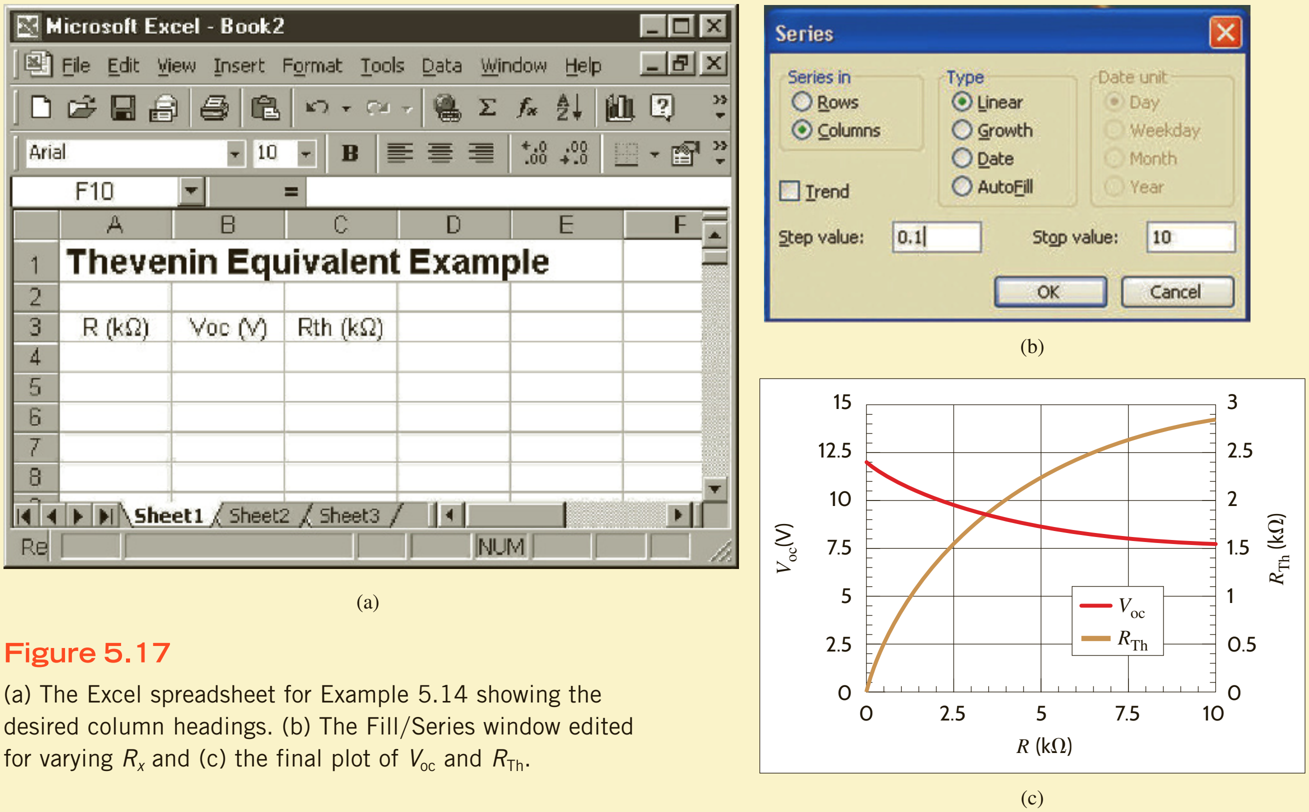

Figure 5.17

(a) The Excel spreadsheet for Example 5.14 showing the desired column headings. (b) The Fill/Series window edited for varying R_x and (c) the final plot of V_{oc} and R_{Th}.

