THE UNIVERSITY OF BRITISH COLUMBIA

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Mini Project 1

University of British Columbia

Electrical and Computer Engineering

ELEC 301

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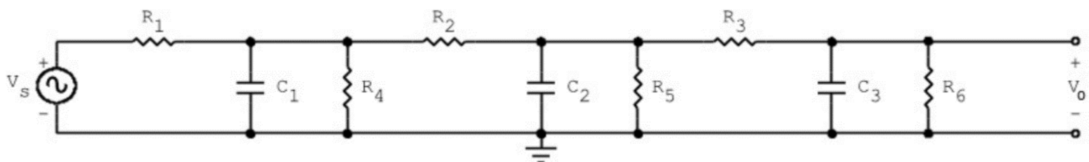
Contents

Part 1

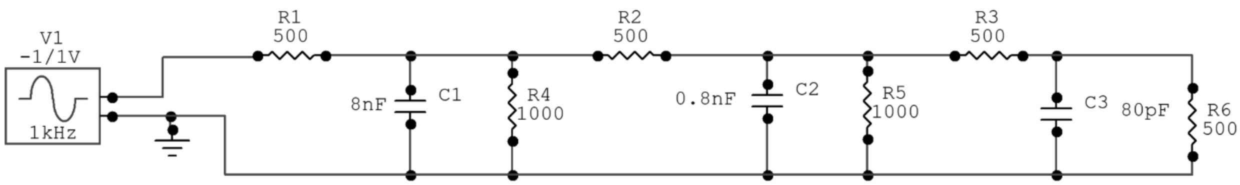
Our circuit and related transfer function in question is shown below:

$$T(s) = \frac{V_o(s)}{V_s(s)} = 0.125 \times \frac{5\times10^5/\text{sec}}{s + 5\times10^5/\text{sec}} \times \frac{5\times10^6/\text{sec}}{s + 5\times10^6/\text{sec}} \times \frac{5\times10^7/\text{sec}}{s + 5\times10^7/\text{sec}}$$



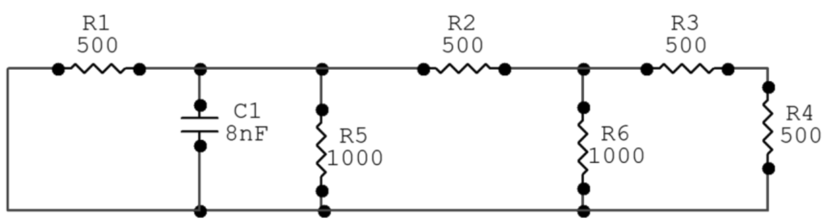
Part A:

Using four 500Ω and two 1000Ω resistors, a low pass circuit shown below was designed from the transfer function above.



In this circuit, we know that $C_1 > C_2 > C_3$, and $\omega_{h1} = 5*10^5$/sec, $\omega_{h2} = 5*10^6$/sec, $\omega_{h3} = 5*10^7$/sec from the transfer function shown above. From the mid-band gain, we know the values of each of the resistors, by inspection. From those values, we can calculate the values of the capacitors above from setting up Short Circuit and Open Circuit time constant equations.

For Capacitor C_1, we can create the circuit to solve for the value by opening the other capacitors. As shown below:



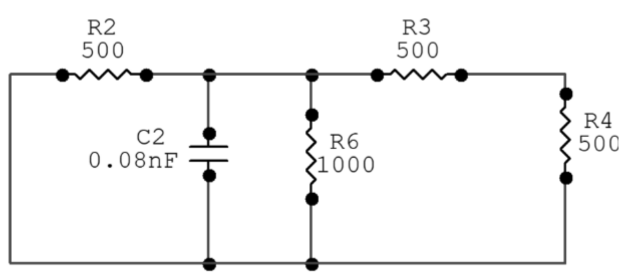
Calculation of Capacitances

$$((((R_3 + R_6)||R_5) + R_2)||R_4)||R_1 = 250\Omega$$

From this, we can calculate C_1:

$$\frac{1}{RC} = \omega \text{ to } \frac{1}{\omega R} = C = 8nF$$

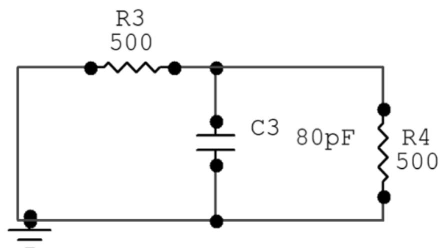
The calculation for C_2 is a similar calculation, however we short C_1 and open C_3, and the equivalent circuit is shown as:



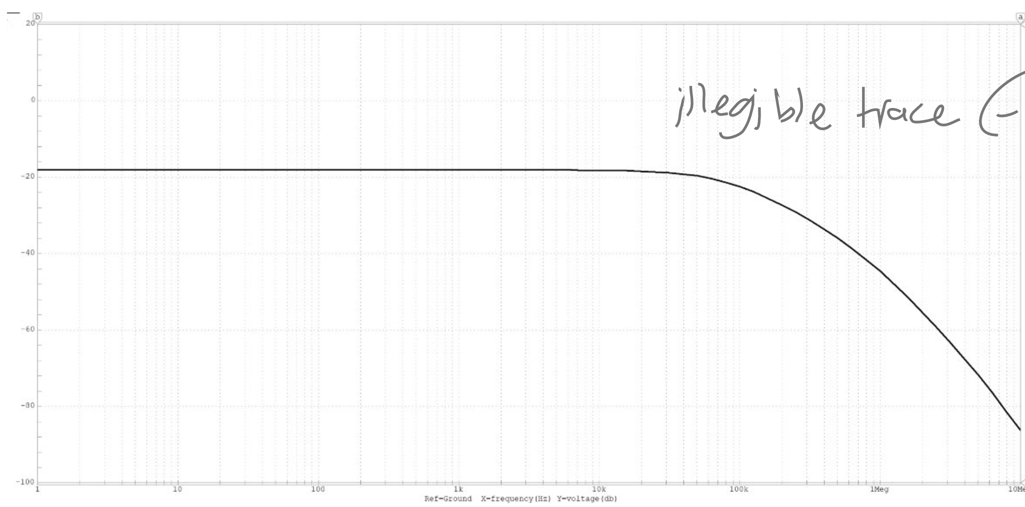
The calculation for the resistance seen by C_1:

$$R_2||((R_3 + R_4)||R_6) = 250\Omega$$

From the same calculation for C_1, we would evaluate C_2=0.08nF.
For C_3, we would calculate the resistance seen by shorting C_1 and C_2. Similarly, the circuit would be:



The equivalent resistance seen by C_3 would be $R_3||R_4 = 250\Omega$. The capacitance calculation follows the previous calculations and evaluates to C_3=80pF.
The Bode Magnitude plot and Phase plot of the above circuit is shown below:



illegible trace (-2)

Figure 1.1 (Magnitude Plot)

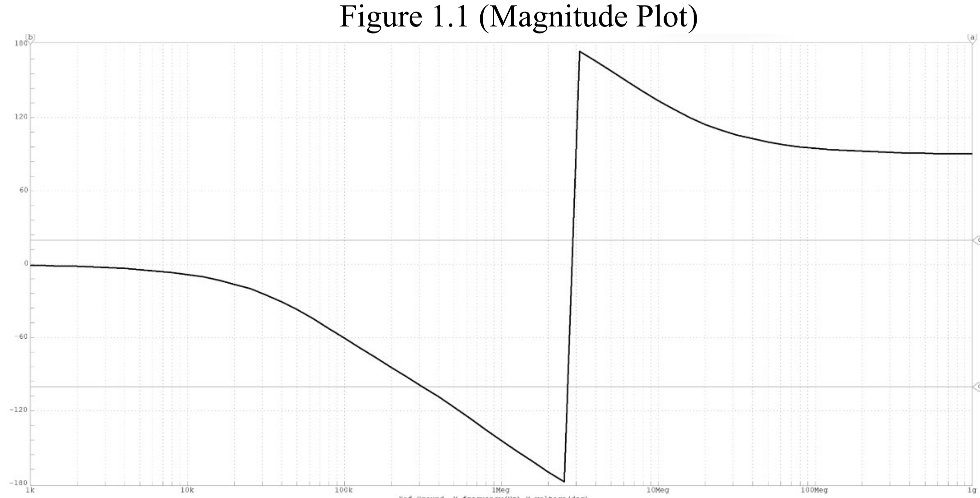
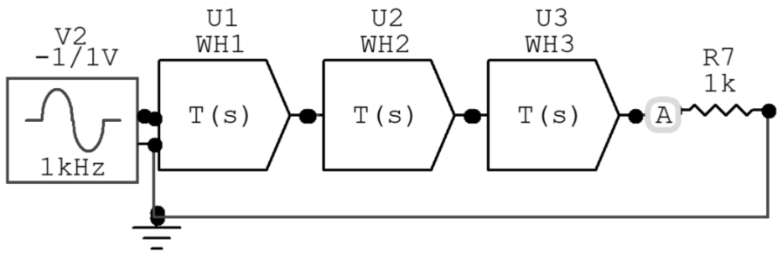


Figure 1.2 (Phase Plot)

Part B

Finding the Bode plots of Transfer function



The bode plot was found from the test point A, as marked in the circuit. Each of the transfer blocks was each of the high frequency pole transfer function cascading onto each other to form the general transfer function for the circuit. The mid-band gain was added to the first transfer block. The plots are as shown below:

illegible trace (-2)

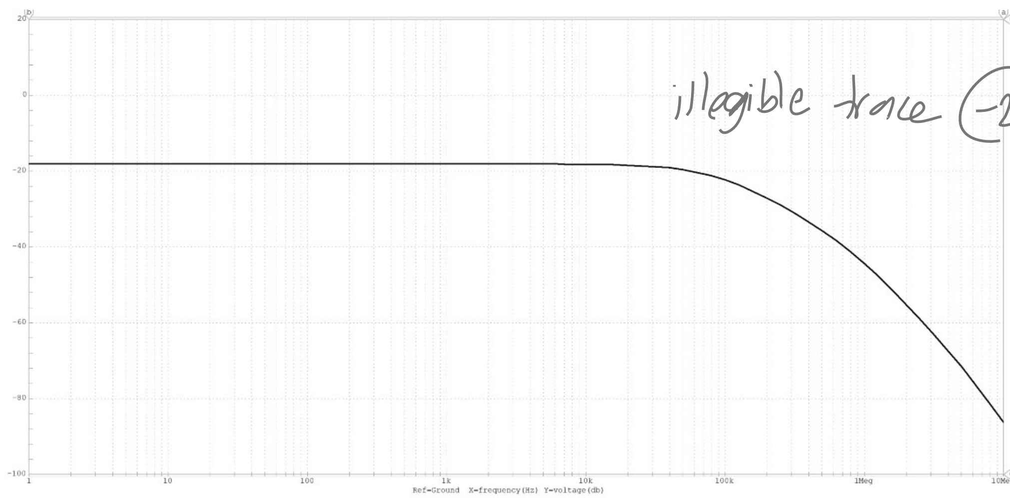


Figure 1.3 (Magnitude Plot)

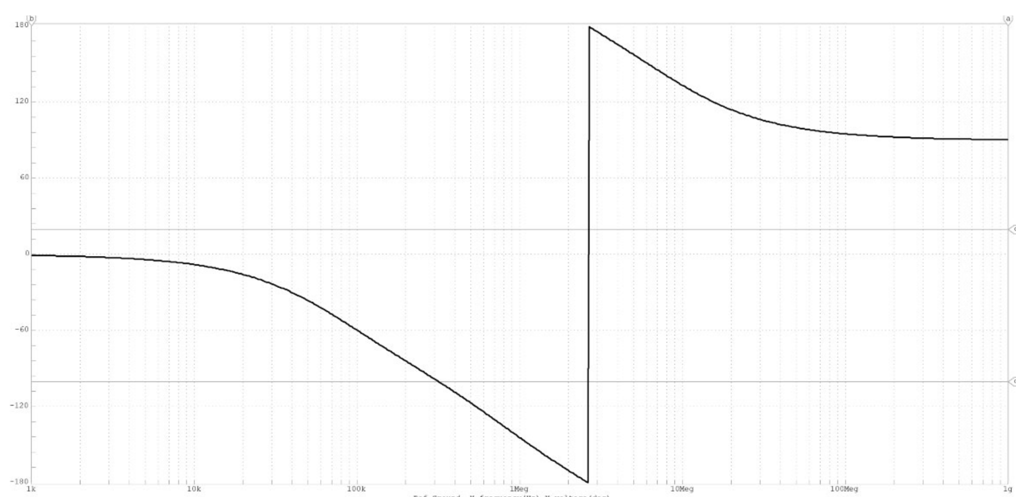


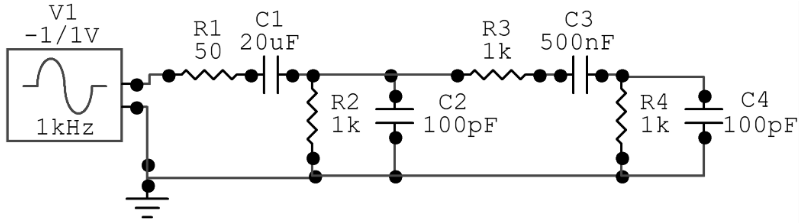
Figure 1.4 (Phase Plot)

From inspecting the ideal plot and the simulated plot, we can state that both are very similar to each other and are almost the same.

