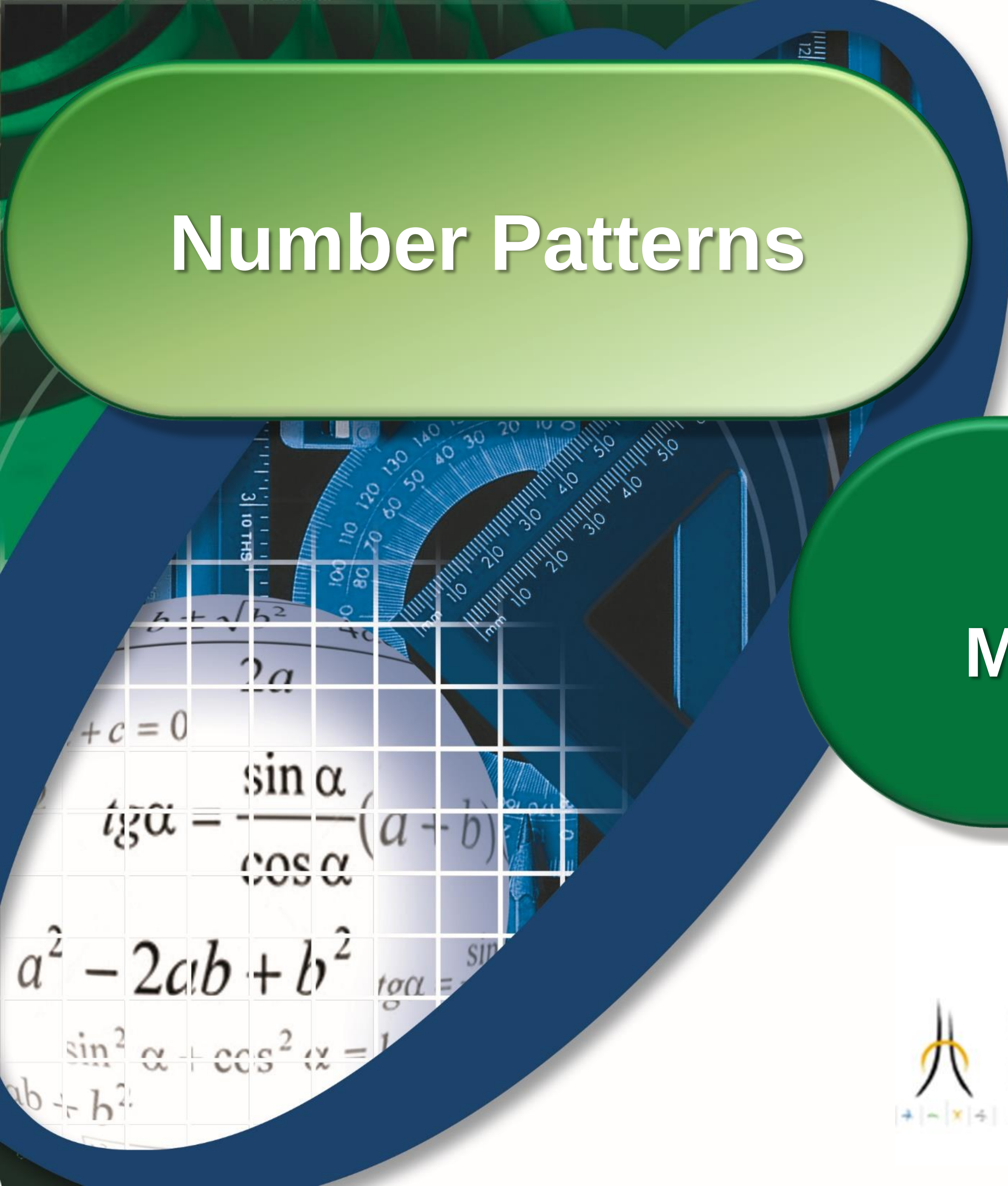
NELSON MANDELA UNIVERSITY

Number Patterns



Grade 10 CAPS Mathematics Series

Govan Mbeki Mathematics Development Centre

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Outcomes for this Topic

In this Topic we will focus on:

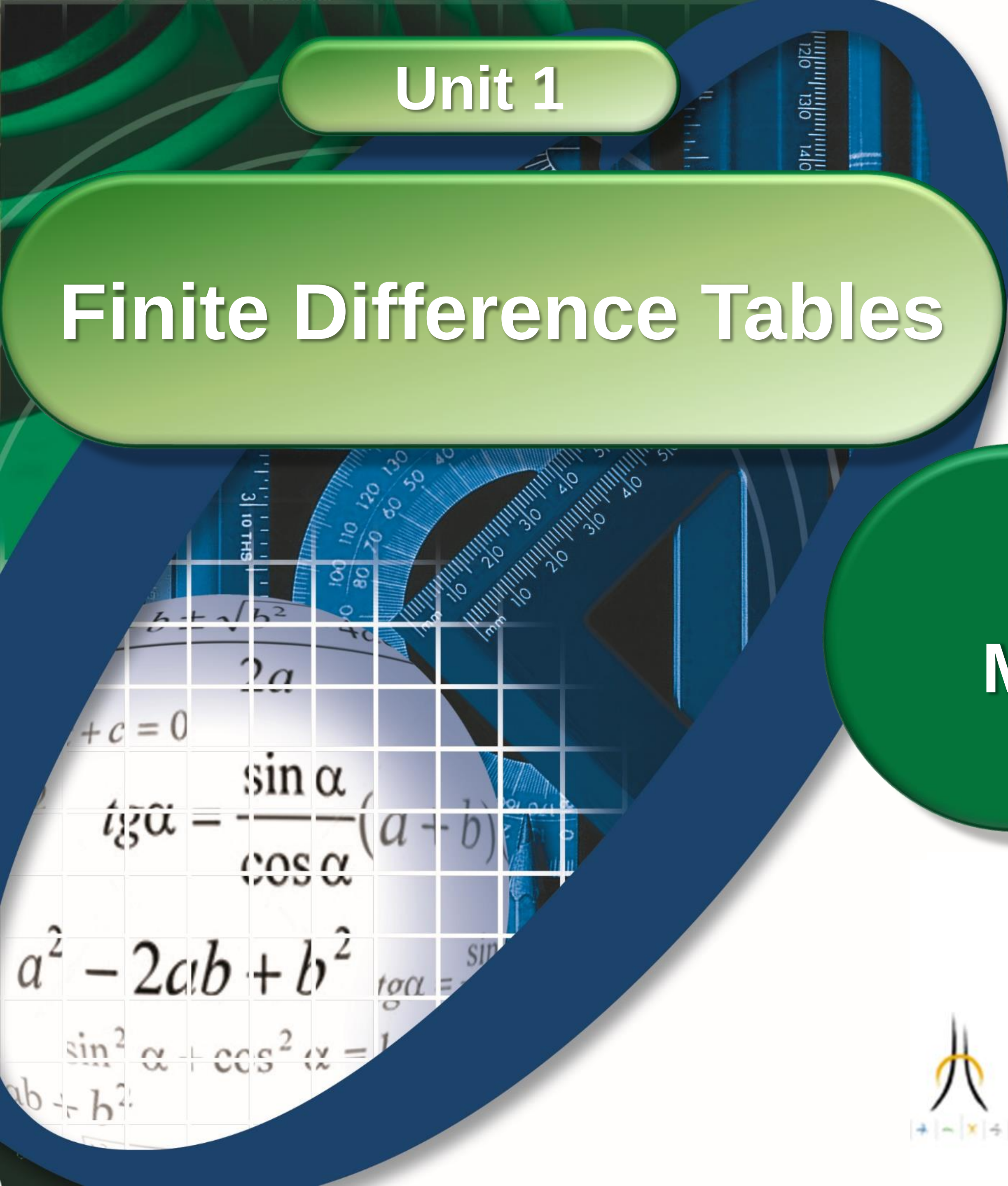
Finite Difference Tables (Unit 1)

Description of Best Number Patterns (Unit 2)

More advanced Number Patterns (Unit 3)

Unit 1

Finite Difference Tables



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Outcomes for this Unit

In this Unit we will:

- **Consider common number patterns linked to the four basic operations.**
- **Mention some special number patterns.**
- **Discuss the implications of constant first, second and third differences.**
- **Discuss methods of finding the generating rule if first differences are constant.**

Common Number Patterns linked to Basic Operations

Can you spot any patterns in the following lists of numbers?

1. $7; 10; 13; 16; 19; 22; \cdots$
2. $17; 15; 13; 11; 9; 7; \cdots$
3. $6; 12; 24; 48; 96; 192; \cdots$
4. $27; 9; 3; 1; \frac{1}{3}; \frac{1}{9}; \cdots$

1. Constant addition of 3.

$$7+3 \to 10+3 \to 13+3 \to \cdots$$

2. Constant subtraction of 2.

$$17-2 \to 15-2 \to 13-2 \to \cdots$$

3. Constant multiplication by 2.

$$6\times 2 \to 12\times 2 \to 24\times 2 \to \cdots$$

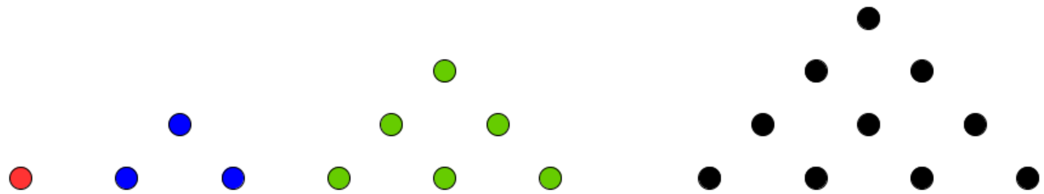
4. Constant division by 3.

$$27\div 3 \to 9\div 3 \to 3\div 3 \to$$

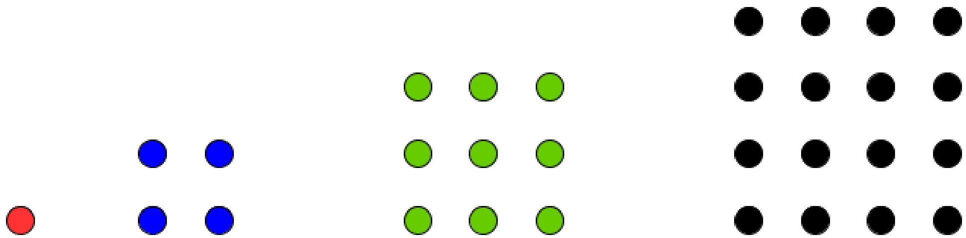
Some Special Number Patterns

Some special number sequences:

Triangular numbers : $1; 3; 6; 10; 15; 21; \cdots$



Square Numbers : $1; 4; 9; 16; 25; 36; \cdots$



Conjecture based on Constant First Differences

In examples 1 and 2 first differences are constant:

1. $7 \quad 10 \quad 13 \quad 16 \quad 19 \quad 22 \cdots$

$$\searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow$$

$$3 \quad 3 \quad 3 \quad 3 \quad 3$$

2. $17 \quad 15 \quad 13 \quad 11 \quad 9 \quad 7 \cdots$

$$\searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow\ \searrow\swarrow$$

$$-2 \quad -2 \quad -2 \quad -2 \quad -2$$

Conjecture : Sequence must be linear of nature with generating rule $T_n = an + b$

