Energy-Efficient Routing Schemes for Wireless Sensor Networks

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***Abstract*— Microsensors operate under severe energy constraints. Depending on the application, sensors can be thrown randomly in an area of interest ("sprinkled in a field") or, in some cases, can be manually placed in specific positions. The sensor network is typically ad hoc, formed by local self-configuration. Data-centric routing is a new useful paradigm for energy-constrained sensor networks. The data coming from different sources are aggregated at the intermediate nodes on the way; that reduces volume of data (eliminating redundancy) and saves transmission energy. In this paper, we design and analyze optimal network configurations and data-centric routing schemes to minimize energy consumption for both random and manual placement of nodes. Specifically, the paper makes the following contributions. We first study energy-optimal network configurations for manual and random placement of nodes under a natural coverage criterion. In particular, we show that in a linear network, energy consumption is minimal when nodes are equally spaced. For a two dimensional network, energy consumptions for various manual uniform arrangements such as triangular, square, and hexagonal array of sensors are analyzed and compared. We also rigorously analyze expected energy consumption under random distribution. We then show that a minimal spanning tree (MST) is the optimal data aggregation tree for energy-efficient routing. We then study energy-efficient distributed algorithms for constructing MSTs. The GHS algorithm to construct MST has an optimal message complexity, but can be energy-expensive. A key contribution of the paper is a new simple algorithm called the Nearest Neighbor Tree (NNT) to build slightly sub-optimal trees, but is very energy-efficient compared to the GHS algorithm. Simulation results shows that NNT gives a close approximation to MST and consumes much less energy compared to GHS in constructing the tree.**

***Index Terms*— Graph theory, Stochastic processes, Optimization, Sensor networks, Data-centric routing.**

I. Introduction and Overview

Advances in integrated circuit technology have enabled mass production of tiny, cost-effective, and energy-efficient wireless sensor devices with on-board processing capabilities. The emergence of mobile and pervasive computing has created new applications for them. Sensor-based applications span a wide range of areas, including remote monitoring of seismic activities, environmental factors (e.g., air, water, soil, wind, chemicals), condition-based maintenance, smart spaces, military surveillance, precision agriculture, transportation, factory instrumentation, and inventory tracking [1], [2].

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A. Sensor Networks

A *microsensor* is a device which is equipped with a sensor module (e.g., an acoustic, a seismic, or an image sensor) capable of sensing some entity in the environment, a digital unit for processing the signals from the sensors and performing network protocol functions, a radio module for communication, and a battery to provide energy for its operation [2]. Microsensors typically have low processing power and slow communication ability. For example, Berkeley mote [3] has a 8-bit Atmel AT90LS8535 microcontroller running at 4 MHz. A low-power radio transceiver MICA2, designed for sensor networks, operates at 916 MHz and provides a data transmission rate of 19.2 Kbps [4]. These parameters ensure limited weight, size, and cost. The size of a MICA2 MPR400CB is $2.25'' \times 1.2'' \times 0.25''$. We use the term sensor to refer to a microsensor.

Networking of the sensors, when deployed in large numbers and embedded deeply within large-scale physical systems, enables to measure aspects of the physical environment in unprecedented detail [1]. There are some similarities between wireless sensor networks and wireless ad-hoc networks. One of the similar characteristics for both of them is multi-hop communications. Some of the power-aware routing protocols [5], [6], [7], [8] proposed for wireless ad-hoc networks can be examined in the context of wireless sensor networks with stringent energy constraints. However, these protocols may not be efficient, effective or feasible, in sensor networks. The nature of applications and routing requirements for the two are significantly different in several aspects [9]. First, the typical mode of communication in a sensor network is from multiple data sources to a data recipient/sink rather than communication between any pair of nodes. Second, since the data being collected by multiple sensors is based on common phenomena, there is likely to be some redundancy in the data being communicated by the various sources in sensor networks. Third, in most envisioned scenarios the sensors are not mobile, so the nature of the dynamics in the two networks is different. Finally, the single

major resource constraint in sensor networks is that of energy. The situation is much worse than in traditional wireless networks, where the communicating devices handled by human users can be replaced or recharged relatively often. The scale of sensor networks and the necessity of unattended operation [10] for months at a time means that energy resources have to be managed even more carefully. This, in turn, precludes high data rate communication and demands energy-efficient routing protocols [9].

B. Data-Centric Routing

Data aggregation has been put forward as a particularly useful paradigm for wireless routing in sensor networks [11], [12], [13]. The idea is to combine the data coming from different sources enroute – eliminating redundancy, minimizing the number of transmissions and thus saving energy. Sensor data is different from data associated with traditional wireless networks since it is not the data itself that is important. Instead, it is the analysis of data, which allows an end-user to determine something about the monitored environment [2]. For example, if sensors are monitoring temperature, the temperatures at different points of a certain area are highly correlated and the end users are only interested in a high-level description of the events occurring. The type of high-level description of data or data aggregation that needs to be performed depends on the monitored events and user requirements. Minimum, maximum, average, count [14], beamforming [15], [16], and functional decomposition [2] are some examples of data aggregating functions and techniques. The advantages, necessities, and opportunities of data aggregation in a sensor network has been confirmed theoretically [17] and experimentally [11]. This paradigm shifts the focus from the traditional *address-centric* approaches to a more *data-centric* approach [17]. In [17], address-centric and data-centric protocols are defined as follows:

Address-centric Protocol (AC): Each source independently sends data along some established path to the sink ("end-to-end routing").

Data-centric Protocol (DC): The sources send data to the sink, but routing nodes enroute look at the content of the data and perform some form of aggregation/consolidation function on the data originating at multiple sources.

The authors in [17], theoretically bound the number of transmissions required in DC protocol and show that DC protocol needs fewer transmissions that that of AC protocol.

Heinzelman et al. [18] presented a simple analysis showing when multi-hop routing is preferable over direct communication with respect to the objective of minimizing energy for transmission.

The above two analyses provide a good theoretical basis for using data-centric multi-hop routing in wireless sensor networks. However, a rigorous theoretical model is needed to estimate energy and to find sensor placement strategy to minimize energy. In this paper, we provide a model for such analysis.

Several schemes have been proposed for data-centric routing in sensor networks. Cluster-based [18], [19], Center at Nearest Source [9], and Shortest Path Tree [9] are important among them.

Cluster-Based Tree (CBT): In this scheme, the sources send data to the associated cluster head. Cluster head aggregate data and send to the sink. Multiple levels of cluster hierarchy [19] can be another option.

Center at Nearest Source (CNS): In this scheme, the source which is nearest to the sink acts as the aggregation point. All other sources send their data directly to this source which then sends the aggregated information on to the sink.

Shortest Paths Tree (SPT): In this scheme, each source sends its information to the sink along the shortest path between the two. When the paths overlap for different sources, they are combined to form the aggregation tree.

SPIN [20], Directed Diffusion [11] , GEAR [21], and Rumor Routing [22] are the recently proposed routing protocols for sensor networks to disseminate query and/or data. These algorithms rely on flooding techniques, and use different heuristics to minimize flooding and setup directed paths.

C. Our Contributions

In this paper, we study energy-optimal network configurations for manual and random placement of nodes under a natural coverage criterion. In particular, we show that in a linear network, energy consumption is minimal when nodes are equally spaced. For a two dimensional network, energy consumptions for various manual uniform arrangements such as triangular, square, and hexagonal array of sensors are analyzed and compared. We also rigorously analyze energy consumption under random distribution.

