LEARNING ASSESSMENT

E2.13 Find the power absorbed by the 6-kΩ resistor in the network in Fig. E2.13.

ANSWER:
$P = 2.67$ mW.

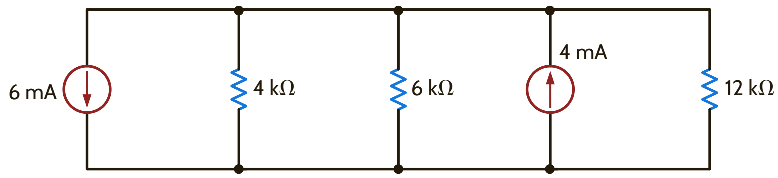


Figure E2.13

2.5 Series and Parallel Resistor Combinations

We have shown in our earlier developments that the equivalent resistance of N resistors in series is

$$R_S = R_1 + R_2 + \cdots + R_N \tag{2.25}$$

and the equivalent resistance of N resistors in parallel is found from

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \tag{2.26}$$

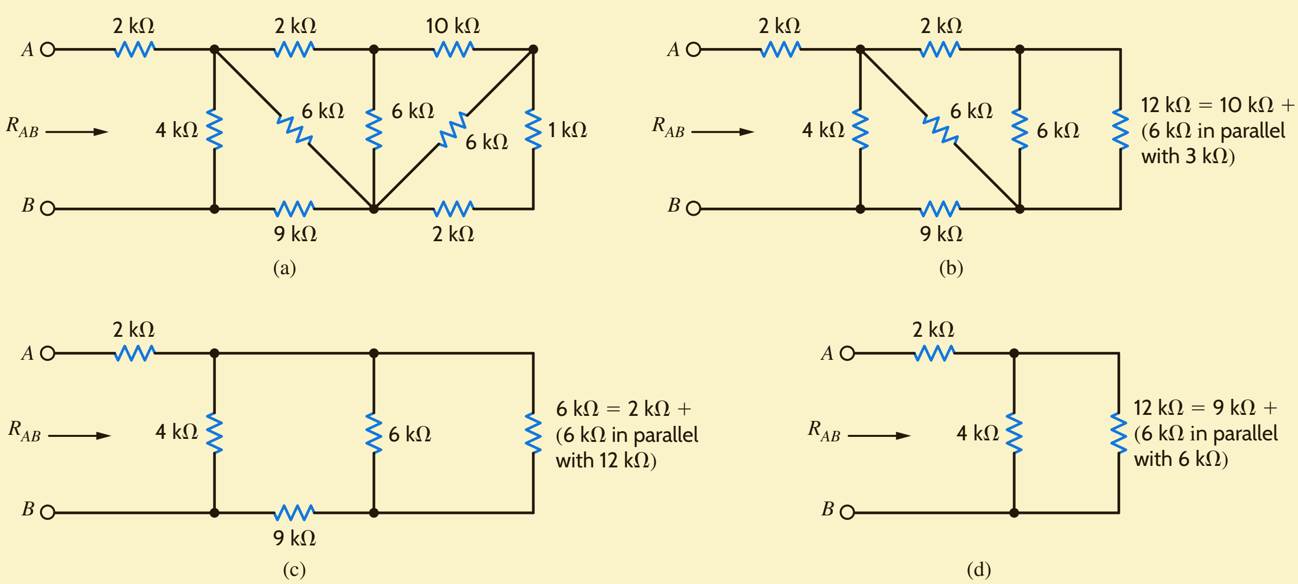
Let us now examine some combinations of these two cases.

EXAMPLE 2.20

We wish to determine the resistance at terminals A-B in the network in Fig. 2.28a.

SOLUTION

Starting at the opposite end of the network from the terminals and combining resistors as shown in the sequence of circuits in Fig. 2.28, we find that the equivalent resistance at the terminals is 5 kΩ.



