Example 9.10 clearly indicates the economic impact of the load's power factor. The cost of producing electricity for a large electric utility can easily be in the billions of dollars. A low power factor at the load means that the utility generators must be capable of carrying more current at constant voltage, and they must also supply power for higher $I^2_{\text{rms}}R_{\text{line}}$ losses than would be required if the load's power factor were high. Since line losses represent energy expended in heat and benefit no one, the utility will insist that a plant maintain a high pf, typically 0.9 lagging, and adjust the rate it charges a customer that does not conform to this requirement. We will demonstrate a simple and economical technique for achieving this power factor correction in a future section.

LEARNING ASSESSMENT

E9.15 An industrial load consumes 100 kW at 0.707 pf lagging. The 60-Hz line voltage at the load is 480 $\angle 0°$ V rms. The transmission-line resistance between the power company transformer and the load is 0.1 Ω. Determine the power savings that could be obtained if the pf is changed to 0.94 lagging.

ANSWER: Power saved is 3.771 kW.

9.6 Complex Power

In our study of ac steady-state power, it is convenient to introduce another quantity, which is commonly called *complex power*. To develop the relationship between this quantity and others we have presented in the preceding sections, consider the circuit shown in Fig. 9.12.

The complex power is defined to be

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} \tag{9.29}$$

where $\mathbf{I}^*_{\text{rms}}$ refers to the complex conjugate of $\mathbf{I}_{\text{rms}}$; that is, if $\mathbf{I}_{\text{rms}} = I_{\text{rms}}\angle\theta_i = I_R + jI_I$, then $\mathbf{I}^*_{\text{rms}} = I_{\text{rms}}\angle{-\theta_i} = I_R - jI_I$. Complex power is then

$$\mathbf{S} = V_{\text{rms}}\angle\theta_v I_{\text{rms}}\angle{-\theta_i} = V_{\text{rms}}I_{\text{rms}}\angle\theta_v - \theta_i \tag{9.30}$$

or

$$\mathbf{S} = V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) + jV_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i) \tag{9.31}$$

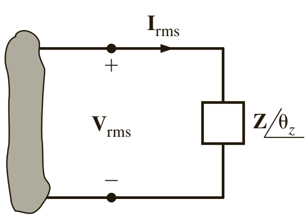

Figure 9.12

Circuit used to explain power relationships.

where, of course, $\theta_v - \theta_i = \theta_{\mathbf{Z}}$. We note from Eq. (9.31) that the real part of the complex power is simply the *real* or *average power*. The imaginary part of **S** we call the *reactive* or *quadrature power*. Therefore, complex power can be expressed in the form

$$\mathbf{S} = P + jQ \tag{9.32}$$

where

$$P = \text{Re}(\mathbf{S}) = V_{\text{rms}}I_{\text{rms}}\cos(\theta_v - \theta_i) \tag{9.33}$$

$$Q = \text{Im}(\mathbf{S}) = V_{\text{rms}}I_{\text{rms}}\sin(\theta_v - \theta_i) \tag{9.34}$$

As shown in Eq. (9.31), the magnitude of the complex power is what we have called the *apparent power*, and the phase angle for complex power is simply the power factor angle. Complex power, like apparent power, is measured in volt-amperes, real power is measured in watts, and to distinguish Q from the other quantities, which in fact have the same dimensions, it is measured in volt-amperes reactive, or var.

Now let's examine the expressions in Eqs. (9.33) and (9.34) in more detail for our three basic circuit elements: R, L, and C. For a resistor, $\theta_v - \theta_i = 0°$, $\cos(\theta_v - \theta_i) = 1$, and $\sin(\theta_v - \theta_i) = 0$. As a result, a resistor absorbs real power ($P > 0$) but does not absorb any reactive power ($Q = 0$). For an inductor, $\theta_v - \theta_i = 90°$ and

$$P = V_{\text{rms}}I_{\text{rms}}\cos(90°) = 0$$

$$Q = V_{\text{rms}}I_{\text{rms}}\sin(90°) > 0$$

An inductor absorbs reactive power but does not absorb real power. Repeating for a capacitor, we get $\theta_v - \theta_i = -90°$ and

$$P = V_{\text{rms}} I_{\text{rms}} \cos(-90°) = 0$$
$$Q = V_{\text{rms}} I_{\text{rms}} \sin(-90°) < 0$$

A capacitor does not absorb any real power; however, the reactive power is now negative. How do we interpret the negative reactive power? Refer to Fig. 9.12 and note that the voltage and current are specified such that they satisfy the passive sign convention. In this case, the product of the voltage and current gives us the power absorbed by the impedance in that figure. If the reactive power absorbed by the capacitor is negative, then the capacitor must be supplying reactive power. The fact that capacitors are a source of reactive power will be utilized in the next section on power factor correction.

