LEARNING ASSESSMENTS

E8.12 Draw a phasor diagram to illustrate all currents and voltages for the network in Fig. E8.12.
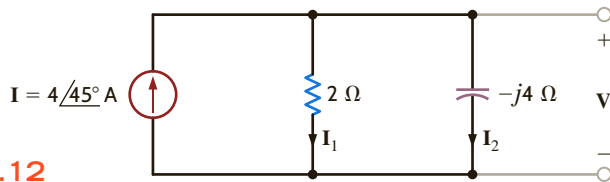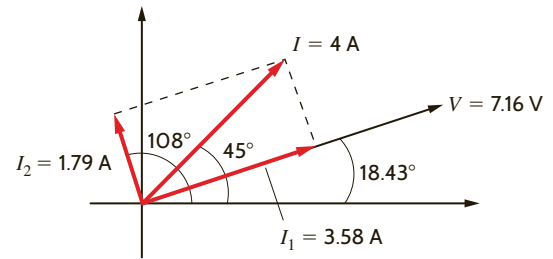
ANSWER:

Figure E8.12

- -

E8.13 Find the value of C such that $v(t)$ and $i(t)$ are in phase in Fig. E8.13.
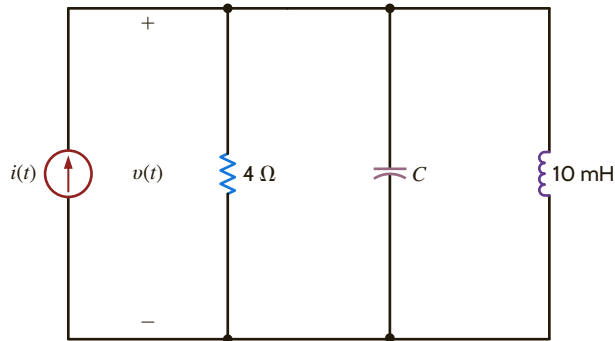
ANSWER:
$C = 400\ \mu F$.

Figure E8.13

8.7

Basic Analysis Using Kirchhoff's Laws

We have shown that Kirchhoff's laws apply in the frequency domain, and therefore they can be used to compute steady-state voltages and currents in ac circuits. This approach involves expressing these voltages and currents as phasors, and once this is done, the ac steady-state analysis employing phasor equations is performed in an identical fashion to that used in the dc analysis of resistive circuits. Complex number algebra is the tool that is used for the mathematical manipulation of the phasor equations, which, of course, have complex coefficients. We will begin by illustrating that the techniques we have applied in the solution of dc resistive circuits are valid in ac circuit analysis also—the only difference being that in steady-state ac circuit analysis the algebraic phasor equations have complex coefficients.

PROBLEM-SOLVING STRATEGY

AC STEADY-STATE ANALYSIS

- For relatively simple circuits (e.g., those with a single source), use
  - Ohm's law for ac analysis—that is, $\mathbf{V} = \mathbf{IZ}$
  - The rules for combining $\mathbf{Z}_s$ and $\mathbf{Y}_p$
  - KCL and KVL
  - Current and voltage division

- For more complicated circuits with multiple sources, use
  - Nodal analysis
  - Loop or mesh analysis
  - Superposition
  - Source exchange
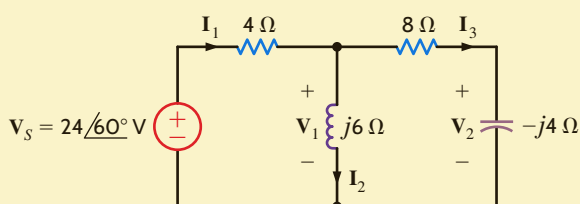 - Thévenin's and Norton's theorems

At this point, it is important for the reader to understand that in our manipulation of algebraic phasor equations with complex coefficients we will, for the sake of simplicity, normally carry only two digits to the right of the decimal point. In doing so, we will introduce round-off errors in our calculations. Nowhere are these errors more evident than when two or more approaches are used to solve the same problem, as is done in the following example.

EXAMPLE 8.14

We wish to calculate all the voltages and currents in the circuit shown in **Fig. 8.16a**.
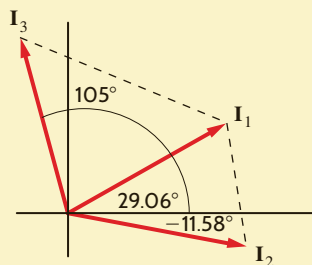
SOLUTION

Our approach will be as follows. We will calculate the total impedance seen by the source $\mathbf{V}_S$. Then we will use this to determine $\mathbf{I}_1$. Knowing $\mathbf{I}_1$, we can compute $\mathbf{V}_1$ using KVL. Knowing $\mathbf{V}_1$, we can compute $\mathbf{I}_2$ and $\mathbf{I}_3$, and so on.