Check : 1. $T_n = 3n + 4$ and 2. $T_n = -2n + 19$

Conjecture based on Constant Second Differences

Normally required in Grade 11

In example 3 second differences are constant:

$$
\begin{array}{cccccccccccc}
3. & 4 & & 9 & & 16 & & 25 & & 36 & & 49 \quad \cdots \\
& & \searrow\swarrow & & \searrow\swarrow & & \searrow\swarrow & & \searrow\swarrow & & \searrow\swarrow & \\
& & \color{blue}{5} & & \color{blue}{7} & & \color{blue}{9} & & \color{blue}{11} & & \color{blue}{13} & \\
& & & \searrow\swarrow & & \searrow\swarrow & & \searrow\swarrow & & \searrow\swarrow & & \\
& & & \color{red}{2} & & \color{red}{2} & & \color{red}{2} & & \color{red}{2} & &
\end{array}
$$

Conjecture : Sequence must be quadratic of nature with generating rule $T_n = an^2 + bn + c$

Check : $T_n = (n+1)^2 = n^2 + 2n + 1$

Conjecture based on Constant Third Differences

Not required in CAPS Syllabus

In example 4 third differences are constant:

4. 8 27 64 125 216 343 $\cdots$

19 37 61 91 127

18 24 30 36

6 6 6

We ask the question why such sequences can be classified as linear, quadratic or cubic.
We will investigate these questions graphically.

Conjecture : Sequence must be cubic of nature

with generating rule $T_n = an^3 + bn^2 + cn + d$

Check : $T_n = (n+1)^3 = n^3 + 3n^2 + 3n + 1$

Graphical View linked to Constant First Differences

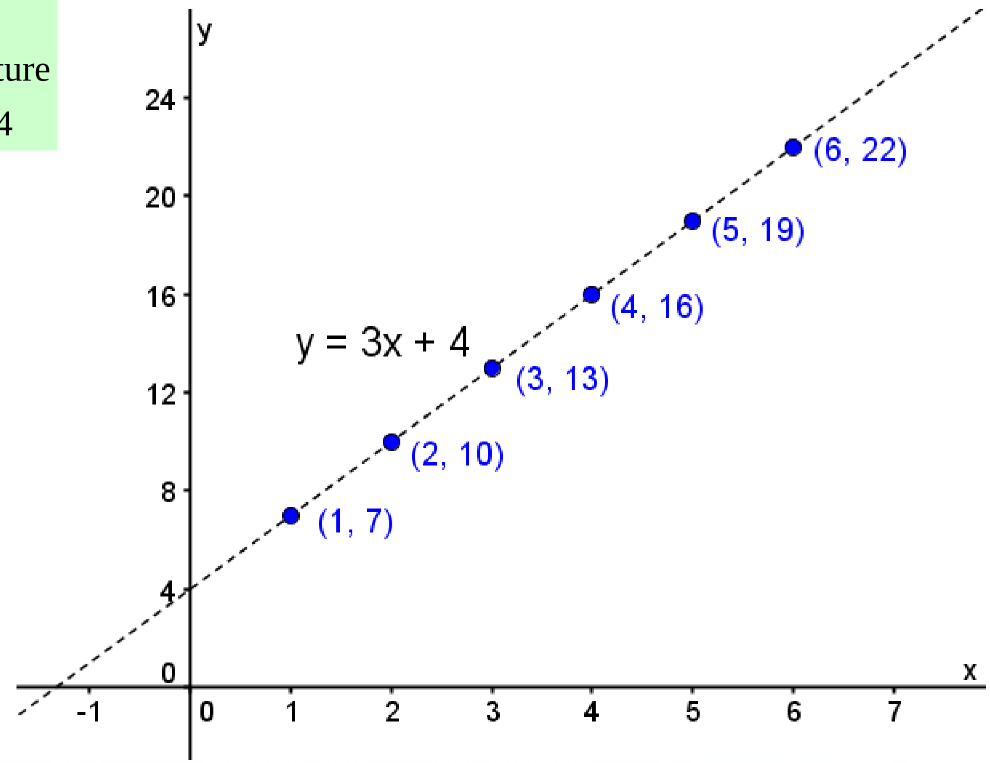
Consider the following examples:

$$\begin{array}{lllllllll} 1. & x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\ & y \rightarrow & 7 & 10 & 13 & 16 & 19 & 22 \cdots \\ & & & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow \\ & & & \color{blue}{3} & \color{blue}{3} & \color{blue}{3} & \color{blue}{3} & \color{blue}{3} \end{array}$$

Conjecture : Sequence must be linear of nature with generating rule $y = 3x + 4$

Note : $\Delta x = 1$ and $\Delta y = 3$

$\therefore$ If $\left[\Delta x \text{ constant} \Rightarrow \Delta y \text{ constant}\right]$ then sequences is linear of nature



