2.1 Ohm's Law

Ohm's law is named for the German physicist Georg Simon Ohm, who is credited with establishing the voltage–current relationship for resistance. As a result of his pioneering work, the unit of resistance bears his name.

Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it. The resistance, measured in ohms, is the constant of proportionality between the voltage and current.

A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in Fig. 2.1a. A resistor is a physical device that can be purchased in certain standard values in an electronic parts store. These resistors, which find use in a variety of electrical applications, are normally carbon composition or wirewound. In addition, resistors can be fabricated using thick oxide or thin metal films for use in hybrid circuits, or they can be diffused in semiconductor integrated circuits. Some typical discrete resistors are shown in Fig. 2.1b.

HINT
The passive sign convention will be employed in conjunction with Ohm's law.

The mathematical relationship of Ohm's law is illustrated by the equation

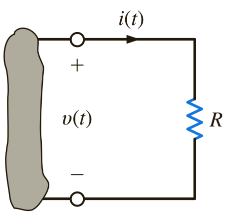
$$v(t) = R\,i(t), \text{ where } R \geqq 0 \tag{2.1}$$

or, equivalently, by the voltage–current characteristic shown in Fig. 2.2a. Note carefully the relationship between the polarity of the voltage and the direction of the current. In addition, note that we have tacitly assumed that the resistor has a constant value and therefore that the voltage–current characteristic is linear.

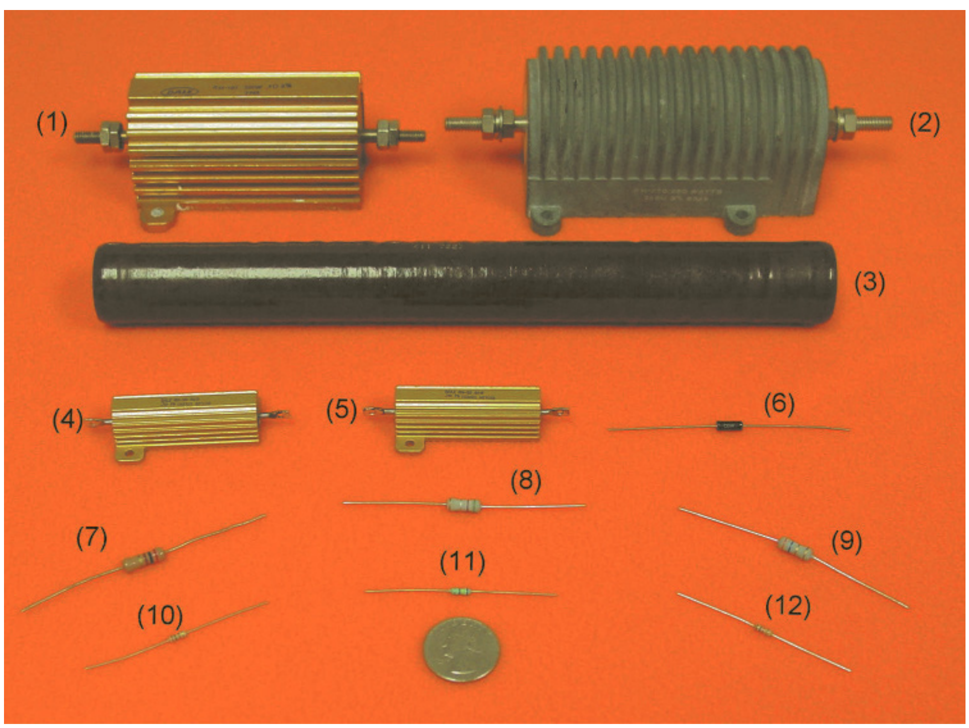
The symbol Ω is used to represent ohms, and therefore,

$$1\ \Omega = 1\ \text{V/A}$$

Although in our analysis we will always assume that the resistors are *linear* and are thus described by a straight-line characteristic that passes through the origin, it is important that readers realize that some very useful and practical elements do exist that exhibit a *nonlinear* resistance characteristic; that is, the voltage–current relationship is not a straight line.



