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Accelerating Sensitivity Encoding Using Compressed Sensing

Dong Liang, Bo Liu and Leslie Ying

Abstract—**The combination of Compressed Sensing (CS) and SENSitivity Encoding (SENSE) for improving MRI acquisition speed and robustness has recently drawn great attentions. However, in the direct combination, the encoding matrix which represents the Fourier transform of channel-specific sensitivity modulation is not guaranteed to be a good CS matrix. In this paper, we propose a different approach that applies CS and SENSE sequentially. The method first uses CS to reconstruct a set of aliased images in each coil, and then applies the basic SENSE on these images to reconstruct the final image. The total reduction factor can achieve the product of the factors of each individual method. The experimental results show that overall performance of our proposed method is superior to the direct combination method with the same reduction factor.**

Index Terms-Compressed Sensing, SENSE, MRI

I. INTRODUCTION

Parallel MRI improves imaging speed by reducing the number of samples simultaneous acquired from multiple channels. SENSitivity Encoding (SENSE) [1, 2] is one of the standard parallel MRI reconstruction methods. The basic SENSE generates a set of aliased images with reduced field-of-view (FOV) from the uniformly downsampled *k*-space data, and then the final image is reconstructed from the aliased images using the coil sensitivity information. Based on the generalized sampling theory [3], its maximum reduction factor should equal to the number of channels under ideal conditions. However, the conditions are rarely met in practice, so the reduction factor is usually far less than the number of channels. The large number of required samples beyond the theory limits the parallel MR imaging speed.

Compressed Sensing (CS) [4, 5] is a new framework for data acquisition and signal recovery. It allows measuring sparse and compressible signals at a rate close to their intrinsic information rate rather than their Nyquist rate. These signals can be reconstructed exactly from very few incoherent measurements by a non-linear procedure [5, 6]. This feature is very desirable in MRI for significant reduction in the number of sampled data, because MRI obeys two key conditions of successful application of CS. The first is that most MR images have a sparse representation in a known transform domain (e.g., image domain or wavelet domain). The second is that MR acquisition is Fourier encoding, which can be design to be incoherent with the sparsifying transform domain by random sampling.

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SparseMRI [7, 8] has successfully applied CS to MRI for reduced Cartesian sampling. Direct extension of the method to parallel MRI (SparseSENSE et al. [9-12]) has shown some success. These methods formulate image reconstruction from randomly sampled multi-channel data as the same constrained nonlinear convex program, except that the Fourier encoding is replaced by the sensitivity encoding. However, the incoherence between the sensitivity encoding and the sparse transformation cannot be guaranteed, which limits the maximum reduction factors.

In this paper, we propose a different approach to apply CS to parallel imaging, named CS-SENSE. The method first uses SparseMRI to reconstruct a set of aliased images from randomly undersampled data in each coil, and then applies the basic SENSE on these images to reconstruct the final image. In this sequential reconstruction, the CS matrix in SparseMRI is Fourier encoding whose incoherence with the sparse transformations has been investigated [7] and successful reconstruction is guaranteed. Our experiment results show that CS-SENSE can achieve a reduction factor being the product of the factors achieved by SparseMRI and SENSE individually, and the image quality is superior to the direct methods.

II. COMPRESSED SENSING AND SENSITIVITY ENCODING

A. Parallel MRI and SENSitivity Encoding

Parallel MRI [1, 2, 13] is a new technique to improve on conventional Fourier encoding for fast imaging. In parallel imaging, *k*-space data are acquired from multiple channels simultaneously such that they can be sampled with a rate lower than the Nyquist sampling rate. The imaging equation can be written in matrix form as

$$\mathbf{Ef} = \mathbf{d} \tag{1}$$

where $\mathbf{d}$ is a concatenation of data from all channels, and $\mathbf{f}$ the desired image to be reconstructed. The entries of the encoding matrix $\mathbf{E}$ are

$$\mathbf{E}_{\{l,m\},n} = e^{-i2\pi k_m \cdot r_n} s_l(r_n) \tag{2}$$

where k_m denotes the *m*-th sampling position in *k*-space, r_n the position of the *n*-th pixel, and s_l the sensitivity profile of the *l*-th receiver channel. The image can be reconstructed by the least-squares solution to Eq. (1).