5.4 Maximum Power Transfer

As we begin our discussion of maximum power transfer, it is instructive to examine what is known as a *load line*. This graphical technique is typically employed in (nonlinear) electronic circuits, and used to determine the operating point of the network. Consider, for example, the network in Fig. 5.18a. The load line is used to represent the relationship between voltage and current in the linear portion of the circuit, represented in this case by the 12-V source and 2-kΩ resistor. This line is shown in Fig. 5.18b and defined by the two points along the axes. The load, which may be linear or nonlinear, has a characteristic curve that defines its voltage/current relationship. If the load is a 4-kΩ resistor, as illustrated in Fig. 5.18c, its characteristic curve will appear as shown in Fig. 5.18b. The operating point is defined as the point

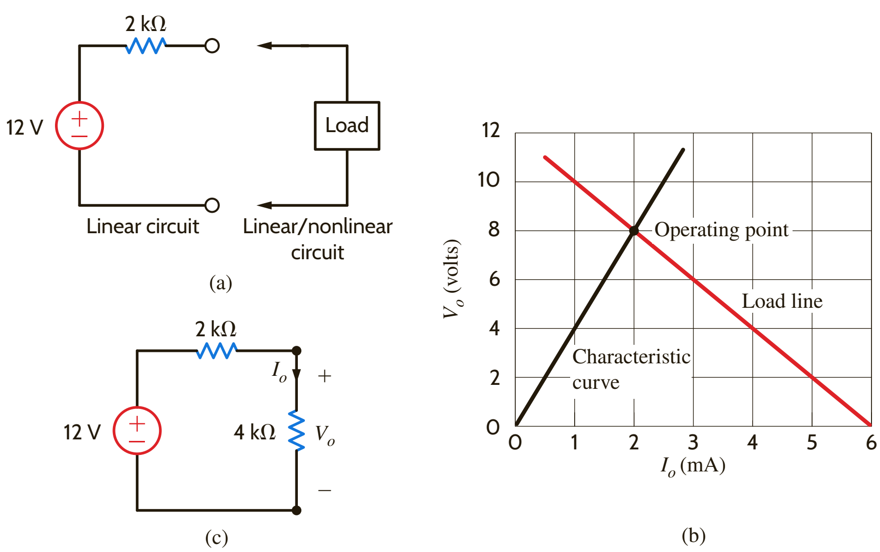

Figure 5.18

Load line analysis.

at which the characteristic curve intersects the load line, because at this point both the voltage and current parameters for each circuit match. Thus, in this example, the operating point is at $V_o = 8$ V and $I_o = 2$ mA. In nonlinear circuits, such as when the load is a diode, the diode's characteristic curve is not a straight line, and this technique provides a useful mechanism for graphically determining the operating point of the circuit.

There are situations in circuit design when we want to select a load so that the maximum power can be transferred to it. We can determine the maximum power that a circuit can supply and the manner in which to adjust the load to effect maximum power transfer by employing Thévenin's theorem.

In circuit analysis, we are sometimes interested in determining the maximum power that can be delivered to a load. By employing Thévenin's theorem, we can determine the maximum power that a circuit can supply and the manner in which to adjust the load to effect maximum power transfer.

Suppose that we are given the circuit shown in Fig. 5.19. The power that is delivered to the load is given by the expression

$$P_{\text{load}} = i^2 R_L = \left(\frac{v}{R + R_L}\right)^2 R_L$$

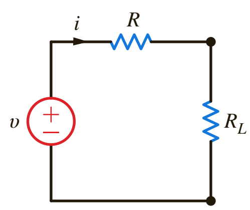

Figure 5.19

Equivalent circuit for examining maximum power transfer.

We want to determine the value of R_L that maximizes this quantity. Hence, we differentiate this expression with respect to R_L and equate the derivative to zero:

$$\frac{dP_{\text{load}}}{dR_L} = \frac{(R + R_L)^2 v^2 - 2v^2 R_L (R + R_L)}{(R + R_L)^4} = 0$$

which yields

$$R_L = R$$

In other words, maximum power transfer takes place when the load resistance $R_L = R$. Although this is a very important result, we have derived it using the simple network in Fig. 5.19. However, we should recall that v and R in Fig. 5.19 could represent the Thévenin equivalent circuit for any linear network.

