Network Science

Lecture 26: October 13, 2009 Lecturer: Achla Marathe

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Social Power in Networks

26.1 Nash Bargaining Solution

Suppose there are two people bargaining for a split of \$1. Also, each person has an *outside option*, x and y, respectively, which means that if they refuse the split, they can each get x and y respectively. Note that $x + y \leq 1$. It is shown in Fig. 26.1.

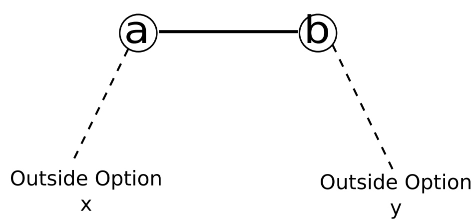

Figure 26.1: Bargaining with outside options.

The Nash bargaining solution to the problem is as follows:

- $\frac{x+1-y}{2}$ to a;
- $\frac{y+1-x}{2}$ to b;

26.2 Ultimatum Game

Ultimatum game is often played in economic experiments, which involves two player to divide a dollar. The procedure is as follows:

- Player A is given a dollar and required to propose a split, i.e., how much to give to player B and how much to keep himself.
- Player B is then given the option to accept or reject the proposal.

- if B accepts the offer, the players will divide the dollar as proposed, if not, each of them gets nothing.

Let's suppose the minimum split unit is 1 cent. Since B will accept any positive offer, so A will give B as little as possible and keep the rest. Here A proposes to give B \$0.01, B can either accept it or reject it. But obtaining 1 cent is better than nothing. So B will accept it. The conclusion in this simple case is that the man with all the power will offer as little as possible, and the one with essentially no power will accept anything offered.

26.2.1 Stable Outcomes

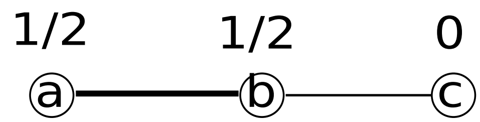

Figure 26.2: Unstable outcome

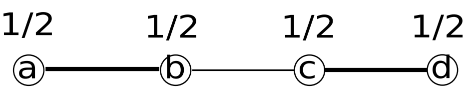

Figure 26.3: Stable outcome

In Fig. 26.3 and 26.2, the bold line between two nodes represents the exchange occurrence. And the number above each node represents how much the node get from the exchange. The number 0 means that the nodes did not involve in the exchange. The total value over two connected nodes should be 1 if the edge between them are bold.

In the sense of "stable outcome", we mean that no node v can "steal" away any other node w from an existing agreement, using an offer that makes both v and w better off. Fig. 26.2 shows a case that c can use an offer that gives b 2/3 while keep 1/3. This offer makes both b and c better off. So the outcome is unstable. However, you can not propose such a divide in Fig. 26.3 to make two of the player better off.
Therefore we define that an outcome of network exchange is stable if it contains no instabilities.

26.2.2 Balanced Outcomes

In case there are many possible stable outcomes for a given network, we define one of them to be balanced, if and only if for each edge in the matching, the split of the money represent the Nash bargaining outcome for the two nodes involved, given the best outside options for each node provide by the values in the rest of the network.