Our key contributions are the analyses and constructions of energy-efficient data-centric routing schemes for sensor networks. We assume that each node waits until it receives data from all of its descendants, then aggregates the data and forwards it to its parent. With this assumption, we show that minimum spanning tree (MST) is the optimal data aggregation (or *routing tree*, as we call). A standard message-optimal distributed algorithm to build MST is the GHS algorithm [23]. The time complexity of this algorithm is $O(n \log n)$ and number of messages need to be exchanged among the nodes is $O(n \log n)$. Later, the time complexity was improved to $O(n)$ in [24], [25] but the number of messages is still $O(n \log n)$ ($\approx 5n \log n + 2|E|$). Exchanging this huge number of messages can consume a prohibitively large amount of energy, which is not be suitable for energy-constrained wireless sensor network specially when the network needs to be reconfigured quite often.

With the objective of minimizing energy consumption for tree configuration as well as in routing data, we build a

nearly-optimal routing tree called *Nearest Neighbor Tree* (NNT). NNT is constructed by exchanging only $O(n)$ ($\approx 3n$) messages. Simulation results shows that about 80% of the edges in NNT are exactly the same edges in MST and energy consumption for data-centric routing using NNT is very close to that using MST while energy consumption in building NNT is less than that in GHS algorithm by a order of magnitude.

The rest of the paper is organized as follow. Modeling assumptions, problems, and definitions are given in Section II. An exact analyses of energy and node placement for a simple linear network are given in Section III. Analysis of two-dimensional network when nodes manually placed is given in Section IV. In Section V, we analyze energy consumption when nodes are randomly distributed. In Section VI, we show that minimum spanning tree is the optimal data routing tree. A sub-optimal routing scheme, NNT, its properties, and an energy-efficient distributed NNT algorithm are described in Section VII. Simulation results are presented in Section VII-B and the conclusions are in Section VIII.

II. Problems and Definitions

Unattended operation and limited energy of the sensor nodes demands a routing scheme which minimizes energy consumptions for routing data from the sources to the sink. The following key questions arise in the context of minimizing energy/cost:

1) How the sensors should be placed (sensor distribution)?

2) How many sensors should be deployed (density)?

3) What is the optimal routing scheme?

4) What is the expected energy consumption in routing data when the sensors are placed randomly?

To answer the above questions in a systematic way, we develop a uniform theoretical framework which makes use of the following terminology. First, we define the energy model used in our analysis.

Energy model: To transmit a signal over a distance r, the required *radiation energy* is proportional to r^m where m is 2 in the free space and ranges up to 4 in environments with multiple-path interferences or local noise [26]. That is, radiation energy to transmit one unit (for some unit) of data to distance r is cr^m for some constant c. Typically, we consider m to be 2. There is a constant (independent of distance) amount of energy e_c, called *electronics energy*, required for each transmission at the sending and receiving end to run the radio electronics and to process data (data aggregation and processing data packets). Thus energy consumption is $e_c + cr^m$. For n transmissions by n nodes, total energy

$$E = \sum_{i=1}^{n} (e_c + cr_i^m) = ne_c + \sum_{i=1}^{n} cr_i^m.$$

When n is constant, total electronics energy ne_c is fixed and we focus on minimizing total radiation energy $\sum_{i=1}^{n} cr_i^m$. When n is variable, our focus is on total energy $ne_c + \sum_{i=1}^{n} cr_i^m$.

Definition 1: Transmission Step. A transmission step is the time duration in which a node begins and completes transmitting data to the next hop.

Definition 2: Transmission Phase. A transmission phase is a collection of successive transmission steps that begins with the step when the sources start sending data and ends with the step when the sink receives data from all of the sources.

To analyze data-centric routing, we have the following framework.

Assumption 1: In one transmission phase, each source produces (by sensing) exactly one unit of data.

Assumption 2: Each node waits until it receives data from all descendants and upon receipt of the data, aggregates them (including its own) reducing size into one unit of data, then sends to the next hop.

Lemma 1: If n be the number of sensors nodes, the total number of transmissions in a transmission phase is n.

Proof: Assumption 1 and 2 lead us to the conclusion that each sensor node needs to transmit exactly once in each transmission phase. That is, the total number of transmissions is n. ■

Definition 3: Coverage. We say, coverage (sensing coverage) by a set of sensor nodes in a region L is d if d is the minimum distance such that every point in the region L has at least one sensor node within distance d. More formally, we say that coverage is d if

$$\forall_{p \in L} \{D(p, N_p) \leq d\} \wedge \exists_{p \in L} \{D(p, N_p) > d - \epsilon\}$$

for any positive number ϵ; or equivalently,

$$d = \sup_{p \in L} \{D(p, N_p)\}$$

where $D(p, N_p)$ is the distance of the nearest sensor node N_p from point p.

Definition 4: Routing Path. A routing path is the path along which a source sends data to the sink.

Definition 5: Routing Tree. The routing paths in a network form a tree when they satisfy the conditions a) a routing path does not contain any cycle and b) if two routing paths merge at some node, they never get separated. This tree is called a *routing tree* or *data aggregation tree*.

Definition 6: Connectivity Graph. A connectivity graph, $G = (V, E)$, is the graph where V is the set of sensor nodes and for any two nodes u and v, weight of the edge (u, v) denoted by $w(u, v)$ is the distance between u and v if u and v are within a specified distance, otherwise $w(u, v) = \infty$.

III. An Exact Analysis

To answer the questions mentioned in Section II, we begin with a simple one-dimensional sensor array shown in Fig. 1.

Theorem 1: Assume that n sensor nodes are placed in a straight line of length R and the sink is placed at one end

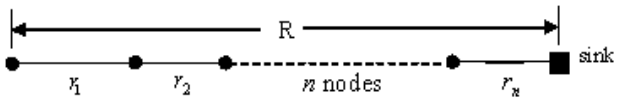


Fig. 1. A simple linear network - one-dimensional sensor array

of the line. The total radiation energy is minimal when nodes are equally spaced, i.e. when the distance between any two neighbors is $\frac{R}{n}$, and the minimum radiation energy $E_{\min} = \frac{cR^2}{n}$.

Proof: Let r_i be the distance from node i to the next hop (Fig. 1). Total radiation energy $E = \sum_{i=1}^{n} cr_i^m$. We minimize $\sum_{i=1}^{n} cr_i^m$ with the constraint $\sum_{i=1}^{n} r_i = R$. An equivalent expression to minimize is

$$L = \sum_{i=1}^{n} cr_i^m - \lambda \left(\sum_{i=1}^{n} r_i - R \right),$$

where λ is a Lagrange's multiplier.

Now, $\frac{\partial L}{\partial r_i} = cmr_i^{m-1} - \lambda = 0$, i.e. $r_i = \left(\frac{\lambda}{cm}\right)^{\frac{1}{m-1}}$.

