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Mini Project 4

University of British Columbia

Electrical and Computer Engineering

ELEC 301

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0   Introduction

For this project, we will be using Multisim to simulate an active filter, a phase shift oscillator, and a feedback circuit. We will also be comparing measured and calculated values on all three circuits.

1   Part A

1.0.1   Part 1

For this part, we will be designing a 2nd order Butterworth low pass active filter using the UA741 operational amplifier. Here is the circuit that we will be using for this part:
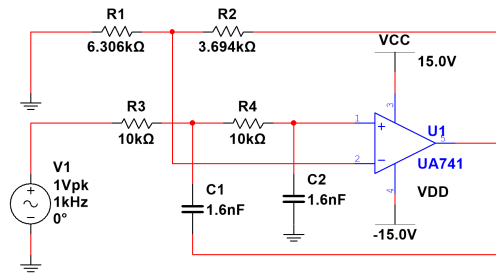
Figure 1: Second Order Butterworth Filter

The calculations to find the resistance R_1,R_2 and capacitance C, we will be using the formulas from the class notes [1]. The formulas can also be found from no. 1 in the Appendix. From the formulas, we can see that

$$R_1 = 6.306k\Omega, R_2 = 3.694k\Omega, C = 1.6nF, A_m = 1.59\frac{V}{V}$$

Below is the phase and magnitude plot for our filter:
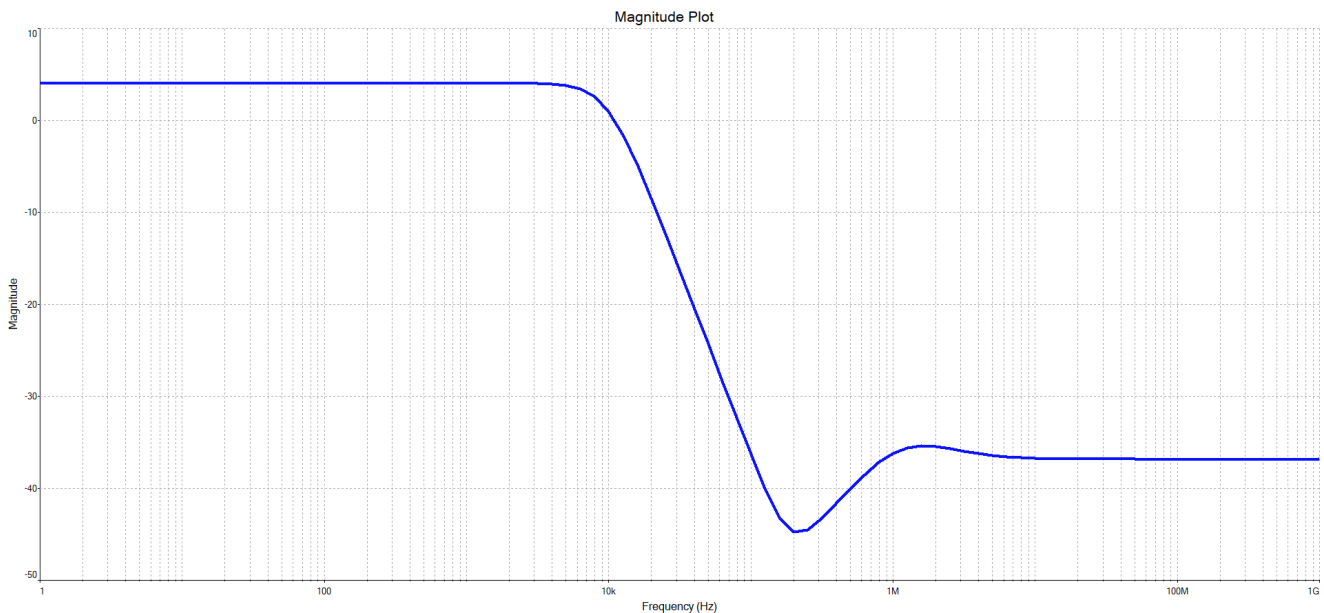
Figure 2: Bode Magnitude Plot
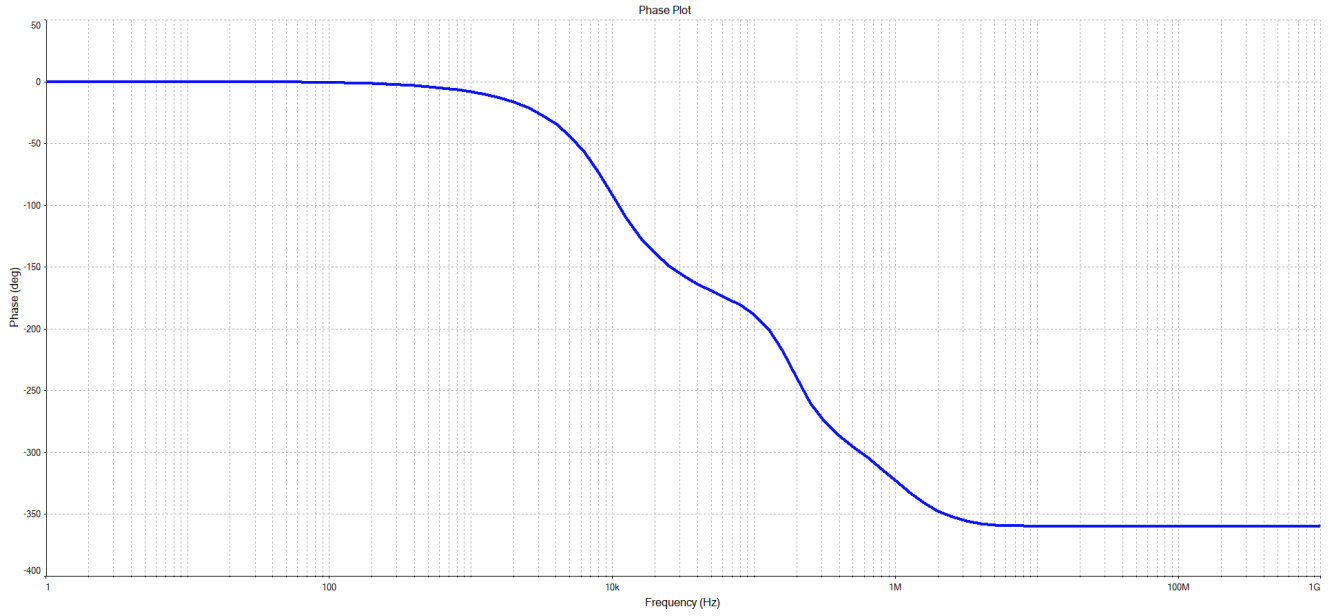
Figure 3: Bode Phase Plot

