ENGG 4280
Final Report

Group 14
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Executive Summary

The purpose of this report is to illustrate the controller to be used on the four-tank system. The system in question uses a pair of pumps connected to a set of 4 tanks. The pumps are connected in opposite corners as the two top tanks drain into the lower two tanks as they both drain into a communal reservoir. Each tank has a pressure sensor able to send data to the computer. Since the control report has been completed the IMC controller has been fully implemented as well as a more advanced model to closer simulate how the plant would respond. This required that noise produced by the sensors and the sampling rate of the system be accounted for.

This system requires that there be no overflow; as such, the system was designed to not overshoot the desired values during standard operation without any major disturbances. It also requires that the inputs be decoupled to allow for easier control of a specific subset of tanks. This resulted in the lower set of tanks being chosen to be decoupled resulting in one setpoint controlling the left one and the other controlling the right. Finally, it was required that the system reach its steady-state value within a minute of it beginning operation.

The final system implemented within this report does not overflow the desired values and reaches most setpoints within 40 seconds of operation. Due to the nature of the decoupling it functions well when both setpoints are relatively near each other though can have difficulties when one large setpoint and a near zero value are chosen. For better disturbance handling in the future a feed-forward IMC could be implemented that would react to changes even quicker.

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1.0 INTRODUCTION

The Four Tank Model project requires controlling a MIMO system of four water tanks and two pumps (Figure 1). This system uses a set of two electrical pumps; these pumps run from a basin beneath the tanks to two tanks each [1]. Each pump sends water to a top tank and a bottom tank with the top tanks draining into a bottom tank each and the bottom tanks draining into the basin. The challenge involved with this setup is to be able to control the height of the water in the bottom two tanks.

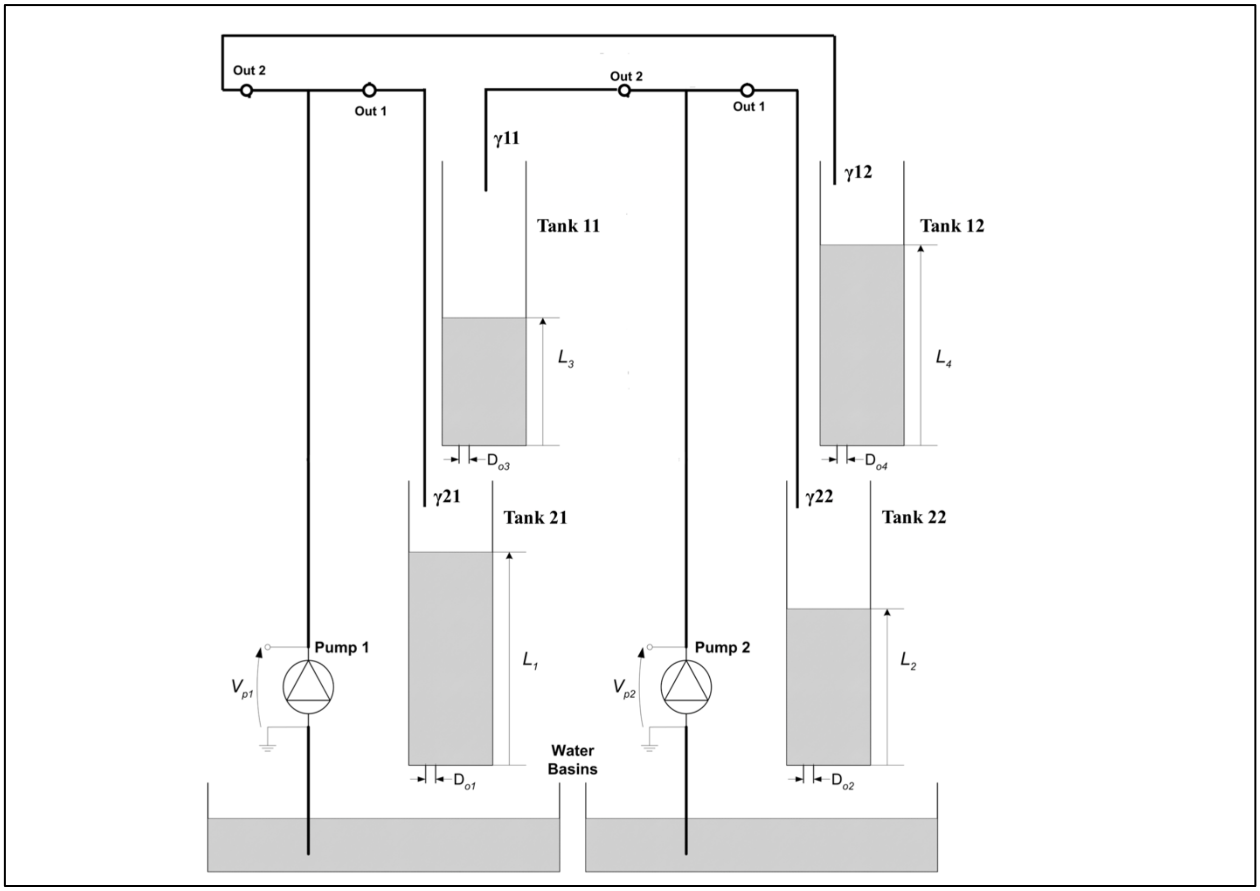

Figure 1 - Four Tank System Diagram

Each tank uses a pressure sensor to detect the height of water in each tank. To complete this project, a controller will be designed to meet criteria that will be determined after testing. This system will first be modeled in Matlab and then be controlled using Matlab to obtain the correct tuning values that are required to run the real-life model and output the results required with minimum error.

2.0 SYSTEM TESTING

2.1 GOALS OF TESTING

To model the system accurately the system is tested several times with varying configurations. The results from these tests are used to determine the plant characteristics so it can be modelled. The goals of testing the system include:

- Finding the Margin of Error of the pressure sensors
- Finding reasonable steady-state voltages that will not overflow the tanks
- Finding reasonable setpoints for the bottom two tanks
- Finding gain factors of the pumps
- Finding the gamma values for each of the orifices

These factors are all important for designing a robust controller for the system.

2.2 PRESSURE SENSOR BASIS TESTING

Before acquiring testing data, it is important to test the accuracy of the sensor readings. To do this, a test was conducted with the pumps powered off (Figure 2). This gives a basis for the sensor readings when they should theoretically be zero.