$\left(\frac{\lambda}{cm}\right)^{\frac{1}{m-1}}$ is a constant (independent of i), that is $r_1 = r_2 = \cdots = r_n = \frac{R}{n}$. Thus, minimum energy, $E_{\min} = \sum_{i=1}^{n} cr_i^m = cn\left(\frac{R}{n}\right)^m$. Considering $m = 2$, $E_{\min} = \frac{cR^2}{n}$. ■

Remarks:

- The routing tree is simply a line for this simple network and the above theorem exactly characterizes the distance of the edges of the tree.
- The constraint in the above optimization implicitly is a coverage constraint - the nodes have to fill the entire line. If we don't have this constraint, then we can place the nodes arbitrarily close to each other which will trivially minimizes energy. In Sections IV and V, we explicitly use a coverage criterion for two dimensional networks.
- The radiation energy is minimal when nodes are equally spaced and we observe that it is a strictly monotone decreasing function of number of nodes. This observation suggests using as many nodes as possible to minimize energy consumption. However, the total electronics energy, ne_c, increases with the number of nodes. Theorem 2 gives the optimal number of sensor nodes for a simple linear network.

Theorem 2: To minimize the total energy consumption, the optimal number of sensors nodes in a simple linear network is $n_{opt} = R\left\{\frac{(m-1)c}{e_c}\right\}^{\frac{1}{m}}$, where e_c is the electronics energy associated with each transmission.

Proof: Total radiation energy is minimized when the nodes are equally spaced [Theorem 1]. Let the distance between two adjacent nodes is r. Radiation energy for n transmissions [Lemma 1] is $ncr^m = \frac{cR^m}{n^{m-1}}$, since $r = \frac{R}{n}$. Distance independent electronics energy = ne_c. Total energy $E = \frac{cR^m}{n^{m-1}} + ne_c$.

Solving $\frac{dE}{dn} = 0$, $n_{opt} = R\left\{\frac{(m-1)c}{e_c}\right\}^{\frac{1}{m}}$.

With m = 2, $n_{opt} = R\sqrt{\frac{c}{e_c}}$. ■

In many applications, we will not be able to position nodes at the exact locations. Instead, we might need to place the sensors at random positions in the area of interest. In such a situation, we are interested to find the expected energy. In the following theorem [Theorem 3], we compute the expected radiation energy for random distribution of the sensor nodes in a linear network. An interesting result to observe is that this expected radiation energy is bounded by $2E_{\min}$, twice the radiation energy when the nodes are equally spaced.

Theorem 3: The expected radiation energy required in a transmission phase in a simple linear network with uniformly randomly distributed n nodes is $E_{\exp} = \frac{2cR^2}{n+1} \leq 2E_{\min}$.

Proof: Consider any arbitrary node N at point A shown in Fig. 2.

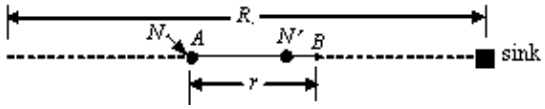


Fig. 2. Randomly distributed sensor nodes in a linear network

Assuming uniform distribution, the probability that a particular node is on the line segment AB of length r is $\frac{r}{R}$. The probability that the next hop N' is within distance $r =$ the probability that at least one of the $n-1$ nodes (except N) on $AB = 1 - \left(1 - \frac{r}{R}\right)^{n-1}$. This is the cumulative distribution function. The derivative of this function is the probability density function $P(r)$. That is,

$$P(r) = \frac{d}{dr}\left\{1 - \left(1 - \frac{r}{R}\right)^{n-1}\right\} = \frac{n-1}{R}\left(1 - \frac{r}{R}\right)^{n-2}$$

Note that $\int_0^R P(r)dr = \int_0^R \frac{n-1}{R}\left(1 - \frac{r}{R}\right)^{n-2}dr = 1$ and the expected (average) distance to the next node is $\int_0^R rP(r)dr = \int_0^R r\frac{n-1}{R}\left(1 - \frac{r}{R}\right)^{n-2}dr = \frac{R}{n}$, which are obvious.

Now, expected radiation energy in one transmission

$$\begin{aligned} E[cr^2] &= \int_0^R cr^2 P(r)dr \\ &= \int_0^R cr^2 \frac{n-1}{R}\left(1 - \frac{r}{R}\right)^{n-2}dr \\ &= \frac{2cR^2}{n(n+1)}. \end{aligned}$$

There are n transmissions in one transmission phase [Lemma 1]. Hence, expected radiation energy in one transmission phase is $E_{\exp} = \frac{2cR^2}{n(n+1)} \times n = \frac{2cR^2}{n+1} \leq \frac{2cR^2}{n} = 2E_{\min}$ [Theorem 1]. ■

In the next two sections, we examine network configuration to minimize energy consumption in routing data in a two dimensional network. First we analyze manual configuration of the network, where we are able to fix the

nodes in the desired locations, followed by analysis of the network with randomly distributed nodes.

IV. Manual Placement of the Sensor Nodes

From the analysis of the simple linear network, we foresee [1] that the energy consumption in a two dimensional network is minimized when nodes are evenly spaced. We need a coverage [Definition 3] criterion to be satisfied such that the whole region of interest is covered by the sensor nodes.

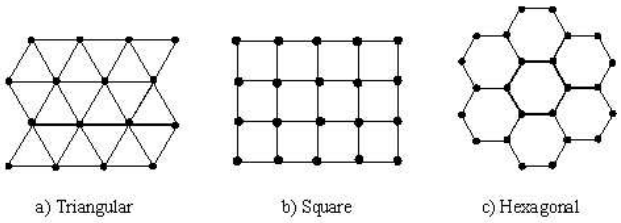


Fig. 3. Even arrangements of the sensor nodes

Equilateral triangle, square, and hexagon are the only possible even arrangements of the nodes such that all nodes have their nearest neighbors at the same distance (Fig. 3). A full angle ($360°$) is divisible by $60°$, $90°$, and $120°$, which are the angles between two adjacent sides of those three regular polygons, respectively. As a result, these three regular polygons can be used to create even arrangements. No other polygon holds this property. In any other arrangement (such as pentagonal or octagonal), some nodes must have their nearest neighbors at a closer distance than others. We restrict ourselves to the analysis of the these three even arrangements.

Let us examine the triangular arrangement. Let the area be A for the region L under consideration. We assume that the required number of nodes is not small and hence ignore the boundary effect.

Let each side of a triangle is r, that is, each node transmits to distance r to the next hop[2]. Area of one triangle is $\frac{1}{2}r.r.\sin\frac{\pi}{3} = \frac{\sqrt{3}}{4}r^2$.

Each node shares 6 triangles (6 triangles meet at one point). Share of a node to one such triangle is $\frac{1}{6}$ of a node. For each triangle, there are 3 nodes at the 3 vertices. Therefore, the number of nodes per triangle is $\frac{1}{6}\times 3 = \frac{1}{2}$; that is, area per sensor node = area of two triangles = $\frac{\sqrt{3}}{2}r^2$. If there are n nodes in area A, $n\frac{\sqrt{3}}{2}r^2 = A$, i.e. $r = \left(\frac{2A}{\sqrt{3}n}\right)^{\frac{1}{2}}$. Radiation energy for n transmissions,

$$e_m = ncr^m = nc\left(\frac{2A}{\sqrt{3}n}\right)^{\frac{m}{2}} = 2^{\frac{m}{2}}3^{-\frac{m}{4}}cA^{\frac{m}{2}}n^{1-\frac{m}{2}}.$$

[1]Obtaining a "clean" theorem under a coverage criterion for two dimensions seems difficult and is left as an open problem.

[2]We assume that the sink is somewhere near the boundary. Every node sends data towards the sink and the routing paths forms a tree. The node nearest to sink is the root of the tree.

The furthest point inside a triangle from its vertices is the centroid of the triangle; that is, the coverage in a triangular arrangement is the distance between a vertex and the centroid as shown in Fig. 4.