1.0.2   Part 2

For this part, we will be grounding the input, and measuring the output of the OpAmp. To determine the value of A_m when the circuit begins to oscillate, we need the transfer fucntion. The function is shown below, where R $= 10k\Omega$ and C is the value found previously:

$$H(s) = A_M \frac{\frac{1}{(RC)^2}}{s^2 + s\frac{3-A_M}{RC} + \frac{1}{(RC)^2}}$$

Changing the values of the resistances, we find that the oscillations occour when the resistor values are around$R_1 = 3k\Omega$ and $R_2 = 7k\Omega$. The oscillation is shown below:
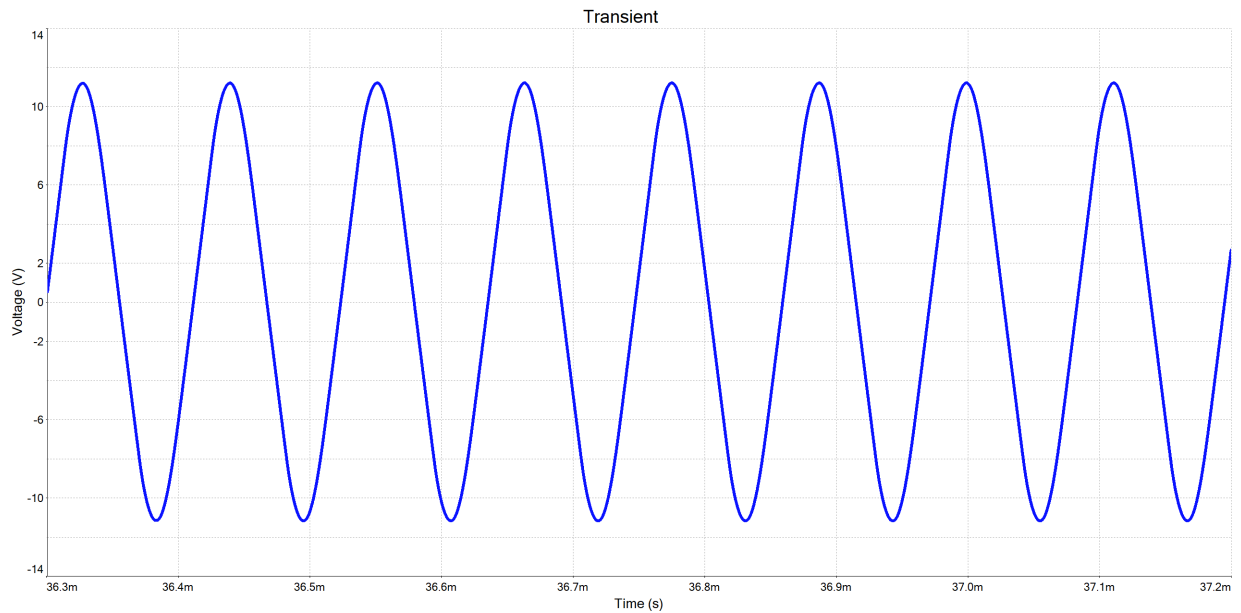
Figure 4: Oscillating Output

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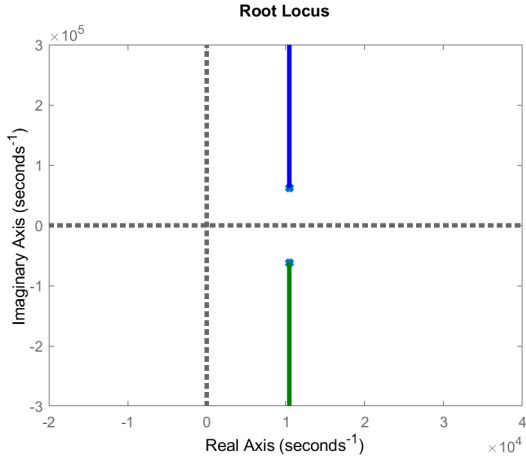
Figure 5: Unstable Root Locus Plot
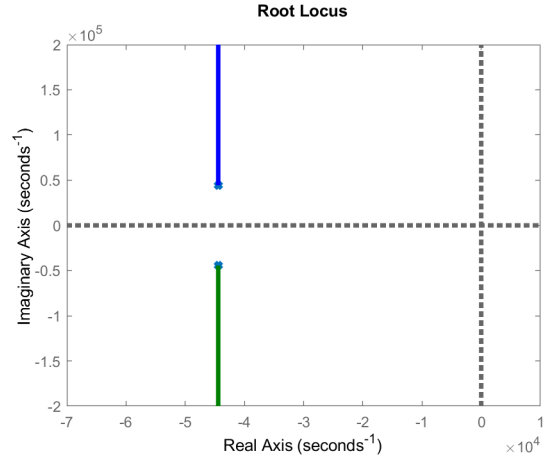
Figure 6: Stable Root Locus Plot

Using the cursors and measuring the differences between the crests of the plot, we find that the oscillating frequency is $\boxed{f_o = 8.93kHz}$. Below is the root locus plot:

The root locus plot where $A_M > 3$ and $A_M < 3$ is Figure 5 and Figure 6 respectively. We can see from the plot that when the oscillations occour, A_M is greater than 3. This would then cause the system to be unstable as shown in Figure 5, since the poles are on the right side on the jw axis. When the poles are on the other side of the jw axis, the system is stable, and doesn't cause the output to oscillate. This happens when $A_M < 3$. The reason why it has to be less than 3 for the system to be stable is because of the characteristic equation in the transfer function. If A_M is equal or greater than 3, it causes one of the coefficients in the characteristic equation to be negative, causing instability in the system and thus, the oscillations occour.