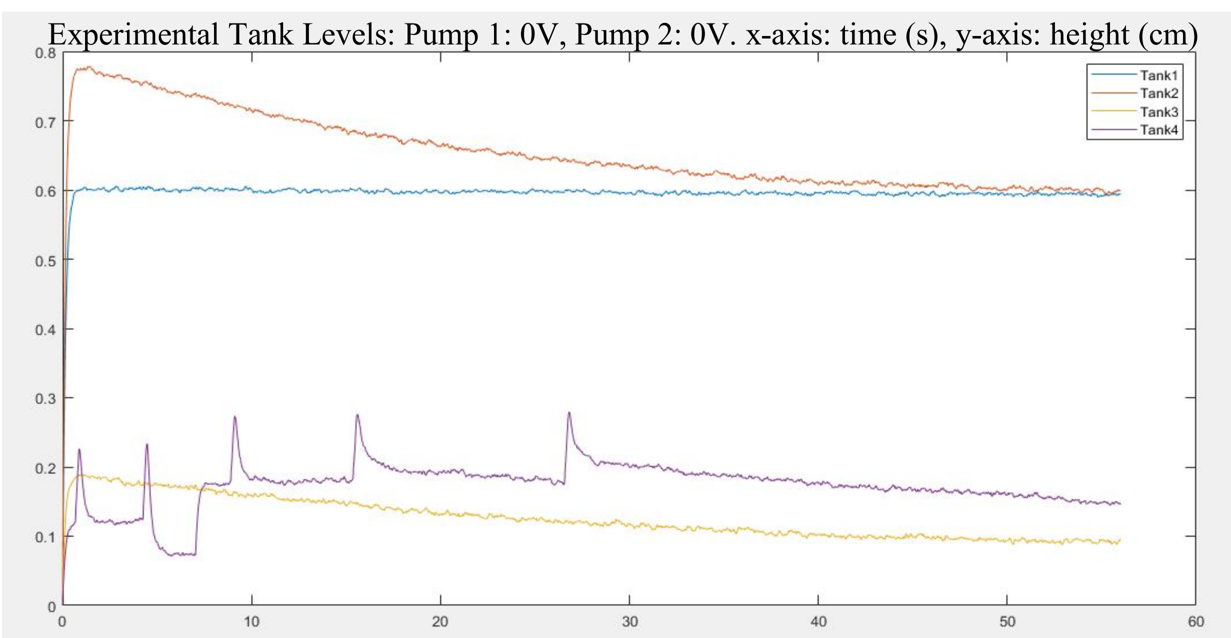

Figure 2 - Experimental Tank Levels. Pump 1: 0V, Pump 2:0V

It can be seen that the values are not zero, and are also not consistent. The Tank 11 sensor remains constant at 0.6 cm, but the other sensors are not stable. This means that the readings of the tank sensors should not be assumed to be 100% accuracy, and there is a degree of error to the results. It should also not be assumed that the errors found in this test will be consistent throughout the rest of testing. By continuing to monitor the subsequent tests, as well as comparing the sensor readings to visual results, the error of the system can be accounted for when discussing the accuracy of the results.

2.3 IMPORTANCE OF TOP TANK LEVELS

To get a general understanding of the systems characteristics, several tests were done without any modifications to the system (i.e. the described system in 1.0 Introduction remains unchanged), while varying the voltage input to the pumps. As it is the goal to set the water level of the bottom two tanks (Tank 21 and Tank 22), the hypothesis is that if the water levels of the top tanks do not exceed those of the bottom tanks, then the bottom tanks will always overflow first before the top tanks. This is very helpful in modelling the controller because it concludes that the level of the top tanks can be neglected. The conclusion from this hypothesis is that the individual pumps will fill their respective top tank quicker than the bottom tank that they are directly filling (Figure 3, Figure 4, Figure 33, Figure 34). By operating the pumps individually, the water levels of the top tanks cannot be ignored.

Experimental Tank Levels: Pump 1: 10V, Pump 2: 0V. x-axis: time (s), y-axis: height (cm)

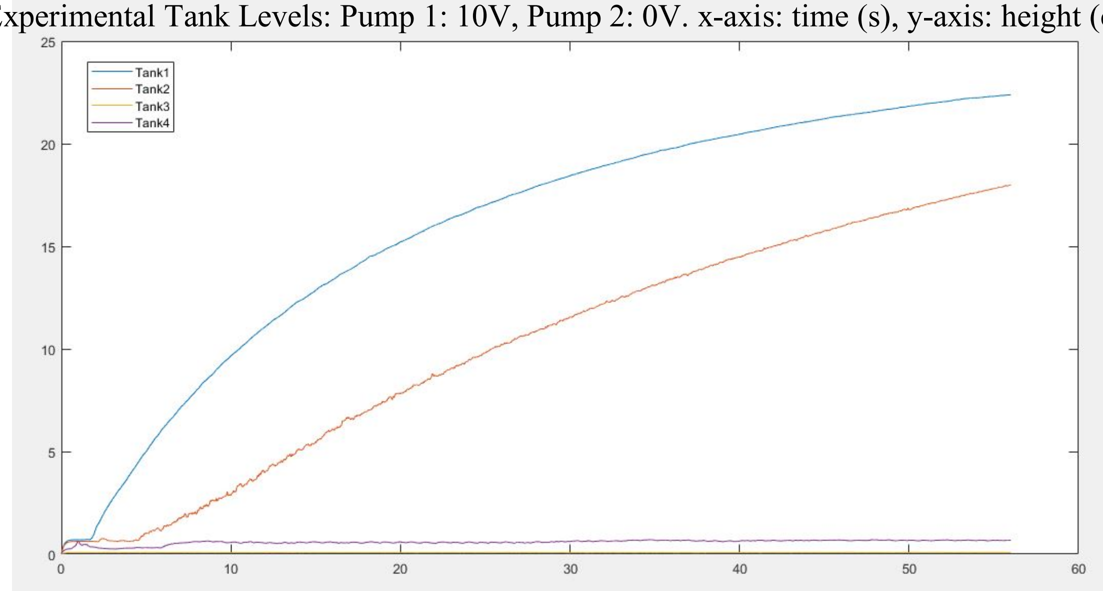

Figure 3 - Experimental Tank Levels. Pump 1: 10V, Pump 2: 0V

Experimental Tank Levels: Pump 1: 0V, Pump 2: 10V. x-axis: time (s), y-axis: height (cm)

