E8.15 Find $\mathbf{V}_o$ in Fig. E8.15.

ANSWER:
$\mathbf{V}_o = 2.98\underline{/-153.43^\circ}$ V.

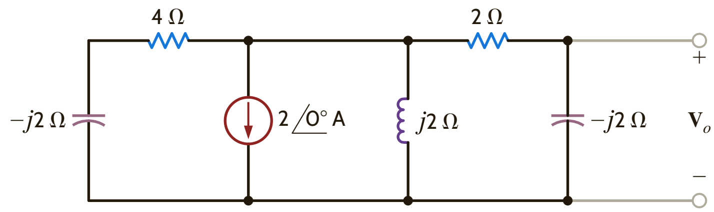


Figure E8.15

E8.16 In the network in Fig. E8.16, $\mathbf{V}_o$ is known to be $8\underline{/45^\circ}$ V. Compute $\mathbf{V}_S$.

ANSWER:
$\mathbf{V}_S = 17.89\underline{/-18.43^\circ}$ V.

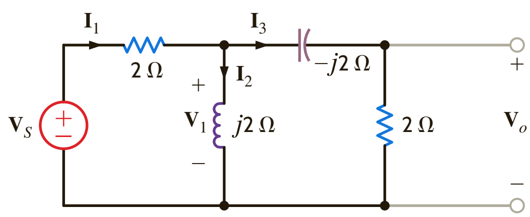


Figure E8.16

8.8 Analysis Techniques

In this section we revisit the circuit analysis methods that were successfully applied earlier to dc circuits and illustrate their applicability to ac steady-state analysis. The vehicle we employ to present these techniques is examples in which all the theorems, together with nodal analysis and loop analysis, are used to obtain a solution.

EXAMPLE 8.15

Let us determine the current $\mathbf{I}_o$ in the network in Fig. 8.17a using nodal analysis, loop analysis, superposition, source exchange, Thévenin's theorem, and Norton's theorem.

SOLUTION

1. ***Nodal Analysis*** We begin with a nodal analysis of the network. The KCL equation for the supernode that includes the voltage source is

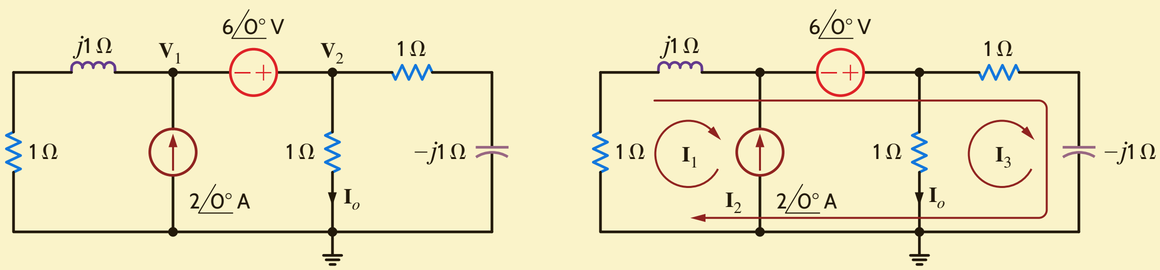
$$\frac{\mathbf{V}_1}{1+j} - 2\underline{/0^\circ} + \frac{\mathbf{V}_2}{1} + \frac{\mathbf{V}_2}{1-j} = 0$$

and the associated KVL constraint equation is

$$\mathbf{V}_1 + 6\underline{/0^\circ} = \mathbf{V}_2$$

HINT

Summing the current, leaving the supernode. Outbound currents have a positive sign.



(a) (b)

Figure 8.17

Circuits used in Example 8.15 for node and loop analysis.

The two equations in matrix form are

$$\begin{pmatrix} 0.5 - 0.5j & 1.5 + 0.5j \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$

The MATLAB solution is then

```
>> Y = [0.5-0.5j 1.5+0.5j; 1 -1]
   Y =
        0.5000-0.5000i       1.5000+0.5000i
        1.0000              -1.0000

>> I = [2; -6]
   I =
         2
        -6

>> V = inv(Y)*I
   V =
        -3.5000-1.5000i
         2.5000-1.5000i

>> abs (V)
         3.8079
         2.9155

>> 180*phase(v)/pi

   Ans =
        -156.8014
         -30.9638
```

And since $\mathbf{I}_0 = \mathbf{V}_2/1$, $\mathbf{I}_0 = 2.9155\underline{/-30.9638^\circ}$ A

```
>> 180*phase(V)/pi

   ans =
        -156.8014
         -30.9638
```

And since $\mathbf{I}_0 = \mathbf{V}_2/1$, $\mathbf{I}_0 = 2.9155\underline{/-30.9638^\circ}$ A.

HINT

Just as in a dc analysis, the loop equations assume that a decrease in potential level is + and an increase is −.

2. *Loop Analysis* The network in Fig. 8.17b is used to perform a loop analysis. Note that one loop current is selected that passes through the independent current source. The three loop equations are

$$\mathbf{I}_1 = -2\underline{/0^\circ}$$

$$1(\mathbf{I}_1 + \mathbf{I}_2) + j1(\mathbf{I}_1 + \mathbf{I}_2) - 6\underline{/0^\circ} + 1(\mathbf{I}_2 + \mathbf{I}_3) - j1(\mathbf{I}_2 + \mathbf{I}_3) = 0$$

$$1\mathbf{I}_3 + 1(\mathbf{I}_2 + \mathbf{I}_3) - j1(\mathbf{I}_2 + \mathbf{I}_3) = 0$$

Combining the first two equations yields

$$\mathbf{I}_2(2) + \mathbf{I}_3(1 - j) = 8 + 2j$$

The third loop equation can be simplified to the form

$$\mathbf{I}_2(1 - j) + \mathbf{I}_3(2 - j) = 0$$

The equations in matrix form are

$$\begin{bmatrix} 2 & 1-j \\ 1-j & 2-j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 8 + 2j \\ 0 \end{bmatrix}$$

The MATLAB solution is then

```
>> Z = [2 1-j; 1-j 2-j]
   Z =
         2.0000              1.0000 - 1.0000i
         1.0000 - 1.0000i    2.0000 - 1.0000i

>> V = [8+2*j; 0]
   V =
         8.0000 + 2.0000i
              0

>> I = inv(Z)*V
   I =
          4.5000 - 1.0000i
         -2.5000 + 1.5000i

>> abs (I)

   ans =
          4.6098
          2.9155

>> 180*phase(I)/pi

   ans =
           -12.5288
           149.0362
```

Therefore, $\mathbf{I}_3 = 2.9155\underline{/149.0362^\circ}$ and $\mathbf{I}_0 = -\mathbf{I}_3 = 2.9155\underline{/-30.9638^\circ}$ A.