(a)



(b)

Figure 2.1

(a) Symbol for a resistor; (b) some practical devices. (1), (2), and (3) are high-power resistors. (4) and (5) are high-wattage fixed resistors. (6) is a high-precision resistor. (7)–(12) are fixed resistors with different power ratings. (Photo courtesy of Mark Nelms and Jo Ann Loden)

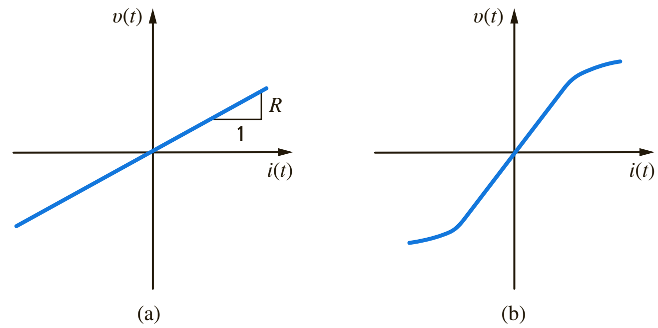


Figure 2.2
Graphical representation of the voltage–current relationship for (a) a linear resistor and (b) a light bulb.

The light bulb from the flashlight in Chapter 1 is an example of an element that exhibits a nonlinear characteristic. A typical characteristic for a light bulb is shown in **Fig. 2.2b**.

Since a resistor is a passive element, the proper current–voltage relationship is illustrated in Fig. 2.1a. The power supplied to the terminals is absorbed by the resistor. Note that the charge moves from the higher to the lower potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat. As indicated in Chapter 1, the rate of energy dissipation is the instantaneous power, and therefore

$$p(t) = v(t)i(t) \qquad \textbf{2.2}$$

which, using Eq. (2.1), can be written as

$$p(t) = Ri^2(t) = \frac{v^2(t)}{R} \qquad \textbf{2.3}$$

This equation illustrates that the power is a nonlinear function of either current or voltage and that it is always a positive quantity.

Conductance, represented by the symbol G, is another quantity with wide application in circuit analysis. By definition, conductance is the reciprocal of resistance; that is,

$$G = \frac{1}{R} \qquad \textbf{2.4}$$

The unit of conductance is the siemens, and the relationship between units is

$$1 \text{ S} = 1 \text{ A/V}$$

Using Eq. (2.4), we can write two additional expressions,

$$i(t) = Gv(t) \qquad \textbf{2.5}$$

and

$$p(t) = \frac{i^2(t)}{G} = Gv^2(t) \qquad \textbf{2.6}$$

Eq. (2.5) is another expression of Ohm's law.

Two specific values of resistance, and therefore conductance, are very important: $R = 0$ and $R = \infty$.

In examining the two cases, consider the network in **Fig. 2.3a**. The variable resistance symbol is used to describe a resistor such as the volume control on a radio or television set. As the resistance is decreased and becomes smaller and smaller, we finally reach a point where the resistance is zero and the circuit is reduced to that shown in **Fig. 2.3b**; that is, the

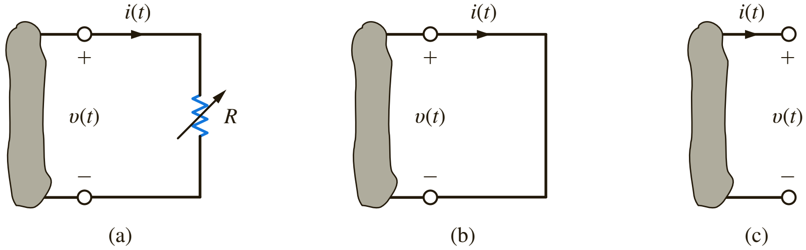


Figure 2.3
Short-circuit and open-circuit descriptions.

resistance can be replaced by a short circuit. On the other hand, if the resistance is increased and becomes larger and larger, we finally reach a point where it is essentially infinite and the resistance can be replaced by an open circuit, as shown in **Fig. 2.3c**. Note that in the case of a short circuit where $R = 0$,

$$\begin{aligned} v(t) &= Ri(t) \\ &= 0 \end{aligned}$$

Therefore, $v(t) = 0$, although the current could theoretically be any value. In the open-circuit case where $R = \infty$,

$$\begin{aligned} i(t) &= v(t)/R \\ &= 0 \end{aligned}$$

Therefore, the current is zero regardless of the value of the voltage across the open terminals.