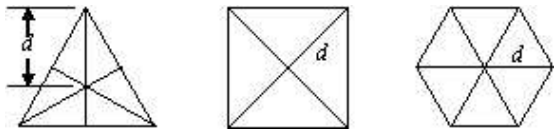


Fig. 4. Coverage, d, in triangular, square, and hexagonal arrangement

Height of the triangle $h = \frac{\sqrt{3}}{2}r$ and coverage,

$$d = \frac{2}{3}h = \frac{1}{\sqrt{3}}r = \left(\frac{2A}{3\sqrt{3}n}\right)^{\frac{1}{2}} = 0.62\sqrt{\frac{A}{n}}. \quad (1)$$

In a similar fashion, we can calculate radiation energy and coverage in square and hexagonal arrangements. The values are shown in Table I.

We see that as the number of sides of the polygon increases (from triangle to square, and from square to hexagon), radiation energy decreases but coverage becomes poorer. Next we analyze radiation energy needed to get a specified coverage in all these three arrangements. Radiation energy and the required number of nodes to have a coverage d is given in Table II. The values are simply re-expressed in terms of d using the relationship between d and n (e.g., Equation 1 for triangular arrangement).

Keeping the coverage constant, as the number of sides of the polygon increases, still radiation energy decreases but required number of nodes increases. Again, total energy consumption in digital and radio electronics increases with number of nodes. That is, radiation energy decreases and electronics energy consumption increases with the number of sides of the polygon. We conclude that there is an optimal arrangement (among the three configurations here) [3]; that is, there are some boundary values and ranges for coverage d, which determine the optimal arrangement to minimize total energy consumption.

Let e_c be the average electronics (digital and radio) energy consumption by a sensor node in one transmission. For triangular arrangement, total energy consumption in one transmission phase (n transmissions),

$$E_{triangle} = ne_c + e_m = \frac{2A}{3\sqrt{3}d^2}e_c + 2\times 3^{\frac{m-3}{2}}cAd^{m-2}.$$

Similarly, $E_{square} = \frac{A}{2d^2}e_c + 2^{\frac{m}{2}-1}cAd^{m-2}$

and $E_{hexagon} = \frac{4A}{3\sqrt{3}d^2}e_c + \frac{4}{3\sqrt{3}}cAd^{m-2}$.

Triangular arrangement is better than square arrangement when $E_{triangle} < E_{square}$, i.e.

[3]Note that there may be a "globally" optimal arrangement which is better than these three for a specified d and boundary conditions; however, finding this might involve solving a complicated non-linear program.

TABLE I

COVERAGE AND RADIATION ENERGY FOR n NODES DEPLOYED IN AREA A.

Arrangement	Coverage d	Energy e_m	Energy with $m=2$	Energy with $m=3$
Triangular	$0.62\sqrt{\frac{A}{n}}$	$2^{\frac{m}{2}}3^{-\frac{m}{4}}cA^{\frac{m}{2}}n^{1-\frac{m}{2}}$	$1.15cA$	$1.24c\sqrt{\frac{A^3}{n}}$
Square	$0.71\sqrt{\frac{A}{n}}$	$cA^{\frac{m}{2}}n^{1-\frac{m}{2}}$	cA	$0.66c\sqrt{\frac{A^3}{n}}$
Hexagonal	$0.88\sqrt{\frac{A}{n}}$	$2^m3^{-\frac{3m}{4}}cA^{\frac{m}{2}}n^{1-\frac{m}{2}}$	$0.77cA$	$0.66c\sqrt{\frac{A^3}{n}}$

TABLE II

REQUIRED NUMBER OF NODES AND ENERGY CONSUMPTIONS TO SATISFY COVERAGE d.

Arrangement	Required n	Energy e_m	Energy with $m=2$	Energy with $m=3$
Triangular	$\frac{2A}{3\sqrt{3}d^2}=0.38\frac{A}{d^2}$	$2\times3^{\frac{m-3}{2}}cAd^{m-2}$	$1.15cA$	$2cAd$
Square	$\frac{A}{2d^2}=0.50\frac{A}{d^2}$	$2^{\frac{m}{2}-1}cAd^{m-2}$	cA	$1.41cAd$
Hexagonal	$\frac{4A}{3\sqrt{3}d^2}=0.77\frac{A}{d^2}$	$\frac{4}{3\sqrt{3}}cAd^{m-2}$	$0.77cA$	$0.77cAd$

$$\frac{2A}{3\sqrt{3}d^2}e_c+2\times3^{\frac{m-3}{2}}cAd^{m-2}<\frac{A}{2d^2}e_c+2^{\frac{m}{2}-1}cAd^{m-2}$$

$$\Leftarrow d<\left(\frac{3\sqrt{3}-4}{3^{\frac{m}{2}}4-2^{\frac{m}{2}}3\sqrt{3}}.\frac{e_c}{c}\right)^{\frac{1}{m}}$$

Similarly, $E_{square}<E_{hexagonal}$, when

$$d<\left(\frac{8-3\sqrt{3}}{2^{\frac{m}{2}}3\sqrt{3}-8}.\frac{e_c}{c}\right)^{\frac{1}{m}}.$$

Let us consider a typical scenario with $m=2$, $c=100$ pJ/bit/m^2, electronics power consumption = 50 mW, and effective data transmission rate = 10 Kbps. Then $e_c=25\times10^{-3}/10^4=25\times10^{-7}$ J/bit. Substituting these values, $E_{triangle}<E_{square}$ if $d<136.38$ m and $E_{square}<E_{hexagonal}$ if $d<171.17$ m. We envision that almost in every practical application of sensors, desired coverage is $d<136.38$ m (which might be the case in many practical situations); we conclude that the triangular arrangement is optimal.

V. RANDOM PLACEMENT OF THE SENSOR NODES

In this section, we analyze data-centric routing and energy consumptions when the sensors are randomly (uniform distribution) placed in a two dimensional region.

Theorem 4: Let n sensor nodes be uniformly randomly distributed in a region L of area A. The expected radiation energy required in one transmission phase using any data-centric routing tree T, $E[e_T]\geq\frac{cA}{\pi}$.

Proof: Let the n sensor nodes be uniformly randomly placed in region L, the shaded region in Fig. 5, and N be an arbitrary sensor node in L.

Consider a re-distribution of the nodes in a circular region L' centered at N such that the area of regions L and L' are equal, i.e. $A=\pi R^2$. The nodes in region $L-(L\cap L')$ are moved to random locations in region $L'-(L\cap L')$. The nodes in region $L\cap L'$ remain in their previous locations.

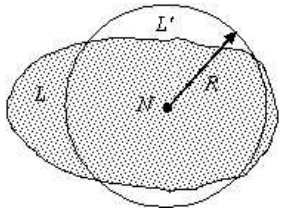


Fig. 5. Originally the sensor nodes are randomly placed in the shaded region L of area A. The nodes are rearranged in a circular region L' such that $A=\pi R^2$.

