15.2 Fourier Transform

The preceding sections of this chapter have illustrated that the exponential Fourier series can be used to represent a periodic signal for all time. We will now consider a technique for representing an aperiodic signal for all values of time.

Suppose that an aperiodic signal $f(t)$ is as shown in Fig. 15.18a. We now construct a new signal $f_p(t)$ that is identical to $f(t)$ in the interval $-T/2$ to $T/2$ but is *periodic* with period T, as shown in Fig. 15.18b. Since $f_p(t)$ is periodic, it can be represented in the interval $-\infty$ to ∞ by an exponential Fourier series

$$f_p(t) = \sum_{n=-\infty}^{\infty} \mathbf{c}_n e^{jn\omega_0 t} \qquad \textbf{15.36}$$

where

$$\mathbf{c}_n = \frac{1}{T}\int_{-T/2}^{T/2} f_p(t)\, e^{-jn\omega_0 t}\, dt \qquad \textbf{15.37}$$

and

$$\omega_0 = \frac{2\pi}{T} \qquad \textbf{15.38}$$

At this point we note that if we take the limit of the function $f_p(t)$ as $T \rightarrow \infty$, the periodic signal in Fig. 15.18b approaches the aperiodic signal in Fig. 15.18a; that is, the repetitious signals centered at $-T$ and $+T$ in Fig. 15.18b are moved to infinity.

The line spectrum for the periodic signal exists at harmonic frequencies ($n\omega_0$), and the incremental spacing between the harmonics is

$$\Delta\omega = (n+1)\omega_0 - n\omega_0 = \omega_0 = \frac{2\pi}{T} \qquad \textbf{15.39}$$

As $T \rightarrow \infty$, the lines in the frequency spectrum for $f_p(t)$ come closer and closer together, $\Delta\omega$ approaches the differential $d\omega$, and $n\omega_0$ can take on any value of ω. Under these conditions, the line spectrum becomes a continuous spectrum. Since as $T \rightarrow \infty$, $\mathbf{c}_n \rightarrow 0$ in Eq. (15.37), we will examine the product $\mathbf{c}_n T$, where

$$\mathbf{c}_n T = \int_{-T/2}^{T/2} f_p(t)\, e^{-jn\omega_0 t}\, dt$$

In the limit as $T \rightarrow \infty$,

$$\lim_{T\to\infty} (\mathbf{c}_n T) = \lim_{T\to\infty} \int_{-T/2}^{T/2} f_p(t)\, e^{-jn\omega_0 t}\, dt$$

which, in view of the previous discussion, can be written as

$$\lim_{T\to\infty} (\mathbf{c}_n T) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

This integral is the Fourier transform of $f(t)$, which we will denote as $\mathbf{F}(\omega)$, and hence

$$\mathbf{F}(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \qquad \textbf{15.40}$$

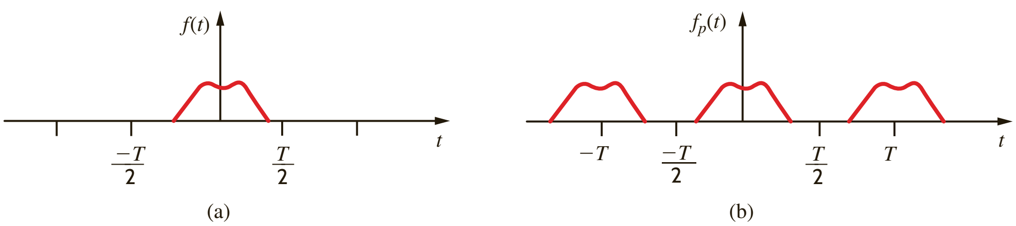


Figure 15.18
Aperiodic and periodic signals.

Similarly, $f_p(t)$ can be expressed as

$$\begin{aligned} f_p(t) &= \sum_{n=-\infty}^{\infty} \mathbf{c}_n e^{jn\omega_0 t} \\ &= \sum_{n=-\infty}^{\infty} (\mathbf{c}_n T) e^{jn\omega_0 t} \frac{1}{T} \\ &= \sum_{n=-\infty}^{\infty} (\mathbf{c}_n T) e^{jn\omega_0 t} \frac{\Delta\omega}{2\pi} \end{aligned}$$

which in the limit as $T \to \infty$ becomes

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{F}(\omega) e^{j\omega t} \, d\omega \qquad \textbf{15.41}$$

Eqs. (15.40) and (15.41) constitute what is called the *Fourier transform pair.* Since $\mathbf{F}(\omega)$ is the Fourier transform of $f(t)$ and $f(t)$ is the inverse Fourier transform of $\mathbf{F}(\omega)$, they are normally expressed in the form

$$\mathbf{F}(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt \qquad \textbf{15.42}$$

$$f(t) = \mathcal{F}^{-1}[\mathbf{F}(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{F}(\omega) e^{j\omega t} \, d\omega \qquad \textbf{15.43}$$

SOME IMPORTANT TRANSFORM PAIRS There are a number of important Fourier transform pairs. In the following material we derive a number of them and then list some of the more common ones in tabular form.