Part II

Part A

The circuit shown below was created with values given, and simulated by CircuitMaker:



Approximation of Pole values

To find the approximations for the pole values, we sketch on the graph and estimate where they are located. We do this by estimating where the slope changes.

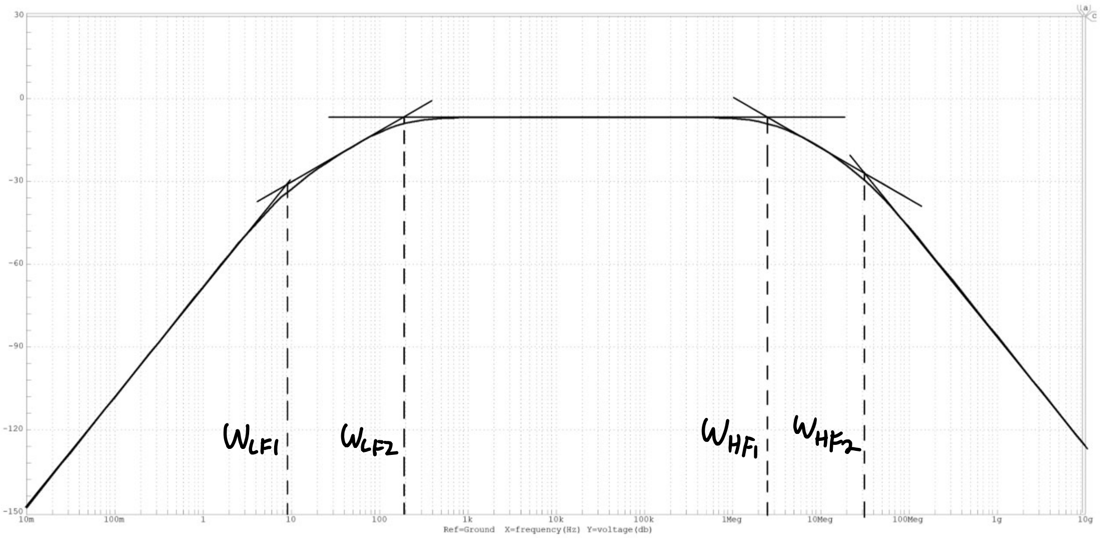


Figure 2.1 (Magnitude Plot)

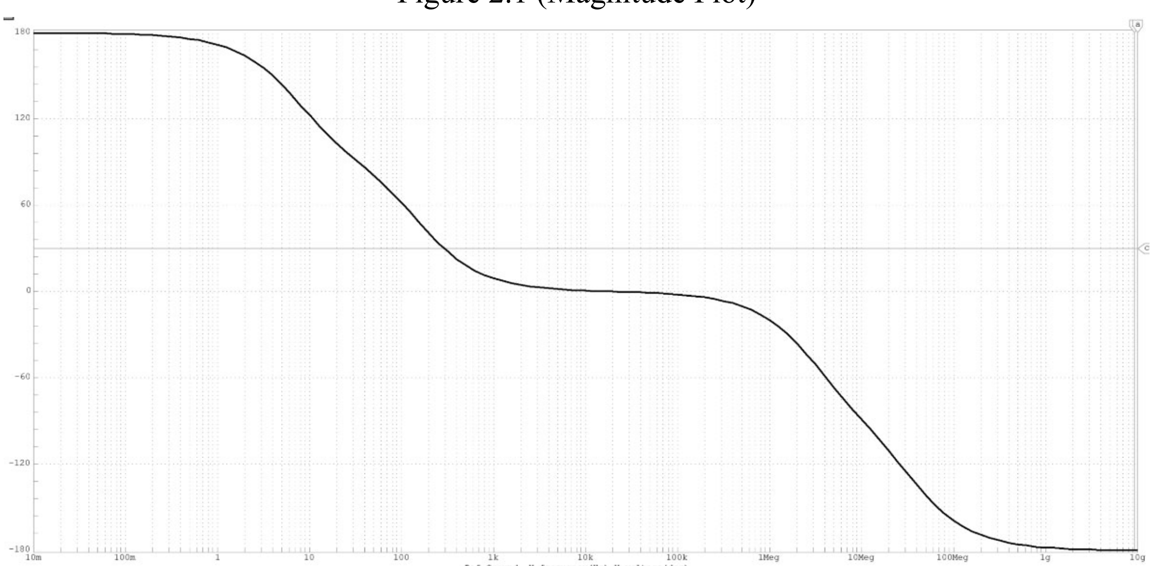


Figure 2.2 (Phase Plot)

The values are found with CircuitMaker cursor tools to find the approximate values of each pole frequency. An example of which is shown below:

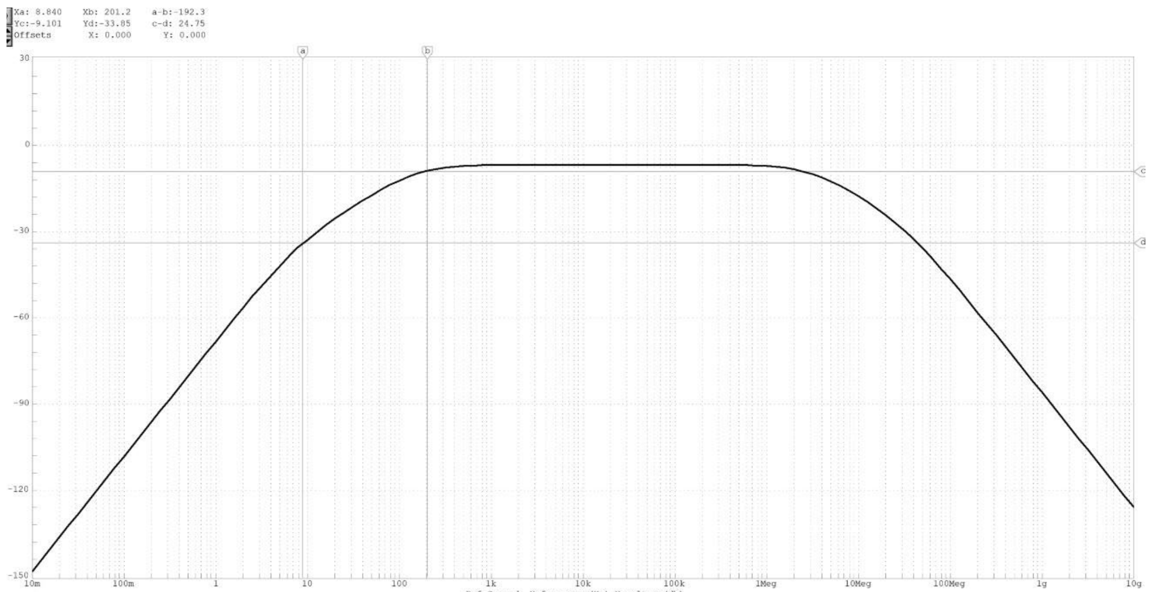


Figure 2.3 (ω Approximations)

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According to the approximation, ω_{LF1} = 8.840*2π rad/s, and ω_{LF2} = 201.2*2π rad/s. Where the cursor 'a' and 'b' were ω_{LF1} and ω_{LF2} respectively. The approximations for ω_{HF1} and ω_{HF2} is shown below:

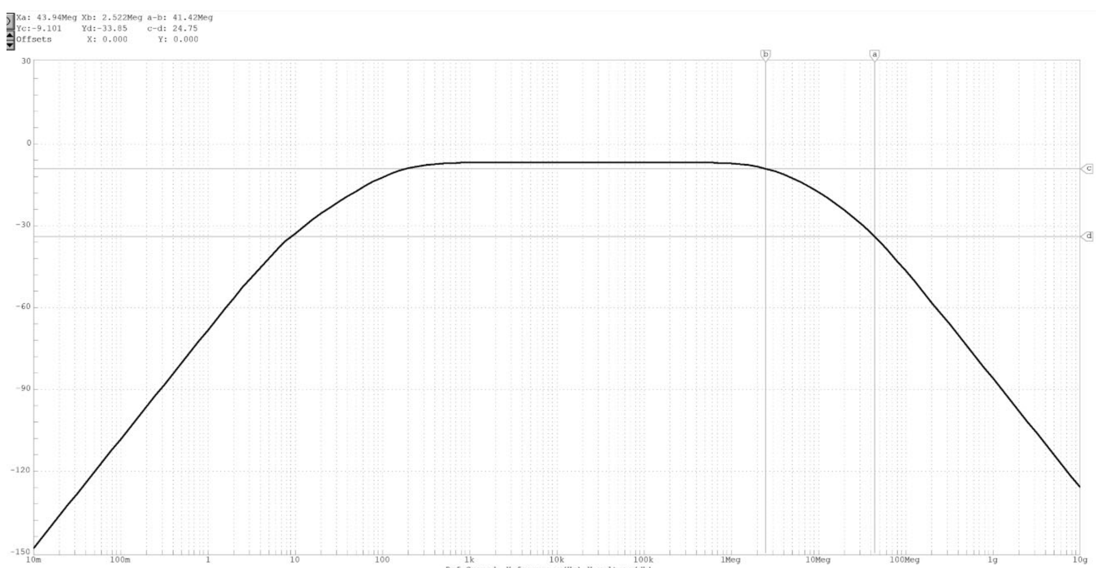


Figure 2.4 (ω Approximations)

31M -1

As shown, the values approximate to ω_{HF1} = 2.522*(2π)*10^6 rad/s, and ω_{HF2}= 43.94*(2π)*10^6 rad/s. Where the cursor 'b' and 'a' were ω_{HF1} and ω_{HF2} respectively.

Approximation of ω3dB

To find the values of ω_{L3dB}, we find the value of the frequency at a point where it is 3 decibels lower than the mid-band gain. The graphical approximation is shown below:

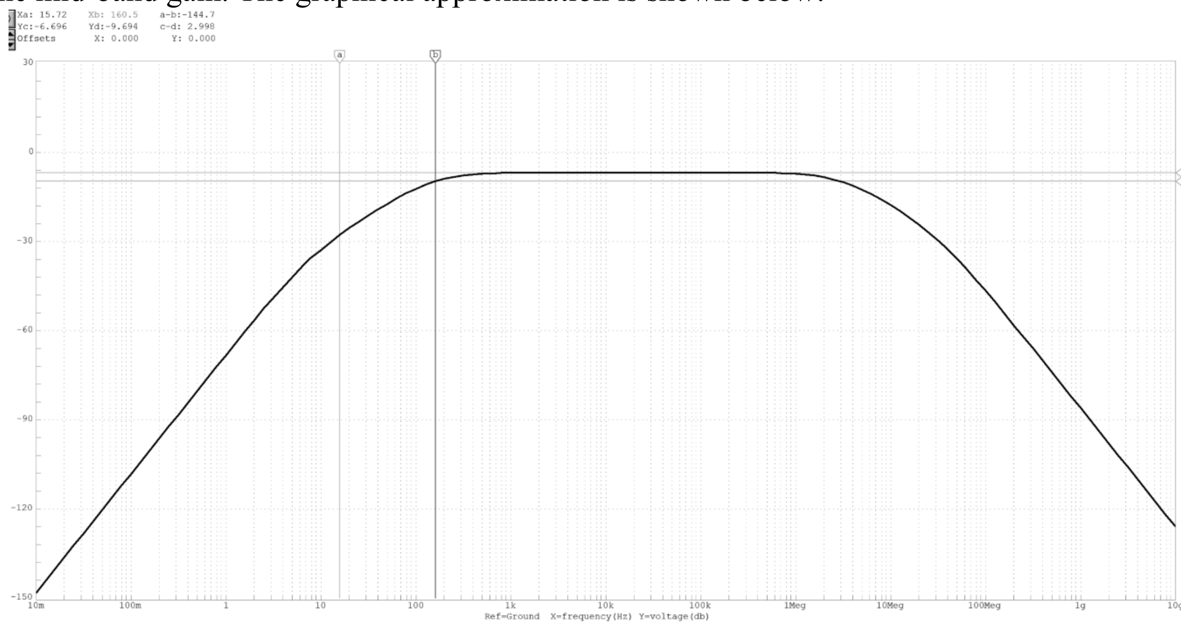


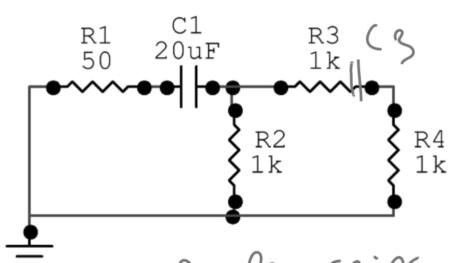
Figure 2.6

From the cursors, we can find that the value of ω_{L3dB} is 160.5*2π rad/s.