We see that resistors absorb only real power, while inductors and capacitors absorb only reactive power. What is a fundamental difference between these elements? Resistors only absorb energy. On the other hand, capacitors and inductors store energy and then release it back to the circuit. Since inductors and capacitors absorb only reactive power and not real power, we can conclude that reactive power is related to energy storage in these elements.

Now let's substitute $\mathbf{V}_{\text{rms}} = \mathbf{I}_{\text{rms}} * \mathbf{Z}$ into Eq. (9.29). Multiplying $\mathbf{I}_{\text{rms}} * \mathbf{I}^*_{\text{rms}} = I_{\text{rms}}\underline{/\theta_i} * I_{\text{rms}}\underline{/-\theta_i}$ yields I^2_{rms}. The complex power absorbed by an impedance can be obtained by multiplying the square of the rms magnitude of the current flowing through that impedance by the impedance.

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = (\mathbf{I}_{\text{rms}}\mathbf{Z})\mathbf{I}^*_{\text{rms}} = \mathbf{I}_{\text{rms}}\mathbf{I}^*_{\text{rms}}\mathbf{Z} = I^2_{\text{rms}}\mathbf{Z} = I^2_{\text{rms}}(R + jX) = P + jQ \qquad \textbf{9.35}$$

Instead of substituting for $\mathbf{V}_{\text{rms}}$ in Eq. (9.29), let's substitute for $\mathbf{I}_{\text{rms}}$:

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \mathbf{V}_{\text{rms}}\left(\frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}}\right)^* = \frac{V^2_{\text{rms}}}{\mathbf{Z}^*} = V^2_{\text{rms}}\mathbf{Y}^* = V^2_{\text{rms}}(G + jB)^* = P + jQ \qquad \textbf{9.36}$$

This expression tells us that we can calculate the complex power absorbed by an admittance by multiplying the square of the rms magnitude of the voltage across the admittance by the conjugate of the admittance. Suppose the box in Fig. 9.12 contains a capacitor. The admittance for a capacitor is $j\omega C$. Plugging into the equation above yields

$$\mathbf{S} = V^2_{\text{rms}}(j\omega C)^* = -j\omega C V^2_{\text{rms}} \qquad \textbf{9.37}$$

Note the negative sign on the complex power. This agrees with our previous statement that a capacitor does not absorb real power but is a source of reactive power.

The diagrams in **Fig. 9.13** further explain the relationships among the various quantities of power. As shown in **Fig. 9.13a**, the phasor current can be split into two components: one that is in phase with $\mathbf{V}_{\text{rms}}$ and one that is 90° out of phase with $\mathbf{V}_{\text{rms}}$. Eqs. (9.33) and (9.34) illustrate that the in-phase component produces the real power, and the 90° component, called the

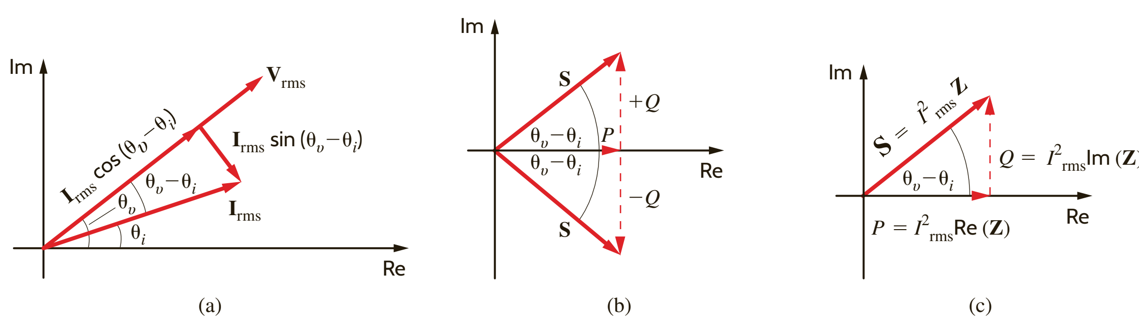

Figure 9.13

Diagram for illustrating power relationships.

quadrature component, produces the reactive or quadrature power. In addition, Eqs. (9.33) and (9.34) indicate that

$$\tan(\theta_v - \theta_i) = \frac{Q}{P} \qquad \textbf{9.38}$$

which relates the pf angle to P and Q in what is called the *power triangle*.