3. *Superposition* In using superposition, we apply one independent source at a time. The network in which the current source acts alone is shown in Fig. 8.18a. By combining the two parallel impedances on each end of the network, we obtain the circuit in Fig. 8.18b, where

HINT
In applying superposition in this case, each source is applied independently, and the results are added to obtain the solution.

$$\mathbf{Z}' = \frac{(1 + j)(1 - j)}{(1 + j) + (1 - j)} = 1\ \Omega$$

Therefore, using current division,

$$\mathbf{I}_0' = 1\underline{/0^\circ}\text{ A}$$

The circuit in which the voltage source acts alone is shown in Fig. 8.18c. The voltage $\mathbf{V}_1''$ obtained using voltage division is

$$\mathbf{V}_1'' = \frac{(6\underline{/0^\circ})\left[\dfrac{1(1 - j)}{1 + 1 - j}\right]}{1 + j + \left[\dfrac{1(1 - j)}{1 + 1 - j}\right]}$$
$$= \frac{6(1 - j)}{4}\text{ V}$$

and hence,

$$\mathbf{I}_0'' = \frac{6}{4}(1 - j)\text{ A}$$

Then

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = 1 + \frac{6}{4}(1 - j) = 2.9155\underline{/-30.9638^\circ}\text{ A.}$$

4. *Source Exchange* As a first step in the source exchange approach, we exchange the current source and parallel impedance for a voltage source in series with the impedance, as shown in Fig. 8.19a.

Figure 8.18

Circuits used in Example 8.15 for a superposition analysis.

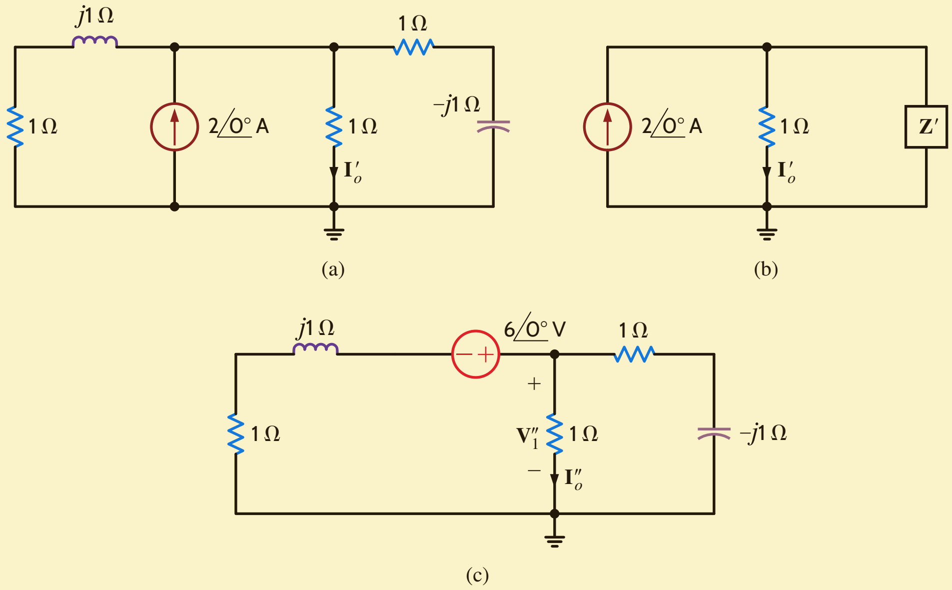
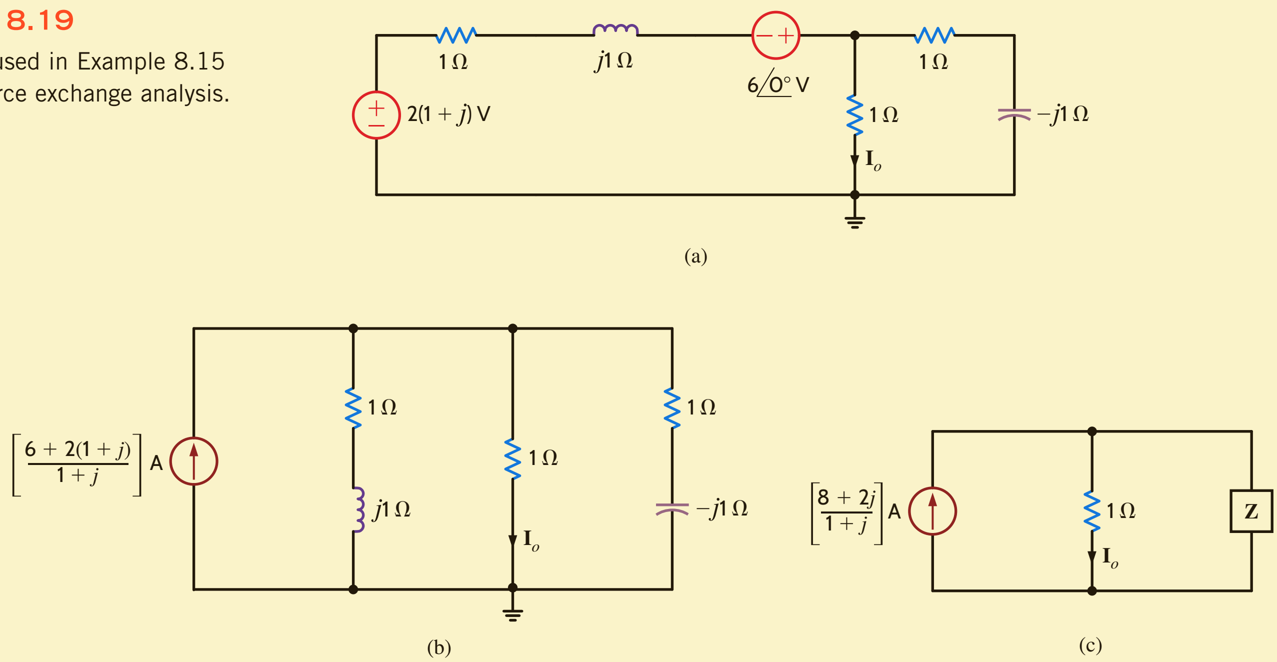