Graphical View linked to Constant Second Differences

Normally required in Grade 11

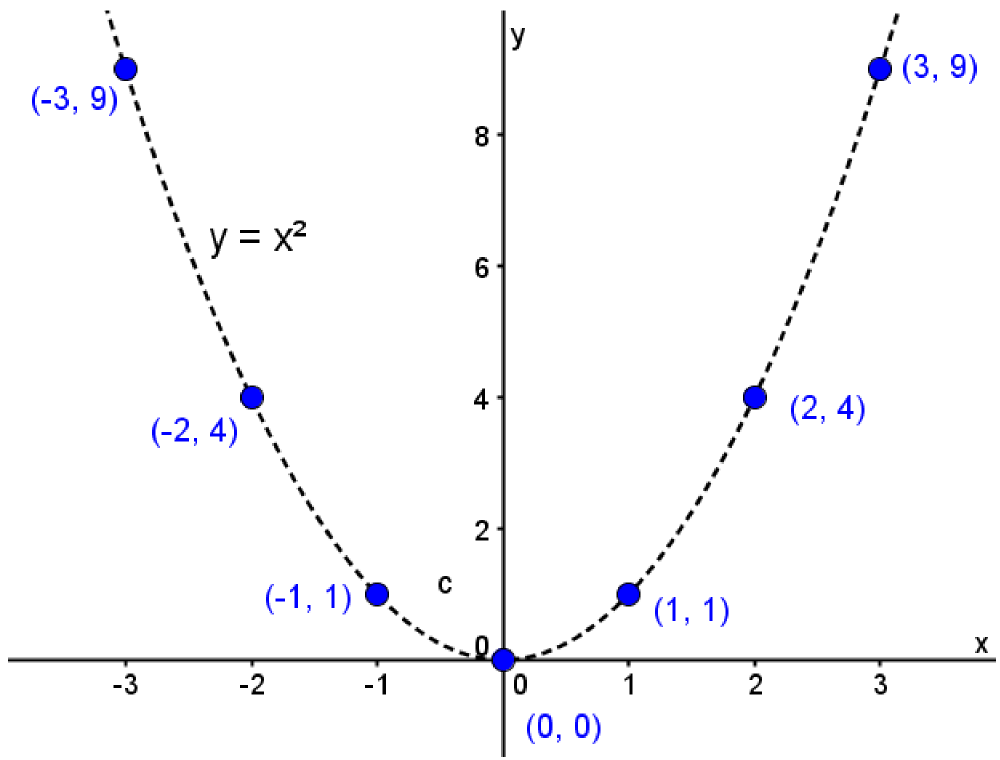
2. $x \rightarrow -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \cdots$

$y \rightarrow 9 \quad 4 \quad 1 \quad 0 \quad 1 \quad 4 \quad 9 \cdots$

$-5 \quad -3 \quad -1 \quad 1 \quad 3 \quad 5 \cdots$

$2 \quad 2 \quad 2 \quad 2 \quad 2 \cdots$

Conjecture: Sequence quadratic of nature with generating rule $y = x^2$



Graphical View linked to Constant Third Differences

Not required in CAPS Syllabus

3. $x \rightarrow\ -3\quad -2\quad -1\quad 0\quad 1\quad 2\quad 3 \cdots$

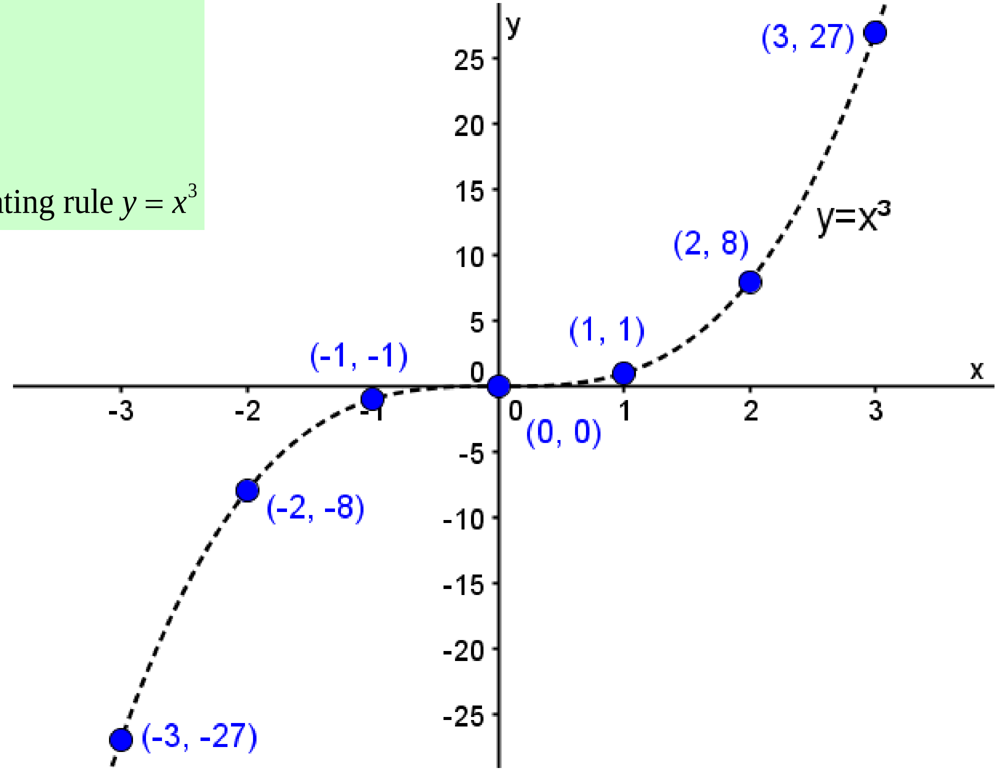
$y \rightarrow\ -27\quad -8\quad -1\quad 0\quad 1\quad 8\quad 27 \cdots$

$19\quad 7\quad 1\quad 1\quad 7\quad 19 \cdots$

$-12\quad -6\quad 0\quad 6\quad 12$

$6\quad 6\quad 6\quad 6$

Conjecture : Sequence cubic of nature with generating rule $y = x^3$



Finding Generating Rule if First Differences are Constant By means of inspection

We will discuss three possible methods:

1. By means of inspection
2. Solving two linear equations simultaneously
3. Using the STATS Mode of Fx82 ES Plus Calculator