B. Compressed Sensing in MRI

The central problem in compressed sensing (CS) is the recovery of a signal $\mathbf{x} \in \mathbb{R}^n$ from its linear measurements $\mathbf{y} \in \mathbb{R}^m$:

$$\mathbf{y} = \mathbf{\Phi x} \tag{3}$$

where m is assumed to be much smaller than n. This underdetermined system of equations makes the recovery of the original vector x impossible without further assumption. However, the vectors x to be recovered usually have sparse representations under certain bases in MRI. Candès et al. [14] showed that under the condition that the encoding matrix $\mathbf{\Phi}$ is incoherent with the sparse transformation $\mathbf{\Psi}$, one can actually recover x from a sample y which is much smaller in size than x by solving a convex program

$$\text{minimize } \|\mathbf{\Psi x}\|_1 \quad s.t. \quad \mathbf{\Phi x} = \mathbf{y} \tag{4}$$

In conventional MRI, the acquired data is the Fourier encoding of the desired image. For most MR images, there exists a sparse representation in a known transform domain (i.e. image domain or wavelet domain). It has been shown that incoherence is satisfied when $\mathbf{\Phi}$ is a random subset of the Fourier basis and $\mathbf{\Psi}$ is the canonical basis or the finer scales of a wavelet transform [15]. Thereby the application of CS has great potential for significant reduction of data samples in MRI.

SparseMRI [7] has been proposed to apply CS to Cartesian MRI with the least modifications on the existing systems. Considering the random sampling of k-space in all dimensions is generally impractical, a practical sampling scheme was designed for conventional Fourier imaging. Specifically, the k-space is randomly undersampled on a Cartesian grid along the phase-encoding direction and fully sampled along the readout direction. In addition, the level of incoherence was measured by the point spread function (PSF). In [7], both wavelet transform and total variation were used as the sparse representations. Taking noise into account, the image is reconstructed by

$$\arg\min_{\mathbf{x}} \{\|\mathbf{b} - \mathbf{F}_u \mathbf{f}\|_2^2 + \lambda_1 \|\mathbf{Wf}\|_1 + \lambda_2 \,\mathrm{TV}(\mathbf{f})\}, \tag{5}$$

where $\mathbf{b}$ is the measured k-space data from scanner, $\mathbf{F}_u$ is the random subset of the rows of the Fourier encoding matrix, $\mathbf{W}$ is the wavelet transform matrix, and $\mathrm{TV}(\cdot)$ is total variation.

Directly extending SpareseMRI to parallel imaging, some work [9] has replaced the Fourier encoding matrix in (5) with the sensitivity encoding matrix $\mathbf{E}$. With the same random sampling pattern, the image is reconstructed by solving

$$\arg\min_{\mathbf{f}} \{\|\mathbf{d} - \mathbf{Ef}\|_2^2 + \lambda_1 \|\mathbf{Wf}\|_1 + \lambda_2 \,\mathrm{TV}(\mathbf{f})\}. \tag{6}$$

In this formulation, the incoherence between the encoding matrix $\mathbf{E}$ and sparse transformation matrix $\mathbf{\Psi}$ has not been validated theoretically. Therefore, the performance is not guaranteed.

