E6.5 The waveform for the current in a 1-nF capacitor is Fig. E6.5. If the capacitor has an initial voltage of -5 V, determine the waveform for the capacitor voltage. How much energy is stored in the capacitor at $t = 6$ ms?
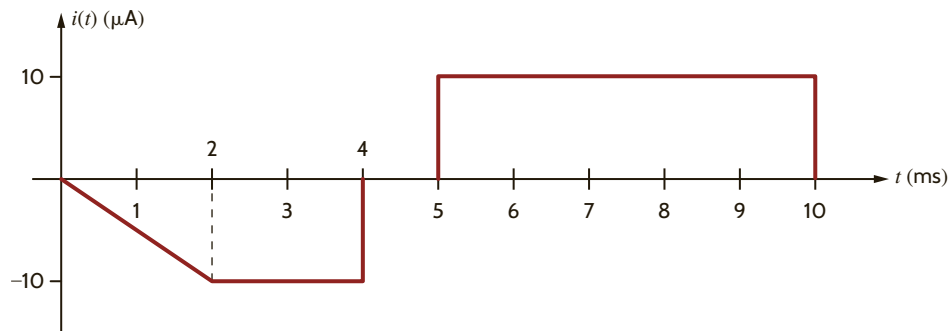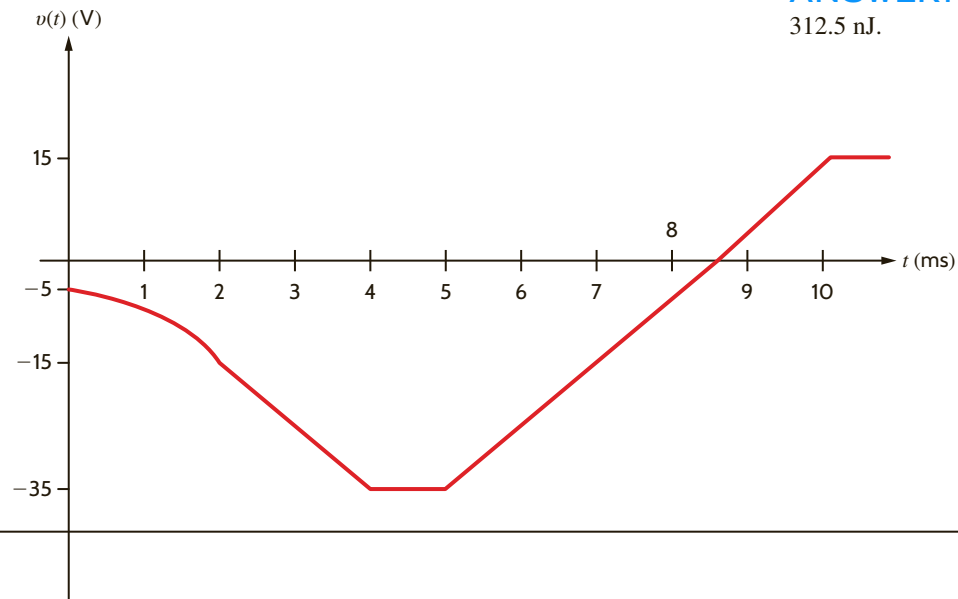

Figure E6.5

ANSWER:
312.5 nJ.

