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A Perspective on K-Space[1]

An essential difference between magnetic resonance (MR) imaging and other medical imaging modalities is the control that the user has over how the data are acquired, manipulated, and reconstructed for viewing. Just by changing some software controls—such as the pulse sequence timing, the order of data acquisition, and the strength and rate of change of auxiliary magnetic fields—the user can modify resolution, field of view (FOV), contrast, speed of acquisition, effect of artifacts, and so on.

The agent of this control is k-space, the abstract platform onto which data are acquired, positioned, and then transformed into the desired image. No such agent exists for x-ray imaging, ultrasonography, or positron emission tomography, and while an analog does exist for computed tomography (CT), access to it is crude and mechanical and lacks the ease and flexibility of the MR techniques. These methods cannot support the rich interaction between the user and the image available with MR imaging.

The price for this rich interaction is the need for an understanding—better yet, an intuitive understanding—of the concepts and mechanisms of k-space manipulation. The problem is that k-space is an abstract notion, not something one can touch or feel or even shake to get a idea of how it works. Even when the contents of k-space are visualized, the data have little meaning and no apparent relationship to the MR image. The mathematical constructs that explain k-space in great detail are fairly sophisticated and do little to give an intuitive feel for what is going on.

In actual fact, the concepts of the k-space are simple and have been known and understood for several centuries. The concept is that of the Fourier transform, which is a series of equations developed in the 1800s to help understand the flow of heat and which has been extended over the past hundred years to explain the flow of current in electric circuits, the design of musical instruments, the spread of microwaves from a satellite antenna, and the creation of an image by passing light through a lens.

It was introduced to MR with the invention of the spin-warp technique by Edelstein et al (1) and has been exploited with ever-increasing sophistication over the past few years.

In this article I will review the concepts of k-space. To avoid dealing with abstract notions, I will take advantage of the fact that the concepts and equations involved with k-space are exactly the same as those involved with the actions of a simple lens—the kind used in telescopes, microscopes, and photographic cameras. Simple concepts such as resolution and FOV, or more sophisticated matters of image wrap, ghosts, motion artifacts, or segmentation are not unique to MR imagers but have been understood and used by lens designers and even camera buffs for centuries. Exploiting this close relationship between how k-space works and what happens in a lens, I will be able to explain the abstract notions of k-space by relating them to the notions understood by everyone who has ever used a camera.

Finally, I should note that there are many excellent and quite detailed discussions of k-space to be found in the literature (2–7). My goal here is to give the reader an intuitive understanding of k-space so that it may be incorporated into everyday practice.

WHY IT IS CALLED K-SPACE

The short answer stems from the fact that "k" is the name of a term found in the general equation that describes the MR signal. That term, which has the units of spatial frequency (cycles per centimeter), has three components, usually called k_x, k_y, and k_z. These three components are coordinates in and actually define a domain, or more colloquially a "space," which we have come to call k-space.

The choice of the letter "k" is based on a tradition among physicists and mathematicians to use that letter to stand for spatial frequency in other similar equations (eg, equations that describe the propagation of light or sound or radio waves) and has no particular significance at all. The first use of the letter k in such equations seems to be lost in history and in any event predates the invention of MR.

The long answer, including a derivation of the general equation, has been relegated to the Appendix.

... AND THERE WAS LIGHT

Consider first the ordinary optical (ie, photographic) image (Fig 1). Creation of such an image is a two-step process. First, light illuminates (excites) the object, and the scattered light is collected by the lens. Second, the lens processes (operates on) the

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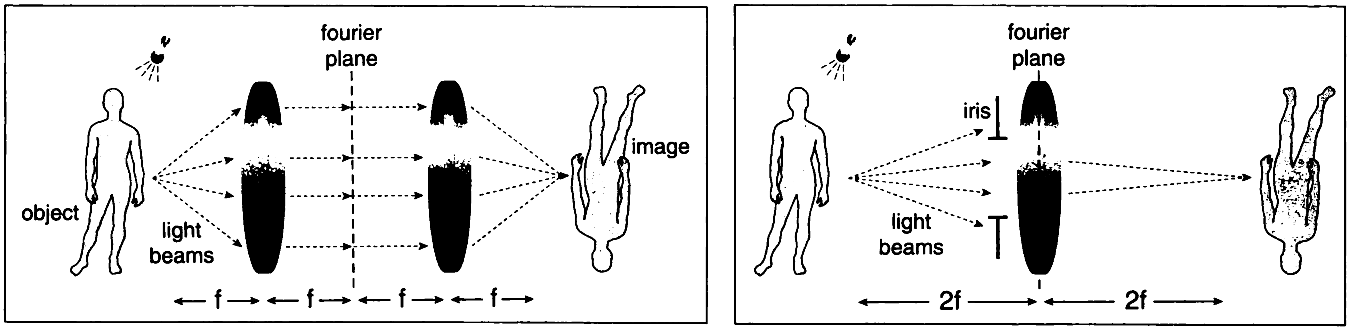
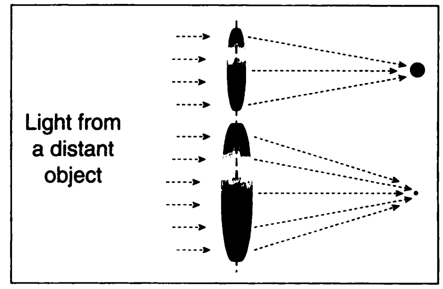
Abbreviations: FOV = field of view, RARE = rapid acquisition with relaxation enhancement, RF = radio frequency, TE = echo time, TR = repetition time.



a.　b.

Figure 1. **(a)** A two-lens system favored for Fourier transfer experiments. With the object placed at the front focal plane of the left lens, the Fourier transfer of the light from that object is formed at its back focal plane (*f*). The second lens performs a Fourier transfer of the light at that plane, which creates the image at its back focal plane. The plane halfway between the lenses is called the Fourier transfer plane. Simple filters, light stops, and apertures placed at the Fourier transform plane can be used to create complex patterns and perform optical functions. **(b)** The simple, single lens is functionally identical to the double lens of **a**. The Fourier transfer plane is inside the lens. Although filters, stops, and so on cannot be placed at the Fourier plane, placement of them close to the lens has similar effects. The iris diameter, for example, affects image resolution by controlling the affected diameter of the Fourier transfer plane. **(c)** In the case of two lenses with the same focal length, the larger lens will have better resolution (ie, will focus the light to a smaller point).



c.

light to create the image. The characteristics of the image, such as its resolution, size, contrast, and so on, are in large measure determined by the processing. As one important example, the maximum resolution of the image is inversely proportional to the size of the lens: The larger the lens, the smaller the detail that can be resolved in the image, as evident to anyone who has seen or used a "close-up" lens. It is instructive to understand this relationship.

The mechanism by which the lens actually does the processing—transforming the light passing through it into an image at a plane some distance behind it—is based on interaction between the lens curvature and the speed of light in the lens (which is slower than that in air). This interaction bends the light passing through the lens, slightly at the center and more at the edges. If the lens has been designed correctly, the light from everywhere on the lens surface will meet at one point. In a perfect lens (that is, one without aberration), the definition (ie, sharpness) of that point will depend on how steep an angle the light makes with the lens (or image plane); the steeper the angle, the more precisely the image point can be defined. An analogy can be made to an ax: One with a wide head can be honed to a sharper, more defined edge than one that is thin. In any event, imagine a lens whose plane is completely filled with light from a distant object: The larger the lens, the greater the angle of light arriving at the image point. Thus, an increase in lens size causes a decrease in the size of the image point and so improves the resolution.

This concept, which applies as well to a "burning" lens used to start a fire as to a photographic lens used to take a picture, was undoubtedly appreciated in very early times but was not really understood until sometime around the 17th and 18th centuries, when several physicists and mathematicians (Huygens, Fresnel, and Young, among others)(8), began to explore the wave nature of light. As part of that exploration, some of them experimented with diffraction gratings and noted that light passing through such a grating will also be bent—somewhat like light passing through a lens—but the amount it is bent is proportional to the spatial frequency (the number of transparent and opaque segments per millimeter) of the grating (Fig 2). They reasoned that a lens could actually be replaced by a collection of gratings with the spatial frequencies that are included proportional to the curvature and radius of the lens and, more important, with the highest spatial frequency proportional to the maximum radius of the lens.

As they continued with this reasoning, it became clear that there was an alternative explanation for the action of a lens. One could imagine that it was *the light* that was actually composed of collections of different spatial frequencies and that the lens was actually a special sort of filter that let more or fewer of the frequencies through. In this construct, the bigger the lens the more frequencies it allowed to pass through it, and the more frequencies that could pass through it the better would be the ultimate image resolution. The lens, of course, has to be more than just a filter—it also must convert and combine the spatial frequencies of the light passing through it into the points of light that make up the image. The value of this alternative model for the lens is that these double functions—filter and conversion—are mathematically described by an algorithm known as the Fourier transform. The essence of a lens, at least as far as the mathematician (and, as we shall see, the radiologist) is concerned, is that it is something that takes data (eg, signals, waves, light) that exist in a plane and performs a Fourier transform on them to create an image in another plane (Fig 3).

The data that the lens operates on, the light at the lens plane is, to repeat, a collection of bundles or packets or patterns of light, each at a different spatial frequency. The collection can be represented as a two-dimensional array (Fig 4). The coordinates of the array are spatial frequencies in the two orthogonal directions, and the brightness at each point in the array is proportional to the energy in the light at those spatial frequencies. Since this is a two-dimensional array, two spatial frequencies are associated with each point—one in x (or the horizontal dimension) and one in y (the verti-

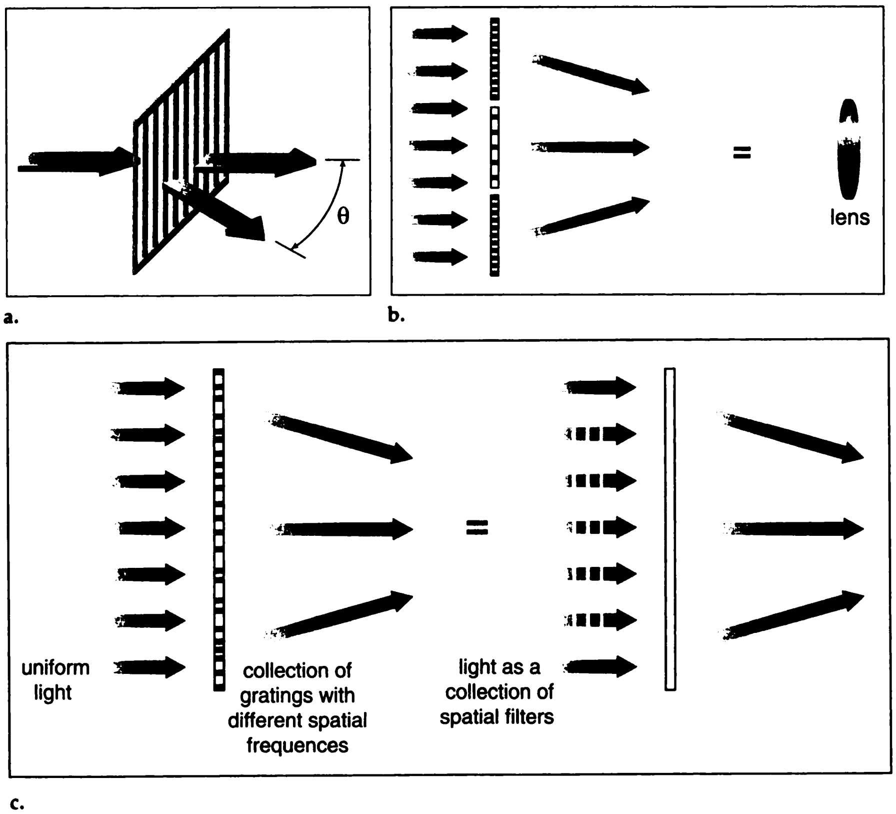


Figure 2. **(a)** Simple optical grating. Part of the light incident from the left is diffracted at an angle θ, where θ is inversely proportional to the line spacing of the grating elements. **(b)** Lens simulated by a collection of gratings. High spatial frequency (ie, small line spacing) of gratings at upper and lower aspects diffracts light at a steep angle, and lower spatial frequency of gratings near center diffracts light at a shallow angle. With careful selection of the range and location of the different spatial frequencies, all the diffracted beams will meet at one point (the focus), which simulates a conventional lens. This arrangement is called a Fresnel lens. **(c)** The Fresnel lens (left) acting on a homogeneous beam of light is itself functionally identical to a homogeneous filter (eg, something as simple as a pane of clear glass) acting on a spatially modulated beam of light. Such spatial modulation is difficult to achieve with light but is very easy to achieve in the MR imaging system.

cal dimension). This plane is called the Fourier transform plane.

Consider now the MR image (Fig 3c). It too is created by means of a two-step process. In the first, radio waves excite the object and the resultant energy from the object is collected. In the second, the data are processed into an image. The details of how the radio waves are generated and acquired are interesting but with few minor exceptions are of little interest to us now. What is of great interest is that the second step of the imaging sequence, the data processing, is a Fourier transform, the very same Fourier transform used in the second step of the optical image sequence. However, this means that there must be a data plane, analogous to the Fourier transform plane in the camera, that consists of signal or data intensity patterns, each at different spatial frequencies, that is operated on by the Fourier transform to create the image. That plane exists; we call it k-space, and it is exactly analogous in function and concept to the two-dimensional array of light at the lens plane (Fig 4).