EXAMPLE 15.11

We wish to derive the Fourier transform for the voltage pulse shown in Fig. 15.19a.

SOLUTION

Using Eq. (15.42), the Fourier transform is

$$\begin{aligned} \mathbf{F}(\omega) &= \int_{-\delta/2}^{\delta/2} V e^{-j\omega t} \, dt \\ &= \frac{V}{-j\omega} e^{-j\omega t} \Big|_{-\delta/2}^{\delta/2} \\ &= V \frac{e^{-j\omega\delta/2} - e^{+j\omega\delta/2}}{-j\omega} \\ &= V\delta \frac{\sin(\omega\delta/2)}{\omega\delta/2} \end{aligned}$$

Therefore, the Fourier transform for the function

$$f(t) = \begin{cases} 0 & -\infty < t \le -\dfrac{\delta}{2} \\ V & -\dfrac{\delta}{2} < t \le \dfrac{\delta}{2} \\ 0 & \dfrac{\delta}{2} < t < \infty \end{cases}$$

is

$$\mathbf{F}(\omega) = V\delta \frac{\sin(\omega\delta/2)}{\omega\delta/2}$$

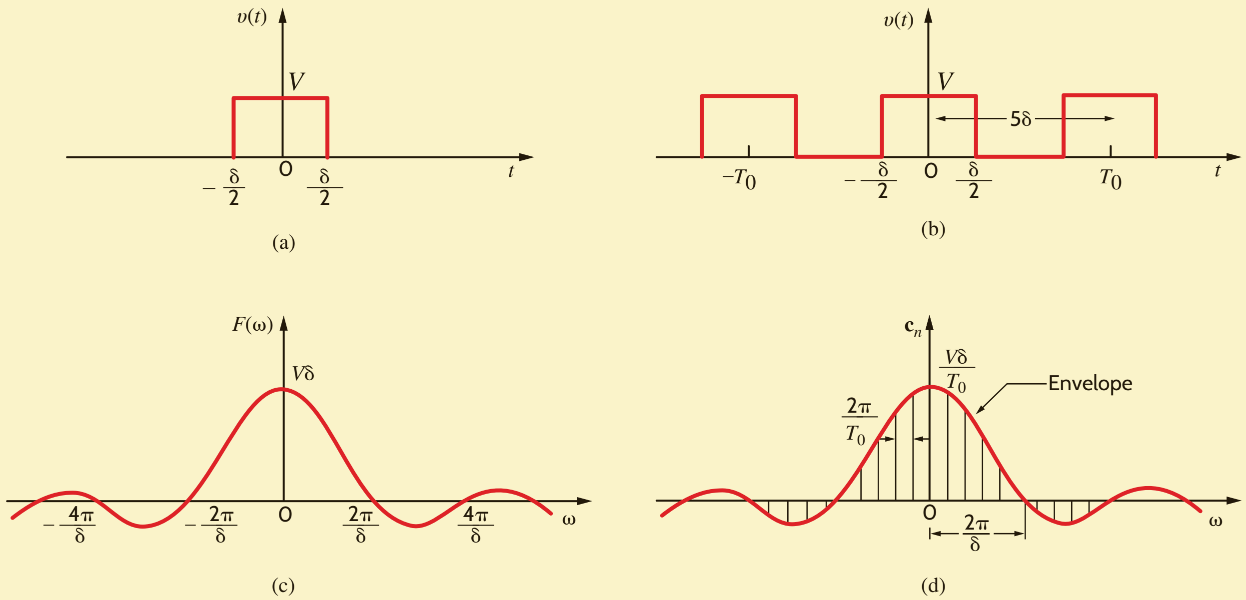


Figure 15.19
Pulses and their spectra.