An *inductor* is a circuit element that consists of a conducting wire usually in the form of a coil. Two typical inductors and their electrical symbol are shown in **Fig. 6.6**. Inductors are typically categorized by the type of core on which they are wound. For example, the core material may be air or any nonmagnetic material, iron, or ferrite. Inductors made with air or nonmagnetic materials are widely used in radio, television, and filter circuits. Iron-core inductors are used in electrical power supplies and filters. Ferrite-core inductors are widely used in high-frequency applications. Note that in contrast to the magnetic core that confines the flux, as shown in **Fig. 6.6b**, the flux lines for nonmagnetic inductors extend beyond the inductor itself, as illustrated in **Fig. 6.6a**. Like stray capacitance, stray inductance can result from any element carrying current surrounded by flux linkages. **Fig. 6.7** shows a variety of typical inductors.
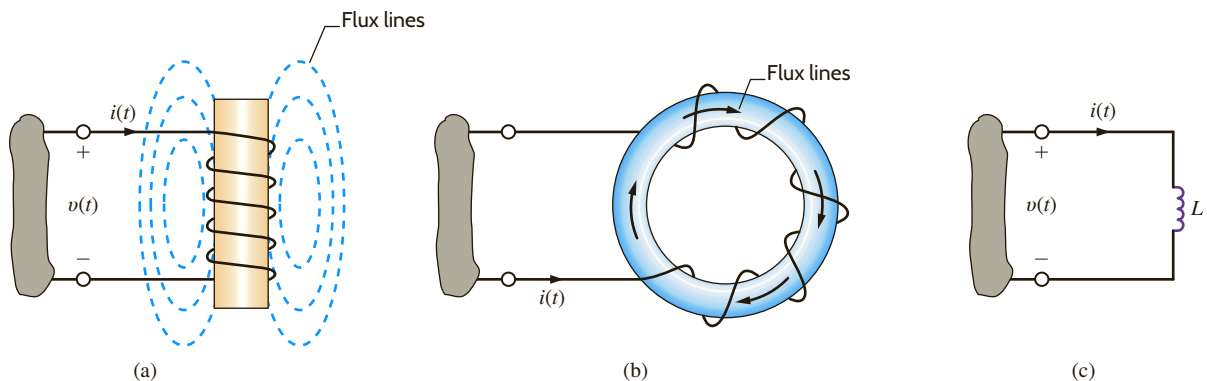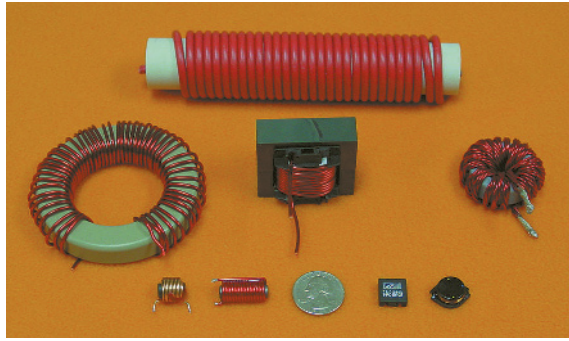
6.2

Inductors

Figure 6.6

Two inductors and their electrical symbol.

(a)   (b)   (c)

Figure 6.7

Some typical inductors
(Courtesy of Mark Nelms
and Jo Ann Loden).

From a historical standpoint, developments that led to the mathematical model we employ to represent the inductor are as follows. It was first shown that a current-carrying conductor would produce a magnetic field. It was later found that the magnetic field and the current that produced it were linearly related. Finally, it was shown that a changing magnetic field produced a voltage that was proportional to the time rate of change of the current that produced the magnetic field; that is,

$$v(t) = L\frac{di(t)}{dt} \qquad \text{6.8}$$

The constant of proportionality L is called the inductance and is measured in the unit *henry*, named after the American inventor Joseph Henry, who discovered the relationship. As seen in Eq. (6.8), 1 henry (H) is dimensionally equal to 1 volt-second per ampere.

Following the development of the mathematical equations for the capacitor, we find that the expression for the current in an inductor is

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)\,dx \qquad \text{6.9}$$

which can also be written as

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(x)\,dx \qquad \text{6.10}$$

The power delivered to the inductor can be used to derive the energy stored in the element. This power is equal to

$$p(t) = v(t)i(t)$$

$$= \left[L\frac{di(t)}{dt}\right]i(t) \qquad \text{6.11}$$

Therefore, the energy stored in the magnetic field is

$$w_L(t) = \int_{-\infty}^{t}\left[L\frac{di(x)}{dx}\right]i(x)\,dx$$

Following the development of Eq. (6.6), we obtain

$$w_L(t) = \frac{1}{2}Li^2(t)\ \text{J} \qquad \text{6.12}$$

Now let's consider the case of a dc current flowing through an inductor. From Eq. (6.8), we see that the voltage across the inductor is directly proportional to the time rate of change of the current flowing through the inductor. A dc current does not vary with time, so the voltage across the inductor is zero. We can say that an inductor is "a short circuit to dc." In analyzing a circuit containing dc sources and inductors, we can replace any inductors with short circuits and calculate voltages and currents in the circuit using our many analysis tools.