Figure 8.19

Circuits used in Example 8.15 for a source exchange analysis.



Adding the two voltage sources and transforming them and the series impedance into a current source in parallel with that impedance are shown in Fig. 8.19b. Combining the two impedances that are in parallel with the 1-Ω resistor produces the network in Fig. 8.19c, where

$$\mathbf{Z} = \frac{(1 + j)(1 - j)}{1 + j + 1 - j} = 1\ \Omega$$

Therefore, using current division,

$$\mathbf{I}_o = \left(\frac{8 + 2j}{1 + j}\right)\left(\frac{1}{2}\right) = \frac{4 + j}{1 + j}$$

$$= 2.9155\underline{/-30.9638^\circ}\ \text{A}$$

HINT

In source exchange, a voltage source in series with an impedance can be exchanged for a current source in parallel with the impedance and vice versa. Repeated application systematically reduces the number of circuit elements.

5. ***Thévenin Analysis*** In applying Thévenin's theorem to the circuit in Fig. 8.17a, we first find the open-circuit voltage, $\mathbf{V}_{oc}$, as shown in **Fig. 8.20a**. To simplify the analysis, we perform a source exchange on the left end of the network, which results in the circuit in **Fig. 8.20b**. Now using voltage division,

$$\mathbf{V}_{oc} = [6 + 2(1 + j)]\left[\frac{1 - j}{1 - j + 1 + j}\right]$$

or

$$\mathbf{V}_{oc} = (5 - 3j)\text{ V}$$

The Thévenin equivalent impedance, $\mathbf{Z}_{Th}$, obtained at the open-circuit terminals when the current source is replaced with an open circuit and the voltage source is replaced with a short circuit, is shown in **Fig. 8.20c** and calculated to be

$$\mathbf{Z}_{Th} = \frac{(1 + j)(1 - j)}{1 + j + 1 - j} = 1\ \Omega$$

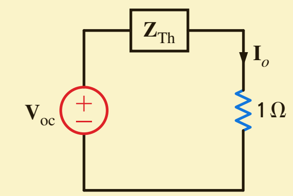
Connecting the Thévenin equivalent circuit to the 1-Ω resistor containing $\mathbf{I}_o$ in the original network yields the circuit in **Fig. 8.20d**. The current $\mathbf{I}_o$ is then

$$\mathbf{I}_o = 2.9155\underline{/-30.9638^\circ}\text{ A}$$

HINT

In this Thévenin analysis,

1. Remove the 1-Ω load and find the voltage across the open terminals, $\mathbf{V}_{oc}$.
2. Determine the impedance $\mathbf{Z}_{Th}$ at the open terminals with all sources made zero.
3. Construct the following circuit and determine $\mathbf{I}_o$.



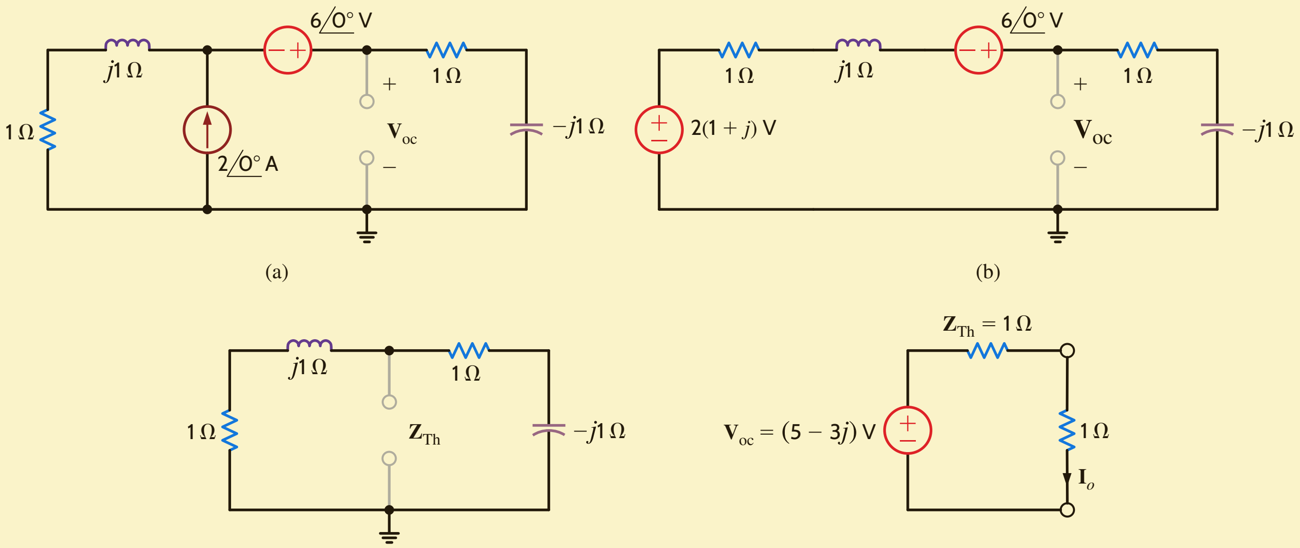


Figure 8.20

Circuits used in Example 8.15 for a Thévenin analysis.