A plot of this function is shown in **Fig. 15.19c**. Let us explore this example even further. Consider now the pulse train shown in **Fig. 15.19b**. Using the techniques that have been demonstrated earlier, we can show that the Fourier coefficients for this waveform are

$$\mathbf{c}_n = \frac{V\delta}{T_0}\,\frac{\sin(n\omega_0\delta/2)}{n\omega_0\delta/2}$$

The line spectrum for $T_0 = 5\delta$ is shown in **Fig. 15.19d**.

The equations and figures in this example indicate that as $T_0 \to \infty$ and the periodic function becomes aperiodic, the lines in the discrete spectrum become denser and the amplitude gets smaller, and the amplitude spectrum changes from a line spectrum to a continuous spectrum. Note that the envelope for the discrete spectrum has the same shape as the continuous spectrum. Since the Fourier series represents the amplitude and phase of the signal at specific frequencies, the Fourier transform also specifies the frequency content of a signal.

EXAMPLE 15.12

Find the Fourier transform for the unit impulse function $\delta(t)$.

SOLUTION

The Fourier transform of the unit impulse function $\delta(t - a)$ is

$$\mathbf{F}(\omega) = \int_{-\infty}^{\infty} \delta(t-a)e^{-j\omega t}\,dt$$

Using the sampling property of the unit impulse, we find that

$$\mathbf{F}(\omega) = e^{-j\omega a}$$

and if $a = 0$, then

$$\mathbf{F}(\omega) = 1$$

Note then that the $\mathbf{F}(\omega)$ for $f(t) = \delta(t)$ is *constant for all frequencies*. This is an important property, as we shall see later.

EXAMPLE 15.13

We wish to determine the Fourier transform of the function $f(t) = e^{j\omega_0 t}$.

SOLUTION

In this case note that if $\mathbf{F}(\omega) = 2\pi\delta(\omega - \omega_0)$, then

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega$$
$$= e^{j\omega_0 t}$$

Therefore, $f(t) = e^{j\omega_0 t}$ and $\mathbf{F}(\omega) = 2\pi\delta(\omega - \omega_0)$ represent a Fourier transform pair.

LEARNING ASSESSMENT

E15.16 If $f(t) = \sin \omega_0 t$, find $\mathbf{F}(\omega)$.

ANSWER:
$\mathbf{F}(\omega) = \pi j[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$.

EXAMPLE 15.14

In AM (amplitude modulation) radio, there are two very important waveforms—the signal, $s(t)$, and the carrier. All of the information we desire to transmit, voice, music, and so on, is contained in the signal waveform, which is in essence transported by the carrier. Therefore, the Fourier transform of $s(t)$ contains frequencies from about 50 Hz to 20,000 Hz. The carrier, $c(t)$, is a sinusoid oscillating at a frequency much greater than those in $s(t)$. For example, the FCC (Federal Communications Commission) rules and regulations have allocated the frequency range 540 kHz to 1.7 MHz for AM radio station carrier frequencies. Even the lowest possible carrier frequency allocation of 540 kHz is much greater than the audio frequencies in $s(t)$. In fact, when a station broadcasts its call letters and frequency, they are telling you the carrier's frequency, which the FCC assigned to that station!

In simple cases, the signal, $s(t)$, is modified to produce a voltage of the form

$$v(t) = [A + s(t)] \cos(\omega_c t)$$

where A is a constant and ω_c is the carrier frequency in rad/s. This voltage, $v(t)$, with the signal "coded" within, is sent to the antenna and is broadcast to the public, whose radios "pick up" a faint replica of the waveform $v(t)$.

Let us plot the magnitude of the Fourier transform of both $s(t)$ and $v(t)$ given that $s(t)$ is

$$s(t) = \cos(2\pi f_a t)$$

where f_a is 1000 Hz, the carrier frequency is 900 kHz, and the constant A is unity.

SOLUTION

The Fourier transform of $s(t)$ is

$$\mathbf{S}(\omega) = \mathcal{F}[\cos(\omega_a t)] = \pi\delta(\omega - \omega_a) + \pi\delta(\omega + \omega_a)$$

and is shown in Fig. 15.20.

The voltage $v(t)$ can be expressed in the form

$$v(t) = [1 + s(t)] \cos(\omega_c t) = \cos(\omega_c t) + s(t)\cos(\omega_c t)$$

The Fourier transform for the carrier is

$$\mathcal{F}[\cos(\omega_c t)] = \pi\delta(\omega - \omega_c) + \pi\delta(\omega + \omega_c)$$

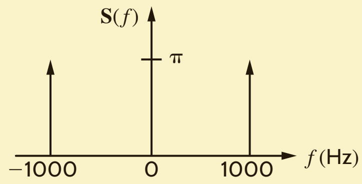


Figure 15.20
Fourier transform magnitude for $s(t)$ versus frequency.