The relationships among **S**, P, and Q can be expressed via the diagrams shown in Figs. 9.13b and c. In Fig. 9.13b we note the following conditions. If Q is positive, the load is inductive, the power factor is lagging, and the complex number **S** lies in the first quadrant. If Q is negative, the load is capacitive, the power factor is leading, and the complex number **S** lies in the fourth quadrant. If Q is zero, the load is resistive, the power factor is unity, and the complex number **S** lies along the positive real axis. Fig. 9.13c illustrates the relationships expressed by Eqs. (9.35) to (9.37) for an inductive load.

In Chapter 1, we introduced Tellegen's theorem, which states that the sum of the powers absorbed by all elements in an electrical network is zero. Based on this theorem, we can also state that complex power is conserved in an ac network—the total complex power delivered to any number of individual loads is equal to the sum of the complex powers delivered to the loads, regardless of how loads are interconnected.

PROBLEM-SOLVING STRATEGY

DETERMINING P OR S

If $v(t)$ and $i(t)$ are known and we wish to find P given an impedance $\mathbf{Z}\underline{/\theta} = R + jX$, two viable approaches are as follows:

STEP 1. Determine **V** and **I** and then calculate

$$P = V_{\text{rms}} I_{\text{rms}} \cos\theta \qquad \text{or} \qquad P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

STEP 2. Use **I** to calculate the real part of **S**—that is,

$$P = R_e(\mathbf{S}) = I^2 R$$

The latter method may be easier to calculate than the former. However, if the imaginary part of the impedance, X, is not zero, then

$$P \neq \frac{V^2}{R}$$

which is a common mistake. Furthermore, the P and Q portions of **S** are directly related to $Z\underline{/\theta}$ and provide a convenient way in which to relate power, current, and impedance. That is,

$$\tan\theta = \frac{Q}{P}$$

$$\mathbf{S} = I^2\mathbf{Z}$$

The following example illustrates the usefulness of **S**.

EXAMPLE 9.11

A load operates at 20 kW, 0.8 pf lagging. The load voltage is $220\underline{/0°}$ V rms at 60 Hz. The impedance of the line is $0.09 + j0.3\ \Omega$. We wish to determine the voltage and power factor at the input to the line.

SOLUTION

The circuit diagram for this problem is shown in Fig. 9.14. As illustrated in Fig. 9.13,

$$S = \frac{P}{\cos\theta} = \frac{P}{\text{pf}} = \frac{20{,}000}{0.8} = 25{,}000\text{ VA}$$

Therefore, at the load

$$\mathbf{S}_L = 25{,}000\angle\theta = 25{,}000\angle 36.87^\circ = 20{,}000 + j15{,}000 \text{ VA}$$

Since $\mathbf{S}_L = \mathbf{V}_L\mathbf{I}_L^*$

$$\begin{aligned}\mathbf{I}_L &= \left[\frac{25{,}000\angle 36.87^\circ}{220\angle 0^\circ}\right]^* \\ &= 113.64\angle -36.87^\circ \text{ A rms}\end{aligned}$$

The complex power losses in the line are

$$\begin{aligned}\mathbf{S}_{\text{line}} &= I_L^2\,\mathbf{Z}_{\text{line}} \\ &= (113.64)^2(0.09 + j0.3) \\ &= 1162.26 + j3874.21 \text{ VA}\end{aligned}$$

As stated earlier, complex power is conserved, and therefore, the complex power at the generator is

$$\begin{aligned}\mathbf{S}_S &= \mathbf{S}_L + \mathbf{S}_{\text{line}} \\ &= 21{,}162.26 + j18{,}874.21 \\ &= 28{,}356.25\angle 41.73^\circ \text{ VA}\end{aligned}$$

Hence, the generator voltage is

$$\begin{aligned}V_S &= \frac{|S_S|}{I_L} = \frac{28{,}356.25}{113.64} \\ &= 249.53 \text{ V rms}\end{aligned}$$

and the generator power factor is

$$\cos(41.73^\circ) = 0.75 \text{ lagging}$$

We could have solved this problem using KVL. For example, we calculated the load current as

$$\mathbf{I}_L = 113.64\angle -36.87^\circ \text{ A rms}$$

Hence, the voltage drop in the transmission line is

$$\begin{aligned}\mathbf{V}_{\text{line}} &= (113.64\angle -36.87^\circ)(0.09 + j0.3) \\ &= 35.59\angle 36.43^\circ \text{ V rms}\end{aligned}$$

Therefore, the generator voltage is

$$\begin{aligned}\mathbf{V}_S &= 220\angle 0^\circ + 35.59\angle 36.43^\circ \\ &= 249.53\angle 4.86^\circ \text{ V rms}\end{aligned}$$

