t)$ must be zero. As we pointed out in Chapter 2, $v_2(t) = 0$ means that the source $v_2(t)$ is replaced with a short circuit. Therefore, to determine the value of $i_1(t)$ due to $v_1(t)$ only, we employ the circuit in Fig. 5.2b and refer to this value of $i_1(t)$ as $i_1'(t)$.

$$i_1'(t) = \frac{v_1(t)}{3k + \dfrac{(3k)(6k)}{3k + 6k}} = \frac{v_1(t)}{5k}$$

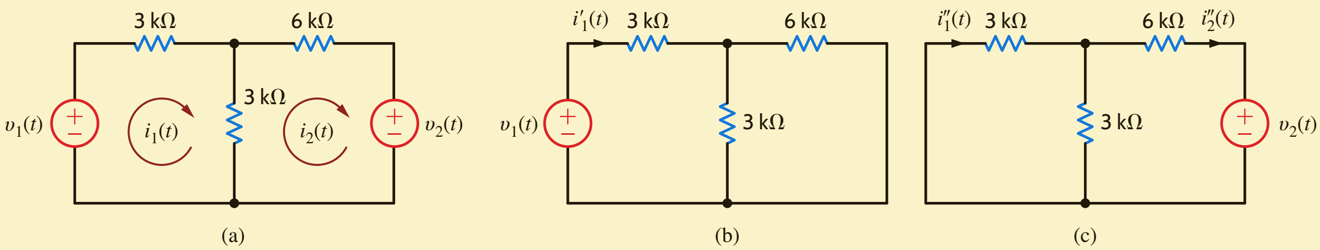
Let us now determine the value of $i_1(t)$ due to $v_2(t)$ acting alone and refer to this value as $i_1''(t)$. Using the network in Fig. 5.2c,

$$i_2''(t) = -\frac{v_2(t)}{6k + \dfrac{(3k)(3k)}{3k + 3k}} = \frac{-2v_2(t)}{15k}$$

Then, using current division, we obtain

$$i_1''(t) = \frac{-2v_2(t)}{15k}\left(\frac{3k}{3k + 3k}\right) = \frac{-v_2(t)}{15k}$$

Figure 5.2
Circuits used to illustrate superposition.



Now, if we add the values of $i_1'(t)$ and $i_1''(t)$, we obtain the value computed directly; that is,

$$i_1(t) = i_1'(t) + i_1''(t) = \frac{v_1(t)}{5\text{k}} - \frac{v_2(t)}{15\text{k}}$$

Note that we have *superposed* the value of $i_1'(t)$ on $i_1''(t)$, or vice versa, to determine the unknown current.

What we have demonstrated in Example 5.2 is true in general for linear circuits and is a direct result of the property of linearity. *The principle of superposition*, which provides us with this ability to reduce a complicated problem to several easier problems—each containing only a single independent source—states that

> In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

When determining the contribution due to an independent source, any remaining voltage sources are made zero by replacing them with short circuits, and any remaining current sources are made zero by replacing them with open circuits.

Although superposition can be used in linear networks containing dependent sources, it is not useful in this case since the dependent source is never made zero.

As the previous example indicates, superposition provides some insight in determining the contribution of each source to the variable under investigation.

We will now demonstrate superposition with two examples and then provide a problem-solving strategy for the use of this technique. For purposes of comparison, we will also solve the networks using both node and loop analyses. Furthermore, we will employ these same networks when demonstrating subsequent techniques, if applicable.

EXAMPLE 5.3

Let us use superposition to find V_o in the circuit in Fig. 5.3a.

