

## Diffusion of Innovation

People's interactions usually occur with a close set of people, friends and colleagues—local interactions. For example, political viewpoints can be adopted from our friends, even if nationally that might be a minority. In this lecture, we are modeling individual decision making in a social setting, where people align their behavior to the close contacts in their networks.

### 19.1 Diffusion of Innovation

New behaviors, practices, opinions, conventions and technologies spread from person to person through social contacts as people are influenced by their friends. The two foremost studies in this area were by Ryan and Gross in 1943 about adoption of hybrid corn and coleman, Katz and Menzel's study of adoption of tetracycline by physicians in the United States. Although, the studies belonged to diverse communities, they had some common elements. Novelty and initial lack of understanding of the innovation made it risky to adopt. The early adopters, in both cases, had a higher socio-economic status and a tendency to travel widely. In both cases, decisions about adopting were influenced by the social structure, and neighbors, friends and colleagues had more influence. There are also a number of reasons why an innovation can fail to spread through a population:

- *Complexity* to understand and implement
- *Observability* to be aware that others are using it
- *Trialability* to mitigate risks by adopting gradually and incrementally, and most crucially,
- *Compatibility* with social system its entering.

The principle of homophily—people tend to interact more with others like themselves—can prevent spread of innovation, as it usually comes from “outside” the social community.

### 19.2 Models for Diffusion through a Network

Models can be built on the basic assumptions about individual decision making, incorporating either *informational* or *direct-benefit* effects. Direct benefit models are based on considering that adopting new behavior is beneficial if more neighbors (in the social network) adopt it. For example, to collaborate with colleagues, it is beneficial if everyone is using the same technology. This idea can be captured by a coordination game.

### 19.2.1 Networked Co-ordination Game

We can represent a game in which  $v$  and  $w$  are the players and  $A$  and  $B$  are possible strategies. In this game, each node has a choice between two possible behaviors  $A$  and  $B$  and if two nodes  $v$  and  $w$  are connected by an edge, they have an incentive to have matching behaviors. The payoffs for the game are defined as follows:

1. if  $v$  and  $w$  both adopt behavior  $A$ , they get a payoff of  $a > 0$
2. if they both adopt  $B$ , they each get a payoff of  $b > 0$ ; and
3. if they adopt opposite behaviors, they each get a payoff of 0.

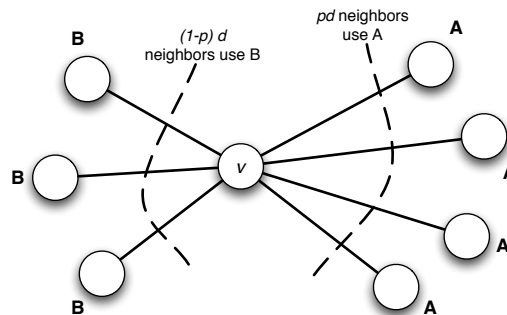


Figure 19.1: Networked coordination game. Neighborhood of  $v$  with the behaviors adopted by its neighbors.

Let  $p$  and  $(1 - p)$  be the fraction of  $v$ 's neighbors having behaviors  $A$  and  $B$ , respectively. If  $v$  has  $d$  total neighbors, then  $pd$  have adopted  $A$  and  $(1 - p)d$  have adopted  $B$ .  $v$ 's payoff for adopting  $A$  is  $pda$  and, payoff for adopting  $B$  is  $(1 - p)db$ . Thus,  $A$  is a better choice if  $pda \geq (1 - p)db$ , or,  $p \geq \frac{b}{a+b}$ . Suppose,  $q$  denotes the expression on the right hand side, then it describes a simple threshold rule. There is a tie breaking question when exactly  $q$  fraction of node's neighbors adopt  $A$ , then we will also adopt  $A$ , rather than  $B$ .

### 19.2.2 Cascading Behavior

In the network coordination game, for a given network, there are 2 equilibria: one in which everyone adopts  $A$ , and everyone adopts  $B$ . An interesting question is, how easy it is to “tip” the network from one to the other. The other question that can be answered relates to the intermediate equilibria.