2 Part B

Below is our phase shift oscillator circuit:
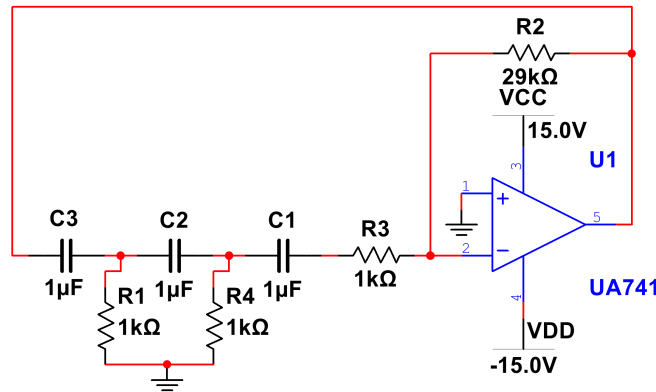
Figure 7: Phase Shift Oscillator Circuit

To find the proper value of the 29R resistor, we will simulate the output response for a very large amount of time (in this case, 1000 seconds was used). Simulating with just $29k\Omega$ we find that

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the output amplitude eventually decays to zero. Increasing the resistor to $29.1k\Omega$ makes the output amplitude to be sustained indefinitely.

Plotting the output of the circuit with the values given in the project document [2] and the increased 29R resistor , we find that the output oscillates as shown:
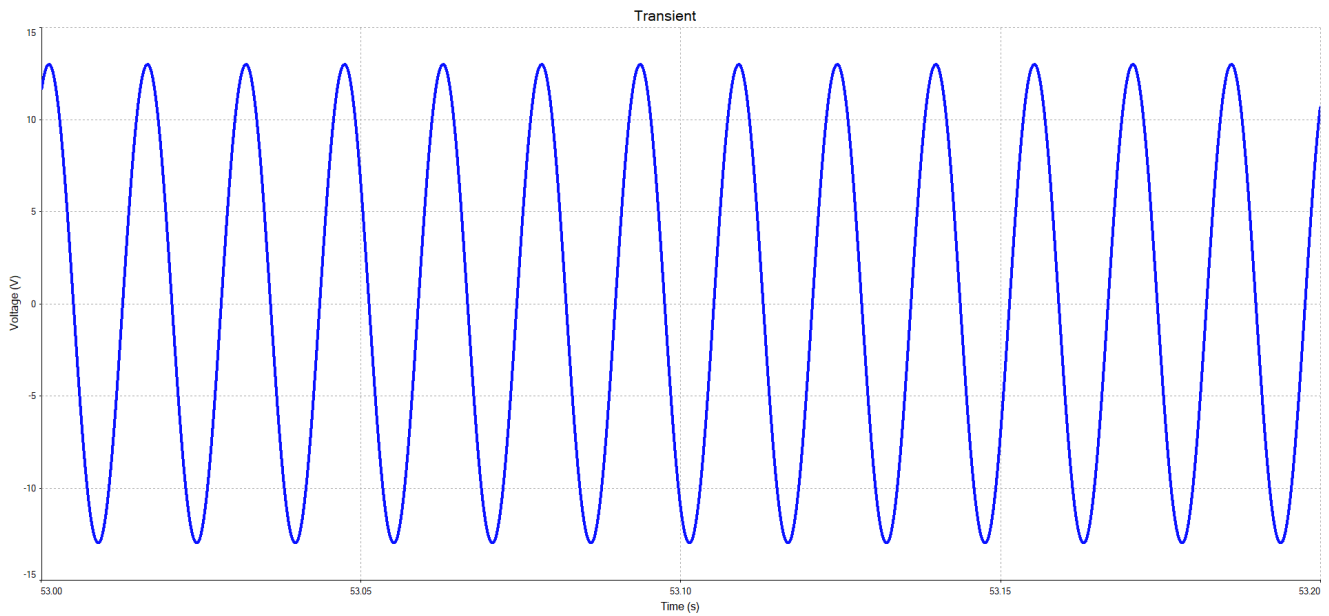
Figure 8: Circuit Output With Unchanged Circuit Elements
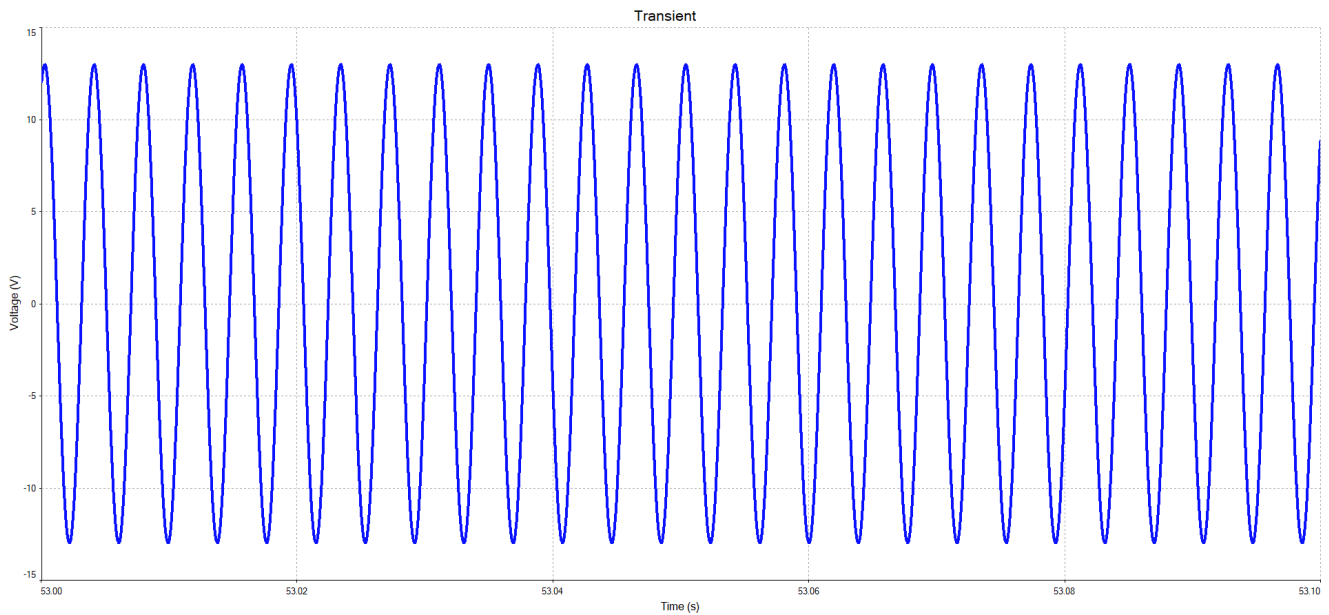
Figure 9: Circuit Output With Half Circuit Elements
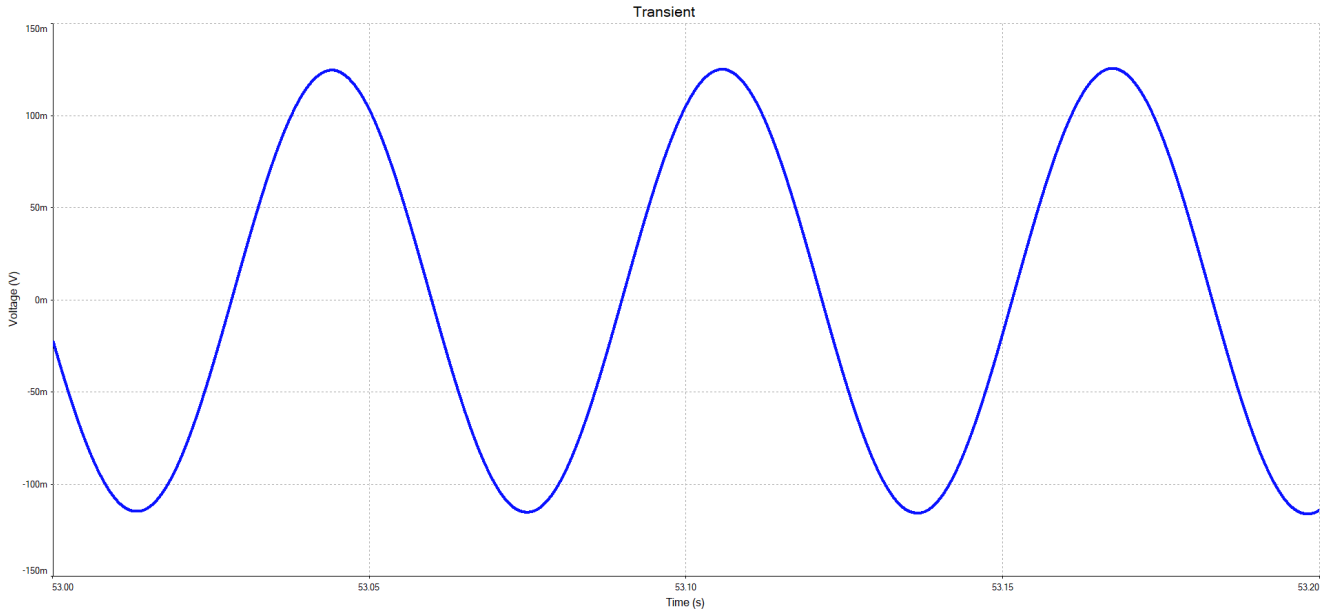
Figure 10: Circuit Output With Double Circuit Elements

