# Statistical Model of the Early Stage in Nucleus-Nucleus collisions

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# Outline

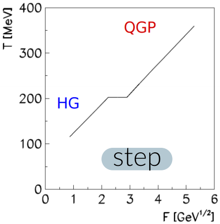
*Acta Phys.Polon. B30 (1999) 2705*
M. Gazdzicki, M. Gorenstein;
"On the Early Stage of Nucleus–Nucleus Collisions";
*arXiv:hep-ph/9803462v3* 30 Oct 1998
circa 400 citations

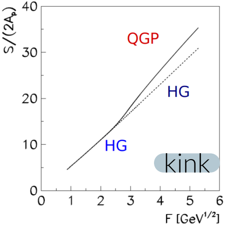
- Brief introduction and motivation.
- Historical sketch.
- Model Asumptions.
- Comment on results.

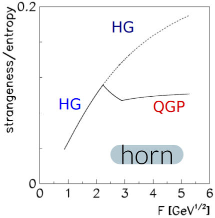
Section 1

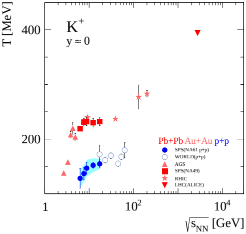
# Introduction

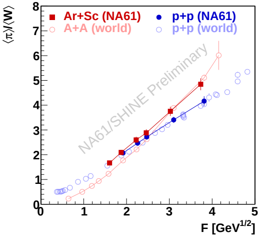
# Flashback

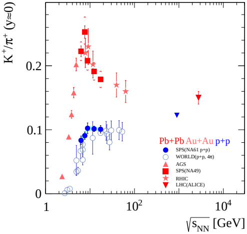



Ion+Ion and proton+proton collisions

# Heavy Ion Collisions

## Heavy Ion Collisions

or more general **High Energy Physics** focuses on studying matter under extreme energy densities. The main scientific objectives in HIC are:

- The investigation of a new state of matter: **quark-gluon plasma** (QGP).
- The study of **color confinement** and asymptotic freedom (or QCD in general).
- The study of the **origin of mass** of hadronic matter.

The work described in this presentation aims to uncover the properties of QGP by introducing a phenomenological model.

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A surprisingly simple model, which happens to surprisingly well reproduce experimental data.

# Motivation

## Baffling questions

- What is the nature of particle creation in strong interactions?
- What is the state of matter created in the collisions of two nuclei? How does it behave?

Under the lack of calculable theory, we need to rely on phenomenological approaches.

**Statistical Model of the Early Stage** (SMES) is the first of such models to introduce the deconfinement of hadrons in the early stage of the nuclei collisions.

Section 2

# History

# Brief historical sketch

| | |
|---|---|
| 1988 | **Strangeness enhancement** in S+S collisions at 200A GeV (NA35) wrt. p+p collisions at the same energy. |
| 1992 | **No strangeness enhancement** in p+A collisions at 200 AGeV. |
| 1994 | **Pion suppression** in central A+A collisions at low energies. |
| 1994 | **Pion enhancement** in central S+S collisions at 200 AGeV. |
| 1995 | **Pion saturation** in central A+A collisions at SPS. |

# Brief historical sketch

| | |
|---|---|
| 1995 | **Energy dependence of strangeness production** in A+A collisions. |
| 1996 | **Strangeness saturation** in central A+A col lisions at SPS. |
| 1997 | Quantitative agreement: **QGP** and A+A results at SPS. |
| 1996-8 | $J/\psi$ **saturation** in Pb+Pb collisions at 158 AGeV. |

Section 3

# Model

# Model Assumptions

1. Particle creation is a **statistical process**. All microscopic states are equally probable, therefore:

$$P(\text{macrostate}) \quad \propto \quad \# \text{ of microstates}$$

   In other formulation:

$$P \propto e^S$$

   , where $S$ is the entropy of the macroscopic state.

2. Particle production process does not produce net baryonic, flavour, nor electric charges. Thus only states with **total baryon, flavor, electric numbers equal to zero** are considered.

# Model Assumptions

3. Properties of created state are dependent entirely on three parameters: **system volume $V$, available energy $E$ and a partition function**.
   For large nuclei the thermodynamical approximation can be used: $E, V \to \epsilon$.
   The state properties can be given as equation of state (EoS).