Part B

Calculation for ω_{LF1}, ω_{LF2}

These calculations are for C_1= 500nF. To find the values of ω_{LF1}, ω_{LF2}, analytically, we can set up SCT and OCT equations. To find ω_{HF1}, we find the resistance seen by C_1, and from knowing that $\frac{1}{RC} = \omega$, we can find the value. The calculation for the first low frequency pole would follow similarly to the calculations in the previous part. The circuit needed for such calculation can be seen below:



R1 + R2 C3:OC 50 rad/s (-2)

The resistance seen by C_1 evaluates to $(\cancel{R_3 + R_4}||R_2)+R_1 = 716.67\Omega$, and $\omega_{LF1} = 69.77\ rad/s$

The second low frequency pole would follow the same calculation, and eventually evaluates to

$$\omega_{LF2} = 976.74\ rad/s$$

After finding the low frequency poles, we can find the calculated ω_{L3dB} using this formula:

$$\sum_{i=1}^{N} \frac{1}{R_i C_{is}} = \omega_{LF1} + \omega_{LF2} = 1046.51 \text{ rad/s.}$$

To find the percentage error we use:

$$\%error = \left| \frac{\omega_{L3dB(calculated)} - \omega_{L3dB(measured)}}{\omega_{L3dB(measured)}} \right| * 100$$

Which evaluates to:

$$\left| \frac{1046.512 - 1008.451}{1008.451} \right| * 100 = 3.77\%$$

Calculations for C_3 = 1uF, 2uF,5uF,10uF

Here are the Magnitude plots of all the different capacitance values (Plots from left to right are 10uF,5uF,2uF,1uF):

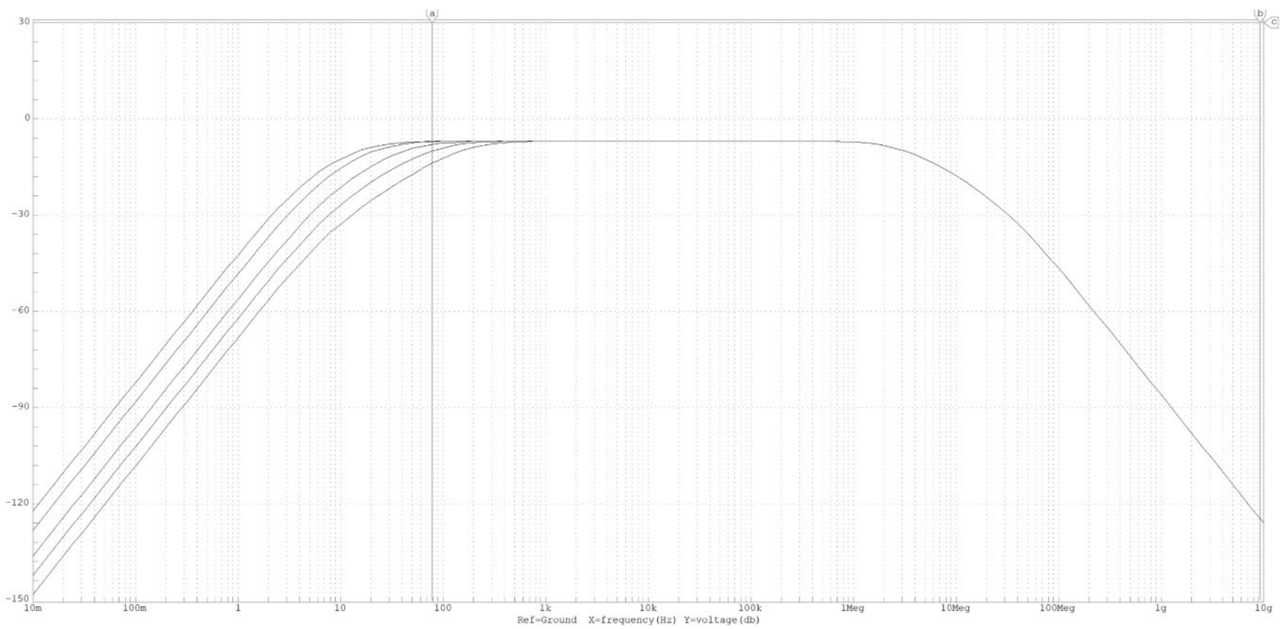


Figure 2.7 (Magnitude Plot)

The calculations for each ω_{L3dB} follows the same calculation as shown previously. The values are evaluated as follows:

	$\omega_{L3dB(calculated)}[\frac{rad}{s}]$	$\omega_{L3dB(measured)}[\frac{rad}{s}]$	% Error
1uF	558.139 490	522.196	6.883
2uF	313.953 248	276.083	13.717
5uF	167.442 108	140.115	19.503
10uF	118.605 68	98.772	20.079

-3

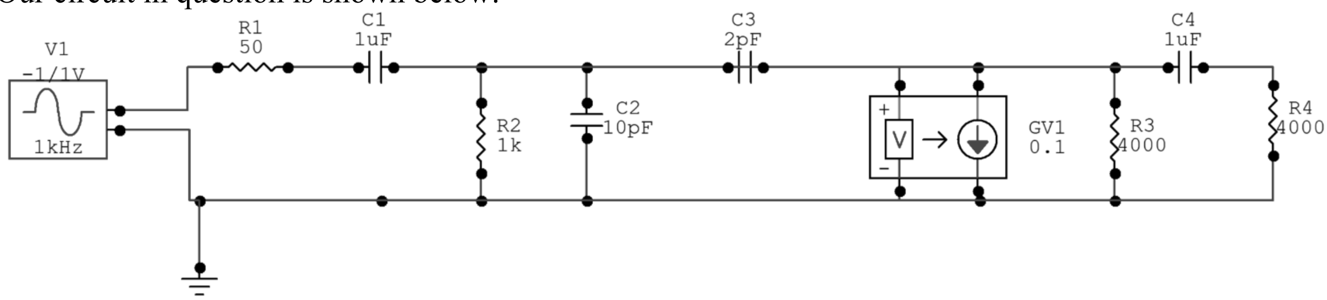
Figure 2.8 (Table of Measured and Calculated Frequencies with percent error)

As the capacitances increase, we can find a correlating increase in error. This is because as we increase the capacitances, they would affect the w_{3dB} values. This is because when we increase the value of the capacitance, it would change the impedance of that capacitor. Doing this will affect where the capacitor filters the signal.