EXAMPLE 5.15

Let us find the value of R_L for maximum power transfer in the network in Fig. 5.20a and the maximum power that can be transferred to this load.

SOLUTION

To begin, we derive the Thévenin equivalent circuit for the network exclusive of the load. V_{oc} can be calculated from the circuit in Fig. 5.20b. The mesh equations for the network are

$$I_1 = 2 \times 10^{-3}$$
$$3\text{k}(I_2 - I_1) + 6\text{k}I_2 + 3 = 0$$

Solving these equations yields $I_2 = 1/3$ mA and, hence,

$$\begin{aligned} V_{\text{oc}} &= 4\text{k}I_1 + 6\text{k}I_2 \\ &= 10 \text{ V} \end{aligned}$$

R_{Th}, shown in Fig. 5.20c, is 6 kΩ; therefore, $R_L = R_{\text{Th}} = 6$ kΩ for maximum power transfer. The maximum power transferred to the load in Fig. 5.20d is

$$P_L = \left(\frac{10}{12\text{k}}\right)^2 (6\text{k}) = \frac{25}{6} \text{ mW}$$

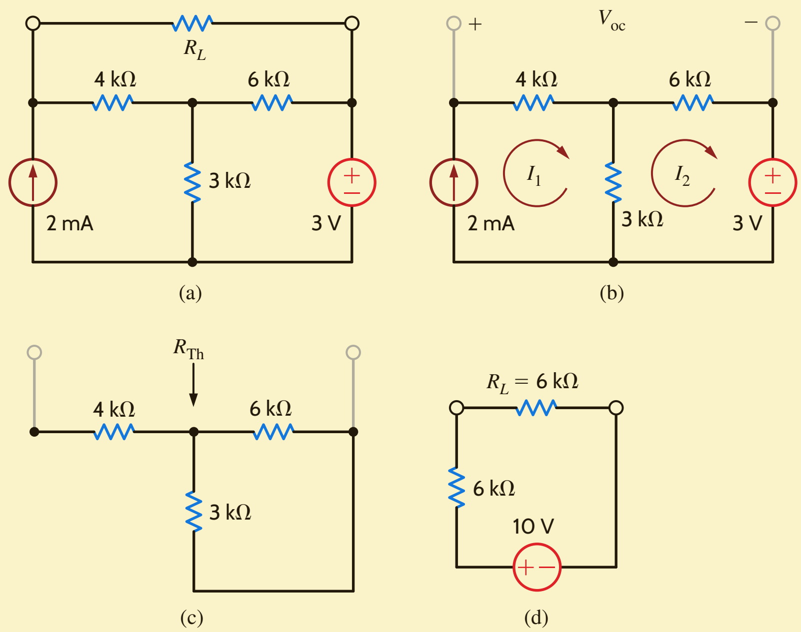

Figure 5.20
Circuits used in Example 5.15.

EXAMPLE 5.16

Let us find R_L for maximum power transfer and the maximum power transferred to this load in the circuit in **Fig. 5.21a**.

SOLUTION

We wish to reduce the network to the form shown in Fig. 5.19. We could form the Thévenin equivalent circuit by breaking the network at the load. However, close examination of the network indicates that our analysis will be simpler if we break the network to the left of the 4-kΩ resistor. When we do this, however, we must realize that for maximum power transfer $R_L = R_{Th} + 4$ kΩ. V_{oc} can be calculated from the network in **Fig. 5.21b**. Forming a supernode around the dependent source and its connecting nodes, the KCL equation for this supernode is

$$\frac{V_{oc} - 2000I_x'}{1\text{k} + 3\text{k}} + (-4 \times 10^{-3}) + \frac{V_{oc}}{2\text{k}} = 0$$

where

$$I_x' = \frac{V_{oc}}{2\text{k}}$$

These equations yield $V_{oc} = 8$ V. The short-circuit current can be found from the network in **Fig. 5.21c**. It is here that we find the advantage of breaking the network to the left of the 4-kΩ resistor. The short circuit shorts the 2-kΩ resistor and, therefore, $I_x'' = 0$. Hence, the circuit is reduced to that in **Fig. 5.21d**, where clearly $I_{sc} = 4$ mA. Then

