Ministère de l'Enseignement Supérieur Et de la Recherche Scientifique
****** Université de Carthage ******

Institut National des Sciences Appliquées et de Technologie

Three-Tank Liquid Level System

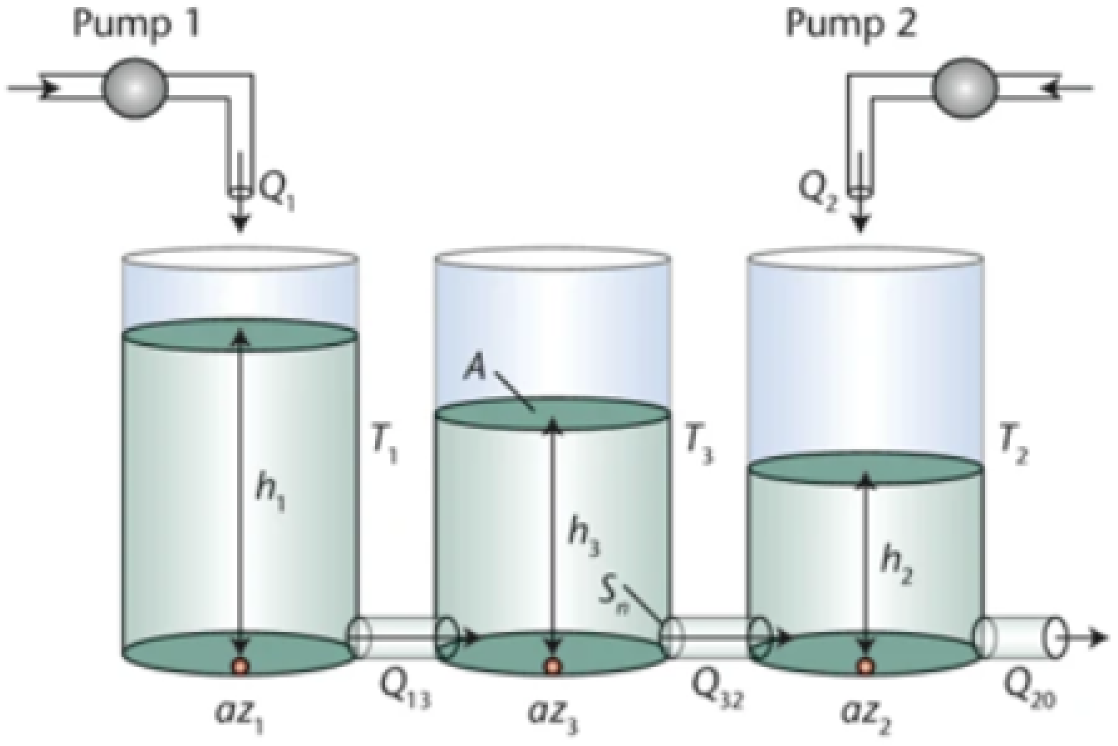
PREPARED BY

- Malek Zitouni
- Mohamed Karim Ben Boubaker
- Maysaa Fezai

Academic Year
2024/2025

Benchmark Description

The three-tank system is an industrial benchmark for studying control challenges in MIMO (Multi-Input Multi-Output) systems. This project aims to model, control, and analyze such a system by integrating PID regulators and simulation tools.



Parameter	Description	Unit
h_i	Water height in tank i (i=1,2,3)	m
Q_j	Pump j flow rate to tank j (j=1,2)	m^3/s
Q_{13}	Flow rate from tank 1 to tank 3	m^3/s
Q_{32}	Flow rate from tank 3 to tank 2	m^3/s
Q_{20}	Flow rate from tank 2 to reservoir	m^3/s
S	Cross-sectional area of tanks 1,2,3	m^2
S_n	Orifice cross-sectional area	m^2
a_i	Flow coefficients (i=1,2,3)	-
g	Gravitational coefficient	m/s^2

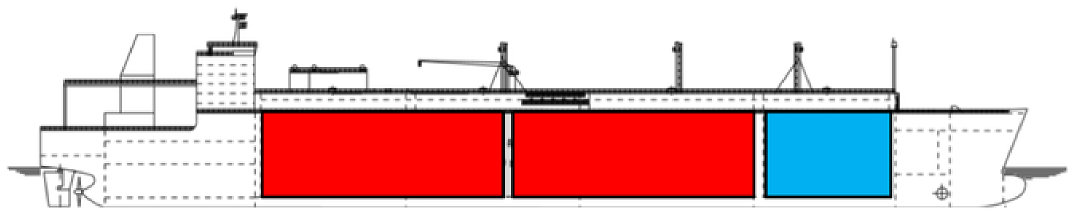
Table T1

Significance of the Three-Tank System in

Industrial Application

- Fluid management systems (water, petroleum, chemical processing).
- Control strategy testing for coupled dynamics and disturbance rejection.
- Energy sector applications (nuclear coolant control, hydroelectric systems, biofuel blending).
- Pharmaceutical/biotech uses (vaccine production, sterile water distribution).

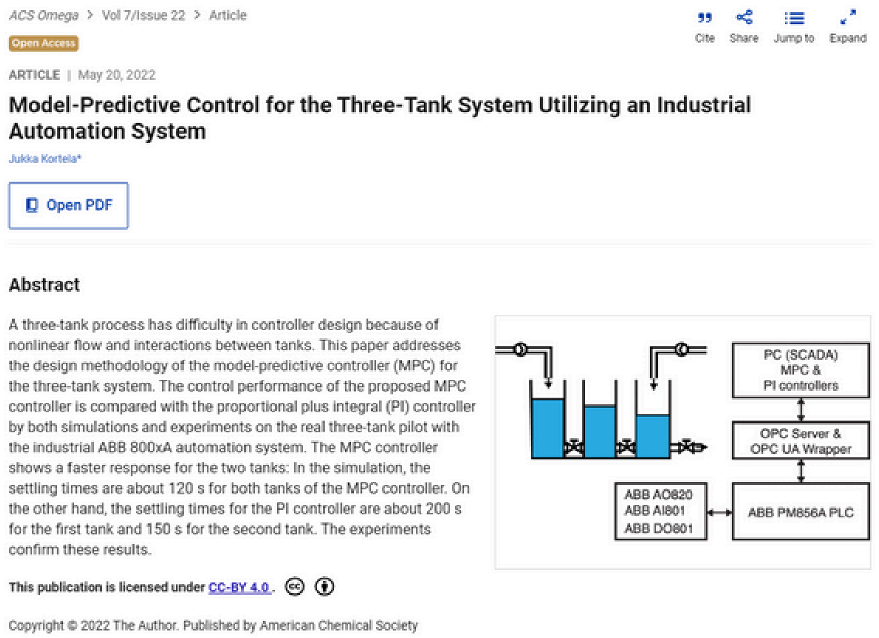
uses a three-tank configuration to improve efficiency, reduce boil-off gas emissions, and lower construction costs.



