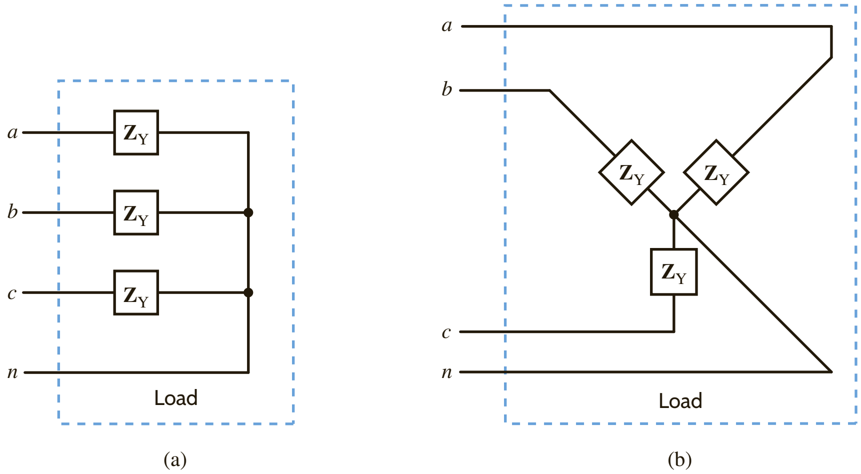


Figure 11.11
Wye (Y)-connected loads.

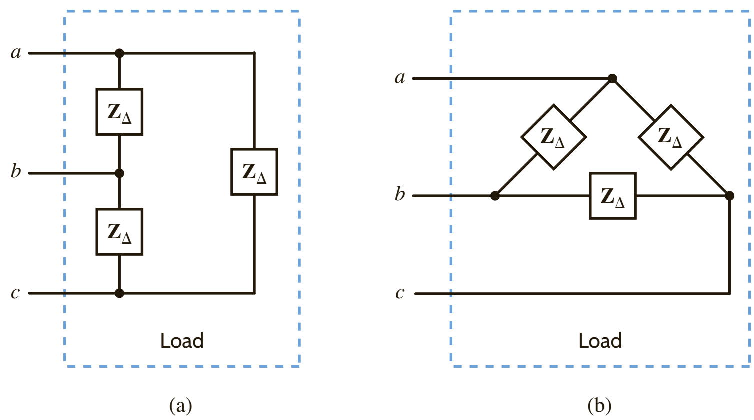


Figure 11.12
Delta (Δ)-connected loads.

11.3 Source/Load Connections

Since the source and the load can each be connected in either Y or Δ, three-phase balanced circuits can be connected Y–Y, Y–Δ, Δ–Y, or Δ–Δ. Our approach to the analysis of all of these circuits will be "Think Y"; therefore, we will analyze the Y–Y connection first.

BALANCED WYE–WYE CONNECTION Suppose now that both the source and load are connected in a wye, as shown in Fig. 11.13. The phase voltages with positive phase sequence are

$$
\begin{aligned}
\mathbf{V}_{an} &= V_p \angle 0^\circ \\
\mathbf{V}_{bn} &= V_p \angle -120^\circ \\
\mathbf{V}_{cn} &= V_p \angle +120^\circ
\end{aligned} \qquad \textbf{11.13}
$$

where V_p, the phase voltage, is the magnitude of the phasor voltage from the neutral to any line. The *line-to-line* voltages or, simply, *line voltages* can be calculated using KVL; for example,

$$
\begin{aligned}
\mathbf{V}_{ab} &= \mathbf{V}_{an} - \mathbf{V}_{bn} \\
&= V_p \angle 0^\circ - V_p \angle -120^\circ \\
&= V_p - V_p\left[-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right] \\
&= V_p\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right] \\
&= \sqrt{3}\, V_p \angle 30^\circ
\end{aligned}
$$

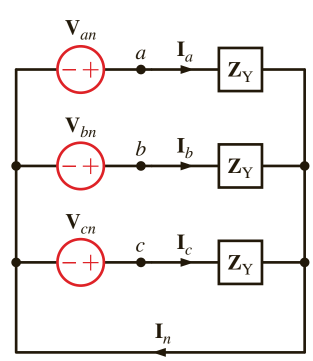


Figure 11.13
Balanced three-phase wye–wye connection.

