Network Science

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Game Theory

3.1 Motivation and Outline

Game theory is frequently used in the network sciences, most notably in economics, as well as in biology, engineering, etc [1]. Game theory mathematically capture player's strategic behavior, in which a player needs to predict choices that will be made by other players in the game. In this lecture, we go through some basic cases in game theory, such as normal-form game, extensive form game, and evolutionary game, etc.

3.2 Basic Aspects of a Game

A game generally has three basic aspects: 1) a set of players, denoted as N, 2) a set of available strategies for each player in the gam, denoted as A_i for player i, 3) the payoff that each player receives based on the strategies chosen by everyone, denoted as π_i for player i.

3.3 Normal-form Game

The normal-formal game associates a payoff for each player with every possible combination of actions. It is usually represented by a matrix showing the players, strategies, and payoffs [1].

For example, in a game, two players wants to split 100 cents (1 dollar). Player 1 puts an proposal of how much player 2 get and then player 2 decides whether to accept this proposal or not. Let the strategy taken by player i as a_i. Mathematically, we have

$$
\begin{aligned}
&N = \{1, 2\},\\
&A_1 = \{x|0 \leq x \leq 100\},\\
&A_2 = \{Accept, Decline\},\\
&\pi_1 = \begin{cases} 100 - x & \text{if } a_2 = Accept \\ 0 & \text{if } a_2 = Decline, \end{cases}\\
&\pi_2 = \begin{cases} x & \text{if } a_2 = Accept \\ 0 & \text{if } a_2 = Decline. \end{cases}
\end{aligned}
$$

3.3.1 Nash Equilibrium

A strategy combination $a^* = (a_1^*, a_2^*, a_2^*, ..., a_n^*)$, is a *Nash equilibrium*, if a_i^* is the best response by player i to other players strategies in the combination. Thus, in the above example, we can see two Nash equilibria $(0, Accept)$ and $(0, Decline)$.

Now, we take a look at another normal form example: *drivers routes game*. In this game, we have three drivers and two routes: one is long, denoted as L and the other is short, denoted as S. Selecting a route r will incur some cost $C(r)$ and obtain some benefit $V(r)$ as well. This problem can be described by the following model,

$$
\begin{aligned}
&N = \{1, 2, 3\},\\
&A_i = \{L, S\}, i \in N\\
&C(L) = 0,\\
&V(L) = 0,\\
&C(S) = 0,\\
&V(S) = \begin{cases} 2 & \text{if 1 or 2 drivers select } S \\ 1/2 & \text{otherwise.} \end{cases}
\end{aligned}
$$

We define the payoff as the net benefit that a driver will receive by choosing the route, i.e. the benefit minus the cost. Thus, we can represent the players, their strategies, and the payoffs by the following matrices. Values inside the matrix represent the payoffs.

(1) $a_3 = L$

		Driver 2	
		L	S
Driver 1	L	0,0,0	0,1,0
	S	1,0,0	1,1,0

(2) $a_3 = S$

		Driver 2	
		L	S
Driver 1	L	0,0,1	0,1,1
	S	1,0,1	-1/2,-1/2,-1/2

There exist three Nash equilibria, including $(L, S, S), (S, L, S), (S, S, L)$.

3.4 Extensive Form Game

The extensive form is an alternative to the normal-form representation [2]. It can model games with a sequence of actions by presenting them in tree structures. In the tree, each node represents a point of choice for a player. The lines out of the node represent one of the strategies for that player. The payoffs are specified at the leaves of the tree [1].

Figure 3.1 illustrates the extensive form representation of the drivers routes game. The numbers besides non-leaf nodes indicate which player is making a decision.

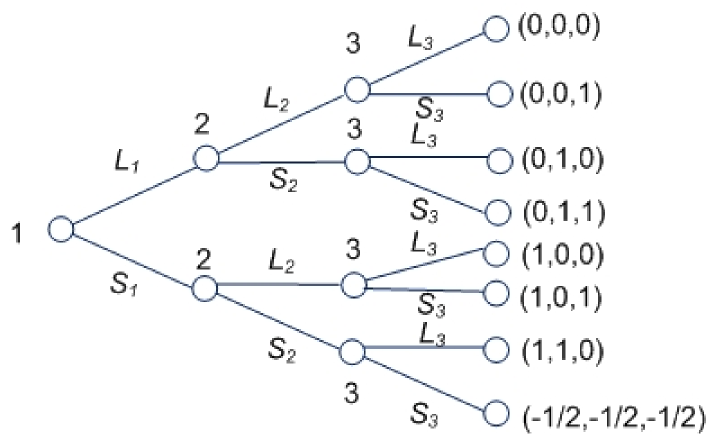

Figure 3.1: Extensive form representation of the driver routes game.

3.5 Evolutionary Game

Evolutionary game theory (EGT) is the application of game theory to interaction dependent strategy evolution in populations [3]. It differs from classical game theory by focusing on the dynamics of strategy change more than the properties of strategy equilibria [3]. EGT is frequently applied in a biological context by defining a framework of strategies in which adaptive features can be modeled [3].

Different from Nash equilibrium, Evolutionarily stable strategy (ESS) is applied in EGT for the evolution of social behavior in animals. It is presumed that the players are individuals with biologically encoded, heritable strategies. The individuals have no control over the strategy they play and need not even be capable of being aware of the game [4]. All players take the same strategy and no mutation can invade.

For instance, the Hawk-Dove game refers to a situation in which there is a competition for a shared resource and the contestants can choose either conciliation or conflict [5]. We define V as the value of the contested resource, and C as the cost of an escalated fight. It is normally assumed that the value of the resource is less than the cost of a fight is, i.e., $C > V > 0$. Supposing $V = 1$, we have the traditional payoff matrix as following.

	Dove	Hawk
Dove	1/2,1/2	0,1
Hawk	1,0	$(1-C)/2, (1-C)/2$

i) Nash Equilibrium If $C < 1$, H dominates D =¿ (H,H) is the NE. If $C > 1$, mixed strategy NE.

Balancing Method:

$$\begin{aligned} E(\pi, (D)) &= q/2; \\ E(\pi, (H)) &= q + (1-C)(1-q)/2. \\ => q/2 &= q + (1-C)(1-q)/2 \\ => q^* &= 1 - 1/C. \end{aligned}$$

NE: $((1-1/C)D + H/C, (1-1/C)D + H/2)$

ii) Check the evolutionary stability

If (b*, b*) is a strict equilibrium, it is an ESS. E.g. Non-strict equilibrium.

$0 < r \leq 1$

r, r	$1, -1$	$-1, 1$
$-1, 1$	r, r	$1, -1$
$1, -1$	$-1, 1$	r, r

=> mixed strategy NE, (1/3,1/3,1/3), $E(\pi_i) = r/3$.

A mutant who uses any of the three pure strategies obtains an expected payoff of $r/3$ when encounters a non-mutant and an $E(\pi) = r$; otherwise, it is not an ESS.

References

[1] *http://en.wikipedia.org/wiki/Game_theory.*

[2] *http://en.wikipedia.org/wiki/Extensive-form_game.*

[3] *http://en.wikipedia.org/wiki/Evolutionary_game_theory.*

[4] *http://en.wikipedia.org/wiki/Evolutionarily_stable_strategy.*

[5] *http://en.wikipedia.org/wiki/Hawk-Dove_game#Hawk-Dove.*