Hence, the generator voltage is 249.53 V rms. In addition,

$$\theta_v - \theta_i = 4.86^\circ - (-36.87^\circ) = 41.73^\circ$$

and therefore,

$$\text{pf} = \cos(41.73^\circ) = 0.75 \text{ lagging}$$

HINT

1. Use the given P_L, $\cos\theta$, and V_L rms to obtain $\mathbf{S}_L$ and $\mathbf{I}_L$ based on Eqs. (9.33) and (9.29), respectively.
2. Use $\mathbf{I}_L$ and $\mathbf{Z}_{\text{line}}$ to obtain $\mathbf{S}_{\text{line}}$ using Eq. (9.35).
3. Use $\mathbf{S}_S = \mathbf{S}_{\text{line}} + \mathbf{S}_L$.
4. $\mathbf{V}_S = \mathbf{S}_S/\mathbf{I}_L^*$ yields V_S and θ_v. Since $\mathbf{V}_S = V_S\angle\theta_v$ and θ_i is the phase of $\mathbf{I}_L$, pf $= \cos(\theta_v - \theta_i)$.

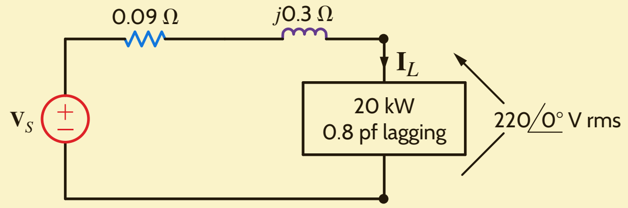

Figure 9.14
Example circuit for power analysis.

EXAMPLE 9.12

Two networks A and B are connected by two conductors having a net impedance of $\mathbf{Z} = 0 + j1\ \Omega$, as shown in Fig. 9.15. The voltages at the terminals of the networks are $\mathbf{V}_A = 120\underline{/30^\circ}$ V rms and $\mathbf{V}_B = 120\underline{/0^\circ}$ V rms. We wish to determine the average power flow between the networks and identify which is the source and which is the load.

SOLUTION

As shown in Fig. 9.15,

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}_A - \mathbf{V}_B}{\mathbf{Z}} \\ &= \frac{120\underline{/30^\circ} - 120\underline{/0^\circ}}{j1} \\ &= 62.12\underline{/15^\circ}\text{ A rms}\end{aligned}$$

The power delivered by network A is

$$\begin{aligned}P_A &= |\mathbf{V}_A||\mathbf{I}|\cos(\theta_{\mathbf{V}_A} - \theta_{\mathbf{I}}) \\ &= (120)(62.12)\cos(30^\circ - 15^\circ) \\ &= 7200.4\text{ W}\end{aligned}$$

The power absorbed by network B is

$$\begin{aligned}P_B &= |\mathbf{V}_B||\mathbf{I}|\cos(\theta_{\mathbf{V}_B} - \theta_{\mathbf{I}}) \\ &= (120)(62.12)\cos(0^\circ - 15^\circ) \\ &= 7200.4\text{ W}\end{aligned}$$

If the power flow had actually been from network B to network A, the resultant signs on P_A and P_B would have been negative.

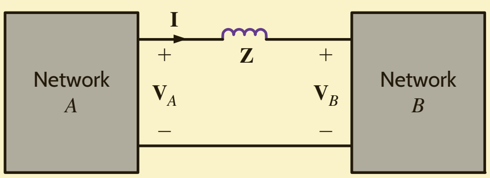

Figure 9.15 Network used in Example 9.12.

LEARNING ASSESSMENTS

E9.16 An industrial load requires 40 kW at 0.84 pf lagging. The load voltage is $220\underline{/0^\circ}$ V rms at 60 Hz. The transmission-line impedance is $0.1 + j0.25\ \Omega$. Determine the real and reactive power losses in the line and the real and reactive power required at the input to the transmission line.

ANSWER: $P_{\text{line}} = 4.685$ kW; $Q_{\text{line}} = 11.713$ kvar; $P_S = 44.685$ kW; $Q_S = 37.55$ kvar.

E9.17 A load requires 60 kW at 0.85 pf lagging. The 60-Hz line voltage at the load is $220\underline{/0^\circ}$ V rms. If the transmission-line impedance is $0.12 + j0.18\ \Omega$, determine the line voltage and power factor at the input.

ANSWER: $\mathbf{V}_{\text{in}} = 284.6\underline{/5.8^\circ}$ V rms; $\text{pf}_{\text{in}} = 0.792$ lagging.