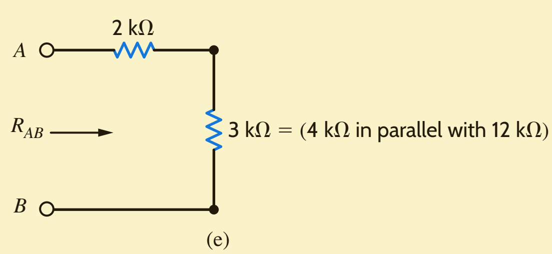


Figure 2.28
Simplification of a resistance network.

LEARNING ASSESSMENT

E2.14 Find the equivalent resistance at the terminals *A-B* in the network in Fig. E2.14.

ANSWER:
$R_{AB} = 22\ \text{k}\Omega$.

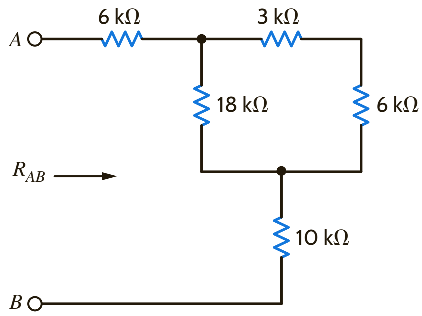


Figure E2.14

PROBLEM-SOLVING STRATEGY

SIMPLIFYING RESISTOR COMBINATIONS

When trying to determine the equivalent resistance at a pair of terminals of a network composed of an interconnection of numerous resistors, it is recommended that the analysis begin at the end of the network opposite the terminals. Two or more resistors are combined to form a single resistor, thus simplifying the network by reducing the number of components as the analysis continues in a steady progression toward the terminals. The simplification involves the following:

STEP 1. ***Resistors in series.*** Resistors R_1 and R_2 are in series if they are connected end to end with one common node and carry exactly the same current. They can then be combined into a single resistor R_S, where $R_S = R_1 + R_2$.

STEP 2. ***Resistors in parallel.*** Resistors R_1 and R_2 are in parallel if they are connected to the same two nodes and have exactly the same voltage across their terminals. They can then be combined into a single resistor R_p, where $R_p = R_1R_2/(R_1 + R_2)$.

These two combinations are used repeatedly, as needed, to reduce the network to a single resistor at the pair of terminals.

LEARNING ASSESSMENTS

E2.15 Find the equivalent resistance at the terminals *A-B* in the circuit in Fig. E2.15.

ANSWER:
$R_{AB} = 3\ \text{k}\Omega$.

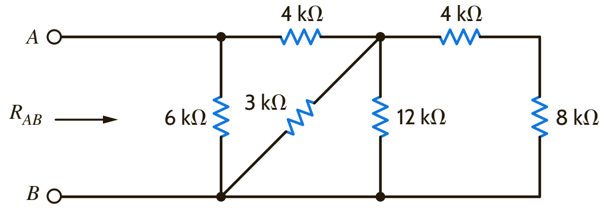


Figure E2.15

E2.16 Find R_{AB} in Fig. E2.16.

ANSWER: $R_{AB} = 12\ \text{k}\Omega$.

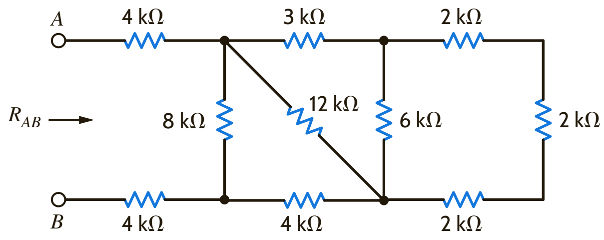


Figure E2.16

EXAMPLE 2.21

A standard dc current-limiting power supply shown in Fig. 2.29a provides 0–18 V at 3 A to a load. The voltage drop, V_R, across a resistor, R, is used as a current-sensing device, fed back to the power supply and used to limit the current I. That is, if the load is adjusted so that the current tries to exceed 3 A, the power supply will act to limit the current to that value. The feedback voltage, V_R, should typically not exceed 600 mV.

If we have a box of standard 0.1-Ω, 5-W resistors, let us determine the configuration of these resistors that will provide $V_R = 600$ mV when the current is 3 A.