From class notes[1], we can calculate the output frequency with:

$$f = \frac{1}{2\pi RC\sqrt{6}}$$

In the table below, the multiplier indicatates the circuit element value (ex. 0.5x indicates $R = 500\Omega$ and C = 0.5μ F). Comparing the calculated and measured frequencies for each modified circuit:

R and C Multiplier	0.5x	1x	2x
Calculated $f_o(Hz)$	259.899	64.975	16.244
Measured $f_o(Hz)$	259.067	64.872	16.221
% Error	0.32	0.15	0.14

Table 1: Calculated and Measured Frequencies

We can observe that the calculated and measured values are very close to each other. Due to the very low percentage error, we can then conculde that our calculations and the simulation are accurate to each other.

The way that a phase shift oscillator works is that a third order RC circuit is connected behind an amplifier with a negative gain[5]. The reasoning behind using a third order circuit is because this is the lowest order circuit that is able to output a phase shifted signal[5]. In this case, the negative feedback is supplied by the 29R resistor, with our OPAMP as our amplifier. The reasoning behind the increase of the 29R resistor, is that our negative gain has to be larger than the gain of the input circuit[5]. The gain of the input in this case was represented by the 29R resistor. We observed that with just 29R, the output was not sustained, and eventaully decayed to zero. Once the resistor was increased, we observed that the output was stable.

