E9.5 Given the network in Fig. E9.5, determine the total average power absorbed or supplied by each element.

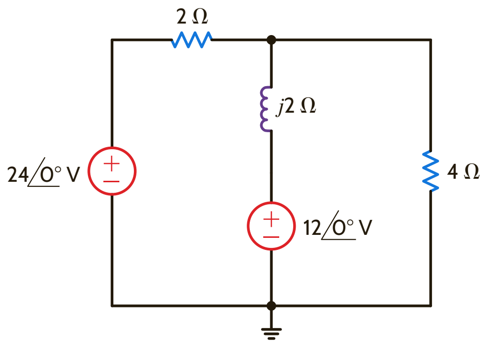


Figure E9.5

ANSWER:
$P_{24/0°} = -55.4$ W;
$P_{12/0°} = 5.5$ W;
$P_{2\Omega} = 22.2$ W;
$P_{4\Omega} = 27.7$ W;
$P_L = 0$.

E9.6 Determine the average power absorbed by the 4-Ω and 3-Ω resistors in Fig. E9.6.

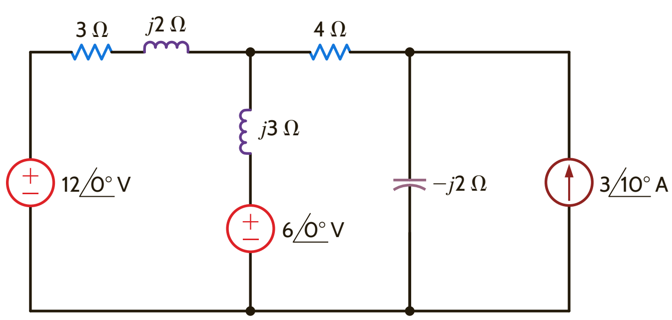


Figure E9.6

ANSWER:
$P_{4\Omega} = 9.86$ W;
$P_{3\Omega} = 0.91$ W.

9.3 Maximum Average Power Transfer

In our study of resistive networks, we addressed the problem of maximum power transfer to a resistive load. We showed that if the network excluding the load was represented by a Thévenin equivalent circuit, maximum power transfer would result if the value of the load resistor was equal to the Thévenin equivalent resistance (i.e., $R_L = R_{\text{Th}}$). We will now reexamine this issue within the present context to determine the load impedance for the network shown in Fig. 9.6 that will result in maximum average power being absorbed by the load impedance $\mathbf{Z}_L$.

The equation for average power at the load is

$$P_L = \frac{1}{2} V_L I_L \cos(\theta_{v_L} - \theta_{i_L}) \tag{9.11}$$

The phasor current and voltage at the load are given by the expressions

$$\mathbf{I}_L = \frac{\mathbf{V}_{oc}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} \tag{9.12}$$

$$\mathbf{V}_L = \frac{\mathbf{V}_{oc}\mathbf{Z}_L}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} \tag{9.13}$$

where

$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}} \tag{9.14}$$

and

$$\mathbf{Z}_L = R_L + jX_L \tag{9.15}$$

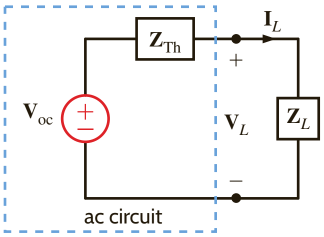


Figure 9.6
Circuit used to examine maximum average power transfer.

HINT

This impedance-matching concept is an important issue in the design of high-speed computer chips and motherboards. For today's high-speed chips with internal clocks running at about 3 GHz and motherboards with a bus speed above 1 GHz, impedance matching is necessary in order to obtain the required speed for signal propagation. Although this high-speed transmission line is based on a distributed circuit (discussed later in electrical engineering courses), the impedance-matching technique for the transmission line is the same as that of the lumped parameter circuit for maximum average power transfer.

The magnitude of the phasor current and voltage are given by the expressions

$$I_L = \frac{V_{oc}}{[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^{1/2}} \qquad 9.16$$

$$V_L = \frac{V_{oc}(R_L^2 + X_L^2)^{1/2}}{[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2]^{1/2}} \qquad 9.17$$