6. ***Norton Analysis*** Finally, in applying Norton's theorem to the circuit in Fig. 8.17a, we calculate the short-circuit current, $\mathbf{I}_{sc}$, using the network in **Fig. 8.21a**. Note that because of the short circuit, the voltage source is directly across the impedance in the left-most branch. Therefore,

$$\mathbf{I}_1 = \frac{6\underline{/0^\circ}}{1 + j}$$

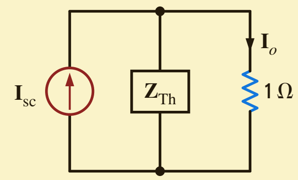
Then, using KCL,

$$\mathbf{I}_{sc} = \mathbf{I}_1 + 2\underline{/0^\circ} = 2 + \frac{6}{1 + j}$$
$$= \left(\frac{8 + 2j}{1 + j}\right)\text{ A}$$

HINT

In this Norton analysis,

1. Remove the 1-Ω load and find the current $\mathbf{I}_{sc}$ through the short-circuited terminals.
2. Determine the impedance $\mathbf{Z}_{Th}$ at the open load terminals with all sources made zero.
3. Construct the following circuit and determine $\mathbf{I}_o$.



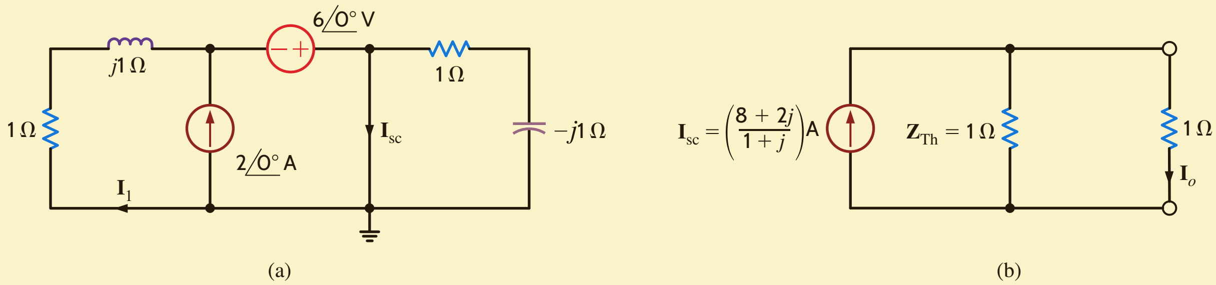


Figure 8.21

Circuits used in Example 8.15 for a Norton analysis.

The Thévenin equivalent impedance, $\mathbf{Z}_{\mathrm{Th}}$, is known to be 1 Ω and, therefore, connecting the Norton equivalent to the 1-Ω resistor containing $\mathbf{I}_o$ yields the network in Fig. 8.21b. Using current division, we find that

$$\mathbf{I}_o = \frac{1}{2}\left(\frac{8 + 2j}{1 + j}\right)$$
$$= 2.9155\angle -30.9638^\circ \text{ A}$$

Let us now consider an example containing a dependent source.

EXAMPLE 8.16

Let us determine the voltage $\mathbf{V}_o$ in the circuit in Fig. 8.22a. In this example we will use node equations, loop equations, Thévenin's theorem, and Norton's theorem. We will omit the techniques of superposition and source transformation. Why?

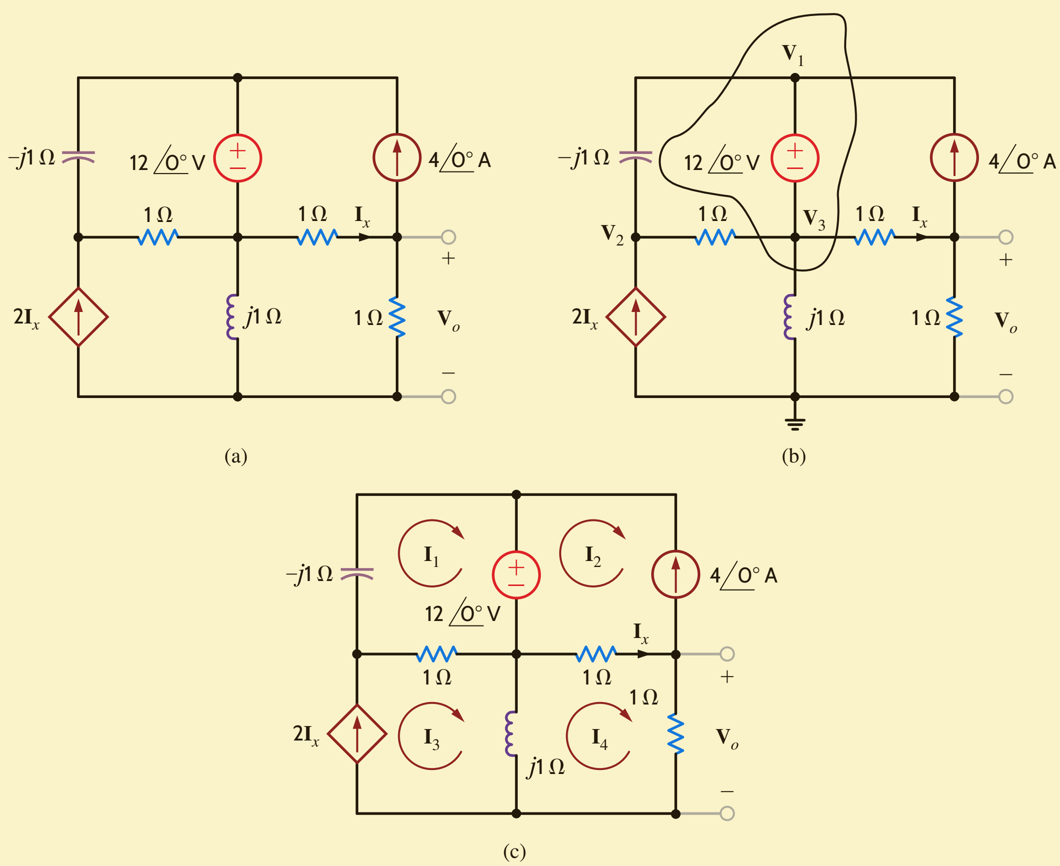
SOLUTION

1. ***Nodal Analysis*** To perform a nodal analysis, we label the node voltages and identify the supernode as shown in Fig. 8.22b. The constraint equation for the supernode is

$$\mathbf{V}_3 + 12\angle 0^\circ = \mathbf{V}_1$$

Figure 8.22

Circuits used in Example 8.16 for nodal and loop analysis.