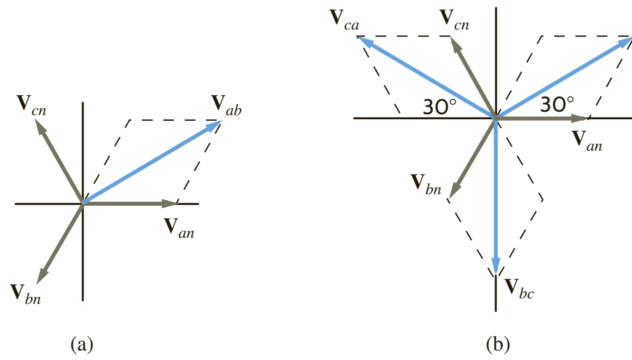


Figure 11.14

Phasor representation of phase and line voltages in a balanced wye–wye system.

The phasor addition is shown in **Fig. 11.14a**. In a similar manner, we obtain the set of line-to-line voltages as

$$\mathbf{V}_{ab} = \sqrt{3}\, V_p \underline{/30^\circ}$$
$$\mathbf{V}_{bc} = \sqrt{3}\, V_p \underline{/-90^\circ} \qquad \textbf{11.14}$$
$$\mathbf{V}_{ca} = \sqrt{3}\, V_p \underline{/-210^\circ}$$

HINT

Conversion rules:

$\underline{/\mathbf{V}_{ab}} = \underline{/\mathbf{V}_{an}} + 30^\circ$

$V_{ab} = \sqrt{3}\, V_{an}$

All the line voltages together with the phase voltages are shown in **Fig. 11.14b**. We will denote the magnitude of the line voltages as V_L, and therefore, for a balanced system,

$$V_L = \sqrt{3}\, V_p \qquad \textbf{11.15}$$

Hence, in a wye-connected system, the line voltage is equal to $\sqrt{3}$ times the phase voltage.

As shown in Fig. 11.13, the line current for the a phase is

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{V_p \underline{/0^\circ}}{\mathbf{Z}_Y} \qquad \textbf{11.16}$$

where $\mathbf{I}_b$ and $\mathbf{I}_c$ have the same magnitude but lag $\mathbf{I}_a$ by 120° and 240°, respectively.

The neutral current $\mathbf{I}_n$ is then

$$\mathbf{I}_n = (\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \qquad \textbf{11.17}$$

Since there is no current in the neutral, this conductor could contain any impedance or it could be an open or a short circuit, without changing the results found previously.

As illustrated by the wye–wye connection in Fig. 11.13, the current in the line connecting the source to the load is the same as the phase current flowing through the impedance $\mathbf{Z}_Y$. Therefore, in a *wye–wye connection*,

$$I_L = I_Y \qquad \textbf{11.18}$$

where I_L is the magnitude of the line current and I_Y is the magnitude of the current in a wye-connected load.

Although we have a three-phase system composed of three sources and three loads, we can analyze a single phase and use the phase sequence to obtain the voltages and currents in the other phases. This is, of course, a direct result of the balanced condition. We may even have impedances present in the lines; however, as long as the system remains balanced, we need analyze only one phase. If the line impedances in lines a, b, and c are equal, the system will be balanced. Recall that the balance of the system is unaffected by whatever appears in the neutral line, and since the neutral line impedance is arbitrary, we assume that it is zero (i.e., a short circuit).

EXAMPLE 11.1

An *abc*-sequence three-phase voltage source connected in a balanced wye has a line voltage of $\mathbf{V}_{ab} = 208\angle{-30°}$ V rms. We wish to determine the phase voltages.

SOLUTION

The magnitude of the phase voltage is given by the expression

$$V_p = \frac{208}{\sqrt{3}}$$
$$= 120 \text{ V rms}$$

The phase relationships between the line and phase voltages are shown in Fig. 11.14. From this figure we note that

$$\mathbf{V}_{an} = 120\angle{-60°} \text{ V rms}$$
$$\mathbf{V}_{bn} = 120\angle{-180°} \text{ V rms}$$
$$\mathbf{V}_{cn} = 120\angle{+60°} \text{ V rms}$$

The magnitudes of these voltages are quite common, and one often hears that the electric service in a building, for example, is three-phase 208/120 V rms.