The implication, and indeed the reality, of this is that the entire process of MR imaging is closely analogous to optical imaging, and concepts from one can be used to explain concepts in the other. The mostly abstract concepts that are basic to MR imaging, especially those that involve k-space, can be explained and understood by reference to the much more familiar and concrete concepts used in photography, microscopy, and other forms of optical imaging. I shall use these analogies to explain how k-space controls some of the basic features of the image such as resolution and FOV, and I shall demonstrate how k-space is manipulated in some of the more sophisticated imaging sequences such as echo-planar imaging, fast spin-echo imaging, segmented k-space imaging, "key hole" imaging, spiral imaging, and others. I hope, through the use of the optical analog, to give the reader an intuitive feel for these and other k-space manipulations so that these manipulations can be incorporated into daily practice.

BASIC IMAGE CONCEPTS

I will start by exploring some of the basic concepts of imaging, whether it is with a camera or with the MR imaging system.

First we must recognize that there is no one-to-one correspondence between a point in k-space and the image, just as there is no point-to-point correspondence between a point in the Fourier transform plane of the camera and the image. Points in the image receive contributions from every point in k-space, and data from every point in k-space contribute to every point in the image.

Pieces of dust or lint on a camera lens (and here I mean the lens itself, not the filter or cover glass up front) do not appear as spots in the image but rather cause a diffuse "haze" in the image due to light scattered over the whole image plane. People with vacuoles or cataracts in the lens of their eyes do not see spots or focal defects but complain of blurriness or decreased contrast. There is no one-to-one correspondence between a spot on the lens and a spot on the film or retina.

This is true not only for discrete spots at the lens but also for large areas. Closing the iris of a lens (whether in a camera, microscope, or eye) does not obscure the outer rim of the image but rather decreases the total brightness and the total sharpness, or resolution, of the image. Every point in the image that was present before the iris was narrowed will be present afterward, the outer points as well as the inner points. However, every point in the image will be affected by the changes at the Fourier transform plane (or k-space)

This brings us to the second point, which is that while every point in the Fourier transform plane (or k-space) contributes to the entire image, the *way* in which it contributes depends on where it is in the Fourier transform plane. In particular, it turns out that points at the edges of the Fourier transform plane contribute to, and indeed determine, image resolution. The farther a point is from the center of the Fourier transform plane, the more it contributes to the sharpness or resolution of the image. The more points there are at the edge, or equivalently the larger the diameter of the lens, the better the resolution. In a telescope, the bigger the lens or mir-

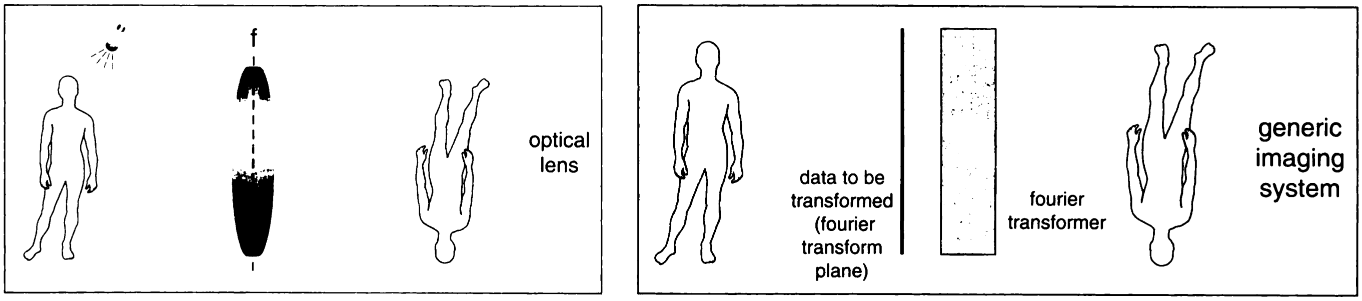
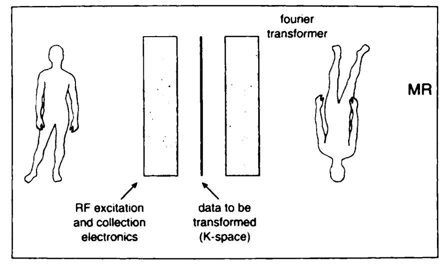


Figure 3. **(a)** Simple optical lens. **(b)** The "effective" lens, which consists of spatially modulated light at or near the Fourier transform plane and a filter that performs a Fourier transfer on the light at the transform plane. **(c)** Lens analog: MR imaging system that consists of two major subsystems. One (left) excites the spins in the patients, collects the echo, and places the data in the Fourier transform plane (ie, k-space). The second is functionally identical to (and is conceptually is the same as) the optical lens described in Figure 2. Data at the transform plane (k-space) are converted to an image by performing a Fourier transform. *RF* = radio frequency.



ror, the smaller the star that can be seen. In a microscope, a larger lens allows visualization of smaller detail. In a camera, use of a bigger lens or opening of the iris increases the resolution.

The reason for this, of course, is that an increase in the diameter of the lens or an increase in the aperture of k-space permits higher spatial frequencies to pass to the image, and it is the higher spatial frequencies (as I just discussed in regard to the lens) that determine sharpness or resolution of the image.

This relationship between lens size and resolution is based on the assumption that there is no aberration. In an actual lens, because of chromatic aberration, coma, astigmatism, and so on, the relationship is valid only up to a point; beyond that point resolution actually deteriorates. Happily, for the most part, these aberrations do not exist in k-space (nonuniform gradient magnetic fields actually will cause aberrations similar to those seen is a glass lens, but it is far easier to shape magnetic fields than glass elements, and most manufacturers have largely eliminated these distortions). We can safely assume for the purposes of this article that k-space has no aberrations.

To illustrate the relationship between resolution and lens aperature, Figure 5 shows a sequence of MR images taken with different apertures in k-space. The first (Fig 5a, 5b) shows the image and the fully opened aperture in k-space, the second (Fig 5c, 5d) shows the image with the aperture diameter reduced by half, the third shows the effect of reducing the aperture by half again, and the fourth shows the effect of reducing the aperture by yet another half (for a total reduction in aperture diameter of one-eighth). The points to note are that each aperture reduction causes a reduction in image sharpness, but that even with such large reductions in the effective area of k-space, the entire image is seen. This, of course, is exactly what happens when one reduces the lens iris on a camera or changes objectives in a microscope and should come as no surprise.

One further thing to note is that in addition to a reduction in resolution, the effect of decreasing the aperture is to create and increase "edge effects" (or ringing) at sharp margins; these are caused by an insufficient number of spatial frequencies. A sharp edge or line contains many (theoretically, an infinite number) of spatial frequencies—both high and low. By decreasing the aperture one eliminates the high spatial frequencies (and so causes blurring or decreased sharpness) but leaves the low frequencies. Some of those low frequencies carry important information about the contrast and width of the line, but some of those low spatial frequencies carry information about edge sharpness—information that was to be shared (ie, compensated for) by the (now eliminated) high spatial frequencies. It is these "uncompensated" low spatial frequencies that give rise to the "edge effects" illustrated in Figure 5.

If the edges of k-space control image resolution, a reasonable question to ask is what is it that the center of k-space controls? It turns out that the center of k-space determines overall contrast, the ratio of light to dark, in the image.

Continuing the experiment described in Figure 5, if we make the aperture (ie, diameter) of k-space very small (1/32 or 3% of the full diameter, as in Fig 6a, 6b) the image contains very little detail but does show, vaguely, the contrast at different parts of the image. By increasing the aperture in stages, one increases the detail (Fig 6c–6f). Taken together, the results of Figures 5 and 6 show a continuum of effects, with data farther from the center controlling resolution and data closer to the center influencing contrast. It is important to note that there is no sharp divide in the sense that points beyond a certain aperture diameter control resolution and points within control contrast; rather, the effects of aperture changes are graceful, with a gradual change in image quality (mostly resolution) with changes in aperture diameter. This, of course, is the experience with photographic cameras, in which even fairly large changes in the aperture (sometimes called f-stop) cause relatively small changes in resolution. (I am ignoring for now the other effect of changing the iris, that of changing the image brightness. The reason for this is that in MR imaging the image brightness is artificially manipulated at the image console or in the laser camera, and there really is no direct relationship between expected image brightness due to changes in k-space aperture and actual image brightness.)

If we accept the conclusion that the location of data in k-space (ie, toward the edge or near the center) affects image resolution and contrast, then the next question to be resolved is how do we in fact control that placement of data in k-space?

FILLING K-SPACE

MR image generation is, as I have said, a two-step process. The last step,

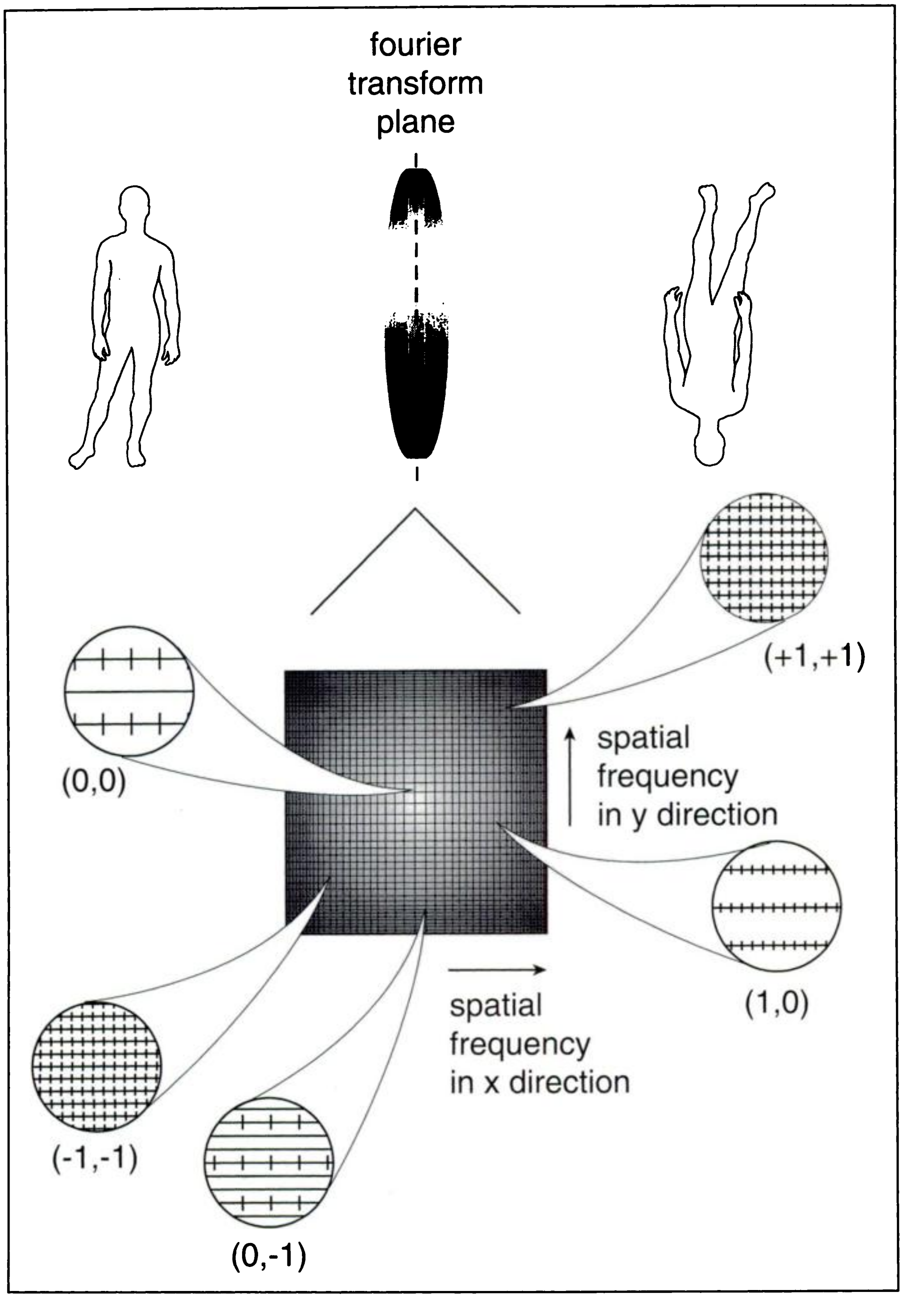


Figure 4. The Fourier transform plane (k-space). The spatial frequencies in the plane vary with location; they are higher at the margins and lower at the center. At the very center (0,0), the spatial frequency is low (hatch marks separated), while at the corners (+1,+1; −1,−1; −1,+1; +1,−1) the spatial frequencies are high (hatch marks close together). At the center of the right margin (+1,0), the spatial frequencies will be high along the x direction and low along the y direction. At the center of the bottom margins, spatial frequencies are high along the y direction and low along the x direction. The important point is that the spatial frequencies in x and y are independent and are determined in each case by means of the distance from the axis.

which I have just described, involves taking data that have been placed in k-space and performing a transform, a Fourier transform, to create the image. It is not important to this step exactly how the data got to k-space. It is important only to the transform that the data be there. It is the task of the first step of the MR process to provide that data and to ensure that the data are properly placed in k-space.

The gradients, and in particular gradient strength, control the placement of data in k-space.

Image data (ie, the radio-frequency [RF] echo from the excited spins in the patient) acquired with (in the presence of) *small* gradients will be placed near the center of k-space, while data acquired with *large* applied gradients will be placed farther from the center. Just by manipulating the gradient strength that is applied while data are acquired, we can control the resolution and contrast of the image. Gradient strength in the MR system is the equivalent of iris diameter in the photographic camera.

There are, in the usual case, two gradients that directly influence image quality—the frequency gradient and the phase-encoding gradient. (The third gradient, the section-selection gradient, controls image location, orientation, and thickness, but none of these affects or is affected by manipulations in k-space and so this gradient will be ignored for now. We simply assume that somehow or another appropriate sections are selected and that we are concerned only with image quality in that section. I will return to this point later.)

