6.1 Capacitors

A *capacitor* is a circuit element that consists of two conducting surfaces separated by a nonconducting, or *dielectric*, material. A simplified capacitor and its electrical symbol are shown in **Fig. 6.1**.

There are many different kinds of capacitors, and they are categorized by the type of dielectric material used between the conducting plates. Although any good insulator can serve as a dielectric, each type has characteristics that make it more suitable for particular applications.

For general applications in electronic circuits (e.g., coupling between stages of amplification), the dielectric material may be paper impregnated with oil or wax, mylar, polystyrene, mica, glass, or ceramic.

Ceramic dielectric capacitors constructed of barium titanates have a large capacitance-to-volume ratio because of their high dielectric constant. Mica, glass, and ceramic dielectric capacitors will operate satisfactorily at high frequencies.

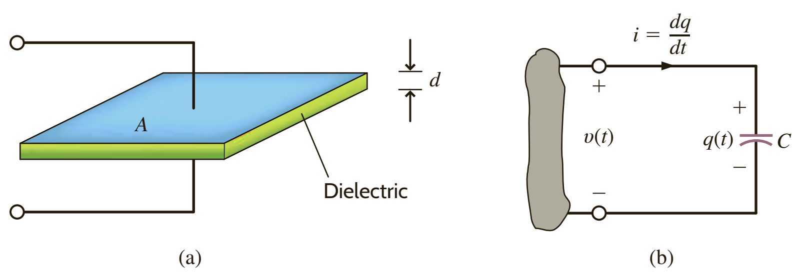
Aluminum electrolytic capacitors, which consist of a pair of aluminum plates separated by a moistened borax paste electrolyte, can provide high values of capacitance in small volumes. They are typically used for filtering, bypassing, and coupling, and in power supplies and motor-starting applications. Tantalum electrolytic capacitors have lower losses and more stable characteristics than those of aluminum electrolytic capacitors. **Fig. 6.2** shows a variety of typical discrete capacitors.

In addition to these capacitors, which we deliberately insert in a network for specific applications, stray capacitance is present any time there is a difference in potential between two conducting materials separated by a dielectric. Because this stray capacitance can cause unwanted coupling between circuits, extreme care must be exercised in the layout of electronic systems on printed circuit boards.

Capacitance is measured in coulombs per volt or farads. The unit *farad* (F) is named after Michael Faraday, a famous English physicist. Capacitors may be fixed or variable and typically range from thousands of microfarads (μF) to a few picofarads (pF).

Figure 6.1

A capacitor and its electrical symbol.



HINT

Note the use of the passive sign convention.

Figure 6.2

Some typical capacitors (Courtesy of Mark Nelms and Jo Ann Loden).

Capacitor technology, initially driven by the modern interest in electric vehicles, is rapidly changing, however. For example, the capacitor on the left in the photograph in Fig. 6.3 is a double-layer capacitor, which is rated at 2.5 V and 100 F. An aluminum electrolytic capacitor, rated at 25 V and 68,000 μF, is shown on the right in this photograph. The electrolytic capacitor can store $0.5 * 6.8 \times 10^{-2} * 25^2 = 21.25$ joules (J). The double-layer capacitor can store $0.5 * 100 * 2.5^2 = 312.5$ J. Let's connect 10 of the 100-F capacitors in series for an equivalent 25-V capacitor. The energy stored in this equivalent capacitor is 3125 J. We would need to connect 147 electrolytic capacitors in parallel to store that much energy.

Figure 6.3

A 100-F double-layer capacitor and a 68,000-μF electrolytic capacitor (Courtesy of Mark Nelms and Jo Ann Loden).