HINT

The phase of

$\mathbf{V}_{an} = \angle \mathbf{V}_{an} = \angle \mathbf{V}_{ab} - 30°$

EXAMPLE 11.2

A three-phase wye-connected load is supplied by an *abc*-sequence balanced three-phase wye-connected source with a phase voltage of 120 V rms. If the line impedance and load impedance per phase are $1 + j1\ \Omega$ and $20 + j10\ \Omega$, respectively, we wish to determine the value of the line currents and the load voltages.

SOLUTION

The phase voltages are

$$\mathbf{V}_{an} = 120\angle{0°} \text{ V rms}$$
$$\mathbf{V}_{bn} = 120\angle{-120°} \text{ V rms}$$
$$\mathbf{V}_{cn} = 120\angle{+120°} \text{ V rms}$$

The per-phase circuit diagram is shown in **Fig. 11.15**. The line current for the *a* phase is

$$\mathbf{I}_{aA} = \frac{120\angle{0°}}{21 + j11}$$
$$= 5.06\angle{-27.65°} \text{ A rms}$$

The load voltage for the *a* phase, which we call $\mathbf{V}_{AN}$, is

$$\mathbf{V}_{AN} = (5.06\angle{-27.65°})(20 + j10)$$
$$= 113.15\angle{-1.08°} \text{ V rms}$$

HINT

$\angle \mathbf{I}_{bB} = \angle \mathbf{I}_{aA} - 120°$

$\angle \mathbf{I}_{cC} = \angle \mathbf{I}_{aA} + 120°$

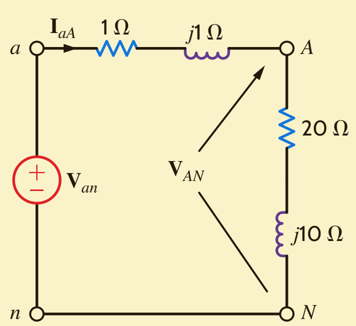


Figure 11.15

Per-phase circuit diagram for the problem in Example 11.2.

The corresponding line currents and load voltages for the b and c phases are

$$\mathbf{I}_{bB} = 5.06\angle{-147.65°} \text{ A rms} \qquad \mathbf{V}_{BN} = 113.15\angle{-121.08°} \text{ V rms}$$
$$\mathbf{I}_{cC} = 5.06\angle{-267.65°} \text{ A rms} \qquad \mathbf{V}_{CN} = 113.15\angle{-241.08°} \text{ V rms}$$

To reemphasize and clarify our terminology, phase voltage, V_p, is the magnitude of the phasor voltage from the neutral to any line, while line voltage, V_L, is the magnitude of the phasor voltage between any two lines. Thus, the values of V_L and V_p will depend on the point at which they are calculated in the system.

LEARNING ASSESSMENTS

E11.1 The voltage for the a phase of an abc-phase-sequence balanced wye-connected source is $\mathbf{V}_{an} = 120\angle{90°}$ V rms. Determine the line voltages for this source.

ANSWER:
$\mathbf{V}_{ab} = 208\angle{120°}$ V rms;
$\mathbf{V}_{bc} = 208\angle{0°}$ V rms;
$\mathbf{V}_{ca} = 208\angle{-120°}$ V rms.

E11.2 An abc-phase-sequence three-phase voltage source connected in a balanced wye has a line voltage of $\mathbf{V}_{ab} = 208\angle{0°}$ V rms. Determine the phase voltages of the source.

ANSWER:
$\mathbf{V}_{an} = 120\angle{-30°}$ V rms;
$\mathbf{V}_{bn} = 120\angle{-150°}$ V rms;
$\mathbf{V}_{cn} = 120\angle{-270°}$ V rms.

E11.3 A three-phase wye-connected load is supplied by an abc-sequence balanced three-phase wye-connected source through a transmission line with an impedance of $1 + j1\ \Omega$ per phase. The load impedance is $8 + j3\ \Omega$ per phase. If the load voltage for the a phase is $104.02\angle{26.6°}$ V rms (i.e., $V_p = 104.02$ V rms at the load end), determine the phase voltages of the source.

