

## Single-Node-Pair Circuits

**CURRENT DIVISION** An important circuit is the single-node-pair circuit. If we apply KVL to every loop in a single-node-pair circuit, we discover that all of the elements have the same voltage across them and, therefore, are said to be connected in parallel. We will, however, apply Kirchhoff's current law and Ohm's law to determine various unknown quantities in the circuit.

Following our approach with the single-loop circuit, we will begin with the simplest case and then generalize our analysis. Consider the circuit shown in Fig. 2.22. Here we have an independent current source in parallel with two resistors.

Since all of the circuit elements are in parallel, the voltage  $v(t)$  appears across each of them. Furthermore, an examination of the circuit indicates that the current  $i(t)$  is into the upper node of the circuit and the currents  $i_1(t)$  and  $i_2(t)$  are out of the node. Since KCL essentially states that what goes in must come out, the question we must answer is how  $i_1(t)$  and  $i_2(t)$  divide the input current  $i(t)$ .

Applying Kirchhoff's current law to the upper node, we obtain

$$i(t) = i_1(t) + i_2(t)$$

and, employing Ohm's law, we have

$$\begin{aligned} i(t) &= \frac{v(t)}{R_1} + \frac{v(t)}{R_2} \\ &= \left( \frac{1}{R_1} + \frac{1}{R_2} \right) v(t) \\ &= \frac{v(t)}{R_p} \end{aligned}$$

where

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \quad 2.16$$



The parallel resistance equation.

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \quad 2.17$$

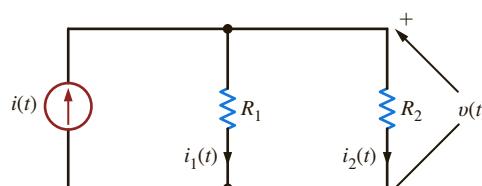
Therefore, the equivalent resistance of two resistors connected in parallel is equal to the product of their resistances divided by their sum. Note also that this equivalent resistance  $R_p$  is always less than either  $R_1$  or  $R_2$ . Hence, by connecting resistors in parallel we reduce the overall resistance. In the special case when  $R_1 = R_2$ , the equivalent resistance is equal to half of the value of the individual resistors.

The manner in which the current  $i(t)$  from the source divides between the two branches is called *current division* and can be found from the preceding expressions. For example,

$$\begin{aligned} v(t) &= R_p i(t) \\ &= \frac{R_1 R_2}{R_1 + R_2} i(t) \end{aligned} \quad 2.18$$

and

$$i_1(t) = \frac{v(t)}{R_1}$$



**Figure 2.22**  
Simple parallel circuit.



**HINT**  
The manner in which current divides between two parallel resistors.

and

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t) \quad 2.19$$

$$\begin{aligned} i_2(t) &= \frac{v(t)}{R_2} \\ &= \frac{R_1}{R_1 + R_2} i(t) \end{aligned} \quad 2.20$$

Eqs. (2.19) and (2.20) are mathematical statements of the current-division rule.

## EXAMPLE 2.17

### SOLUTION

Given the network in Fig. 2.23a, let us find  $I_1$ ,  $I_2$ , and  $V_o$ .

First, it is important to recognize that the current source feeds two parallel paths. To emphasize this point, the circuit is redrawn as shown in Fig. 2.23b. Applying current division, we obtain

$$\begin{aligned} I_1 &= \left[ \frac{40\text{k} + 80\text{k}}{60\text{k} + (40\text{k} + 80\text{k})} \right] (0.9 \times 10^{-3}) \\ &= 0.6 \text{ mA} \end{aligned}$$

and

$$\begin{aligned} I_2 &= \left[ \frac{60\text{k}}{60\text{k} + (40\text{k} + 80\text{k})} \right] (0.9 \times 10^{-3}) \\ &= 0.3 \text{ mA} \end{aligned}$$

Note that the larger current flows through the smaller resistor, and vice versa. In addition, note that if the resistances of the two paths are equal, the current will divide equally between them. KCL is satisfied since  $I_1 + I_2 = 0.9 \text{ mA}$ .

