Faster Imaging with Randomly Perturbed, Undersampled Spirals and |L|_1 Reconstruction

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Introduction

We propose a fast imaging method based on undersampled k-space spiral sampling and non-linear reconstruction. Our approach is inspired by theoretical results in sparse signal recovery [1,2] showing that compressible signals can be completely recovered from randomly undersampled frequency data. Since random sampling in frequency space is impractical for MRI hardware, we develop a practical strategy allowing 50% undersampling by adapting spiral MR imaging. We introduce randomness by perturbing our spiral trajectories. We reconstruct by minimizing the L_1 norm of a transformed image subject to data fidelity constraints. Simulations and experimental results show good reconstruction particularly from heavily undersampled k-space data where conventional methods fail. This method can be used with other imaging/reconstruction methods such as SENSE[8].

Theory

The goal of partial k-space reconstruction is to reconstruct an image from incomplete Fourier data -- a highly under-determined problem. Medical images often have a sparse representation in some domain (such as finite differences, wavelets, etc.), where the number of coefficients needed to describe the image accurately is significantly smaller than the number of pixels in the image. We exploit sparsity by constraining our reconstruction to have a sparse representation and be consistent with the measured k-space data. Surprisingly, if the underlying true object has a sparse representation (see [1,2,3] for details) and the sampling in frequency is uniformly and randomly distributed, we can recover the signal accurately by solving the following constrained optimization problem,

$$\begin{aligned} &\text{minimize} \;\; ||\Psi(m)||_1 \qquad (1) \\ &\text{s.t} \;\; ||Fm - y||_2 < \varepsilon \end{aligned}$$

Here, m is the image, Ψ transforms the image into a sparse representation, F is an undersampled Fourier matrix, y is the measured k-space data and ε controls fidelity of the reconstruction to the measured data. ε is usually set to the noise level. The objective enforces sparsity whereas the constraint enforces data consistency. Eq. 1 is a convex quadratic program (QP)[4] that can be solved efficiently by interior point methods. Note that the non-linearity of the L_1 norm is crucial [4]. Our approach can also be used with SENSE reconstruction; simply substitute in place of the Fourier matrix an encoding matrix that includes both Fourier and coil sensitivity matrices.

Methods

We propose two different sparsifying transforms, the wavelet transform and finite differences -- both widely used in image processing [7]. For finite differences, the objective becomes the total variation TV = $\Sigma_x\Sigma_y\,|\nabla m(x,y)|$ where $\nabla m(x,y)$ is the spatial gradient of the image, computed by finite differences. Random sampling, as advocated in [1,2,3] is not feasible in MR because of hardware limitations. However, spiral trajectories are a good candidate for approximating random sampling. They span k-space uniformly but on the other hand they are far from being as regular as a Cartesian grid. Furthermore spiral imaging is fast and time-efficient. To introduce more randomness we perturbed the individual spiral trajectories, slightly deviating from the deterministic spiral along each interleave; the interleave angles are also perturbed by a small random angle. To validate our approach we considered a 34 interleave perturbed spiral trajectory, designed for a 16 cm FOV 1 mm resolution. We undersampled by 50% by acquiring data only on a subset of 17 out of the 34 interleaves using a GRE sequence (TE=1.3ms, TR=8.24, RO=3ms, α=30°, ST= 4mm). The experiment was conducted on a 1.5T GE Signa scanner with gradients capable of 40mT/m and 150mT/m/ms maximum slew rate. The image was reconstructed by TV reconstruction implemented with finite derivatives, and with L_1 wavelet (Daubechies 4) reconstruction. Results were compared to gridding and minimum-norm reconstructions. Our reconstructions used a primal-dual interior point solver [4] with min-max nuFFT [5,6].

Results and discussion

Fig 1. illustrates the results of four reconstruction algorithms. As expected, the gridding and minimum norm reconstructions exhibit severe aliasing artifacts due to undersampling. On the other hand, the L_1 reconstructions removed most aliasing artifacts while preserving resolution. TV penalization performs slightly better than the L_1/wavelet penalty. This difference is attributable to the object's being piece wise constant and so being sparser for the finite difference operator than for the wavelet transform. Fine structures that are severely corrupted by aliasing are well recovered by the L_1 reconstructions. Note that the Fourier transform of all the reconstructed images in Fig.1 is the same (up to noise level) at the spiral sample points. The L_1 method was able to recover the information because the correct image representation is sparse, and sparsity is being imposed.

Conclusion

In conclusion, L_1 –penalized image reconstruction outperforms conventional linear reconstruction, recovering the image even with severe undersampling. The non-linearity of the L_1 norm is the key; however our method is more computationally intensive than traditional linear methods. In the current, rather inefficient Matlab™ implementation we are able to reconstruct a 256x256 2D image in a matter of several minutes. Our simulations show that using perturbed spirals offers better reconstruction than just by uniformly undersampling k-space. This type of reconstruction can be used to speed up acquisition whenever there is sparsity to exploit. Applications such as angiography, time-resolved and contrast enhanced imaging are perfect candidates as such images can be have a very sparse representation.

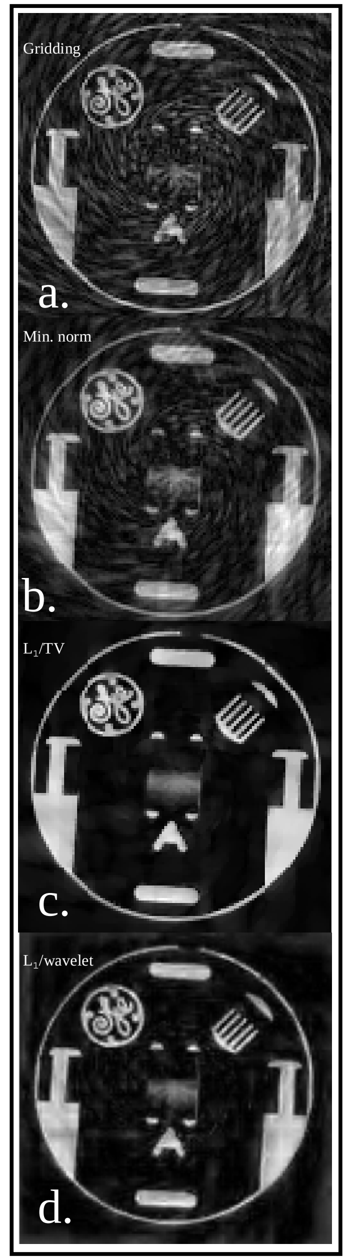

Figure 1: *Various reconstructions from 50% k-space undersampling. a) gridding b) Minimum norm c) Total variation d) L_1 wavelet Note that structures that are severely corrupted by aliasing are recovered by the TV and L_1 /wavelet reconstructions.*

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