Since the initial value for the inductor $i(0) = 0$, this equation becomes

$$\mathscr{L}[v_S(t)] = \mathbf{V}_S(s) = L[s\mathbf{I}(s)] + R\mathbf{I}(s)$$

Now the circuit is represented not by a time-domain differential equation, but rather by an algebraic expression in the s-domain. Solving for $\mathbf{I}(s)$, we can write

$$\mathbf{I}(s) = \frac{\mathbf{V}_S(s)}{sL + R} = \frac{1}{s[sL + R]}$$

We find $i(t)$ using the inverse Laplace transform. First, let us express $\mathbf{I}(s)$ as a sum of partial products:

$$\mathbf{I}(s) = \frac{1/L}{s\left[s + \frac{R}{L}\right]} = \frac{1}{sR} - \frac{1}{R\left[s + \frac{R}{L}\right]}$$

The inverse transform is simply

$$i(t) = \frac{1}{R}(1 - e^{-Rt/L})$$

Given the circuit element values in Fig. 14.1, the current is

$$i(t) = 10(1 - e^{-1000t})u(t) \text{ mA}$$

which is exactly the same as that obtained using the differential equation approach. Note carefully that the solution using the Laplace transform approach yields the entire solution in one step.

We have shown that the Laplace transform can be used to transform a differential equation into an algebraic equation. Since the voltage–current relationships for resistors, capacitors, and inductors involve only constants, derivatives, and integrals, we can represent and solve any circuit in the s-domain.

14.2 Circuit Element Models

The Laplace transform technique employed earlier implies that the terminal characteristics of circuit elements can be expressed as algebraic expressions in the s-domain. Let us examine these characteristics for the resistor, capacitor, and inductor.

The voltage–current relationship for a resistor in the time domain using the passive sign convention is

$$v(t) = Ri(t) \tag{14.2}$$

Using the Laplace transform, we find that this relationship in the s-domain is

$$\mathbf{V}(s) = R\mathbf{I}(s) \tag{14.3}$$

Therefore, the time-domain and complex frequency-domain representations of this element are as shown in **Fig. 14.2a**.

The time-domain relationships for a capacitor using the passive sign convention are

$$v(t) = \frac{1}{C}\int_0^t i(x)\,dx + v(0) \tag{14.4}$$

$$i(t) = C\frac{dv(t)}{dt} \tag{14.5}$$

The s-domain equations for the capacitor are then

$$\mathbf{V}(s) = \frac{\mathbf{I}(s)}{sC} + \frac{v(0)}{s} \tag{14.6}$$

$$\mathbf{I}(s) = sC\mathbf{V}(s) - Cv(0) \tag{14.7}$$

and hence the s-domain representation of this element is as shown in **Fig. 14.2b**.

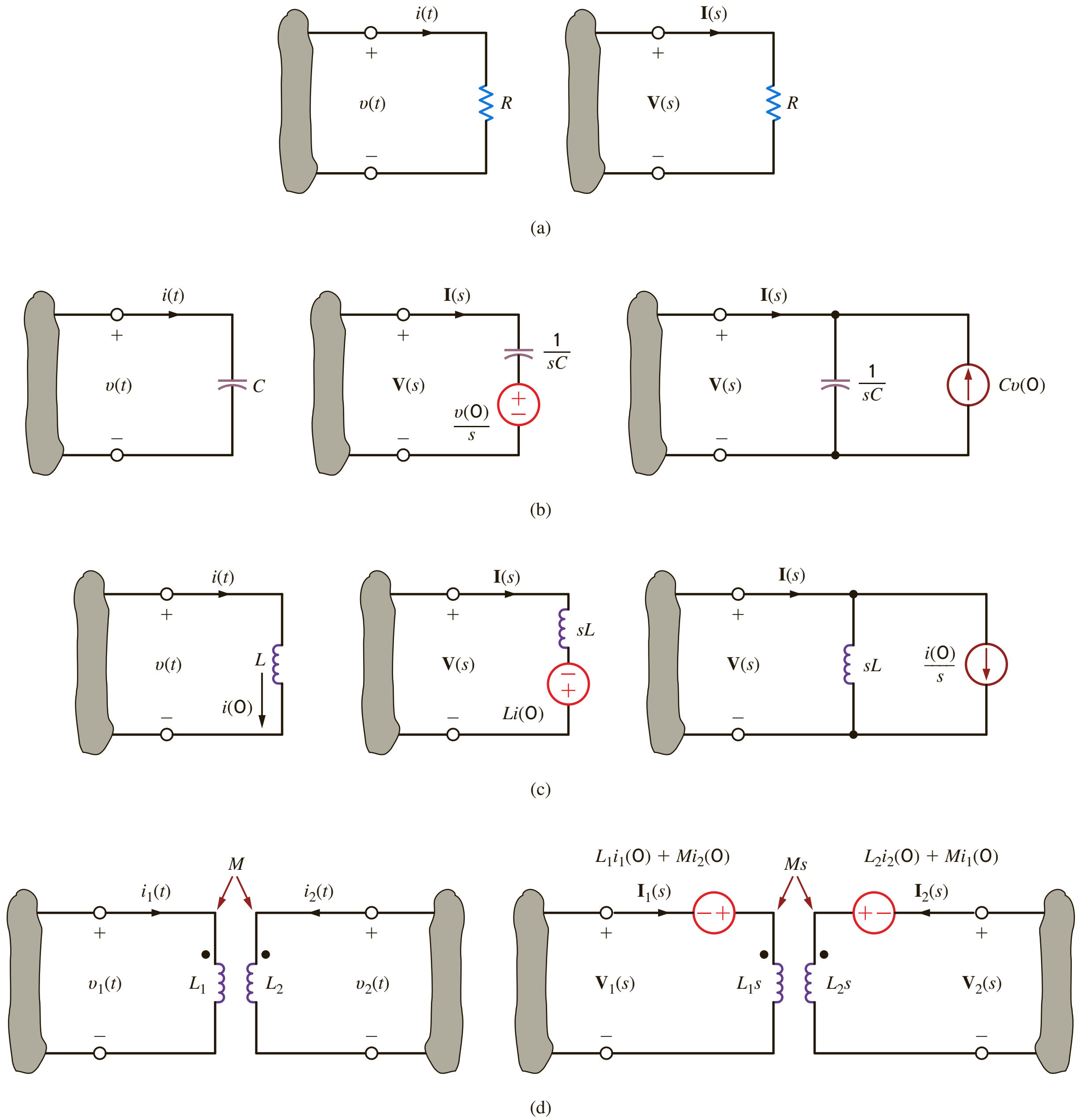

Figure 14.2

Time-domain and *s*-domain representations of circuit elements.

For the inductor, the voltage–current relationships using the passive sign convention are

$$v(t) = L\frac{di(t)}{dt} \tag{14.8}$$

$$i(t) = \frac{1}{L}\int_0^t v(x)\,dx + i(0) \tag{14.9}$$

The relationships in the *s*-domain are then

$$\mathbf{V}(s) = sL\mathbf{I}(s) - Li(0) \tag{14.10}$$

$$\mathbf{I}(s) = \frac{\mathbf{V}(s)}{sL} + \frac{i(0)}{s} \tag{14.11}$$