The voltage  $V_o$  can be derived using Ohm's law as

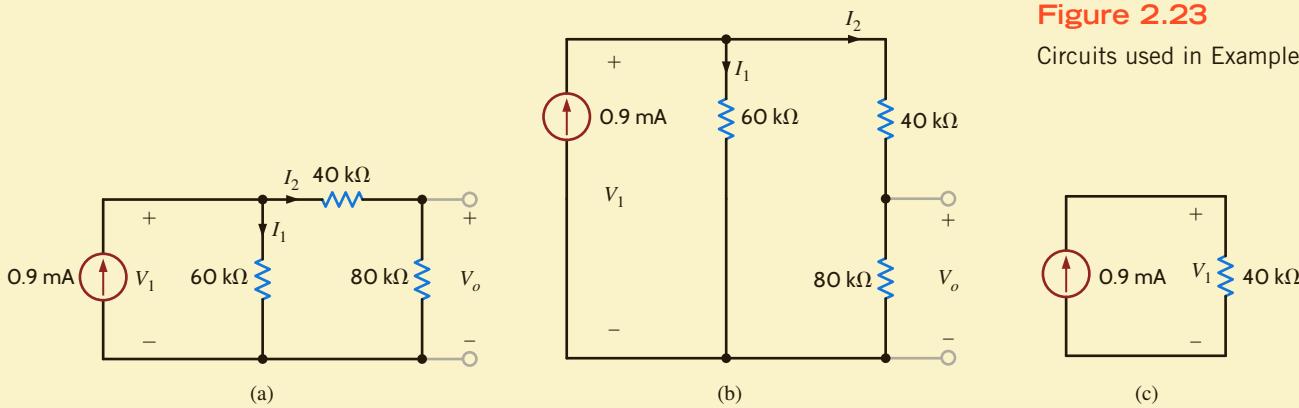
$$\begin{aligned} V_o &= 80\text{k}I_2 \\ &= 24 \text{ V} \end{aligned}$$

The problem can also be approached in the following manner. The total resistance seen by the current source is  $40 \text{ k}\Omega$ ; that is,  $60 \text{ k}\Omega$  in parallel with the series combination of  $40 \text{ k}\Omega$  and  $80 \text{ k}\Omega$ , as shown in Fig. 2.23c. The voltage across the current source is then

$$\begin{aligned} V_1 &= (0.9 \times 10^{-3}) 40\text{k} \\ &= 36 \text{ V} \end{aligned}$$

Now that  $V_1$  is known, we can apply voltage division to find  $V_o$ :

$$\begin{aligned} V_o &= \left( \frac{80\text{k}}{80\text{k} + 40\text{k}} \right) V_1 \\ &= \left( \frac{80\text{k}}{120\text{k}} \right) 36 \\ &= 24 \text{ V} \end{aligned}$$

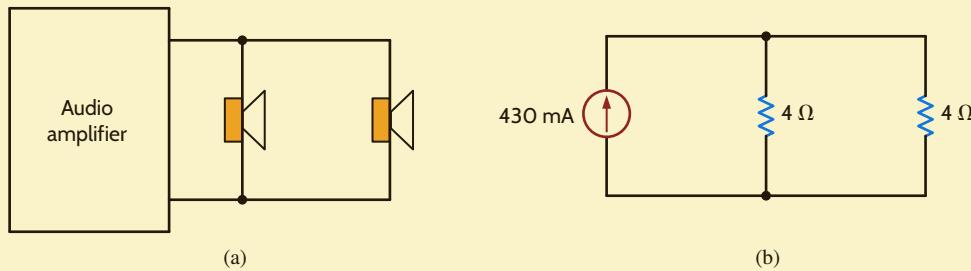
**Figure 2.23**

Circuits used in Example 2.17.