The phase angles for the phasor current and voltage are contained in the quantity $(\theta_{v_L} - \theta_{i_L})$. Note also that $\theta_{v_L} - \theta_{i_L} = \theta_{\mathbf{Z}_L}$ and, in addition,

$$\cos\theta_{\mathbf{Z}_L} = \frac{R_L}{(R_L^2 + X_L^2)^{1/2}} \qquad 9.18$$

Substituting Eqs. (9.16) to (9.18) into Eq. (9.11) yields

$$P_L = \frac{1}{2}\frac{V_{oc}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} \qquad 9.19$$

which could, of course, be obtained directly from Eq. (9.16) using $P_L = \frac{1}{2}I_L^2 R_L$. Once again, a little forethought will save us some work. From the standpoint of maximizing P_L, V_{oc} is a constant. The quantity $(X_{Th} + X_L)$ absorbs no power, and therefore any nonzero value of this quantity only serves to reduce P_L. Hence, we can eliminate this term by selecting $X_L = -X_{Th}$. Our problem then reduces to maximizing

$$P_L = \frac{1}{2}\frac{V_{oc}^2 R_L}{(R_L + R_{Th})^2} \qquad 9.20$$

However, this is the same quantity we maximized in the purely resistive case by selecting $R_L = R_{Th}$. Therefore, for maximum average power transfer to the load shown in Fig. 9.6, $\mathbf{Z}_L$ should be chosen so that

$$\mathbf{Z}_L = R_L + jX_L = R_{Th} - jX_{Th} = \mathbf{Z}_{Th}^* \qquad 9.21$$

Finally, if the load impedance is purely resistive (i.e., $X_L = 0$), the condition for maximum average power transfer can be derived via the expression

$$\frac{dP_L}{dR_L} = 0$$

where P_L is the expression in Eq. (9.19) with $X_L = 0$. *The value of R_L that maximizes P_L under the condition $X_L = 0$ is*

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} \qquad 9.22$$

PROBLEM-SOLVING STRATEGY

MAXIMUM AVERAGE POWER TRANSFER

STEP 1. Remove the load $\mathbf{Z}_L$ and find the Thévenin equivalent for the remainder of the circuit.

STEP 2. Construct the circuit shown in Fig. 9.6.

STEP 3. Select $\mathbf{Z}_L = \mathbf{Z}_{Th}^* = R_{Th} - jX_{Th}$, and then $\mathbf{I}_L = \mathbf{V}_{oc}/2R_{Th}$ and the maximum average power transfer $= \frac{1}{2}\mathbf{I}_L^2 R_{Th} = \mathbf{V}_{oc}^2/8R_{Th}$.

EXAMPLE 9.5

Given the circuit in Fig. 9.7a, we wish to find the value of $\mathbf{Z}_L$ for maximum average power transfer. In addition, we wish to find the value of the maximum average power delivered to the load.

SOLUTION

To solve the problem, we form a Thévenin equivalent at the load. The circuit in Fig. 9.7b is used to compute the open-circuit voltage

$$\mathbf{V}_{oc} = \frac{4\underline{/0^\circ}\,(2)}{6 + j1}(4) = 5.26\underline{/-9.46^\circ}\text{ V}$$

The Thévenin equivalent impedance can be derived from the circuit in Fig. 9.7c. As shown in the figure,

$$\mathbf{Z}_{Th} = \frac{4(2 + j1)}{6 + j1} = 1.41 + j0.43\ \Omega$$

Therefore, $\mathbf{Z}_L$ for maximum average power transfer is

$$\mathbf{Z}_L = 1.41 - j0.43\ \Omega$$

With $\mathbf{Z}_L$ as given previously, the current in the load is

$$\mathbf{I} = \frac{5.26\underline{/-9.46^\circ}}{2.82} = 1.87\underline{/-9.46^\circ}\text{ A}$$

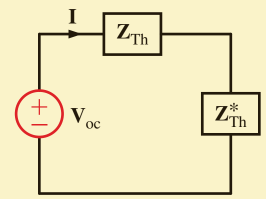
Therefore, the maximum average power transferred to the load is

$$P_L = \frac{1}{2}I_M^2 R_L = \frac{1}{2}(1.87)^2(1.41) = 2.47\text{ W}$$

HINT

In this Thévenin analysis,

1. Remove $\mathbf{Z}_L$ and find the voltage across the open terminals, $\mathbf{V}_{oc}$.
2. Determine the impedance $\mathbf{Z}_{Th}$ at the open terminals with all independent sources made zero.
3. Construct the following circuit and determine $\mathbf{I}$ and P_L.