Revolutionizing LNG Shipping: GTT's Three-Tank Carrier Design and Its Implications for the Industry

Rod S.
Founder at LNG Cluster ™ & IOXN Group ™ | Author | PhD Management, DBA | Public and Private Sector Consulting Leader

December 6, 2024



ACS Omega > Vol 7/Issue 22 > Article

Cite Share Jump to Expand

Open Access

ARTICLE | May 20, 2022

Model-Predictive Control for the Three-Tank System Utilizing an Industrial Automation System

Jukka Kortela*

Open PDF

Abstract

A three-tank process has difficulty in controller design because of nonlinear flow and interactions between tanks. This paper addresses the design methodology of the model-predictive controller (MPC) for the three-tank system. The control performance of the proposed MPC controller is compared with the proportional plus integral (PI) controller by both simulations and experiments on the real three-tank pilot with the industrial ABB 800xA automation system. The MPC controller shows a faster response for the two tanks: In the simulation, the settling times are about 120 s for both tanks of the MPC controller. On the other hand, the settling times for the PI controller are about 200 s for the first tank and 150 s for the second tank. The experiments confirm these results.

This publication is licensed under CC-BY 4.0 .

Copyright © 2022 The Author. Published by American Chemical Society

Modeling Using Torricelli's Law

To demonstrate :

$$h1^{\cdot} = \frac{1}{S}(Q1 - Q13)$$
$$h2^{\cdot} = \frac{1}{S}(Q2 - Q32 - Q20)$$
$$h3^{\cdot} = \frac{1}{S}(Q13 - Q32)$$

Modeling Using Torricelli's Law

To demonstrate :

$$\dot{h1} = \frac{1}{S}(Q1 - Q13)$$

$$\dot{h2} = \frac{1}{S}(Q2 - Q32 - Q20)$$

$$\dot{h3} = \frac{1}{S}(Q13 - Q32)$$

With consideration of :

$$Q13 = a1Sn \cdot sgn(h1 - h3) \cdot \sqrt{(2g|h1 - h3|)}$$

$$Q20 = a2Sn \cdot \sqrt{(2gh2)}$$

$$Q32 = a3Sn \cdot sgn(h3 - h2) \cdot \sqrt{(2g|h3 - h2|)}$$

Modeling Using Torricelli's Law

To demonstrate :

$$\dot{h1} = \frac{1}{S}(Q1 - Q13)$$
$$\dot{h2} = \frac{1}{S}(Q2 - Q32 - Q20)$$
$$\dot{h3} = \frac{1}{S}(Q13 - Q32)$$

With consideration of :

$Q13 = a1Sn \cdot sgn(h1 - h3) \cdot \sqrt{(2g|h1 - h3|)}$
$Q20 = a2Sn \cdot \sqrt{(2gh2)}$
$Q32 = a3Sn \cdot sgn(h3 - h2) \cdot \sqrt{(2g|h3 - h2|)}$

Demonstration :

Water volume variation rate V for each tank :

$$\frac{dV}{dt} = Qin - Qout$$

$$V = S \times h$$

$$\frac{dh}{dt} = \frac{1}{S}(Qin - Qout)$$

Modeling Using Torricelli's Law

To demonstrate :

$$\dot{h1} = \frac{1}{S}(Q1 - Q13)$$
$$\dot{h2} = \frac{1}{S}(Q2 - Q32 - Q20)$$
$$\dot{h3} = \frac{1}{S}(Q13 - Q32)$$

	Tank 1	**Tank 2**	**Tank 3**
Qin	$Q1$	$Q2$	[illegible]
	$Q13$	[illegible]	$Q32$

In and Out tank flows

With consideration of :

$Q13 = a1Sn \cdot sgn(h1 - h3) \cdot \sqrt{(2g|h1 - h3|)}$
$Q20 = a2Sn \cdot \sqrt{(2gh2)}$
$Q32 = a3Sn \cdot sgn(h3 - h2) \cdot \sqrt{(2g|h3 - h2|)}$

Demonstration :

Water volume variation rate V for each tank :

$$\frac{dV}{dt} = Qin - Qout$$

$$V = S \times h$$

$$\frac{dh}{dt} = \frac{1}{S}(Qin - Qout)$$

Then :

$$\dot{h1} = \frac{1}{S}(Q1 - Q13)$$
$$\dot{h2} = \frac{1}{S}(Q2 - Q32 - Q20)$$
$$\dot{h3} = \frac{1}{S}(Q13 - Q32)$$

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

- Square root dependence of flow rates on height differences.

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

- Square root dependence of flow rates on height differences.
- Sign function (sgn) introducing a discontinuity when height differences change sign.

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

- Square root dependence of flow rates on height differences.
- Sign function (sgn) introducing a discontinuity when height differences change sign.
- Strong coupling between tanks via flow rates Q13 and Q32: Changes in one tank propagate to others.

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

- Square root dependence of flow rates on height differences.
- Sign function (sgn) introducing a discontinuity when height differences change sign.
- Strong coupling between tanks via flow rates Q13 and Q32: Changes in one tank propagate to others.
- Dynamic interdependence, where variations in one tank affect the others.

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

- Square root dependence of flow rates on height differences.
- Sign function (sgn) introducing a discontinuity when height differences change sign.
- Strong coupling between tanks via flow rates Q13 and Q32: Changes in one tank propagate to others.
- Dynamic interdependence, where variations in one tank affect the others.
- Different behavior depending on flow direction ($h1>h3$h1>h3 vs. $h1<h3$h1<h3).