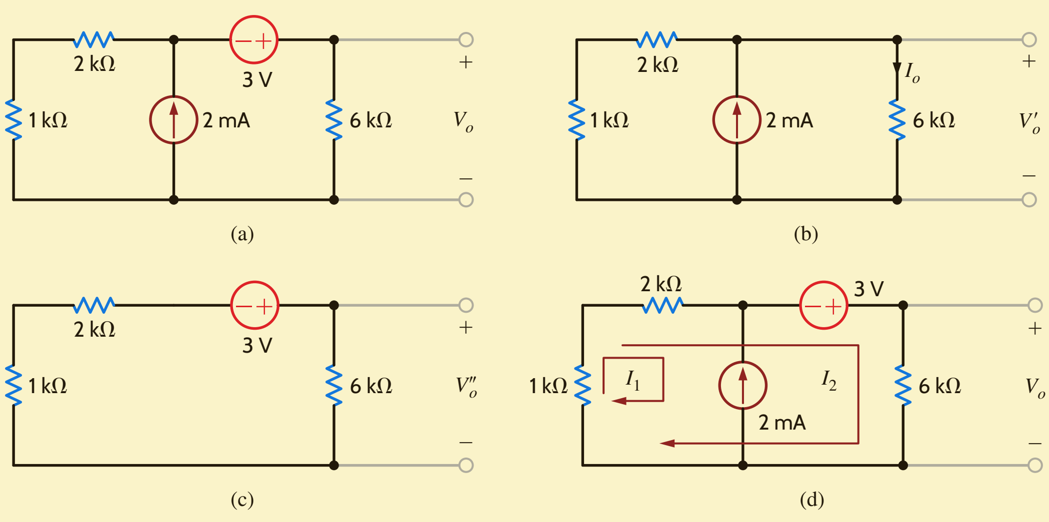


Figure 5.3 Circuits used in Example 5.3.

SOLUTION

The contribution of the 2-mA source to the output voltage is found from the network in Fig. 5.3b, using current division

$$I_o = (2 \times 10^{-3})\left(\frac{1\text{k} + 2\text{k}}{1\text{k} + 2\text{k} + 6\text{k}}\right) = \frac{2}{3}\text{ mA}$$

and

$$V_o' = I_o(6\text{k}) = 4\text{ V}$$

The contribution of the 3-V source to the output voltage is found from the circuit in Fig. 5.3c. Using voltage division,

$$V_o'' = 3\left(\frac{6\text{k}}{1\text{k} + 2\text{k} + 6\text{k}}\right)$$
$$= 2 \text{ V}$$

Therefore,

$$V_o = V_o' + V_o'' = 6 \text{ V}$$

Although we used two separate circuits to solve the problem, both were very simple.

If we use nodal analysis and Fig. 5.3a to find V_o and recognize that the 3-V source and its connecting nodes form a supernode, V_o can be found from the node equation

$$\frac{V_o - 3}{1\text{k} + 2\text{k}} - 2 \times 10^{-3} + \frac{V_o}{6\text{k}} = 0$$

which yields $V_o = 6$ V. In addition, loop analysis applied as shown in Fig. 5.3d produces the equations

$$I_1 = -2 \times 10^{-3}$$

and

$$3\text{k}(I_1 + I_2) - 3 + 6\text{k}I_2 = 0$$

which yield $I_2 = 1$ mA and hence $V_o = 6$ V.

EXAMPLE 5.4

Consider now the network in Fig. 5.4a. Let us use superposition to find V_o.