It is interesting to calculate the dimensions of a simple equivalent capacitor consisting of two parallel plates each of area A, separated by a distance d as shown in Fig. 6.1. We learned in basic physics that the capacitance of two parallel plates of area A, separated by distance d, is

$$C = \frac{\varepsilon_o A}{d}$$

where ε_o, the permitivity of free space, is 8.85×10^{-12} F/m. If we assume the plates are separated by a distance in air of the thickness of one sheet of oil-impregnated paper, which is about 1.016×10^{-4} m, then

$$100 \text{ F} = \frac{(8.85 \times 10^{-12})A}{1.016 \times 10^{-4}}$$

$$A = 1.148 \times 10^9 \text{ m}^2$$

and since 1 square mile is equal to 2.59×10^6 square meters, the area is

$$A \approx 443 \text{ square miles}$$

which is the area of a medium-sized city! It would now seem that the double-layer capacitor in the photograph is much more impressive than it originally appeared. This capacitor is actually constructed using a high surface area material such as powdered carbon that is adhered to a metal foil. There are literally millions of pieces of carbon employed to obtain the required surface area.

Suppose now that a source is connected to the capacitor shown in Fig. 6.1; then positive charges will be transferred to one plate and negative charges to the other. The charge on the capacitor is proportional to the voltage across it such that

$$q = Cv \tag{6.1}$$

where C is the proportionality factor known as the capacitance of the element in farads.

The charge differential between the plates creates an electric field that stores energy. Because of the presence of the dielectric, the conduction current that flows in the wires that connect the capacitor to the remainder of the circuit cannot flow internally between the plates. However, via electromagnetic field theory it can be shown that this conduction current is equal to the displacement current that flows between the plates of the capacitor and is present any time that an electric field or voltage varies with time.

Our primary interest is in the current–voltage terminal characteristics of the capacitor. Since the current is

$$i = \frac{dq}{dt}$$

then for a capacitor

$$i = \frac{d}{dt}(Cv)$$

which for constant capacitance is

$$i = C\frac{dv}{dt} \tag{6.2}$$

Eq. (6.2) can be rewritten as

$$dv = \frac{1}{C} i \, dt$$

Integrating this expression from $t = -\infty$ to some time t and assuming $v(-\infty) = 0$ yields

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\, dx \tag{6.3}$$

where $v(t)$ indicates the time dependence of the voltage. Eq. (6.3) can be expressed as two integrals, so that

$$\begin{aligned} v(t) &= \frac{1}{C}\int_{-\infty}^{t_0} i(x)\, dx + \frac{1}{C}\int_{t_0}^{t} i(x)\, dx \\ &= v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\, dx \end{aligned} \tag{6.4}$$

where $v(t_0)$ is the voltage due to the charge that accumulates on the capacitor from time $t = -\infty$ to time $t = t_0$.

The energy stored in the capacitor can be derived from the power that is delivered to the element. This power is given by the expression

$$p(t) = v(t)i(t) = Cv(t)\frac{dv(t)}{dt} \tag{6.5}$$

and hence, the energy stored in the electric field is

$$\begin{aligned} w_C(t) &= \int_{-\infty}^{t} Cv(x)\frac{dv(x)}{dx}\, dx = C\int_{-\infty}^{t} v(x)\frac{dv(x)}{dx}\, dx \\ &= C\int_{v(-\infty)}^{v(t)} v(x)\, dv(x) = \frac{1}{2}Cv^2(x)\Big|_{v(-\infty)}^{v(t)} \end{aligned}$$

$$= \frac{1}{2}Cv^2(t) \text{ J} \tag{6.6}$$

since $v(t = -\infty) = 0$. The expression for the energy can also be written using Eq. (6.1) as

$$w_C(t) = \frac{1}{2}\frac{q^2(t)}{C} \tag{6.7}$$

Eqs. (6.6) and (6.7) represent the energy stored by the capacitor, which, in turn, is equal to the work done by the source to charge the capacitor.

Now let's consider the case of a dc voltage applied across a capacitor. From Eq. (6.2), we see that the current flowing through the capacitor is directly proportional to the time rate of change of the voltage across the capacitor. A dc voltage does not vary with time, so the current flowing through the capacitor is zero. We can say that a capacitor is "an open circuit to dc" or "blocks dc." Capacitors are often utilized to remove or filter out an unwanted dc voltage. In analyzing a circuit containing dc voltage sources and capacitors, we can replace the capacitors with an open circuit and calculate voltages and currents in the circuit using our many analysis tools.