A typical car stereo consists of a 2-W audio amplifier and two speakers represented by the diagram shown in **Fig. 2.24a**. The output circuit of the audio amplifier is in essence a 430-mA current source, and each speaker has a resistance of  $4\ \Omega$ . Let us determine the power absorbed by the speakers.

The audio system can be modeled as shown in **Fig. 2.24b**. Since the speakers are both  $4\ \Omega$  devices, the current will split evenly between them, and the power absorbed by each speaker is

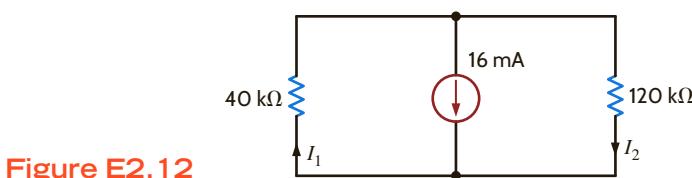
$$\begin{aligned} P &= I^2R \\ &= (215 \times 10^{-3})^2(4) \\ &= 184.9 \text{ mW} \end{aligned}$$

**EXAMPLE 2.18****SOLUTION****Figure 2.24**

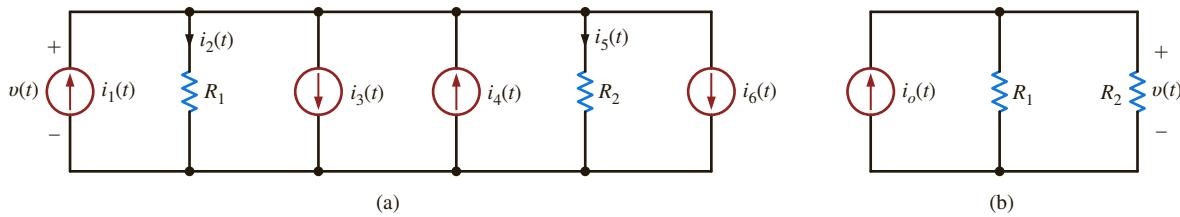
Circuits used in Example 2.18.

**LEARNING ASSESSMENT**

**E2.12** Find the currents  $I_1$  and  $I_2$  and the power absorbed by the  $40\text{-k}\Omega$  resistor in the network in Fig. E2.12.

**Figure E2.12****ANSWER:**

$I_1 = 12\text{ mA}$ ,  
 $I_2 = -4\text{ mA}$ , and  
 $P_{40\text{-k}\Omega} = 5.76\text{ W}$ .



**Figure 2.25**

## Equivalent circuits.

**MULTIPLE-SOURCE/RESISTOR NETWORKS** Let us now extend our analysis to include a multiplicity of current sources and resistors in parallel. For example, consider the circuit shown in Fig. 2.25a. We have assumed that the upper node is  $v(t)$  volts positive with respect to the lower node. Applying Kirchhoff's current law to the upper node yields

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

or

$$i_1(t) - i_3(t) + i_4(t) - i_6(t) = i_2(t) + i_5(t)$$

The terms on the left side of the equation all represent sources that can be combined algebraically into a single source; that is,

$$i_o(t) = i_1(t) - i_3(t) + i_4(t) - i_6(t)$$

which effectively reduces the circuit in Fig. 2.25a to that in **Fig. 2.25b**. We could, of course, generalize this analysis to a circuit with  $N$  current sources. Using Ohm's law, we can express the currents on the right side of the equation in terms of the voltage and individual resistances so that the KCL equation reduces to

$$i_o(t) = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) v(t)$$

Now consider the circuit with  $N$  resistors in parallel, as shown in Fig. 2.26a. Applying Kirchhoff's current law to the upper node yields

$$i_o(t) = i_1(t) + i_2(t) + \dots + i_N(t) \\ = \left( \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \right) v(t) \quad \text{2.21}$$