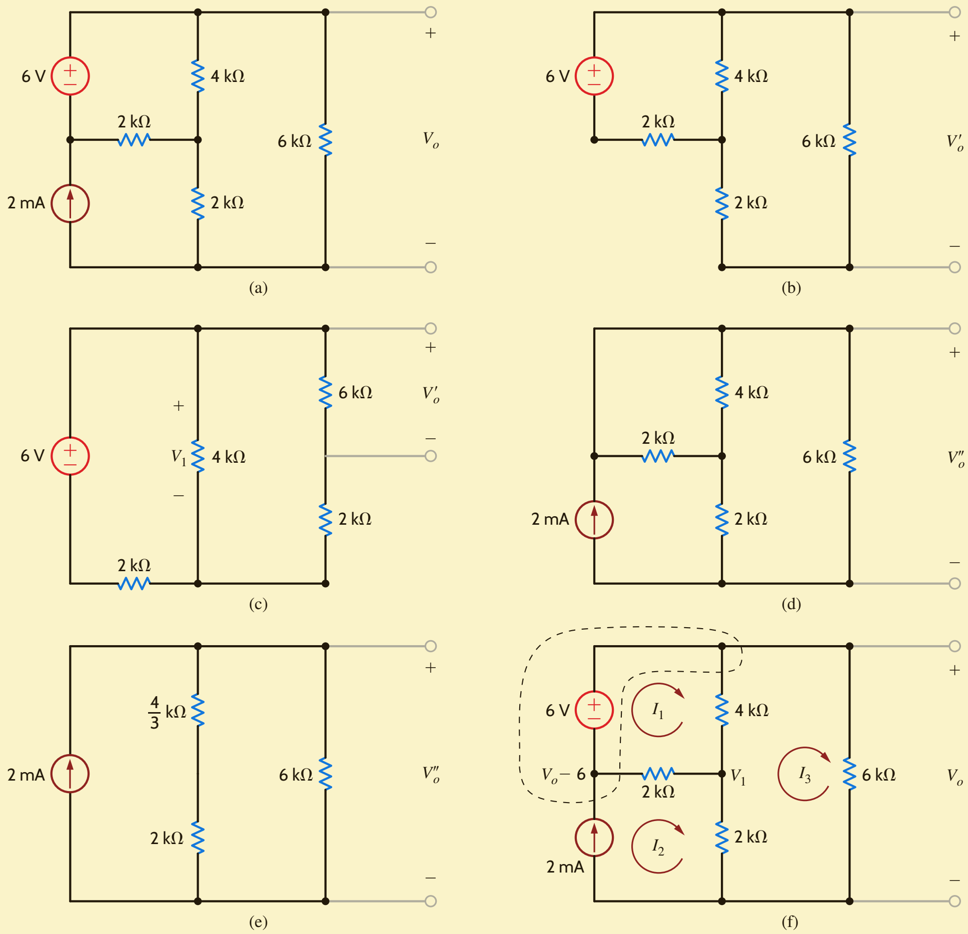


Figure 5.4 Circuits used in Example 5.4.

SOLUTION

The contribution of the 6-V source to V_0 is found from the network in Fig. 5.4b, which is redrawn in Fig. 5.4c. The 2 kΩ + 6 kΩ = 8-kΩ resistor and 4-kΩ resistor are in parallel, and their combination is an 8/3-kΩ resistor. Then, using voltage division,

$$V_1 = 6\left(\frac{\frac{8}{3}\text{k}}{\frac{8}{3}\text{k} + 2\text{k}}\right) = \frac{24}{7}\text{ V}$$

Applying voltage division again,

$$V_o' = V_1\left(\frac{6\text{k}}{6\text{k} + 2\text{k}}\right) = \frac{18}{7}\text{ V}$$

The contribution of the 2-mA source is found from Fig. 5.4d, which is redrawn in Fig. 5.4e. V_o'' is simply equal to the product of the current source and the parallel combination of the resistors; that is,

$$V_o'' = (2 \times 10^{-3})\left(\frac{10}{3}\text{k}//6\text{k}\right) = \frac{30}{7}\text{ V}$$

Then

$$V_o = V_o' + V_o'' = \frac{48}{7}\text{ V}$$

A nodal analysis of the network can be performed using Fig. 5.4f. The equation for the supernode is

$$-2 \times 10^{-3} + \frac{(V_o - 6) - V_1}{2\text{k}} + \frac{V_o - V_1}{4\text{k}} + \frac{V_o}{6\text{k}} = 0$$

The equation for the node labeled V_1 is

$$\frac{V_1 - V_o}{4\text{k}} + \frac{V_1 - (V_o - 6)}{2\text{k}} + \frac{V_1}{2\text{k}} = 0$$

Solving these two equations, which already contain the constraint equation for the supernode, yields $V_o = 48/7$ V.