The term $s(t)\cos(\omega_c t)$ can be written as

$$s(t)\cos(\omega_c t) = s(t)\left\{\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right\}$$

Using the property of modulation as given in Table 15.3, we can express the Fourier transform of $s(t)\cos(\omega_c t)$ as

$$\mathcal{F}[s(t)\cos(\omega_c t)] = \frac{1}{2}\{\mathbf{S}(\omega - \omega_c) + \mathbf{S}(\omega + \omega_c)\}$$

Employing $\mathbf{S}(\omega)$, we find

$$\begin{aligned}\mathcal{F}[s(t)\cos(\omega_c t)] &= \mathcal{F}[\cos(\omega_a t)\cos(\omega_c t)] \\ &= \frac{\pi}{2}\{\delta(\omega - \omega_a - \omega_c) + \delta(\omega + \omega_a - \omega_c) \\ &\quad + \delta(\omega - \omega_a + \omega_c) + \delta(\omega + \omega_a + \omega_c)\}\end{aligned}$$

Finally, the Fourier transform of $v(t)$ is

$$\begin{aligned}\mathbf{V}(\omega) = \frac{\pi}{2}\{&2\delta(\omega - \omega_c) + 2\delta(\omega + \omega_c) + \delta(\omega - \omega_a - \omega_c) \\ &+ \delta(\omega + \omega_a - \omega_c) + \delta(\omega - \omega_a + \omega_c) + \delta(\omega + \omega_a + \omega_c)\}\end{aligned}$$

which is shown in **Fig. 15.21**. Notice that $\mathbf{S}(\omega)$ is centered about the carrier frequency. This is the effect of modulation.

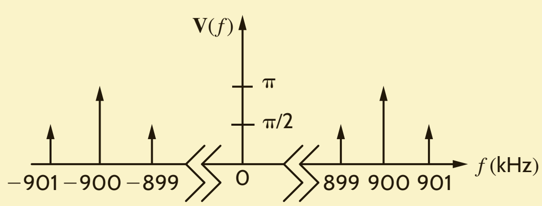


Figure 15.21
Fourier transform of the transmitted waveform $v(t)$ versus frequency.

A number of useful Fourier transform pairs are shown in Table 15.2.

TABLE 15.2 Fourier transform pairs

$f(t)$	$\mathbf{F}(\omega)$
$\delta(t - a)$	$e^{-j\omega a}$
A	$2\pi A\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$
$\sin\omega_0 t$	$j\pi\delta(\omega + \omega_0) - j\pi\delta(\omega - \omega_0)$
$e^{-at}u(t),\ a > 0$	$\dfrac{1}{a + j\omega}$
$e^{-\alpha\lvert t\rvert},\ a > 0$	$\dfrac{2a}{a^2 + \omega^2}$
$e^{-at}\cos\omega_0 t u(t),\ a > 0$	$\dfrac{j\omega + a}{(j\omega + a)^2 + \omega_0^2}$
$e^{-at}\sin\omega_0 t u(t),\ a > 0$	$\dfrac{\omega_0}{(j\omega + a)^2 + \omega_0^2}$

TABLE 15.3 Properties of the Fourier transform

$f(t)$	$\mathbf{F}(\omega)$	PROPERTY
$Af(t)$	$A\mathbf{F}(\omega)$	Linearity
$f_1(t) \pm f_2(t)$	$\mathbf{F}_1(\omega) \pm \mathbf{F}_2(\omega)$	
$f(at)$	$\frac{1}{a}\mathbf{F}\left(\frac{\omega}{a}\right),\ a > 0$	Time-scaling
$f(t - t_0)$	$e^{-j\omega t_0}\mathbf{F}(\omega)$	Time-shifting
$e^{j\omega t_0}f(t)$	$\mathbf{F}(\omega - \omega_0)$	Modulation
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n\mathbf{F}(\omega)$	
$t^n f(t)$	$(j)^n\frac{d^n\mathbf{F}(\omega)}{d\omega^n}$	Differentiation
$\int_{-\infty}^{\infty} f_1(x)f_2(t - x)\,dx$	$\mathbf{F}_1(\omega)\mathbf{F}_2(\omega)$	
$f_1(t)f_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{F}_1(x)\mathbf{F}_2(\omega - x)\,dx$	Convolution

SOME PROPERTIES OF THE FOURIER TRANSFORM The Fourier transform defined by the equation

$$\mathbf{F}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

has a number of important properties. Table 15.3 provides a short list of a number of these properties.

