Brief Announcement: A Fast Distributed Approximation Algorithm for Minimum Spanning Trees in the SINR Model

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Abstract. We study the *minimum spanning tree* (MST) construction problem in wireless networks under the physical interference model based on SINR constraints. We develop the first distributed (randomized) $O(\mu)$-approximation algorithm for MST, with the running time of $O(D \log n)$ (with high probability) where D denotes the diameter of the disk graph obtained by using the maximum possible transmission range, and $\mu = \log \frac{d_{max}}{d_{min}}$ denotes the "distance diversity" w.r.t. the largest and smallest distances between two nodes. (When $\frac{d_{max}}{d_{min}}$ is n-polynomial, $\mu = O(\log n)$.)

The Physical Interference Model. In recent years, the physical interference model based on SINR (Signal to Interference and Noise Ratio) constraints, referred to as the SINR model, has been found to be a more realistic model of wireless interference. In this model, a subset $L' \subseteq L$ of links can make successful transmission simultaneously if and only if the following condition holds for each $l \in L'$: $\frac{P(l)/d^{\alpha}(l)}{\sum_{l' \in L' \setminus \{l\}} P(l')/d^{\alpha}(l', l) + N} \geq \beta$, where $\alpha > 2$ is the "path-loss exponent", $\beta > 1$ is the minimum SINR required, N is the background noise, and $\phi > 0$ is a constant (note that α, β, ϕ and N are all constants). Here, $P(l)$ denotes the transmission power and $d(l)$ denotes the length of l, and $d(l', l)$ denotes the distance between the sender of l' and the receiver of l.

MST-SINR: The Minimum Spanning Tree Problem under the SINR Model. Given a set V of wireless nodes with a sink node s, the goal is to find a spanning tree T which minimizes $cost(T) = \sum_{(u,v) \in T} d(u, v)$, in a distributed manner. In particular, we are interested in ensuring that the communication at each step can be implemented in the SINR model. The spanning tree needs to be constructed implicitly, so that each node $u \in V$ only needs to find and know the id of its parent node $par(u)$.

Problem Background. The physical interference model is deemed as more realistic and much more challenging than graph-based ones for research in wireless networks. In recent years, distributed algorithms have been developed for a few fundamental "local" problems under this model, such as for maximum independent set, coloring, maximum dominating set problems. However, classical "global" problems such as minimum spanning tree, shortest path problems require an algorithm to "traverse" the entire network, and therefore more challenging for distributed solutions. Such problems remain open in a distributed setting, where the network diameter D is an inherent lower bound.

SINR-based Distributed Computing Model. Let $r_{max} = (P_{max}/c)^{1/\alpha}$ denote the maximum transmission range of any node at the maximum power level. We summarize the main aspects of our distributed model in the context of physical interference as follows: (1) The network is synchronized with unit slots. (2) The network is connected w.r.t. a range r_{max}/c for some constant c; (3) Nodes have a common estimate of n, within a polynomial factor. (4) Nodes share a common estimate of $d_{min} = 1$ and d_{max}, the minimum and maximum distances between nodes. (5) We assume nodes are equipped with software-defined radios and can transmit at any power level $P \in [1, P_{max}]$. (6) The success of communication is determined by SINR.

Distributed Algorithm for MST. Our distributed algorithm is based on the Nearest Neighbor Tree (NNT) scheme [1] which consists of two steps: (1) each node first chooses a unique *rank*; (2) each node connects to the *nearest* node of higher rank. Our algorithm involves two stages: "bottom-up" and "top-down."

(1) During the bottom-up stage of $\Theta(\mu \log n)$ slots, for some range $r'_{max} < r_{max}$, we run $\log r'_{max} \leq \mu$ phases, ranging from $i = 1, \ldots, \log r'_{max}$. In the ith phase,
   (i) a subset S_i of nodes participate, and the edges chosen so far form a forest rooted at nodes in S_i. The nodes in S_i transmit at power level of $c \cdot d_i^{\alpha}$ for a constant c, where $d_i = 2^i$.
   (ii) each node $v \in S_i$ approximates the NNT scheme by connecting to a "close-by" node in S_i within distance $c' \cdot d_i$ of higher rank, if there exists one, for a constant c'. The nodes which are not able to connect continue to phase $i + 1$.

   At the end of the bottom-up stage, we obtain a forest.
(2) During the top-down stage of $\Theta(D \log n)$ slots, we first form a constant-density dominating set Dom with some range so that
   (i) each node $v \notin Dom$ is within distance r'_{max} of some node in Dom, and
   (ii) for each node $u \in Dom$, the number of nodes within range r'_{max} is "small".

   We then assign ranks from the sink to the periphery of the network using local broadcast at each step (taking advantage of the constant density). This leads to connecting all the forests produced in the first stage.

The details and analysis can be found in the full paper [2]. Theorem 1 summarizes the performance of our algorithm. Our algorithm's running time is essentially optimal (upto a logarithmic factor), since computing *any* spanning tree takes $\Omega(D)$ time.

Theorem 1. *There exists a distributed algorithm that produces a spanning tree of cost $O(\mu)$ times the optimal in time $\Theta(D \log n)$,* w.h.p., *in the SINR model.*

References

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2. Khan, M., Kumar, V.S.A., Pandurangan, G., Pei, G.: A fast distributed approximation algorithm for minimum spanning trees in the sinr model (2012) `http://arxiv.org/abs/1206.1113`.