Once again, referring to the network in Fig. 5.4f, the mesh equations for the network are

$$-6 + 4\text{k}(I_1 - I_3) + 2\text{k}(I_1 - I_2) = 0$$
$$I_2 = 2 \times 10^{-3}$$
$$2\text{k}(I_3 - I_2) + 4\text{k}(I_3 - I_1) + 6\text{k}I_3 = 0$$

Solving these equations, we obtain $I_3 = 8/7$ mA and, hence, $V_o = 48/7$ V.

PROBLEM-SOLVING STRATEGY

APPLYING SUPERPOSITION

STEP 1. In a network containing multiple independent sources, each source can be applied independently with the remaining sources turned off.

STEP 2. To turn off a voltage source, replace it with a short circuit, and to turn off a current source, replace it with an open circuit.

STEP 3. When the individual sources are applied to the circuit, all the circuit laws and techniques we have learned, or will soon learn, can be applied to obtain a solution.

STEP 4. The results obtained by applying each source independently are then added together algebraically to obtain a solution.

Superposition can be applied to a circuit with any number of dependent and independent sources. In fact, superposition can be applied to such a network in a variety of ways. For example, a circuit with three independent sources can be solved using each source acting alone, as we have just demonstrated, or we could use two at a time and sum the result with that obtained from the third acting alone. In addition, the independent sources do not have to assume their actual value or zero. However, it is mandatory that the sum of the different values chosen add to the total value of the source.

Superposition is a fundamental property of linear equations and, therefore, can be applied to any effect that is linearly related to its cause. In this regard it is important to point out that although superposition applies to the current and voltage in a linear circuit, it cannot be used to determine power because power is a nonlinear function.

LEARNING ASSESSMENTS

E5.3 Compute V_o in the circuit in Fig. E5.3 using superposition.

ANSWER: $V_o = \frac{4}{3}$ V.

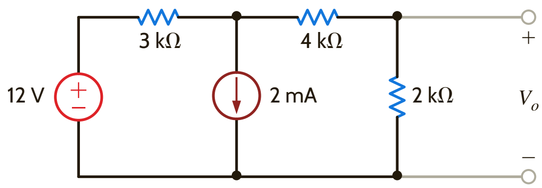


Figure E5.3

E5.4 Find V_0 in Fig. E5.4 using superposition.

ANSWER: $V_0 = 5.6$ V.

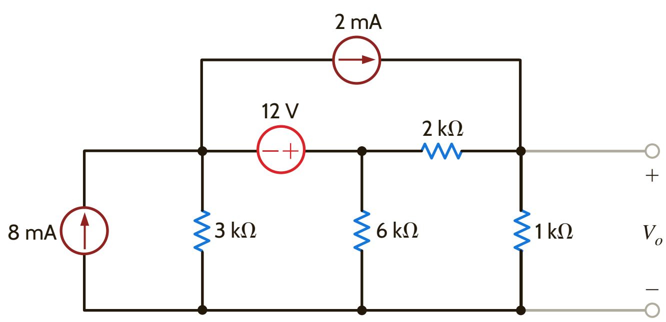


Figure E5.4

E5.5 Find I_0 in Fig. E5.5 using superposition.

ANSWER: $I_0 = -2/3$ mA.

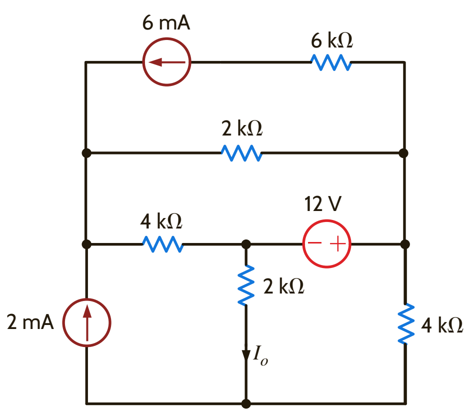


Figure E5.5