The proofs of these properties are generally straightforward; however, as an example we will demonstrate the time convolution property.

If $\mathcal{F}[f_1(t)] = \mathbf{F}_1(\omega)$ and $\mathcal{F}[f_2(t)] = \mathbf{F}_2(\omega)$, then

$$\begin{aligned}\mathcal{F}\left[\int_{-\infty}^{\infty} f_1(x)f_2(t-x)\,dx\right] &= \int_{t=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f_1(x)f_2(t-x)\,dx\,e^{-j\omega t}\,dt \\ &= \int_{x=-\infty}^{\infty} f_1(x)\int_{t=-\infty}^{\infty} f_2(t-x)\,e^{-j\omega t}\,dt\,dx\end{aligned}$$

If we now let $u = t - x$, then

$$\begin{aligned}\mathcal{F}\left[\int_{-\infty}^{\infty} f_1(x)f_2(t-x)\,dx\right] &= \int_{x=-\infty}^{\infty} f_1(x)\int_{u=-\infty}^{\infty} f_2(u)\,e^{-j\omega(u+x)}\,du\,dx \\ &= \int_{x=-\infty}^{\infty} f_1(x)e^{-j\omega x}\int_{u=-\infty}^{\infty} f_2(u)\,e^{-j\omega u}\,du\,dx \\ &= \mathbf{F}_1(\omega)\mathbf{F}_2(\omega)\end{aligned} \qquad \textbf{15.44}$$

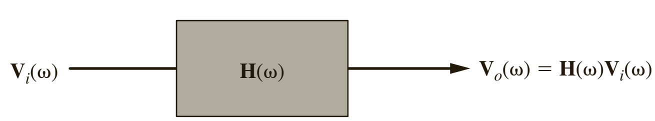
We should note very carefully the time convolution property of the Fourier transform. With reference to Fig. 15.22, this property states that if $\mathbf{V}_i(\omega) = \mathcal{F}[v_i(t)]$, $\mathbf{H}(\omega) = \mathcal{F}[h(t)]$, and $\mathbf{V}_o(\omega) = \mathcal{F}[v_o(t)]$, then:

$$\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\mathbf{V}_i(\omega) \qquad \textbf{15.45}$$

where $\mathbf{V}_i(\omega)$ represents the input signal, $\mathbf{H}(\omega)$ is the network transfer function, and $\mathbf{V}_o(\omega)$ represents the output signal. Eq. (15.45) tacitly assumes that the initial conditions of the network are zero.

Figure 15.22

Representation of the time convolution property.



LEARNING ASSESSMENTS

E15.17 Determine the output $v_o(t)$ in Fig. E15.17 if the signal $v_i(t) = e^{-t}u(t)$ V, the network impulse response $h(t) = e^{-2t}u(t)$, and all initial conditions are zero.

ANSWER:
$v_o(t) = (e^{-t} - e^{-2t})u(t)$ V.

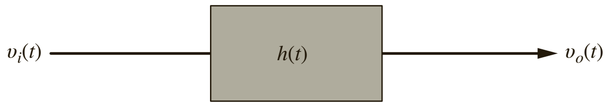


Figure E15.17

E15.18 Use the transform technique to find $v_o(t)$ in Fig. E15.18 if $v(t) = 15 \cos 10t$ V.

ANSWER:
$v_o(t) = 4.12 \cos (10t + 74°)$ V.

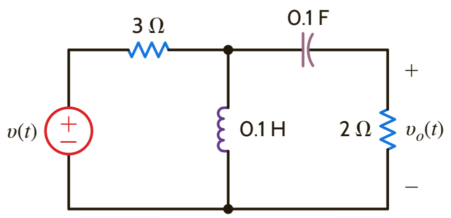


Figure E15.18

PARSEVAL'S THEOREM A mathematical statement of Parseval's theorem is

$$\int_{-\infty}^{\infty} f^2(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathbf{F}(\omega)|^2\, d\omega \qquad \textbf{15.46}$$

