Network Science	
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Network Dynamical Process: SDS IV

16.1 SDS for Threshold functions

For every SDS S such that all local functions are threshold functions, the following holds:
1. Threshold functions have only fixed points.
(i) Length of transients is $O(m+n)$.
(ii) Potential function argument.

2. NOR SDS do not have fixed points and have transients of length 1.

Proof:
Given S and a C such that $\phi(C') = C$, then for v with $s_v(C) = 1$ we have $s_{v'}(C') = 0 \; \forall \; v' \in N(v)$. Additionally, if $\exists$ v' with $s_{v'}(C') = 1$, then $s_v(C) = 0 \; \forall \; v \in N(v')$. As a result, NOR-SDS do not have fixed points. ■

Applications: Cycles in a NOR SDS can be used for random number generation.

3. The predecessor existence problem (PRE): Given a configuration C, does there exist another configurations C' such that $\phi(C') = C$? PRE is a NP-Complete problem. For an SDS with only AND and OR functions, the PRE is in P.

Proof:
For AND-SDS, $v \ni s_v(C) = 1 \Rightarrow s_{v'}(C') = 1 \; \forall \; v' \in N(v')$. So we can find the predecessor of system state C in polynomial time.
Similarly, for OR-SDS, $v \ni s_v(C) = 0 \Rightarrow s_{v'}(C') = 0 \; \forall \; v' \in N(v')$. So we can find the predecessor of system state C in polynomial time. ■

Application: Problems on network and routing modeled with SDSs.

16.2 References

[1] C. Barrett, H. Hunt III, M. Marathe, S. Ravi, D. Rosenkrantz, R. Stearns, Analysis problems for sequential dynamical systems and communicating state machines, in: Proc. International Sym-

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[3] Chris Barrett, Harry B. Hunt III, Madhav V. Marathe, S.S. Ravi, Daniel J. Rosenkrantz, Richard E. Stearns, Mayur Thakur, Predecessor existence problems for finite discrete dynamical systems, Theoretical Computer Science, Volume 386, Issues 1-2, 28 October 2007, Pages 3-37