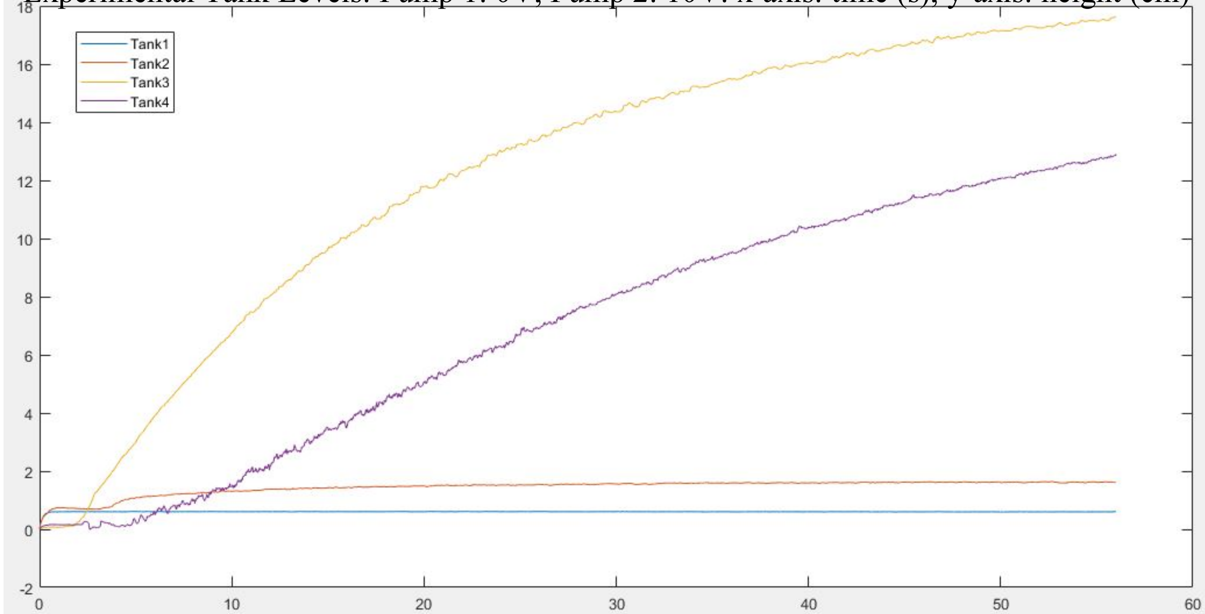

Figure 4 - Experimental Tank Levels. Pump 1: 0V, Pump 2: 10V

The next tests were done with equal voltage for each of the pumps. The results from these tests showed that the bottom tanks filled faster than their respective top tank (i.e. the bottom left tank filled faster than the top left tank) (Figure 5, Figure 35). By analyzing this data, it can be concluded that as long as the voltage inputs to each of the pumps is similar (assume +/- 1V at steady state), then the water level of the top tanks can be neglected.

Experimental Tank Levels: Pump 1: 10V, Pump 2: 10V. x-axis: time (s), y-axis: height (cm)

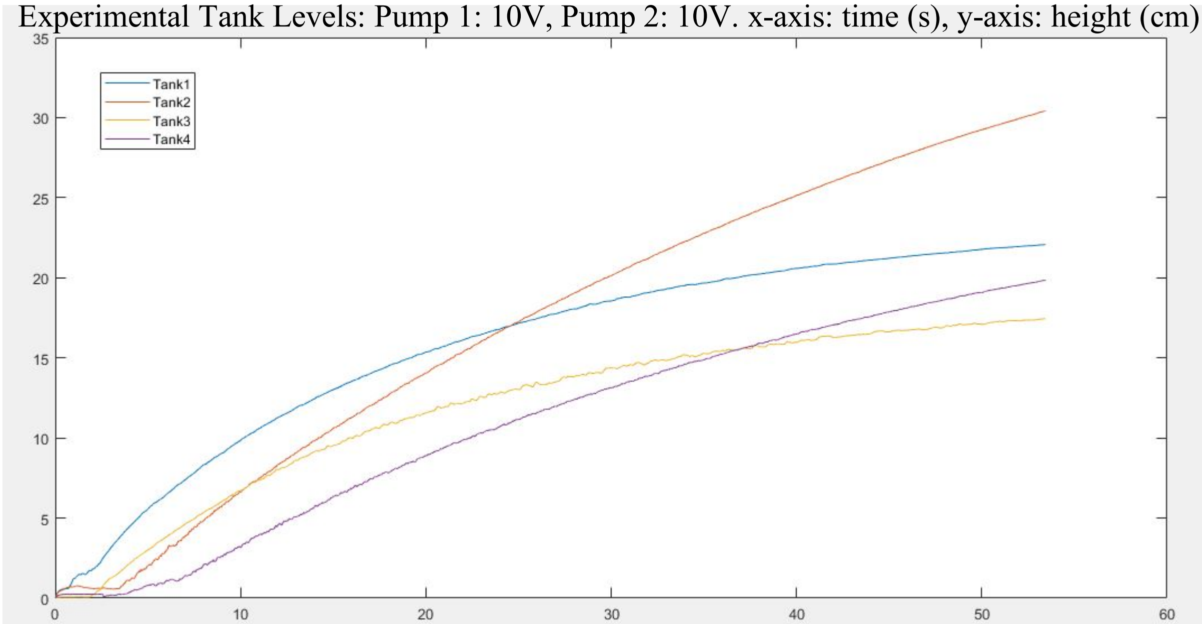

Figure 5 - Experimental Tank Levels. Pump 1: 10V, Pump 2: 10V

2.4 DETERMINING THE SETPOINTS OF THE BOTTOM TANKS

The information gathered from these tests provided the basis for deciding the setpoints for the bottom two tanks. To begin, the setpoints are chosen to correspond with voltages that will have similar steady-state voltages so that the water levels of the top two tanks can be neglected. By analyzing the tests done with equal voltages for both pumps, setpoints of 12 cm and 5.5 cm are chosen for Tank 21 and Tank 22 respectively. These setpoint values correspond closely to a 5V input for both pumps (Figure 35).

2.5 FINDING GAIN FACTORS OF THE SYSTEM

Finding the gain factors of the system is an important aspect of modelling the system. The first important gain required is the sensor gain. The sensor gain is the characteristic that accounts for the error of the sensor readings. By comparing the readings from the sensors to visual results, this ratio is recognized as the sensor gain (Figure 6, Figure 7). This gain is then multiplied by the sensor readings to provide a more accurate representation of the water levels.