Note that the power absorbed by a capacitor, given by Eq. (6.5), is directly proportional to the time rate of change of the voltage across the capacitor. What if we had an instantaneous change in the capacitor voltage? This would correspond to $dv/dt = \infty$ and infinite power. In Chapter 1, we ruled out the possibility of any sources of infinite power. Since we only have finite power sources, the voltage across a capacitor cannot change instantaneously. This will be a particularly helpful idea in the next chapter when we encounter circuits containing switches. This idea of "continuity of voltage" for a capacitor tells us that the voltage across the capacitor just after a switch moves is the same as the voltage across the capacitor just before that switch moves.

The polarity of the voltage across a capacitor being charged is shown in Fig. 6.1b. In the ideal case, the capacitor will hold the charge for an indefinite period of time, if the source is removed. If at some later time an energy-absorbing device (e.g., a flash bulb) is connected across the capacitor, a discharge current will flow from the capacitor and, therefore, the capacitor will supply its stored energy to the device.

EXAMPLE 6.1

If the charge accumulated on two parallel conductors charged to 12 V is 600 pC, what is the capacitance of the parallel conductors?

SOLUTION

Using Eq. (6.1), we find that

$$C = \frac{Q}{V} = \frac{(600)\left(10^{-12}\right)}{12} = 50 \text{ pF}$$

EXAMPLE 6.2

The voltage across a 5-μF capacitor has the waveform shown in Fig. 6.4a. Determine the current waveform.

SOLUTION

Note that

$$\begin{aligned} v(t) &= \frac{24}{6 \times 10^{-3}} t && 0 \leq t \leq 6 \text{ ms} \\ &= \frac{-24}{2 \times 10^{-3}} t + 96 && 6 \leq t < 8 \text{ ms} \\ &= 0 && 8 \text{ ms} \leq t \end{aligned}$$

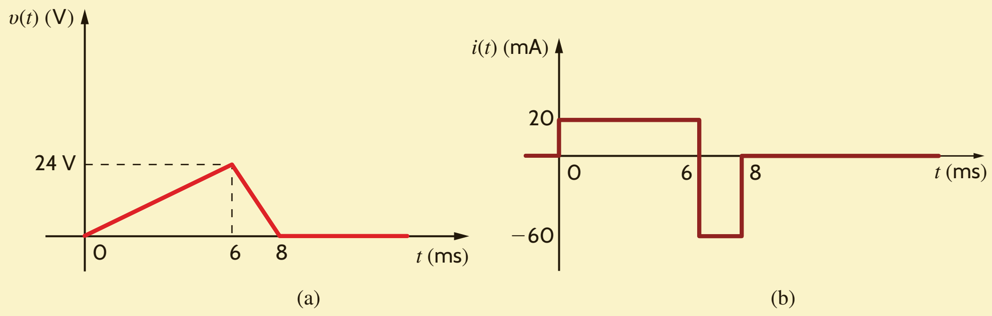


Figure 6.4 Voltage and current waveforms for a 5-μF capacitor.

Using Eq. (6.2), we find that

$$\begin{aligned} i(t) &= C\frac{dv(t)}{dt} \\ &= 5 \times 10^{-6}(4 \times 10^{3}) && 0 \leq t \leq 6 \text{ ms} \\ &= 20 \text{ mA} && 0 \leq t \leq 6 \text{ ms} \\ i(t) &= 5 \times 10^{-6}(-12 \times 10^{3}) && 6 \leq t \leq 8 \text{ ms} \\ &= -60 \text{ mA} && 6 \leq t < 8 \text{ ms} \end{aligned}$$

and

$$i(t) = 0 \qquad 8 \text{ ms} \leq t$$

Therefore, the current waveform is as shown in Fig. 6.4b, and $i(t) = 0$ for $t > 8$ ms.