While there is no correlation between points in k-space and points in the image, there is a direct one-to-one relationship between points in k-space and gradient strength. Image data acquired with the phase-encoding gradient equal to +Gp will be along the rightmost margin in k-space (line *a* in Fig 7), and image data acquired with the phase-encoding gradient equal to 0 will be along the central vertical axis in k-space (line *b* in Fig 7). The line in k-space at the leftmost margin (line *c* in Fig 7) will be filled when image data are acquired with the phase-encoding gradient set to its maximum negative value (-G_P). By simply adjusting the gradient strength (and sense, ie, whether positive or negative), we precisely determine placement of data in k-space.

Which points in k-space are filled and the order in which they are filled are determined by the pulse sequence. The remarkable thing about MR imaging is that the pulse sequence designer—and through the designer, the radiologist—has complete control over image quality by virtue of the control over how and when k-space is filled. If k-space is made large (ie, a large-diameter aperture), then the image will have high resolution. If only the central portions are filled, then the image will have lower resolution. If, for some reason, only the edges but not the center is filled, then the image will have high resolution but poor contrast (it will, in fact be an edge-enhanced image, as seen in Fig 6b).

One might wonder that since the goal of imaging is usually to produce a picture with the highest quality possible, why would the pulse sequence designer not ensure that k-space be made as large as possible (commensu-

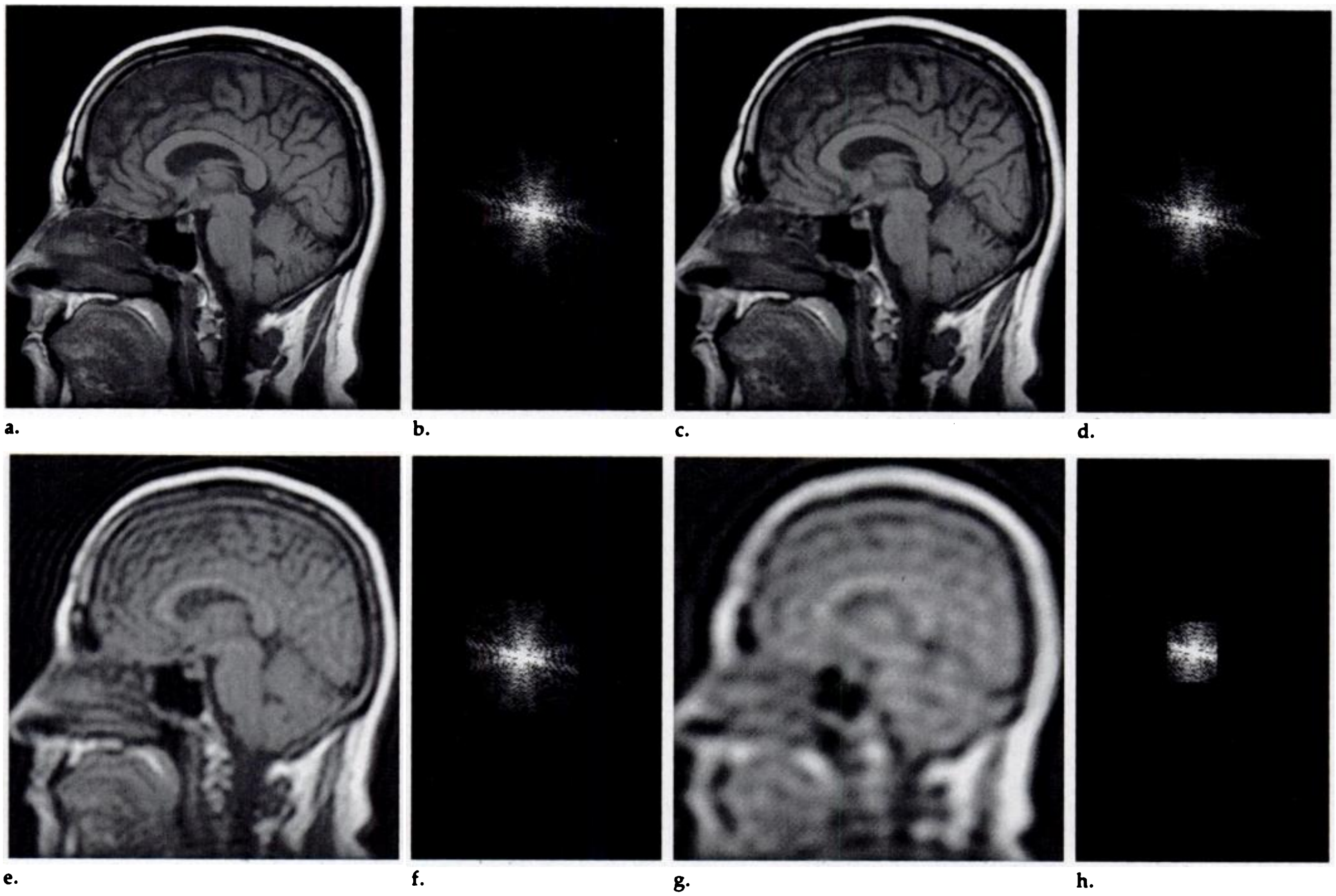


Figure 5. The effect of changing the size of k-space. **(a)** A 256 × 256 sagittal MR image and **(b)** its associated k-space data (because of the limited dynamic range of the photograph, the peripheral aspects of the k-space data are difficult to see). Serial reductions in the size of k-space cause progressive loss of detail. The size of k-space is reduced by one-half in **c** and **d**, by another one-half in **e** and **f**, and by another one-half (to a total of one-eighth) in **g** and **h**. Note that in addition to a reduction in resolution, there is an increase in the "edge effects" (or ringing) at sharp margins caused by an insufficient number of spatial frequencies.

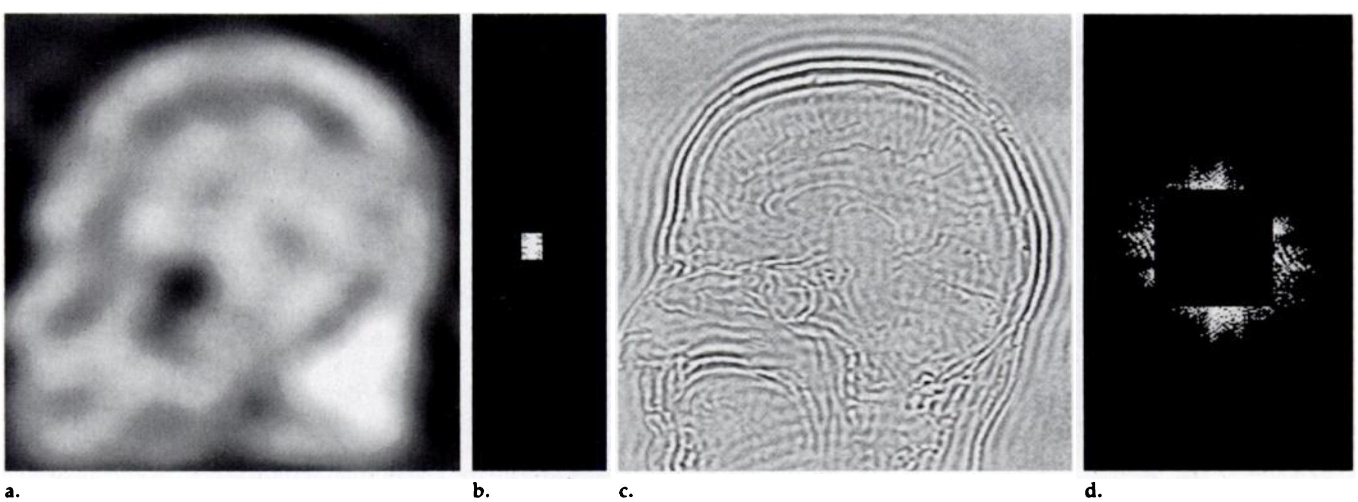


Figure 6. **(a)** MR image and **(b)** associated k-space data demonstrate that reduction of k-space to the extreme (1/32 of its original size) results in almost total loss of detail but preservation of contrast **(a ,b)**. **(c, d)** The obverse situation, with the center of k-space obliterated but the remainder preserved, results in an image with preserved detail but loss of contrast. This is an extreme case of edge enhancement.

rate with the maximum gradients available) and that all of k-space be filled all the time? One answer is simply that filling k-space takes time and that it makes no sense to take a lot of time filling a large k-space if it is likely that the patient (or at least part of the patient) will move during the acquisition. Patient motion can blur and distort the image (just as with a photographic camera) more than increased resolution can enhance it.

A second answer is that increasing the size and number of lines in k-space decreases the size of each image element. Since there are only a finite number of hydrogen atoms per unit volume, reducing the image ele-

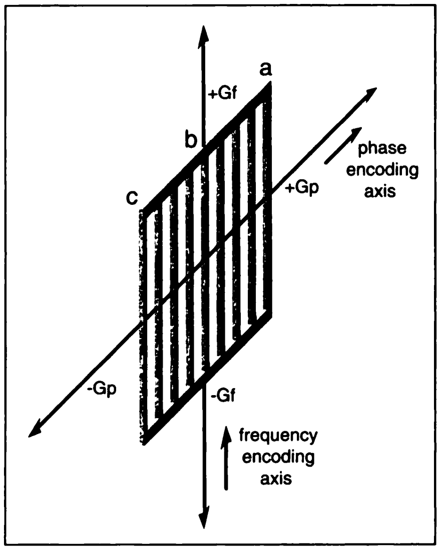


Figure 7. Filling of k-space. Line *a* is filled when the phase-encoding gradient amplitude is large and positive. Line *b* is filled when the phase-encoding gradient amplitude is zero. Line *c* is filled when the phase-encoding gradient amplitude is large but negative. $+Gf$ indicates maximal positive and $-Gf$ indicates maximal negative spatial frequencies along the frequency-encoding directions. $+Gp$ indicates maximal positive and $-Gp$ indicates maximal negative spatial frequencies along the phase-encoding directions.

ment size reduces the number of hydrogen atoms contained in each element, which leads to a reduction in signal and therefore a reduction in signal-to-noise ratio. In situations in which signal-to-noise ratio is limited, the system designer and radiologist may have to compromise on resolution.

A third answer involves limits on the length of each line (and hence the resolution) along the read-out direction. That length is proportional to the product of the read-out gradient strength and its duration (ie, how long it is left on). Increasing the gradient can put unsustainable stresses on the gradient amplifiers, which leads to inhomogeneities in the magnetic field. Increases in gradient strength also lead to increases in read-out amplifier bandwidth (a consequence of the Larmor equation, which relates frequency to magnetic field), which ultimately leads to decreases in signal-to-noise ratio. Increasing the duration can lead to undesirable increases in echo time (TE) and, because of susceptibility (T2*) effects, further decreases in signal intensity.

The real goal of pulse sequence design then is to fill as much of k-space as quickly as possible while trying to balance image quality and speed.

In the following sections I will explore several approaches to image acquisition and display that illustrate different methods of k-space manipulation. Some of the examples employ pulse sequences that stress speed at the expense of image quality (echo planar, keyhole, spiral), while others aim for high resolution in reasonable acquisition times (fast spin echo, segmented k-space). Along the way I will stop to examine some additional features and limitations of k-space manipulation.

SPIN-ECHO TECHNIQUE

The basic "spin-warp" technique fills k-space one line at a time, at a rate of one line per 90° RF pulse (Fig 8) (1). The length of the line (along the frequency-encoding direction) is proportional to the maximum strength of the read-out (which is the same as the frequency-encoding) gradient and to the duration of that gradient. The position of the line is determined by the value of the phase-encoding gradient. Figure 8 shows eight lines in k-space and the corresponding pulse sequences used to position those lines. Since only one line is placed in k-space for each 90° RF pulse, the time to fill k-space is given by $N \times TR$, where N is the number of phase encoding lines and TR is the pulse repetition time. If, for example, we use 256 lines to fill k-space (and we shall see in a moment what determines how many lines are actually needed), and if TR is 2 seconds, then it would take 512 seconds (over 8 minutes) to fill k-space.

Notice that k-space is rectangular rather than round like most camera lenses, and while that does not really change the analogy to the camera, it does give k-space manipulators some extra freedom and flexibility. For example, consider the pulse sequence designer who wants to decrease the image acquisition time of the spin-echo sequence. Since each line of k-space along the phase-encoding direction takes TR seconds to fill, a reduction in the number of phase-encoding lines would give a proportional reduction in time. If, in the example above, only 64 lines of k-space were filled, the time would decrease from 8½ minutes to about 2 minutes, a considerable savings. The consequence to the image of such a reduction depends on how k-space is actually filled. There are two possibilities.

If we keep the spacing between each line of k-space constant (by keeping the increment in magnetic strength between each phase-encoding gradient constant), then when we decrease the number of lines in k-space by four, and the size of k-space along the phase-encoding direction must also decrease by a factor of four (from 256 to 64). All the lines will be bunched around the center of k-space. Since the size of k-space controls image resolution, however, and since the size of k-space that is filled has decreased, it follows that the detail or resolution in the image must also decrease. Notice that we have decreased the size along only one dimension of k-space (the phase-encoding direction), and as a result we will decrease the resolution only along one dimension (axis) of the image. The remarkable thing is that while the resolution of the image decreases, the size of the image does not change. If we assume for the moment that the number of lines in the image is equal to the number of lines in k-space, then what happens is that each line in the image gets "blurrier" or fatter by just the factor of four necessary to maintain image size.

Alternatively, the pulse sequence designer (or radiologist) could decide to keep the size of k-space constant and choose to increase the spacing between the lines along the phase-encoding direction instead. The resolution in the image, which is determined by the size of k-space, would not change, but the image size would definitely have to change and would decrease by the factor of four (I will explain why in just a moment).

These are very different results, which were determined entirely by means of how k-space is filled.

