PROBLEM-SOLVING STRATEGY

POWER FACTOR CORRECTION

STEP 1. Find Q_{old} from P_L and θ_{old}, or the equivalent pf_{old}.

STEP 2. Find θ_{new} from the desired pf_{new}.

STEP 3. Determine $Q_{\text{new}} = P_{\text{old}} \tan \theta_{\text{new}}$.

STEP 4. $Q_{\text{new}} - Q_{\text{old}} = Q_{\text{cap}} = -\omega C V^2$ rms.

LEARNING ASSESSMENTS

E9.20 Compute the value of the capacitance necessary to change the power factor in Learning Assessment E9.16 to 0.95 lagging.

ANSWER: $C = 773\ \mu\text{F}$.

E9.21 Find the value of capacitance to be connected in parallel with the load in Fig. E9.21 to make the source power factor 0.95 leading, $f = 60$ Hz.

ANSWER: $C = 546.2\ \mu\text{F}$.

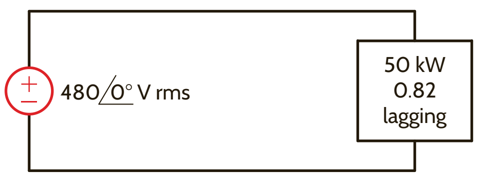

Figure E9.21

9.8 Single-Phase Three-Wire Circuits

The single-phase three-wire ac circuit shown in **Fig. 9.21** is an important topic because it is the typical ac power network found in households. Note that the voltage sources are equal; that is, $\mathbf{V}_{an} = \mathbf{V}_{nb} = \mathbf{V}$. Thus, the magnitudes are equal and the phases are equal (single phase). The line-to-line voltage $\mathbf{V}_{ab} = 2\mathbf{V}_{an} = 2\mathbf{V}_{nb} = 2\mathbf{V}$. Within a household, lights and small appliances are connected from one line to *neutral n*, and large appliances such as hot water heaters and air conditioners are connected line to line. Lights operate at about 120 V rms and large appliances operate at approximately 240 V rms.

Let us now attach two identical loads to the single-phase three-wire voltage system using perfect conductors as shown in **Fig. 9.21b**. From the figure we note that

$$\mathbf{I}_{aA} = \frac{\mathbf{V}}{\mathbf{Z}_L}$$

and

$$\mathbf{I}_{bB} = -\frac{\mathbf{V}}{\mathbf{Z}_L}$$

KCL at point N is

$$\begin{aligned}\mathbf{I}_{nN} &= -(\mathbf{I}_{aA} + \mathbf{I}_{bB}) \\ &= -\left(\frac{\mathbf{V}}{\mathbf{Z}_L} - \frac{\mathbf{V}}{\mathbf{Z}_L}\right) \\ &= 0\end{aligned}$$

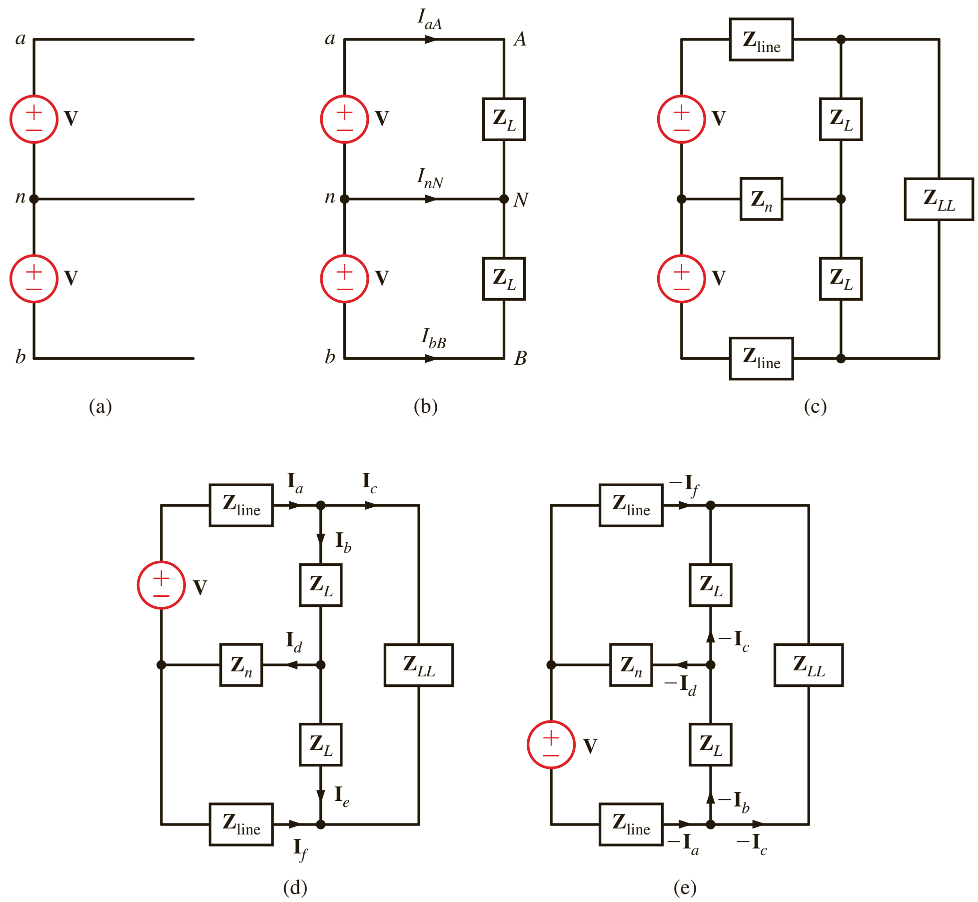

Figure 9.21 Single-phase three-wire system.