In the new region L', the probability that a particular node (other than N) is within distance r from node N is $\frac{\pi r^2}{\pi R^2}=\frac{r^2}{R^2}$. The probability that the nearest neighbor of N is within distance r, $C(r)=$ the probability that at least one of the $n-1$ nodes is within distance r, i.e.

$$C(r)=1-\left(1-\tfrac{r^2}{R^2}\right)^{n-1}.$$

The probability density function

$$P(r)=\tfrac{d}{dr}C(r)=\tfrac{(n-1)2r}{R^2}\left(1-\tfrac{r^2}{R^2}\right)^{n-2}.$$

Expected radiation energy to transmit to the nearest neighbor in region L' by node N,

$$\begin{aligned}E[e'_N]&=E[cr^2]\\&=\textstyle\int_0^R cr^2P(r)\,dr\\&=\textstyle\int_0^R cr^2\frac{(n-1)2r}{R^2}\left(1-\frac{r^2}{R^2}\right)^{n-2}dr\\&=\tfrac{cR^2}{n}.\end{aligned}$$

For node N, distance to the nearest neighbor in region $L \geq$ distance to the nearest neighbor in region L'. Therefore, in the original region L, expected energy $E[e_N] \geq E[e'_N]$. In any routing scheme, a node cannot send data to a node closer than its nearest neighbor. Thus, total energy for n transmissions [Lemma 1] by n nodes $E[e_T] \geq nE[e_N] \geq n\frac{cR^2}{n} = \frac{cA}{\pi}$. ■

The above theorem gives the radiation energy requirement for a given number of nodes. However, to find the total energy needed under a coverage criterion, we need the number of nodes needed to have a coverage of d. The following theorem regarding coverage for randomly distributed nodes in a unit square has been proven in [27]:[4]

Theorem 5—[27]: Let n nodes be uniformly distributed in a unit square and let d (in general, can be a function of n) be the radius of coverage of a node. Then, given any two constants $c_1 > 1/4 > c_0$, there is full coverage asymptotically almost surely (i.e., every point in the region is within a distance of d from any node with probability tending to 1 as $n \to \infty$) if $d \geq \sqrt{\frac{c_1 \log n}{n}}$ and no full coverage if $d \leq \sqrt{\frac{c_0 \log n}{n}}$.

Using the above theorem and theorem 4 we can show the following theorem.

Theorem 6: To have a fixed coverage d under a uniform random distribution in area A, the (total) expected energy needed is at least $E > An_d e_c + \frac{cA}{\pi}$, where n_d is the solution of the equation $\frac{n}{\log n} = \frac{1}{4d^2}$.

The above theorem enables us to compare (say, by numerical methods) the energy requirements of random placement with other configurations such as the ones in Section IV.

VI. Optimal Data-Centric Routing

In this section we show that a minimum spanning tree (MST) is an optimal routing tree for data-centric routing [Theorem 7] and analyze energy requirements to construct such a tree.

Theorem 7: Let $G = (V, E)$ be the connectivity graph of the sensor nodes as defined in Definition 6. A routing tree with minimum energy consumption is a minimum spanning tree on G.

Proof: Since total electronics energy consumption ne_c (for n sensor nodes) is same for all possible routing trees, it is sufficient to show that in the case of MST, the required radiation energy is minimal.

Let $w(u, v)$ be the weight of edge (u, v) in G. Let G' be the graph with the same vertices and edges as in G but weight for edge(u, v), $w'(u, v) = \infty$ if $w(u, v) = \infty$, otherwise $w'(u, v) = cw^m(u, v)$, which is radiation energy required for one transmission from u to v. A minimum spanning tree T' on G' minimizes $\sum_{(u,v)\in T'} w'(u, v)$, which is $\sum_{(u,v)\in T'} cw^m(u, v)$, that is, T' minimizes radiation energy for a transmission phase.

Now we show that for all u and v, $(u, v) \in T'$ if and only if $(u, v) \in T$, where T is an MST on G. Consider Kruskal's algorithm [28] to find MST: the edges are sorted by non-decreasing weight, and then, edges are added one by one from the sorted list with the condition that the added edges do not form a cycle. An edge (u, v) is added to the tree if u and v are not connected using the edges already added. For any two edges (u_1, v_1) and (u_2, v_2), $w'(u_1, v_1) \geq w'(u_2, v_2) \Leftrightarrow cw^m(u_1, v_1) \geq cw^m(u_2, v_2) \Leftrightarrow w(u_1, v_1) \geq w(u_2, v_2)$, that is, the both set of weights $w'$and w produce the same sorted order of the edges. As a result, the set of edges in T' is equal to the set of edges in T. Since T' minimizes radiation energy, hence T does so. ■

Energy requirements in building MST: A distributed algorithm to construct an MST, called GHS algorithm, was proposed in [23]. In the GHS algorithm, initially each node is considered to be a fragment (or a connected component). As the edges are added, the fragments grow by combing smaller fragments. In each "round" of the algorithm, each fragment finds its minimum length outgoing edge (MOE) - which is guaranteed to be in an MST - and uses this edge to combine fragments. Each fragment elects its leader (this is known to every node in the fragment) to manage the combining operation. To find the MOE, the leaders of two nodes, which are adjacent to the edge added immediately in the previous step, send initiate message (relayed by the intermediate nodes) to the members of the fragment. Upon receipt of initiate message, each node tests its adjacent edges by exchanging test/accept/reject messages to check if the node at the other end is in same fragment. Thus, each member node finds its outgoing edge and reports it to the leaders. Upon receipts of reports, the leaders select a new leader - the node which is adjacent to the MOE for the entire fragment and this begins a new round.

Thus a relatively large number of messages needs to be exchanged to find MOEs, for leader election, and to perform the combining operations; thus, the amount of energy consumed in configuring MST can become prohibitively large. Also as fragments grow, parallelism of the operations reduces (more sequential operations) requiring longer time[5] to terminate the algorithm. The required number of messages can be shown to be $2|E| + 5n \log n$ and time complexity is $O(n \log n)$, where $|E|$ is the number of edges in the connectivity graph and n is the number of nodes. The time complexity has been improved to $O(n)$ in [24], [25], but GHS was shown to be optimal in terms of number of messages.

In the next section, we propose a sub-optimal routing tree, which requires much less energy to build than MST.

[4]We omit the proof of this theorem (which is not directly interesting here) for lack of space. It will appear in the full version of the paper.

[5]But, here we are more concerned about the number of messages (rather than time) as these directly translates to more energy consumption.

VII. An Energy-efficient Construction

Although MST is the optimal routing tree [Theorem 7] for data-centric routing, building such tree in a distributed fashion is highly energy intensive as discussed in the previous Section. Since the sensors are typically deployed in large numbers in an ad-hoc fashion, the nodes must configure the routing tree by themselves after deployment. Reconfiguration of the tree is also a common event in sensor networks due to node failures and environmental dynamics. Consequently, it is desirable to minimize energy consumption in tree configuration phase.

We propose a simple sub-optimal routing tree algorithm called nearest neighbor tree (NNT) (specifically, the degree-NNT defined below), which we show to be significantly less expensive in terms of energy consumption than the GHS algorithm.

A. Distributed NNT Algorithm

The following definitions are needed to describe the algorithm and its properties. We then describe the algorithm and prove its properties. The complete distributed algorithm to construct a degree-NNT is given in Algorithm 1. The algorithm is executed by all nodes simultaneously.

Definition 7: Neighbor-Set. The neighbor-set of node p is denoted by $\text{NE}(p)$. $x \in \text{NE}(p)$ if and only if $x \neq p$ and node x is in the circle centered at p and with a specified radius r (called *initial broadcast radius*). $|\text{NE}(p)|$ is the *degree* of node p.