The reason the image size would have to change in this last case, and the basis for our assumption that the number of lines in k-space is equal to the number of lines in the image, can be explained by referring back to our optical analog.

In the usual imaging process, data in the Fourier transform plane are continuous, not segmented into separate lines. The effect of segmenting the data into discrete lines can be represented by superimposing a grid or grating onto the Fourier transform plane. This has a dramatic effect, because the grating scatters the light and causes the creation of multiple copies (technically referred to as higher orders) of the original image that are symmetrically placed to either side of that image (Fig 9). (If interested, the reader can see this by placing a nylon stocking or pantyhose over the head, or at least in front of the eyes, and looking at a bright object against a dim background. Multiple copies of the bright object will be seen oriented along the warp and woof of the stocking. It is best to do this at night and by looking at a street light. It might also be good to make sure no neighbors are around to take in the sight of a radiologist wandering around at night with a stocking over his or her head.)

The spacing between the higher orders is inversely proportional to the spacing of the grating lines (which is the same as the spacing of the lines in k-space). Placing the lines close together spreads out the orders, while increasing the line spacing moves the orders close together. (The adventurous radiologist can demonstrate this, as well, by stretching the stocking and noting that the spacing between the orders changes.) If the orders come too close together, they will overlap and thus cause the so-called wrap that is commonly seen in clinical studies when the FOV is too big, the number of phase-encoding lines is too small, or the patient is malpositioned in the magnet. In fact, the image is not "wrapping" around at all. What we see are the diffraction orders overlapping (Fig 10a).

One way to understand this is to imagine that the monitor at the operator's console is really a window cut into the center of an opaque screen. All of the diffracted orders project onto the inside of that opaque screen, but only the images that project onto the window can be seen (Fig 10b). In the ideal case, only the zeroth order projects onto the window and the higher orders hit the opaque screen and cannot be seen. If, however, the higher orders are not diffracted far enough away, they too will project onto the window, with part of the +1 order on one side and part of the −1 order on the other side. It will look like "wrap," but of couse it is not. It is just the diffraction orders overlapping.

Figure 10c is an example of such "wrap." The central image (between the vertical black lines) is what would appear on the console window (and be recorded on film). The images to either side of center are simulations of the +1 and −1 diffracted orders (as they would be seen if we looked at the opaque screen that we imagine surrounds the console window). The nose pushing into the back of the patient's head is not actually wrapping around from the front but is really caused by overlap of the diffracted order on to the central order. The spacing between phase-encoding lines is too wide to diffract the higher orders away from the central image. It

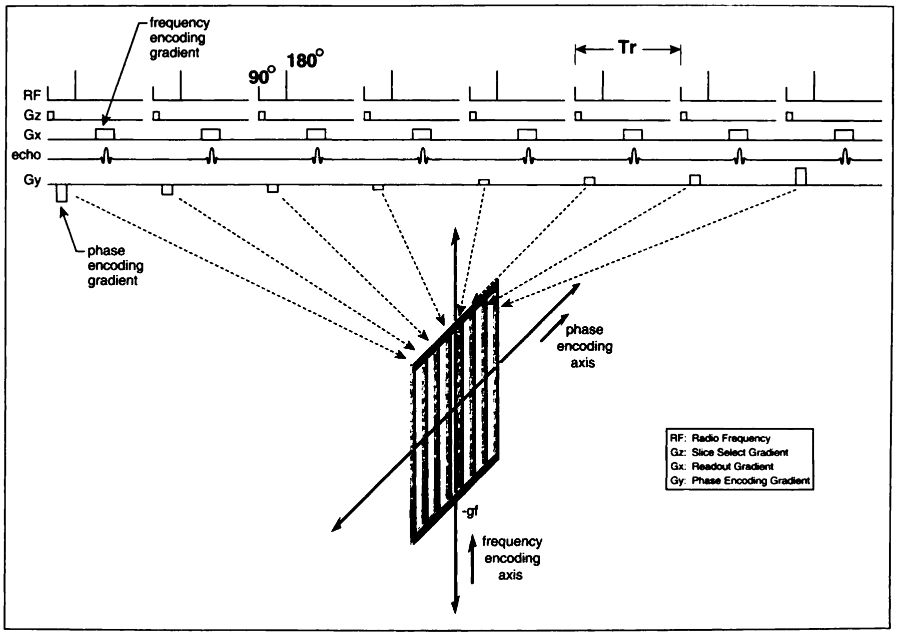


Figure 8. Filling of k-space with the classic spin-echo sequence. One line is filled following each 90° pulse (ie, one line per TR interval). The location of each line depends on the phase-encoding gradient pulse. A large negative pulse fills a line at the left of k-space (large negative spatial frequency). A large positive pulse fills a line at the right of k-space (large positive spatial frequency). Intermediate strength phase-encoding gradients (positive and negative) fill intermediate locations (at corresponding positive and negative spatial frequencies). The pulse sequence is conventional. A section-select gradient is turned on concurrent with the 90° RF pulse. A readout gradient is turned on concurrent with the appearance of the echo. The phase-encoding gradient is placed between the 90° and 180° RF pulses. The strength of the phase-encoding gradients is sequentially stepped from large negative to large positive values; the number of steps determines how many lines will be in k-space, the value of the steps determines how close the lines will be, and the maximum value (positive or negative) determines the maximum size of k-space and thus the image resolution. $-gf$ is the maximal negative spatial frequency along the frequency-encoding direction.

is like our friend with the stocking over his or her head pulling too hard and spreading out the stocking mesh, which brings the diffracted orders too close together. We could ask our friend to release the tension, but what we actually do can be illustrated by a better example. We imagine a transparent elastic lens, one that can be stretched or squeezed along its radius. We further imagine that grid lines (corresponding to the phase-encoding lines we have been talking about) are etched in its surface. When we stretch the lens to make it bigger the image resolution gets better, but if the grid lines get separated too far we get overlap, just like we see in Figure 10c. If we squeeze the lens to make it smaller the grid lines get closer together, the diffracted orders spread apart, and we eliminate the overlap, just as shown in Figure 10d. The problem with this solution is that the image gets smaller and resolution decreases (we did reduce the lens size, which is the same as decreasing the maximum spatial frequency in k-space), but that is a compromise we have to make to get rid of "wrap." Another solution, which involves a different compromise, would be to maintain the lens size and decrease the grid line spacing by increasing the number of lines etched on its surface. This would correspond to increasing the number of lines of k-space (9,10). The problem with this is that acquisition time is increased in proportion to the number of lines added. It would be better to arrange data in k-space to avoid "wrap" in the first place.

To avoid overlap, the FOV of the image must be no larger than the separation between the orders, which means that as the spacing between the lines in k-space gets bigger, the FOV of the image should get smaller. The inverse is also true—if the FOV is to increase, the spacing between lines can decrease. This relationship between FOV and spacing between the lines in k-space (D) can be written as

$$\text{FOV} \propto 1/D. \qquad (1)$$

In this relationship, we assume that the number of lines in k-space does not change, so that if the spacing between the lines gets bigger, k-space itself (actually the maximum spatial frequency) gets bigger.

We previously found that the relationship between aperture size in k-space (which is the same as maximum spatial frequency k_{max}) and line spacing in the image (d) is

$$k_{max} \propto 1/d. \qquad (2)$$

If we remember that the line spacing d in the image can be calculated as $d = \mathrm{FOV}/N_i$ and line spacing in k-space D can be calculated as $D = k_{max}/N_k$ (where N_i is the number of lines in the image and N_k is the number of lines in k-space), then we can show that

$$N_k = N_i. \qquad (3)$$

The consequence of the desire to prevent "wrap" in the image is that the number of lines in k-space is equal to the number of lines in the image. Thus, in the example above in which the pulse sequence designer maintained the size of k-space but reduced the number of lines (and so increased D), the number of lines in the image as well as the image size (FOV) was reduced proportionally. This had to happen to maintain resolution. In the prior example, in which both the number of lines and the size of k-space was reduced together, D was not changed and so image size was maintained, but image resolution was reduced.

Equations (1), (2), and (3) summarize the basic relationships between k-space and the image.

HALF-FOURIER AND HALF-ECHO

As we have seen, it is possible to reduce image acquisition time by simply reducing the number of lines in the image, but this speed improvement usually comes with a proportional reduction in image resolution or image size. There is one exception to this, and it stems from the symmetry and inherent redundancy of data in k-space. Data stored at positive spatial frequencies are closely related to data stored at negative spatial frequencies and are not at all independent. Knowing the value of data at one positive spatial frequency, we can predict the value at a symmetric negative spatial frequency. An intuitive sense that this must be so can be gained by considering a simple optical experiment (again, the lens plane is the analog of k-space).

Imagine the basic imaging system such as in Figure 1b. With the iris full open we would get an image, and we could measure, or at least note, image quality, resolution, number of lines, and so on. Now imagine (or actually get out your camera and do it) that we cover half the lens—say the left half (as before, we cover half the *lens*, not the cover glass nearby, for the analogy to work). We notice that the image is now half as bright (after all, we are blocking half the light), but the entire image is still seen, just as we discussed before. If we were careful to include the center line—the diameter (which corresponds to the very central line or zero frequency of k-space)—then we would notice that the contrast in the image has not changed. If we look very carefully we might notice that the resolution in the vertical direction has not changed, but the resolution in the horizontal direction has changed and is now half what it was before we covered half the lens. (This makes sense too, since we have reduced the number of lines, or the bandwidth, in k-space.)

Now imagine (or go back to your camera and do it) that we remove the cover that is over the left half of the lens and place it over the *right* half. Nothing changed! The image is still half as bright, the entire image is still visible (not just the right half), contrast is preserved, and the resolution (half as good in the horizontal as in the vertical direction) has not changed. Either we did not do anything or the data in the left half of the lens plane (k-space) are the same as those in the right half. It is the latter, of course, because of the inherent symmetry of a lens (and of k-space).

The actual symmetry is a *point symmetry*, which means that points in k-space are symmetric about the central point of k-space—points in the lower left, for example, are symmetric with (ie, the same as) points in the upper right. However, regardless of the actual type of symmetry, the implication is that half the data are redundant. Once we have acquired half of the data—that is, filled half of k-space—we really do not have to acquire any more because we know what it will be!

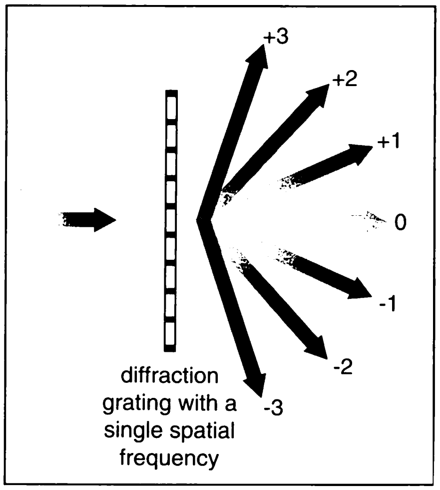
And that, of course, is just what half-Fourier (sometimes called half-NEX [number of excitations]) is. Only half of k-space is filled (so for a 128-line image only 64 acquisitions would be needed) and the computer would fill in the rest of k-space (taking data from the lower left and copying it to points at the upper right, and data from the upper left and copying it to the lower right and so on). We would have to take care to include the center line to ensure contrast was preserved. Once k-space was filled, the Fourier transform would be performed and the entire image, with full resolution, would be generated as if we had spent twice the time to fill all of k-space!

There is a cost for this dramatic improvement in speed (there is no free lunch), and the cost here is a reduction in signal-to-noise ratio. As I mentioned, image brightness halved. It turns out that when we cover half the lens the noise *power* also halves, with the result that the signal-to-noise ratio amplitude in the image is reduced by 40% (ie, $\sqrt{2}$). Other problems exist, as well, including those that have to do with the increased burden on the computer by making it copy the data, inhomogeneities in the magnetic field, and attenuation of the RF field, for example, but these are of less consequence (at least to us). The bottom line is that if there is enough signal-to-noise ratio to spare, then half-Fourier technique is an excellent way to reduce image acquisition time. If there is not enough to spare, then one could compromise—say 3/4-NEX—and accept more modest time savings with less reduction (in this case only 15%) in signal-to-noise ratio.