The total impedance seen by the source $\mathbf{V}_S$ is

$$\mathbf{Z}_{eq} = 4 + \frac{(j6)(8 - j4)}{j6 + 8 - j4}$$

$$= 4 + \frac{24 + j48}{8 + j2}$$
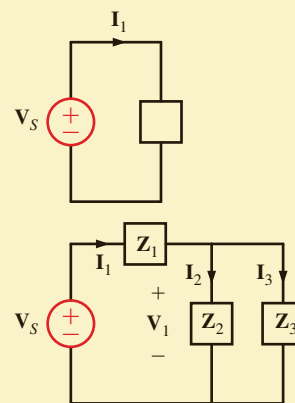
$$= 4 + 4.24 + j4.94$$

$$= 9.61\underline{/30.94°} \ \Omega$$

Figure 8.16

(a) Example ac circuit, (b) phasor diagram for the currents (plots are not drawn to scale).

Then

$$\mathbf{I}_1 = \frac{\mathbf{V}_S}{\mathbf{Z}_{eq}} = \frac{24\underline{/60°}}{9.61\underline{/30.94°}}$$

$$= 2.5\underline{/29.06°} \ \text{A}$$

$\mathbf{V}_1$ can be determined using KVL:

$$\mathbf{V}_1 = \mathbf{V}_S - 4\mathbf{I}_1$$

$$= 24\underline{/60°} - 10\underline{/29.06°}$$

$$= 3.26 + j15.93$$

$$= 16.26\underline{/78.43°} \ \text{V}$$

Note that $\mathbf{V}_1$ could also be computed via voltage division:

$$\mathbf{V}_1 = \frac{\mathbf{V}_S \dfrac{(j6)(8 - j4)}{j6 + 8 - j4}}{4 + \dfrac{(j6)(8 - j4)}{j6 + 8 - 4}} \ \text{V}$$

HINT

Technique

1. Compute $\mathbf{I}_1$.

2. Determine $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1\mathbf{Z}_1$

Then $\mathbf{I}_2 = \dfrac{\mathbf{V}_1}{\mathbf{Z}_2}$ and $\mathbf{I}_3 = \dfrac{\mathbf{V}_1}{\mathbf{Z}_3}$

Current and voltage division are also applicable.

which from our previous calculation is

$$\mathbf{V}_1 = \frac{(24\underline{/60°})(6.51\underline{/49.36°})}{9.61\underline{/30.94°}}$$

$$= 16.26\underline{/78.42°} \text{ V}$$

Knowing $\mathbf{V}_1$, we can calculate both $\mathbf{I}_2$ and $\mathbf{I}_3$:

$$\mathbf{I}_2 = \frac{\mathbf{V}_1}{j6} = \frac{16.26\underline{/78.43°}}{6\underline{/90°}}$$

$$= 2.71\underline{/-11.58°} \text{ A}$$

and

$$\mathbf{I}_3 = \frac{\mathbf{V}_1}{8 - j4}$$

$$= 1.82\underline{/105°} \text{ A}$$

Note that $\mathbf{I}_2$ and $\mathbf{I}_3$ could have been calculated by current division. For example, $\mathbf{I}_2$ could be determined by

$$\mathbf{I}_2 = \frac{\mathbf{I}_1 (8 - j4)}{8 - j4 + j6}$$

$$= \frac{(2.5\underline{/29.06°})(8.94\underline{/-26.57°})}{8 + j2}$$

$$= 2.71\underline{/-11.55°} \text{ A}$$

Finally, $\mathbf{V}_2$ can be computed as

$$\mathbf{V}_2 = \mathbf{I}_3 (-j4)$$

$$= 7.28\underline{/15°} \text{ V}$$

This value could also have been computed by voltage division. The phasor diagram for the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ is shown in **Fig. 8.16b** and is an illustration of KCL.

Finally, the reader is encouraged to work the problem in reverse; that is, given $\mathbf{V}_2$, find $\mathbf{V}_S$. Note that if $\mathbf{V}_2$ is known, $\mathbf{I}_3$ can be computed immediately using the capacitor impedance. Then $\mathbf{V}_2 + \mathbf{I}_3$ (8) yields $\mathbf{V}_1$. Knowing $\mathbf{V}_1$ we can find $\mathbf{I}_2$. Then $\mathbf{I}_2 + \mathbf{I}_3 = \mathbf{I}_1$, and so on. Note that this analysis, which is the subject of Learning Assessment E8.16, involves simply a repeated application of Ohm's law, KCL, and KVL.

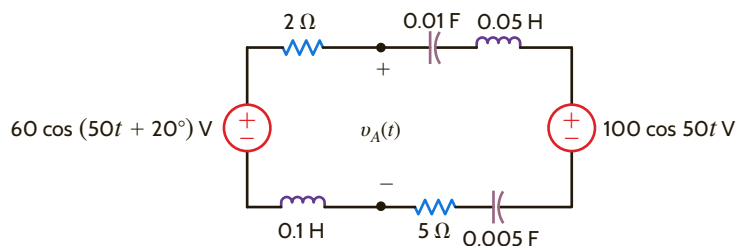# LEARNING ASSESSMENTS

E8.14 Find $v_A(t)$ in Fig. E8.14.

Figure E8.14

ANSWER:
$v_A(t) = 95.83 \cos(50t + 24.1°)$ V.