Note from Eq. (6.11) that an instantaneous change in inductor current would require infinite power. Since we don't have any infinite power sources, the current flowing through an inductor cannot change instantaneously. This will be a particularly helpful idea in the next chapter when we encounter circuits containing switches. This idea of "continuity of current" for an inductor tells us that the current flowing through an inductor just after a switch moves is the same as the current flowing through an inductor just before that switch moves.

Find the total energy stored in the circuit of **Fig. 6.8a**.
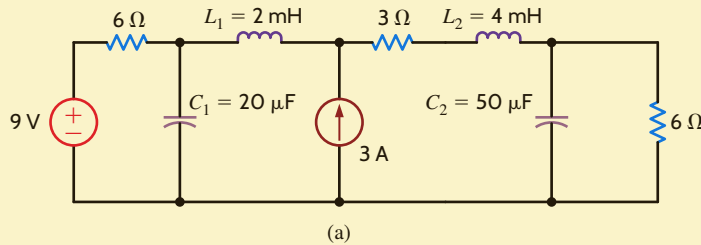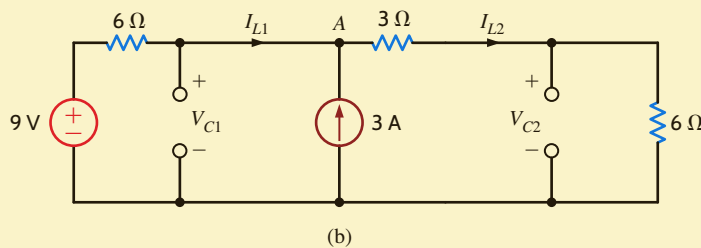
EXAMPLE 6.5

Figure 6.8

Circuits used in Example 6.5.

(a)

(b)

This circuit has only dc sources. Based on our earlier discussions about capacitors and inductors and constant sources, we can replace the capacitors with open circuits and the inductors with short circuits. The resulting circuit is shown in **Fig. 6.8b**.

SOLUTION

This resistive circuit can now be solved using any of the techniques we have learned in earlier chapters. If we apply KCL at node A, we get

$$I_{L2} = I_{L1} + 3$$

Applying KVL around the outside of the circuit yields

$$6I_{L1} + 3I_{L2} + 6I_{L2} = 9$$

Solving these equations yields $I_{L1} = -1.2$ A and $I_{L2} = 1.8$ A. The voltages V_{C1} and V_{C2} can be calculated from the currents:

$$V_{C1} = -6I_{L1} + 9 = 16.2 \text{ V}$$
$$V_{C2} = 6I_{L2} = 6(1.8) = 10.8 \text{ V}$$

The total energy stored in the circuit is the sum of the energy stored in the two inductors and two capacitors:

$$w_{L1} = \frac{1}{2}(2 \times 10^{-3})(-1.2)^2 = 1.44 \text{ mJ}$$

$$w_{L2} = \frac{1}{2}(4 \times 10^{-3})(1.8)^2 = 6.48 \text{ mJ}$$

$$w_{C1} = \frac{1}{2}(20 \times 10^{-6})(16.2)^2 = 2.62 \text{ mJ}$$

$$w_{C2} = \frac{1}{2}(50 \times 10^{-6})(10.8)^2 = 2.92 \text{ mJ}$$

The total stored energy is 13.46 mJ.

The inductor, like the resistor and capacitor, is a passive element. The polarity of the voltage across the inductor is shown in Fig. 6.6.

Practical inductors typically range from a few microhenrys to tens of henrys. From a circuit design standpoint it is important to note that inductors cannot be easily fabricated on an integrated circuit chip, and therefore chip designs typically employ only active electronic devices, resistors, and capacitors that can be easily fabricated in microcircuit form.

EXAMPLE 6.6

The current in a 10-mH inductor has the waveform shown in **Fig. 6.9a**. Determine the voltage waveform.