and the KCL equations for the nodes of the network are

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j1} + \frac{\mathbf{V}_3 - \mathbf{V}_2}{1} - 4\underline{/0°} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{1} + \frac{\mathbf{V}_3}{j1} = 0$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j1} + \frac{\mathbf{V}_2 - \mathbf{V}_3}{1} - 2\left(\frac{\mathbf{V}_3 - \mathbf{V}_o}{1}\right) = 0$$

$$4\underline{/0°} + \frac{\mathbf{V}_o - \mathbf{V}_3}{1} + \frac{\mathbf{V}_o}{1} = 0$$

HINT

How does the presence of a dependent source affect superposition and source exchange?

The matrix equation is

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ j & -(1+j) & 2-j & -1 \\ -j & 1+j & -3 & 2 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_0 \end{bmatrix} = \begin{bmatrix} -12 \\ 4 \\ 0 \\ -4 \end{bmatrix}$$

The MATLAB solution is then

```
>> Y = [-1 0 1 0; j -(1+j) 2-j -1; -j 1+j -3 2;0 0 -1 2]
   Y =
        -1.0000                   0                1.0000                   0
              0 + 1.0000i   -1.0000 - 1.0000i    2.0000 - 1.0000i   -1.0000
              0 - 1.0000i    1.0000 + 1.0000i   -3.0000              2.0000
              0                   0             -1.0000              2.0000

>> I = [-12; 4; 0; -4]
   I =
           -12
             4
             0
            -4

>> V = inv(Y)*I
   V =
        9.6000 +  4.8000i
        6.8000 + 12.4000i
       -2.4000 +  4.8000i
       -3.2000 +  2.4000i

>> abs (V)
   ans =
        10.7331
        14.1421
         5.3666
         4.0000

>> 180*phase(V)/pi
   ans =
         26.5651
         61.2602
        116.5651
        143.1301
```

And therefore,

$$\mathbf{V}_o = 4\underline{/143.13°}\text{ V}$$

2. ***Loop Analysis*** The mesh currents for the network are defined in Fig. 8.22c. The constraint equations for the circuit are

$$\mathbf{I}_2 = -4\angle 0^\circ$$
$$\mathbf{I}_x = \mathbf{I}_4 - \mathbf{I}_2 = \mathbf{I}_4 + 4\angle 0^\circ$$
$$\mathbf{I}_3 = 2\mathbf{I}_x = 2\mathbf{I}_4 + 8\angle 0^\circ$$

The KVL equations for mesh 1 and mesh 4 are

$$-j1\mathbf{I}_1 + 1(\mathbf{I}_1 - \mathbf{I}_3) = -12\angle 0^\circ$$
$$j1(\mathbf{I}_4 - \mathbf{I}_3) + 1(\mathbf{I}_4 - \mathbf{I}_2) + 1\mathbf{I}_4 = 0$$

Note that if the constraint equations are substituted into the second KVL equation, the only unknown in the equation is $\mathbf{I}_4$. This substitution yields

$$\mathbf{I}_4 = 4\angle 143.13^\circ \text{ A}$$

and hence,

$$\mathbf{V}_o = 4\angle 143.13^\circ \text{ V}$$

3. ***Thévenin's Theorem*** In applying Thévenin's theorem, we will find the open-circuit voltage and then determine the Thévenin equivalent impedance using a test source at the open-circuit terminals. We could determine the Thévenin equivalent impedance by calculating the short-circuit current; however, we will determine this current when we apply Norton's theorem.

The open-circuit voltage is determined from the network in Fig. 8.23a. Note that $\mathbf{I}'_x = 4\angle 0^\circ$ A and since $2\mathbf{I}'_x$ flows through the inductor, the open-circuit voltage $\mathbf{V}_{oc}$ is

$$\mathbf{V}_{oc} = -1(4\angle 0^\circ) + j1(2\mathbf{I}'_x)$$
$$= -4 + j8 \text{ V}$$

To determine the Thévenin equivalent impedance, we turn off the independent sources, apply a test voltage source to the output terminals, and compute the current leaving the test source. As shown in Fig. 8.23b, since $\mathbf{I}''_x$ flows in the test source, KCL requires that the current in the inductor be $\mathbf{I}''_x$ also. KVL around the mesh containing the test source indicates that