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

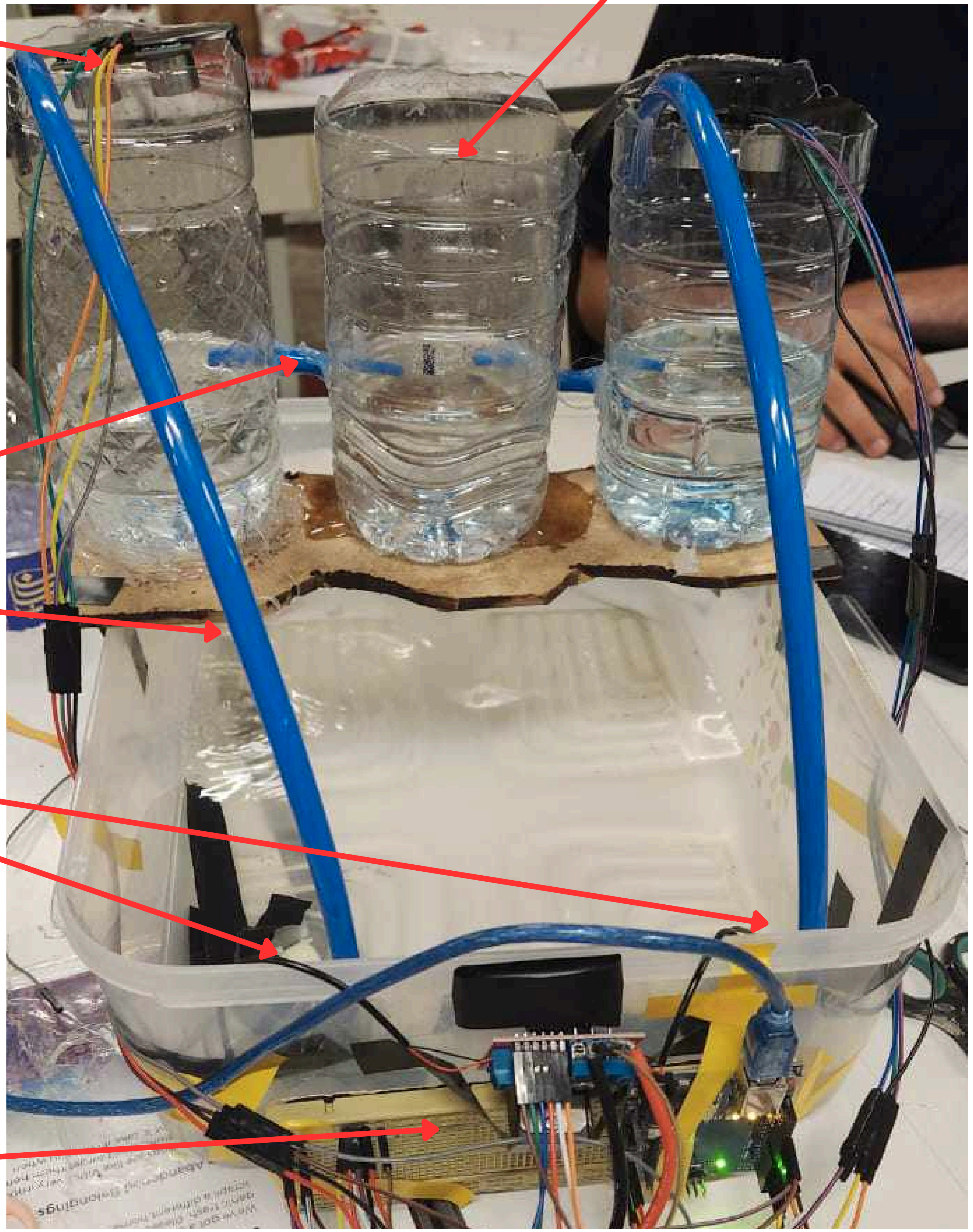
- Square root dependence of flow rates on height differences.
- Sign function (sgn) introducing a discontinuity when height differences change sign.
- Strong coupling between tanks via flow rates Q13 and Q32: Changes in one tank propagate to others.
- Dynamic interdependence, where variations in one tank affect the others.
- Different behavior depending on flow direction ($h1>h3$h1>h3 vs. $h1<h3$h1<h3).
- Control Design: Requires nonlinear methods (e.g., feedback linearization, sliding mode control, or adaptive control).

Complex Nonlinearities

This system exhibits several sources of complex nonlinearities:

Real-World Behavior: Explains why such systems exhibit oscillations, slow settling times, or sensitivity to initial conditions.

Benchmark on Physical Scale Model



LabVIEW Implementation

Front Panel

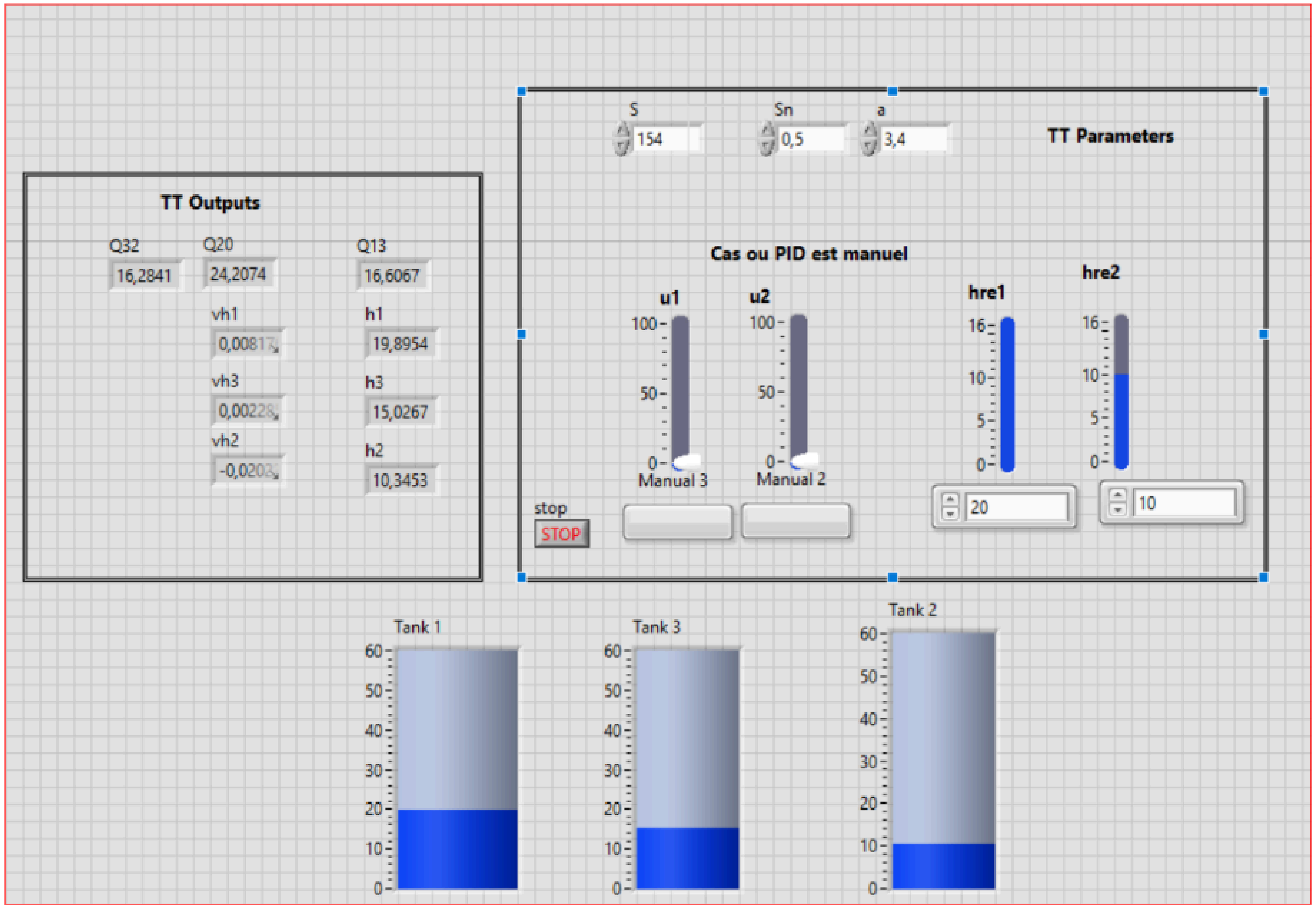
- Controls: Setpoints href, PID Parameters
- Indicators: Level graphs, Flow rates (Q1, Q2)