Method 1: By means of Inspection

- Generating rule is of the form $y = ax + b$
- First difference is 3 $\Rightarrow a = 3 \Rightarrow y = 3x + b$
- Substitute $x = 1$ and $y = 7$ into $y = 3x + b$

  or any other ordered pair
- Thus $7 = 3 \times 1 + b \Rightarrow b = 7 - 3 = 4$
- Therefor the generating rule is $y = 3x + 4$
- Test correctness: Use any other ordered pair

  $13 = 3 \times 3 + 4 \Rightarrow (3;13)$ also satisfy $y = 3x + 4$

Example

$$\begin{array}{cccccccc} x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ y \rightarrow & 7 & 10 & 13 & 16 & 19 & 22 & \cdots \\ & & 3 & 3 & 3 & 3 & 3 & \end{array}$$

Finding Generating Rule if First Differences are Constant By solving two linear equations simultaneously

Example

$$\begin{array}{lcccccc} x\rightarrow & 1 & 2 & 3 & 4 & 5 & 6\ \cdots \\ y\rightarrow & 7 & 10 & 13 & 16 & 19 & 22\ \cdots \end{array}$$

$$3 \quad 3 \quad 3 \quad 3 \quad 3$$

Method 2: Solve two linear equations simultaneously

- Generating rule $y = ax + b \Rightarrow$ Need to find values of a and b
- Use any two ordered pairs and set up two linear equations:

  $(2;10) \Rightarrow 10 = 2a + b \cdots \boxed{1}$ and $(3;13) \Rightarrow 13 = 3a + b \cdots \boxed{2}$

- Solve equations $\boxed{1}$ and $\boxed{2}$ simultaneously:

  $\boxed{2} - \boxed{1} \Rightarrow a = 3$

  Back Substitute $a = 3$ in either $\boxed{1}$ or $\boxed{2}$

  $\therefore 10 = 2 \times 3 + b \Rightarrow b = 10 - 6 = 4$

  or $13 = 3 \times 3 + b \Rightarrow b = 13 - 9 = 4$

- Therefor the generating rule is $y = 3x + 4$

Finding Generating Rule if First Differences are Constant By means of the Casio Fx 82 ES Plus Calculator

Example

$$
\begin{array}{lcccccc}
x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\
y \rightarrow & 7 & 10 & 13 & 16 & 19 & 22 \cdots \\
 & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \\
 & 3 & 3 & 3 & 3 & 3 &
\end{array}
$$

Method 3: By means of Casio Fx 82 ES Plus Calculator

- Switch to STATS Mode and select linear option: $y = Bx + A$
- Enter at least two (Why?) sets of input-output values.
- Recall Stats Variable values: $B = 3$ and $A = 4$
- Therefor the generating rule is $y = 3x + 4$

Tutorial 1: Determine Generating Rule of a number sequence

For the sequence below:

$$
\begin{array}{lcccccc}
x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\
y \rightarrow & 17 & 15 & 13 & 11 & 9 & 7 \cdots \\
 & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \\
 & -2 & -2 & -2 & -2 & -2 &
\end{array}
$$

PAUSE Topic

- Do Tutorial 1
- Then View Solutions

1. Determine the generating rule by means of inspection
2. Setup and solve two linear equations simultaneously to determine the generating rule
3. Utilize your calculator in STATS Mode to determine the generating rule
4. Use a sequence value to test the correctness of your generating rule

Tutorial 1: Problem 1: Suggested Solution

For the sequence below:

$$
\begin{array}{lcccccc}
x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\
y \rightarrow & 17 & 15 & 13 & 11 & 9 & 7 \cdots \\
 & \multicolumn{6}{c}{\searrow\swarrow\searrow\swarrow\searrow\swarrow\searrow\swarrow\searrow\swarrow} \\
 & \multicolumn{6}{c}{-2 \quad -2 \quad -2 \quad -2 \quad -2}
\end{array}
$$

1. Determine the generating rule by means of inspection.

- Generating rule is of the form $y = ax + b$
- First difference is $-2 \Rightarrow a = -2 \Rightarrow y = -2x + b$
- Substitute $x = 1$ and $y = 17$ into $y = -2x + b$
- Thus $17 = -2 \times 1 + b \Rightarrow b = 17 + 2 = 19$
- Therefor the generating rule is $y = -2x + 19$

Tutorial 1: Problem 2: Suggested Solution

For the sequence below:

$x \rightarrow 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \cdots$

$y \rightarrow 17 \quad 15 \quad 13 \quad 11 \quad 9 \quad 7 \cdots$

$\searrow\swarrow \; \searrow\swarrow \; \searrow\swarrow \; \searrow\swarrow \; \searrow\swarrow$

$-2 \quad -2 \quad -2 \quad -2 \quad -2$

2. Setup and solve two linear equations simultaneously to determine the generating rule.

- Generating rule $y = ax + b \Rightarrow$ Need to find values of a and b
- Use any two ordered pairs and set up two linear equations:

  $(2;15) \Rightarrow 15 = 2a + b \cdots \boxed{1}$ and $(3;13) \Rightarrow 13 = 3a + b \cdots \boxed{2}$
- Solve equations $\boxed{1}$ and $\boxed{2}$ simultaneously:

  $\boxed{2} - \boxed{1} \Rightarrow a = -2$

  Back Substitute $a = -2$ in $\boxed{1}$:

  $\therefore 15 = 2 \times (-2) + b \Rightarrow b = 15 + 4 = 19$
- Therefor the generating rule is $y = -2x + 19$