EXAMPLE 6.3

Determine the energy stored in the electric field of the capacitor in Example 6.2 at $t = 6$ ms.

SOLUTION

Using Eq. (6.6), we have

$$w(t) = \frac{1}{2}Cv^2(t)$$

At $t = 6$ ms,

$$\begin{aligned} w(6\text{ ms}) &= \frac{1}{2}(5 \times 10^{-6})(24)^2 \\ &= 1440\ \mu\text{J} \end{aligned}$$

LEARNING ASSESSMENT

E6.1 A 10-μF capacitor has an accumulated charge of 500 nC. Determine the voltage across the capacitor.

ANSWER: 0.05 V.

EXAMPLE 6.4

The current in an initially uncharged 4-μF capacitor is shown in Fig. 6.5a. Let us derive the waveforms for the voltage, power, and energy and compute the energy stored in the electric field of the capacitor at $t = 2$ ms.

SOLUTION

The equations for the current waveform in the specific time intervals are

$$\begin{aligned} i(t) &= \frac{16 \times 10^{-6}t}{2 \times 10^{-3}} && 0 \le t \le 2\text{ ms} \\ &= -8 \times 10^{-6} && 2\text{ ms} \le t \le 4\text{ ms} \\ &= 0 && 4\text{ ms} < t \end{aligned}$$

Since $v(0) = 0$, the equation for $v(t)$ in the time interval $0 \le t \le 2$ ms is

$$v(t) = \frac{1}{(4)(10^{-6})}\int_0^t 8(10^{-3})x\,dx = 10^3t^2$$

and hence,

$$v(2\text{ ms}) = 10^3(2 \times 10^{-3})^2 = 4\text{ mV}$$

In the time interval $2\text{ ms} \le t \le 4$ ms,

$$\begin{aligned} v(t) &= \frac{1}{(4)(10^{-6})}\int_{2(10^{-3})}^t -(8)(10^{-6})dx + (4)(10^{-3}) \\ &= -2t + 8 \times 10^{-3} \end{aligned}$$

The waveform for the voltage is shown in Fig. 6.5b.

Since the power is $p(t) = v(t)i(t)$, the expression for the power in the time interval $0 \le t \le 2$ ms is $p(t) = 8t^3$. In the time interval $2\text{ ms} \le t \le 4$ ms, the equation for the power is

$$\begin{aligned} p(t) &= -(8)(10^{-6})(-2t + 8 \times 10^{-3}) \\ &= 16(10^{-6})t - 64(10^{-9}) \end{aligned}$$