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = 2\text{ k}\Omega$$

Connecting the Thévenin equivalent to the remainder of the original circuit produces the network in **Fig. 5.21e**. For maximum power transfer $R_L = R_{Th} + 4$ kΩ $= 6$ kΩ, and the maximum power transferred is

$$P_L = \left(\frac{8}{12\text{k}}\right)^2 (6\text{k}) = \frac{8}{3}\text{ mW}$$

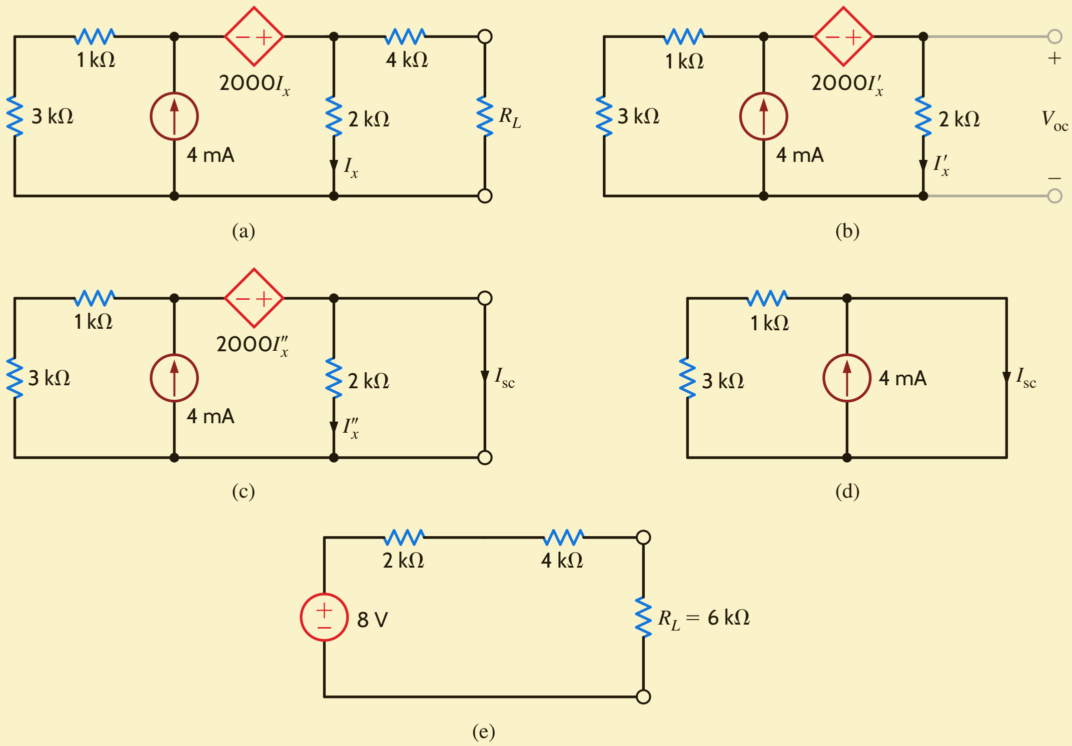

Figure 5.21

Circuits used in Example 5.16.

LEARNING ASSESSMENTS

E5.16 Given the circuit in Fig. E5.16, find R_L for maximum power transfer and the maximum power transferred.

ANSWER:
$R_L = 6$ kΩ;
$P_L = \frac{2}{3}$ mW.

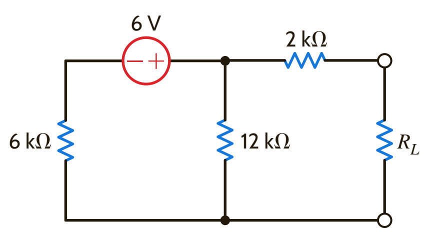

Figure E5.16

E5.17 Find R_L for maximum transfer and the maximum power transferred to R_L in Fig. E5.17.