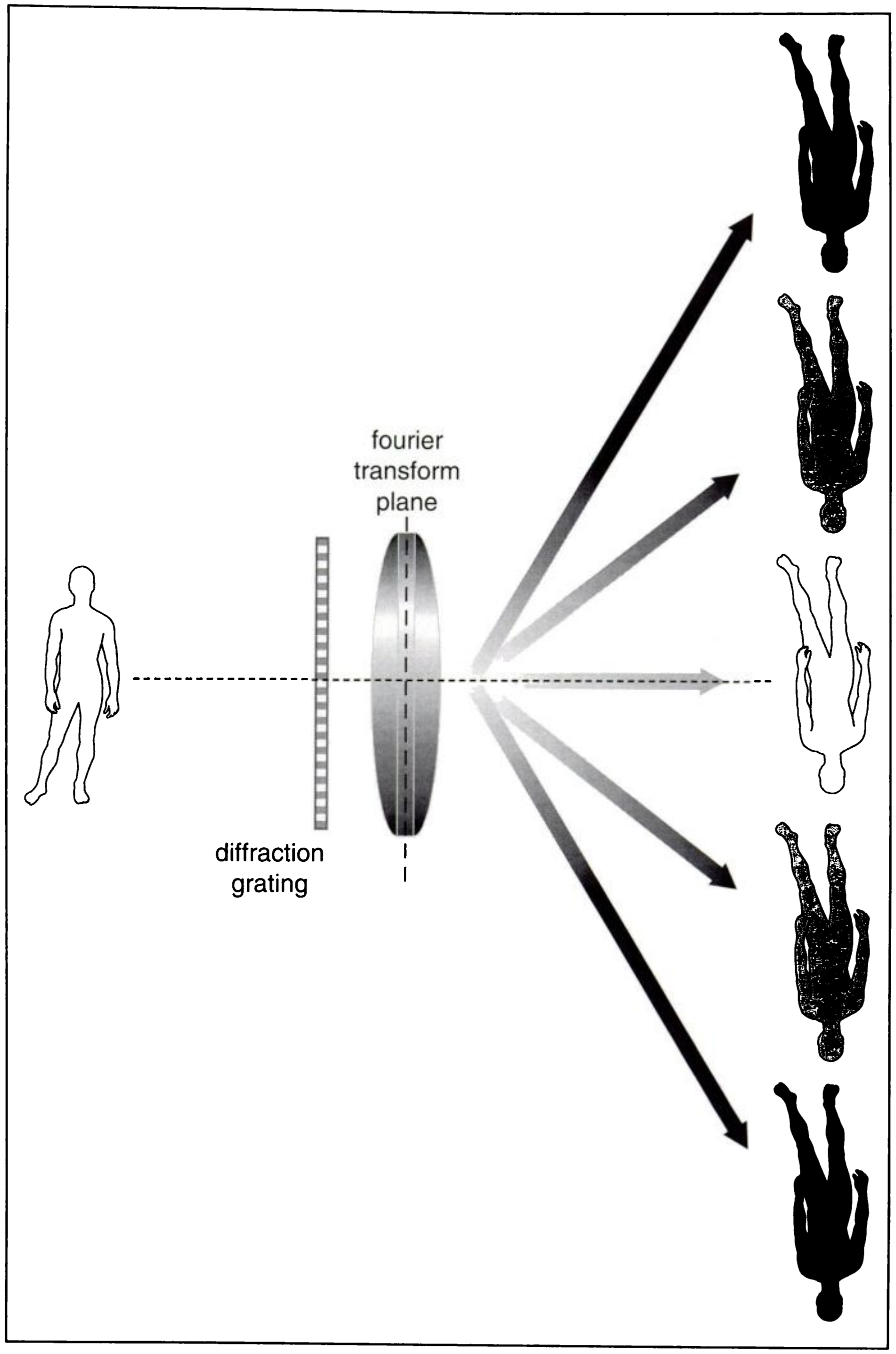
We can take the symmetry argument one step further by noting that if there is left-to-right symmetry, there must also be top-to-bottom symmetry. Thus, if we had performed the experiment by covering up first the top and then the bottom of the lens, we would have gotten the same results. In k-space, however, this would mean covering up (or more accurately, not acquiring) half the readout line. That is precisely what is done in the half-echo technique. Instead of acquiring the entire echo during the readout for a line in k-space, the acquisition is aborted halfway through and the acquisition for next line is started. All the considerations regarding image quality discussed above pertain, and the only cost to this technique is the same 40% loss in signal-to-noise ratio. The benefit is a reduction in TE, which can be important in reducing susceptibility artifacts. The more general term for this approach is partial-echo technique, when other than half the echo is acquired.

FAST SPIN-ECHO TECHNIQUE

The basic idea of this approach, first suggested by Hennig et al (11–13) and Mulkern et al (14) and subsequently modified by Melki and colleagues (15–18), is to fill more than one line of k-space at each 90° RF pulse. The pulse sequence (Fig 11) has a 90° pulse followed by many 180° pulses, and instead of the phase-encoding



a.



b.

Figure 9. **(a)** Optical grating shows multiple diffracted beams (orders). The angular spacing between each order is inversely proportional to the grating spatial frequency (ie, line spacing). In theory, an infinite number of orders are generated. **(b)** If a grating is placed at or near the Fourier transform plane, multiple images will be created, one for each diffraction order. As before, the separation between each image (order) is inversely proportional to the line spacing (spatial frequency) of the gratings. With small spaces between gratings (high spatial frequency), the images will be well separated and the higher orders can be ignored. The central image has the highest intensity and is the one displayed at the console.

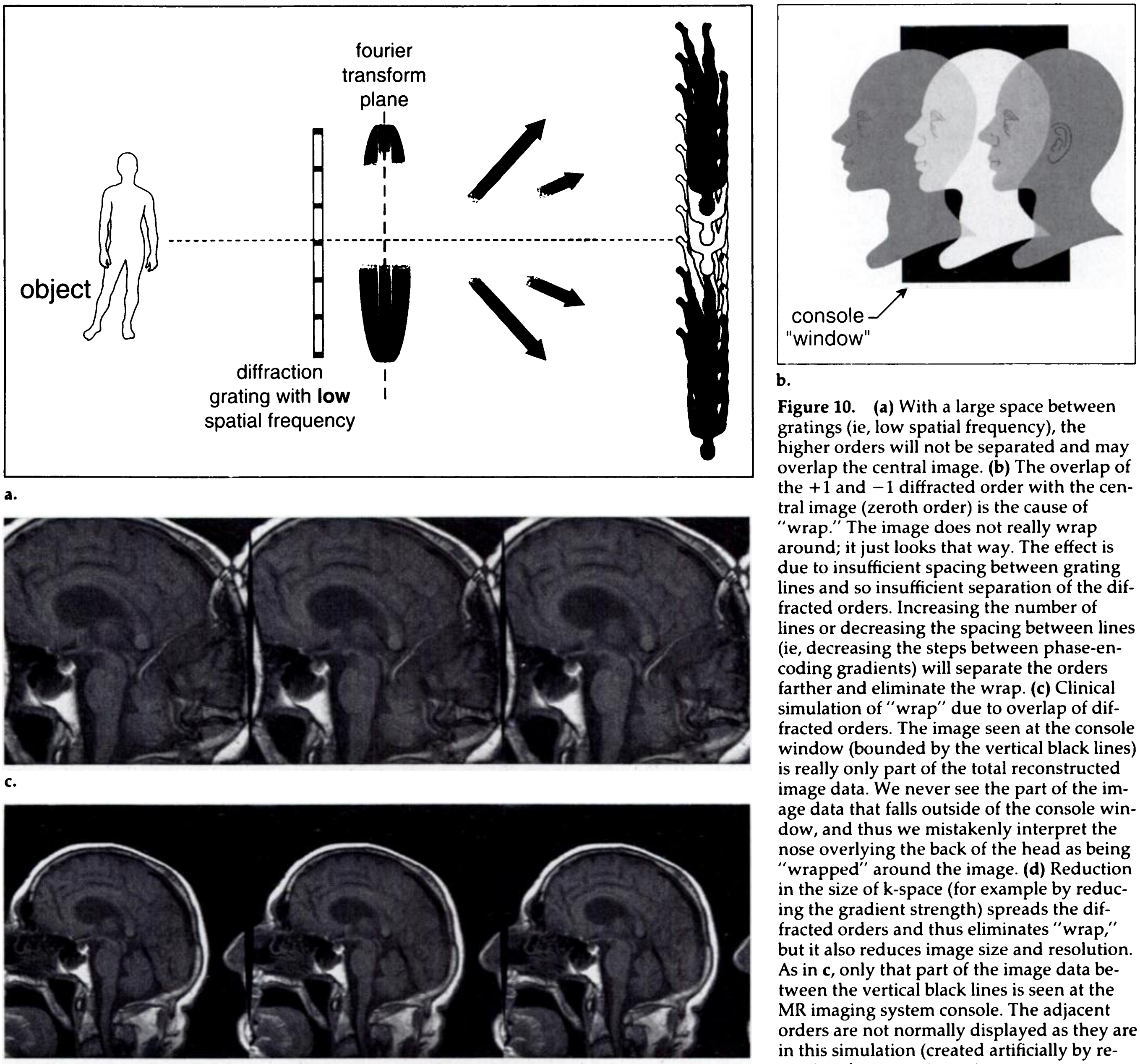
pulse being incremented merely once per 90° pulse, it is incremented with every 180° pulse. Since a line in k-space is created and filled with each phase-encoding pulse, this approach fills many lines in k-space with each 90° RF pulse. The savings in image acquisition time can be considerable. In the extreme case it is possible to fill all of k-space with one 90° pulse followed by 256 sequential 180° RF pulses, which would allow total image acquisition in just a few seconds.

The problem with such an approach is that there is T2 decay of excited spins. Each line of k-space will be smaller than the one acquired just before it by the ratio $e^{-\mathrm{TE}/\mathrm{T2}}$, with the amplitudes of each successive line in k-space decreasing in monotonic fashion until finally they vanish below the noise level.

This is very different from the situation in the classic spin-echo technique, in which every line of k-space (acquired at the same TE) has the same amplitude, and this difference has dramatic effects on image quality. The actual effect on image quality depends on exactly how we decide to fill k-space.

We remember that the desired contrast in an MR image depends on the number of protons in each tissue and on the intrinsic tissue parameters T1 and T2, but we also remember that we can modify that contrast by the values we choose for TE and TR. In the spin-echo pulse sequence, every line of k-space is acquired with the same TE and TR, and so any or all lines contribute equally to image contrast. In actual fact, as I have discussed above, most contributions to image contrast come from lines near the center of k-space, while data near the edges of k-space determine image resolution. The important point however, is that any data line, no matter when it is acquired in the spin-echo imaging sequence, could be placed anywhere in k-space without affecting image quality. In this new sequence, each line of data is different, and it matters very much where it is placed.

We could, for example, fill k-space in the manner of the classic spin-echo sequence—from one edge to the other—but then the edges (or at least one edge) would have high signal and the center would have relatively low signal intensity. In the case of a 256 echo train with a TE of 20 msec for the first echo, for a tissue with a T2



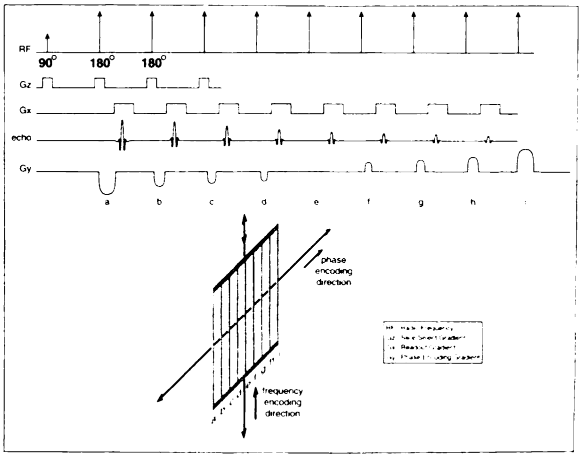
a. b. c. d.

Figure 10. **(a)** With a large space between gratings (ie, low spatial frequency), the higher orders will not be separated and may overlap the central image. **(b)** The overlap of the +1 and −1 diffracted order with the central image (zeroth order) is the cause of "wrap." The image does not really wrap around; it just looks that way. The effect is due to insufficient spacing between grating lines and so insufficient separation of the diffracted orders. Increasing the number of lines or decreasing the spacing between lines (ie, decreasing the steps between phase-encoding gradients) will separate the orders farther and eliminate the wrap. **(c)** Clinical simulation of "wrap" due to overlap of diffracted orders. The image seen at the console window (bounded by the vertical black lines) is really only part of the total reconstructed image data. We never see the part of the image data that falls outside of the console window, and thus we mistakenly interpret the nose overlying the back of the head as being "wrapped" around the image. **(d)** Reduction in the size of k-space (for example by reducing the gradient strength) spreads the diffracted orders and thus eliminates "wrap," but it also reduces image size and resolution. As in **c**, only that part of the image data between the vertical black lines is seen at the MR imaging system console. The adjacent orders are not normally displayed as they are in this simulation (created artificially by repeating the center image).

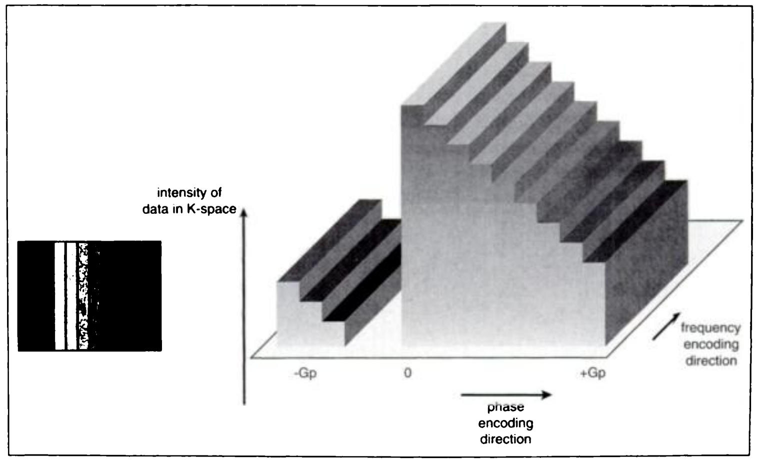
value of 50 msec (muscle for example), the lines at the center of k-space would be much more than 1,000 trillion times smaller (actually more than 10^{21} times smaller) than the value at the edge of k-space and would appear to have an effective TE of 2,560 msec (ie, 20 × 128). (By effective TE I mean that the contrast would be similar to that obtained with a conventional spin-echo sequence with a TE of 2,560 msec). This would be an exceptionally T2-weighted sequence indeed!

It would actually be more complicated than that. The effect of the high signal intensity at one side of k-space would be to create an edge-enhanced image, which is similar in some respects to my previous example of a camera or telescope with the center of the lens blocked (Fig 6b). However, the effect of the T2 decay would be to make that very edge-enhanced, very T2-weighted image practically invisible!

