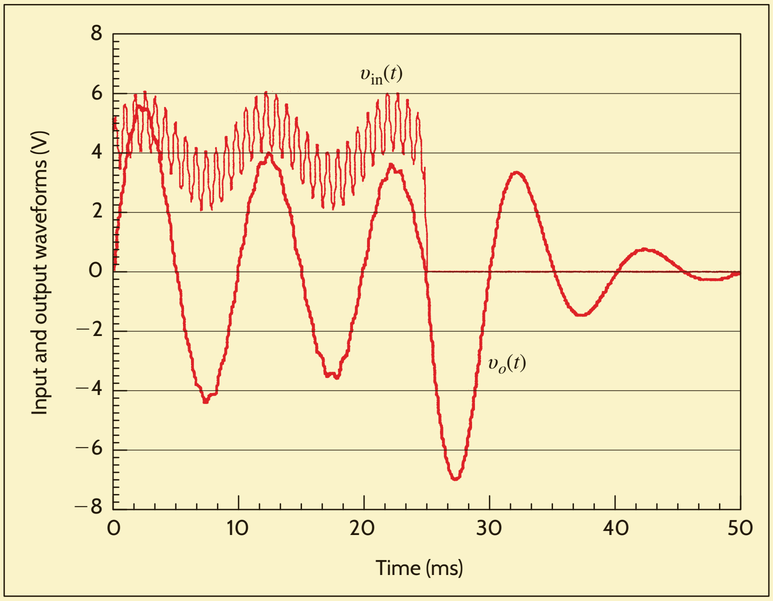

Figure 13.4

Plots of input and output waveforms reveal the nature of the band-pass filter—particularly, attenuation of dc and higher-frequency components.

Plots of the resulting $v_o(t)$ and $v_{in}(t)$ are shown in **Fig. 13.4**. An examination of the output waveform indicates that the 100-Hz component of $v_{in}(t)$ is amplified, whereas the dc and 1234-Hz components are attenuated. That is, $v_o(t)$ has an amplitude of approximately 3 V and an average value of near zero. Indeed, the circuit performs as a band-pass filter. Remember that these waveforms are not measured; they are simulation results obtained from our model, $h(t)$.

13.7 Initial-Value and Final-Value Theorems

Suppose that we wish to determine the initial or final value of a circuit response in the time domain from the Laplace transform of the function in the s-domain without performing the inverse transform. If we determine the function $f(t) = \mathscr{L}^{-1}[\mathbf{F}(s)]$, we can find the initial value by evaluating $f(t)$ as $t \to 0$ and the final value by evaluating $f(t)$ as $t \to \infty$. It would be very convenient, however, if we could simply determine the initial and final values from $\mathbf{F}(s)$ without having to perform the inverse transform. The initial- and final-value theorems allow us to do just that.

The *initial-value theorem* states that

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\mathbf{F}(s) \qquad \mathbf{13.31}$$

provided that $f(t)$ and its first derivative are transformable.

The proof of this theorem employs the Laplace transform of the function $df(t)/dt$:

$$\int_0^\infty \frac{df(t)}{dt} e^{-st}\,dt = s\mathbf{F}(s) - f(0)$$

Taking the limit of both sides as $s \to \infty$, we find that

$$\lim_{s\to\infty} \int_0^\infty \frac{df(t)}{dt} e^{-st}\,dt = \lim_{s\to\infty} [s\mathbf{F}(s) - f(0)]$$

and since

$$\int_0^\infty \frac{df(t)}{dt} \lim_{s\to\infty} e^{-st}\,dt = 0$$

then

$$f(0) = \lim_{s\to\infty} s\mathbf{F}(s)$$

which is, of course,

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} s\mathbf{F}(s)$$

The *final-value theorem* states that

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\mathbf{F}(s) \qquad \textbf{13.32}$$

provided that $f(t)$ and its first derivative are transformable and that $f(\infty)$ exists. This latter requirement means that the poles of $\mathbf{F}(s)$ must have negative real parts with the exception that there can be a simple pole at $s = 0$.

The proof of this theorem also involves the Laplace transform of the function $df(t)/dt$:

$$\int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt = s\mathbf{F}(s) - f(0)$$

Taking the limit of both sides as $s \to 0$ gives us

$$\lim_{s\to 0} \int_0^\infty \frac{df(t)}{dt} e^{-st}\, dt = \lim_{s\to 0} [s\mathbf{F}(s) - f(0)]$$

Therefore,

$$\int_0^\infty \frac{df(t)}{dt}\, dt = \lim_{s\to 0} [s\mathbf{F}(s) - f(0)]$$

and

$$f(\infty) - f(0) = \lim_{s\to 0} s\mathbf{F}(s) - f(0)$$

and hence,

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} s\mathbf{F}(s)$$

EXAMPLE 13.13

Let us determine the initial and final values for the function

$$\mathbf{F}(s) = \frac{10(s+1)}{s(s^2 + 2s + 2)}$$

and corresponding time function

$$f(t) = 5 + 5\sqrt{2}\, e^{-t} \cos(t - 135°)u(t)$$

SOLUTION

Applying the initial-value theorem, we have

$$\begin{aligned} f(0) &= \lim_{s\to\infty} s\mathbf{F}(s) \\ &= \lim_{s\to\infty} \frac{10(s+1)}{s^2 + 2s + 2} \\ &= 0 \end{aligned}$$

The poles of $\mathbf{F}(s)$ are $s = 0$ and $s = -1 \pm j1$, so the final-value theorem is applicable. Thus,

$$\begin{aligned} f(\infty) &= \lim_{s\to\infty} s\mathbf{F}(s) \\ &= \lim_{s\to 0} \frac{10(s+1)}{s^2 + 2s + 2} \\ &= 5 \end{aligned}$$

Note that these values could be obtained directly from the time function $f(t)$.