or

$$i_o(t) = \frac{v(t)}{R_o} \quad 2.22$$

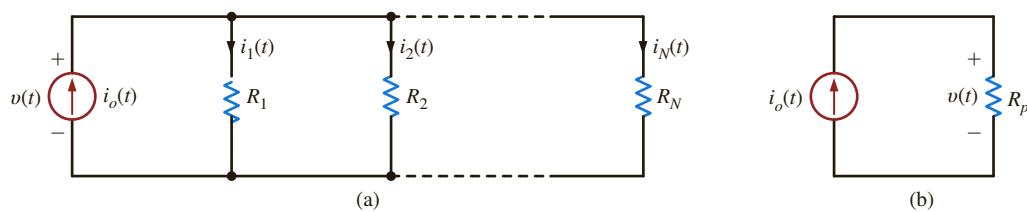
where

$$\frac{1}{R_p} = \sum_{i=1}^N \frac{1}{R_i} \quad 2.23$$

so that as far as the source is concerned, Fig. 2.26a can be reduced to an equivalent circuit, as shown in **Fig. 2.26b**.

**Figure 2.26**

## Equivalent circuits



The current division for any branch can be calculated using Ohm's law and the preceding equations. For example, for the  $j$ th branch in the network of Fig. 2.26a,

$$i_j(t) = \frac{v(t)}{R_j}$$

Using Eq. (2.22), we obtain

$$i_j(t) = \frac{R_p}{R_j} i_o(t) \quad 2.24$$

which defines the current-division rule for the general case.

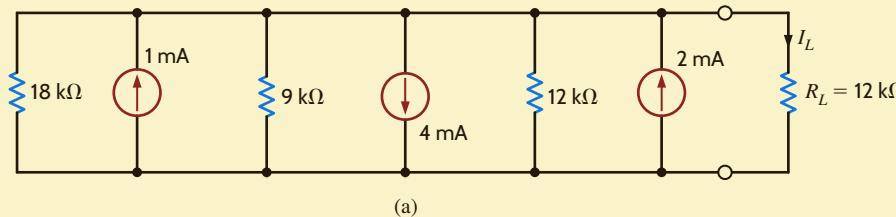
Given the circuit in **Fig. 2.27a**, we wish to find the current in the  $12\text{-k}\Omega$  load resistor.

To simplify the network in Fig. 2.27a, we add the current sources algebraically and combine the parallel resistors in the following manner:

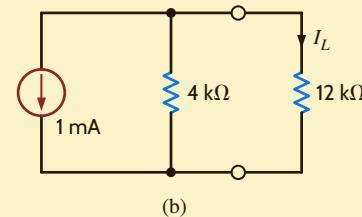
$$\begin{aligned} \frac{1}{R_p} &= \frac{1}{18\text{k}} + \frac{1}{9\text{k}} + \frac{1}{12\text{k}} \\ R_p &= 4\text{ k}\Omega \end{aligned}$$

Using these values we can reduce the circuit in Fig. 2.27a to that in **Fig. 2.27b**. Now, applying current division, we obtain

$$\begin{aligned} I_L &= -\left[\frac{4\text{k}}{4\text{k} + 12\text{k}}\right](1 \times 10^{-3}) \\ &= -0.25\text{ mA} \end{aligned}$$



(a)



(b)

**Figure 2.27**

Circuits used in Example 2.19.

## EXAMPLE 2.19

### SOLUTION



## PROBLEM-SOLVING STRATEGY

- STEP 1.** Define a voltage  $v(t)$  between the two nodes in this circuit. We know from KVL that there is only one voltage for a single-node-pair circuit. A polarity is assigned to the voltage such that one of the nodes is assumed to be at a higher potential than the other node, which we will call the reference node.
- STEP 2.** Using Ohm's law, define a current flowing through each resistor in terms of the defined voltage.
- STEP 3.** Apply KCL at one of the two nodes in the circuit.
- STEP 4.** Solve the single KCL equation for  $v(t)$ . If  $v(t)$  is positive, then the reference node is actually at a lower potential than the other node; if not, the reference node is actually at a higher potential than the other node.

## SINGLE-NODE-PAIR CIRCUITS