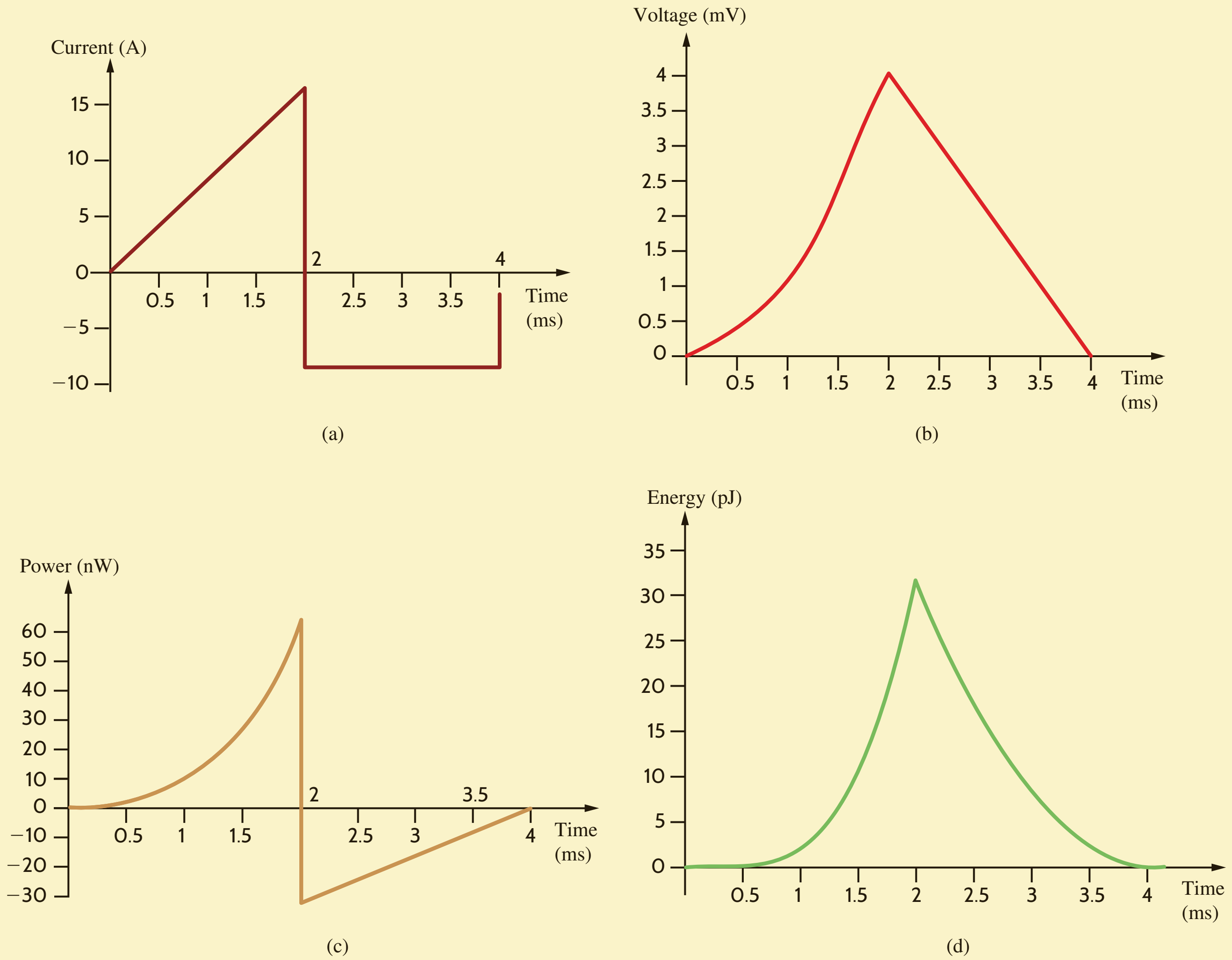


Figure 6.5

Waveforms used in Example 6.4.

The power waveform is shown in **Fig. 6.5c**. Note that during the time interval $0 \le t \le 2$ ms, the capacitor is absorbing energy and during the interval 2 ms $\le t \le 4$ ms, it is delivering energy.

The energy is given by the expression

$$w(t) = \int_{t_0}^{t} p(x)dx + w(t_0)$$

In the time interval $0 \le t \le 2$ ms,

$$w(t) = \int_{0}^{t} 8x^3 dx = 2t^4$$

Hence,

$$w(2\text{ ms}) = 32\text{ pJ}$$

In the time interval $2 \le t \le 4$ ms,

$$\begin{aligned} w(t) &= \int_{2\times 10^{-3}}^{t} [(16 \times 10^{-6})x - (64 \times 10^{-9})]\, dx + 32 \times 10^{-12} \\ &= [(8 \times 10^{-6})x^2 - (64 \times 10^{-9})x]_{2\times 10^{-3}}^{t} + 32 \times 10^{-12} \\ &= (8 \times 10^{-6})t^2 - (64 \times 10^{-9})t + 128 \times 10^{-12} \end{aligned}$$

From this expression we find that $w(2\text{ ms}) = 32$ pJ and $w(4\text{ ms}) = 0$. The energy waveform is shown in **Fig. 6.5d**.

LEARNING ASSESSMENTS

E6.2 The voltage across a 2-μF capacitor is shown in Fig. E6.2. Determine the waveform for the capacitor current.

