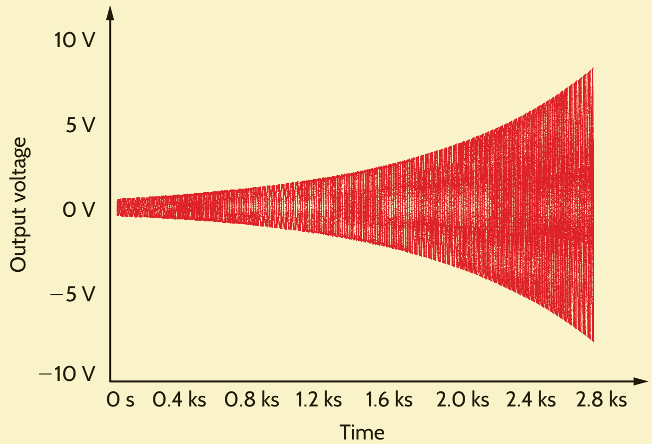

Figure 14.33
Tacoma Narrows Bridge simulation at 42-mph wind speed and one degree of initial twist.

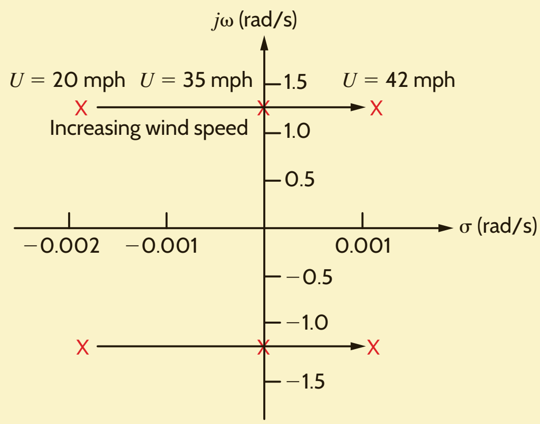

Figure 14.34
Polo-zero plot for Tacoma Narrows Bridge second-order model at wind speeds of 20, 35, and 42 mph.

The dependency of the damping ratio on wind speed can also be demonstrated by investigating how the system poles change with the wind. The characteristic equation for the system is

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

or

$$s^2 + (0.01156 - 0.00033U)s + 1.579 = 0$$

The roots of the characteristic equation yield the pole locations. **Fig. 14.34** shows the system poles at wind speeds of 20, 35, and 42 mph. Note that at 20 mph, the stable situation is shown in Fig. 14.31, and the poles are in the left half of the s-plane. At 35 mph ($\zeta = 0$) the poles are on the $j\omega$ axis and the system is oscillatory, as shown in Fig. 14.32. Finally, at 42 mph, we see that the poles are in the right half of the s-plane, and from Fig. 14.33 we know this is an unstable system. This relationship between pole location and transient response is true for all systems—right-half plane poles result in unstable systems.

14.5 Steady-State Response

In Section 14.3 we have demonstrated, using a variety of examples, the power of the Laplace transform technique in determining the complete response of a network. This complete response is composed of transient terms, which disappear as $t \to \infty$, and steady-state terms, which are present at all times. Let us now examine a method by which to determine the steady-state response of a network directly. Recall from previous examples that the network response can be written as

$$\mathbf{Y}(s) = \mathbf{H}(s)\mathbf{X}(s) \tag{14.24}$$

where $\mathbf{Y}(s)$ is the output or response, $\mathbf{X}(s)$ is the input or forcing function, and $\mathbf{H}(s)$ is the network function or transfer function defined in Section 12.1. The transient portion of the response $\mathbf{Y}(s)$ results from the poles of $\mathbf{H}(s)$, and the steady-state portion of the response results from the poles of the input or forcing function.

As a direct parallel to the sinusoidal response of a network as outlined in Section 8.2, we assume that the forcing function is of the form

$$x(t) = X_M e^{j\omega_0 t} \tag{14.25}$$

which by Euler's identity can be written as

$$x(t) = X_M \cos \omega_0 t + jX_M \sin \omega_0 t \tag{14.26}$$

The Laplace transform of Eq. (14.25) is

$$\mathbf{X}(s) = \frac{X_M}{s - j\omega_0} \tag{14.27}$$

and therefore,

$$\mathbf{Y}(s) = \mathbf{H}(s)\left(\frac{X_M}{s - j\omega_0}\right) \tag{14.28}$$

At this point, we tacitly assume that $\mathbf{H}(s)$ does not have any poles of the form $(s - j\omega_k)$. If, however, this is the case, we simply encounter difficulty in defining the steady-state response.

Performing a partial fraction expansion of Eq. (14.28) yields

$$\mathbf{Y}(s) = \frac{X_M \mathbf{H}(j\omega_0)}{s - j\omega_0} + \text{terms that occur due to the poles of } \mathbf{H}(s) \tag{14.29}$$

The first term to the right of the equal sign can be expressed as

$$\mathbf{Y}(s) = \frac{X_M \left|\mathbf{H}(j\omega_0)\right| e^{j\phi(j\omega_0)}}{s - j\omega_0} + \cdots \tag{14.30}$$

since $\mathbf{H}(j\omega_0)$ is a complex quantity with a magnitude and phase that are a function of $j\omega_0$.