This relationship can be easily derived as follows:

$$\begin{aligned}
\int_{-\infty}^{\infty} f^2(t)\, dt &= \int_{-\infty}^{\infty} f(t)\, \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{F}(\omega)e^{j\omega t}\, d\omega\, dt \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{F}(\omega)\int_{-\infty}^{\infty} f(t)e^{-j(-\omega)t}\, dt\, d\omega \\
&= \int_{-\infty}^{\infty} \frac{1}{2\pi}\mathbf{F}(\omega)\mathbf{F}(-\omega)\, d\omega \\
&= \int_{-\infty}^{\infty} \frac{1}{2\pi}\mathbf{F}(\omega)\mathbf{F}^*(\omega)\, d\omega \\
&= \int_{-\infty}^{\infty} \frac{1}{2\pi}|\mathbf{F}(\omega)|^2\, d\omega
\end{aligned}$$

The importance of Parseval's theorem can be seen if we imagine that $f(t)$ represents the current in a 1-Ω resistor. Since $f^2(t)$ is power and the integral of power over time is energy, Eq. (15.46) shows that we can compute this 1-Ω energy or normalized energy in either the time domain or the frequency domain.

EXAMPLE 15.15

Using the transform technique, we wish to determine $v_o(t)$ in Fig. 15.23 if (a) $v_i(t) = 5e^{-2t}u(t)$ V and (b) $v_i(t) = 5\cos 2t$ V.

SOLUTION

a. In this case since $v_i(t) = 5e^{-2t}u(t)$ V, then

$$\mathbf{V}_i(\omega) = \frac{5}{2 + j\omega}\,\text{V}$$

$\mathbf{H}(\omega)$ for the network is

$$\mathbf{H}(\omega) = \frac{R}{R + j\omega L} = \frac{10}{10 + j\omega}$$

From Eq. (15.45),

$$\mathbf{V}_o(\omega) = \mathbf{H}(\omega)\mathbf{V}_i(\omega) = \frac{50}{(2 + j\omega)(10 + j\omega)} = \frac{50}{8}\left(\frac{1}{2 + j\omega} - \frac{1}{10 + j\omega}\right)$$

Hence, from Table 15.2, we see that

$$v_o(t) = 6.25[e^{-2t}u(t) - e^{-10t}u(t)] \text{ V}$$

b. In this case, since $v_i(t) = 5\cos 2t$,

$$\mathbf{V}_i(\omega) = 5\pi\delta(\omega - 2) + 5\pi\delta(\omega + 2) \text{ V}$$

The output voltage in the frequency domain is then

$$\mathbf{V}_o(\omega) = \frac{50\pi[\delta(\omega - 2) + \delta(\omega + 2)]}{(10 + j\omega)}$$

Using the inverse Fourier transform gives us

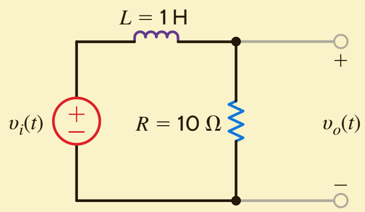
$$v_o(t) = \mathcal{F}^{-1}[\mathbf{V}_o(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} 50\pi\,\frac{\delta(\omega - 2) + \delta(\omega + 2)}{10 + j\omega}\,e^{j\omega t}\,d\omega$$

Employing the sampling property of the unit impulse function, we obtain

$$v_o(t) = 25\left(\frac{e^{j2t}}{10 + j2} + \frac{e^{-j2t}}{10 - j2}\right) = 25\left(\frac{e^{j2t}}{10.2e^{j11.31°}} + \frac{e^{-j2t}}{10.2e^{-j11.31°}}\right) = 4.90\cos(2t - 11.31°) \text{ V}$$

This result can be easily checked using phasor analysis.

Figure 15.23
Simple *RL* circuit.



EXAMPLE 15.16

Consider the network shown in Fig. 15.24a. This network represents a simple low-pass filter, as shown in Chapter 12. We wish to illustrate the impact of this network on the input signal by examining the frequency characteristics of the output signal and the relationship between the 1-Ω or normalized energy at the input and output of the network.