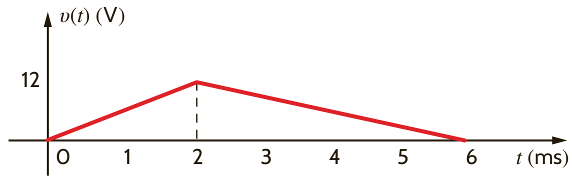
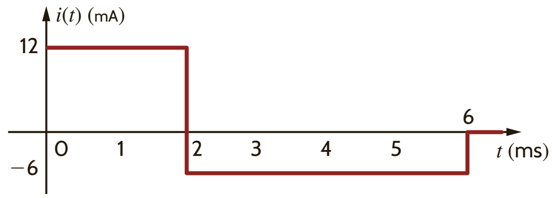


Figure E6.2

ANSWER:



E6.3 Compute the energy stored in the electric field of the capacitor in Learning Assessment E6.2 at $t = 2$ ms.

ANSWER:
$w = 144$ μJ.

E6.4 The voltage across a 5-μF capacitor is shown in Fig. E6.4. Find the waveform for the current in the capacitor. How much energy is stored in the capacitor at $t = 4$ ms?

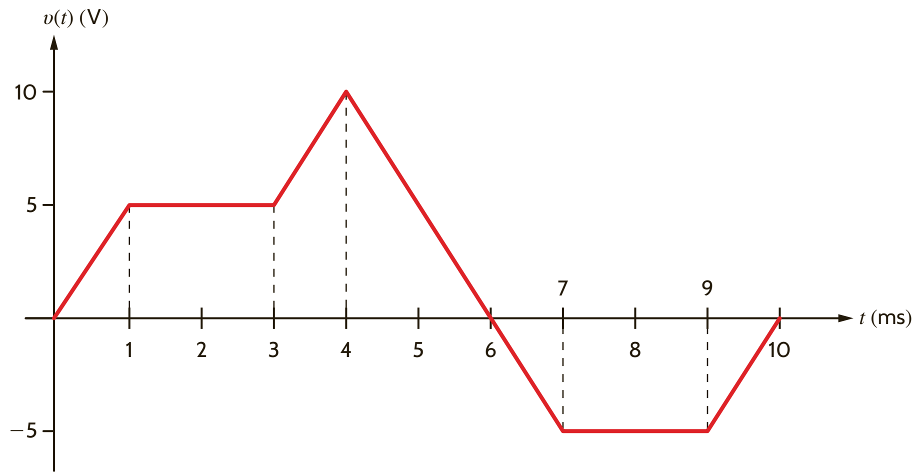
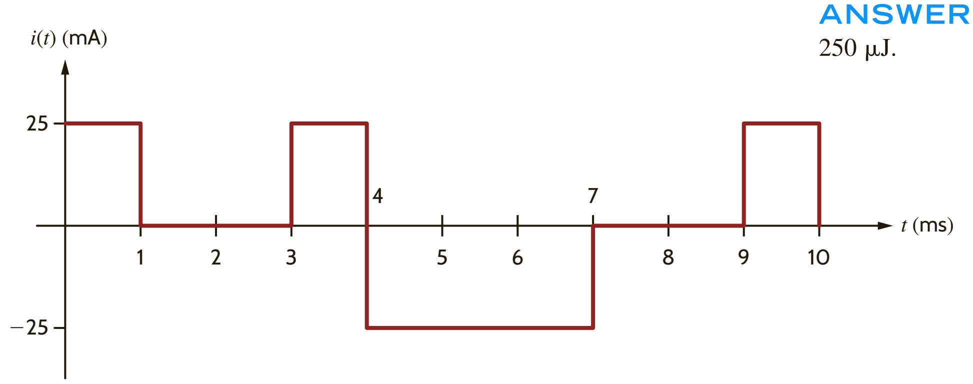


Figure E6.4



ANSWER:
250 μJ.