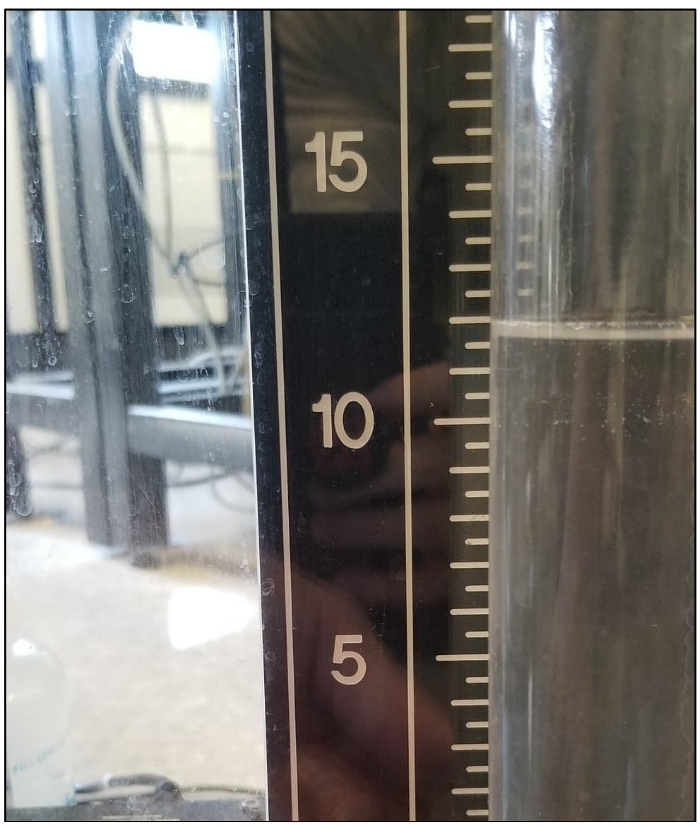

Figure 6 - Final Water Level Visualized, Tank 21, used for Determining Sensor Gain

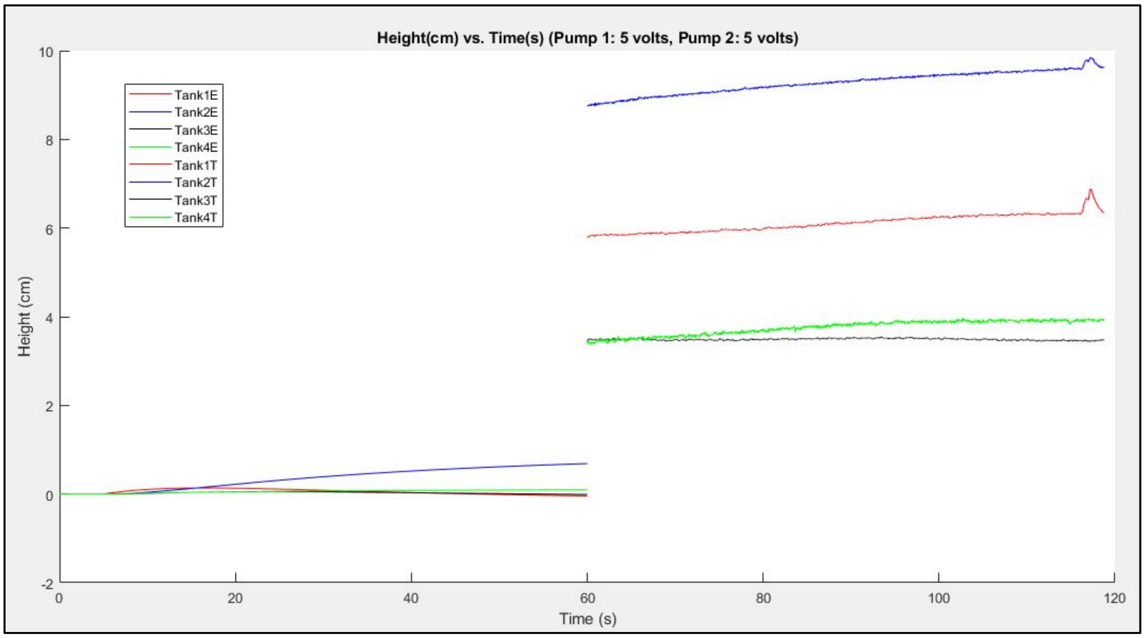

Figure 7 - Final Water Level Sensor Readings, used for Determining Sensor Gain

Another gain required to model the system is the pump flow into each of the tanks. This is the rate at which water is pumped directly into each of the tanks. To conduct this test, the outflow orifice of each tank is plugged, and the voltage input for each of the pumps is set to 5V, the approximate steady-state voltage required to meet the desired setpoints. The result of this test shows a linear relationship between time and water level (Figure 8). The slope of these results illustrates the value for these gain constants.

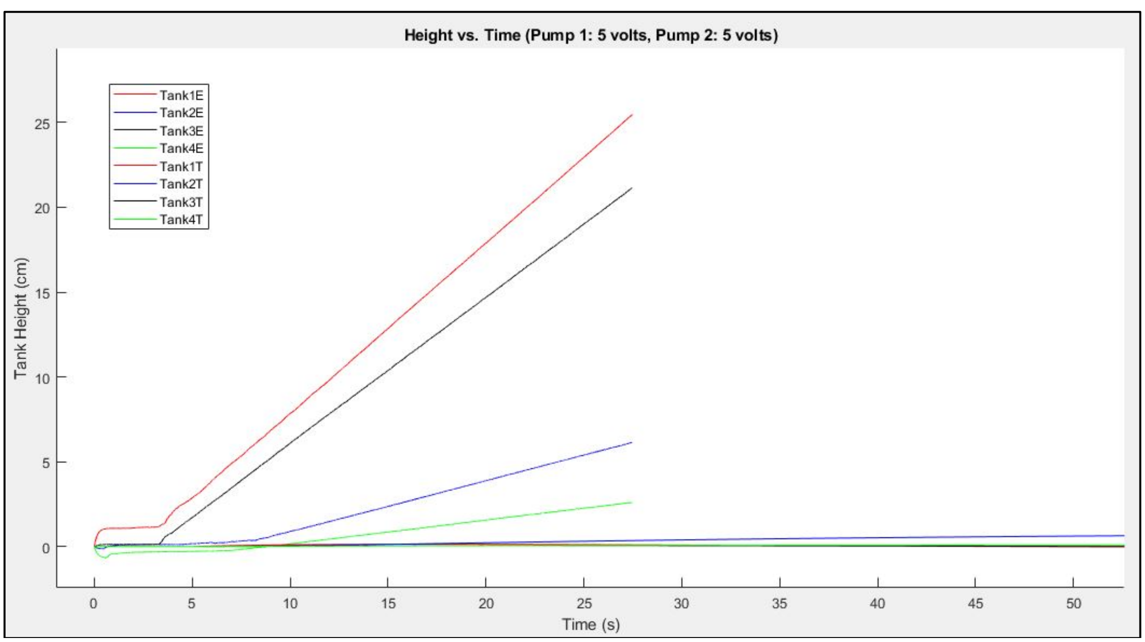

Figure 8 - Experimental Water Level. Outlet Orifices Plugged. Pump 1: 5V, Pump 2: 5V