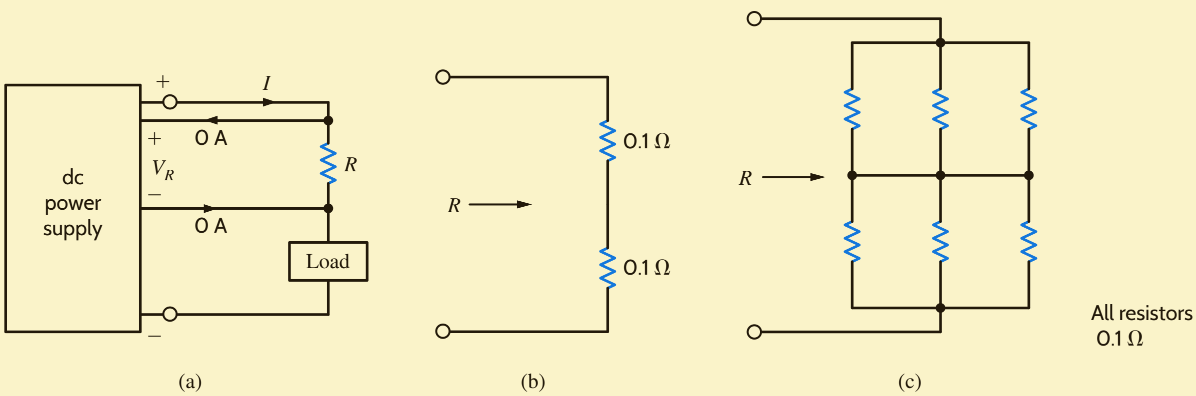


Figure 2.29
Circuits used in Example 2.21.

SOLUTION Using Ohm's law, the value of R should be

$$R = \frac{V_R}{I} = \frac{0.6}{3} = 0.2\ \Omega$$

Therefore, two 0.1-Ω resistors connected in series, as shown in Fig. 2.29b, will provide the proper feedback voltage. Suppose, however, that the power supply current is to be limited to 9 A. The resistance required in this case to produce $V_R = 600$ mV is

$$R = \frac{0.6}{9} = 0.0667\ \Omega$$

We must now determine how to interconnect the 0.1-Ω resistor to obtain $R = 0.0667\ \Omega$. Since the desired resistance is less than the components available (i.e., 0.1-Ω), we must connect the resistors in some type of parallel configuration. Since all the resistors are of equal value, note that three of them connected in parallel would provide a resistance of one-third their value, or 0.0333 Ω. Then two such combinations connected in series, as shown in Fig. 2.29c, would produce the proper resistance.

Finally, we must check to ensure that the configurations in Figs. 2.29b and c have not exceeded the power rating of the resistors. In the first case, the current $I = 3$ A is present in each of the two series resistors. Therefore, the power absorbed in each resistor is

$$\begin{aligned} P &= I^2R \\ &= (3)^2(0.1) \\ &= 0.9 \text{ W} \end{aligned}$$

which is well within the 5-W rating of the resistors.

In the second case, the current $I = 9$ A. The resistor configuration for R in this case is a series combination of two sets of three parallel resistors of equal value. Using current division, we know that the current I will split equally among the three parallel paths and, hence, the current in each resistor will be 3 A. Therefore, once again, the power absorbed by each resistor is within its power rating.

EXAMPLE 2.22

We wish to find all the currents and voltages labeled in the ladder network shown in Fig. 2.30a.