III. Proposed Method

In Cartesian SENSE, the sensitivity encoding in Eq. (1) can be decomposed to two sequential linear operations. The first one is a number of sensitivity modulations,

$$\mathbf{Cf}_\Omega = \mathbf{f}_\Omega^A \tag{7}$$

where $\mathbf{C}$ is the sensitivity modulation matrix, vector $\mathbf{f}_\Omega$ the pixels of desired image $\mathbf{f}$ that are superimposed in the aliased image, and vector $\mathbf{f}_\Omega^A$ the pixels at same position of the aliased images from all channels; the second one is Fourier transform of all pixels

$$\mathbf{Ff}_l^A = \mathbf{d}_l \tag{8}$$

where $\mathbf{F}$ the Fourier operator, $\mathbf{f}_l^A$ the aliased image of the l-th coil with reduced FOV, $\mathbf{d}_l$ the reduced k-space data from the l-th coil. Based on the fact that Eq. (8) is the same as the conventional Fourier encoding formulation, we can apply SparseMRI directly to the aliased image from each channel without compromising the incoherence condition. In our proposed CS-SENSE method, we randomly undersample the k-space of the aliased images with reduced FOV, which is represented as

$$\mathbf{F}_u \mathbf{f}_l^A = \mathbf{b}_l. \tag{9}$$

In generating the random sampling pattern, similar to [7], variable-density sampling schemes with denser sampling near the center of k-space are applied, taking into account the fact that the energy of MR images in concentrated in the central k-space[12]. The aliased image $\mathbf{f}_l^A$ is then reconstructed by

$$\arg\min_{\mathbf{f}_l^A} \left\{\|\mathbf{b}_l - \mathbf{F}_u \mathbf{f}_l^A\|_2^2 + \lambda_1 \|\mathbf{Wf}_l^A\|_1 + \lambda_2 \,\mathrm{TV}\left(\mathbf{f}_l^A\right)\right\}, \tag{10}$$

which is solved numerically using nonlinear conjugate gradients [7]. With the aliased images, we can then generated the desired full FOV image $\mathbf{f}$ pixel by pixel using the least squares solution to Eq. (7)

$$\mathbf{f}_\Omega = (\mathbf{C}^\mathbf{H}\mathbf{C})^{-1}\mathbf{C}^\mathbf{H}\mathbf{f}_\Omega^A. \tag{11}$$

Please note, For CS-SENSE, the net reduction factor R is equal to the product of the reduction factor R_1 in SparseMRI for aliased images and the reduction factor R_2 in SENSE, i.e., $R = R_1 \times R_2$.

IV. Experimental Results and Discussion

A. Experiments Settings

The feasibility of the proposed method was validated on two datasets. A T1-weighted phantom scan was performed using a 2D spin echo sequence on a 3T commercial scanner (GE Healthcare, Waukesha, WI) equipped with an 8-channel torso coil (TE/TR = 11/300 ms, 18cm FOV, 8 slices, 256×256 matrix). The second is a set of in vivo brain data, which were acquired on a 3T commercial scanner (GE Healthcare, Waukesha, WI) with an 8-channel head coil (Invivo, Gainesville, FL). A healthy volunteer was scanned with a 2D T_1-weighted spin echo protocol (axial plane, TE/TR = 11/700 ms, 22cm FOV, 10 slices, 256×256 matrix). Informed consent was obtained from the volunteer in accordance with the institutional review board policy. The sensitivity information of each coil was obtained by pre-scanning.

All methods are implemented in MATLAB (Mathworks, Natick, MA). The sum-of-square (SoS) reconstructions from the fully sampled data of all channels are shown in Fig. 1 as the reference for comparison, where the left is the phantom image and the right is the brain image. Both the proposed CS-SENSE and the direct combination methods were used to reconstruct the desired image with different reduction factors. All images are normalized and shown in the same scale. We label the method and reduction factor on the top-left and top-right corners of each reconstructed image, respectively.

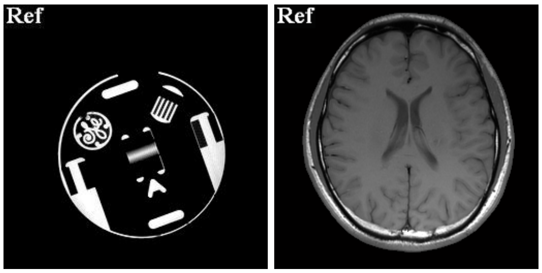

Fig. 1 The reference images denoted by “Ref” on the top-left corner.