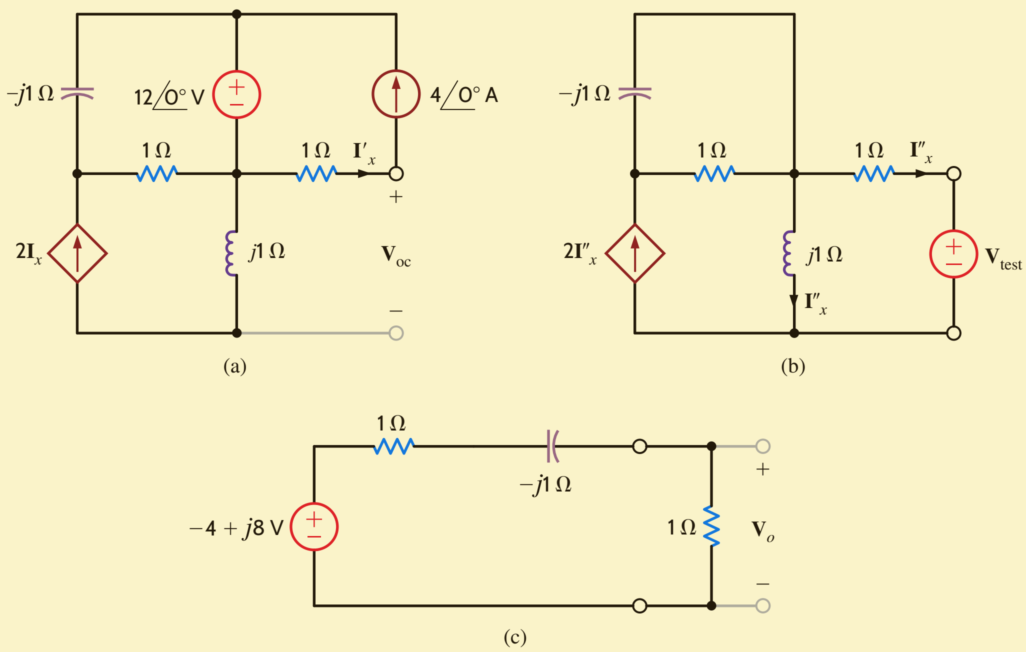


Figure 8.23 Circuits used in Example 8.16 when applying Thévenin's theorem.

$$j1\mathbf{I}''_x - 1\mathbf{I}''_x - \mathbf{V}_{\text{test}} = 0$$

Therefore,

$$\mathbf{I}''_x = \frac{-\mathbf{V}_{\text{test}}}{1 - j}$$

Then

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_{\text{test}}}{-\mathbf{I}''_x}$$
$$= 1 - j\,\Omega$$

If the Thévenin equivalent network is now connected to the load, as shown in **Fig. 8.23c**, the output voltage $\mathbf{V}_o$ is found to be

$$\mathbf{V}_o = \frac{-4 + 8j}{2 - j1}(1)$$
$$= 4\underline{/143.13^\circ}\text{ V}$$

4. ***Norton's Theorem*** In using Norton's theorem, we will find the short-circuit current from the network in **Fig. 8.24a**. Once again, using the supernode, the constraint and KCL equations are

$$\mathbf{V}_3 + 12\underline{/0^\circ} = \mathbf{V}_1$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j1} + \frac{\mathbf{V}_2 - \mathbf{V}_3}{1} - 2\mathbf{I}'''_x = 0$$

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j1} + \frac{\mathbf{V}_3 - \mathbf{V}_2}{1} - 4\underline{/0^\circ} + \frac{\mathbf{V}_3}{j1} + \mathbf{I}'''_x = 0$$

$$\mathbf{I}'''_x = \frac{\mathbf{V}_3}{1}$$

The matrix equation is

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ -j & (1+j) & -1 & -2 \\ j & -(1+j) & 1-j & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{I}'''_x \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

The MATLAB solution is then

```
>> Y = [-1 0 1 0; -j 1+j -1 -2; j -(1+j) 1-j 1; 0 0 -1 1]
   Y =
        -1.0000                     0                1.0000                   0
              0 - 1.0000i    1.0000 + 1.0000i    -1.0000             -2.0000
              0 + 1.0000i   -1.0000 - 1.0000i     1.0000 - 1.0000i    1.0000
              0                     0            -1.0000              1.0000

>> I = [-12; 0; 4; 0]
   I =
           -12
             0
             4
             0

>> V = inv(Y)*I
   V =
       10.0000 +  2.0000i
        4.0000 + 12.0000i
       -2.0000 +  2.0000i
       -2.0000 +  2.0000i
```

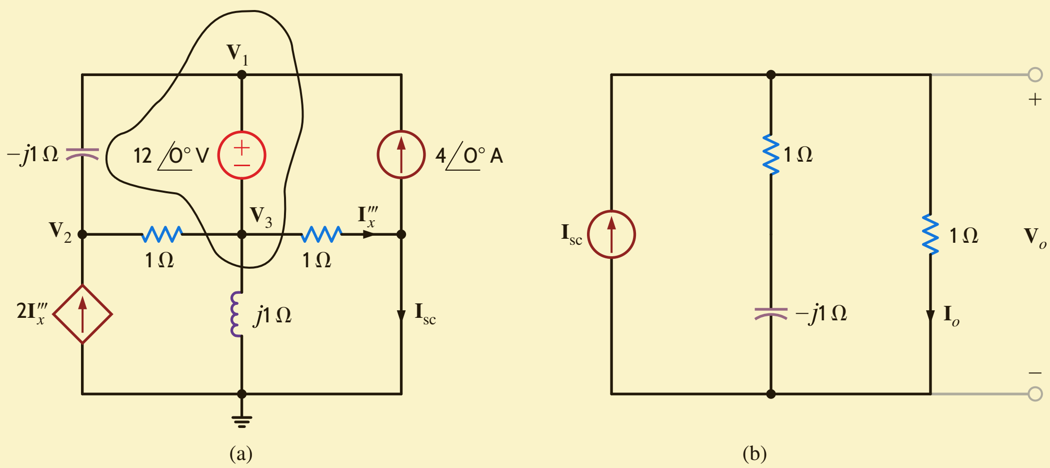


Figure 8.24

Circuits used in Example 8.16 when applying Norton's theorem.

```
>> abs (V)
   ans =
          10.1980
          12.6491
           2.8284
           2.8284
>> 180*phase(V)/pi
   ans =
          11.3099
          71.5651
         135.0000
         135.0000
```

or

$$\mathbf{I}_x''' = 2.8284\underline{/135^\circ}\ \text{A}$$

The KCL equation at the right-most node in the network in Fig. 8.24a is

$$\mathbf{I}_x''' = 4\underline{/0^\circ} + \mathbf{I}_{sc}$$

Solving for $\mathbf{I}_{sc}$, we obtain

$$\mathbf{I}_{sc} = 6.3245\underline{/161.57^\circ}\ \text{A}$$

The Thévenin equivalent impedance was found earlier to be

$$\mathbf{Z}_{Th} = 1 - j\ \Omega$$

Using the Norton equivalent network, the original network is reduced to that shown in Fig. 8.24b. The voltage $\mathbf{V}_o$ is then

$$\begin{aligned}\mathbf{V}_o &= \mathbf{I}_{sc}\{(1)(1 - j)/(1 + 1 - j)\}\ \text{V}\\ &= 4\underline{/143.13^\circ}\ \text{V}\end{aligned}$$

LEARNING ASSESSMENTS

E8.17 Use nodal analysis to find $\mathbf{V}_o$ in the network in Fig. E8.17.