4. **Production volume** is Lorentz contracted nucleus' volume:

$$V = \frac{V_0}{\gamma}$$

$V_0 = \frac{4}{3}\pi r_0^3$ and $\gamma = \sqrt{s_{NN}/(2m_N)}$.

$r_0 = 1.30$ in order to fit mean $\mu_B$ of nucleus $(= 0.11 fm^{-3})$

# Model Assumptions

5. Only a fraction of the total energy in A+A collision is transformed into the **energy of created particles**:

$$E = \eta(\sqrt{s_{NN}} - m_N)A$$

$\eta = 0.67$ assumed independent on on $E$ and $A$.

6. Large nucleus volume allows us to use the **grand canonical approximation**.

# Model Assumptions

7 The most **elementary particles** are quarks and gluons.

- Considered **quarks**: $u,d,s,c$ and $\bar{u},\bar{d},\bar{s},\bar{c}$.
  Resulting internal degrees of freedom = 6 (3 colors × 2 spins).
  $m_u = m_d = m_{\bar{u}} = m_{\bar{d}} = 0$, $m_s = m_{\bar{s}} = 175$MeV,
  $m_c = m_{\bar{c}} = 1.5$GeV.
- **Gluon** internal degrees of freedom = 16 (8 colors × 2 spins).

8 **EoS** for creation of quarks and gluons is assumed to be the ideal gas eq. modified by the Bag constant:

$$p = p^{id} - B, \quad \epsilon = \epsilon^{id} + B$$

Bag constant (=600 MeV/fm$^3$) accounts for strong interactions between quarks, gluons and QCD vacuum.

# Model Assumptions

9 In the freeze-out stage: **the degrees of freedom are hadrons**.
Due to their finite proper volume hadrons can exist only at low energy density: $\epsilon < 0.1 \div 0.4$ GeV/fm$^3$.

10 At collision energies lower than the QGP treshold the early stage d.o.f. can be approximated by point-like colourless bosons (**White state**).
The nonstrange d.o.f. are taken to be massless.
The internal number of d.o.f. in W-state is about 3 times lower than the internal d.o.f. for the QGP:

$$g_Q = g^b + \frac{7}{8}g^f = g^b + \frac{7}{8} \cdot 2_{q,\bar{q}} \cdot g_q, \quad g_q = 6n_f$$

$$g_Q = 2 \cdot 8 + \frac{7}{8} \cdot 2 \cdot 6 \cdot 3 \approx 48; \;\; g_W = 48/3 = 16$$

.

# Model Assumptions

11. Mass of the strange d.o.f. = 500 MeV (mass of the kaons).
12. The number of strange internal d.o.f. = 14 (fit to AGS data).
13. The only process **changing the entropy** of the system during expansion, hadronization and freeze-out in an **equilibration** with the baryonic subsystem (absorption of $\pi$ mezons or $\Delta$'s).
14. The total number of $s$ and $\bar{s}$ quarks is conserved during expansion, hadronization and freeze-out.

# Calculations

## Simplified Model

EoS defined by in terms of the pressure function $\boldsymbol{p} = \boldsymbol{p}(\boldsymbol{T})$:

$$s(T) = \frac{dp}{dT}, \quad \epsilon(T) = T\frac{dp}{dT} - p$$

The pressure of particle species '$\boldsymbol{j}$' is given by (ideal gas EoS, additive):

$$p^j(T) = \frac{g^j}{2\pi^2}\int_0^\infty k^2 dk \frac{k^2}{3(k^2+m_j^2)^{1/2}} \frac{1}{\exp\frac{\sqrt{k^2+m_j^2}}{T} \pm 1}$$

# Analytical Calculations

## Simplified Model

Simplification: all **d.o.f are massless**. Then:

$$p^j = \frac{\sigma^j}{3}T^4; \quad \sigma_{bosons} = \pi^2 g^j/30, \quad \sigma_{fermions} = \frac{7}{8}\pi^2 g^j/30.$$

with the effective d.o.f given by: $g = g^b + \frac{7}{8}g^f \quad \rightarrow \quad g_W \text{ and } g_Q$.