We could alternatively fill the center lines of k-space first (Fig 12). In that case, since for the most part it is the center of k-space that determines contrast, and since we are filling that center with data acquired with a short TE, the contrast would be similar to that obtained with the spin-echo sequence with a short TE and long TR (what some would call a "proton-density–weighted image"). While the contrast would be similar to that in a conventional image, however, the resolution would not. Since each successive line in k-space is less intense than the one before, as we move away from the center the lines of k-space get dimmer and dimmer until, like before, they vanish below the noise. The point where they vanish becomes the effective size of k-space, and this can be very small indeed. Take for example the case in which the T2 of the material being imaged is 100 msec and the TE is 20 msec. After only 15 lines the intensity would be just about at the point where it equals the noise, and all successive lines would effectively vanish below that noise. (Note that 15 × 20 = 300 msec, which equals 3 × T2. To a good engi-



11.



12.

Figures 11, 12. **(11)** Multiecho imaging (RARE). In this sequence the 90° excitation pulse is followed by many 180° echoes that form pulses. The data at each successive echo are used to fill a line in k-space. The strength of each phase-encoding gradient (one is placed after each 180° RF pulse) determines data line position in k-space. The mechanism for k-space filling is identical to that of the classic spin-echo sequence, but in the RARE technique multiple (up to 256) lines are filled with each 90° RF excitation pulse, whereas in classic spin-echo imaging only one line is filled with each 90° RF excitation pulse. The RARE technique can be up to 256 times faster then the classic spin-echo sequence. **(12)** RARE problems. Sequential lines in k-space acquired after each successive echo have successively decreased amplitudes. This is very different from the classic spin-echo situation, in which every line in k-space (acquired at the same TE) has the same intensity. The effect is to decrease the effective width of k-space (with effective width limited to the number of lines with intensity greater than background noise). Image contrast is variable and depends mainly on where each line is placed. The insert at the left is the view of k-spaces seen en face, with the gray scale proportional to the intensity of each line of k-space. The larger central view shows in perspective the variable amplitudes of the k-space lines used in the RARE technique. −Gp indicates negative and +Gp indicates negative spatial frequencies along the phase-encoding direction.

neering approximation, the signal is effectively zero for times greater than 3 × T2.) The effect on the resolution would be dramatic, similar to that of a lens with a very narrow aperture (only 15 of 256 lines wide, or less than 6% open). The resolution will be very poor, and the image will be quite blurry (at least along the phase-encoding direction).

This imaging technique is unique in that the material being imaged determines the resolution. If we had chosen a material with a longer T2, the number of lines in k-space that would have been filled before the signal effectively vanished would have been greater and the resolution would have been better. The result is that the resolution is fairly good for long T2 materials (eg, cerebrospinal fluid) and quite poor for short T2 materials (eg, muscle).

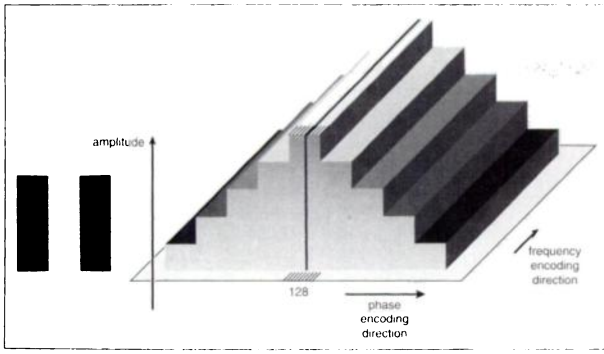
One very nice feature, however, is that the contrast in the image (ie, the amount of T2 weighting) is controllable by the user. Sequential filling from the center of k-space gives a short TE, long TR type image. Sequential filling from one edge of k-space gives a very long TE, long TR type image. Filling from intermediate positions can yield images with intermediate contrast. Again, if we assume a TE of 20 msec for the first echo and a k-space that is 256 lines wide, and if we were to start filling k-space at line 123 (five lines before the center), the contrast would approximate that of an image acquired with a TE of 100 msec. Simple movement of the line from which the start of k-space is filled, a process that Hennig et al called scrolling (11), allows the user to choose any arbitrary T2 weighting in the image without changing any timing parameters.

One important problem with this technique, at least in the case in which k-space is filled from some place near the center, is the large asymmetric discontinuity it creates in the k-space intensity pattern. The situation is reminiscent of a lens with a big black gouge in it, and in fact, just like that lens, this pattern creates artifacts (consisting of streaking, ghosts, and a strong background haze) that degrade the image.

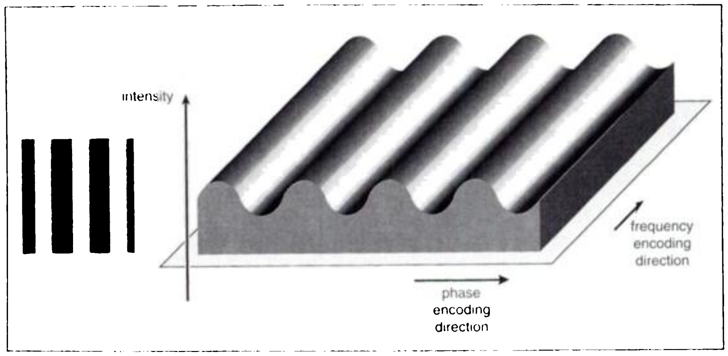
Melki and colleagues (15,16) described two modifications to this technique that overcame the problems with resolution and image artifact.

The first was to take a less aggressive stance on the number of lines filled with each 90° pulse (which they called a "shot"). Recognizing that for many materials, collection of more than 16 or so lines with each shot was not useful (either because of loss of contrast or loss of resolution), they proposed a limit on the number of lines (four to 16) filled at each shot and then use of several such sequences (64 to 16) to fill k-space. Consider the case in which 16 lines are filled at each shot (which they would call an echo train length of 16), and assume a TE of 20 msec (which they called the echo spacing). Sixteen shots would be needed to fill 256 lines of k-space, and each shot would acquire data with an echo at 20 msec, 40 msec, 60 msec, 80 msec,. . ., 260 msec, 280 msec, 300 msec, and 320 msec. They would then collect and rearrange the data into 16 lines of k-space with a TE of 20 msec, 16 lines with a TE of 40 msec, another 16 lines with a TE of 60 msec, and so on until a final grouping of 16 lines with a TE of 320 msec. If we assume a TR of 2 seconds (2,000 msec), the total image acquisition time would be a quite respectable 32 seconds (ie, 16 shots at 2 seconds per shot).

The second modification was to depart from the sequential approach to filling k-space and use instead a more symmetric arrangement. Thus, for example, in the case of the 16 shots used above, data from the first acquisitions from each shot would be placed in the 16 lines at and near the center of k-space (in the case of a k-space that is 256 lines wide, the first line of data from the first shot would be put at line number 128 [which would be done by acquiring that piece of data with no phase-encoding pulse applied]), the first line of data from the second shot would be put at line number 129 (done by acquiring that piece of data with a small positive phase-encoding gradient—for



13.



14.

Figures 13, 14. **(13)** Fast spin-echo technique. K-space is filled with data acquired with several short echo trains rather than the single long train of the RARE technique. In this example, we imagine that 16 echo trains (also called shots), each containing eight echoes, are used. We show only the portion of k-space filled by the first five echoes of each train. The center portion of k-space is filled by the first echo of each train. There will be 16 lines near the center of k-space, each acquired with the same TE. The second echo of each train will be placed next, with eight to one side of the center span and eight to the other side. These will all have been acquired with twice the TE of the first echo, and so the group of eight lines to each side will have a lower amplitude than the group of 16 lines in the center. The third echo of each train, each acquired at 3 × TE (eg, at 60 msec if TE was 20 msec), will be used to fill the next 16 lines, symmetrically placed eight to each side as before. This group will have a still lower amplitude because of the longer TE. Data from successive echoes will fill k-space in a similar fashion, which will form the pyramid structure of decreasing intensity illustrated here. The inset shows the intensity of the lines of k-space seen en face. **(14)** Intensity modulation of k-space due to periodic motion in the image. If the motion is slow relative to the duration of the frequency-encoding pulse (which is usually the case) and is repetitive, then it will impart some type of periodic modulation in the phase-encoding direction. It may take the form of amplitude modulation (as shown here) or it could take the form of phase modulation (which would be difficult to illustrate), but the effect would be the same: the creation of ghosts. Note that the number or waves or "ripples" is proportional to the ratio of TR (the pulse repetition period) to the period of the disturbance, as discussed in the text.

example, +1 G), the first line of data from the third shot would be put at line number 127 (accomplished by acquiring it with a small negative phase-encoding pulse—for example, −1 G), and so on until lines 120–136 were filled. Each of these first lines would have been acquired with a TE of 20 msec. The second lines from each shot (those with a TE of 40 msec) would be placed symmetrically about the center, grouped with eight to one side (lines 137–144) and eight to the other side (lines 112–119). Proper placement is accomplished by using appropriate phase-encoding gradients during acquisition—for example, +4 G for the data to be placed at line 137, −4 G for the data to be placed at line 119, +5 G for the data to be placed at line 138, and so on. The process continues in this symmetric fashion until all of k-space is filled (Fig 13). The large discontinuities that occur when k-space is filled sequentially are averted, and attendant artifacts are reduced.

As with the original rapid acquisition with relaxation enhancement (RARE) technique, the lines in k-space can be scrolled to simulate any desired T2 weighting.

The use of limited echo train lengths (between four and 16) and symmetric filling of k-space removes many of the limitations of the original RARE technique, with the result that image quality approaches that of the classic spin-echo technique but with much greater speed. Even so, some limitations persist that have caused some to question its use in some applications.

First, because of symmetric filling, the intensity of data at the edges of k-space is decreased compared with that in the center. Such shading is a commonly used "special effect" in photographic imagery, where it is called apodization and serves to create a "softer" image with slightly reduced resolution and smoother edges. (It is sometimes used by professionals in the motion picture industry, for example, for photographing older actors and actresses, because it can reduce the prominence of wrinkles.)

Second, the decrease or change in signal intensity from grouping to grouping (ie, from the group with a TE of 20 msec to the groups with a TE of 40 msec) is not smooth but rather is stepwise with finite discontinuities. Although not as severe as in the initial sequential arrangement of the RARE technique, with the single large "gouge," these discontinuities cause haze and artifacts in the image, much like scratches in a lens (or even streaks in a car's windshield) might. The regularity of the defects causes "ghost" images (the situation is exactly like the example of the grating discussed earlier, except the grating lines are farther apart and weaker, so the higher order are closer together and dimmer) that can sometimes be seen as dim overlapping images (3) but that more often manifest as a faint haze over the image.

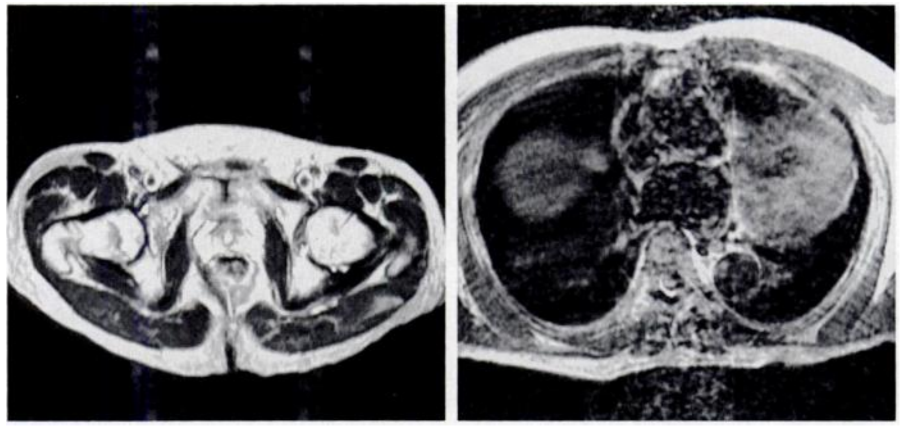
Both of these effects serve to lessen the contrast, sometimes reducing the conspicuity of lesions. Debate continues over the importance of these and other effects on clinical utility and on the best areas of applicability of this approach to fast image acquisition (2,19).

MOTION AND RESPIRATORY COMPENSATION

Rearrangement of the order of lines in k-space can also be used to minimize some motion artifacts (20–26).

Imagine MR image acquisition of a slowly, periodically moving object such as a chest or abdomen with respiratory motion. Each line of k-space will be affected by that motion to a greater or lesser amount, depending on the degree and rate of motion as that line is acquired. (I will not dwell on the nature of that effect, for it is not important to us now. What is important is that there is some effect and that it is proportional to the motion.) In the case in which the spin-echo pulse sequence is used, with each line of k-space acquired at a regular rate (say five lines per second, or a TR of 200 msec) and with k-space filled in order from one end to the other, k-space will be modulated by the effect of the periodic motion. Figure 14 schematically illustrates the situation of respiratory motion at a rate of 12 per minute, where I have assumed, for purposes of illustration, that the motion simply affects the amplitude of each line of k-space. In this case, k-space looks somewhat like the corrugated surface of a washboard. The important feature, for our purposes, is the periodicity of the corrugations.

Just like the case of a grid (or stocking) placed just in front of our eyes, or a grating placed at the lens (or Fourier) plane of a camera, the periodic corrugations act like a diffraction grating and create multiple images (higher-order diffractions) that are copies of the original images. Since the corrugations have a lower periodicity than each line of k-space, the higher orders overlap the original image (they are not scattered out of the FOV) and appear as ghosts of the original image. In the case in which only part of the image moves, such as the beating aorta or femoral artery, multiple copies will be seen above and below the true image of the vessel (diffracted along the phase-encoding direction). In the case in which the whole object moves, such as with respiratory motion, ghosts of the entire object will be seen diffracted above and below the true image (Fig 15).



a. b.

