The standard form for the quadratic factor is

$$(j\omega\tau)^2 + 2\zeta\omega\tau j + 1$$

where $\tau = 1/\omega_0$, and hence in general the quadratic factor can be written as

$$\frac{(j\omega)^2}{\omega_0^2} + \frac{2\zeta\omega}{\omega_0}j + 1 \qquad \textbf{12.51}$$

If we now compare this form of the quadratic factor with the denominator of $\mathbf{Y}(j\omega)$, we find that

$$\omega_0^2 = \frac{1}{LC}$$

$$\frac{2\zeta}{\omega_0} = CR$$

and therefore,

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

However, from Eq. (12.13),

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

and hence,

$$Q = \frac{1}{2\zeta} \qquad \textbf{12.52}$$

To illustrate the significance of this equation, consider the Bode plot for the function $\mathbf{Y}(j\omega)$. The plot has an initial slope of ± 20 dB/decade due to the zero at the origin. If $\zeta > 1$, the poles represented by the quadratic factor in the denominator will simply roll off the frequency response, as illustrated in Fig. 12.12a, and at high frequencies the slope of the composite characteristic will be −20 dB/decade. If $0 < \zeta < 1$, the frequency response will peak as shown in Fig. 12.12a, and the sharpness of the peak will be controlled by ζ. If ζ is very small, the peak of the frequency response is very narrow, the Q of the network is very large, and the circuit is very selective in filtering the input signal. Eq. (12.52) and Fig. 12.23 illustrate the connections among the frequency response, the Q, and the ζ of a network.

12.4 Scaling

Throughout this book we have employed a host of examples to illustrate the concepts being discussed. In many cases the actual values of the parameters were unrealistic in a practical sense, even though they may have simplified the presentation. In this section we illustrate how to *scale* the circuits to make them more realistic.

There are two ways to scale a circuit: *magnitude* or *impedance scaling* and *frequency scaling*. To magnitude scale a circuit, we simply multiply the impedance of each element by a scale factor K_M. Therefore, a resistor R becomes $K_M R$. Multiplying the impedance of an inductor $j\omega L$ by K_M yields a new inductor $K_M L$, and multiplying the impedance of a capacitor $1/j\omega C$ by K_M yields a new capacitor C/K_M. Therefore, in magnitude scaling,

HINT
Magnitude or impedance scaling.

$$R' \rightarrow K_M R$$
$$L' \rightarrow K_M L \qquad \textbf{12.53}$$
$$C' \rightarrow \frac{C}{K_M}$$

since

$$\omega_0' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_M LC/K_M}} = \omega_0$$

and Q' is

$$Q' = \frac{\omega_0 L'}{R'} = \frac{\omega_0 K_M L}{K_M R} = Q$$

The resonant frequency, the quality factor and, therefore, the bandwidth are unaffected by magnitude scaling.

In frequency scaling the scale factor is denoted as K_F. The resistor is frequency independent and, therefore, unaffected by this scaling. The new inductor L', which has the same impedance at the scaled frequency ω_1', must satisfy the equation

$$j\omega_1 L = j\omega_1' L'$$

where $\omega_1' = k_F \omega_1$. Therefore,

$$j\omega_1 L = jK_F \omega_1 L'$$

Hence, the new inductor value is

$$L' = \frac{L}{K_F}$$

Using a similar argument, we find that

HINT
Frequency scaling.

$$C' = \frac{C}{K_F}$$

Therefore, to frequency scale by a factor K_F,

$$\begin{aligned} R' &\rightarrow R \\ L' &\rightarrow \frac{L}{K_F} \\ C' &\rightarrow \frac{C}{K_F} \end{aligned} \qquad \textbf{12.54}$$

Note that

$$\omega_0' = \frac{1}{\sqrt{(L/K_F)(C/K_F)}} = K_F \omega_0$$

and

$$Q' = \frac{K_F \omega_0 L}{RK_F} = Q$$

and therefore,

$$\text{BW}' = K_F(\text{BW})$$

Hence, the resonant frequency and bandwidth of the circuit are affected by frequency scaling.

EXAMPLE 12.17

If the values of the circuit parameters in Fig 12.37 are $R = 2\ \Omega$, $L = 1$ H, and $C = 1/2$ F, let us determine the values of the elements if the circuit is magnitude scaled by a factor $K_M = 10^2$ and frequency scaled by a factor $K_F = 10^2$.

SOLUTION

The magnitude scaling yields

$$\begin{aligned} R' &= 2K_M = 200\ \Omega \\ L' &= (1)K_M = 100\ \text{H} \\ C' &= \frac{1}{2}\frac{1}{K_M} = \frac{1}{200}\ \text{F} \end{aligned}$$

Applying frequency scaling to these values yields the final results:

$$\begin{aligned} R'' &= 200\ \Omega \\ L'' &= \frac{100}{K_F} = 100\ \mu\text{H} \\ C'' &= \frac{1}{200}\frac{1}{K_F} = 0.005\ \mu\text{F} \end{aligned}$$