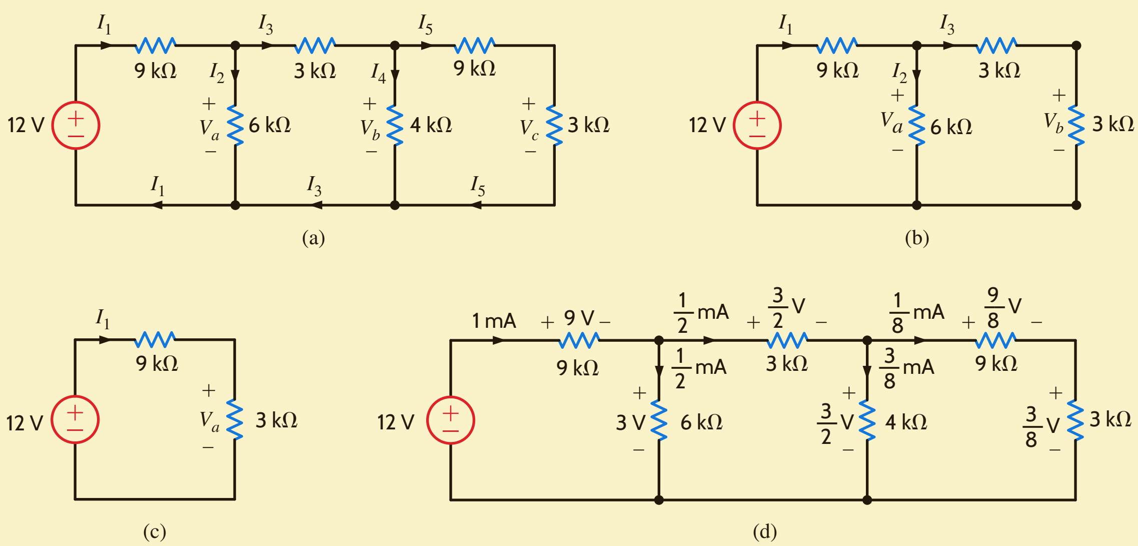


Figure 2.30 Analysis of a ladder network.

SOLUTION

To begin our analysis of the network, we start at the right end of the circuit and combine the resistors to determine the total resistance seen by the 12-V source. This will allow us to calculate the current I_1. Then employing KVL, KCL, Ohm's law, and/or voltage and current division, we will be able to calculate all currents and voltages in the network.

At the right end of the circuit, the 9-kΩ and 3-kΩ resistors are in series and, thus, can be combined into one equivalent 12-kΩ resistor. This resistor is in parallel with the 4-kΩ resistor, and their combination yields an equivalent 3-kΩ resistor, shown at the right edge of the circuit in Fig. 2.30b. In Fig. 2.30b the two 3-kΩ resistors are in series, and their combination is in parallel with the 6-kΩ resistor. Combining all three resistances yields the circuit shown in Fig. 2.30c.

Applying Kirchhoff's voltage law to the circuit in Fig. 2.30c yields

$$I_1(9\text{k} + 3\text{k}) = 12$$
$$I_1 = 1 \text{ mA}$$

V_a can be calculated from Ohm's law as

$$V_a = I_1(3\text{k})$$
$$= 3 \text{ V}$$

or, using Kirchhoff's voltage law,

$$V_a = 12 - 9\text{k}I_1$$
$$= 12 - 9$$
$$= 3 \text{ V}$$

Knowing I_1 and V_a, we can now determine all currents and voltages in Fig. 2.30b. Since $V_a = 3$ V, the current I_2 can be found using Ohm's law as

$$I_2 = \frac{3}{6\text{k}}$$
$$= \frac{1}{2} \text{ mA}$$

Then, using Kirchhoff's current law, we have

$$I_1 = I_2 + I_3$$
$$1 \times 10^{-3} = \frac{1}{2} \times 10^{-3} + I_3$$
$$I_3 = \frac{1}{2} \text{ mA}$$

Note that the I_3 could also be calculated using Ohm's law:

$$V_a = (3\text{k} + 3\text{k})I_3$$
$$I_3 = \frac{3}{6\text{k}}$$
$$= \frac{1}{2} \text{ mA}$$

Applying Kirchhoff's voltage law to the right-hand loop in Fig. 2.30b yields

$$V_a - V_b = 3\text{k}I_3$$
$$3 - V_b = \frac{3}{2}$$
$$V_b = \frac{3}{2} \text{ V}$$

or, since V_b is equal to the voltage drop across the 3-kΩ resistor, we could use Ohm's law as

$$V_b = 3\text{k}I_3$$
$$= \frac{3}{2} \text{V}$$

We are now in a position to calculate the final unknown currents and voltages in Fig. 2.30a. Knowing V_b, we can calculate I_4 using Ohm's law as

$$V_b = 4kI_4$$
$$I_4 = \frac{\frac{3}{2}}{4\text{k}}$$
$$= \frac{3}{8} \text{mA}$$