B. Results and Discussion

In the case of phantom data, the reduction factor takes $R = 2 \times 2$, 3×2, 4×2 and 6×2 for CS-SENSE, and $R = 4$, 6, 8 and 12 for direct combination method. R_2 is fixed to 2 in CS-SENSE to avoid noise amplification due to the ill-conditioning at large reduction factors in SENSE. Fig. 2 shows the reconstructions from the scanned phantom data. The first and third rows show the reconstructed images of CS-SENSE method and direct combination method respectively. The second and fourth rows show the corresponding zoomed region containing the “comb” to reveal details.

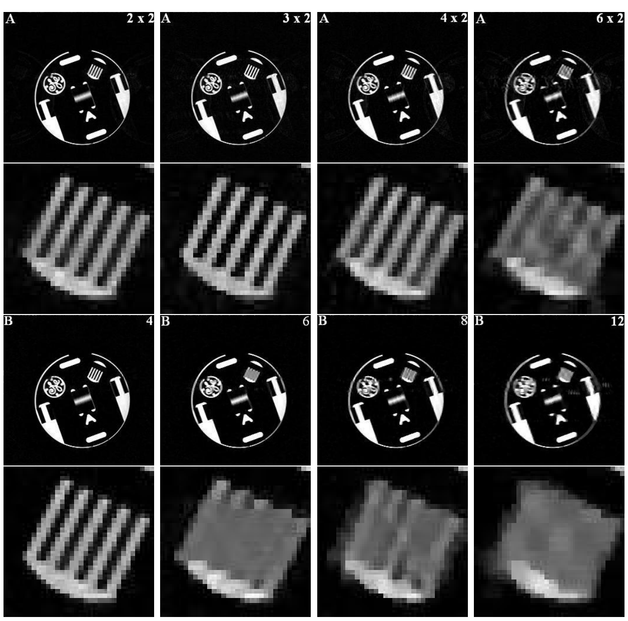

Fig. 2. The reconstructions (first and third rows) and the zoomed “comb” regions (second and fourth rows) from the scanned phantom data with reduction factor R=4, 6, 8 and 12 shown on the top-right corners. The proposed CS-SENSE is denoted as method “A” (top two rows) and the direct combination is denoted as method “B” (bottom two rows) on the top-left corners.

We can see that for moderate reduction factor (R=4), the reconstructions using both methods are visually almost the same as the reference image. As the reduction factor becomes larger (R=6), the superior performance of the proposed method is more visible; the reconstructed image is less blurry with more details as seen in the zoomed “comb” region. For the reduction factor equal to the number of channels (R=8), the direct reconstruction has aliasing artifacts with most details lost. In comparison, the proposed CS-SENSE method has little artifacts and preserves much more details. When reduction factor is larger than the number of channels (R=12), the reconstructed image of CS-SENSE method shows obvious degradation because too few measurements were used in reconstructing aliased coil images but still preserves more details than the direct combination method.

Fig. 3 shows the reconstructions and corresponding error images with different reduction factors for human brain data. For better visualization, the error images were amplified 1000 times and then any values greater than 255 were truncated. We took reduction factors of R = 4, 6, and 8. Different combinations of R_1 =2, 3 and 4, R_2 =2 and R_1=2, R_2 =4 were used.