SOLUTION   Using Eq. (6.8) and noting that

$$i(t) = \frac{20 \times 10^{-3}t}{2 \times 10^{-3}} \qquad 0 \le t \le 2 \text{ ms}$$
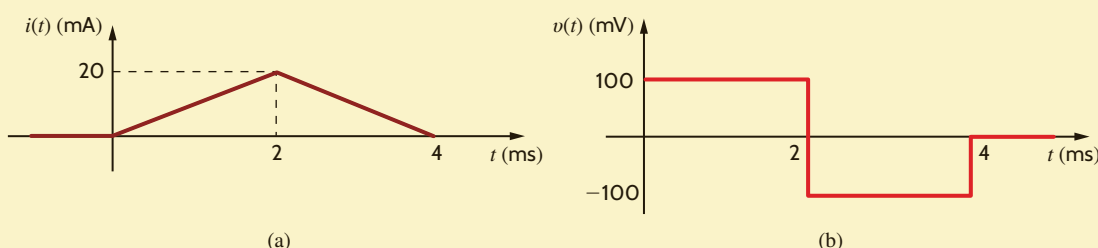
$$i(t) = \frac{-20 \times 10^{-3}t}{2 \times 10^{-3}} + 40 \times 10^{-3} \qquad 2 \le t \le 4 \text{ ms}$$

and

$$i(t) = 0 \qquad 4 \text{ ms} < t$$

Figure 6.9

Current and voltage waveforms for a 10-mH inductor.

(a)   (b)

we find that

$$v(t) = (10 \times 10^{-3}) \frac{20 \times 10^{-3}}{2 \times 10^{-3}} \qquad 0 \le t \le 2 \text{ ms}$$

$$= 100 \text{ mV}$$

and

$$v(t) = (10 \times 10^{-3}) \frac{-20 \times 10^{-3}}{2 \times 10^{-3}} \qquad 2 \le t \le 4 \text{ ms}$$

$$= -100 \text{ mV}$$

and $v(t) = 0$ for $t > 4$ ms. Therefore, the voltage waveform is shown in **Fig. 6.9b**.

EXAMPLE 6.7

The current in a 2-mH inductor is

$$i(t) = 2 \sin 377t \text{ A}$$

SOLUTION   Determine the voltage across the inductor and the energy stored in the inductor.

From Eq. (6.8), we have

$$v(t) = L\frac{di(t)}{dt}$$

$$= (2 \times 10^{-3})\frac{d}{dt}(2 \sin 377t)$$

$$= 1.508 \cos 377t \text{ V}$$

and from Eq. (6.12),

$$w_L(t) = \frac{1}{2}Li^2(t)$$

$$= \frac{1}{2}(2 \times 10^{-3})(2 \sin 377t)^2$$

$$= 0.004 \sin^2 377t \text{ J}$$

EXAMPLE 6.8

The voltage across a 200-mH inductor is given by the expression

$$v(t) = (1 - 3t)e^{-3t} \text{ mV} \qquad t \geq 0$$

$$= 0 \qquad\qquad t < 0$$

Let us derive the waveforms for the current, energy, and power.

SOLUTION

The waveform for the voltage is shown in **Fig. 6.10a**. The current is derived from Eq. (6.10) as

$$i(t) = \frac{10^3}{200}\int_0^t (1 - 3x)e^{-3x}\,dx$$

$$= 5\left\{\int_0^t e^{-3x}\,dx - 3\int_0^t xe^{-3x}\,dx\right\}$$

$$= 5\left\{\frac{e^{-3x}}{-3}\Big|_0^t - 3\left[-\frac{e^{-3x}}{9}(3x + 1)\right]_0^t\right\}$$

$$= 5te^{-3t} \text{ mA} \qquad t \geq 0$$

$$= 0 \qquad\qquad t < 0$$

A plot of the current waveform is shown in **Fig. 6.10b**.

The power is given by the expression

$$p(t) = v(t)i(t)$$

$$= 5t(1 - 3t)e^{-6t} \text{ μW} \qquad t \geq 0$$

$$= 0 \qquad\qquad\qquad t < 0$$

The equation for the power is plotted in **Fig. 6.10c**.