Figure 15. **(a)** Widely spaced ghosts due to relatively rapid femoral artery pulsations and **(b)** "fine" thoracic wall ghosts due to slow respiration. The relatively rapid repetitive motion of the femoral artery causes a "fine" modulation in k-space, which leads to large diffraction angles of the higher order (and so wide separation of the ghosts). The slower repetitions of respiratory motion lead to widely spaced modulation in k-space and thus close spacing (less diffraction) of the ghosts. The major determinant of the spacing and strength of the "ghosts" is the ratio of TR to the periodicity of the moving structure.

The crucial factor that gives rise to the ghosts is the periodicity of the modulation of k-space, which makes it act like an optical grating. If the periodicity could be reduced, the ghosts would vanish. The periodicity is proportional to the ratio of TR to the period of the motion, a relationship that can be at least intuitively appreciated by inspection of Figure 14. It is that ratio that determines the number of cycles of the "corrugation," and it is the "corrugation" or grating spacing that determines how widely the diffraction orders are scattered, as well as how intense they are. The number of "corrugations" can be calculated and is actually proportional to $(TR \times NSA)/T$, where NSA is the number of signals averaged and T is the period of the motion. With more cycles (achieved by either increasing TR, increasing NSA, or decreasing T), the orders ("ghosts") are diffracted farther away and will disappear (be diffracted out of our FOV) when the periodicity of the "corrugation" equals TR (or when $TR \times NSA = T$). This condition is called "pseudogating."

There is a better, more reliable way to minimize the effect of motion.

Bailes et al (21) proposed the reordering of lines of k-space to remove that periodicity. Instead of just filling lines of k-space at a regular rate from one end to the other, they would choose where to place the lines depending on the motion at the time a line was being acquired. Thus, for example, if the motion was strong and negative (say end expiration), they would put that line at the bottom of k-space. If the motion was strong and positive while a line was acquired, that line would be put at the top of k-space. Lines acquired while the motion was moderate would be put near the middle of k-space. This rearrangement would transform the periodic modulation of k-space into a slow monotonic modulation (Fig 16), which would completely eliminate the possibility of higher-order ghost images. Since all the spatial frequencies of the original image are still acquired and used for reconstruction, the original image quality is still maintained; only the ghosts are lost. This technique, which was one of the first demonstrations of rearranging lines of k-space, demonstrates the enormous power and flexibility of k-space manipulation that is unique to MR imaging.

SEGMENTED K-SPACE IMAGING

Rearrangement of lines in k-space is an adequate way to compensate for slow, regular respiratory motion, but it does not work well for irregular or complex motion (eg, irregular breathing, sighing) or for the very fast motion seen with cardiac activity. The problem is that even if the lines are rearranged, all of k-space must still be filled. If the sequence being used has a relatively long TR, then it will take a relatively long time to fill k-space ($N \times TR$, where N is the number of lines in k-space). With long image acquisition times the probability that patient motion remains nicely periodic decreases, and so does image quality. One could use sequences with a very short TR (<7 msec, for example, as used in the fast low-angle shot technique)(27), which would enable filling a 128-line k-space in less than 1 second; this approach, however, would still be too slow for cardiac imaging and can give relatively poor contrast even for stationary or slowly moving structures. The limited contrast is due to the fact that, by itself, a sequence with very short TR has little T1 or T2 weighting. Preparatory pulses that control contrast (eg, a 180° inversion pulse for T1 contrast or a driven equilibrium sequence for T2 contrast) can be added just before the short TR sequence, but because of T1 and T2 decay these lose their effect even with acquisition times as short as 1 second. Rearrangement of lines in k-space so that the early acquired lines are put near the center of k-space helps maintain contrast, but then relaxation decay decreases the amplitude at the edges of k-space, which compromises resolution.

Atkinson and Edelman (28), Chien et al (29), and Edelman et al (30) proposed a means of overcoming some of these problems that involved use of a method that did not fill k-space all at once but rather filled it a few lines at a time. If, for example, we filled eight lines at a time, then it would take 16 acquisitions (each acquiring eight lines) to fill a 128-line k-space. We have many choices as to where to put these lines, but the simplest is simply to fill every 16th line with each acquisition. Thus (as illustrated in Fig 17),

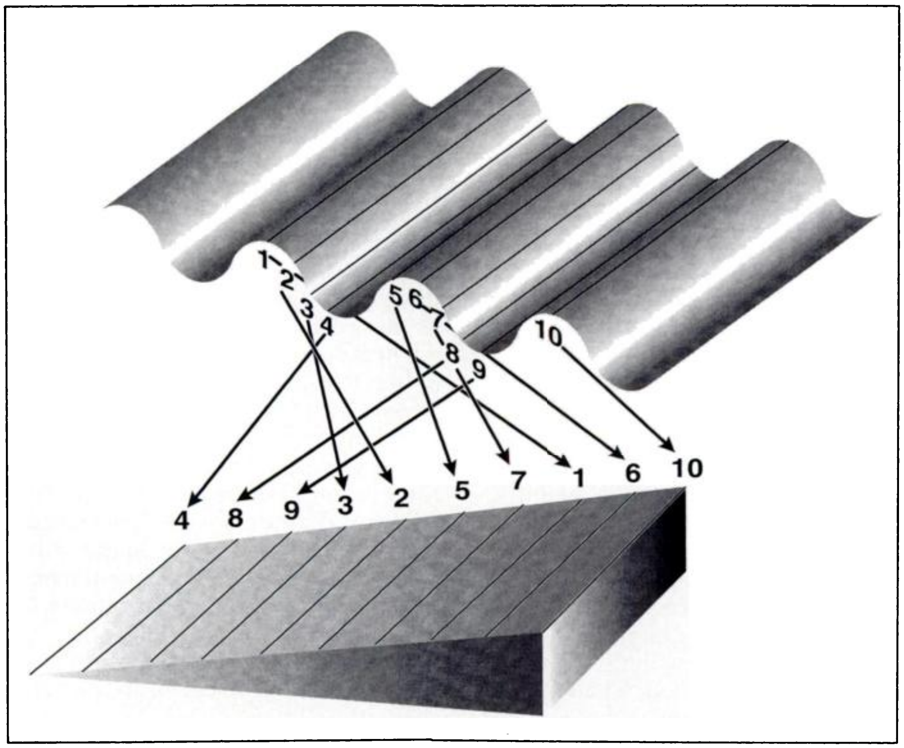


Figure 16. Respiratory-ordered phase encoding. K-space is modulated by the respiratory motion. The lines in k-space are rearranged so that lines where the modulation is "negative" (eg, lines 4, 8, and 9) are placed to the left of k-space and lines where the modulation is "positive" (eg, lines 1, 6, and 10) are placed to the right of k-space. Intermediate lines (eg, lines 3, 2, 5, and 7) are put in intermediate positions. The effect is to replace the periodically modulated k-space with one that has a linear modulation, which eliminates the ghosts. The linear modulation will have minimal effect on the image (at most it will cause slight edge enhancement that in practice will usually not be noticeable).

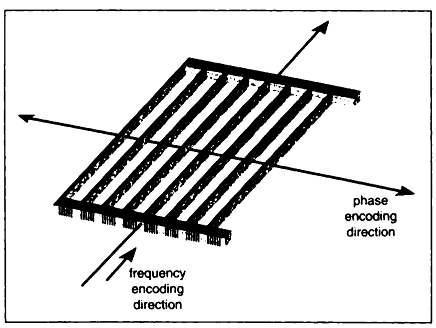


Figure 17. Segmented k-space. In this illustration, k-space is filled eight lines at a time, and all eight lines are acquired nearly simultaneously (typically in just 50 msec or so). The lines are spaced to span k-space. It would take 16 such "eight at a time" acquisitions to completely fill a 128-line k-space. The small vertical lines on the "front" of k-space indicate the positions of the next six acquisitions. In a typical cardiac application, each group of eight would be acquired once per heart beat, at the same relative time after the QRS complex, so that a complete image could be acquired in just 16 heart beats.

lines 1, 17, 33, 49, 65, 81, 97, and 113 would be filled first, then lines 2, 18, 34, 50, 66, 82, 98, and 114 would be filled with the second acquisition, and so on until with the 16th acquisition, all the lines of k-space would be filled. A particularly useful application of this technique is cardiac imaging, in which each acquisition is synchronized to the cardiac rhythm (ie, triggered by the electrocardiogram so that each acquisition occurs once per heart beat, at the same point in the cardiac cycle). K-space would be filled in just 16 heart beats, and with a TR of 6.2 msec, each acquisition would take just 50 msec (8 × 6.2). The acquisition time is short enough to "stop" heart motion, and the total time required to fill all of k-space (16 heart beats) is short enough that most people can hold their breath for the entire sequence. As with fast low-angle shot imaging, preparatory pulses are needed to set image contrast (eg, a 180° RF pulse for T1 contrast), but this pulse is delivered with each eight-line acquisition, and in the very short time required for each acquisition there is no remarkable T1 or T2 decay.

This technique completely eliminates all motion artifact while preserving image quality and shows such exquisite detail in the heart that it is even possible to acquire remarkably good images of the coronary arteries (31,32).

KEYHOLE IMAGING

While at present it is not widely used, "keyhole imaging" (33) is a clever technique for rapid imaging and nicely summarizes some of the methods I have been talking about. The technique was designed for cine-type imaging, in which images are acquired in rapid sequence in order to illustrate dynamic changes of some structures in the image field. The usual problem with such rapid imaging is achievement of high speed, good contrast, and high resolution. To achieve high resolution one needs a large k-space, but it takes time to fill k-space with many lines. Decreasing the size of k-space decreases the time required for the study but also decreases the resolution. Use of very short TRs can decrease the time required but may also decrease the contrast. One approach that I have just discussed is the segmenting of k-space—filling all of k-space but only part at a time. Another, the keyhole technique, fills *all* of k-space once, but then updates only a small, central part of k-space to create the cine acquisition.

The method takes advantage of two phenomena, one physical and the other psychological. First, the method takes advantage of the fact that a very rapidly changing structure will nearly always be blurred to some extent unless acquisition is performed in zero time, which is effectively impossible. There will always be some compromise between resolution and speed. Second, when we look at a field with a stationary background and a changing foreground, it is not actually necessary to update all the background to appreciate the changes. Further, if the background has high resolution and overall contrast is maintained, the eye will usually not notice that the changing structures have lower resolution. Thus, if a detailed image is obtained initially (perhaps before the change starts) by filling all of k-space, and if subsequently only a relatively few lines in the center of k-space are acquired at a rapid rate and used to replace just the previously acquired central lines, the changes will be seen and the entire image will appear to the viewer to have high resolution. By adjusting just how many lines near the center of k-space are acquired with the rapid sequence, the operator can adjust the compromise between speed and true resolution.

The changes I have been referring to can either be changes in contrast, as would be seen after gadolinium or other contrast agent was injected, or be changes in position, such as might be seen with images of a moving joint. Most applications to date have involved imaging changes following admininstration of contrast material (34–36).

SPIRAL IMAGING

All the methods I have described so far utilize a rectangular k-space that is filled one line at a time. This is a direct consequence of the fact that the phase- and frequency-encoding gradients are separated in time. First, the phase-encoding gradient is pulsed on to set a spatial frequency (a line in k-space) and then the frequency-encoding pulse comes on to fill that line. There is no law of nature, however, that specifies it must be this way. All that nature (or at least the MR radiologist) requires is that k-space be filled. No one really cares too much about exactly how it is filled, for once it is filled we will just go ahead and perform a two-dimensional Fourier transform on k-space and ignore the details about how the data got there. The pulse sequence designer has considerable freedom in choosing when and how to apply the encoding gradients, and some interesting things happen if one decides to let the phase- and frequency-encoding gradients occur at the same time.

Imagine first that the two gradients were just turned on at the same time and for the same duration. In that case we would again fill a line in k-space, but that line would be at 45° and would cut a diagonal through k-space (with every increment in the frequency-encoding direction there would be an equal increment in the phase-encoding direction tracing a line that bisects the two axes, ie, at 45°). This would be interesting, and we could indeed fill k-space with a collection of 45° lines, but it would be complicated—they would have to have to be of unequal length for one thing, and there would be no particular advantage.

Imagine next that we apply gradients with sinusoidal amplitudes. (Nowhere is it written that the gradients must have constant amplitudes. They can just as well have amplitudes that vary with time, and the most generic varying amplitude is the sine wave.) In that case, it turns out, we will fill a circle in k-space, and the diameter of the circle will be proportional to the amplitude of the gradients, if we assume the amplitudes are equal (if they are not equal, then we will actually fill an ellipse). By successively varying the amplitudes, we would fill concentric rings of k-space (ie, a bulls-eye pattern). This would work and be more interesting than filling k-space with a collection of lines at some angle, but it too would be complicated (for example, the duration of the gradients would have to vary as we filled circles farther from the center) and would not offer many advantages.

Imagine now (as did Macovsky [37], Meyer et al [38], and Shenberg and Macovski [39]) that we again use gradients with sinusoidal amplitudes, but this time we let the amplitudes increase with time (Fig 18). This time we will fill k-space with a spiral pattern, starting at the center (both gradients small) and spiraling farther and farther away as the amplitudes continuously increase. This actually does have some advantages, particularly for very fast imaging. For one thing we could, if we wished, fill all of k-space in one spiral (by using gradient-echo pulses with very short TRs and TEs). More important, however, is the fact that even if we fill k-space quickly, little strain would be put on the circuitry generating the gradients, for instead of changing from one value to another quickly as they might in a line at a time scheme, they need only ramp up gently as the sine waves gradually build in amplitude. This is no small advantage and has stimulated considerable interest in the application of the spiral imaging approach to cardiac, neurofunctional, and other high-speed MR imaging techniques.

THREE-DIMENSIONAL FOURIER IMAGING

There is one important difference between optical imaging and MR imaging that I have ignored throughout this discussion: Optical systems image a plane, whereas MR imaging systems can, and do, image a volume. I have avoided that difference by focusing attention onto one plane or section in the three-dimensional volume and by purposely disregarding how that section came to be selected. It is now time to address, briefly, how sections are selected.

