Network Science		
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Introduction to Network Science:

- System with N components, each with k states $\Rightarrow k^N$ configurations.
- Functions assigning relative probabilites to each configurations.
- The partition functions normalizes the probability

22.1 Ising Model

$\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow$ when $N \longrightarrow$.

The up and down arrows signifies the two different states/configurations of magnets. Consider there are N magnets with different configurations, some are pointing up and others are pointing down, i,e $\sigma_i \in \{-1, +1\}$. $p(c) =^{-\beta E(c)}$ where $E(c) = \sum_i [\sigma_i \sigma_{i+1} + \frac{\alpha}{\beta}\sigma_i]$. β is the inverse temperature. Here $k = 2$.

$$p(c) \propto^{-\beta E(c)}$$

where $p(c) \equiv$ relative probability of configuration C., $E(c) \equiv energyofconfigC$ and $\beta = \frac{1}{RT}$ where T is temparature.

$$E = \Sigma_i(\sigma_i \sigma_{i+1} + \frac{\alpha}{\beta}\sigma_i)$$

.

Z = Partition Function = Σ_i(relative probabilty (C)).

$Z^N(\alpha, \beta) = \Sigma_C e^{\beta(\Sigma_i(\sigma_i\sigma_{i+1} + \frac{\alpha}{\beta}\sigma_i))} = Z^{(N)}(\alpha, \beta)$.

There are k^N terms in the above sum, where each term is easy to compute.

For high temperature, $\beta \to 0$ and for low temperature, $\beta \to \infty$.

22.2 Renormalization Group

-powerful technique, -connection to computer science;

$$Z^(2)(\alpha,\beta) = 2e^{-2\beta}cosh(2\alpha) + 2e^{2\beta}$$

.

$$Z^(4)(\alpha,\beta) = 2e^{-4\beta}cosh(4\alpha) + 8cosh(2\alpha) + 4 + 2e^{2\beta}$$

$$\partial(lnZ)/\partial\alpha = N < \sigma >$$

where $< \sigma >$ is the average value over all σ.

$$\frac{\partial^2(lnZ)}{\partial\alpha^2} = \text{variance of}\sigma,$$

$$\frac{\partial(lnZ)}{\partial\beta} = N < \sigma_i\sigma_{i+1} >$$

$$\frac{\partial^2(lnZ)}{\partial\beta^2} = \text{variance of} < \sigma_i\sigma_{i+1} >$$

.

Suppose A intersects only with B and C.

$$\begin{aligned} p(C = A) &= p(A|B, C, D, E, ...) \\ &= p(A|B, C)p(B, C, D, E, ...) \end{aligned}$$

$$\begin{aligned} Z &= \sum_C e^{-\beta E(C)} \\ &= \sum_C^{\prime} e^{-\beta E(C')} \sum \{e^{\beta[\sigma_1\sigma_2+\sigma_2\sigma_3+(\alpha/\beta)\sigma_2]}\} \\ &= \sum_C^{\prime} e^{-\beta E(C')} \{e^{-\beta[\sigma_1+\sigma_3+\frac{\alpha}{\beta}]} + e^{\beta[\sigma_1+\sigma_3+\frac{\alpha}{\beta}]}\} \end{aligned}$$

where $\sigma = \{-1, +1\}$.

$$f(\sigma) = F_1 + F_2(\sigma)$$

, for $\sigma \in \{-1, +1\}$.

where

$$F_1 = \frac{[f(1) + f(-1)]}{2}$$

and

$$F_2 = \frac{[f(1) - f(-1)]}{2}$$

$$2cosh(\beta(\sigma_1 + \sigma_3) + \alpha) = e^{-\beta' \sigma_1 \sigma_3 - \alpha' \sigma_1 - \alpha' \sigma_3}.$$

$$\begin{aligned} \ln(2cosh(\beta(\sigma_1 + \sigma_3) + \alpha)) &= -\beta' \sigma_1 \sigma_3 - \alpha' \sigma_1 - \alpha' \sigma_3 \\ &= F_1(\sigma_3) + F_2(\sigma_3)\sigma_1 \\ &= F_{11} + F_{12}\sigma_3 + F_{21}\sigma_1 + F_{22}\sigma_3\sigma_1 \end{aligned}$$

Q: Can we connect this idea to clusering ? Can we use to achieve a heirarchical decomposition of the network ?