The expression for the energy is

$$w(t) = \frac{1}{2}Li^2(t)$$

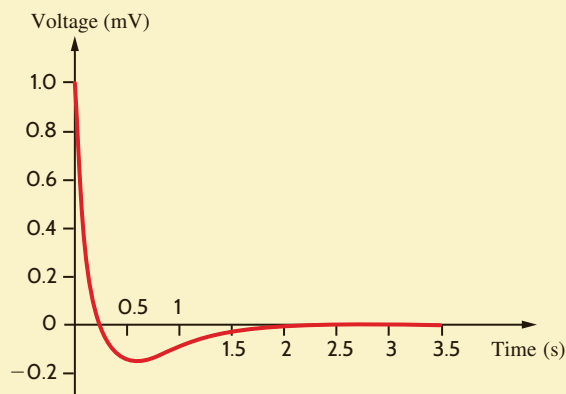$$= 2.5t^2e^{-6t} \text{ μJ} \qquad t \geq 0$$

$$= 0 \qquad\qquad t < 0$$

This equation is plotted in **Fig. 6.10d**.

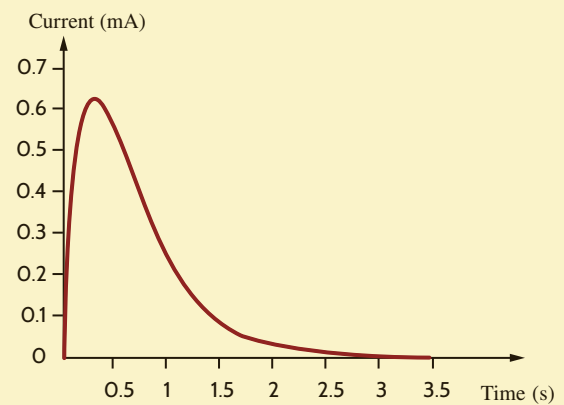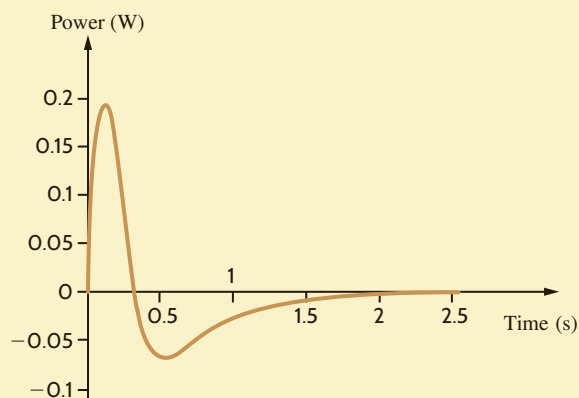**Figure 6.10**

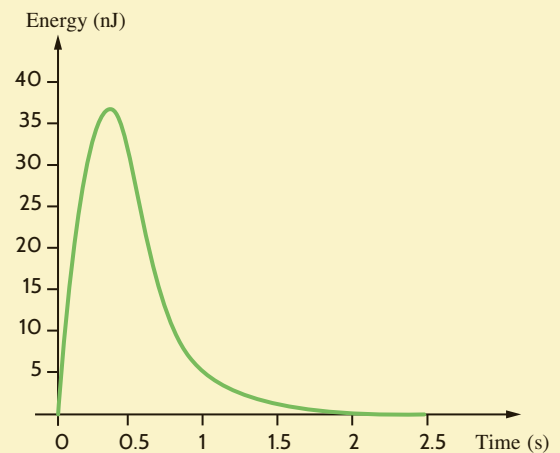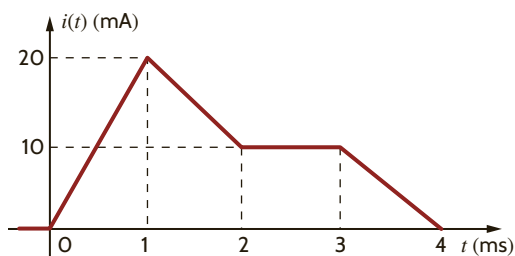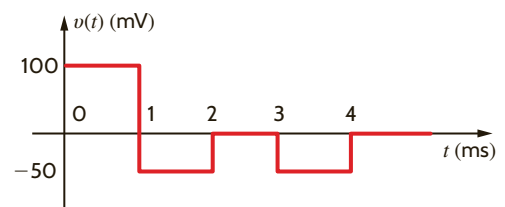Waveforms used in Example 6.8.

(a)

(b)

(c)

(d)

LEARNING ASSESSMENTS

E6.6 The current in a 5-mH inductor has the waveform shown in Fig. E6.6. Compute the waveform for the inductor voltage.