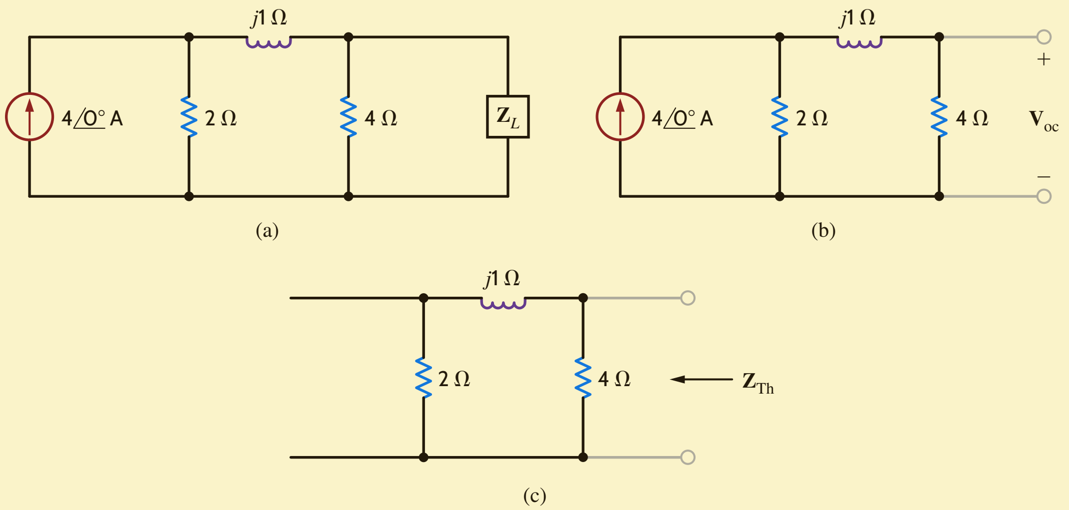


Figure 9.7 Circuits for illustrating maximum average power transfer.

EXAMPLE 9.6

For the circuit shown in Fig. 9.8a, we wish to find the value of $\mathbf{Z}_L$ for maximum average power transfer. In addition, let us determine the value of the maximum average power delivered to the load.

SOLUTION

We will first reduce the circuit, with the exception of the load, to a Thévenin equivalent circuit. The open-circuit voltage can be computed from Fig. 9.8b. The equations for the circuit are

$$\mathbf{V}_x' + 4 = (2 + j4)\mathbf{I}_1$$
$$\mathbf{V}_x' = -2\mathbf{I}_1$$

HINT

When there is a dependent source, both $\mathbf{V}_{oc}$ and $\mathbf{I}_{sc}$ must be found and $\mathbf{Z}_{Th}$ computed from the equation

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}}$$

Solving for $\mathbf{I}_1$, we obtain

$$\mathbf{I}_1 = \frac{1\underline{/-45^\circ}}{\sqrt{2}}$$

The open-circuit voltage is then

$$\begin{aligned}\mathbf{V}_{oc} &= 2\mathbf{I}_1 - 4\underline{/0^\circ} \\ &= \sqrt{2}\,\underline{/-45^\circ} - 4\underline{/0^\circ} \\ &= -3 - j1 \\ &= +3.16\underline{/-161.57^\circ}\text{ V}\end{aligned}$$

The short-circuit current can be derived from Fig. 9.8c. The equations for this circuit are

$$\begin{aligned}\mathbf{V}_x'' + 4 &= (2 + j4)\mathbf{I} - 2\mathbf{I}_{sc} \\ -4 &= -2\mathbf{I} + (2 - j2)\mathbf{I}_{sc} \\ \mathbf{V}_x'' &= -2(\mathbf{I} - \mathbf{I}_{sc})\end{aligned}$$

Solving these equations for $\mathbf{I}_{sc}$ yields

$$\mathbf{I}_{sc} = -(1 + j2)\text{ A}$$

The Thévenin equivalent impedance is then

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{3 + j1}{1 + j2} = 1 - j1\ \Omega$$

Therefore, for maximum average power transfer the load impedance should be

$$\mathbf{Z}_L = 1 + j1\ \Omega$$

The current in this load $\mathbf{Z}_L$ is then

$$\mathbf{I}_L = \frac{\mathbf{V}_{oc}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} = \frac{-3 - j1}{2} = 1.58\underline{/-161.57^\circ}\text{ A}$$

Hence, the maximum average power transferred to the load is

$$\begin{aligned}P_L &= \frac{1}{2}(1.58)^2(1) \\ &= 1.25\text{ W}\end{aligned}$$