Then, from Kirchhoff's current law, we have

$$I_3 = I_4 + I_5$$
$$\frac{1}{2} \times 10^{-3} = \frac{3}{8} \times 10^{-3} + I_5$$
$$I_5 = \frac{1}{8}\text{ mA}$$

We could also have calculated I_5 using the current-division rule. For example,

$$I_5 = \frac{4\text{k}}{4\text{k} + (9\text{k} + 3\text{k})} I_3$$
$$= \frac{1}{8}\text{ mA}$$

Finally, V_c can be computed as

$$V_c = I_5(3\text{k})$$
$$= \frac{3}{8}\text{ V}$$

V_c can also be found using voltage division (i.e., the voltage V_b will be divided between the 9-kΩ and 3-kΩ resistors). Therefore,

$$V_c = \left[\frac{3\text{k}}{3\text{k} + 9\text{k}}\right] V_b$$
$$= \frac{3}{8}\text{ V}$$

Note that Kirchhoff's current law is satisfied at every node and Kirchhoff's voltage law is satisfied around every loop, as shown in **Fig. 2.30d**.

The following example is, in essence, the reverse of the previous example in that we are given the current in some branch in the network and are asked to find the value of the input source.

EXAMPLE 2.23

Given the circuit in **Fig. 2.31** and $I_4 = 1/2$ mA, let us find the source voltage V_o.

SOLUTION

If $I_4 = 1/2$ mA, then from Ohm's law, $V_b = 3$ V. V_b can now be used to calculate $I_3 = 1$ mA. Kirchhoff's current law applied at node y yields

$$I_2 = I_3 + I_4$$
$$= 1.5\text{ mA}$$

Then, from Ohm's law, we have

$$V_a = (1.5 \times 10^{-3})(2\text{k})$$
$$= 3\text{ V}$$

Since $V_a + V_b$ is now known, I_5 can be obtained:

$$I_5 = \frac{V_a + V_b}{3\text{k} + 1\text{k}}$$
$$= 1.5\text{ mA}$$

Applying Kirchhoff's current law at node x yields

$$I_1 = I_2 + I_5$$
$$= 3\text{ mA}$$

Now KVL applied to any closed path containing V_o will yield the value of this input source. For example, if the path is the outer loop, KVL yields

$$-V_o + 6kI_1 + 3kI_5 + 1kI_5 + 4kI_1 = 0$$

Since $I_1 = 3$ mA and $I_5 = 1.5$ mA,

$$V_o = 36 \text{ V}$$

If we had selected the path containing the source and the points x, y, and z, we would obtain

$$-V_o + 6kI_1 + V_a + V_b + 4kI_1 = 0$$

Once again, this equation yields

$$V_o = 36 \text{ V}$$

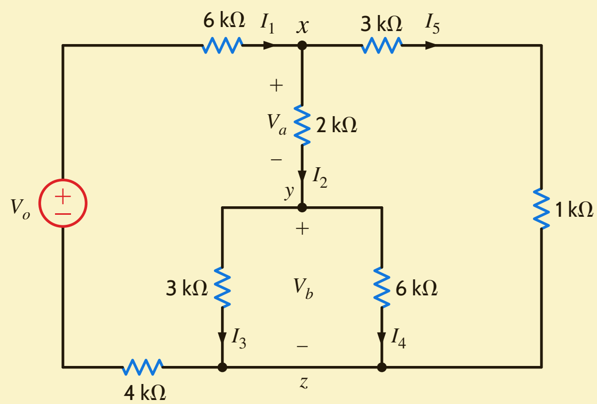


Figure 2.31
Example circuit for analysis.