ANSWER:

Figure E6.6

E6.7 Compute the energy stored in the magnetic field of the inductor in Learning Assessment E6.6 at $t = 1.5$ ms.

ANSWER:
$W = 562.5$ nJ.

E6.8 The current in a 2-H inductor is shown in Fig. E6.8. Find the waveform for the inductor voltage. How much energy is stored in the inductor at $t = 3$ ms?
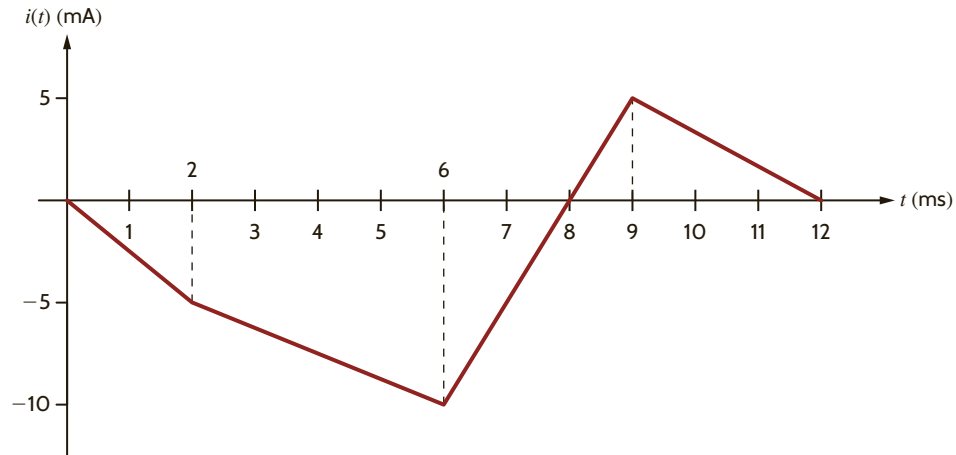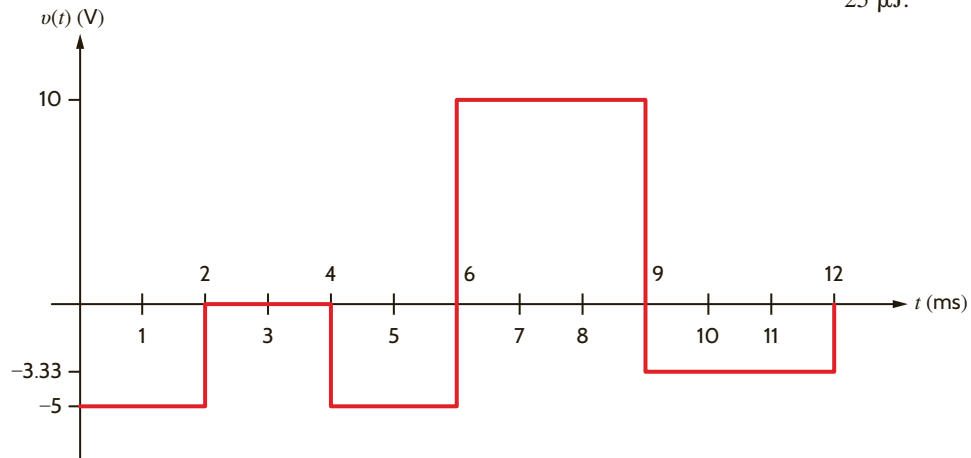

Figure E6.8

ANSWER:
25 μJ.