3 Part C

Below is our Feedback Circuit:
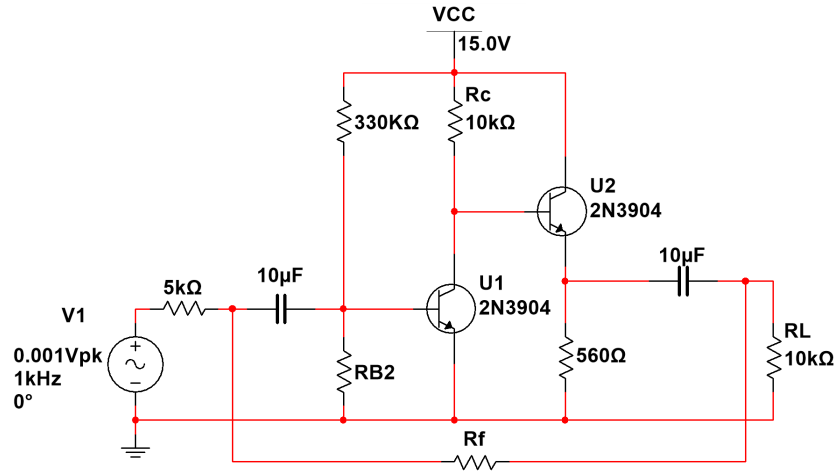
Figure 11: Feedback Circuit

Since we want to find the value of R_{B2} that creates the largest open-loop gain at 1kHz, we will be setting our input voltage at 1kHz at 1mV. To find the value, we will be doing a parameter sweep with a transient response to find which resistance value results in the largest gain. We find that after doing the parameter sweep, the value R_{B2} to be around $\boxed{20k\Omega}$. Resistance values above $20k\Omega$, we find that the output amplitude decreases. The parameter sweep is shown below:
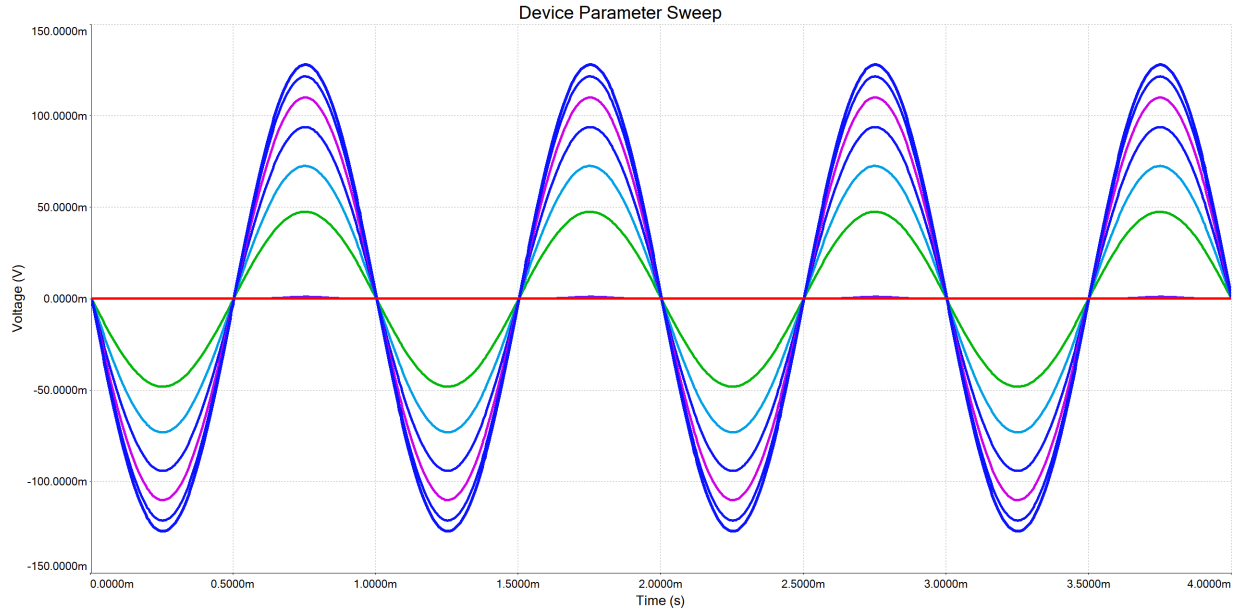
Figure 12: Parameter Sweep with R_{B2} and Transient Response

3.1   Part 1

To measure the DC Operating points, we will be using the DC Circuit below:
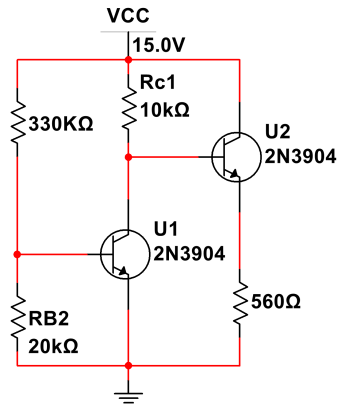
Figure 13: Feedback DC Circuit

For this project, we will be assuming that $V_T = 0.025$V. We can use these formulas to calculate our transistor parameters:

$$r_\pi = \frac{V_T}{I_B} \ , \beta = \frac{I_C}{I_B} \text{ and } g_m = \frac{\beta}{r_\pi}$$

Here are the DC Operating points, with our measured r_π, g_m, and h_{FE}:

	I_C	I_B	I_E	V_C	V_B	V_E	g_m	r_π	h_{FE}
Q1	1.29mA	10.8uA	1.31mA	1.90V	0.654V	0V	0.05	2.315kΩ	119
Q2	2.19mA	15.4uA	2.21mA	15V	1.90V	1.23V	0.09	1.623kΩ	142

