	Network Science	
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Introduction to Network Science

- N elements , each with state $s_i \in \{1, 2, \cdots, k\}$.
- A configuration $C = \{s_1, s_2, \cdots, s_N\}$.
- A measurement function $\mu : C \to Z$ (Z is the set of integers).
- μ is many to one function
- Ergodic Theory: relates time averages to ensemble averages.
- Ensembles
- Ensembles: micro-canonical : completely isolated system
- Ensembles: canonical: systems in contact with a heat reservior
- Ensembles: Grand-canonical: N varies.
- Assumption: Probability is uniformly distributed across configurations in $\mu^{-1}(m)$ in equilibrium.

If there is a fluctuation of size ∇E, then # configs of heat reserviour $\sim e^{\nabla E} \Rightarrow P(\mu^{-1}(E_0 + \nabla E)) \sim e^{\nabla E}$ since total number of configuration is constant.

Therefor, for a canonical ensemble, $p(C) = e^{-\beta E}$. $p(C) = \frac{1}{configs}$; $p(C) = \frac{1}{configswithenergyE}$.

Master Equation.

p$_{t+1}(C') = \sum_i p(C'_{t+1}/C_i^{(i)})p(C_t^{(i)}$

p$_{t+1}(C') = \sum_{i \neq j} [p(C^j/C^i)p(C^i) - p(C^i/C^j)p(C^j)]$
$= \sum_{i \neq j} [\tau_{ji}p(C^i) - \tau_{ij}p(C^j)]$

Partion Function.

Z$(\beta) = \sum_C e^{-\beta E}$

Z can be used as a generating function.

-dln $Z/d\beta = \frac{-1}{Z}\frac{dZ}{d\beta} + \sum_C E(C)\frac{e^{-(C)}}{Z} = \overline{E}$.