Part 3

Part A

Our circuit in question is shown below:

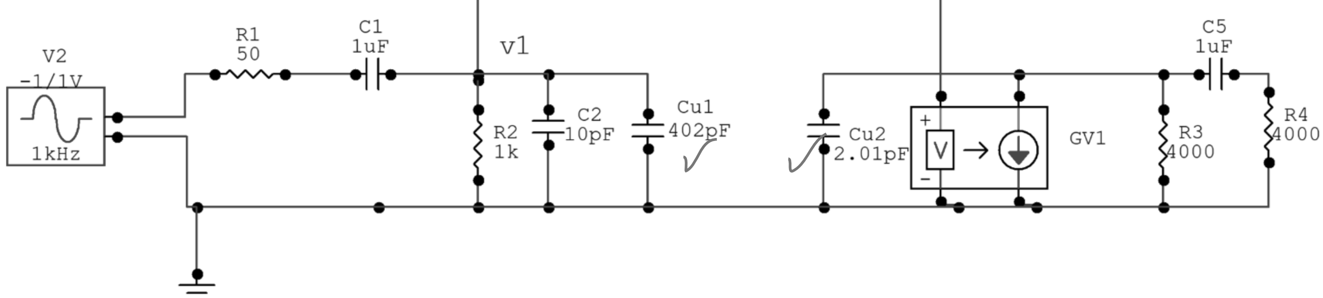


Miller's Theorem Analysis:

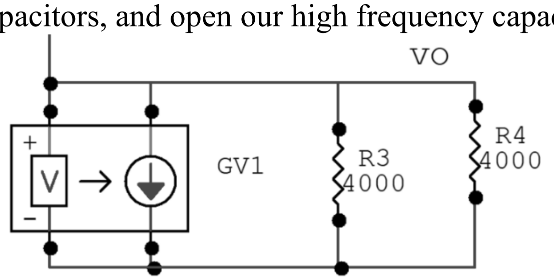
In the case of this analysis, we perform miller's theorem on capacitor c_3

To perform Miller's theorem, we first find the miller gain *k*, and the equivalent miller capacitances with these formulas and the following circuit:

$$k = \frac{v_o}{v_1} \qquad c_{\mu 1} = c(1-k) \qquad c_{\mu 2} = c(1-\frac{1}{k})$$



To find *k* we first find the expression for v_o in terms of v_1. To do this, we look only at the output stage and short our low frequency capacitors, and open our high frequency capacitors:



With this circuit, we first find the current across R_4 (I_o). We can do this using a current divider:

$$I_o = -Gv_1\frac{R_3}{R_4+R_3}$$

Then, we can find the voltage across the resistor using:

$$R_4(I_o) = (-0.1)(v_1)(\tfrac{4000}{4000+4000}) = -200v_1$$

Therefore:

$$k = \frac{200v_1}{v_1} = -200$$

With this information, we can find $c_{\mu1}$ and $c_{\mu2}$ with the formulas stated previously. They evaluate to:

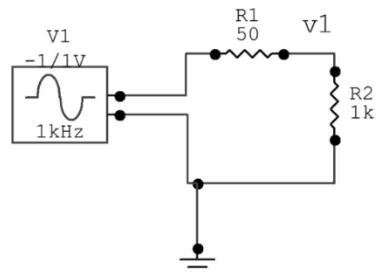
$$c_{\mu1} = c_3(1-k) = 402pF \qquad c_{\mu2} = c_3(1-\frac{1}{k}) = 2.01pF$$

Mid-band Gain Calculation:

With all our components found, we can now calculate our mid-band gain. We can use this equation for the mid-band gain:

$$A_m = \frac{v_1}{v_s}\frac{v_o}{v_1}$$

We need to find an equation for $v_1$in to find our mid-band gain. However, to find it, we also need use SCT and OCT equations. In this case, we open the capacitors in parallel and short the capacitors in series:



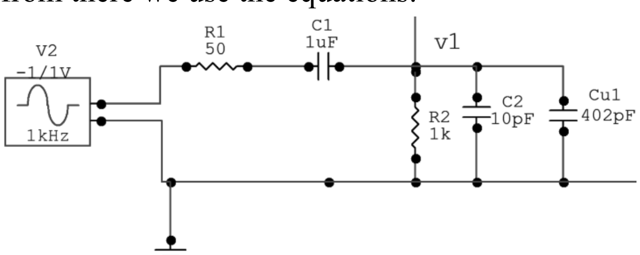
We can use a voltage divider to get the equation for v_1 in terms of v_s:

$$v_{1=}v_s(\frac{R_2}{R_2+R_1})$$

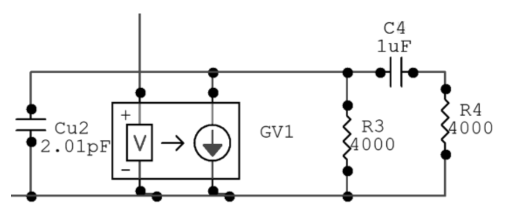
Therefore, mid-band gain is:

$$A_m = \frac{v_1}{v_s}\frac{v_o}{v_1} = \frac{v_s(\frac{R_2}{R_2+R_1})}{v_s}\frac{-200v_1}{v_1} = -190.476$$

To calculate the poles, we can use similar calculations as shown in the previous parts to find them (SC time constant and OC time constant equations). We split the input output stage as follows, and from there we use the equations:



Input Stage



Output Stage

Using the SC time constant and OC time constant calculations, similar to Part 1, the equations are:

$$\omega_{lf1} = \frac{1}{c_1(R_1+R_2)} \quad \omega_{lf2} = \frac{1}{c_4(R_4+R_3)} \quad \omega_{hf1} = \frac{1}{(c_2+c_{\mu1})(R_1||R_2)} \quad \omega_{hf2} = \frac{1}{c_{\mu2}(R_3||R_4)}$$

Evaluates to:

	ω_{lf1}	ω_{lf2}	ω_{hf1}	ω_{hf2}
ω[rad/s]	952.381	125	50.971*10^6	248.756*10^6

Figure 3.1 (Table of calculated Frequencies)

With these values, the transfer function of this circuit results to:

$$T(s) = (-190.476)\frac{s}{s+952.381} * \frac{s}{s+125} * \frac{50.971 * 10^6}{s+50.971 * 10^6} * \frac{248.756 * 10^6}{s+248.756 * 10^6}$$

Part B

Below are the magnitude and phase plots for the simulated circuit:

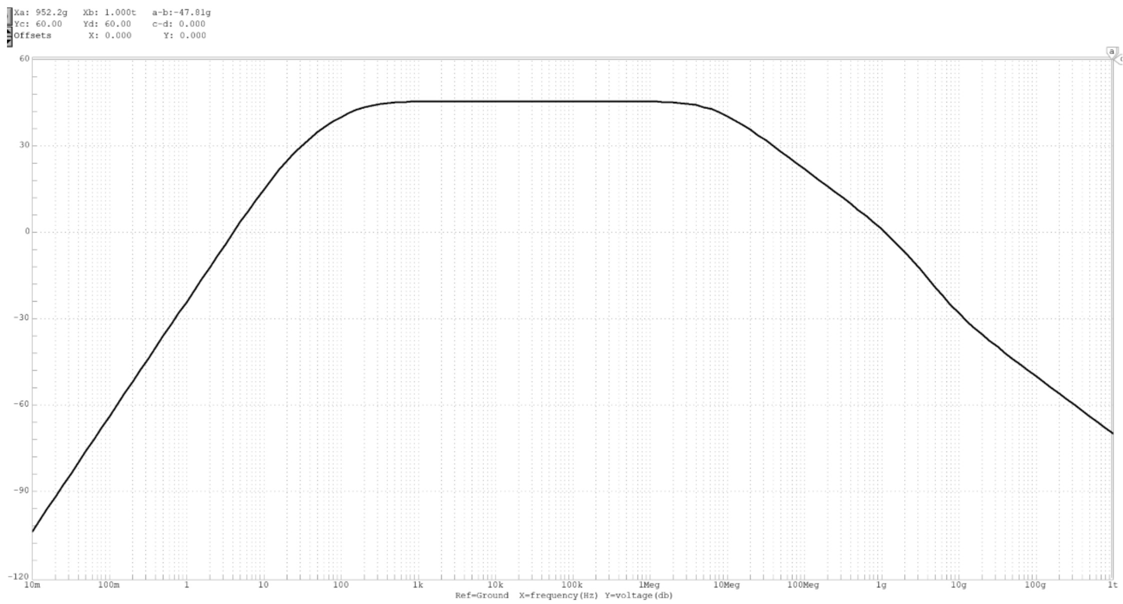


Figure 3.2 (Magnitude Plot)

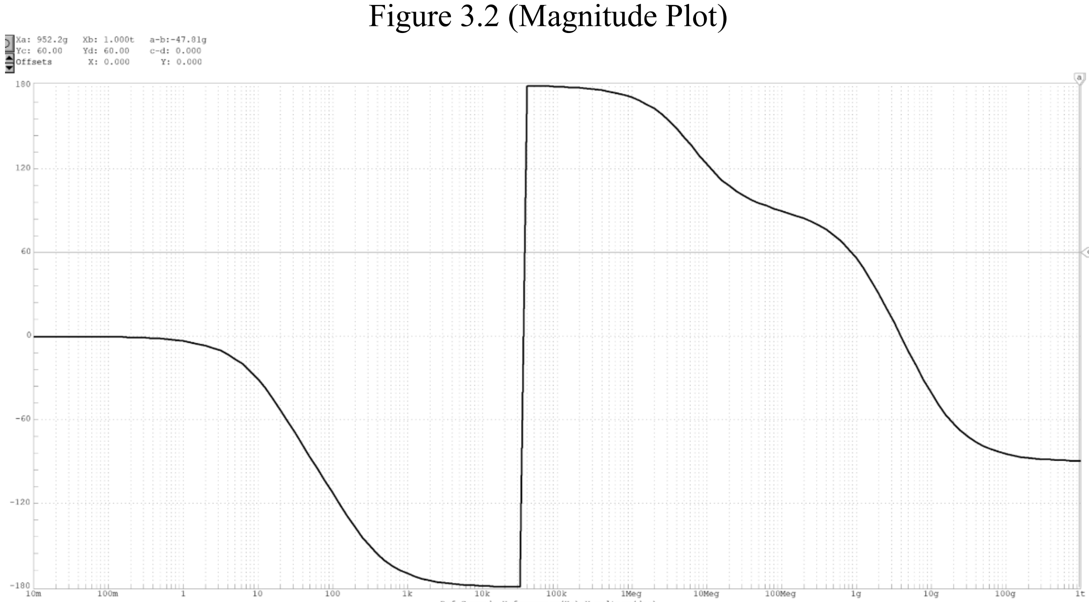


Figure 3.3 (Phase Plot)

Approximation of Lower 3dB Frequency:

To approximate the lower 3dB frequency ω_{L3dB}, we follow the same process as previously shown, where we measure a point in the plot where it is 3 decibels lower than the mid-band gain:

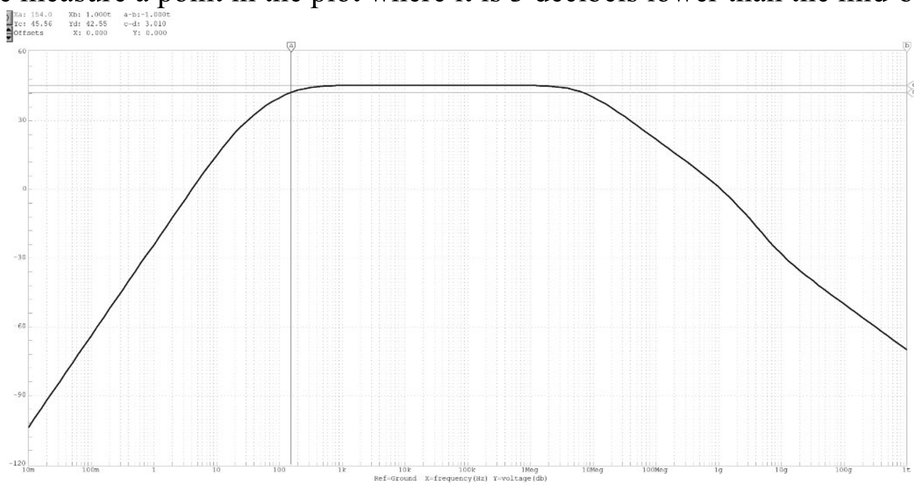


Figure 3.4(ω_{L3dB} Approximation)

Above, the cursor was used to approximate the point in the graph, leading to ω_{L3dB} = 154*2π rad/s.