SOLUTION The network transfer function is

$$\mathbf{H}(\omega) = \frac{1/RC}{1/RC + j\omega} = \frac{5}{5 + j\omega} = \frac{1}{1 + 0.2j\omega}$$

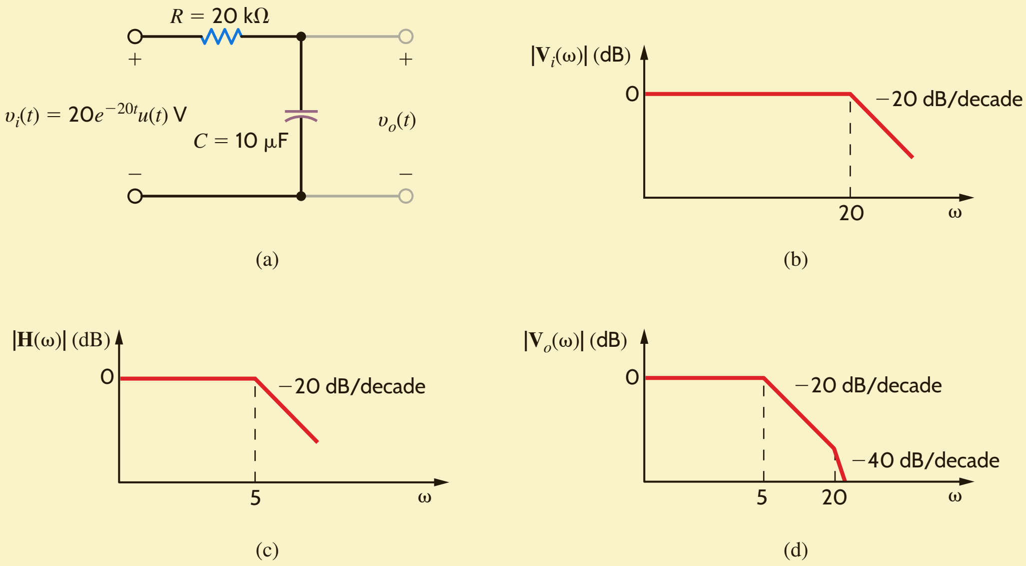


Figure 15.24 Low-pass filter, its frequency characteristic, and its output spectra.

The Fourier transform of the input signal is

$$\mathbf{V}_i(\omega) = \frac{20}{20 + j\omega} = \frac{1}{1 + 0.05j\omega}$$

Then, using Eq. (15.45), the Fourier transform of the output is

$$\mathbf{V}_o(\omega) = \frac{1}{(1 + 0.2j\omega)(1 + 0.05j\omega)}$$

Using the techniques of Chapter 12, we note that the straight-line log-magnitude plot (frequency characteristic) for these functions is shown in **Figs. 15.24b–d**. Note that the low-pass filter passes the low frequencies of the input signal but attenuates the high frequencies.

The normalized energy at the filter input is

$$\begin{aligned} W_i &= \int_0^\infty (20e^{-20t})^2\, dt \\ &= \frac{400}{-40} e^{-40t} \Big|_0^\infty \\ &= 10 \text{ J} \end{aligned}$$

The normalized energy at the filter output can be computed using Parseval's theorem. Since

$$\mathbf{V}_o(\omega) = \frac{100}{(5 + j\omega)(20 + j\omega)}$$

and

$$|\mathbf{V}_o(\omega)|^2 = \frac{10^4}{(\omega^2 + 25)(\omega^2 + 400)}$$

$|\mathbf{V}_o(\omega)|^2$ is an even function, and therefore

$$W_o = 2\left(\frac{1}{2\pi}\right)\int_0^\infty \frac{10^4\, d\omega}{(\omega^2 + 25)(\omega^2 + 400)}$$

However, we can use the fact that

$$\frac{10^4}{(\omega^2 + 25)(\omega^2 + 400)} = \frac{10^4/375}{\omega^2 + 25} - \frac{10^4/375}{\omega^2 + 400}$$

Then

$$\begin{aligned} W_o &= \frac{1}{\pi}\left(\int_0^\infty \frac{10^4/375}{\omega^2 + 25}\, d\omega - \int_0^\infty \frac{10^4/3\,75}{\omega^2 + 400}\, d\omega\right) \\ &= \frac{10^4}{375}\left(\frac{1}{\pi}\right)\left[\frac{1}{5}\left(\frac{\pi}{2}\right) - \frac{1}{20}\left(\frac{\pi}{2}\right)\right] \\ &= 2.0 \text{ J} \end{aligned}$$

LEARNING ASSESSMENTS

E15.19 Compute the total 1-Ω energy content of the signal $v_i(t) = e^{-2t}u(t)$ V using both the time-domain and frequency-domain approaches.

ANSWER: $W = 0.25$ J.

E15.20 Compute the 1-Ω energy content of the signal $v_i(t) = e^{-2t}u(t)$ V in the frequency range from 0 to 1 rad/s.

ANSWER: $W = 0.07$ J.