Definition 8: Available-for-Connection Set or AC-set. If node p is allowed to get connected to node x, we say x is available to p for connection. The set of nodes, which are available to p for connection is the AC-set of p and denoted by $\text{AC}(p)$. We define $x \in \text{AC}(p)$, if and only if $p \prec x$ for some irreflexive and transitive binary relation $\prec$. Such ordering of the nodes ensures that the connections among nodes do not create any cycle.

Definition 9: Available Neighbors. If x is available for connection to as well as a neighbor of p, x is called a available neighbor of p. $\text{AN}(p)$ is the set of all available neighbors of p. $\text{AN}(p) = \text{NE}(p) \cap \text{AC}(p)$.

Definition 10: Dead End. If $\text{AN}(p) = \phi$, i.e. node p has no available neighbor, p is said to be in dead end.

Next, we describe how ordering of the nodes can be defined such that each node can determine its relative order with respect to its neighbors locally and show that if every node gets connected to any member of its AC-set, there is no cycle in the resulting graph.

One such simple ordering heuristic is as follows. Every node generates a random number independently (between say 0 and 1) and broadcasts this number along with its ID, *identification number*, up to a pre-specified broadcast radius r. Each node collects random number-ID pairs of its neighbors and determines its order with respect to the neighbors according to the definition below.

Let R_p be the random number generated by node p and $\text{ID}(p)$ denotes the identification number of p. We assume that every node is given a unique ID before deployment.

Definition 11: Random Order $\prec_r$. For any two nodes p and q, $p \prec_r q$ if and only if either

a) $R_p < R_q$ or
b) $R_p = R_q$ and $\text{ID}(p) < \text{ID}(q)$.

The proposed sub-optimal routing tree can also be built using another ordering heuristic called degree order[6], which is determined by two rounds of messages. In the first round, each node broadcasts a message called "active" containing its ID up to the pre-specified initial broadcast radius r. Appropriate value for initial broadcast radius can be determined through a simulation process before deploying the sensors. Details about initial broadcast radius is discussed later in Section VII-B. Upon receipt of "active" messages from its neighbors, each node builds its neighbor list NE. If node p hears the message broadcasted by q, p considers q as a neighbor. In the second round, each node broadcasts another message called "count" containing its number of neighbors $|\text{NE}(p)|$ and ID. Based on the number of neighbors, each node determines its order as defined below.

Definition 12: Degree Order $\prec_d$. For any two nodes p and q, $p \prec_d q$ if and only if either

a) $|\text{NE}(p)| < |\text{NE}(q)|$ or
b) $|\text{NE}(p)| = |\text{NE}(q)|$ and $\text{ID}(p) < \text{ID}(q)$.

Lemma 2: Using degree order (or random order), if each node p gets connected to only one node $x \in \text{AC}(p)$ if $\text{AC}(p) \neq \phi$, there is no cycle in the resulting graph.

Proof: Assume that there exists a cycle $\langle p_0, p_1, p_2, \ldots, p_n, p_0 \rangle$. Since p_0 is connected to p_1, $p_1 \in \text{AC}(p_0)$, i.e. $p_0 \prec_d p_1$ [Definition 8]. Similarly, $p_1 \prec_d p_2$ and so on. Using Definition 12, it is easy to show that the relation $\prec_d$ is transitive. Therefore, $p_0 \prec_d p_0$. That is, either $|\text{NE}(p_0)| < |\text{NE}(p_0)|$ or $\text{ID}(p_0) < \text{ID}(p_0)$, which is absurd. Therefore, there is no cycle in the resulting graph. ■

The algorithm consists of essentially (at most) two steps as following. First step: after exchanging the "active" and "count" messages, each node p selects the nearest node q, if any, such that $q \in \text{NE}(p)$ and $p \prec q$, and sends a "connect" message to p. We assume that distance can be inferred from signal strength. Second step: if p is not able to connect to some other node, that is, if p is in dead-end (i.e. $\text{AN}(p) = \phi$), it increases broadcast radius from the specified initial value r to l to cover the whole region, where l is the maximum possible distance between any two nodes. For example, in a rectangular or square region, l is the length of the diagonal. Then p broadcasts a message called "deadend" containing its id and degree up to this new radius l. When a node, say q, receives a "deadend" message from another node, say p, it sends back an "available" message to p if $p \prec q$. If p receives "available" message from more than one node, it selects the nearest one for connection and send a "connect" message. Thus every node selects the nearest node from its AC-set for connection. Such connections create a tree and

[6] results show that this heuristic performs better than random order.

Algorithm 1 Distributed algorithm for degree-NNT. The algorithm is executed by each node p.

/* message is written in the format $\langle$message name, sender, [recipient], [other information]$\rangle$. When a message is broadcasted, the recipients are not specified. Initial broadcast radius $r \leq l$; l is the maximum possible distance between any two sensor nodes. */

First step:
NE$(p) \leftarrow \phi$ /* neighbor list */
Broadcast $\langle$active, $p\rangle$
For all q, upon receipt of $\langle$active, $q\rangle$ do
  NE$(p) \leftarrow$ NE$(p) \cup \{q\}$
  $distance[q] \leftarrow \frac{1}{s_{p,q}}$
  /* $s_{p,q}$ is strength of the signal received by p from q */
Broadcast $\langle$count, p, |NE(p)|$\rangle$
For each q, upon receipt of $\langle$count, q, |NE(q)|$\rangle$ do
  $ncount[q] \leftarrow$ |NE(q)|

/* find the available nearest neighbor if any */
$minnode \leftarrow$ NONE
$mindist \leftarrow \infty$
For each $q \in$ NE(p) do
  if $distance[q] < mindist$ and $p \prec_d q$
    $minnode \leftarrow q$
    $mindist \leftarrow distance[q]$

if $minnode \neq$ NONE
  send $\langle$connect, $p, q\rangle$ to q
else /* p is in dead end */
Second step:
  increase broadcast radius to l
  broadcast $\langle$deadend, p, $|NE(p)|\rangle$
For each q, upon receipt of $\langle$deadend, q, $|NE(q)|\rangle$ do
  if $q \prec_d p$,
    send $\langle$available, $p, q\rangle$ to q
If p is in dead end and receives one or more "available" messages
  select the nearest node q from the senders
  send $\langle$connect, $p, q\rangle$ to q

/* creating list of nodes connected to p */
$childrenlist_p \leftarrow \phi$
For each q, upon receipt of $\langle$connect, $q, p\rangle$ do
  $childrenlist_p \leftarrow childrenlist_p \cup \{q\}$

one connected component of all nodes as is shown in Theorem 8.

Lemma 3: There is at most one (in fact, exactly one) sensor node p such that AC$(p) = \phi$.

Proof: Assume that there are more than one node with an empty AC-set. Let p and q are two such nodes, that is, AC$(p) =$ AC$(q) = \phi$. Using Definition 11 (or 12 similarly), it is easy to show that for any two nodes p and q, $p \not\prec_d q \Rightarrow q \prec_d p$. That is either $p \prec_d q$ or $q \prec_d p$, i.e. $q \in$ AC(p) or $p \in$ AC(q), which contradicts with the assumption. Hence there is at most one node with empty AC-set. ■

Theorem 8: When each node p gets connected to only one node $x \in$ AC(p) if AC$(p) \neq \phi$, the resulting graph is a singly connected component and it is a tree.