Note that there is no current in the neutral wire, and therefore it could be removed without affecting the remainder of the system; that is, all the voltages and currents would be unchanged. One is naturally led to wonder just how far the simplicity exhibited by this system will extend. For example, what would happen if each line had a line impedance, if the neutral conductor had an impedance associated with it, and if there were a load tied from line to line? To explore these questions, consider the circuit in Fig. 9.21c. Although we could examine this circuit using many of the techniques we have employed in previous chapters, the symmetry of the network suggests that perhaps superposition may lead us to some conclusions without having to resort to a brute-force assault. Employing superposition, we consider the two circuits in Figs. 9.21d and e. The currents in Fig. 9.21d are labeled arbitrarily. Because of the symmetrical relationship between Figs. 9.21d and e, the currents in Fig. 9.21e correspond directly to those in Fig. 9.21d. If we add the two *phasor* currents in each branch, we find that the neutral current is again zero. A neutral current of zero is a direct result of the symmetrical nature of the network. If either the line impedances $\mathbf{Z}_{\text{line}}$ or the load impedances $\mathbf{Z}_L$ are unequal, the neutral current will be nonzero. We will make direct use of these concepts when we study three-phase networks in Chapter 11.

EXAMPLE 9.15

A three-wire single-phase household circuit is shown in Fig. 9.22a. Use of the lights, stereo, and range for a 24-hour period is demonstrated in Fig. 9.22b. Let us calculate the energy use over the 24 hours in kilowatt-hours. Assuming that this represents a typical day and that our utility rate is \$0.08/kWh, let us also estimate the power bill for a 30-day month.

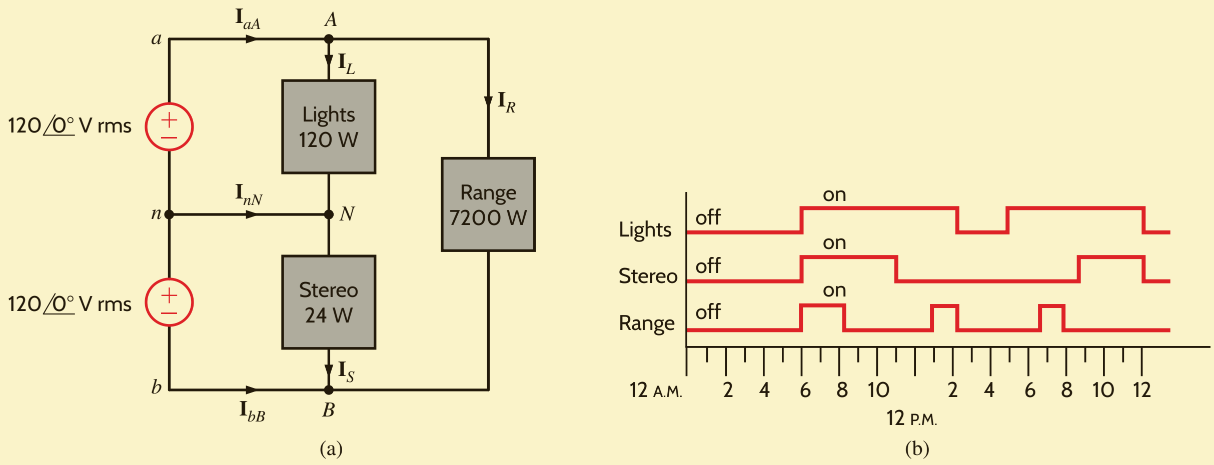

Figure 9.22

Household three-wire network and appliance usage.

SOLUTION Applying nodal analysis to Fig. 9.22a yields

$$\mathbf{I}_{aA} = \mathbf{I}_L + \mathbf{I}_R$$
$$\mathbf{I}_{bB} = -\mathbf{I}_S - \mathbf{I}_R$$
$$\mathbf{I}_{nN} = \mathbf{I}_S - \mathbf{I}_L$$

The current magnitudes for each load can be found from the corresponding power levels as follows:

$$I_L = \frac{P_L}{V_{an}} = \frac{120}{120} = 1 \text{ A rms}$$

$$I_S = \frac{P_S}{V_{nb}} = \frac{24}{120} = 0.2 \text{ A rms}$$

$$I_R = \frac{P_R}{V_{ab}} = \frac{7200}{240} = 30 \text{ A rms}$$

The energy used is simply the integral of the power delivered by the two sources over the 24-hour period. Since the voltage magnitudes are constants, we can express the energy delivered by the sources as

$$E_{an} = V_{an} \int I_{aA}\, dt$$

$$E_{nb} = V_{nb} \int -I_{bB}\, dt$$

The integrals of I_{aA} and I_{bB} can be determined graphically from Fig. 9.22b.

$$\int_{12\text{A.M.}}^{12\text{A.M.}} I_{aA}\, dt = 4I_R + 15I_L = 135$$

$$\int_{12\text{A.M.}}^{12\text{A.M.}} -I_{bB}\, dt = 8I_S + 4I_R = 121.6$$

Therefore, the daily energy for each source and the total energy is

$$E_{an} = 16.2 \text{ kWh}$$
$$E_{nb} = 14.6 \text{ kWh}$$
$$E_{\text{total}} = 30.8 \text{ kWh}$$

Over a 30-day month, a \$0.08/kWh utility rate results in a power bill of

$$\text{Cost} = (30.8)(30)(0.08) = \$73.92$$