Table 2: DC Operating Points

3.2   Part 2

3.2.1   Finding Measured Open Loop Frequency Responce and I/O Impedance

Here is the open loop frequency response plot:
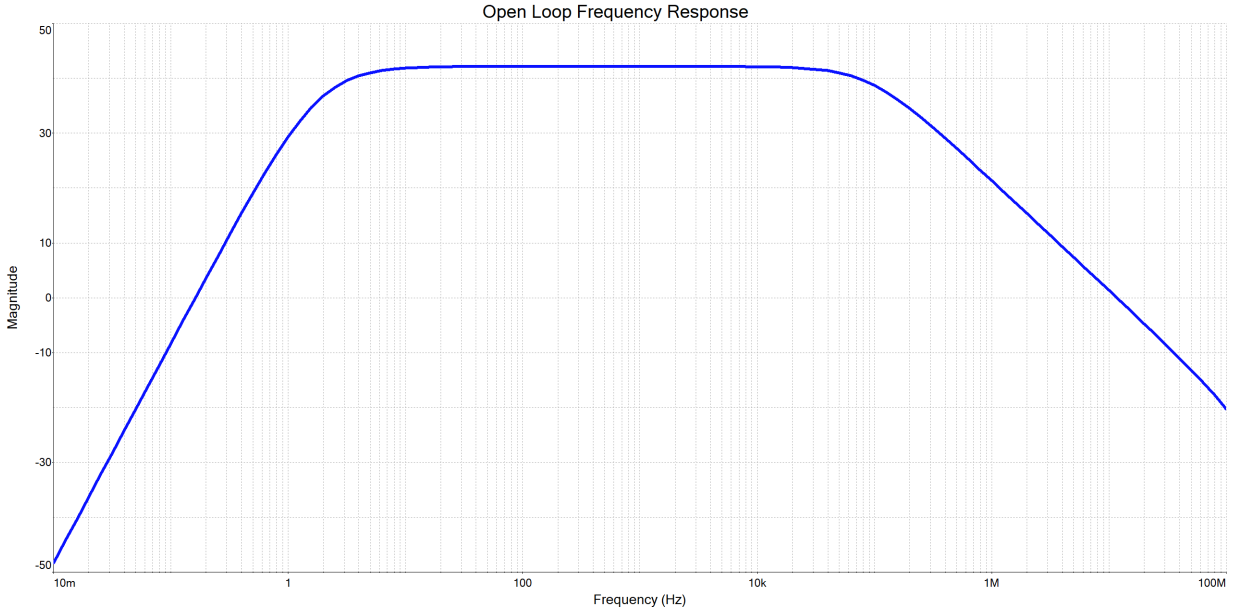
Figure 14: Open Loop Frequency Response

Measuring with the cursors in our simulation software, we find that the 3dB points are:

$$w_{L3dB} = 2.9219 * 2\pi[\frac{rad}{s}], w_{H3dB} = 91.3083 * 2\pi * 10^3[\frac{rad}{s}]$$

We can also find our mid-band gain $\boxed{A_m = -127.557\frac{V}{V}}$ from the frequency response plot. In order to find the input and output resistance at 1kHz, we will be measuring the voltage and current at the input, and we will be adding a test source at the output and grounding the input for the output impedance measurements. After measuring, we find:

$$R_{in} = \frac{240\mu V}{93.4nA} = \boxed{2.569k\Omega}, \ R_{out} = \frac{707\mu V}{11.4\mu A} = \boxed{62.017\Omega}$$

3.2.2 Predicted Closed Loop Frequency Response

For this section, we will need to calculate our circuit parameters and input and output resistance at 1kHz with $R_f = 100k\Omega$. To do this, we need to know our circuit's topology. Since the output is sampled by R_f and mixes the current from the output into the input, we can determine that our feedback circuit uses shunt-shunt topology. Due to this, we will be using y-parameters to perform our calculations. Below is the y-parameter matrix we will be using:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

To find the value of y_{11}, we set $V_2 = 0$. Then, we find that $\boxed{y_{11} = \frac{1}{R_f}}$ for our y_{12} parameter, we set $V_1 = 0$. We also know that $\beta = y_{12}$. It is found as $\boxed{y_{12} = \beta = \frac{-1}{R_f}}$.For $y21$, we find that it's effect is small compared relative to the other gain values, so we will not consider it in calculations. For y_{22}, we set $V_1 = 0$ and find that $\boxed{y_{22} = \frac{1}{R_f}}$.

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3.2.3   Gain Calculations

Since our circuit is in shunt-shunt topology, our open-loop will be $\frac{V_{out}}{I_{in}}$. Therefore, we need to convert our input to the amplifier from a voltage source to a current source. To do this, we will be transforming our source from a voltage source in series with our source resistance to a current source in parallel to our source resistance. Doing this, our resulting input current is $\boxed{I_{in} = \dfrac{V_s}{5k\Omega}}$. So we find our open loop gain to be:

$$A' = \frac{V_{out}}{I_{in}} = \frac{V_{out}}{\frac{V_s}{5k\Omega}} = \frac{V_{out}}{V_s}(5k\Omega) = -127.557\frac{V}{V} * (5k\Omega) = \boxed{-637.786k\frac{V}{A}}$$

Now, we are able to find our gain with feedback. The formula used can be found in the class notes[1]. It is found as:

$$A_f = \frac{A'}{1 + \beta A'} = -86.446k\frac{V}{A}$$

Since the units we need is $\frac{V}{V}$ we convert our input to a voltage source, and this can be done by dividing A_f by $5k\Omega$. After doing this, we find $\boxed{A = -17.289\dfrac{V}{V}}$. Which is our closed loop gain.

3.2.4   3dB Calculations

To calculate the w_{3dBf} (3dB frequencies with feedback), we can use the following formulas from the class notes[1]. The frequencies are found to be:

$$w_{Hf3dB} = (w_{H3dB})(1 + A'\beta) = (91.308k * 2\pi)(1 + \frac{-1}{100k\Omega} * -637.786k\frac{V}{A}) = \boxed{4233.189k\frac{rad}{s}}$$

$$w_{Lf3dB} = \frac{w_{L3dB}}{1 + A'\beta} = \frac{2.9219 * 2\pi}{1 + \frac{-1}{100k\Omega}(-637.786k\frac{V}{A})} = \boxed{2.488\frac{rad}{s}}$$

3.2.5   Input and Output Resistance Calculations

To calculate our input and output resitance values with feedback, we can use our open loop resistance values and formulas[1] to calculate the values we eventually find:

$$R_{if} = \frac{R_{in}}{1 + A'\beta} = \frac{2.569k\Omega}{1 + \frac{-1}{100k\Omega}(-637.786k\frac{V}{A})} = \boxed{348.204\Omega}$$

$$R_{of} = \frac{R_{out}}{1 + A'\beta} = \frac{62.017\Omega}{1 + \frac{-1}{100k\Omega}(-637.786k\frac{V}{A})} = \boxed{8.406\Omega}$$

