9.4 Effective or rms Values

In the preceding sections of this chapter, we have shown that the average power absorbed by a resistive load is directly dependent on the type, or types, of sources that are delivering power to the load. For example, if the source was dc, the average power absorbed was I^2R, and if the source was sinusoidal, the average power was $1/2I_M^2R$. Although these two types of waveforms are extremely important, they are by no means the only waveforms we will encounter in circuit analysis. Therefore, a technique by which we can compare the *effectiveness* of different sources in delivering power to a resistive load would be quite useful.

To accomplish this comparison, we define what is called the *effective value of a periodic waveform*, representing either voltage or current. Although either quantity could be used, we will employ current in the definition. Hence, we define the effective value of a periodic current as a constant or dc value, which as current would deliver the same average power to a resistor R. Let us call the constant current I_{eff}. Then the average power delivered to a resistor as a result of this current is

$$P = I_{\text{eff}}^2 R$$

Similarly, the average power delivered to a resistor by a periodic current $i(t)$ is

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)R\,dt$$

Equating these two expressions, we find that

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\,dt} \qquad \textbf{9.23}$$

Note that this effective value is found by first determining the *square* of the current, then computing the average or *mean* value, and finally, taking the square *root*. Thus, in "reading" the mathematical Eq. (9.23), we are determining the root mean square, which we abbreviate as rms, and therefore I_{eff} is called I_{rms}.

Since dc is a constant, the rms value of dc is simply the constant value. Let us now determine the rms value of other waveforms. The most important waveform is the sinusoid, and therefore, we address this particular one in the following example.

EXAMPLE 9.7

We wish to compute the rms value of the waveform $i(t) = I_M \cos(\omega t - \theta)$, which has a period of $T = 2\pi/\omega$.

SOLUTION

Substituting these expressions into Eq. (9.23) yields

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T I_M^2 \cos^2(\omega t - \theta)\,dt\right]^{1/2}$$

Using the trigonometric identity

$$\cos^2\phi = \frac{1}{2} + \frac{1}{2}\cos 2\phi$$

we find that the preceding equation can be expressed as

$$I_{\text{rms}} = I_M\left\{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t - 2\theta)\right]dt\right\}^{1/2}$$

Since we know that the average or mean value of a cosine wave is zero,

$$\begin{aligned} I_{\text{rms}} &= I_M\left(\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{1}{2}\,dt\right)^{1/2} \\ &= I_M\left[\frac{\omega}{2\pi}\left(\frac{t}{2}\right)\Bigg|_0^{2\pi/\omega}\right]^{1/2} = \frac{I_M}{\sqrt{2}} \end{aligned} \qquad \textbf{9.24}$$

Therefore, the rms value of a sinusoid is equal to the maximum value divided by $\sqrt{2}$. Hence, a sinusoidal current with a maximum value of I_M delivers the same average power to a resistor R as a dc current with a value of $I_M/\sqrt{2}$. Recall that earlier a phasor **X** was defined as $X_M\angle\theta$ for a sinusoidal wave of the form $X_M \cos(\omega t + \theta)$. This phasor can also be represented as $X_M/\sqrt{2}\angle\theta$ if the units are given in rms. For example, $120\angle 30°$ V rms is equivalent to $170\angle 30°$ V.

On using the rms values for voltage and current, the average power can be written, in general, as

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \tag{9.25}$$

The power absorbed by a resistor R is

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R} \tag{9.26}$$

In dealing with voltages and currents in numerous electrical applications, it is important to know whether the values quoted are maximum, average, rms, or what. We are familiar with the 120-V ac electrical outlets in our home. In this case, the 120 V is the rms value of the voltage in our home. The maximum or peak value of this voltage is $120\sqrt{2} = 170$ V. The voltage at our electrical outlets could be written as $170 \cos 377t$ V. The maximum or peak value must be given if we write the voltage in this form. There should be no question in our minds that this is the peak value. It is common practice to specify the voltage rating of ac electrical devices in terms of the rms voltage. For example, if you examine an incandescent light bulb, you will see a voltage rating of 120 V, which is the rms value. For now we will add an rms to our voltages and currents to indicate that we are using rms values in our calculations.