ANSWER:
$\mathbf{V}_{an} = 120\angle{30°}$ V rms;
$\mathbf{V}_{bn} = 120\angle{-90°}$ V rms;
$\mathbf{V}_{cn} = 120\angle{-210°}$ V rms.

E11.4 A positive-sequence balanced three-phase wye-connected source with a phase voltage of 277 V rms supplies power to a balanced wye-connected load. The per-phase load impedance is $60 - j40\ \Omega$. Determine the line currents in the circuit if the phase angle of $\mathbf{V}_{an} = 0°$.

ANSWER:
$\mathbf{I}_{aA} = 3.84\angle{33.69°}$ A rms;
$\mathbf{I}_{bB} = 3.84\angle{-86.31°}$ A rms;
$\mathbf{I}_{cC} = 3.84\angle{153.69°}$ A rms.

E11.5 An abc-sequence set of voltages feeds a balanced three-phase wye–wye system. The line and load impedances are $0.5 + j0.75\ \Omega$ and $20 - j24\ \Omega$, respectively. If the load voltage of the a phase is $\mathbf{V}_{AN} = 125\angle{10°}$ V rms, find the line voltages of the input.

ANSWER:
$\mathbf{V}_{ab} = 214.8\angle{41.6°}$ V rms;
$\mathbf{V}_{bc} = 214.8\angle{-78.4°}$ V rms;
$\mathbf{V}_{ca} = 214.8\angle{161.6°}$ V rms.

E11.6 In a balanced three-phase wye–wye system, the total power in the lines is 650 W. $\mathbf{V}_{AN} = 117\angle{15°}$ V rms and the power factor of the load is 0.88 leading. If the line impedance is $1 + j2\ \Omega$, determine the load impedance.

ANSWER:
$\mathbf{Z}_L = 7 - j3.78\ \Omega$.

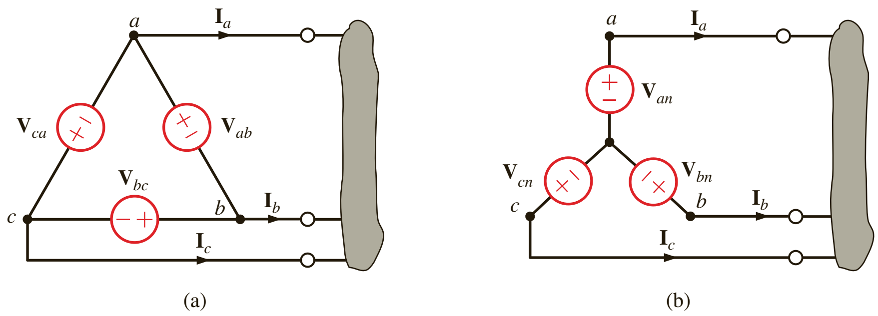


Figure 11.16

Sources connected in delta and wye.

The previous analysis indicates that we can simply treat a three-phase balanced circuit on a per-phase basis and use the phase relationship to determine all voltages and currents. Let us now examine the situations in which either the source or the load is connected in Δ.

DELTA-CONNECTED SOURCE Consider the delta-connected source shown in **Fig. 11.16a**. Note that the sources are connected line to line. We found earlier that the relationship between line-to-line and line-to-neutral voltages was given by Eq. (11.14) and illustrated in Fig. 11.14 for an *abc*-phase sequence of voltages. Therefore, if the delta sources are

$$\begin{aligned} \mathbf{V}_{ab} &= V_L \angle 0^\circ \\ \mathbf{V}_{bc} &= V_L \angle -120^\circ \\ \mathbf{V}_{ca} &= V_L \angle +120^\circ \end{aligned} \qquad \textbf{11.19}$$

where V_L is the magnitude of the phase voltage. The equivalent wye sources shown in **Fig. 11.16b** are

$$\begin{aligned} \mathbf{V}_{an} &= \frac{V_L}{\sqrt{3}} \angle -30^\circ = V_p \angle -30^\circ \\ \mathbf{V}_{bn} &= \frac{V_L}{\sqrt{3}} \angle -150^\circ = V_p \angle -150^\circ \\ \mathbf{V}_{cn} &= \frac{V_L}{\sqrt{3}} \angle -270^\circ = V_p \angle +90^\circ \end{aligned} \qquad \textbf{11.20}$$

where V_p is the magnitude of the phase voltage of an equivalent wye-connected source. Therefore, if we encounter a network containing a delta-connected source, we can easily convert the source from delta to wye so that all the techniques we have discussed previously can be applied in an analysis.