Tutorial 1: Problem 3: Suggested Solution

For the sequence below:

$$
\begin{array}{lcccccc}
x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\
y \rightarrow & 17 & 15 & 13 & 11 & 9 & 7 \cdots
\end{array}
$$

$-2 \quad -2 \quad -2 \quad -2 \quad -2$

3. Utilize your calculator in STATS Mode to determine the generating rule.

- Switch to STATS Mode and select linear option: $y = Bx + A$
- Enter at least two (Why?) sets of input-output values.
- Recall Stats Variable values: $B = -2$ and $A = 19$
- Therefor the generating rule is $y = -2x + 19$

Tutorial 1: Problem 4: Suggested Solution

For the sequence below:

$$
\begin{array}{lcccccc}
x \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \cdots \\
y \rightarrow & 17 & 15 & 13 & 11 & 9 & 7 \cdots \\
 & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \searrow\swarrow & \\
 & -2 & -2 & -2 & -2 & -2 &
\end{array}
$$

4. Use a sequence value to test the correctness of your generating rule.

- Generating rule is $y = -2x + 19$
- Test correctness: Use any other ordered pair

$$-2 \times 3 + 19 = -6 + 19 = 13 \Rightarrow (3;13) \text{ also satisfies } y = -2x + 19$$

Unit 2

Description of Best Number Patterns

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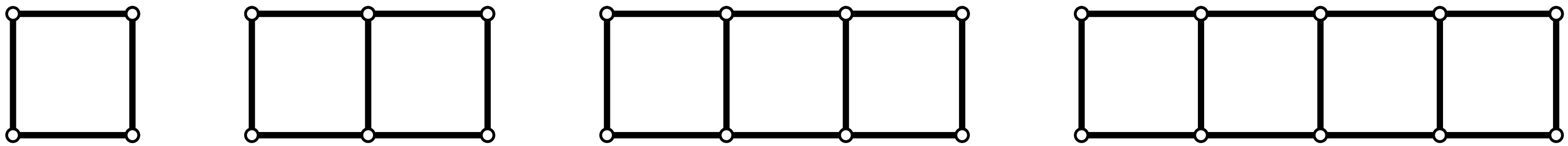
Outcomes for this Unit

In this Unit we will focus on:

- **How to construct an input-output table for a given number pattern.**
- **The extension of number patterns.**
- **What it means to find the best number pattern.**
- **The generalization of number patterns.**
- **The usefulness of generalized number patterns.**

Example of extending a Number Pattern

Matchsticks are used to make squares as shown below.



Observations from these sketches can be recorded in an input-output table:

Number of squares: s	1	2	3	4
Number of matches: $M(s)$	4	7	10	13

This number pattern can be extended:

Number of squares: s	1	2	3	4	5	6	7
Number of matches: $M(s)$	4	7	10	13	$13+3=16$	$16+3=19$	$19+3=22$

Finding the best number pattern

Finding the best number pattern implies:

- More than predicting the next term
- It means a general prediction of the output from the input
- Input is number of squares: s
- Output is the number of corresponding matches: $M(s)$
- Need to identify what is **constant** and what is **changing**

Find the best pattern in terms of problem conditions:

Number of squares: s	1	2	3	4	5	6	7
Number of matches: $M(s)$	4	7	10	13	16	19	22
First pattern: $M(s)$	4	$4+3$	$7+3$	$10+3$	$13+3$	$16+3$	$19+3$
Better pattern: $M(s)$	$4+0\times3$	$4+1\times3$	$4+2\times3$	$4+3\times3$	$4+4\times3$	$4+5\times3$	$4+6\times3$
Best pattern: $M(s)$	$1+1\times3$	$1+2\times3$	$1+3\times3$	$1+4\times3$	$1+5\times3$	$1+6\times3$	$1+7\times3$

Generalizing a number pattern

To generalize a number pattern implies:

- Finding the best output pattern in terms of the input pattern
- Identifing what is **constant** and what is **changing**
- Formulating a general rule (conjecture) or generating rule

Find the best output pattern in terms of input pattern:

Number of squares: s	1	2	3	4	5	6	7
Best pattern: $M(s)$	$1+1\times3$	$1+2\times3$	$1+3\times3$	$1+4\times3$	$1+5\times3$	$1+6\times3$	$1+7\times3$

Note in ouput framework $1+\square\times3$ only input changes.

Generalize output in terms of input: $M(s)=1+s\times3=1+3s$

Utilize the generalization of a number pattern

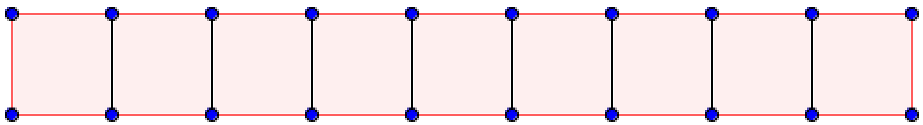
Generalization can be utilized to:

- Predict output for any input
- Predict input for any suitable output
- Correctness of predictions can be checked against problem conditions

Generating Rule: $M(s) = 1 + s \times 3 = 1 + 3s$

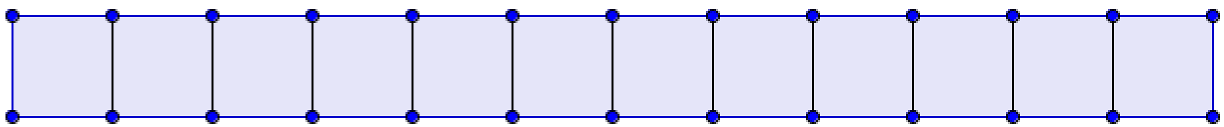
1. How many matches are needed to construct a term in the sequence consisting of 9 square cells?