EXAMPLE 9.8

Determine the rms value of the current waveform in Fig. 9.9 and use this value to compute the average power delivered to a 2-Ω resistor through which this current is flowing.

SOLUTION

The current waveform is periodic with a period of $T = 4$ s. The rms value is

$$\begin{aligned} I_{\text{rms}} &= \left\{ \frac{1}{4} \left[\int_0^2 (4)^2\, dt + \int_2^4 (-4)^2\, dt \right] \right\}^{1/2} \\ &= \left[\frac{1}{4} \left(16t \Big|_0^2 + 16t \Big|_2^4 \right) \right]^{1/2} \\ &= 4 \text{ A} \end{aligned}$$

The average power delivered to a 2-Ω resistor with this current is

$$P = I_{\text{rms}}^2 R = (4)^2(2) = 32 \text{ W}$$

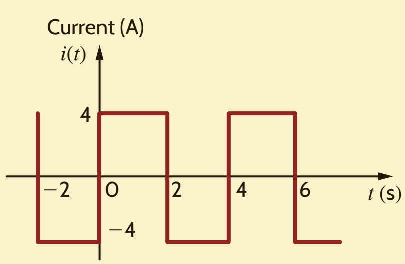


Figure 9.9
Waveform used to illustrate rms values.

EXAMPLE 9.9

We wish to compute the rms value of the voltage waveform shown in Fig. 9.10.

SOLUTION

The waveform is periodic with period $T = 3$ s. The equation for the voltage in the time frame $0 \leq t \leq 3$ s is

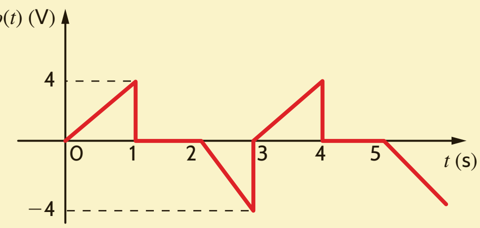
$$v(t) = \begin{cases} 4t \text{ V} & 0 < t \leq 1 \text{ s} \\ 0 \text{ V} & 1 < t \leq 2 \text{ s} \\ -4t + 8 \text{ V} & 2 < t \leq 3 \text{ s} \end{cases}$$

The rms value is

$$\begin{aligned} V_{\text{rms}} &= \left\{ \frac{1}{3} \left[\int_0^1 (4t)^2\, dt + \int_1^2 (0)^2\, dt + \int_2^3 (8 - 4t)^2\, dt \right] \right\}^{1/2} \\ &= \left[\frac{1}{3} \left(\frac{16t^3}{3} \Big|_0^1 + \left(64t - \frac{64t^2}{2} + \frac{16t^3}{3} \right) \Big|_2^3 \right) \right]^{1/2} \\ &= 1.89 \text{ V} \end{aligned}$$

Figure 9.10

Waveform used to illustrate rms values.



LEARNING ASSESSMENTS

E9.11 The current waveform in Fig. E9.11 is flowing through a 4-Ω resistor. Compute the average power delivered to the resistor.

ANSWER:
$P = 32$ W.

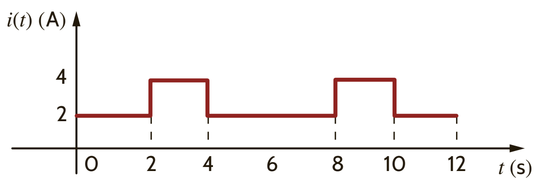


Figure E9.11

E9.12 The current waveform in Fig. E9.12 is flowing through a 10-Ω resistor. Determine the average power delivered to the resistor..

ANSWER:
$P = 80$ W.

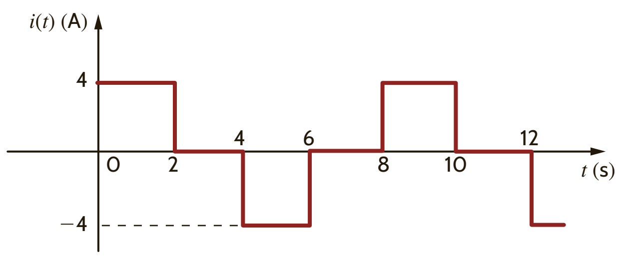


Figure E9.12