PROBLEM-SOLVING STRATEGY

THREE-PHASE BALANCED AC POWER CIRCUITS

STEP 1. Convert the source/load connection to a wye–wye connection if either the source, load, or both are connected in delta since the wye–wye connection can be easily used to obtain the unknown phasors.

STEP 2. Only the unknown phasors for the *a* phase of the circuit need be determined since the three-phase system is balanced.

STEP 3. Finally, convert the now-known phasors to the corresponding phasors in the original system.

EXAMPLE 11.3

Consider the network shown in Fig. 11.17a. We wish to determine the line currents and the magnitude of the line voltage at the load.

SOLUTION

The single-phase diagram for the network is shown in Fig. 11.17b. The line current $\mathbf{I}_{aA}$ is

$$\mathbf{I}_{aA} = \frac{(208/\sqrt{3})\angle -30^\circ}{12.1 + j4.2}$$
$$= 9.38\angle -49.14^\circ \text{ A rms}$$

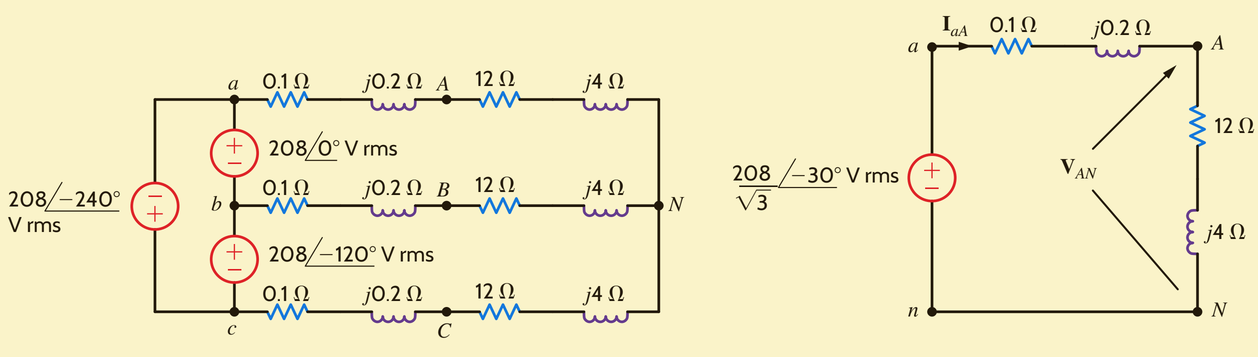
and thus $\mathbf{I}_{bB} = 9.38\angle -169.14^\circ$ V rms and $\mathbf{I}_{cC} = 9.38\angle 70.86^\circ$ V rms. The voltage $\mathbf{V}_{AN}$ is then

$$\mathbf{V}_{AN} = (9.38\angle -49.14^\circ)(12 + j4)$$
$$= 118.65\angle -30.71^\circ \text{ V rms}$$

Therefore, the magnitude of the line voltage at the load is

$$V_L = \sqrt{3}(118.65)$$
$$= 205.51 \text{ V rms}$$

The phase voltage at the source is $V_p = 208/\sqrt{3} = 120$ V rms, while the phase voltage at the load is $V_p = 205.51/\sqrt{3} = 118.65$ V rms. Clearly, we must be careful with our notation and specify where the phase or line voltage is taken.



(a) (b)

Figure 11.17 Delta–wye network and an equivalent single-phase (*a*-phase) diagram.

LEARNING ASSESSMENTS

E11.7 Consider the network shown in Fig. E11.7. Compute the magnitude of the line voltages at the load.

ANSWER: $V_L = 205.2$ V rms.