Here is our closed loop frequency response curve:
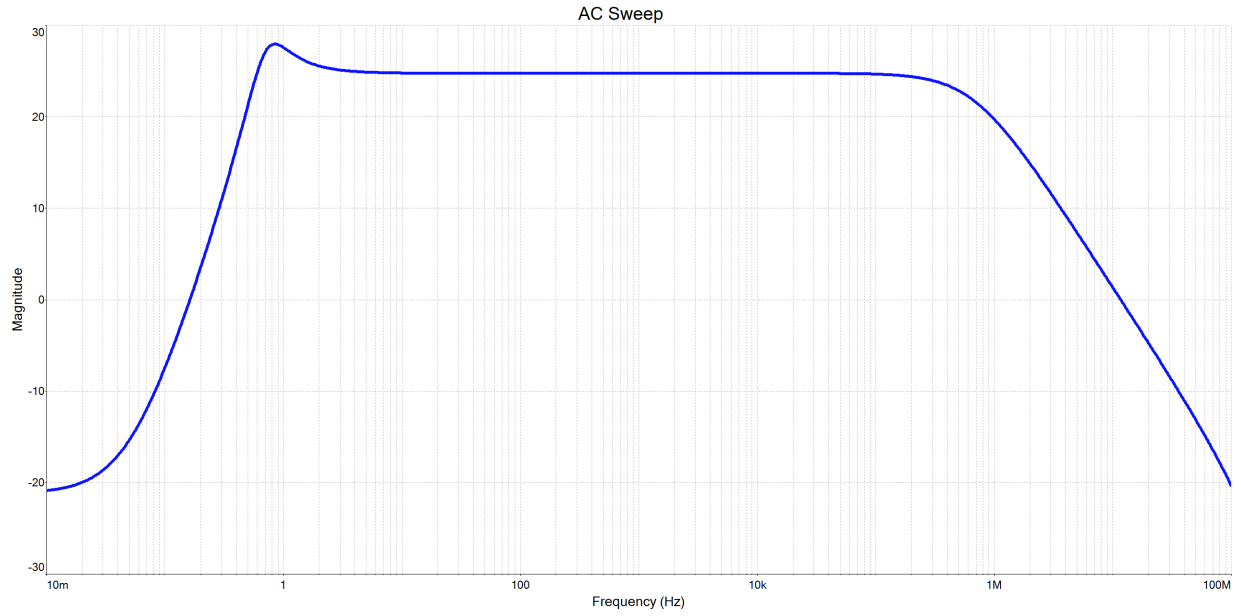
Figure 15: Closed Loop Frequency Response

From this, we can measure the following values:

	w_{L3dB}	w_{H3dB}	R_{in}	R_{out}	A_f
Measured	$3.221[\frac{rad}{s}]$	$4257.361\text{k}[\frac{rad}{s}]$	240.7Ω	8.456Ω	$17.248\frac{V}{V}$

Table 3: Measured Feedback Values

Comparing the measured values and the calculated values, we can see that the calculated values are accurate to the measured values.

3.3 Part 3

From the bottom most trace, the values of R_f are increasing for each increasing curve. Here is all of the different bode plots of various R_f values:
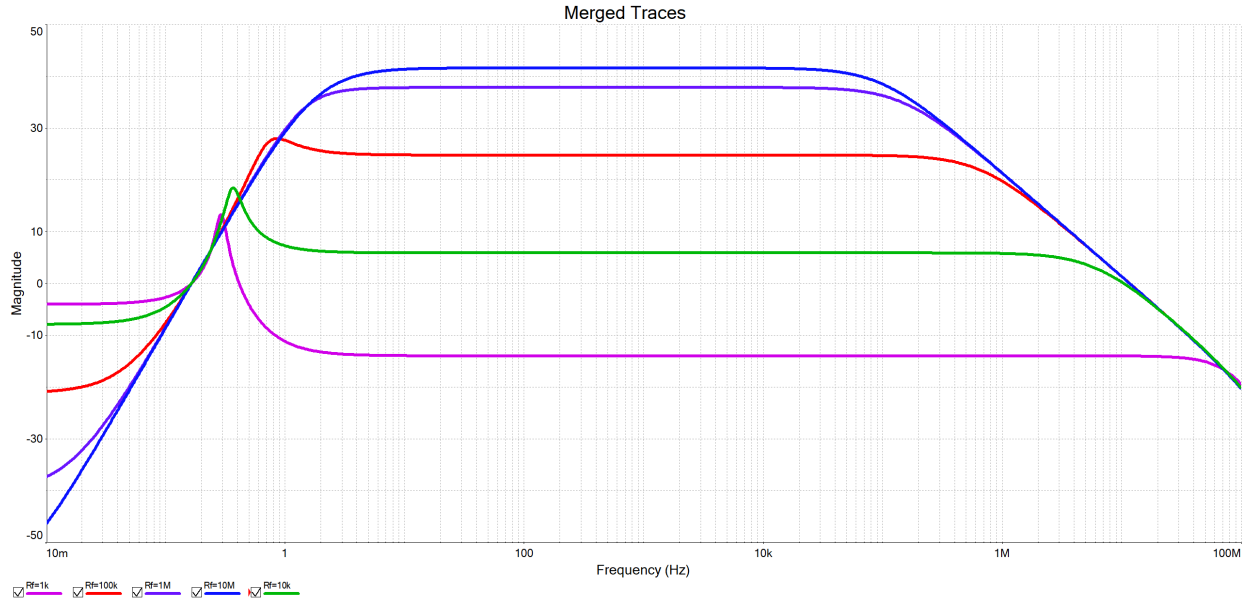
Figure 16: Closed Loop Frequency Response with Different R_f Values