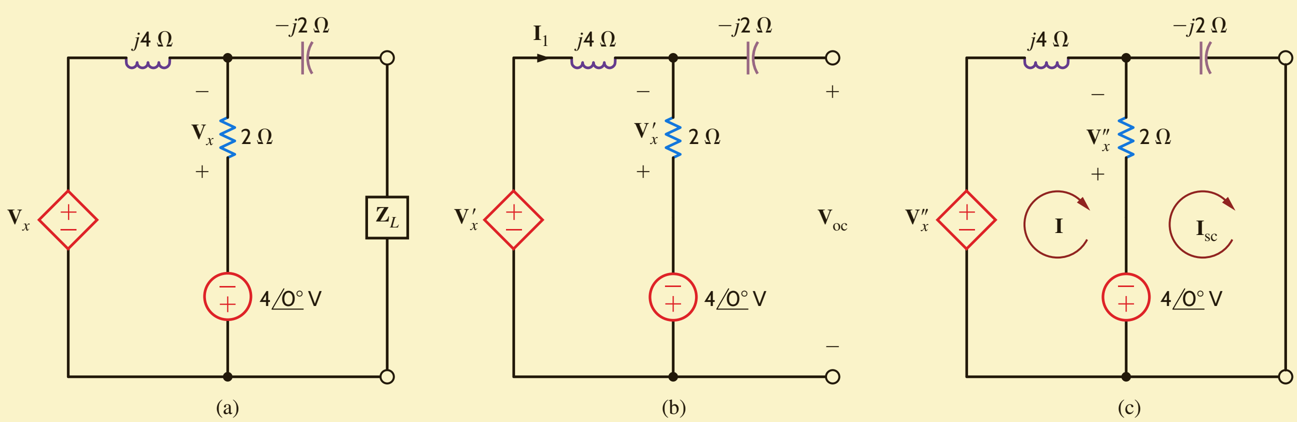


Figure 9.8

Circuits for illustrating maximum average power transfer.

LEARNING ASSESSMENTS

E9.7 Given the network in Fig. E9.7, find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred to the load.

ANSWER:
$\mathbf{Z}_L = 1 + j1\ \Omega$;
$P_L = 45$ W.

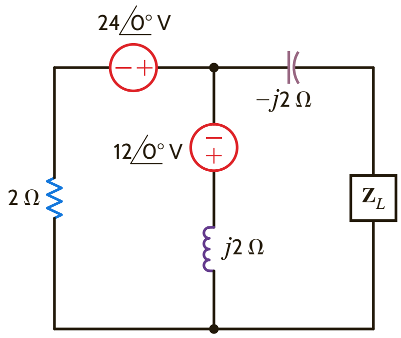


Figure E9.7

E9.8 Find $\mathbf{Z}_L$ for maximum average power transfer and the maximum average power transferred to the load in the network in Fig. E9.8.

ANSWER:
$\mathbf{Z}_L = 2 - j2\ \Omega$;
$P_L = 45$ W.

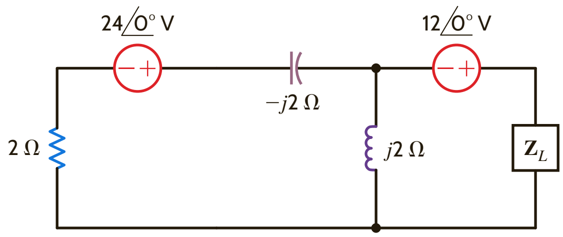


Figure E9.8

E9.9 Determine $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ in Fig. E9.9.

ANSWER:
$\mathbf{Z}_L = 4.79 - j1.68\ \Omega$;
$P_L = 14.26$ W.

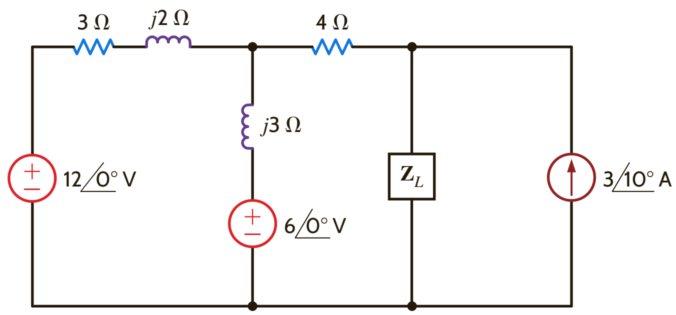


Figure E9.9

E9.10 Find $\mathbf{Z}_L$ for maximum average power transfer and the value of the maximum average power transferred to $\mathbf{Z}_L$ in Fig. E9.10.

ANSWER:
$\mathbf{Z}_L = 5.67 - j2.2\ \Omega$;
$P_L = 9.29$ W.

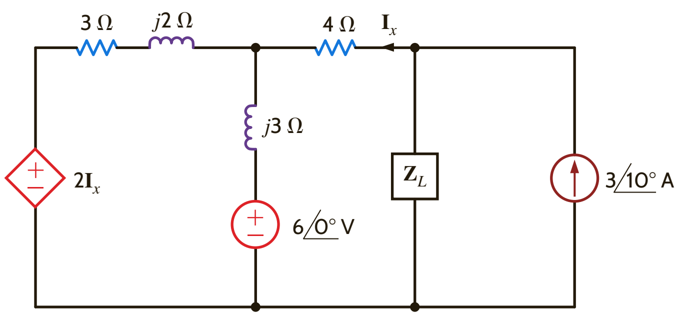


Figure E9.10