Performing the inverse transform of Eq. (14.30), we obtain

$$\begin{aligned} y(t) &= X_M \left|\mathbf{H}(j\omega_0)\right| e^{j\omega_0 t} e^{j\phi(j\omega_0)} + \cdots \\ &= X_M \left|\mathbf{H}(j\omega_0)\right| e^{(j\omega_0 t + \phi(j\omega_0))} + \cdots \end{aligned} \tag{14.31}$$

and hence the steady-state response is

$$y_{ss}(t) = X_M \left|\mathbf{H}(j\omega_0)\right| e^{j(\omega_0 t + \phi(j\omega_0))} \tag{14.32}$$

HINT
The transient terms disappear in steady state.

Since the actual forcing function is $X_M \cos \omega_0(t)$, which is the real part of $X_M e^{j\omega_0 t}$, the steady-state response is the real part of Eq. (14.32):

$$y_{ss}(t) = X_M \left|\mathbf{H}(j\omega_0)\right| \cos\left[\omega_0 t + \phi(j\omega_0)\right] \tag{14.33}$$

In general, the forcing function may have a phase angle θ. In this case, θ is simply added to $\phi(j\omega_0)$ so that the resultant phase of the response is $\phi(j\omega_0) + \theta$.

EXAMPLE 14.13

For the circuit shown in Fig. 14.35a, we wish to determine the steady-state voltage $v_{oss}(t)$ for $t > 0$ if the initial conditions are zero.

SOLUTION

As illustrated earlier, this problem could be solved using a variety of techniques, such as node equations, mesh equations, source transformation, and Thévenin's theorem. We will employ node equations to obtain the solution. The transformed network using the impedance values for the parameters is shown in Fig. 14.35b. The node equations for this network are

$$\left(\frac{1}{2} + \frac{1}{s} + \frac{s}{2}\right)\mathbf{V}_1(s) - \left(\frac{s}{2}\right)\mathbf{V}_o(s) = \frac{1}{2}\mathbf{V}_i(s)$$

$$-\left(\frac{s}{2}\right)\mathbf{V}_1(s) + \left(\frac{s}{2} + 1\right)\mathbf{V}_o(s) = 0$$

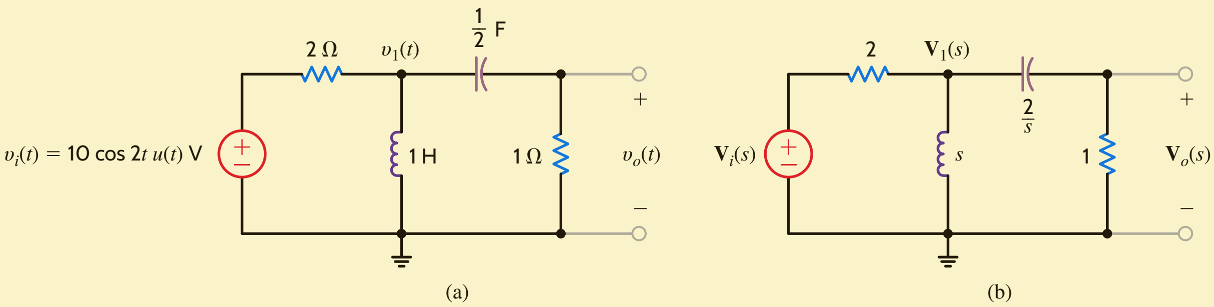

Figure 14.35
Circuits used in Example 14.13.

Solving these equations for $\mathbf{V}_o(s)$, we obtain

$$\mathbf{V}_o(s) = \frac{s^2}{3s^2 + 4s + 4}\mathbf{V}_i(s)$$

Note that this equation is in the form of Eq. (14.24), where $\mathbf{H}(s)$ is

$$\mathbf{H}(s) = \frac{s^2}{3s^2 + 4s + 4}$$

Since the forcing function is $10 \cos 2t\, u(t)$, then $V_M = 10$ and $\omega_0 = 2$. Hence,

$$\begin{aligned}\mathbf{H}(j2) &= \frac{(j2)^2}{3(j2)^2 + 4(j2) + 4}\\ &= 0.354\,\underline{/45^\circ}\end{aligned}$$

Therefore,

$$\begin{aligned}|\mathbf{H}(j2)| &= 0.354\\ \phi(j2) &= 45^\circ\end{aligned}$$

and, hence, the steady-state response is

$$\begin{aligned}v_{oss}(t) &= V_M|\mathbf{H}(j2)|\cos[2t + \phi(j2)]\\ &= 3.54 \cos(2t + 45^\circ)\text{ V}\end{aligned}$$

The complete (transient plus steady-state) response can be obtained from the expression

$$\begin{aligned}\mathbf{V}_o(s) &= \frac{s^2}{3s^2 + 4s + 4}\mathbf{V}_i(s)\\ &= \frac{s^2}{3s^2 + 4s + 4}\left(\frac{10s}{s^2 + 4}\right)\\ &= \frac{10s^3}{(s^2 + 4)(3s^2 + 4s + 4)}\end{aligned}$$

Determining the inverse Laplace transform of this function using the techniques of Chapter 13, we obtain

$$v_o(t) = 3.54 \cos(2t + 45^\circ) + 1.44e^{-(2/3)t}\cos\left(\frac{2\sqrt{2}}{3}t - 55^\circ\right)\text{ V}$$

Note that as $t \to \infty$ the second term approaches zero, and thus the steady-state response is

$$v_{oss}(t) = 3.54 \cos(2t + 45^\circ)\text{ V}$$

which can easily be checked using a phasor analysis.