In the so-called two-dimensional Fourier transform technique, a section is selected by applying the RF excitation pulse (a 90° pulse in the case of spin-echo imaging, a smaller flip angle pulse in the case of gradient-echo imaging) in the presence of a gradient—the section-select gradient. This gradient (a small magnetic field that is larger on one side or along one axis of the volume than the other) changes the resonant (ie, precessing) frequency of the spins along that axis in direct proportion to the strength of the gradient—higher at one side than the other. The RF excitation pulse, which is applied to the entire volume, will excite only one plane that is oriented perpendicular to the gradient direction. The location of the section (ie, how close to the center or an edge) is determined by the frequency of that RF pulse. The width of the selected (or excited) section is proportional to the gradient strength and the bandwidth of the RF pulse. Narrow sections require narrow bandwidths and large gradients. Limitations on the narrowness of the bandwidth in practical systems, imposed in part by the desire for faster acquisitions, put a premium on the ability of a system to deliver large gradients. Once a section is selected, all the discussions on image quality and k-space that we have just gone through apply.

In the three-dimensional Fourier transform technique, the RF excitation pulse is again applied in the presence of a gradient, but the gradient strength and RF bandwidth are chosen to excite a thick slab (the slab can be as thick as the patient or smaller). The slab is subdivided into individual sections by applying phase-encoding gradients in the section-select direction. Just as the phase-encoding gradients described above were applied to fill lines in a two-dimensional k-space, these new phase-encoding gradients are applied to fill *partitions* in a three-dimensional k-space. K-space now is a "cube" made up of lines in three dimensions: one set along the frequency-encoding direction and the other two along orthogonal phase-encoding directions (which we can call x and y). Image space is also a cube, and image data are along three orthogonal directions (ie, height, width, and depth). Just as with the two-dimensional case, the image space is reconstructed by means of Fourier transform, but now a three-dimensional Fourier transform is used. This simply means that three separate Fourier transforms are performed, one after the other, and each works on the data transformed by the one before (except that the first one operates on the raw data). The first two transforms construct the data in a section, just as before, with the same considerations regarding resolution, quality, artifacts, and so on. The third transform creates the sections (to fill

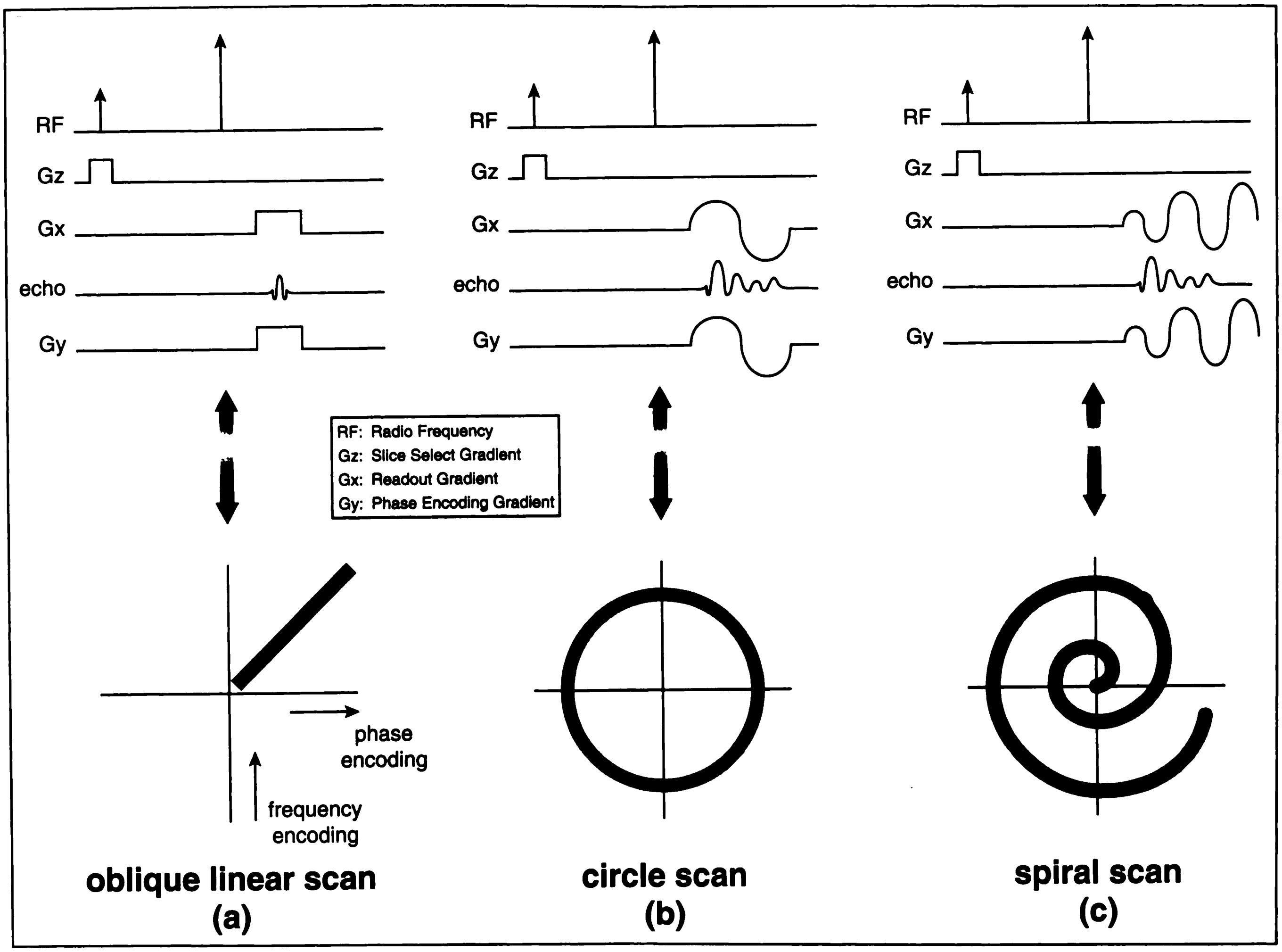


Figure 18. Simultaneous application of phase- and frequency-encoding gradients. *a,* When the phase- and frequency-encoding gradients are constant and applied simultaneously, data are placed along an oblique line through k-space. If both gradients are equal, the line is at 45° to the x axis. *b,* When both gradients have an equal but sinusoidal amplitude, data will be placed along a circle in k-space. A change in the amplitudes of the gradients produce a change in the radius of the circle. If the amplitudes are not equal, the path in k-space will be elliptical. *c,* Sinusoidal gradients with linearly increasing amplitudes will result in a spiral trajectory through k-space.

out the "depth" of the image space), and it obeys the same criteria that I discussed for the first two transforms. Some of the consequences are as follows:

1. The number of sections is equal to the number of phase-encoding pulses (partitions) in the section-select direction, just as the number of lines in the image is equal to the number of phase-encoding pulses as described previously in my initial discussions of k-space.

2. The thickness of each section is equal to the slab thickness divided by the number of partitions, just as the thickness of each line in the image is equal to the image size divided by the number of lines (or phase-encoding pulses). By appropriate choice of slab thickness and number of partitions, it is possible to arrange that the dimensions of each image element (now called a voxel) are the same in all directions. This special case of equal image resolution in all directions is called *isotropic* imaging (and so by comparison all other cases are called anisotropic imaging).

3. Each section is contiguous, just as each line in the image is contiguous.

4. Aliasing artifacts will be created if the number of partitions is insufficient to move the higher orders (now in the section-select direction) away from the image volume. These artifacts affect primarily the end sections (just as the aliasing artifacts, or "wrap," primarily affect the edges of the image), and some manufacturers do not even bother to reconstruct those end sections (they show, for example, only 28 sections when 32 partitions were acquired).

It is interesting to note that the computer that performs the three-dimensional Fourier transform does not take into account sections and lines, and in fact the data can be reconstructed and presented (sliced and diced) in any orientation; the choices can be made to fit the viewing habits of the viewer and are ideally optimized to fit the orientation of the anatomy.

Finally, it is worth repeating that regardless of whether two-dimensional or three-dimensional techniques are used, the considerations of image quality and resolution in any image plane are those that I have discussed before.

CONCLUSION

A major strength of MR imaging lies in the ability to manipulate k-space. Unlike operators of any other medical imaging modality, the MR imaging operator can manipulate data to control contrast, resolution, FOV, and imaging time. By simply changing the order with which k-space is filled, one can control artifacts or trade off contrast and resolution. By shaping k-space, the operator can tailor the imaging sequence to the structure being imaged (for example, by using a rectangular FOV with different resolutions in x and y to image the thoracic spine) to optimize both speed and resolution. K-space can be so easily cut and rearranged to fashion an image that it the operator is almost like a tailor, cutting cloth to create a new suit.

As designers and radiologists gain experience with the methods and power of k-space manipulation, the capabilities and applications of MR imaging are being extended at an ever-increasing pace. There is no reason to expect that an ultimate limit has been reached. The real limits now seem to be the limits of the imagination of those who build and use the systems.

The purpose of this review is to serve as an aid to that imagination and to give structure and body to what may sometimes be abstract notions. If we remember the close relationship between a lens and k-space and then consider what might happen in a simple camera, we may be able to give form to a hazy notion and help ease the path from thought to deed.

APPENDIX: HOW K-SPACE GOT ITS NAME

Consider a collection of hydrogen atoms, such as might be found in a bucket of water or a piece of fat, that happened to be placed in an MR imaging system. The MR signal that would emanate from that collection in response to a pulse excitation (eg, by a 90° RF pulse) would be given by the equation $\mathbf{S} = e^{-t/\mathrm{T2}}\cos(\gamma Bt)$, where t = time, T2 is the spin-spin relaxation time, γ is the gyromagnetic ratio, and B is the magnetic field. If we note that $\mathbf{S}$ is a real function and if we will consider only the real part of the following equation, this can be written in more general terms as

$$\mathbf{S} = e^{-t/\mathrm{T2}}e^{-j2\pi(\gamma Bt)}.$$

When a gradient (G) is applied, the magnetic field can be written as $B = B_0 + \mathbf{G} \cdot \mathbf{x}$, where use of bold characters indicates that the variable is a vector with components along orthogonal projections, for example the x, y, and z directions. In general, $\mathbf{G}$ can be time varying, but I will assume, for the purposes of this discussion, that it is constant (as is the case with much of MR imaging—spiral imaging being one notable exception). We can then write

$$\mathbf{S} = e^{-t/\mathrm{T2}}e^{-j2\pi(\gamma B_0 t)}e^{-j2\pi(\gamma \mathbf{G}t\cdot\mathbf{x})}.$$

Now, in the more interesting case in which we image a person and not just a bucket of fat or water, the magnetization will vary from point to point in the field, in part because of differences in the number of protons and in part because of differences in the relaxation times. In that case, the equation for the total MR signal becomes

$$\mathbf{S} = \int\int\int e^{-t/\mathrm{T2}}e^{-j2\pi(\gamma B_0 t)} \cdot \mathrm{f}(\mathbf{x})e^{-j2\pi(\gamma \mathbf{G}t\cdot\mathbf{x})}\,\mathrm{d}\mathbf{x},$$

where $\mathrm{f}(\mathbf{x})$ is the spatially varying magnetization, $\mathrm{d}\mathbf{x}$ is a small (differential) volume element in the patient, and $\int\int\int$ indicates integration (summation) over the volume of the patient.

The term $e^{-t/\mathrm{T2}}e^{-j2\pi(\gamma B_0 t)}$ does not vary with location (ie, $\mathbf{x}$) and so is constant over the patient. It can be replaced, for purposes of this discussion, by a constant, say A, placed in front of the integral signs. The general equation for the MR signal emanating from a patient after an RF pulse excitation becomes (5)

$$\mathbf{S} = A\int\int\int \mathrm{f}(\mathbf{x})e^{-j2\pi(\gamma \mathbf{G}t\cdot\mathbf{x})}\,\mathrm{d}\mathbf{x}.$$

The equation can be made even more compact by defining the relationship $\mathbf{k} = \gamma\mathbf{G}t$, so that the general equation now becomes

$$\mathbf{S} = A\int\int\int \mathrm{f}(\mathbf{x})e^{-j2\pi(\mathbf{k}\cdot\mathbf{x})}\,\mathrm{d}\mathbf{x}.$$

This equation is identical in form and function to the equation that defines the Fourier transform of a function $\mathrm{f}(\mathbf{x})$. To illustrate, examine the usual form of the Fourier transform of some function of time $\mathrm{f}(t)$. This equation, which is very common in electrical engineering work (40), is

$$\mathrm{F}(\omega) = \int \mathrm{f}(t)e^{-j2\pi\omega t}\,\mathrm{d}t,$$

where ω indicates frequency and t indicates time. Except for the fact that time (t) and frequency (ω) in this equation are one-dimensional scalars, while $\mathbf{k}$ and $\mathbf{x}$ in the previous equation are three-dimensional vectors, the equations are identical. By comparison then, $\mathbf{S}$ represents the Fourier transform of the spatial magnetization in the patient $\mathrm{f}(\mathbf{x})$ and is a function of spatial frequency, and $\mathbf{k}$ can be understood to describe that spatial frequency.

The vector $\mathbf{k}$ has three components, which for the case of constant gradients can be defined as $\mathbf{k}_x = \gamma\mathbf{G}_x t$, $\mathbf{k}_y = \gamma\mathbf{G}_y t$, and $\mathbf{k}_z = \gamma\mathbf{G}_z t$. These three components are in fact the coordinates that define the domain of $\mathbf{S}$, or, as we usually call it, the k-space. ■

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