3.0 MODEL DESCRIPTION/VALIDATION

The model used is a basic statically decoupled system before an IMC controller. This allows the height of the bottom two tanks to be controlled without overflowing the top two tanks. This model uses the values found in section 2.5 Finding Gain Factors of the System to determine what values to be used in each of the parts. Due to the constraints of no longer being able to interact with the four-tank system blocks were added to the Matlab model to simulate how a real-world system would be sampled.

Equation 1 -State Space Equations

$$\dot{h} = \begin{bmatrix} \frac{-a\sqrt{2g}}{2A\sqrt{ho11}} & 0 & 0 & 0 \\ 0 & \frac{-a\sqrt{2g}}{2A\sqrt{ho12}} & 0 & 0 \\ \frac{a\sqrt{2g}}{2A\sqrt{ho11}} & 0 & \frac{-a\sqrt{2g}}{2A\sqrt{ho21}} & 0 \\ 0 & \frac{a\sqrt{2g}}{2A\sqrt{ho12}} & 0 & \frac{-a\sqrt{2g}}{2A\sqrt{ho22}} \end{bmatrix} h + \begin{matrix} \frac{kp11}{A} & 0 \\ 0 & \frac{kp12}{A} \\ 0 & \frac{kp21}{A} \\ \frac{kp22}{A} & 0 \end{matrix} u$$

$$y = \begin{matrix} 0 & 0 & sensorGain21 & 0 \\ 0 & 0 & 0 & SensorGain22 \end{matrix}$$

The model was created using a state-space equation. The four desired set point values are used via equations inside of the A matrix of a standard state-space equation. This uses the outlet and tank areas that can be found within [1] combined with earth’s gravitational constant. This is then added with flow constants tested for within 2.5 Finding Gain Factors of the System, in the B matrix. Finally, the sensor gains are added into the C matrix to transfer the readings into usable values for the system. This can be viewed above in Equation 1 along with the equations used. The variables are as follows: a = inlet area, A = tank area, g = force of gravity, hoij = equilibrium height, kpij = flow gain.

Due to the process of IMC design having been followed from [1] and [2] it is likely that the model used will be very close to what is required. Unfortunately, it is impossible to tell to what degree the model follows the physical system due to the inability to test the model against it. As such, the decoupling uses simple fluid dynamics equations to model the process through which water drains into lower tanks and is thus likely very close to reality the rest of the model cannot be validated at this time. Therefore ADCs, DACs, and noise blocks were added to the system stand-in to simulate how the system would have responded and is therefore exactly like the model used in the IMC. Once it is possible to access the plant again the outputs of the modelled plant can be compared with the model and tweaked so that they more closely follow the plots of each other.

4.0 SYSTEM DECOUPLING

4.1 GOAL

The goal of decoupling a MIMO system is so that two separate controllers can be designed that will have independent effects of one another. There are several methods of decoupling that can be done. Since for this system there are constant desired setpoints, the system can be statically decoupled using a linearized model around 5V inputs for each of the pumps. In this case, though the water is pumped into four outputs, the model can be simplified to only consider the bottom two tanks. This means that though control of the height of the top two tanks is lost, the height of the water in the bottom two tanks can be more measured precisely. This can be done because as mentioned in 2.3 Importance of Top Tank Levels, the bottom tanks will be the first to overflow, and therefore the top tanks are safe to disregard.

4.2 LINEARIZING BERNOULLI EQUATIONS

Equation 2 - Bernoulli Equations

$$\frac{dh11}{dt} = \frac{gamma11 * kp11}{A} \times v1 - \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h11}$$
$$\frac{dh12}{dt} = \frac{gamma12 * kp12}{A} \times v2 - \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h12}$$
$$\frac{dh21}{dt} = \frac{gamma21 * kp21}{A} \times v2 - \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h21} + \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h11}$$
$$\frac{dh22}{dt} = \frac{gamma22 * kp22}{A} \times v1 - \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h22} + \left(\frac{a}{A}\right)\sqrt{2g}\sqrt{h12}$$

These are the equations we were given from the lab manual, with all a values being the same and all A values being the same, which is why they are unspecific for each equation.

Any constants not given in the lab manual (or that we thought might be inaccurate) were measured experimentally. The gammas were found by plugging all the tanks and comparing heights reached after a certain amount of time, while the other gains were gathered by leaving the plugs open until steady state was reached then measuring the heights. These tests were repeated twice and an average was reached.

Note that in the original equations, kp only has two forms. During our experimental findings we found kp values for each tank, and realized that these kp values also double as our gammas (which are ratios of tank flow rate from one pump to two tanks). As such, the gamma value is not actually used for the remaining calculations as kp does the same thing.

For example calculations of kp, and sensorGain,, examine lines 11-20 of Appendix C – Matlab Code

With that said, here are our simplified differential equations:

Equation 3 - Simplified Differential Equations

$$c11 = \frac{kp11}{A} \text{ and } b = \left(\frac{a}{A}\right)\sqrt{2g}$$
$$\frac{dh11}{dt} = c11 \times v1 - b\sqrt{h11}$$
$$\frac{dh12}{dt} = c12 \times v2 - b\sqrt{h12}$$
$$\frac{dh21}{dt} = c21 \times v2 - b\sqrt{h21} + b\sqrt{h11}$$
$$\frac{dh22}{dt} = c22 \times v1 - b\sqrt{h22} + b\sqrt{h12}$$

Finding the output ‘y’:

$$y1 = kc1 \times x1 \text{ and } y2 = kc2 \times x2$$

where kc is sensor gain for the corresponding tank of the system.