Proof: Let n be the number of nodes. Initially, there is no edge in the graph and, as a result, there are n components, each containing exactly one node. Since the connections do not create any cycle [Lemma 2], each connection adds an edge to the graph that connects two nodes in different components reducing the number of components by one. From Lemma 3, we conclude that there are at least (in fact, exactly) $n - 1$ such edges. Therefore, the resulting graph is a singly connected component. Since there is no cycle, the graph is a tree. ■

Definition 13: Nearest Neighbor Tree (NNT). When each node p, if AC$(p) \neq \phi$, connects itself to a nearest node $x \in$ AC(p), the resulting tree is called a nearest neighbor tree or, in short, NNT. When degree order is used to determine availability for connection, the tree is called a *degree-NNT*. When random order is used, the tree is called a *random-NNT*.

B. Simulation Results

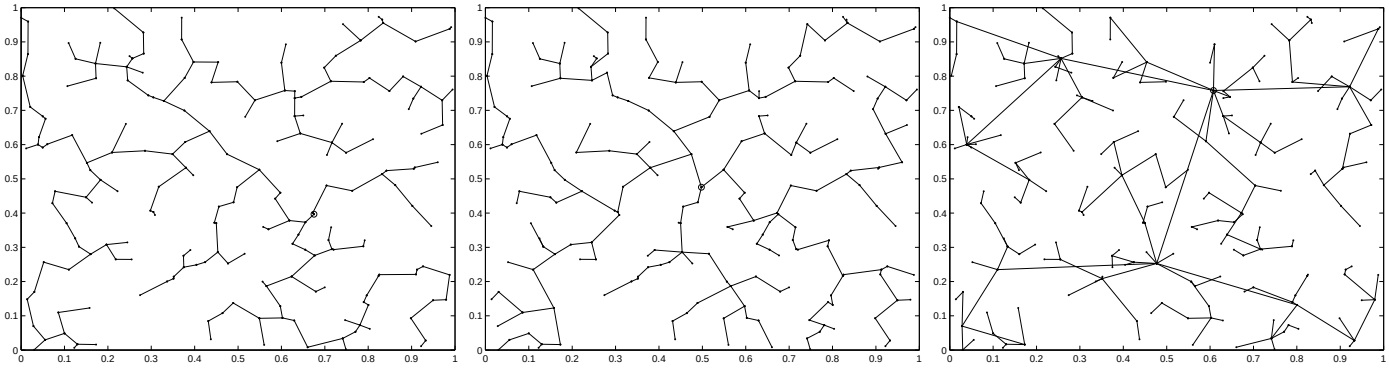
NNT and MST are simulated by generating random nodes in a unit square (1 m $\times$ 1 m). To study the effect of the number of nodes, the experiments are repeated for 100 to 1000 (in steps of 100) nodes. For the sake of fairness, every measured parameter is computed by averaging 100 different random distributions of the nodes. MST and NNT built from the same set of 200 random nodes are shown in Fig. 6. In this section, we restrict ourselves only in studying uniform random distributions of the nodes through simulation.

Total electronics energy consumption ne_c for n nodes in one transmission phase does not vary from one routing tree to another. Therefore, we compare the performance of MST and NNT in minimizing the total radiation energy, which is directly proportional to the sum of the squared edges[7] of the tree. We see that the sum of the squared edges in degree-NNT can be very close to that of MST (Fig. 7).

Degree-NNT as a close-approximation to MST:

1) One of the reasons for degree-NNT to be close to MST is that it selects the "nearest" from the nodes which are available for connection. This is also true for random-NNT. Simulation results show that on the average 63% of the edges in random-NNT and 80% in degree-NNT are exactly the same edges as in MST. We provide a heuristic explanation for this phenomenon. For any two arbitrary nodes p and q, $\Pr\{p \prec q\} = 0.5$, i.e. on the average, 50% of the nodes are able to select their minimum outgoing edges, all of which are included in a MST as well (the minimum outgoing edge of each node will always be in an MST). From the rest of the 50%, 25% nodes are able to select their second minimum outgoing edges, some of which are most likely to be in MST. As a result, just for selecting the nearest available node, 63% of the edges in MST also become a part of NNT. Next we give more heuristic arguments as to how degree-NNT improves this further to about 80%.

[7]We assume $m = 2$; the results are essentially the same for other values of m.



(a) MST (b) Degree-NNT (c) Random-NNT

Fig. 6. MST, degree-NNT, and random-NNT built from the same set of 200 nodes, which are randomly distributed in a unit square.

2) Another reason for degree-NNT to be close to MST is that a node with fewer number of neighbors gets preference over nodes with larger number of neighbors. Let p and q be two nodes such that $|\text{NE}(p)| < |\text{NE}(q)|$ and (p, q) is the minimum outgoing edges for both p and q. The algorithm allows p to use the edge (p, q) for connection but q is not allowed. Since q has more edges to select from, it has more chance to be able to select another edge and avoid a dead end. If p does not get preference over q, it has greater chance to run out of edges and be in dead end, and that forces p to get connected to a far distant node. Thus giving preference to p, chances of having long distance connections are reduced. As we can see in Fig. 6, random-NNT has a few larger edges but degree-NNT has none. This heuristic is most effective for the nodes at the boundary and the nodes in the sparse region. The boundary nodes can have very few neighbors (can be as low as 1 or 2 for some nodes). If they do not get preference, they will run out of edges and will be in dead end. The algorithm allows the nodes at the boundaries to connect first. Thus, by making the connections starting at the boundaries and progressing towards the center (a more dense region), the degree-NNT algorithm reduces the number of dead ends and avoids larger edges.

3) Again, let p and q be two nodes such that $|\text{NE}(p)| < |\text{NE}(q)|$ and (p, q) be the minimum outgoing edge for both p and q. (p, q) is an edge in MST as well as in degree-NNT (either p or q use this edge for connection since either $p \prec_d q$ or $q \prec_d p$). The edge with minimum length among the edges, other than (p, q), adjacent to p and q is also in MST. Let this edge be E_1. Now consider the case: q uses edge (p, q) for connection. Then p has to select an edge other than (p, q). $\Pr\{E_1$ is adjacent to $p\} = \frac{\text{NE}(p)-1}{|\text{NE}(p)|+|\text{NE}(q)|-2}$ and the probability that p is allowed to select E_1 is 0.5 and thus the probability that E_1 is included in NNT is $\frac{0.5(\text{NE}(p)-1)}{|\text{NE}(p)|+|\text{NE}(q)|-2}$. Similarly, if p uses the edge (p, q), the probability that E_1 is included in NNT is $\frac{0.5(\text{NE}(q)-1)}{|\text{NE}(p)|+|\text{NE}(q)|-2}$. Since $|\text{NE}(p)| < |\text{NE}(q)|$, allowing p instead of q to use the edge (p, q) increases the probability to include edge E_1, the minimum outgoing edge for the fragment formed by nodes p and q. Thus, further, degree-NNT becomes closer to MST.

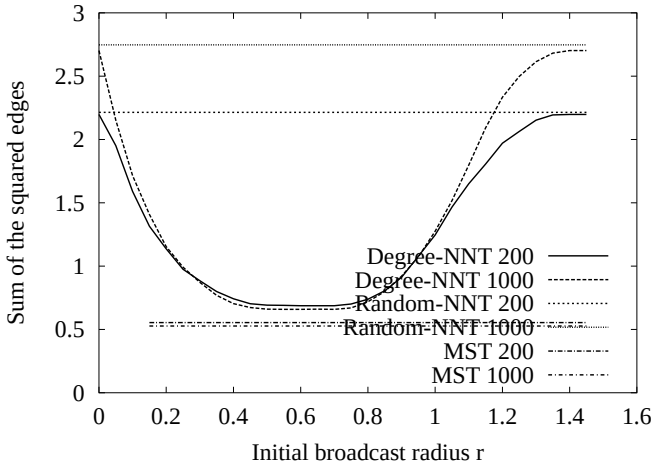


Fig. 7. Sum of the squared edges for MST, degree-NNT, and random-NNT of 200 and 1000 nodes randomly distributed in a unit square.