Block Diagram :

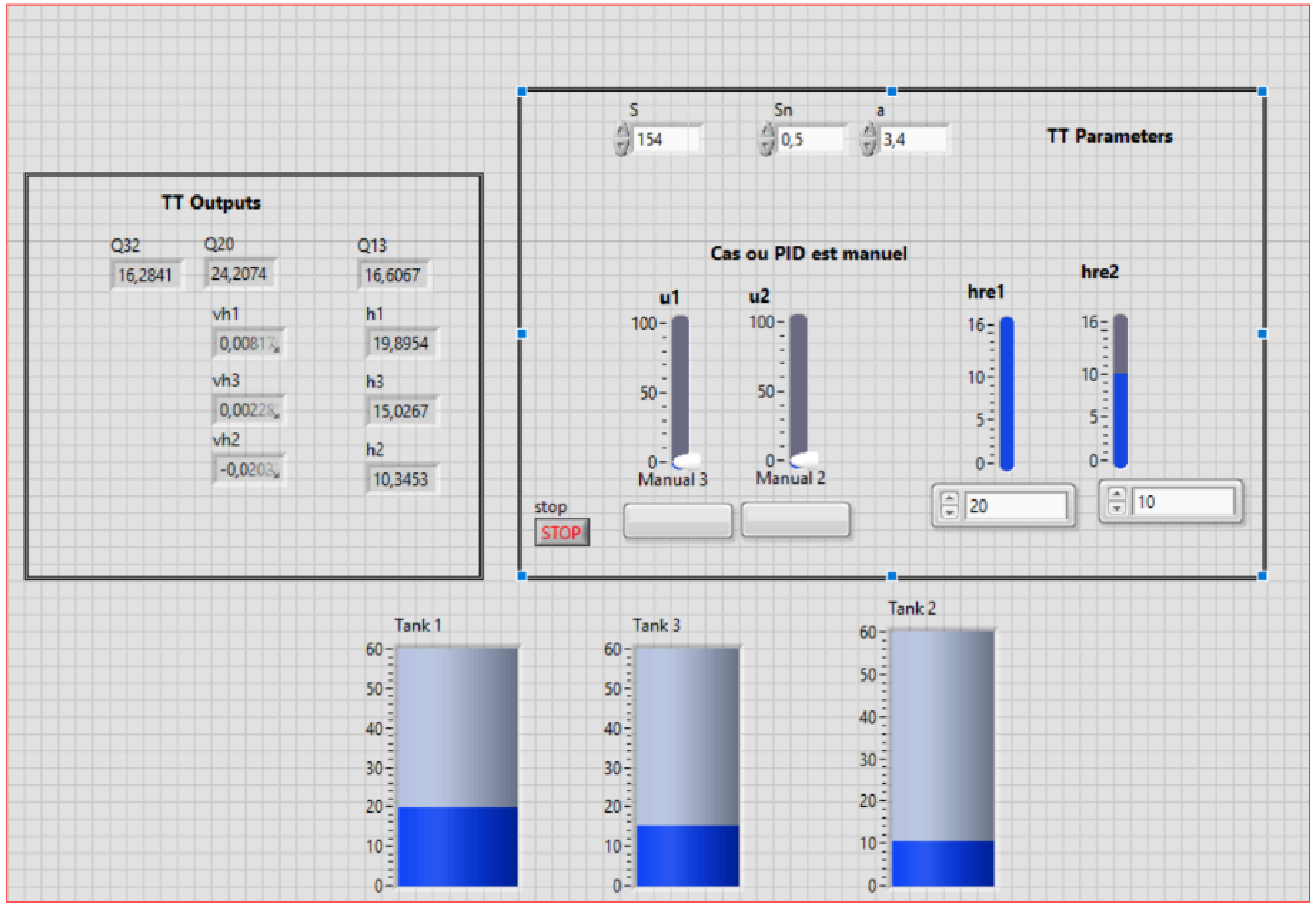
- PID loops implemented with PID Advanced.vi