EXAMPLE 2.1

In the circuit in **Fig. 2.4a**, determine the current and the power absorbed by the resistor.

SOLUTION

Using Eq. (2.1), we find the current to be

$$I = V/R = 12/2\text{k} = 6 \text{ mA}$$

Note that because many of the resistors employed in our analysis are in kΩ, we will use k in the equations in place of 1000. The power absorbed by the resistor is given by Eq. (2.2) or (2.3) as

$$\begin{aligned} P &= VI = (12)(6 \times 10^{-3}) = 0.072 \text{ W} \\ &= I^2R = (6 \times 10^{-3})^2(2\text{k}) = 0.072 \text{ W} \\ &= V^2/R = (12)^2/2\text{k} = 0.072 \text{ W} \end{aligned}$$

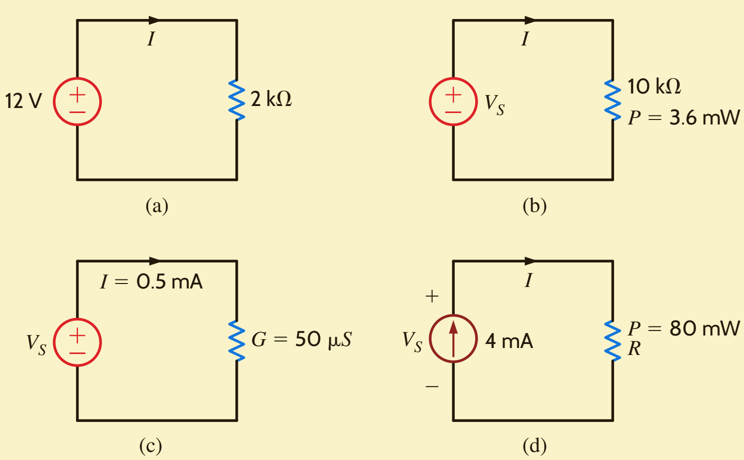


Figure 2.4
Circuits for Examples 2.1 to 2.4.

EXAMPLE 2.2

The power absorbed by the 10-kΩ resistor in **Fig. 2.4b** is 3.6 mW. Determine the voltage and the current in the circuit.

SOLUTION

Using the power relationship, we can determine either of the unknowns:

$$V_S^2/R = P$$
$$V_S^2 = (3.6 \times 10^{-3})(10\text{k})$$
$$V_S = 6 \text{ V}$$

and

$$I^2R = P$$
$$I^2 = (3.6 \times 10^{-3})/10\text{k}$$
$$I = 0.6 \text{ mA}$$

Furthermore, once V_S is determined, I could be obtained by Ohm's law, and likewise once I is known, then Ohm's law could be used to derive the value of V_S. Note carefully that the equations for power involve the terms I^2 and V_S^2. Therefore, $I = -0.6$ mA and $V_S = -6\ V$ also satisfy the mathematical equations and, in this case, the direction of *both* the voltage and current is reversed.

EXAMPLE 2.3

Given the circuit in **Fig. 2.4c**, we wish to find the value of the voltage source and the power absorbed by the resistance.

SOLUTION

The voltage is

$$V_S = I/G = (0.5 \times 10^{-3})/(50 \times 10^{-6}) = 10 \text{ V}$$

The power absorbed is then

$$P = I^2/\text{G} = (0.5 \times 10^{-3})^2/(50 \times 10^{-6}) = 5 \text{ mW}$$

Or we could simply note that

$$R = 1/G = 20 \text{ k}\Omega$$

and therefore

$$V_S = IR = (0.5 \times 10^{-3})(20\text{k}) = 10 \text{ V}$$

and the power could be determined using $P = I^2R = V_S^2/R = V_S I$.