Effect of initial broadcast radius r:

If r is large, such as the maximum possible distance between any two nodes or so, almost every node is a neighbor of everyone else. Thus number of neighbors for all nodes are same and there is no desired effect of using number of neighbors in ordering the nodes. As a result, degree-NNT behaves like a random-NNT (Fig. 7). The same argument is valid when r is very low (0 or close to 0), every node has almost no neighbor or very few neighbors resulting in essentially the same number of neighbors for all nodes. Simulation shows that in a unit square, when r is between 0.35 and 0.85 (Fig. 7), degree-NNT is very close to MST. Another interesting observation is that for these values of r, sum of the squared edges of degree-NNT is (almost) same for both 200 and 1000 nodes; that is, sum of the squared edges is constant with respect to the number of nodes. This is also a property of an MST as we

see in Fig. 7, which is observed by R. Bland earlier and studied in [29].

The number of dead ends and connectivity (as defined below) of the tree are also affected by the initial broadcast radius.

Connectivity: If the initial broadcast radius is very small, the nodes have very few neighbors. As a result, in the first step of the algorithm, a significant number of nodes do not have any available neighbor, i.e. there are more nodes are in dead ends.These nodes are forced to increase their broadcast radius to l causing higher energy consumption. If we use a slightly larger initial broadcast radius, number of dead ends reduces significantly. When r is 0.3, in a unit square, degree-NNT is completely connected having only one node in dead end. There is always one node in dead end, which can be considered as the root of the tree. The number of dead ends is equal to the number of fragments in the graph built in the first step of the algorithm. We define *connectivity* as the inverse of the number of dead ends. The maximum value of connectivity is 1, when there is only one node in dead end, i.e. the network is fully connected. The simulation results for connectivity of NNT is shown in Fig. 8. If we choose $r \approx 0.4$, degree-NNT become a close approximation to MST (Fig. 7) as well as the tree gets fully connected in the first step of the algorithm avoiding long-distance communication of the second step. The appropriate value for r can be determined before deploying the sensors by simulation[8] for the particular setting. Using the simulation results, an optimal value for r can be chosen such that NNT gets closest to MST and the sensors can be equipped with this value.

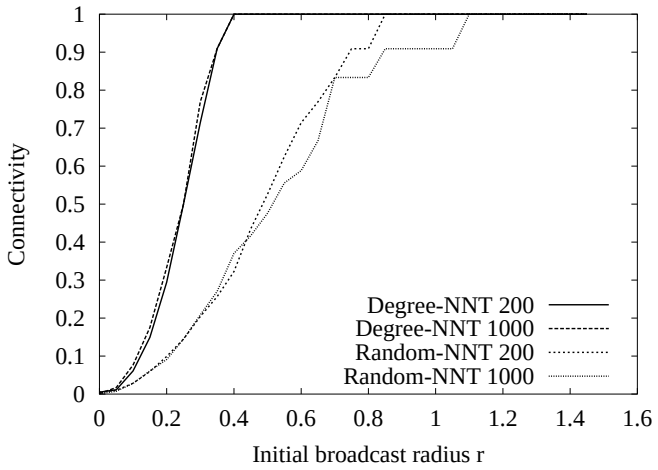


Fig. 8. Connectivity of degree-NNT and random-NNT with 200 and 1000 nodes randomly distributed in a unit square.

Number of messages to construct degree-NNT: The simulated result for the number of messages transmitted by the nodes to construct degree-NNT is shown in Fig. 9.

[8]A theoretical proof for connectivity and developing a mathematical formula for an appropriate r are left for future work.

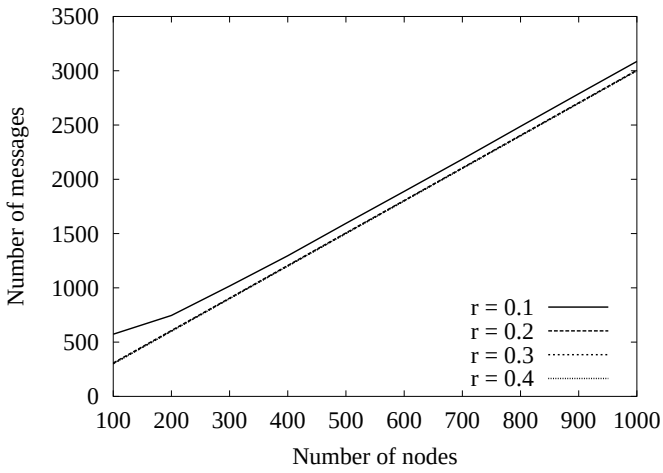


Fig. 9. Total number of messages exchanged by the nodes to construct degree-NNT using initial broadcast radius r = 0.1, 0.2, 0.3, and 0.4. The nodes are randomly distributed in a unit square. In the figure, the lines for r = 0.2, 0.3, and 0.4 have merged with each other.

We see the number of messages increases linearly with the number nodes. From this simulation result, we conclude that when initial broadcast radius $r \geq 0.2$ (considering unit square), number of messages grows approximately as $3n$.

Comparison between GHS and NNT algorithm:

To find energy consumption in running the algorithms, we simulate a deployment area of 200 m $\times$ 200 m with 200 sensor nodes randomly distributed in the area. Various power and energy related specifications are collected from [18], [30], [31]. The specifications are: digital electronics power is 11 mW, radio receiver electronics power is 13.5 mW, radio idle listening power is 13.5 mW, radio trans. electronics power is 24.8 mW, radio path loss is 100 pJ/bit/m^2, and effective transmission rate is 10 Kbps.

We found that GHS algorithm consumes six times larger amount of energy than NNT. For these settings, GHS algorithm consumes 0.64 J of energy, while NNT algorithm consumes 0.09 J. The reason is due to the fact that to build the tree, GHS needs (in an order of magnitude) more messages than NNT.

VIII. Conclusion and Further Work

In this paper, we have tried to systematically study, under a unified theoretical framework, configurations and routing schemes for the data-centric paradigm in sensor networks. In the first part of the paper, we rigorously computed energy requirements for both manual (specific configurations) and random placement of nodes. In the second part, we focused on optimal routing trees for data-centric routing and showed that the minimum spanning tree is energy-optimal. Then we addressed the important problem of constructing MST (or a good approximation of MST) in an energy-efficient and distributed manner.

Several open problems for future work emerge in our framework:

- What is the optimal (with respect to energy consumption) placement in two dimensions (and three dimensions) given a specific number of nodes and a coverage criterion in a given region?
- An important goal in designing energy-efficient distributed algorithms is reducing the message complexity, even at the cost of getting slightly suboptimal solution. NNT is a first step in designing such an algorithm for MST and we studied its performance by simulation. We are currently working on theoretically analyzing the performance of NNT algorithm to better understand its properties.
- Designing even better energy-efficient algorithms to construct MSTs; in this context, it is most interesting to theoretically characterize the trade-off between optimality (i.e., how close is the approximation to MST) and energy consumed.

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