ANSWER: $\mathbf{V}_o = 2.12\underline{/75°}$ V.

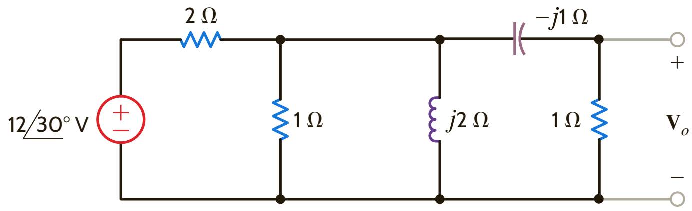


Figure E8.17

E8.18 Find $\mathbf{I}_1$ in Fig. E8.18 using nodal analysis.

ANSWER: $\mathbf{I}_1 = 0.7781\underline{/-161.9°}$ A.

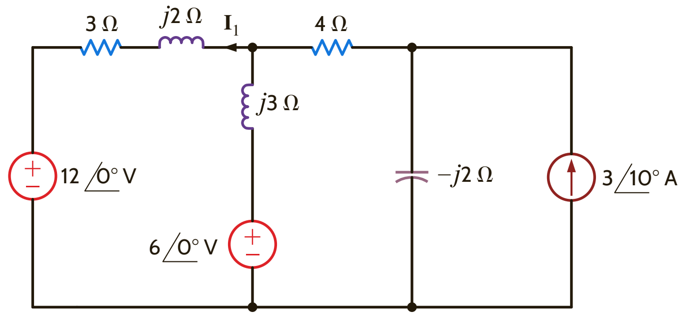


Figure E8.18

E8.19 Find $\mathbf{V}_x$ in Fig. E8.19 using (a) nodal analysis and (b) mesh analysis.

ANSWER: $\mathbf{V}_x = 17.4\underline{/-21.62°}$ V.

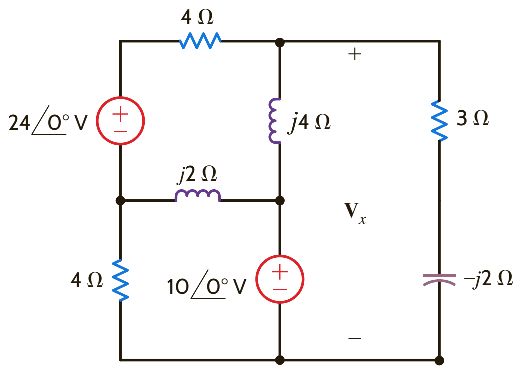


Figure E8.19

E8.20 Use (a) mesh equations and (b) Thévenin's theorem to find $\mathbf{V}_o$ in the network in Fig. E8.20.

ANSWER: $\mathbf{V}_o = 10.88\underline{/36°}$ V.

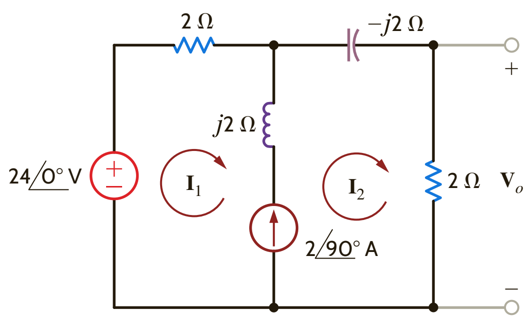


Figure E8.20

E8.21 Find $\mathbf{V}_o$ in Fig. E8.21 using mesh analysis.

ANSWER:
$\mathbf{V}_o = 1.4654\underline{/-12.34^\circ}$ V.

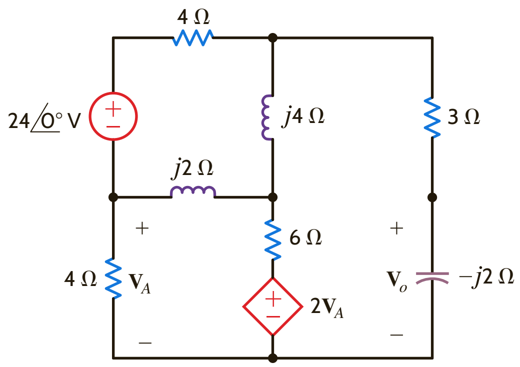


Figure E8.21

E8.22 Find $\mathbf{I}_1$ in Fig. E8.18 using superposition.

ANSWER:
$\mathbf{I}_1 = 0.7781\underline{/-161.9^\circ}$ V.

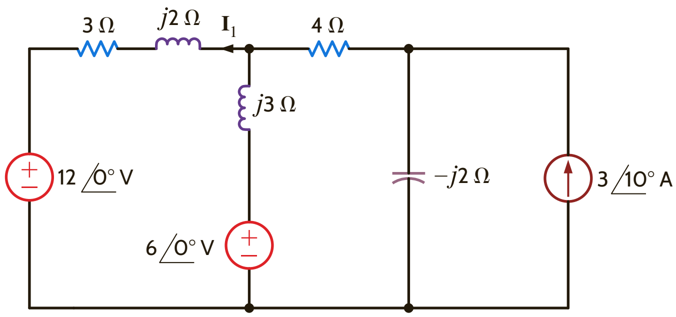


Figure E8.22

E8.23 Use (a) superposition, (b) source transformation, and (c) Norton's theorem to find $\mathbf{V}_o$ in the network in Fig. E8.23.

ANSWER:
$\mathbf{V}_o = 12\underline{/90^\circ}$ V.