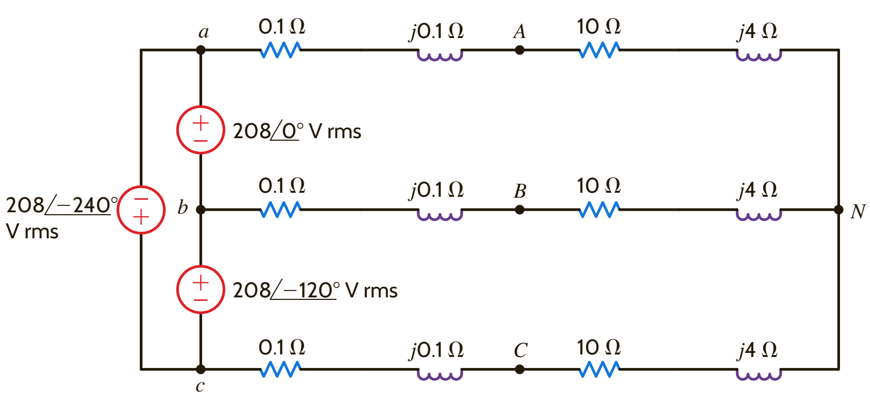


Figure E11.7

E11.8 Find the magnitude of the line voltage at the load in Fig. E11.8.

ANSWER:
$V_L = 209.2$ V rms.

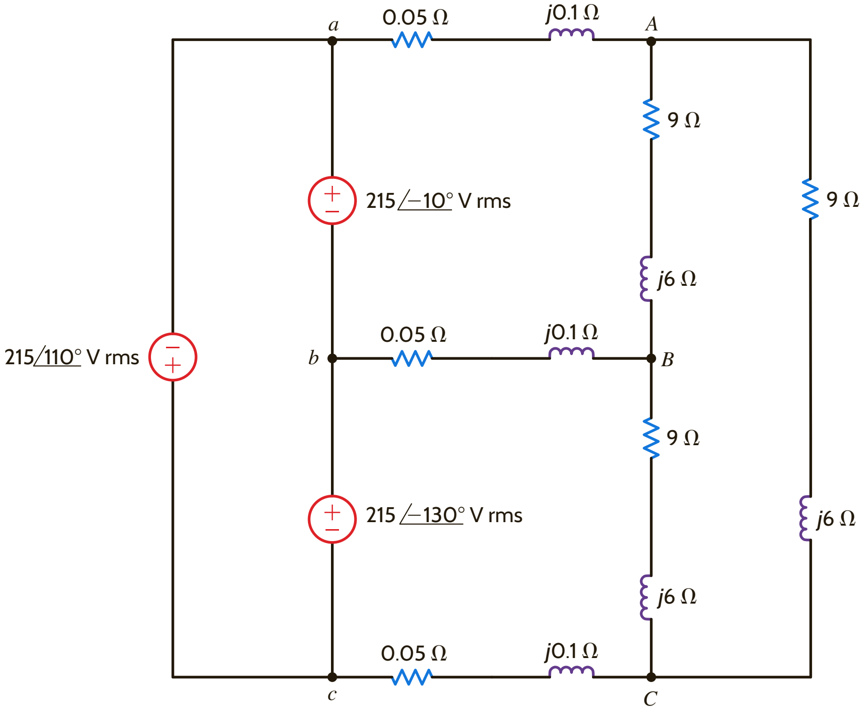


Figure E11.8

DELTA-CONNECTED LOAD Consider now the Δ-connected load shown in Fig. 11.18. Note that in this connection the line-to-line voltage is the voltage across each load impedance.

If the phase voltages of the source are

$$\begin{aligned} \mathbf{V}_{an} &= V_p\underline{/0^\circ} \\ \mathbf{V}_{bn} &= V_p\underline{/-120^\circ} \\ \mathbf{V}_{cn} &= V_p\underline{/+120^\circ} \end{aligned} \tag{11.21}$$

then the line voltages are

$$\begin{aligned} \mathbf{V}_{ab} &= \sqrt{3}\, V_p\underline{/30^\circ} = V_L\underline{/30^\circ} = \mathbf{V}_{AB} \\ \mathbf{V}_{bc} &= \sqrt{3}\, V_p\underline{/-90^\circ} = V_L\underline{/-90^\circ} = \mathbf{V}_{BC} \\ \mathbf{V}_{ca} &= \sqrt{3}\, V_p\underline{/-210^\circ} = V_L\underline{/-210^\circ} = \mathbf{V}_{CA} \end{aligned} \tag{11.22}$$