We start with a network with everyone using a particular behavior (say,  $B$ ). Now, a small number of initial adopters change their behavior and decide to use  $A$ . This change is not due to the payoffs. Given this fact that some have changed their behavior, other people in the network might change their behavior causing a cascade of the changes in the network. The number of people that switch behavior is dependent on the network structure, the choice of initial adopters and the value of threshold  $q$ . Figure 19.2 shows the steps taken to change behaviors in a sample network.

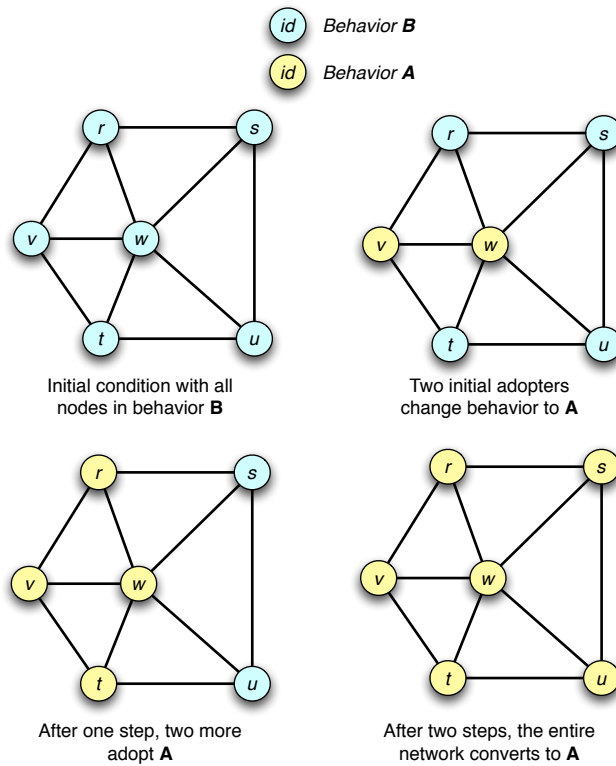


Figure 19.2: Coordination game is setup such that  $a = 3$  and  $b = 2$ . Nodes will switch when the fraction of neighbors  $q = 2/5$  use behavior A.  $v$  and  $w$  are chosen as the initial adopters of A, while everyone uses B.

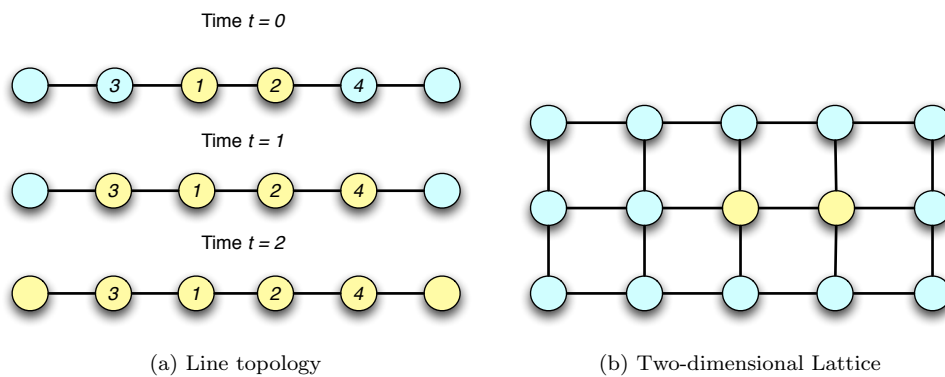


Figure 19.3: Diffusion of information over different topologies. Game setup as  $a = 3$  and  $b = 2$  and  $q = 2/5$ . (a). Everyone switches from behavior B to the new behavior A when nodes a and b switch; (b). In an 2-dimensional lattice, no one switches.

It is also important to consider situations where the adoptions continue for some time but, then stops. Consider the network in Figure 19.4a. Starting with 7 and 8 as initial adopters, in a coordination game with  $a = 3$  and  $b = 2$ , we have, at  $t = 0$ , 5 and 10;  $t = 1$ , 4 and 9; and  $t = 3$ , 6 adopting A. But, this process does not continue further and none of the other nodes adopt A. The outcome is shown in Figure 19.4b.