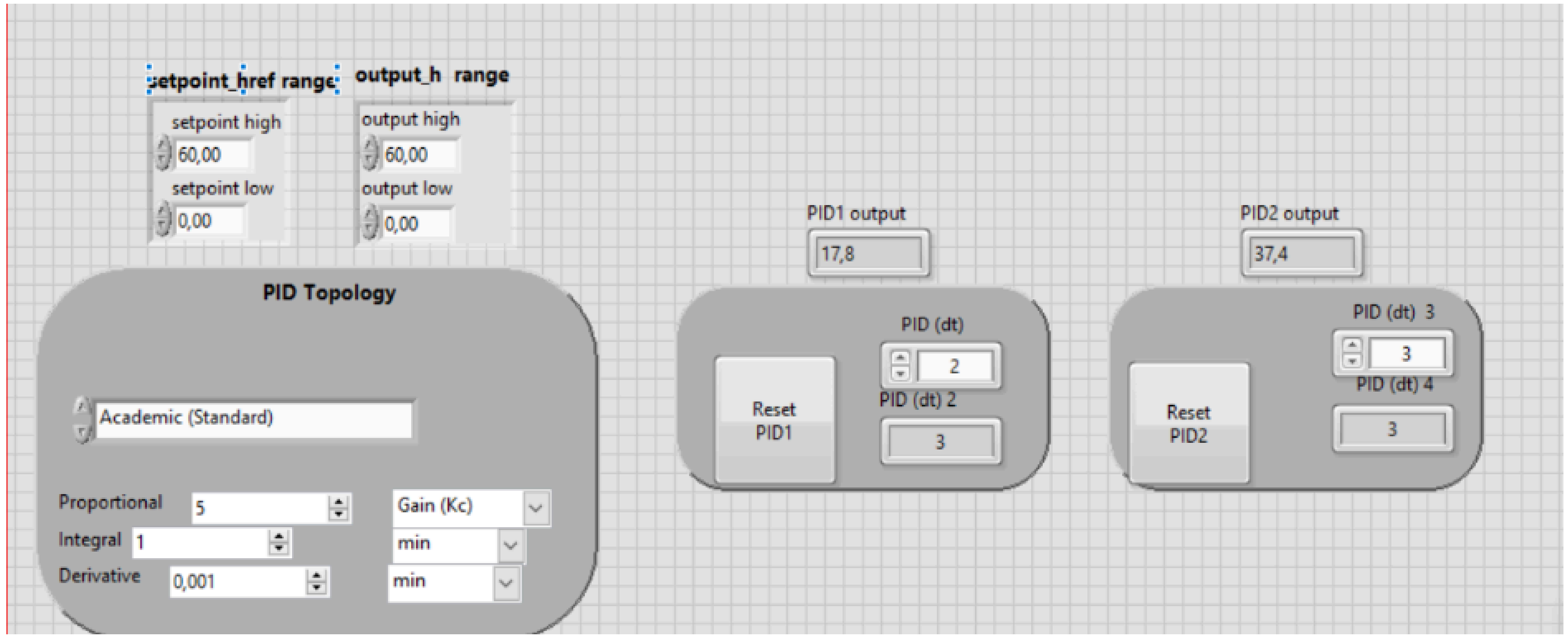
LabVIEW Implementation

Front Panel



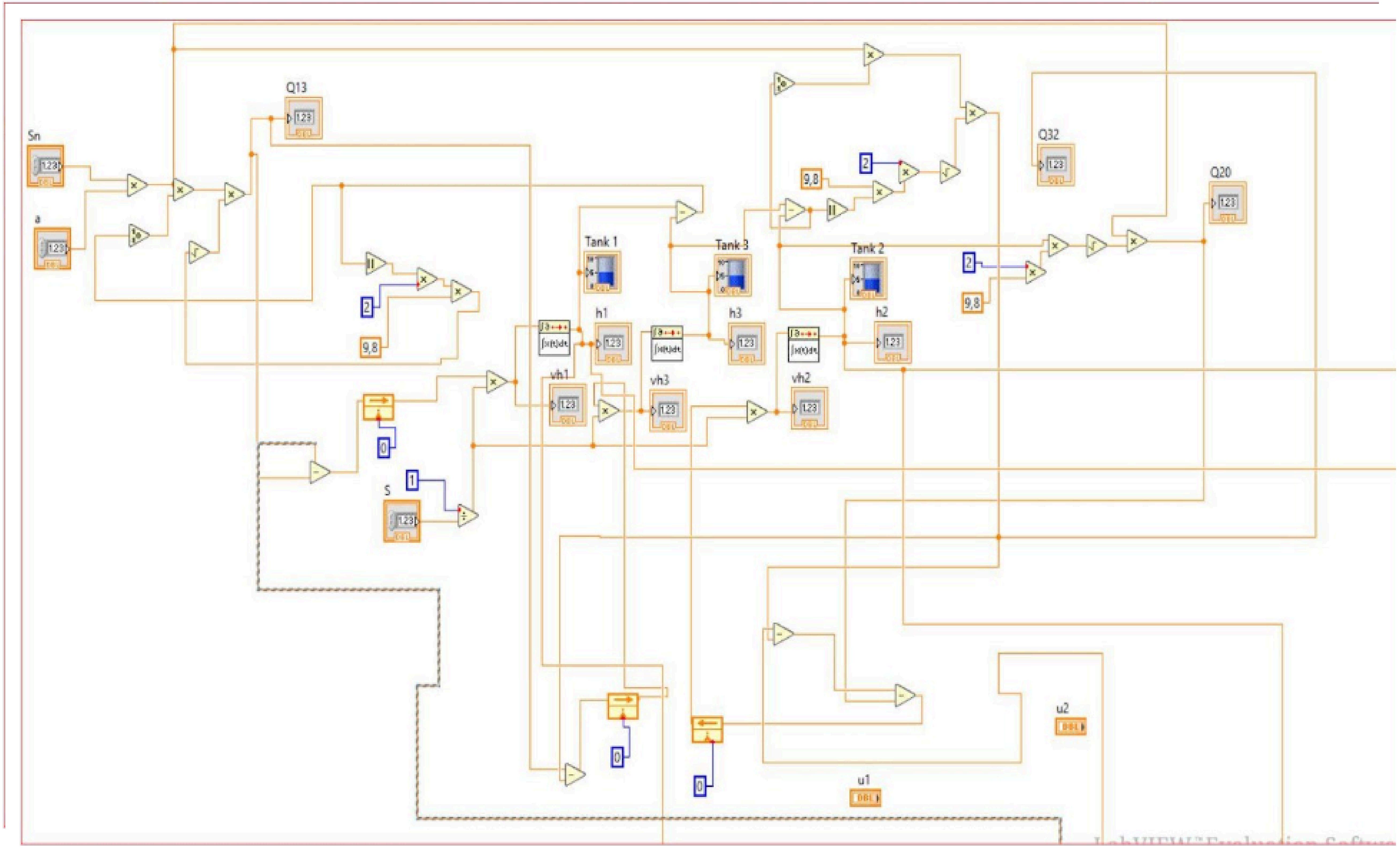
LabVIEW Implementation



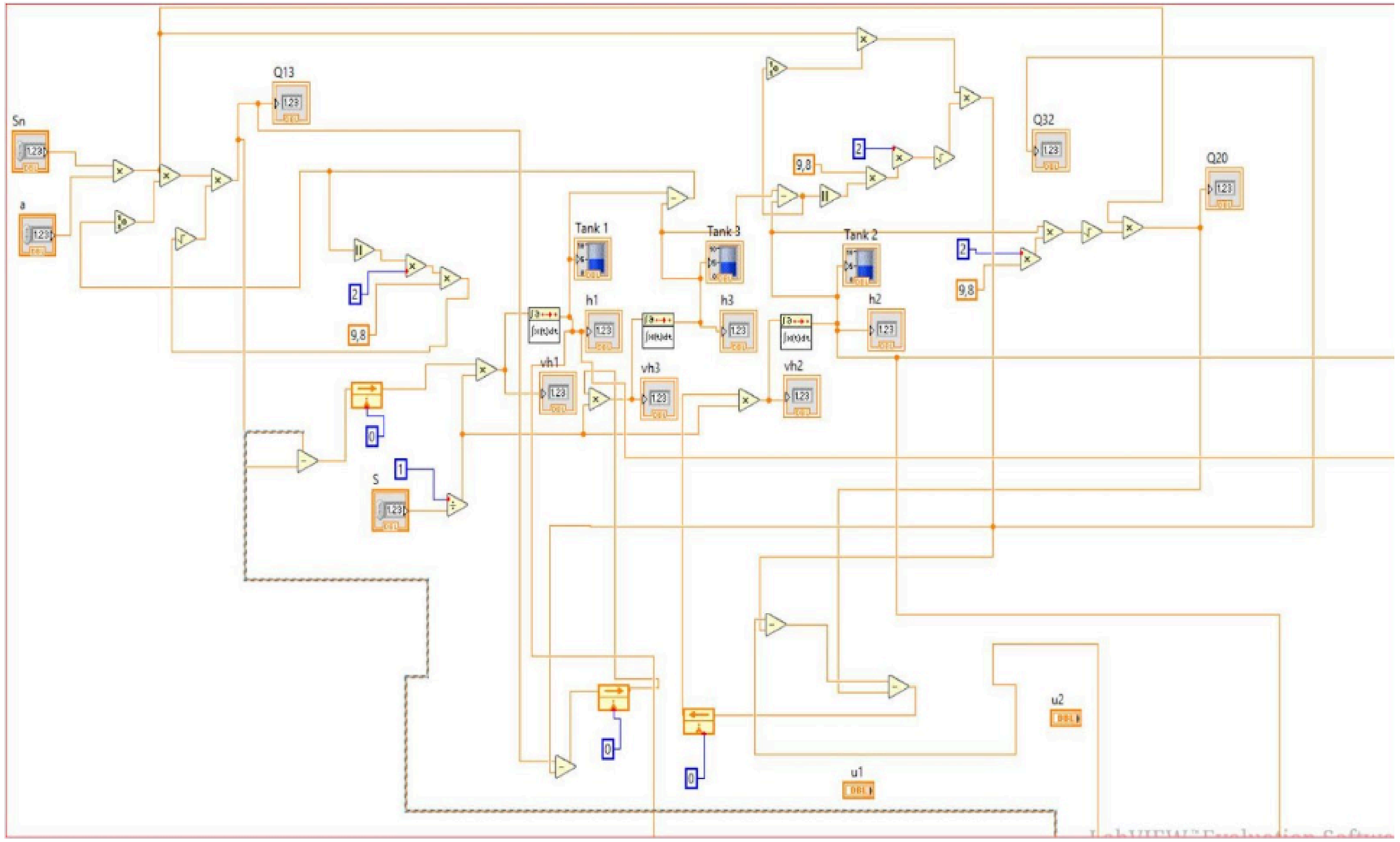


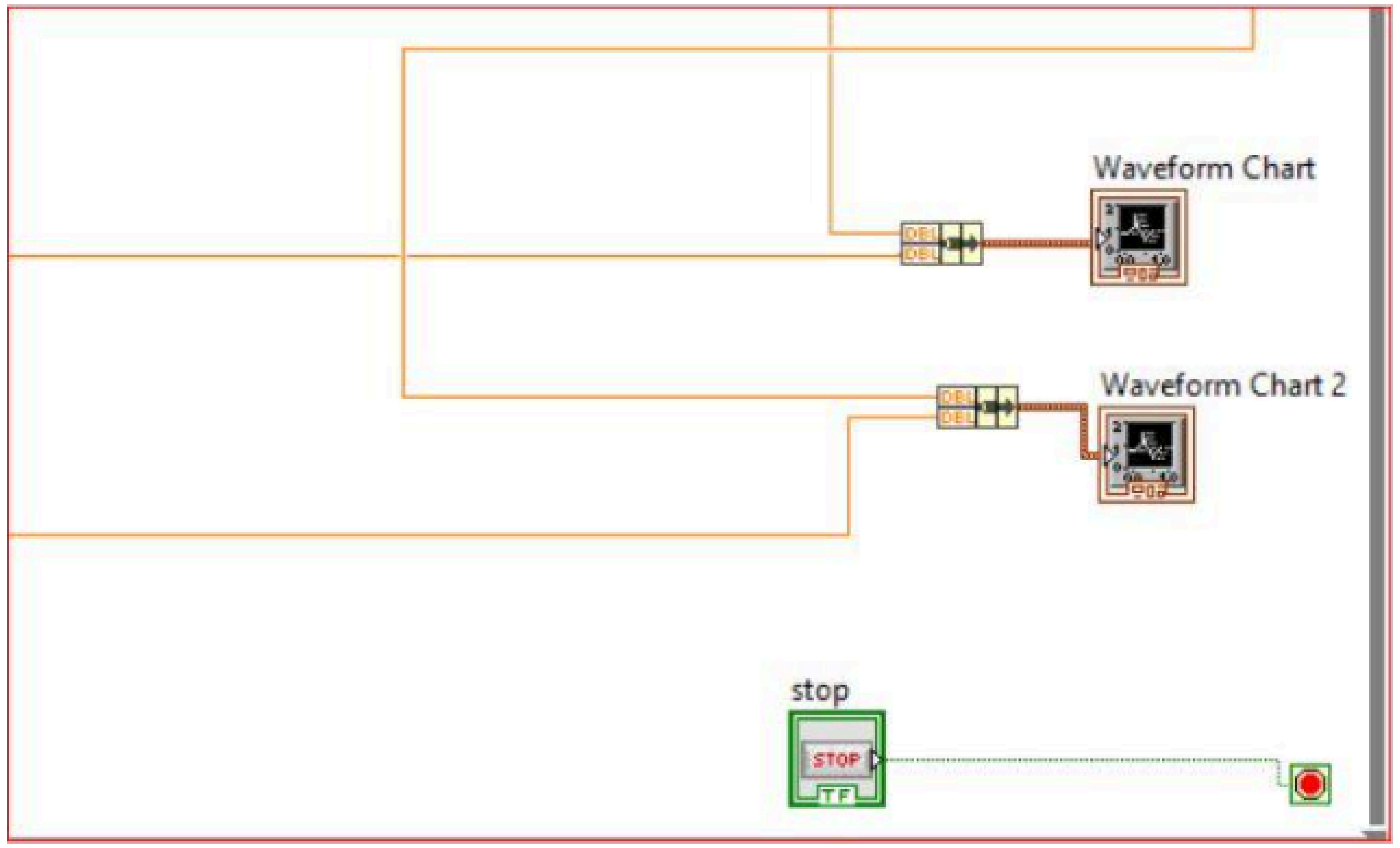
LabVIEW Implementation

Block Diagram

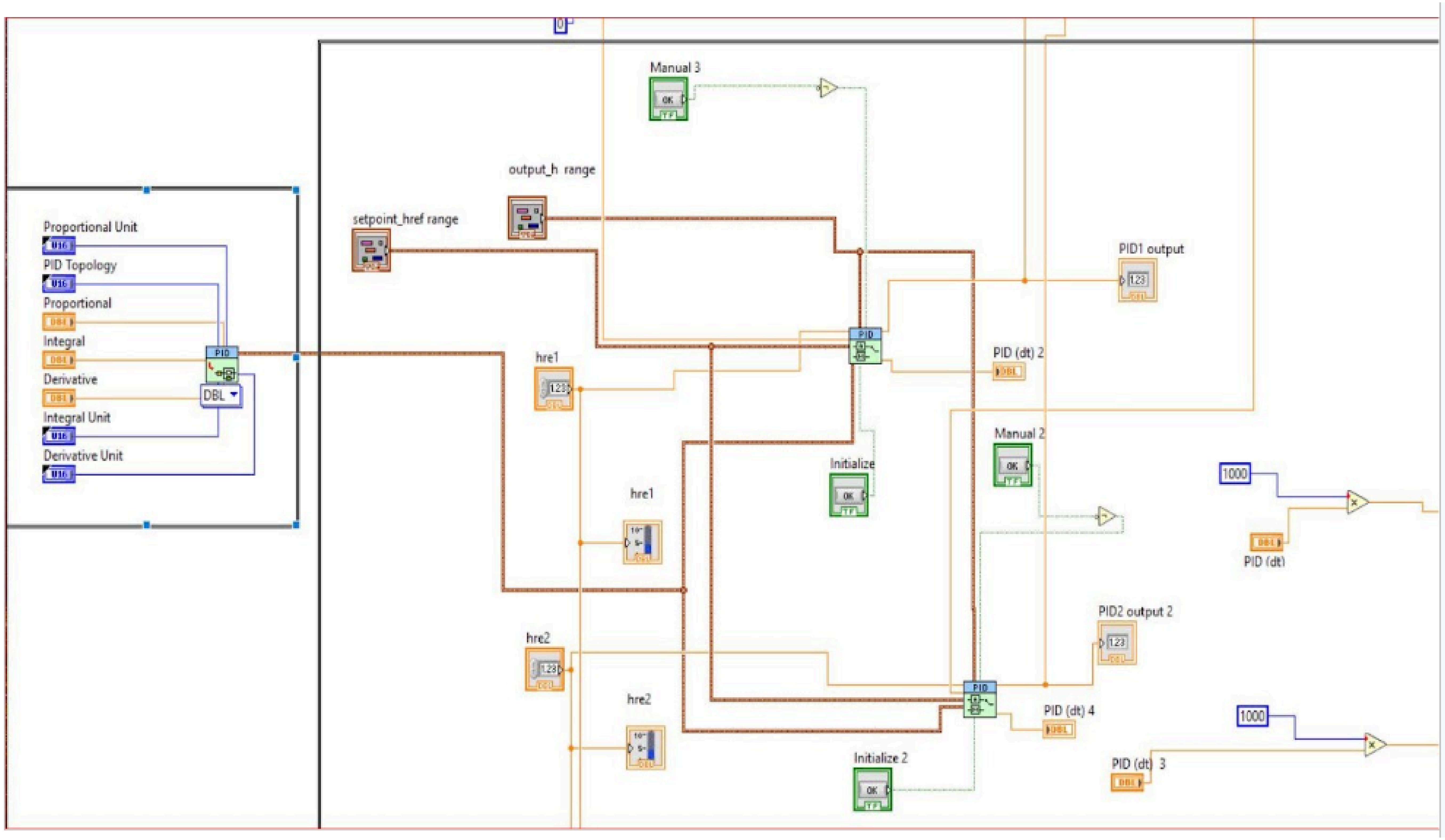


LabVIEW Implementation





LabVIEW Implementation

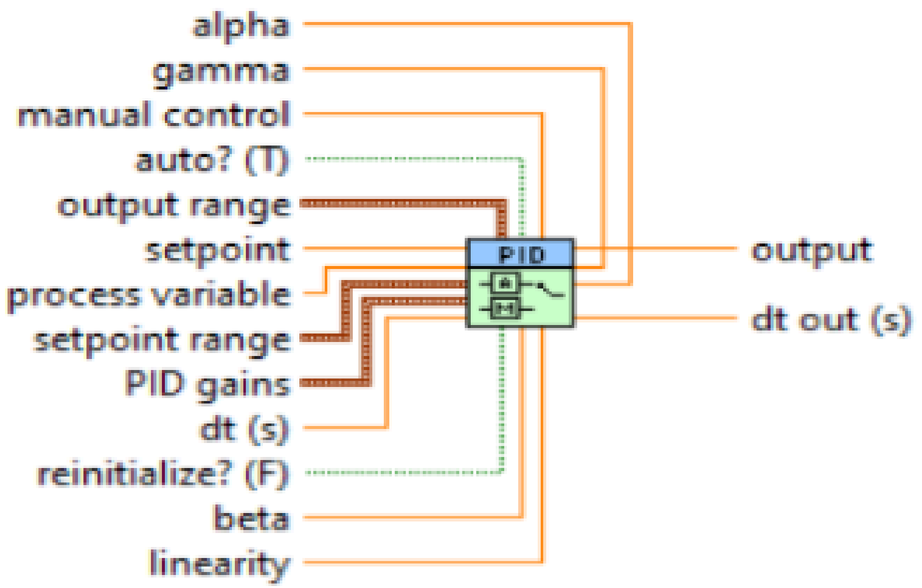


PID Control

PID advanced VI

Context Help

NI_PID_pid.lvlib:PID Advanced.vi



Implements a PID controller using a PID algorithm with advanced optional features. The advanced PID algorithm includes the features of the algorithm the PID VI uses, as well as manual mode control with bumpless manual-to-automatic transitions, nonlinear