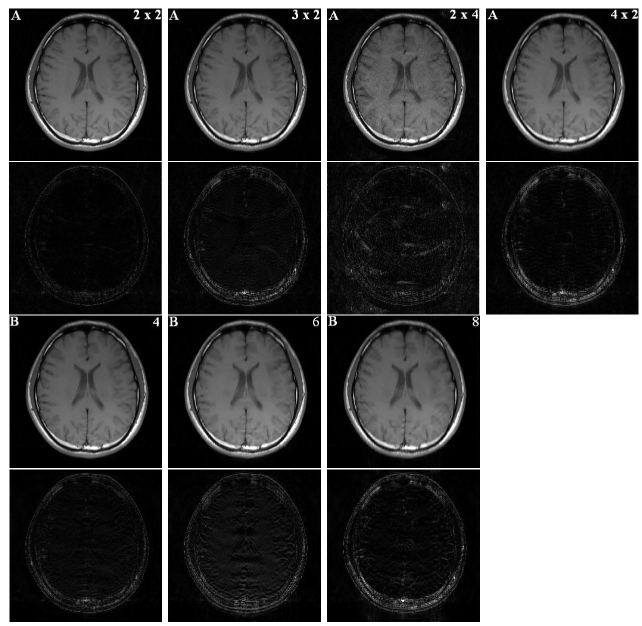

Fig. 3. The reconstructions (first and third rows) and the corresponding error images (second and fourth rows) from the human brain data with reduction factor R=4, 6 and 8 shown on the top-right corners. The proposed CS-SENSE is denoted as method “A” (top two rows) and the direct combination is denoted as method “B” (bottom two rows) on the top-left corners. The error images were amplified 1000 times and truncated at 255.

Similarly, for moderate reduction factor (R=4), both methods can reconstruct images visually almost the same as the reference image. The CS-SENSE has slightly higher noise level at some locations, and the direct reconstruction has more aliasing artifacts near the skull. As the reduction factor increases to 6, the direct reconstruction has more aliasing artifacts, and the image is more blurry and loses more details. For high reduction factor equal to the number of channels (R=8), the proposed method with R_2=4 for SENSE is much noisier due to the ill-conditioning problem, but preserves much more details, while the direct reconstruction has serious aliasing artifacts with most details lost. The proposed method with R_2=2 for SENSE is visibly superior to the above two reconstructions with a good compromise between noise, aliasing artifacts, and blurriness. The results suggest that a larger R_1 will result in more aliasing artifacts and a larger R_2 will result in more noise amplification. Depending on the noise level and the condition of SENSE reconstruction, this observation could be a guideline for selection of the reduction factors R_1 and R_2.

Table 1. Comparison of NMSEs

NMSE ($\times 10^{-002}$)	R=4	R=6	R=8	
			R=2x4	R=4x2
A	0.25	0.83	1.04	1.17
B	0.59	1.06	1.25	

The normalized mean square error (NMSE) provides a combined metric for both image noise and artifacts. The NMSE between the reference and reconstructed images for human brain data were also computed to evaluate the reconstruction performance, given in Table 1. In terms of NMSE, the proposed CS-SENSE method is superior to the direct combination method with the same reduction factor. This may be due to the improved incoherence between the encoding matrix and the sparse representation matrix in CS-SENSE.

The current implementation of CS-SENSE method needs longer time than the direct combination method. This is due to the repetition in solving Eq. (10) for all channels and the extra computation of Eq. (11) in CS-SENSE method, while the direct combination method only solves Eq. (6) once. However, the running time could be reduced by parallel solving Eq. (10) for all channels using multiprocessors or dedicated hardware systems, because the *k*-space data are acquired from multiple channels simultaneously in pMRI and reconstruction of the aliased image for each channel is independent of each other. In this way, compared to SparseSENSE, CS-SENSE only needs an extra computation for SENSE, which is non-iterative and efficient [2].

V. Conclusion

In this paper, a method for further accelerating parallel imaging using CS is proposed. This method uses CS in replace of Fourier transform to reconstruct the aliased images with reduced FOV for basic SENSE reconstruction. The experimental results demonstrate the proposed method is able to accelerate conventional MRI by a large factor being the product of the factors achieve by SparseMRI and SENSE individually.

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