### 19.2.3 Cascading Communities and “Viral Marketing”

We can make a few observations from the network in Figure 19.4a. A counter-intuitive conclusion is that tightly knit communities in a network can hinder the spread of innovation. Behavior A was able to reach all the well connected nodes in the network, but could not jump along links 3–2, 8–14 and 9–11, respectively. Suppose, we consider A and B as technologies, it becomes clear that in order for people to accept one technology over the other, it is important to increase the payoff to make it acceptable to people. For example, in the network of Figure 19.4a, simply changing payoff from  $a = 3$  to  $a = 4$  ensures that the technology spreads to the entire network (changes the threshold  $q$  from  $2/5$  to  $1/3$ ).

The other way to make people adopt technology A over B is to select the initial adopters, intelligently. If the marketer were to make nodes 12 or 13 to switch to A, then the cascading effects of the switching would start again. This shows that the question of how to choose the individual nodes for switching to a new product is subtle and is based on the positions of the nodes in the underlying network.

### 19.2.4 Cascades and Clusters

In this section, we formalize the cascading behavior of the simple model. We define a cluster of density  $f$  as the set of nodes such that each node in the set has at least a fraction  $f$  of the network neighbors in the set. Figure 19.5 shows four-node clusters each of density  $2/3$ . There is some level of internal “cohesion” among the nodes in the cluster, i.e., they are connected to other nodes that are in the same cluster.

From Figure 19.4a, we see two communities each of density  $2/3$  that remain unaffected by adoption of behavior A. We now formulate a result that essentially says that the adoption of new behavior stops when it runs into a deeper cluster.

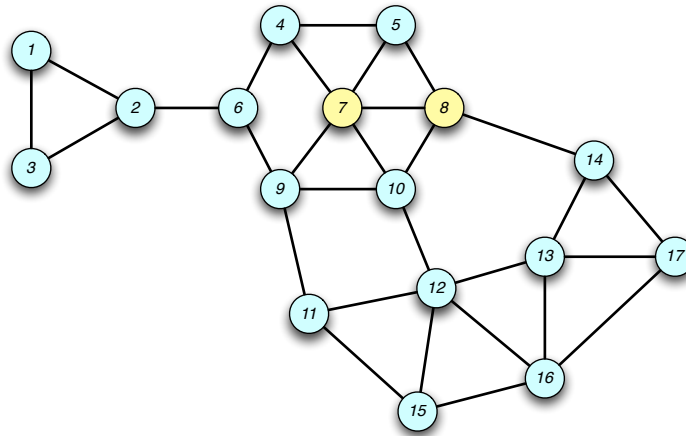
**Claim 1** *Innovation can only spread around the network if-and-only-if, there are clusters of density  $(1 - q)$ .*

- (i). *If the remaining network consists of clusters of density greater than  $(1 - q)$ , then the set of initial adopters will not cause a cascade.*
- (ii). *Whenever a set of initial adopters do not cause a cascade with threshold  $q$ , the remaining network must consist of clusters of density greater than  $(1 - q)$ .*

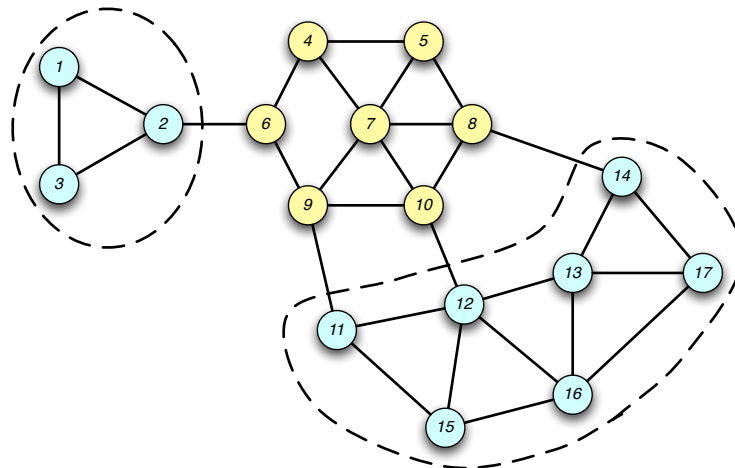
**Proof:**

(i). **Clusters are obstacles to cascades**

*By Contra-positive*



(a) Initial condition



(b) Final condition

Figure 19.4: Diffusion of information over other topologies. Game setup as  $a = 3$  and  $b = 2$  and  $q = 2/5$ . **(a)**. Initial condition of the network with nodes 7 and 8 adopting behavior **A** at time  $t = 0$ ; **(b)**. At  $t = 3$  the condition of the network when nodes 7, 8, 5, 10, 9, 4, 6 have all switched to behavior **A**. No more switches take place in the network.