PROBLEM-SOLVING STRATEGY

ANALYZING CIRCUITS CONTAINING A SINGLE SOURCE AND A SERIES-PARALLEL INTERCONNECTION OF RESISTORS

STEP 1. Systematically reduce the resistive network so that the resistance seen by the source is represented by a single resistor.

STEP 2. Determine the source current for a voltage source or the source voltage if a current source is present.

STEP 3. Expand the network, retracing the simplification steps, and apply Ohm's law, KVL, KCL, voltage division, and current division to determine all currents and voltages in the network.

LEARNING ASSESSMENTS

E2.17 Find V_o in the network in Fig. E2.17.

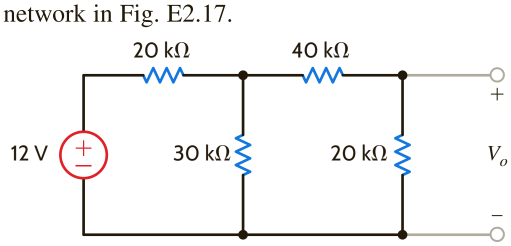


Figure E2.17

ANSWER:
$V_o = 2$ V.

E2.18 Find V_S in the circuit in Fig. E2.18.

ANSWER:
$V_S = 9$ V.

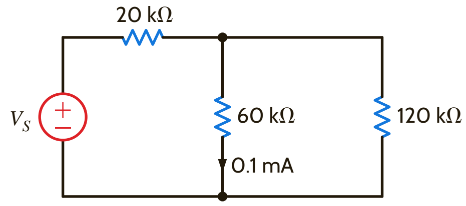


Figure E2.18

E2.19 Find I_S in the circuit in Fig. E2.19.

ANSWER:
$I_S = 0.3$ mA.

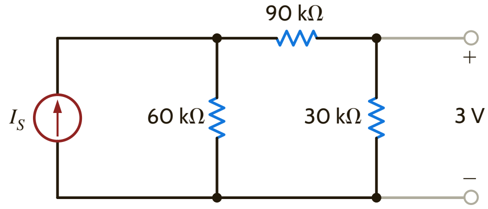


Figure E2.19

E2.20 Find V_1 in Fig. E2.20.

ANSWER:
$V_1 = 12$ V.

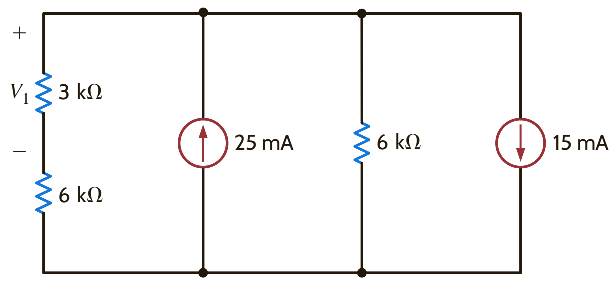


Figure E2.20

E2.21 Find I_0 in Fig. E2.21.

ANSWER:
$I_0 = -4$ mA.

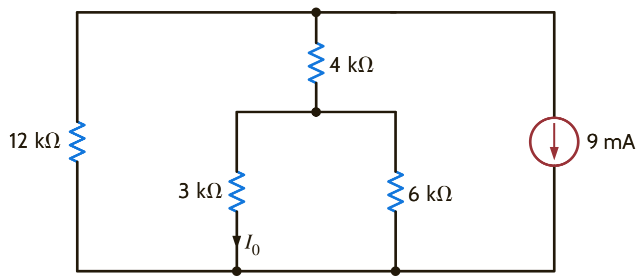


Figure E2.21

E2.22 Find V_o, V_1, and V_2 in Fig. E2.22.

ANSWER:
$V_o = 3.33$ V,
$V_1 = -4$ V, and $V_2 = 4$ V.