integral action, two degree-of-freedom control, and error-squared control. Use the DBL instance of this VI to implement a single control loop. Use the DBL Array instance to implement parallel multi-loop control.

Detailed help

PID Control

Objective: Achieve $h = h\text{ref}$ (maintain liquid levels at reference setpoints).
with varying of Qi

Manual Mode (Q1 and Q2 are fixed by the user) :

PID Control

Objective: Achieve $h = h_{ref}$ (maintain liquid levels at reference setpoints).
with varying of Qi

Manual Mode (Q1 and Q2 are fixed by the user) :

- Liquid levels evolve naturally according to the system's Torricelli-based dynamics (square-root flow laws).

PID Control

Objective: Achieve $h = h_{ref}$ (maintain liquid levels at reference setpoints).
with varying of Qi

Manual Mode (Q1 and Q2 are fixed by the user) :

- Liquid levels evolve naturally according to the system's Torricelli-based dynamics (square-root flow laws).
- No feedback correction is applied—the error $e = h_{ref} - h$ is ignored.

PID Control

Objective: Achieve $h = h_{ref}$ (maintain liquid levels at reference setpoints).
with varying of Qi

Manual Mode (Q1 and Q2 are fixed by the user) :

- Liquid levels evolve naturally according to the system's Torricelli-based dynamics (square-root flow laws).
- No feedback correction is applied—the error $e = h_{ref} - h$ is ignored.
- The user must manually guess and set fixed flow rates to reach desired levels.