$$p(T) = \frac{\pi^2 g}{90}T^4\,(-B), \quad \epsilon(T) = \frac{\pi^2 g}{30}T^4\,(+B), \quad s(T) = \frac{\pi^2 g_W}{45}T^3,$$

First order phase transition at (Gibbs criterion):

$$p_W(T_C) = p_Q(T_C)$$

$T_C$ adjusted to AGS data $= 200$MeV sets the Bag constant.

$$T_C = \left[\frac{90B}{\pi^2(g_Q - g_W)}\right]^{\frac{1}{4}}$$

# Analytical Calculations

## Simplified Model

At $T = T_C$ there is a mixed phase:

$$\epsilon_{mix} = (1-\xi)\epsilon_W^C + \xi\epsilon_Q^C\,, \quad s_{mix} = (1-\xi)s_W^C + \xi s_Q^C$$

$\xi$ and $(1-\xi)$ are relative volumes.
The early stage energy density:

$$\epsilon = \frac{E}{V} = \frac{\eta\rho_0(\sqrt{s_{NN}} - 2m_N)\sqrt{s_{NN}}}{2m_N}$$

The entropy density:

$$s_W(\epsilon) = \frac{4}{3}\left(\frac{\pi^2 g_W}{30}\right)^{\frac{1}{4}} \epsilon^{\frac{3}{4}}$$

$$s_Q(\epsilon) = \frac{4}{3}\left(\frac{\pi^2 g_Q}{30}\right)^{\frac{1}{4}} (\epsilon - B)^{\frac{3}{4}}$$

# Analytical Calculations

## Simplified Model

Entropy density in the mixed phase:

$$s_{min}(\epsilon) = \frac{\epsilon_Q^C s_W^C - \epsilon_W^C s_Q^C}{4B} + \frac{s_Q^C - s_W^C}{4B}\epsilon \equiv a\epsilon + b$$

The total entropy per wounded nucleon produced in A+A collisions:

$$\frac{S}{2A_p} = \frac{V\,s}{2A_p} = \frac{m_N s}{\rho_0\sqrt{s_{NN}}}$$

Pure W-state:

$$\left(\frac{S}{2A_p}\right)_W = C g_W^{1/4} F$$

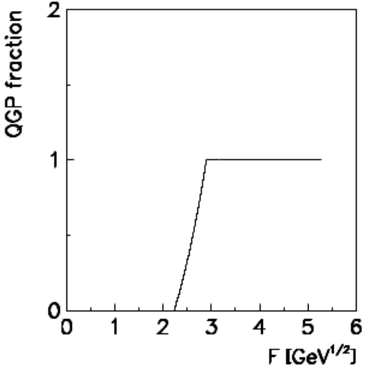
Pure Q-state:

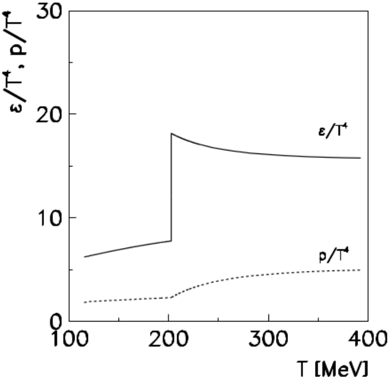
$$\left(\frac{S}{2A_p}\right)_Q = C g_Q^{1/4} F \left(1 - \frac{3 m_N B}{2\eta\rho_0 F^4}\right)$$

$C = \frac{2}{3}\left(\frac{\pi^2 m_N}{15\rho_0}\right)^{1/4}\eta^{3/4}$ and Fermi variable: $F = \frac{(\sqrt{s_{NN}} - 2m_N)^{3/4}}{(\sqrt{s_{NN}})^{1/4}}$

# Analytical Calculations

## Simplified Model



# Section 4

## Results

# Main Results

## Assumptions validation

- It was tested, that the grand canonical approximation is applicable for Si+Al collisions (AGS) and produces credible estimations of entropy and strangeness.
- Another tested set of data are S+S collisions (SPS). The model reproduced well the charm production.

# Main Results

## Consequences

- Entropy, strangeness and charm yields per participating nucleon should be independent of the size of colliding nuclei.
- Model predicts 3 seperate regions of states produced in the early stage: pure W-state (low energies), pure Q-state (high energies) and a mixed state in the intermediate energy region.
  For chosen parameters the mixed phase emerges at $p_{LAB} = 30 \div 65$AGeV
- The transistion from W-state to Q-state should manifest itself as a rapid increase of pion multiplicity and non-monotonic energy dependence of the strangeness to pion ratio.