   Solution: $M(9) = 1 + 3 \times 9 = 28$ matches



2. How many square cells will there be in a sequence if 37 matches are used in the construction thereof?

   Solution: $M(s) = 37 = 1 + 3s \Rightarrow 3s = 36 \Rightarrow s = 12 \Rightarrow$ this term in the sequence will consist of 12 squares



Role of Calculator in finding the generating rule

Number of squares: s	1	2	3	4
Number of matches: $M(s)$	4	7	10	13
First Differences	$7-4=3$	$10-7=3$	$13-10=3$	

$\therefore$ First differences are constant and equal to 3

If first differences are constant then relationship between input and output values is linear and can be written as $y=mx+c$

Input: x	1	2	3	4
Output: y	$m+c$	$2m+c$	$3m+c$	$4m+c$
First Differences	$(2m+c)-(m+c)=m$	$(3m+c)-(2m+c)=m$	$(4m+c)-(3m+c)=m$	

$\therefore$ First differences are constant and equal to m.

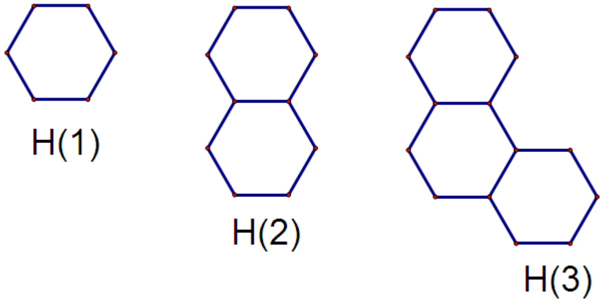
In this example:

- Calculator in STATS MODE can be utilize to determine the general rule

  Calculator formula: $y = Bx + A \Rightarrow$ Problem formula: $M(s) = 3s+1$
- Predict outputs for given inputs: Can show that $\hat{y}(9)=28 \Rightarrow M(9)=28$
- Predict inputs for given outputs: Can show that $\hat{x}(37)=12 \Rightarrow s=12$ if $M(s)=37$

Tutorial 2: Description of Best Number Patterns

Toothpicks are used to construct sequences of hexagons cells (Hexa means six) as shown in the sketches below.



1. Extend and complete the table below:

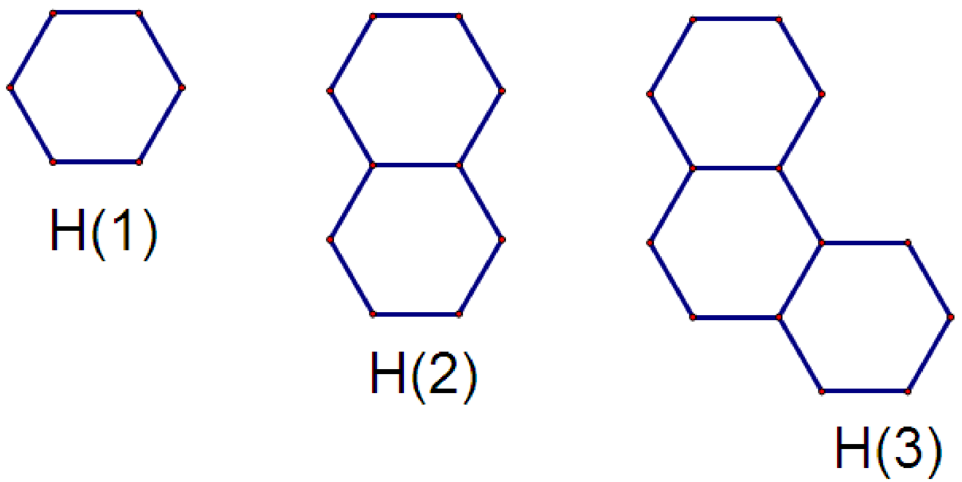
Input : Number of cells: n	1	2	3	4	5	6
Output : Number of toothpicks: $H(n)$						
Best Pattern						

2. Use your calculator to generalize the relationship between $H(n)$ and n.
3. Use your calculator to determine how many tootpicks are needed to construct a term in the sequence consisting of 120 hexagon cells.
4. Utilize the general formula obtained in 2. to check the correctness of your calculator answer in 3.
5. Use your calculator to determine how many hexagon cells will there be in a sequence term if 2 316 toothpicks are used in the construction thereof?
6. Utilize the general formula obtained in 2. to check the correctness of your calculator answer in 5.

PAUSE Topic

- Do Tutorial 2
- Then View Solutions

Tutorial 2: Problem 1: Suggested Solution



1. Extend and complete the table below:

Input : Number of cells: n	1	2	3	4	5	6
Output : Number of toothpicks: $H(n)$	6	11	16	21	26	31
Best Pattern	$1+1\times5$	$1+2\times5$	$1+3\times5$	$1+4\times5$	$1+5\times5$	$1+6\times5$

Tutorial 2: Problem 2: Suggested Solution

2. Use your calculator to generalize the relationship between $H(n)$ and n.

Input: Number of cells: n	1	2	3	4	5	6
Output: Number of toothpicks: $H(n)$	6	11	16	21	26	31

- Note that first differences are constant namely 5.
- Select STATS MODE and Linear option: $y = Bx + A$
- Enter input and output values

- Recall Stats Variable values for A and B.
- Record values: $A = 1$ and $B = 5$
- Write down the general rule for problem: $H(n) = 5n + 1$
- Use TABLE Mode to extend table above

Tutorial 2: Problems 3 and 4: Suggested Solutions

3. Use your calculator to determine how many toothpicks are needed to construct a term in the sequence consisting of 120 hexagon cells.

Use calculator to show that $\hat{y}(120) = 601$.