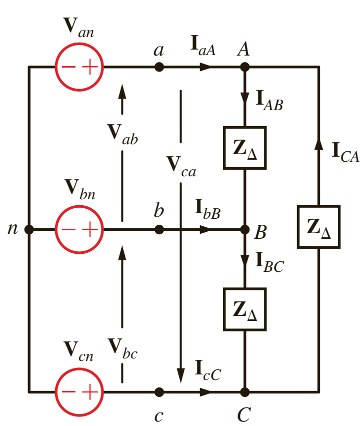


Figure 11.18
Balanced three-phase wye–delta system.

where V_L is the magnitude of the line voltage at both the delta-connected load and at the source since there is no line impedance present in the network.

From Fig. 11.18 we note that if $\mathbf{Z}_\Delta = Z_\Delta \underline{/\theta}$, the phase currents at the load are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} \tag{11.23}$$

where $\mathbf{I}_{BC}$ and $\mathbf{I}_{CA}$ have the same magnitude but lag $\mathbf{I}_{AB}$ by 120° and 240°, respectively. KCL can now be employed in conjunction with the phase currents to determine the line currents. For example,

$$\begin{aligned}\mathbf{I}_{aA} &= \mathbf{I}_{AB} + \mathbf{I}_{AC}\\ &= \mathbf{I}_{AB} - \mathbf{I}_{CA}\end{aligned}$$

However, it is perhaps easier to simply convert the balanced Δ-connected load to a balanced Y-connected load using the Δ–Y transformation. This conversion is possible since the wye–delta and delta–wye transformations outlined in Chapter 2 are also valid for impedance in the frequency domain. In the balanced case, the transformation equations reduce to

$$\mathbf{Z}_\text{Y} = \frac{1}{3}\mathbf{Z}_\Delta$$

and then the line current $\mathbf{I}_{aA}$ is simply

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\text{Y}}$$

Finally, using the same approach as that employed earlier to determine the relationship between the line voltages and phase voltages in a Y–Y connection, we can show that the relationship between the *magnitudes* of the phase currents in the Δ-connected load and the line currents is

$$I_L = \sqrt{3}\, I_\Delta \tag{11.24}$$

EXAMPLE 11.4

A balanced delta-connected load contains a 10-Ω resistor in series with a 20-mH inductor in each phase. The voltage source is an *abc*-sequence three-phase 60-Hz, balanced wye with a voltage $\mathbf{V}_{an} = 120\underline{/30^\circ}$ V rms. We wish to determine all Δ currents and line currents.

SOLUTION

The impedance per phase in the delta load is $\mathbf{Z}_\Delta = 10 + j7.54\ \Omega$. The line voltage $\mathbf{V}_{ab} = 120\sqrt{3}\underline{/60^\circ}$ V rms. Since there is no line impedance, $\mathbf{V}_{AB} = \mathbf{V}_{ab} = 120\sqrt{3}\underline{/60^\circ}$ V rms. Hence,

$$\begin{aligned}\mathbf{I}_{AB} &= \frac{120\sqrt{3}\underline{/60^\circ}}{10 + j7.54}\\ &= 16.60\underline{/+22.98^\circ}\text{ A rms}\end{aligned}$$

If $\mathbf{Z}_\Delta = 10 + j7.54\ \Omega$, then

$$\begin{aligned}\mathbf{Z}_\text{Y} &= \frac{1}{3}\mathbf{Z}_\Delta\\ &= 3.33 + j2.51\ \Omega\end{aligned}$$

Then the line current

$$\begin{aligned}\mathbf{I}_{aA} &= \frac{\mathbf{V}_{an}}{\mathbf{Z}_\text{Y}} = \frac{120\underline{/30^\circ}}{3.33 + j2.51}\\ &= \frac{120\underline{/30^\circ}}{4.17\underline{/37.01^\circ}}\\ &= 28.78\underline{/-7.01^\circ}\text{ A rms}\end{aligned}$$

