Dictionary Design for Compressed Sensing MRI

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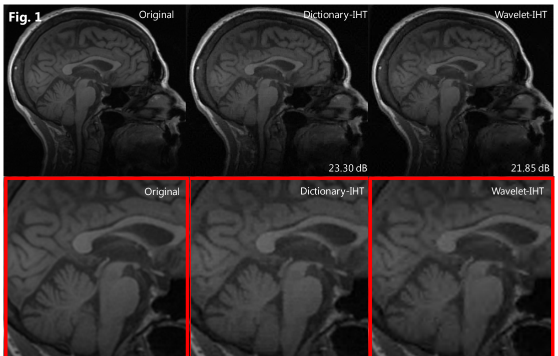
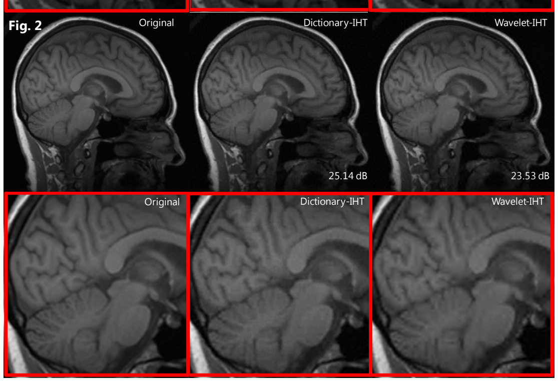
Introduction: The recently introduced Compressed Sensing (CS) theory promises to accelerate data acquisition in magnetic resonance imaging (MRI) [1-3]. One of the important requirements in CS MRI is that the image has a sparse representation. This sparse representation is crucial for successful recovery in CS. Generally speaking, sparser representations yield improved performance in terms of either further reduction in the number of measurements necessary for successful recovery or improved reconstruction quality for a given number of measurements. In this work, we propose a framework for designing and utilizing sparse dictionaries in CS MRI applications. Reconstruction results demonstrate that the proposed technique can yield significantly improved image quality compared to commonly used sparsity transforms in CS MRI.

Theory: Let $\mathbf{D}$ denote a sparsity dictionary, $\mathbf{F}_u$ the undersampled Fourier measurement matrix, and $\mathbf{b}$ the acquired *k*-space data. Under the assumption that the image has a sparse representation in dictionary $\mathbf{D}$, the CS MRI can be represented by the following minimization problem: $\min_{\gamma} \|\mathbf{b} - \mathbf{A}\boldsymbol{\gamma}\|_2^2$ s.t. $\|\boldsymbol{\gamma}\|_0 \le M$, where γ is the sparse representation of the image in dictionary $\mathbf{D}$, and $\mathbf{A} = \mathbf{F}_u\mathbf{D}^H$. Numerous algorithms have been proposed to solve this CS MRI problem. A simple and effective CS technique is Iterative Hard Thresholding (IHT) [4]. In IHT, the solution at iteration $n+1$ (denoted $\boldsymbol{\gamma}^{n+1}$) is obtained using: $\boldsymbol{\gamma}^{n+1} = H_M(\boldsymbol{\gamma}^n + \mu\mathbf{A}^H(\mathbf{b} - \mathbf{A}\boldsymbol{\gamma}^n))$, where H_M is a nonlinear operator that retains the M largest coefficients and μ is the step size. In practice, the sparsity dictionary in CS MRI is often selected from well-studied classes of transforms where natural images are known to be sparse. Examples of such transforms used in previous CS MRI studies include wavelet transforms, discrete cosine transforms, and finite differences (i.e. total variation). While these transforms have been proven to work well for the general class of natural images, they are not optimized for each MRI application. In MRI, significant prior information exists about the object being imaged and this prior information can be used to design more efficient sparsity dictionaries for CS MRI. For example, prior to image reconstruction, it is known that the data was acquired on a particular anatomy using a particular pulse sequence with known parameters. Thus, a sparsity dictionary that was designed using this prior information can yield significantly enhanced sparsity. Recently, an algorithm (referred to as K-SVD) for training a dictionary for sparse signal representation was proposed [5]. Given a set of signal samples $\mathbf{X} = [\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}]$, the K-SVD algorithm aims to find a dictionary $\mathbf{D}$ and a sparse matrix $\boldsymbol{\Gamma}$ which minimizes the representation error by solving $\arg\min_{\boldsymbol{\Psi},\mathbf{D}} \|\mathbf{X} - \mathbf{D}\boldsymbol{\Gamma}\|_F^2$ subject to $\|\gamma_i\|_0 \le T, \forall i$ where $\boldsymbol{\gamma}_i$ are columns of $\boldsymbol{\Gamma}$, and T is the desired sparsity. The goal in this work is to make use of the K-SVD algorithm for designing sparsity dictionaries for a restricted class of MR images and utilizing these dictionaries for CS recovery.





Methods: Experiments were carried out using 69 *in vivo* fully-sampled 3D (248x256x256 pixels) brain datasets. One of the datasets was selected at random and used for training and the remaining 68 datasets were used during testing. Slice 124 of the training data was used to train a dictionary using K-SVD. The dictionary was trained on 4x4 image patches and contained 324 atoms. Both the designed dictionary and an orthonormal wavelet transform (Daubechies wavelet with 3 vanishing moments) were then used in IHT to recover slice 124 of the test images. The *k*-space data for each test slice were obtained by sampling *k*-space along 130 radial views with 256 points along each view. The stopping criterion for IHT was set to the *l*-2 norm of residual being less than 10^{-7}.

Results: Fig. 1 shows the reconstruction results obtained for one of the test images. While the image reconstructed using the wavelet transform contains residual wavelet artifacts, the image obtained using the trained dictionary is much closer to the original image. The Signal-to-Noise Ratios (SNRs) calculated between the original image and the reconstructed images also favor the trained dictionary method. A second sample slice is illustrated in Fig. 2. Similar observations can be made about the remaining test slices as well.

Conclusions: A novel sparse reconstruction technique that incorporates prior information through dictionary training is introduced for CS MRI. Reconstruction results demonstrate that the proposed technique can yield significantly improved image quality compared to commonly used sparsity transforms in CS MRI.

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