E6.9 The voltage across a 0.1-H inductor is shown in Fig. E6.9. Compute the waveform for the current in the inductor if $i(0) = 0.1$ A. How much energy is stored in the inductor at $t = 7$ ms?
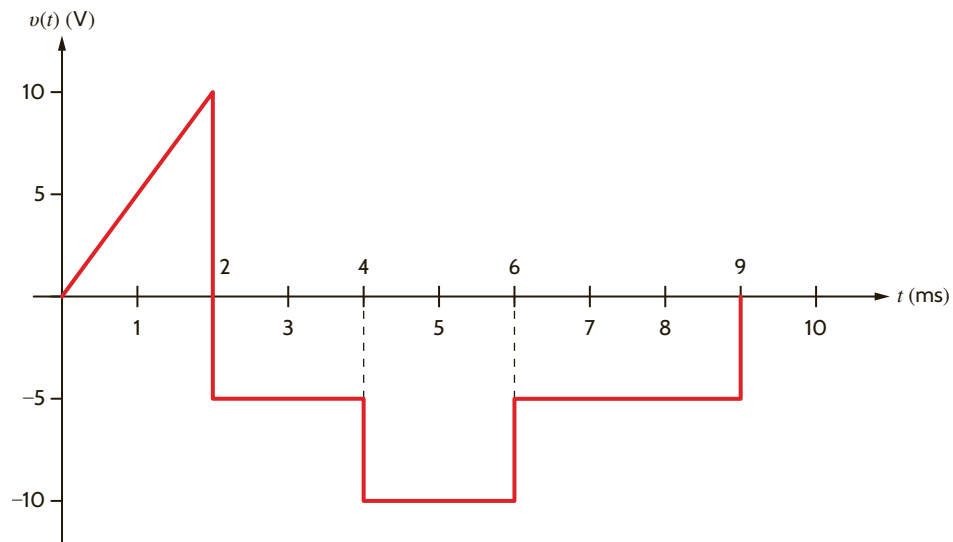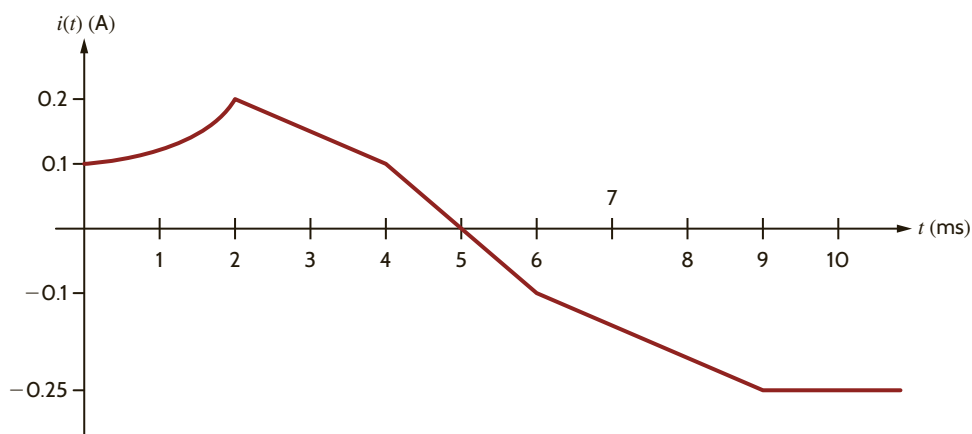

Figure E6.9

ANSWER:
1.125 mJ.

E6.10 Find the energy stored in the capacitor and inductor in Fig. E6.10.

ANSWER:
$0.72\ \mu\text{J}; 0.5\ \mu\text{J}.$
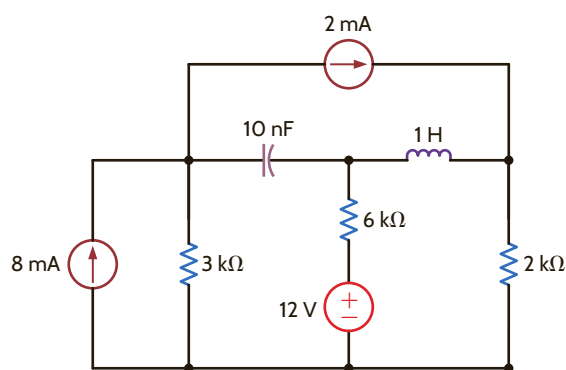

Figure E6.10

EXAMPLE 6.9

We wish to find the possible range of capacitance values for a 51-mF capacitor that has a tolerance of 20%.