E15.21 Determine the total 1-Ω energy content of the output $v_o(t)$ in Fig. E15.21 if $v_i(t) = 5e^{2t}u(t)$ V.

ANSWER: $W = 5.21$ J.

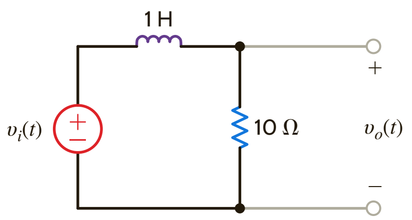


Figure E15.21

Example 15.16 illustrates the effect that $\mathbf{H}(\omega)$ has on the frequency spectrum of the input signal. In general, $\mathbf{H}(\omega)$ can be selected to shape that spectrum in some prescribed manner. As an illustration of this effect, consider the *ideal* frequency spectrums shown in Fig. 15.25. Fig. 15.25a shows an ideal input magnitude spectrum $|\mathbf{V}_i(\omega)|$. $|\mathbf{H}(\omega)|$ and the output magnitude spectrum $|\mathbf{V}_o(\omega)|$, which are related by Eq. (15.45), are shown in Figs. 15.25b–e for *ideal* low-pass, high-pass, band-pass, and band-elimination filters, respectively.

We note that by using Parseval's theorem we can compute the total energy content of a signal using either a time-domain or frequency-domain approach. However, the frequency-domain approach is more flexible in that it permits us to determine the energy content of a signal within some specified frequency band.

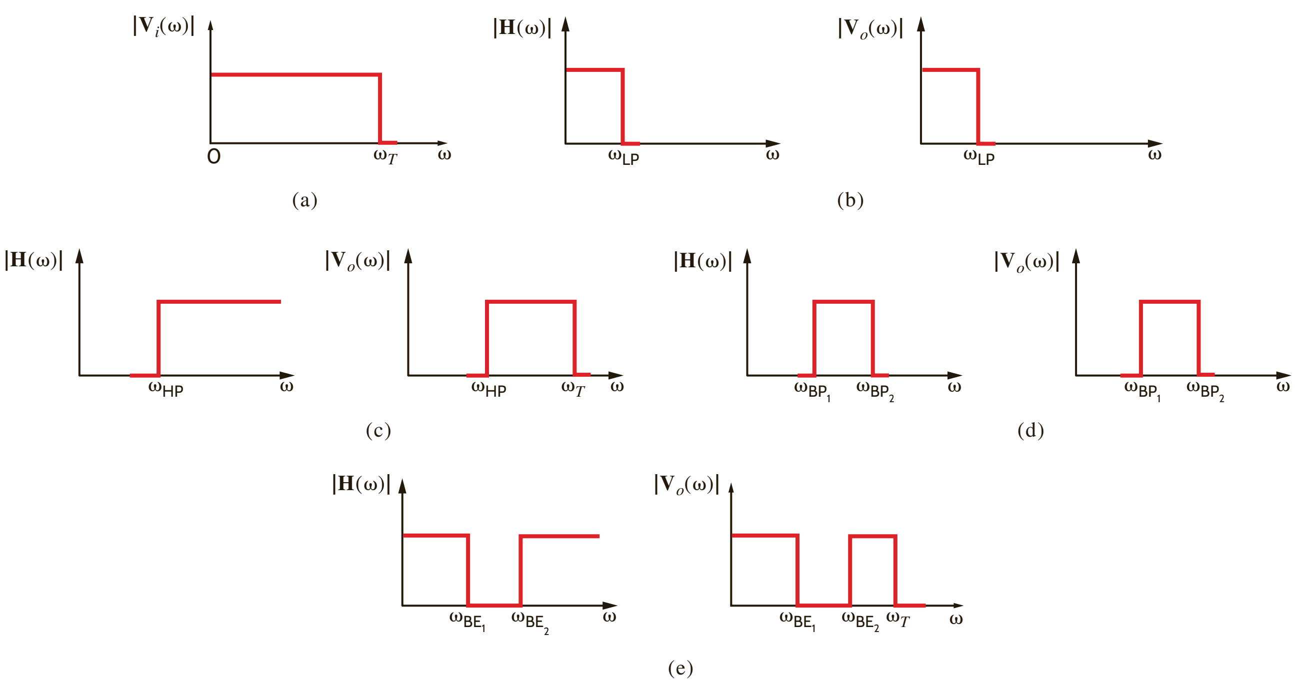


Figure 15.25

Frequency spectra for the input and output of ideal low-pass, high-pass, band-pass, and band-elimination filters.