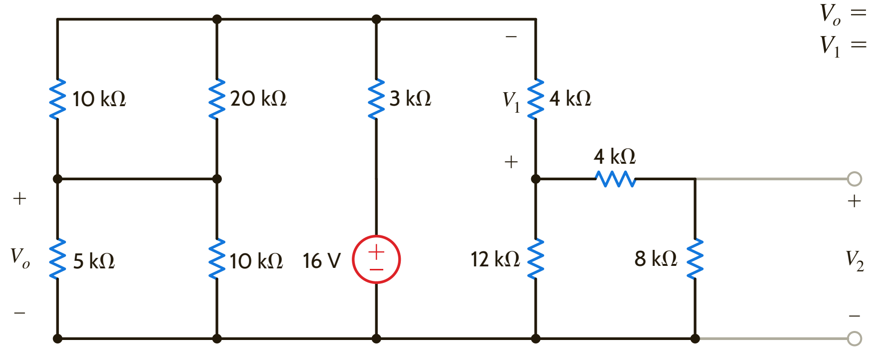


Figure E2.22

E2.23 Find V_o and V_1 in Fig. E2.23.

ANSWER:
$V_o = -60$ V;
$V_1 = 10$ V.

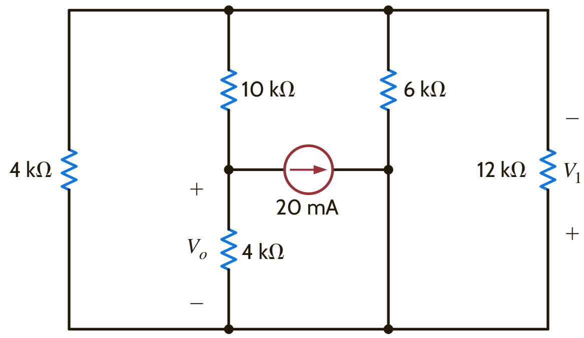


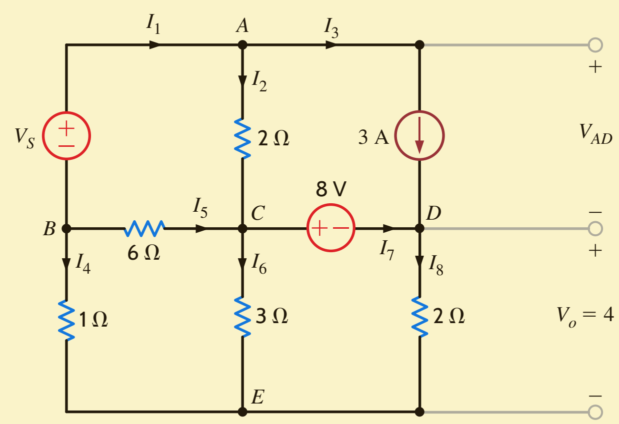
Figure E2.23

EXAMPLE 2.24

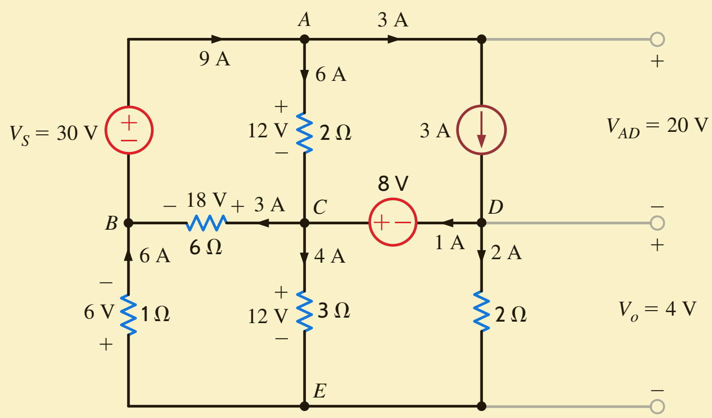
Consider the network in Fig. 2.32a. Given that $V_{DE} = V_o = 4$ V, find the value of the voltage source V_S and the voltage across the current source V_{AD}.

By using Kirchhoff's laws and Ohm's law, we can calculate the desired quantities. Since $V_{DE} = 4$ V, using Ohm's law we obtain $I_8 = 2$ A. Applying Kirchhoff's current law

Figure 2.32 Example circuit containing a current source.



(a)



(b)