SOLUTION   The minimum capacitor value is $0.8C = 40.8$ mF, and the maximum capacitor value is $1.2C = 61.2$ mF.
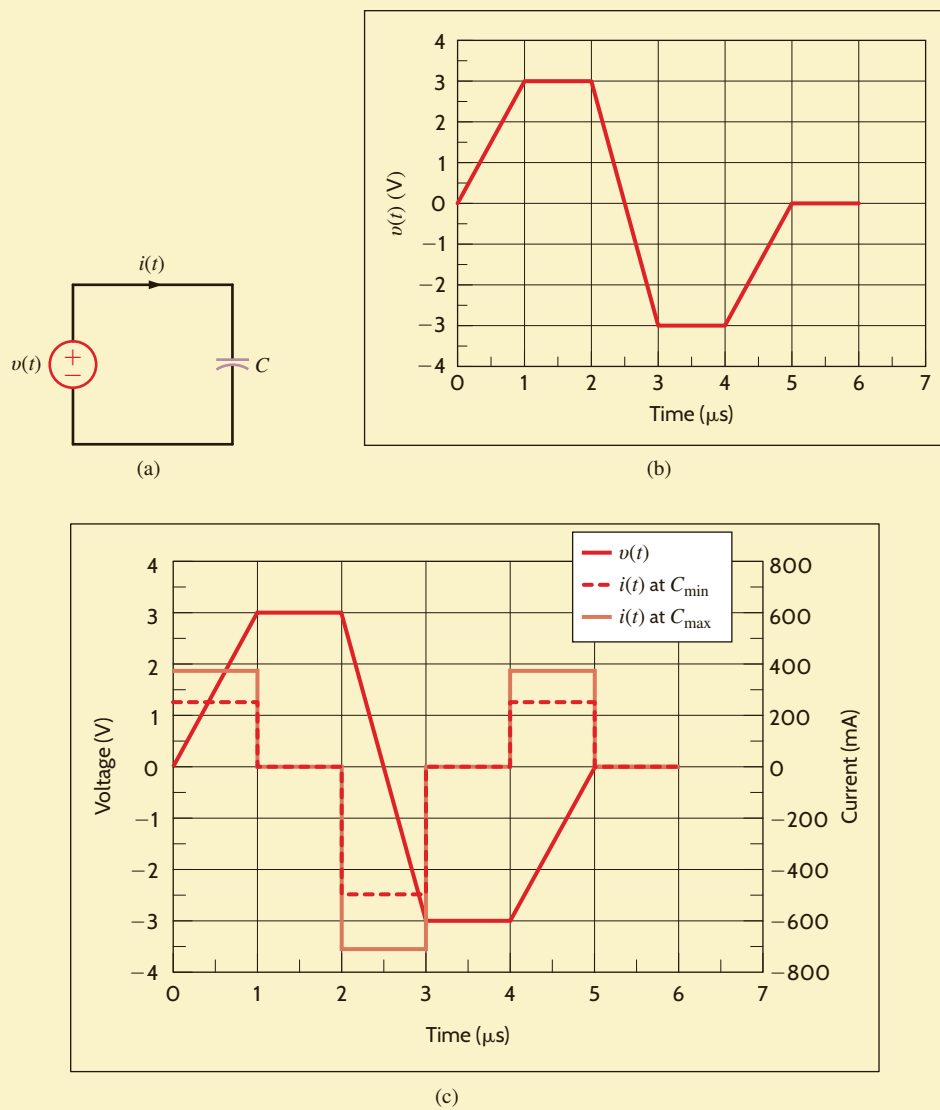

EXAMPLE 6.10

The capacitor in **Fig. 6.11a** is a 100-nF capacitor with a tolerance of 20%. If the voltage waveform is as shown in **Fig. 6.11b**, let us graph the current waveform for the minimum and maximum capacitor values.

SOLUTION   The maximum capacitor value is $1.2C = 120$ nF, and the minimum capacitor value is $0.8C = 80$ nF. The maximum and minimum capacitor currents, obtained from the equation

$$i(t) = C\frac{dv(t)}{dt}$$

are shown in **Fig. 6.11c**.

Figure 6.11

Circuit and graphs used in Example 6.10.

The inductor in **Fig. 6.12a** is a 100-μH inductor with a tolerance of 10%. If the current waveform is as shown in **Fig. 6.12b**, let us graph the voltage waveform for the minimum and maximum inductor values.

EXAMPLE 6.11

The maximum inductor value is $1.1L = 110$ μH, and the minimum inductor value is $0.9L = 90$ μH. The maximum and minimum inductor voltages, obtained from the equation

SOLUTION

$$v(t) = L\frac{di(t)}{dt}$$

are shown in **Fig. 6.12c**.