# Comparison with data

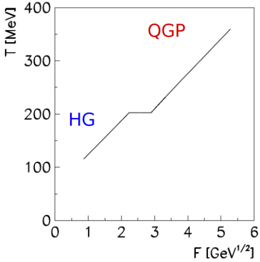
Early stage entropy (unit of pion entropy):

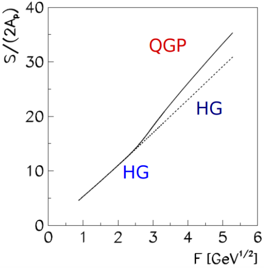
$$\langle S_\pi \rangle = \langle \pi \rangle + \kappa \left\langle K + \bar{K} \right\rangle + \alpha \langle N_P \rangle$$

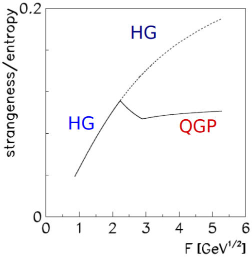
Total strangeness production as ratio to pions:

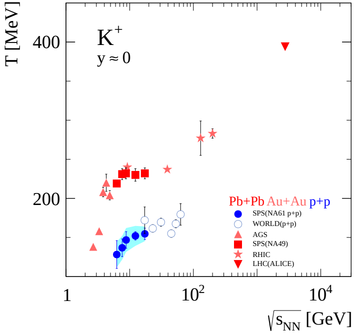
$$E_s = \frac{\langle \Lambda \rangle + \left\langle K + \bar{K} \right\rangle}{\langle \pi \rangle}$$

# Results – "step"

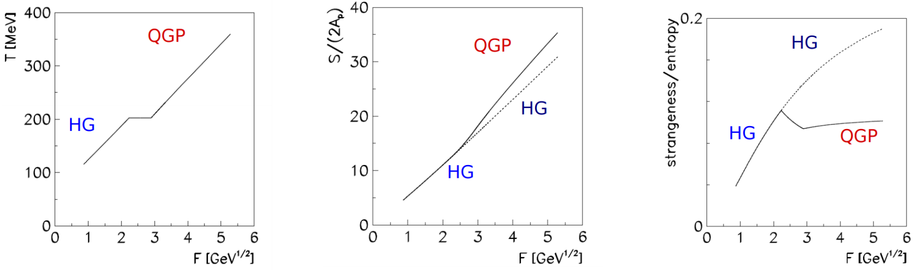


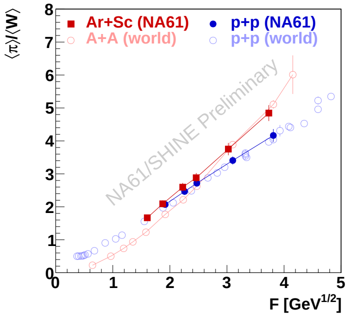



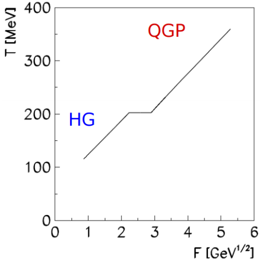



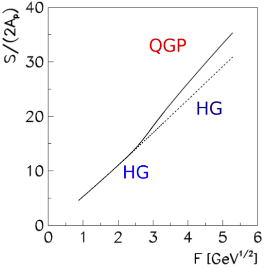
# Results – "kink"

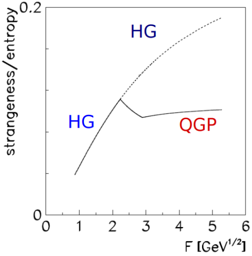


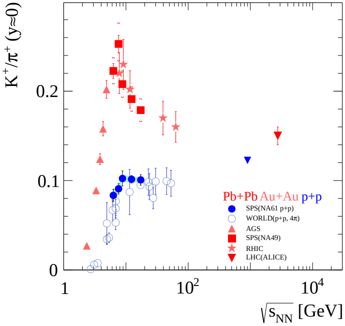



# Results – "horn"



Thank you for your attention!