The goal is to rearrange this system of equations into the state-space form $x' = Ax + Bu$ and $y = Cx + Du$, where A, B, C, and D are matrices. The D matrix is all zeroes in our case so it is ignored. But to do this, we need entirely linear functions and $\sqrt{h_{ij}}$ is not a linear function. We solved this by using the first-order Taylor series approximation shown below:

Equation 4 - First-Order Taylor Series Linear Approximation

$$\text{First Order Taylor Series Linear Approximation} = \sum_{n=0}^{\infty} \frac{f^{(n)}(x0)}{n!}(x - x_0)^n = f(x_0) + \left(\frac{f'(x_0)}{1!}\right)(x - x_0)$$

x_0 is hoij shown below. We centered this approximation around inputs of 5Volts

```
%ho = desired equilibrium height (measured steady state values at 1p05 2p05)
ho11 = 5.0;
ho12 = 3.0;
ho21 = 12.0;
ho22 = 5.5;
vo1 = 5; %(voltage)
vo2 = 5; %(voltage)
```

$\boldsymbol{f(x) = \sqrt{x}\,, f'(x) = \frac{1}{2\sqrt{x}}}$

4.3 CREATING STATE SPACE MATRICES

With the equations simplified and linearized, the differential equations above can use another variable that simplifies the equations further, then can be made into the A and B matrices.

$$el_{11} = \left(\frac{1}{2}\right) * \frac{b}{sqrt(ho11)}$$

Then with those sample calculations out of the way, here's what the A, B, and C (with D being a 2x2 matrix of zeroes) matrices wound up being (shown in Figure 37 lines 42 - 58):

```
A = [-el_11 0 0 0;
     0 -el_12 0 0;
     el_11 0 -el_21 0;
     0 el_12 0 -el_22];

B = [c11 0;
     0 c12;
     0 c21;
     c22 0];

C =[sensorGain11 0 0 0;
     0 sensorGain12 0 0;
     0 0 sensorGain21 0;
     0 0 0 sensorGain22];
```

The matrix form for C above could have been used, an $I \times sensorGain_{ij}$ matrix, but we only care about the measurements of two tanks, so y is a 1x2 vector not a 1x4 which this C-matrix would create. Instead we chose this truncated form:

```
C = [0 0 sensorGain21 0;
     0 0 0 sensorGain22];
```

This means the only y outputs are for h21 and h22, the bottom two tanks shown in the diagram below (Figure 9).

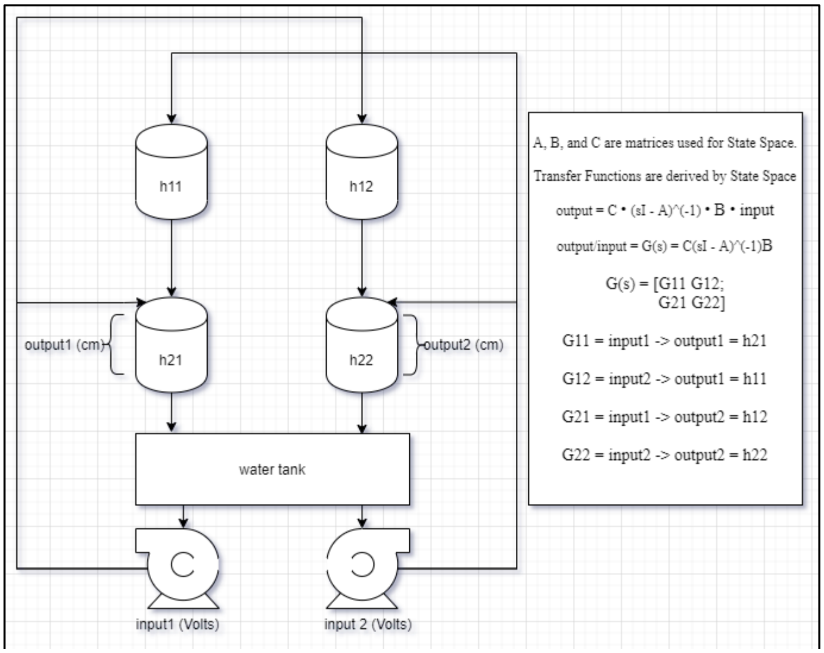

Figure 9 - Reference for Labelling Conventions between Tank and Plant Transfer Function Labels

$$\boldsymbol{ytf} = \boldsymbol{G}(\boldsymbol{s}) = \boldsymbol{C}(\boldsymbol{sI} - \boldsymbol{A})^{-1}\boldsymbol{B}$$

Using this equation to find the 2x2 transfer function matrix for the plant (ytf)

Ytf=G(s)=Plant TF=

```
  From input 1 to output...
           0.01801
1:  -------------------------
  s^2 + 0.1871 s + 0.008345
           0.02031
       2:  ----------
          s + 0.1084
  From input 2 to output...
           0.04722
       1:  -----------
          s + 0.07339
           0.01844
2:  ------------------------
  s^2 + 0.2552 s + 0.01591
```

The line:

```
PlantStatic = evalfr(ytf,0)
```

finds the resultant matrix when s = 0. Then, pinv() is a Moore-Penrose pseudo inverse algorithm, used to approximate the decoupling matrix.

```
DecoupleStatic = pinv(PlantStatic)*C

DecoupleStatic =
[0          0    0.3829   -0.1964;
0          0   -0.0619    0.6588]

DecoupleMatrixShortened =
[0.3829   -0.1964;
-0.0619    0.6588]
```

This is the Decoupling matrix that we wound up using, just by truncating the columns so it was square, like the Plant matrix.

To confirm that the decoupling matrix will work with the plant, we test by doing:

$$ResultantC = DecoupleMatrixShortened \times PlantStatic$$

```
initialC =
[0          0    0.7867         0;
0          0         0    0.7265]

resultantC =
[0.7897    0.0188;
-0.0102    0.7235]
```

As can be seen, the values are very similar to the initial C matrix if it was truncated. This was acceptable to the group and we proceeded to convert the D matrix and plant matrix into blocks in Simulink, shown in the next section.

4.4 DECOUPLED MODEL

The final statically decoupled system provides a foundation to design two independent controllers for controlling each of the bottom tanks (Figure 10, Figure 11, Figure 12). Since it is not possible to create ideal decoupling in this system, the resultant model is slightly less accurate than the original model. Also, since it is not ideally decoupled, the two controllers are not perfectly independent from each other. The decoupling constants in Figure 11 illustrates the effectiveness of the decoupler. In a perfectly decoupled system, D11 and D22 would be 1, and D21 and D12 would be 0. This would symbolize that the input from two has no effect on the output of one. In this case, the constant values are theoretically the most decoupled that this system can achieve (Figure 11). As a result, the system response of this system is not exactly equivalent to the actual plant (Figure 13).