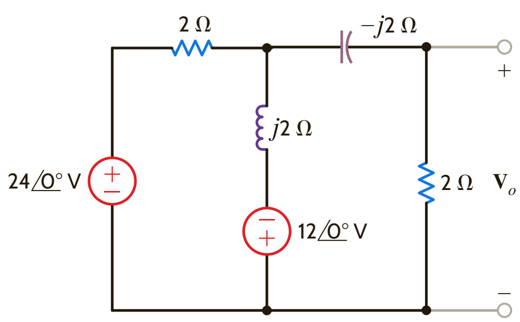


Figure E8.23

E8.24 Find $\mathbf{V}_o$ in Fig. E8.24 using Thevenin's theorem.

ANSWER:
$\mathbf{V}_o = 1.4654\underline{/-12.34^\circ}$ V.

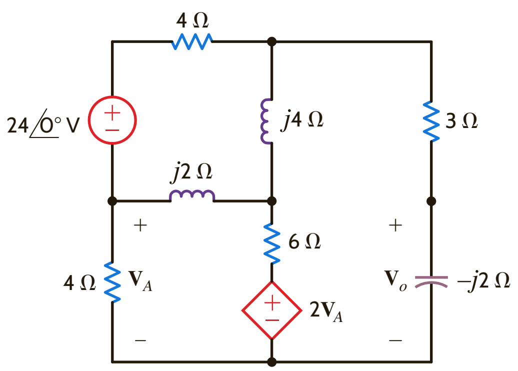


Figure E8.24

EXAMPLE 8.17

Let's solve for the current $i(t)$ in the circuit in Fig. 8.25. At first glance, this appears to be a simple single-loop circuit. A more detailed observation reveals that the two sources operate at different frequencies. The radian frequency for the source on the left is 10 rad/s, while the source on the right operates at a radian frequency of 20 rad/s. If we draw a frequency-domain circuit, which frequency do we use? How can we solve this problem?

SOLUTION

Recall that the principle of superposition tells us that we can analyze the circuit with each source operating alone. The circuit responses to each source acting alone are then added together to give us the response with both sources active. Let's use the principle of superposition to solve this problem. First, calculate the response $i'(t)$ from the source on the left using the circuit shown in Fig. 8.26a. Now we can draw a frequency-domain circuit for $\omega = 10$ rad/s, as shown in Fig. 8.26b.

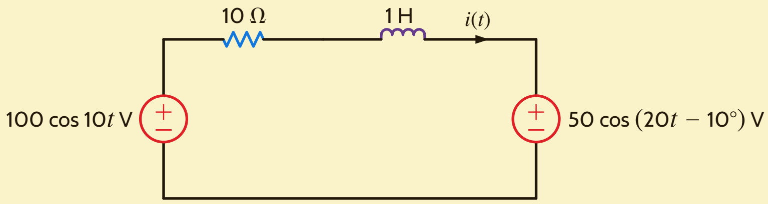


Figure 8.25 Circuit used in Example 8.17.

Then $\mathbf{I}' = \dfrac{100\angle 0^\circ}{10 + j10} = 7.07\angle -45^\circ$ A. Therefore, $i'(t) = 7.07\cos(10t - 45^\circ)$ A.

The response due to the source on the right can be determined using the circuit in Fig. 8.27a. Note that $i''(t)$ is defined in the opposite direction to $i(t)$ in the original circuit. The frequency-domain circuit for $\omega = 20$ rad/s is also shown in Fig. 8.27b.

The current $\mathbf{I}'' = \dfrac{50\angle -10^\circ}{10 + j20} = 2.24\angle -73.43^\circ$ A. Therefore, $i''(t) = 2.24\cos(20t - 73.43^\circ)$ A.

The current $i(t)$ can now be calculated as $i'(t) - i''(t) = 7.07\cos(10t - 45^\circ) - 2.24\cos(20t - 73.43^\circ)$ A.

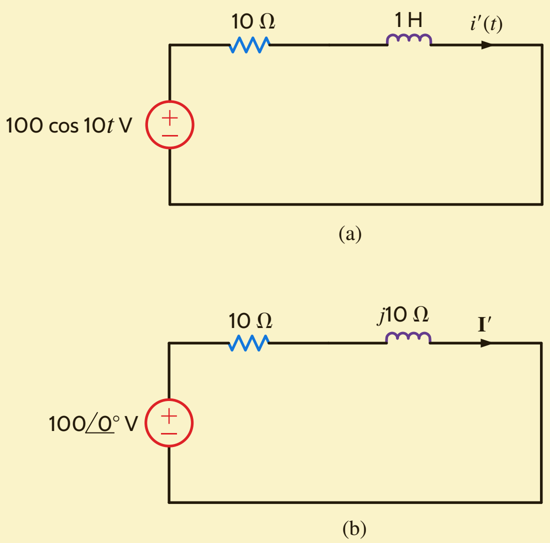


Figure 8.26 Circuits used to illustrate superposition.

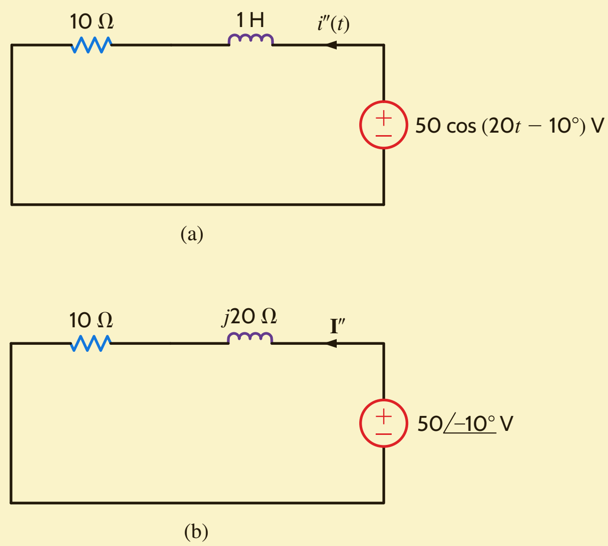


Figure 8.27 Circuits used to illustrate superposition.