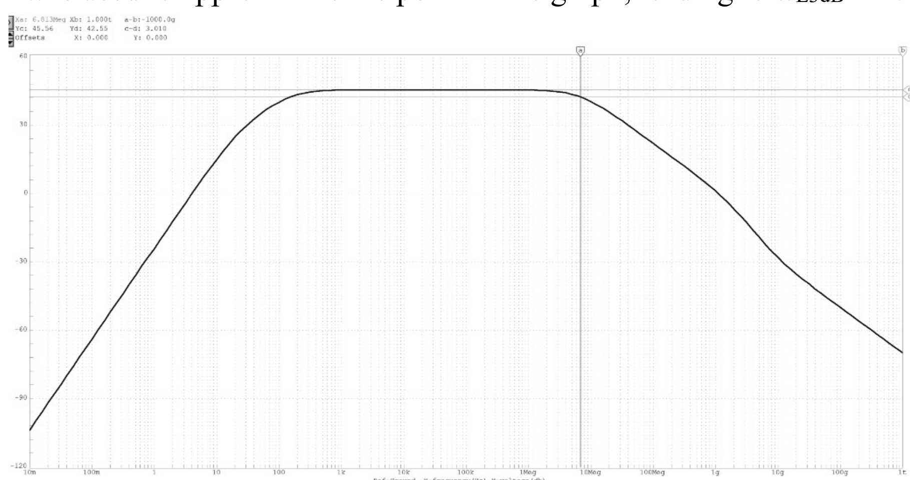


Figure 3.5 (ω_{H3dB} Approximation)

Similarly, for the plot shown above, we can approximate $\omega_{H3dB} = 6.813*10^6*2\pi$ rad/s.

Pole Frequency Approximation:

To estimate the poles and zeros on the plot, we can sketch slopes on the plot, and finding where the slopes change can estimate approximately where the poles and zeros are, as shown below:

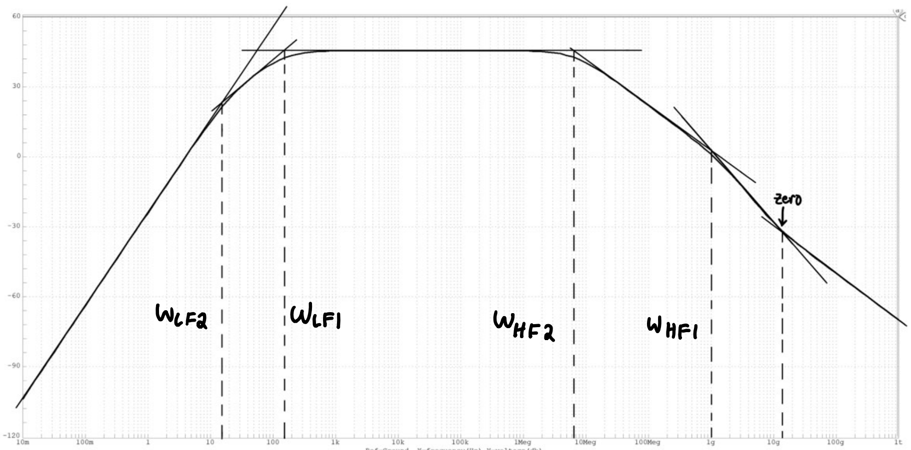


Figure 3.6 (Pole Frequency Estimation)

The approximated poles and zeros are as follows:

	ω_{lf1}	ω_{lf2}	ω_{hf1}	ω_{hf2}	zero
$\omega[\frac{rad}{s}]$	1000.028	100.28	$40.351*10^6$	$6.87*10^9$	$50.35*10^9$

Figure 3.7 (Table of Frequencies)

Comparing each of the poles with the calculated values, we find that the estimated low frequency poles are quite close to the calculation, as the difference between each of the estimated and calculated poles are within about 100 rad/s of each other. As with the estimated and calculated high frequency poles, they are extremely far from each other to the point where they are not reasonable estimates of each other. The difference between the calculated ω_{hf1} and the measured value is about 10MHz. Doing the same calculation on ω_{hf2} leads to about a 6600MHz difference. The reason why the low frequency poles have a good estimation on the plot is because of the logarithmic scale. In the area of the plot around where the low frequency poles are, the plot is in the 1-100Hz range, making graphical estimates to be reasonable. In the area where the high frequency poles are, the scale of the x-axis is in the 10MHz-1GHz range, making the estimates graphically extremely inaccurate. This is because each small increment of the x-axis in that range can jump in the MHz range. This would make measurements be very difficult to perform accurately.

Calculating Percent Error:
For calculating the percent error in the ω_{3dB}, we can use similar calculations as we did in Part 2. First, we need calculated ω_{L3dB}:

$$\sum_{i=1}^{N} \frac{1}{R_i C_{is}} = \omega_{lf1} + \omega_{lf2} = 1077.381\ rad/s$$

This formula is for calculated ω_{H3dB}:

$$\frac{1}{\sum_{i=1}^{M} C_i R_{is}} = \frac{1}{\tau_{hf1} + \tau_{hf2}} = \frac{1}{c_{\mu2}(R_3||R_4) + (c_2 + c_{\mu1})(R_1||R_2)} = 42.302 * 10^6\ rad/s$$

For percent error:

$$\%error = \left| \frac{\omega_{3dB(calculated)} - \omega_{3dB(measured)}}{\omega_{3dB(measured)}} \right| * 100$$

For ω_{L3dB}:

$$\left| \frac{1077.38095238 - 967.610537306}{967.610537306} \right| * 100 = 11.344\%\ error$$

For ω_{H3dB}:

$$\left| \frac{42.302 * 10^6 - 42.807 * 10^6}{42.807 * 10^6} \right| * 100 = 1.178\%\ error$$

The reason why the 3dB calculations were relatively accurate was because the measurement was reliant on the y-axis. Once we find the decibel level on where the 3dB point should be, it was relatively easy to line up the cursors on CircuitMaker to find the exact point on where the 3dB point was on the x-axis.

References

1. CircuitMaker™ Manual
2. ELEC 301 Class Notes
3. Mini Project 1 Document