Therefore, the remaining phase and line currents are

$$\mathbf{I}_{BC} = 16.60\angle{-97.02°} \text{ A rms} \qquad \mathbf{I}_{bB} = 28.78\angle{-127.01°} \text{ A rms}$$
$$\mathbf{I}_{CA} = 16.60\angle{+142.98°} \text{ A rms} \qquad \mathbf{I}_{cC} = 28.78\angle{+112.99°} \text{ A rms}$$

In summary, the relationships between the line voltage and phase voltage and the line current and phase current for both the Y and Δ configurations are shown in **Fig. 11.19**. The currents and voltages are shown for one phase. The two remaining phases have the same magnitude but lag by 120° and 240°, respectively.

Careful observation of Table 11.1 indicates that the following rules apply when solving problems in balanced three-phase systems:

- The phase of the voltages and currents in a Δ connection is 30° ahead of those in a Y connection.
- The magnitude of the line voltage or, equivalently, the Δ-connection phase voltage, is $\sqrt{3}$ times that of the Y-connection phase voltage.
- The magnitude of the line current or, equivalently, the Y-connection phase current, is $\sqrt{3}$ times that of the Δ-connection phase current.
- The load impedance in the Y connection is one-third of that in the Δ-connection, and the phase is identical.

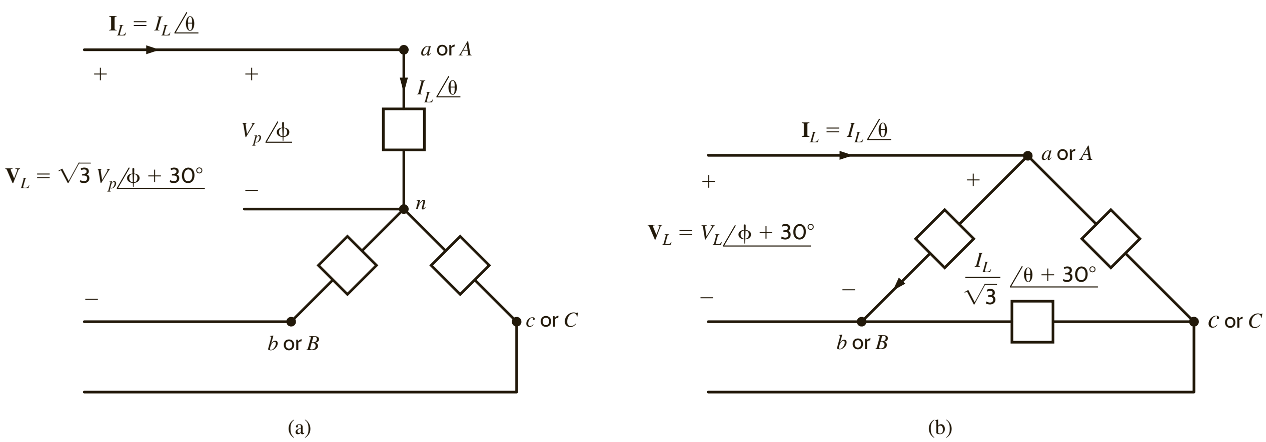


Figure 11.19

Voltage and current relationships for Y and Δ configurations.

TABLE 11.1 The voltage, current, and impedance relationships for Y and Δ configurations

	Y	Δ
Line voltage	$\sqrt{3}\, V_p\angle\phi + 30°$	$V_L\angle\phi + 30°$
($\mathbf{V}_{ab}$ or $\mathbf{V}_{AB}$)	$= V_L\angle\phi + 30°$	
Line current $\mathbf{I}_{aA}$	$I_L\angle\theta$	$I_L\angle\theta$
Phase voltage	$V_p\angle\phi$ ($\mathbf{V}_{an}$ or $\mathbf{V}_{AN}$)	$\sqrt{3}\, V_p\angle\phi + 30°$
Phase current	$I_L\angle\theta$	$\frac{I_L}{\sqrt{3}}\angle\theta + 30°$
Load impedance	$Z_Y\angle\phi - \theta$	$3Z_Y\angle\phi - \theta$