PART 5: Trajectory generation and control

Exercise 5.1: quintic splines

Consider the following 1-D quintic spline:

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5.$$

Find the constants a_i, $i = 0, 1, 2, 3, 4, 5$, to respect the constraints $x(0) = 0,\ \dot{x}(0) = 0,\ \ddot{x}(0) = 0,\ x(1) = 1,\ \dot{x}(1) = 0$, and $\ddot{x}(1) = 0$.

Exercise 5.2: B-splines

Given the *Cox-de Boor recusion formula*:

$$\begin{aligned}
N_{i,0}(t) &= \begin{cases} 1 & \text{if } t_i \le t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \\
N_{i,p}(t) &= \frac{t - t_i}{t_{i+p} - t_i} N_{i,p-1}(t) + \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} N_{i+1,p-1}(t),
\end{aligned}$$

Compute the two first order basis functions $N_{0,1}(t)$ and $N_{1,1}(t)$ with $t_i = i$ and combine them linearly to interpolate between the two points $(1, 2)$ and $(2, 3)$.

Exercise 5.3: computation of a DCM ⋆

In Mellinger and Kumar (2011),

Mellinger D, Kumar V. Minimum snap trajectory generation and control for quadrotors. In2011 IEEE International Conference on Robotics and Automation 2011 May 9 (pp. 2520-2525). IEEE.

a Direction Cosine Matrix (DCM) is found knowing the upward direction of the drone z_B and the yaw angle ψ:

$$R_B = \begin{bmatrix} x_B & y_B & z_B \end{bmatrix},\ y_B = \frac{z_B \times x_C}{||z_B \times x_C||},\ x_C = \begin{bmatrix} \cos\psi \\ \sin\psi \\ 0 \end{bmatrix},\ x_B = y_B \times z_B.$$

However, this is based on the Z-X-Y Euler angles convention, and the convention we are using in this course is Z-Y-X, i.e. $R_B = R_z(\psi)R_y(\theta)R_x(\phi)$ with the roll, pitch and yaw angles ϕ, θ and ψ (respectively), and

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix},\ R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \text{ and } R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Compute R_B based on z_B and ψ in the Z-Y-X Euler angles convention.