EXAMPLE 2.4

Given the network in **Fig. 2.4d**, we wish to find R and V_S.

SOLUTION

Using the power relationship, we find that

$$R = P/I^2 = (80 \times 10^{-3})/(4 \times 10^{-3})^2 = 5 \text{ k}\Omega$$

The voltage can now be derived using Ohm's law as

$$V_S = IR = (4 \times 10^{-3})(5\text{k}) = 20 \text{ V}$$

The voltage could also be obtained from the remaining power relationships in Eqs. (2.2) and (2.3).

Before leaving this initial discussion of circuits containing sources and a single resistor, it is important to note a phenomenon that we will find to be true in circuits containing many sources and resistors. The presence of a voltage source between a pair of terminals tells us precisely what the voltage is between the two terminals regardless of what is happening in the balance of the network. What we do not know is the current in the voltage source. We must apply circuit analysis to the entire network to determine this current. Likewise, the presence of a current source connected between two terminals specifies the exact value of the current through the source between the terminals. What we do not know is the value of the voltage across the current source. This value must be calculated by applying circuit analysis to the entire network. Furthermore, it is worth emphasizing that when applying Ohm's law, the relationship $V = IR$ specifies a relationship between the voltage *directly across* a resistor R and the current that is *present* in this resistor. Ohm's law does not apply when the voltage is present in one part of the network and the current exists in another. This is a common mistake made by students who try to apply $V = IR$ to a resistor R in the middle of the network while using a V at some other location in the network.

LEARNING ASSESSMENTS

E2.1 Given the circuits in Fig. E2.1, find (a) the current I and the power absorbed by the resistor in Fig. E2.1a, and (b) the voltage across the current source and the power supplied by the source in Fig. E2.1b.

ANSWER:
(a) $I = 0.3$ mA,
$P = 3.6$ mW;
(b) $V_S = 3.6$ V,
$P = 2.16$ mW.

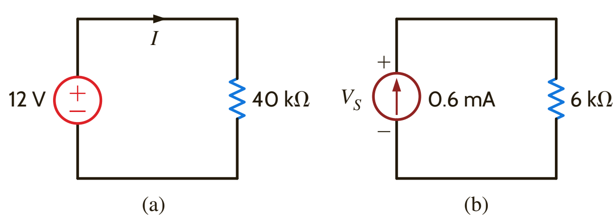


Figure E2.1

E2.2 Given the circuits in Fig. E2.2, find (a) R and V_S in the circuit in Fig. E2.2a, and (b) find I and R in the circuit in Fig. E2.2b.

ANSWER:
(a) $R = 10$ kΩ, $V_S = 4$ V;
(b) $I = 20.8$ mA, $R = 576$ Ω.

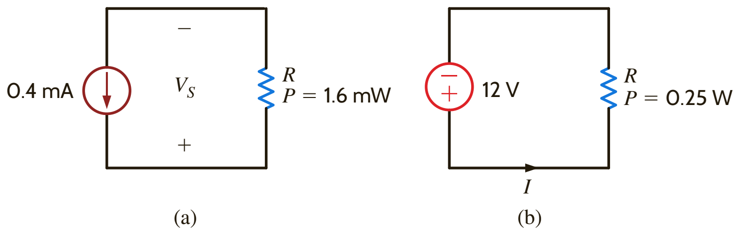


Figure E2.2

E2.3 The power absorbed by G_x in Fig. E2.3 is 50 mW. Find G_x.

ANSWER:
$G_x = 500$ μS.

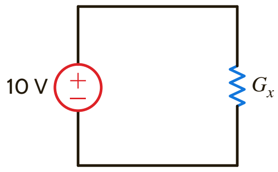


Figure E2.3