Network Science

Lecture 17: September 10, 2009 — Lecturer: Madhav Marathe

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Network Dynamical Process: SDS IV *cont.*

17.1 PRE for {AND, OR}-SDS

Theorem 1 *For {AND, OR}-SDS, PRE is NP-complete.*

Proof: Start with a known "hard" problem.

Definition 1 *3SAT*
Let,
variables $V = \{x_1, x_2, ..., x_n\}, x_i \in 0, 1, and$
clauses $C = \{c_1, c_2, ..., c_n\}$,
where, a clause is a disjunction of three variables.
For example, $C = \{x_1 \vee x_2 \vee x_n\}$.
A conjunctive normal form (CNF) is of the form,
$F_1 = C_1 \wedge C_2 \wedge C_2 \wedge ... \wedge C_m$
3SAT problem: Given $F_1(C, V)$, does there exist a $\{0, 1\}$ arrangement to all variables that evaluates to be true?

Lemma 1 *3SAT problem is NP-hard.*

If $F(C, V)$ is transformed to a function of $(s(G(V, E), F), C)$, then
3SAT is satisfiable $\iff$ S has a configuration C' such that $F(C') = C$.

Definition 2 *Gold's Monotone 3SAT*
All clauses are strictly of length 3 and monotone, that is, each has either only positive variables or only negative variables.

For example,

$$\begin{aligned} C_1 &= (x_1 + x_2 + x_3) \\ C_2 &= (x_1 + x_4 + x_5) \\ C_3 &= (\bar{x}_1 + \bar{x}_2 + \bar{x}_4) \\ C_4 &= (\bar{x}_2 + \bar{x}_3 + \bar{x}_5) \end{aligned} \tag{17.1}$$

where, C_i are clauses and x_i are variables.

It can be shown that the {AND, OR}-SDS problem can be mapped onto a Gold's Monotone 3SAT problem, implying that it is NP-hard and hence NP-complete. ■

17.2 References

[1] Christopher L. Barrett , Harry B. Hunt, III , Madhav V. Marathe , S. S. Ravi , Daniel J. Rosenkrantz , Richard E. Stearns, Complexity of reachability problems for finite discrete dynamical systems, Journal of Computer and System Sciences, v.72 n.8, p.1317-1345, December, 2006