$\therefore H(120) = 601$

$\Rightarrow$ 601 toothpicks are needed to construct this term

4. Utilize the general formula obtained in 2. to check the correctness of your calculator answer in 3.

Use the formula: $H(n) = 5n + 1$ to calculate $H(120)$.

$\therefore H(120) = 5 \times 120 + 1 = 601$

$\Rightarrow$ 601 toothpicks are needed to construct this term

Tutorial 2: Problem 5 and 6: Suggested Solutions

5. Use your calculator to determine how many hexagon cells will there be in a sequence term if 2 316 toothpicks are used in the construction thereof?

Use calculator to show that $\hat{x}(2316) = 463.$

$\therefore H(463) = 2316$

$\Rightarrow$ There will be 463 hexagon cells in this term.

6. Utilize the general formula obtained in 2. to check the correctness of your calculator answer in 5.

Use the formula: $H(n) = 5n + 1$ to calculate n if $H(n) = 2316.$

$\therefore H(n) = 5n + 1 = 2316 \Rightarrow 5n = 2315 \Rightarrow n = 2315 \div 5 = 463$

$\Rightarrow$ In this term there will be 463 hexagon cells.

Unit 3

More Advanced Number Patterns

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Outcomes for this Unit

In this Unit we will focus on:

- **Number patterns in recurring letter sequences.**
- **Some more advanced number patterns.**

Number Patterns in Recurring Letter Sequences

Consider the following letter sequence:

TEATEATEATEA$\cdots$

Comment on which letter will appear in positions 510 and 673.

$$\begin{cases} \text{T appears in positions } 1,\ 4,\ 7,\ \cdots \text{ or } 3n-2 \text{ where } n \in \mathbb{N} \\ \text{E appears in positions } 2,\ 5,\ 8,\ \cdots \text{ or } 3n-1 \text{ where } n \in \mathbb{N} \\ \text{A appears in positions } 3,\ 6,\ 9,\ \cdots \text{ or } 3n \text{ where } n \in \mathbb{N} \end{cases}$$

$\because$ $510 = 3 \times 170$ the letter A will appear in position 510 for the 170th time in this sequence.

$\because$ $673 = 3 \times 225 - 2$ the letter T will appear in position 673 for the 225th time in this sequence.

More advanced number pattern: Finding the remainder

Problem : What is the remainder if 3^{8972} is divided by 13?

Use problem solving strategies of (1) Construct a table and (2) Look for a pattern.

Index of 3 (I)	Remainder (R)
1	3
2	9
3	1
4	3
5	9
6	1
$\vdots$	$\vdots$

Pattern

$$\begin{cases} R=1 \Leftrightarrow I=3n;\ n\in\mathbb{N} \\ R=9 \Leftrightarrow I=3n-1;\ n\in\mathbb{N} \\ R=3 \Leftrightarrow I=3n-2;\ n\in\mathbb{N} \end{cases}$$

$8972=3\times 2\ 991-1$

$\therefore$ The remainder if 3^{8972} is divided by 13 is 9.

Tutorial 3: More advanced Number Patterns

1. Consider the following letter sequence:

SOUPSOUPSOUPSOUP$\cdots$

Comment on which letter will appear in positions 749 and 847.

2. What is the units digit in 7^{399}?

PAUSE Topic

- Do Tutorial 3
- Then View Solutions

Tutorial 3: Problem 1: Suggested Solution

1. Consider the following letter sequence:

SOUPSOUPSOUPSOUP$\cdots$

Comment on which letter will appear in positions 749 and 847.

$$\begin{cases} \text{S appears in positions } 1,\ 5,\ 9,\ \cdots \text{ or } 4n-3 \text{ where } n \in \mathbb{N} \\ \text{O appears in positions } 2,\ 6,\ 10,\ \cdots \text{ or } 4n-2 \text{ where } n \in \mathbb{N} \\ \text{U appears in positions } 3,\ 7,\ 11,\ \cdots \text{ or } 4n-1 \text{ where } n \in \mathbb{N} \\ \text{P appears in positions } 4,\ 8,\ 12,\ \cdots \text{ or } 4n \text{ where } n \in \mathbb{N} \end{cases}$$

$\because$ $749 = 4 \times 188 - 3$ the letter S will appear in position 749 for the 188th time in this sequence.

$\because$ $847 = 4 \times 212 - 1$ the letter U will appear in position 847 for the 212th time in this sequence.

Tutorial 3: Problem 2: Suggested Solution

2. What is the units digit in 7^{399}?

Use problem solving strategies of:

1. Construct a table $\left(\text{Use calculator to construct a table for } y = 7^x\right)$ and

2. Look for a pattern.

Index of 7 (I)	Units Digit (U)
0	1
1	7
2	9
3	3
4	1
5	7
6	9
7	3
8	1
$\vdots$	$\vdots$

Pattern

$$\begin{cases} U = 1 \Leftrightarrow I = 4n;\ n \in \mathbb{N}_0 \\ U = 7 \Leftrightarrow I = 4n+1;\ n \in \mathbb{N}_0 \\ U = 9 \Leftrightarrow I = 4n+2;\ n \in \mathbb{N}_0 \\ U = 3 \Leftrightarrow I = 4n+3;\ n \in \mathbb{N}_0 \end{cases}$$

$399 = 4 \times 99 + 3$

$\therefore$ The units digit for 7^{399} is 3.

End of Topic Slides on Number Patterns

REMEMBER!

- Consult text-books and past exam papers and memos for additional examples.
- Attempt as many as possible other similar examples on your own.
- Compare your methods with those that were discussed in these Topic slides.
- Repeat this procedure until you are confident.
- Do not forget: **Practice makes perfect!**