PID Control

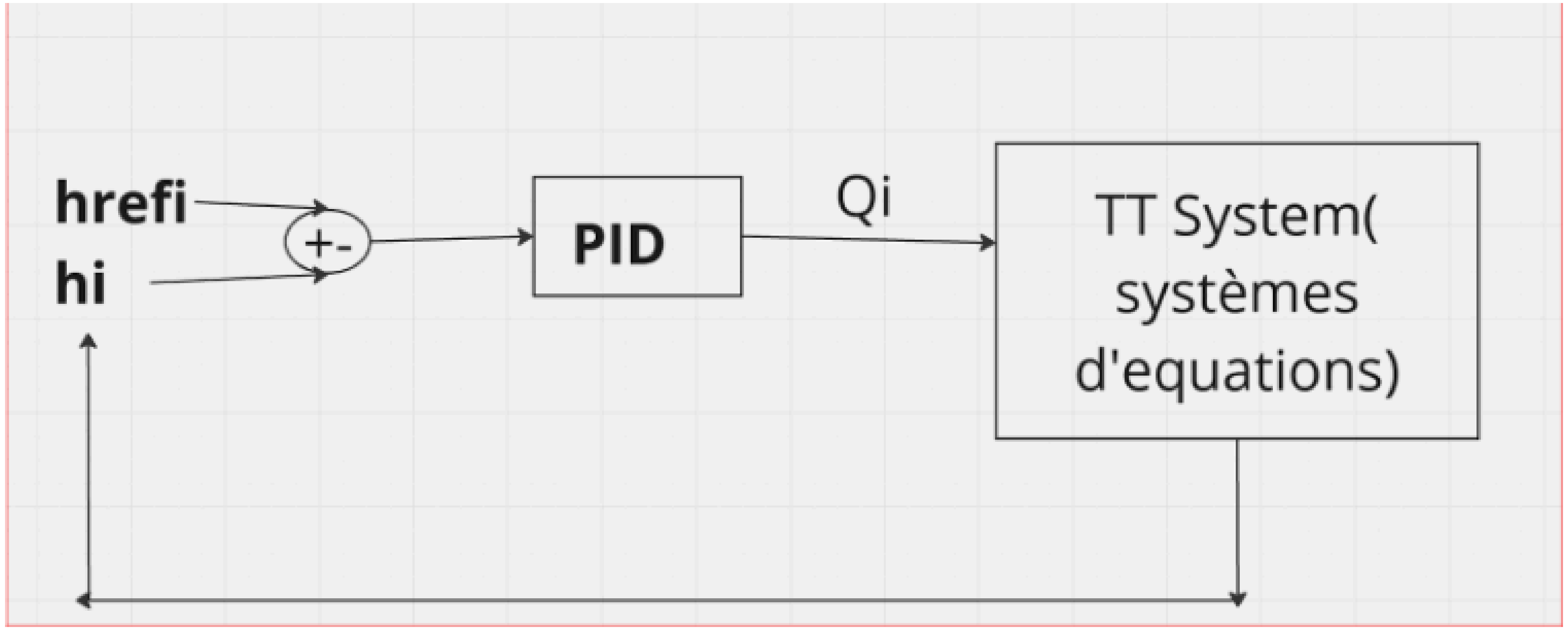
Objective: Achieve $h = h_{ref}$ (maintain liquid levels at reference setpoints).
with varying of Qi

Manual Mode (Q1 and Q2 are fixed by the user) :

- Liquid levels evolve naturally according to the system's Torricelli-based dynamics (square-root flow laws).
- No feedback correction is applied—the error $e = h_{ref} - h$ is ignored.
- The user must manually guess and set fixed flow rates to reach desired levels.
- No stability guarantee: Levels may diverge or oscillate unpredictably due to:
  - Nonlinear flow coupling between tanks.
  - Sensitivity to initial conditions.

Automatic Mode (PID Regulation) :

The PID controller provides closed-loop feedback control, dynamically adjusting pump flow rates (Q1,Q2) to maintain liquid levels ($h1$,$h2$,$h3$) at their desired setpoints (href) :



PID controller

Automatic Mode (PID Regulation) :

Kp **: Proportional Gain**

- Role: Reacts to the current error $e(t) = h_{ref} - h(t)$
- **Effect:**
  - **High** Kp **: Faster response but risks overshoot/oscillations.**
  - **Low** Kp **: Slower convergence (may not correct errors quickly)**

Automatic Mode (PID Regulation) :

Kd : **Derivative Gain**

- **Role: Anticipates future errors by reacting to the error's rate of change** $\frac{de(t)}{dt}$
- **Effect:**
  - **High** Kd **: Dampens oscillations but amplifies sensor noise.**
  - **Low** Kd **: Poor damping (levels may oscillate around** h**ref).**

Automatic Mode (PID Regulation) :

Ki : **Integral Gain**

- **Role**: Eliminates **steady-state error** by integrating past errors $\int e(t)dt$.
- **Effect**:
    - **High** Ki :Faster elimination of small residual errors but may cause **integral windup** (unstable growth).
    - **Low** Ki :Persistent small deviations (e.g., never quite reaching $h$ref).

Automatic Mode (PID Regulation) :

In this case, the flow of every tank can be expressed by the following equation :

$$Qi(t) = Kp \cdot ei(t) + Ki\int ei(\tau)d\tau + Kd\frac{de(t)}{dt}$$

Automatic Mode (PID Regulation) :

- **Nonlinear flows**: Kp must compensate for $hi-hj$ effects.
- **Coupling**: Kd mitigates interactions between tanks.
- **Setpoint Changes**: Ki ensures no residual error after transitions.

PID Advantages :

- **Stability:**
  Handles **strong coupling** (Tank 1 ↔ Tank 3 ↔ Tank 2).
  Rejects **unexpected disturbances** (e.g., sudden flow changes).

- **Precision:**

  Integral action eliminates steady-state error (unlike manual mode).

  Robust to Torricelli's nonlinearity ($\sqrt{(h_i - h_j)}$)

- Speed: **Proportional + Derivative** terms ensure fast, oscillation-free response.

PID Regulation Process

1. Initialization

- Set the reference level ($h$ref) for each tank.
- Switch to automatic mode to enable PID control of flow rates (Qi).

2. Real-Time Control Loop

- Measurement: LabVIEW reads liquid heights (hi) from sensors.
- Error Calculation: Computes tracking error:

$$ei(t)=h\text{ref},i-hi(t)$$

3. PID Action: Adjusts pump flows (Qi) to minimize error:

$$Qi(t)=Kp\ ei(t)+Ki\int ei(t)dt+Kd\,\frac{de(t)}{dt}$$

PID Regulation Process

4. Optimization: Tune *Ki* to:

- **Eliminate steady-state error (e.g., residual deviation).**
- **Avoid integral windup (use anti-windup in PID Advanced.vi).**

Key Adjustments :

- For slow response: ↑ ***Kp*** **or** ***Ki***.
- For oscillations: ↑ ***Kd*** or ↓ ***Kp***.

Conclusion

The implementation of a PID control system on the physical three-tank model validated the effectiveness of automatic control compared to manual operation. By integrating LabVIEW software and advanced PID loops, the system successfully maintained water levels at their setpoints despite nonlinearities and complex interactions between the tanks. The results confirm the importance of proper tuning of PID gains (Kp, Ki, Kd) to ensure stability, fast response, and elimination of steady-state error. However, achieving optimal performance requires precise fine-tuning of PID parameters. In some cases, it may also be advisable to consider a more advanced controller (such as predictive or adaptive control) to better manage the system's nonlinearities and dynamic couplings.