For the calculated values, the calculations were performed the same way it was done in the previous part. Since we need to measure our β value however, we can use the following equation to use the measured values to solve for our measured β:

$$\beta_{measured} = \frac{A_f - A^{'}}{A_f A^{'}}$$

Since our open loop gain is the same value as found in the previous section, we only need to measure our A_f values from the bode plot. However, once we measure the gain from the bode plot, we need to convert it's values from $\frac{V}{V}$ to $\frac{V}{A}$. We can do this by multiplying the measured value by $5k\Omega$, similar to the way it was converted in the previous section. Once we have A_f in $\frac{V}{V}$, we can then use that value to calculate our $\beta_{measured}$. The measured and calculated values are shown below:

$R_f(\Omega)$	Measured $A_f(\frac{V}{A})$	Measured β(S)	Calculated $A_f(\frac{V}{A})$	Calculated β(S)
1k	-995.5	-1.003e-3	-998.435	-0.001
10k	-9.820k	-1.0027e-4	-9.845k	-0.0001
100k	-86.240k	-1.0028e-5	-86.446k	-0.00001
1M	-388.995k	-1.0028e-6	-389.420k	-0.000001
10M	-599.425k	-1.0034e-7	-681.2342k	-0.0000001

Table 4: Measured and Calculated Feedback Values

Comparing the calculated values and the measured values, we can see that for each value of R_f, the calculated and measured values are mostly accurate to the simulated values. We can see that at $R_f = 10M\Omega$ that the measured and calculated values are not as close to the simulated values. This could be due to inaccuracies in the simulation, or assumptions in the calculation.

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3.3.1 Part 4

To find the input and output resistance of the amplifier at different values of R_f and at 1kHz, we will be using the same method as done previously to measure each of those values. To find the calculated values of the input and output resistance for each R_f, we will be using the calculated values of β for the corresponding R_f value. We will be using the same formula as done previously to find the measured and calculated values of R_{of} and R_{if}. For the "amount of feedback", we will be abbreviating it on our table as "A.o.F.". To calculate it, we will be using $1+A'\beta$. This is because this is the way that feedback is accounted for in the calculations for the input and output resistance calculations as shown below:

$$R_{of/if} = \frac{R_{in/out}}{1+A'\beta}$$

Therfore the "amount of feedback is calculated by the denominator of the above equation." The measured values are shown below:

$R_f(\Omega)$	Measured $R_{if}(\Omega)$	Measured $R_{of}(\Omega)$	Calculated $R_{if}(\Omega)$	Calculated $R_{of}(\Omega)$
10k	26.763	1.117	39.658	0.957
100k	266.393	8.457	348.204	8.406
1M	2.663k	37.606	1.568k	37.866

Table 5: Measured and Calculated R_{of} and R_{if} Values

For the estimated value, we will be using the measured values from Table 5. For the predicted values, we will be calculating it with the β and A' values that we already know (values from parts 2 and 3).

Below is the Amount of feedback measured and calculated values:

$R_f(\Omega)$	Predicted A.o.F	Measured A.o.F (from R_{if})	Measured A.o.F (from R_{of})
10k	64.7786	95.991	55.521
100k	7.37786	9.644	7.333
1M	1.637786	0.965	1.649

Table 6: Measured and Predicted "Amount of Feedback"

We can see that comparing the A.o.F. between the measured and the predicted values, they are more accurate to each other as we increase the R_f value. We can determine that because we can see that the differences between the measured and predicted continue to decrease as R_f increases. We can also observe that the A.o.F. from the output resistance is more accurate than the value calculated from the input resistance. However, the observation that each value of A.o.F. continues to be more accurate as R_f increases is still accurate to both methods of calculating A.o.F.. This could happen due to assumptions in the calculation that is not accounted for compared to the simulation.

We can also see from table 5 that the measured and calculated values of R_{of} are very accurate to the measured value. However, we can see that the input impedance calculations are not as accurate to the measured values. This could also be due to inaccuracies in the simulation, or assumptions in the calculation.

3.3.2 Part 5

From the class notes[1], we find that the de-sensitivity factor is found by $1+A\beta$. If $R_f = \infty$, then, our de-sensitivity factor is 1 since the $A\beta$ portion equates to 0 because in this case, $\beta = \frac{1}{\infty}$. Simulating for $R_f = \infty$, we find:

$R_f(\Omega)$	Measured Gain($\frac{V}{V}$)
9.9k	-126.848
10k	-127.553
10.1k	-128.201

Table 7: Measured Output with $R_f = \infty$

Simulating for $R_f = 100k\Omega$, we find:

$R_f(\Omega)$	Measured Gain($\frac{V}{V}$)
9.9k	-17.235
10k	-17.248
10.1k	-17.260

Table 8: Measured Output with $R_f = 100k\Omega$

With these values, we can find the de-sensitivity factor for $R_f = 100k\Omega$. The values of the gain does not change much for the different values of R_c. The de-sensitivity factor is found to be:

$R_f(\Omega)$	De-Sensitivity Factor
9.9k	7.3424
10k	7.378
10.1k	7.410

Table 9: De-Sensitivity Factor

Comparing the gain values from parts 2 and 3, we find that the gain values are very similar to each other. Therefore, we find that this circuit is desensitized. For smaller values of R_f, an observation can be made that the gain is larger at mid band. This could occour because the de-sensitivity factor decreases with a smaller R_f value. With the lower de-sensitivity factor, the mid band gain would then be larger; compared to the gain from a larger R_f value.

4   Appendix

$$k = 3 - \sqrt{2} = 1 + \frac{R_2}{R_1} \tag{1}$$

$$A_m = 1 + \frac{R_2}{R_1} \tag{2}$$

$$R_2 + R_1 = 10k\Omega \tag{3}$$

$$R_1 = \frac{10000}{-\sqrt{2} + 3}\Omega \tag{4}$$

$$R_2 = \frac{1}{7}(-10000\sqrt{2} + 40000)\Omega \tag{5}$$

5   References

1. ELEC 301 Class notes
2. Mini Project 4 Document
3. Standard Resistor and Capacitor Values (Canvas)
4. Circuit Maker SPICE Model
5. Sedra, Smith - Microelectronic Circuits 5th Edition