ANSWER:
14/9 kΩ;
2/7 mW.

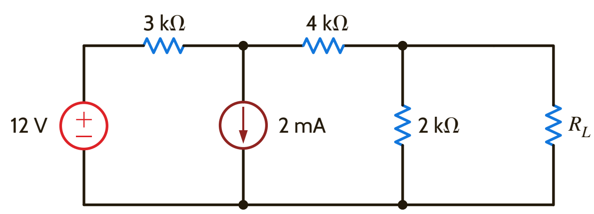

Figure E5.17

E5.18 Find R_L for maximum transfer and the maximum power transferred to R_L in Fig. E5.18.

ANSWER:
24/13 kΩ; 27/26 mW.

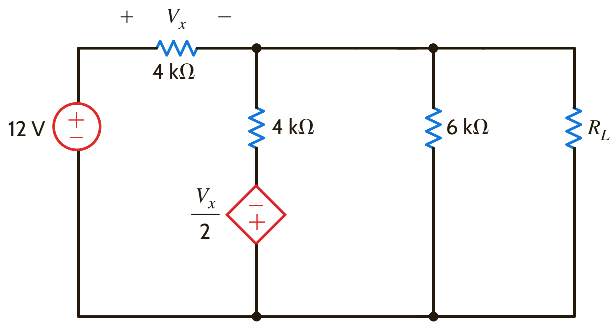

Figure E5.18

EXAMPLE 5.17

Given the network in Fig. 5.22 with $V_{in} = 5$ V and $R_1 = 2\ \Omega$, let us graphically examine a variety of aspects of maximum power transfer by plotting the parameters V_{out}, I, P_{out}, P_{in}, and the efficiency $= P_{out}/P_{in}$ as a function of the resistor ratio R_2/R_1.

SOLUTION

The parameters to be plotted can be determined by simple circuit analysis techniques. By voltage division

$$V_{out} = \left[\frac{R_2}{R_1 + R_2}\right]V_{in} = \left[\frac{R_2}{2 + R_2}\right](5)$$

From Ohm's law

$$I = \frac{V_{in}}{R_1 + R_2} = \frac{5}{2 + R_2}$$

The input and output powers are

$$P_{in} = IV_{in} = \frac{V_{in}^2}{R_1 + R_2} = \frac{25}{2 + R_2} \qquad P_{out} = IV_{out} = R_2\left[\frac{V_{in}}{R_1 + R_2}\right]^2 = R_2\left[\frac{5}{2 + R_2}\right]^2$$

Finally, the efficiency is

$$\text{efficiency} = \frac{P_{out}}{P_{in}} = \frac{R_2}{R_1 + R_2} = \frac{R_2}{2 + R_2}$$

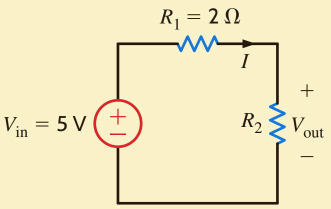

Figure 5.22
Circuit used in maximum power transfer analysis.

The resulting plots of the various parameters are shown in Fig. 5.23 for R_2 ranging from $0.1R_1$ to $10R_1$. Note that as R_2 increases, V_{out} increases toward V_{in} (5 V) as dictated by voltage division. Also, the current decreases in accordance with Ohm's law. Thus, for small values of R_2, V_{out} is small, and when R_2 is large, I is small. As a result, the output power (the product of these two parameters) has a maximum at $R_2/R_1 = 1$ as predicted by maximum power transfer theory.

Maximum power does not correspond to maximum output voltage, current, or efficiency. In fact, at maximum power transfer, the efficiency is always 0.5, or 50%. If you are an electric utility supplying energy to your customers, do you want to operate at maximum power transfer? The answer to this question is an obvious "No" because the efficiency is only 50%. The utility would only be able to charge its customers for one-half of the energy produced. It is not uncommon for a large electric utility to spend billions of dollars every year to produce electricity. The electric utility is more interested in operating at maximum efficiency.