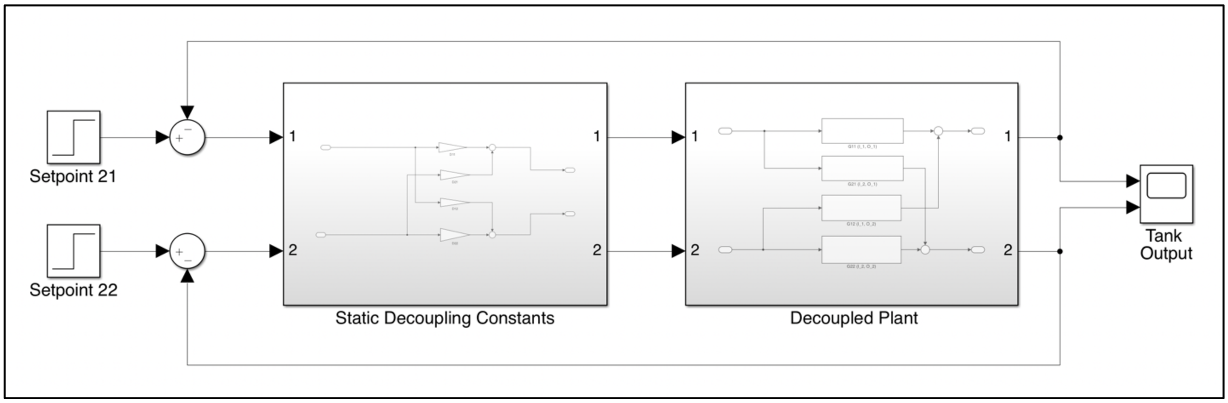

Figure 10 - Decoupled System

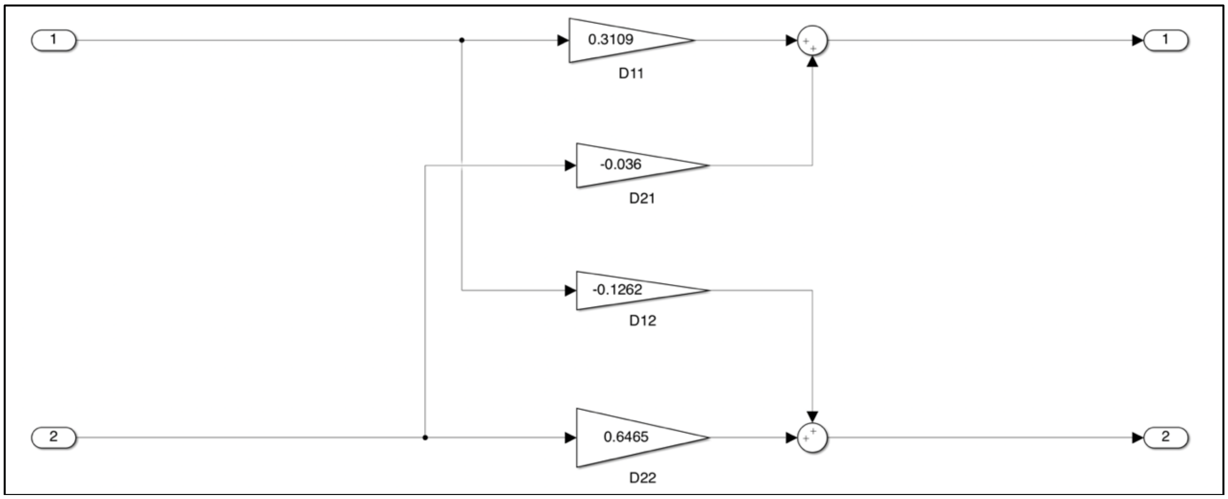

Figure 11 - Decoupling Constants Subsystem

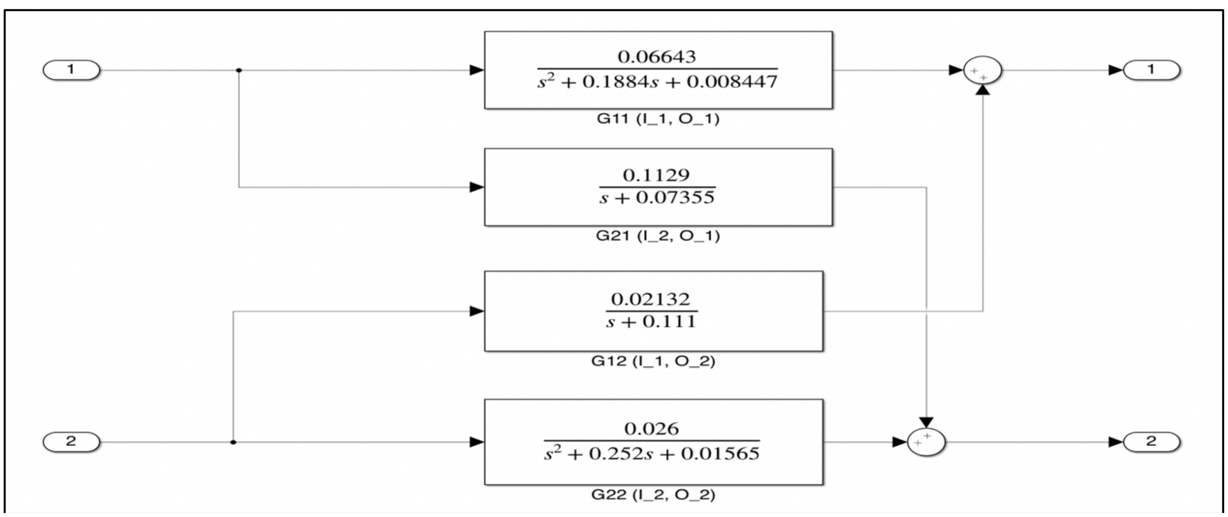

Figure 12 - Plant Transfer Functions Subsystem

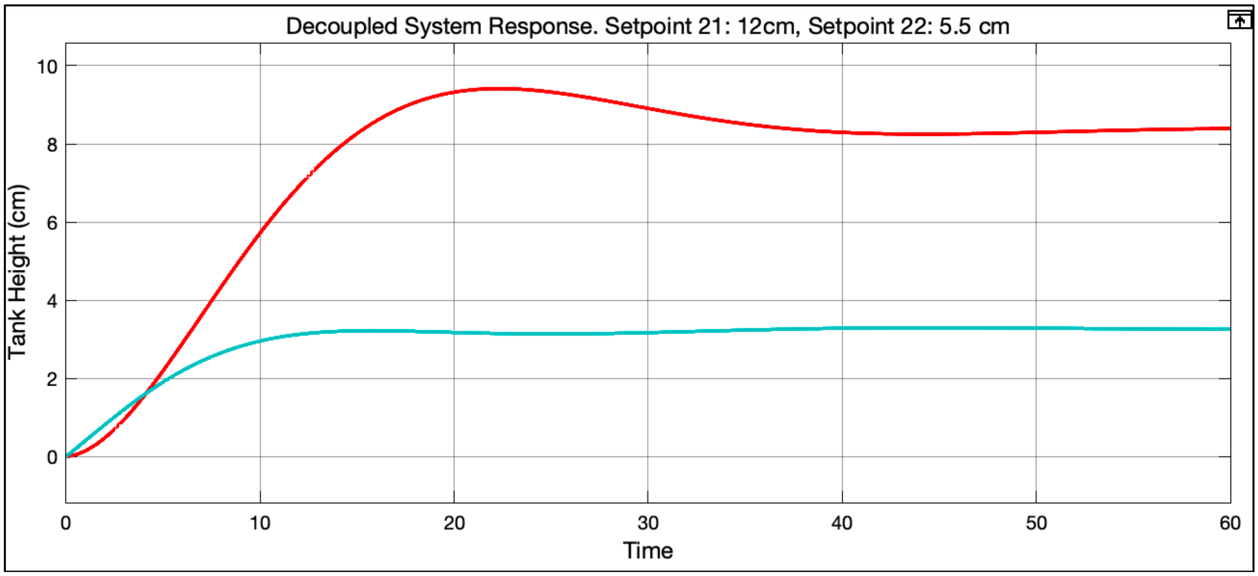

Figure 13 - Decoupled System Response. Setpoint 21: 12 cm, Setpoint 22: 5.5 cm

5.0 ALTERNATIVE CONTROLLER DESIGN

5.1 PID CONTROLLER USING ZIEGLER-NICHOLS TUNING

The controller specification for this plant have been changed from the proposal due to unrealistic goals where we set the settling to time to be less than 30 seconds. The new specification of the system needs to have a settling time less than 60 seconds, a low overshoot percentage less than 15%, and lastly for the system not to overflow (Maximum tank height of 30 cm). These specifications can be acquired with the use of a PID controller. PID will allow the group to tune the system and make it reach the set point while still being within the plant specification limits.