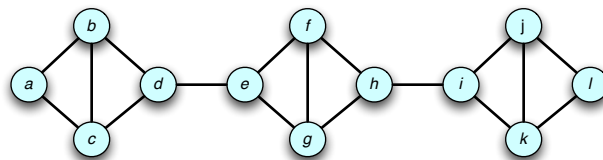


Figure 19.5: A collection of 4-node clusters with each cluster having a density of  $2/3$ .

Consider an arbitrary network in which behavior  $A$  is spreading with threshold  $q$ , starting with a set of initial adopters. Suppose the remaining part of the network contains a cluster of density greater than  $(1 - q)$ .

Lets assume the opposite — some node inside the cluster adopts  $A$ . Let  $v$  be that node., and it adopts  $A$  at time  $t$ . For this to happen, there was a decision at  $(t - 1)$  that created the condition for this to occur at  $t$ . Since, no other node in the cluster adopted  $A$ , it is clear that the only node that switched is outside the cluster. Since the cluster has a density of  $(1 - q)$ , more than  $(1 - q)$  fraction of  $v$ 's neighbors are within the cluster. This means that  $v$  has  $q$  fraction of nodes outside the cluster. Since these are the only neighbors of  $v$  that can cause the switch in  $v$ 's behavior, there is a contradiction.

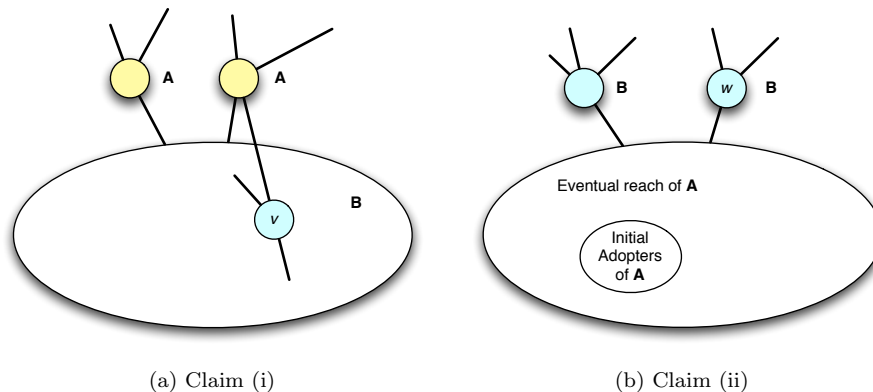


Figure 19.6: **(a)**. The spread of new behaviors when nodes have threshold  $q$  stops on reaching a cluster of density greater than  $(1 - q)$ ; **(b)**. If the cascade of  $A$  stops before spreading to the entire network, then what remains of the network form a cluster of density greater than  $(1 - q)$ .

## (ii). Clusters are the only obstacles to cascades

This part will prove that not only do clusters stop cascading of behaviors, but are in fact the only obstacles. We show this by proving that whenever initial adopters fail to cause a cascade with threshold  $q$ , the remaining network is a cluster of density  $(1 - q)$ . Consider running the process of adopting behavior  $A$  starting from initial adopters, until it stops (it stops because there are nodes that have adopted  $B$  and cannot be converted). Let  $S$  denote the set of nodes using  $B$ . Consider a node  $w$  in  $S$ . Since  $w$  does not want to switch, it must have a fraction of neighbors using  $A$  less than those using  $B$ . Thus the fraction of nodes using  $B$  is greater than  $(1 - q)$ . But the only nodes using  $B$  belong to the set  $S$ , implying that  $S$  is a cluster of density greater than  $(1 - q)$ .