There is multiple PID tuning methods, the one that will be done for this controller is Ziegler-Nichols tuning and specifically the no overshoot formulas. The first step into tuning a PID is to set the I & D gains to zero, then start increasing the P gain until neutral stability is acquired as shown in Figure 14. This P gain is now called $K_{U\ \text{tank 21}} = 8.62$ & $K_{U\ \text{tank 22}} = 8.62$. Next step is to start calculating the Ziegler-Nichols values using the table shown above. Repeat this step for each of the controllers.

Table 1 - Ziegler-Nichols PID Equations

Control Type	**K_P**	**Ti**	**T_d**		**$K_I = K_P/Ti$**	**$K_D = T_d\, K_P$**
PID no%OS	0.2 K_U	$T_U/2$	$T_U/3$		(2/5) K_U/T_U	(1/15) K_U/T_U

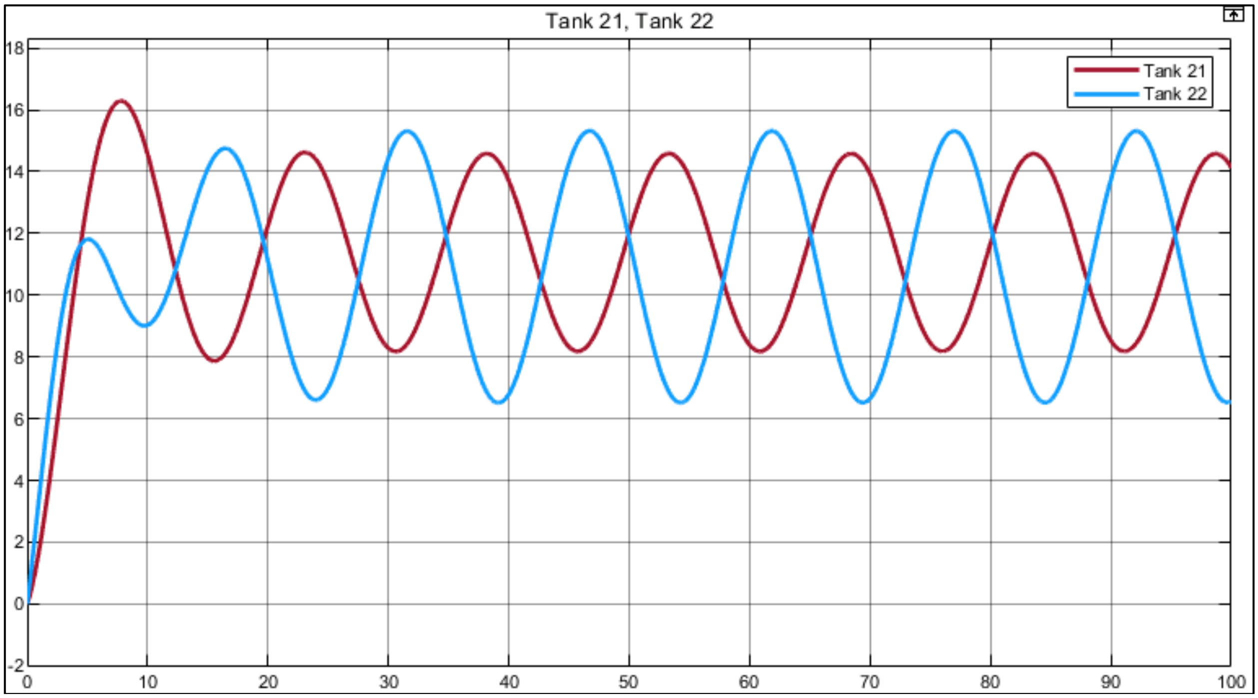

Figure 14 - Neutral Stability @ $K_u = 8.62$

Table 2 - Ziegler-Nichols Tuning Values Prior to Tweaking

PID	K_p	T_i	T_d		$K_I = K_p/T_i$	$K_D = T_dK_p$
Tank 1	1.724	7.912	5.274		0.2179	9.093
Tank 2	1.724	7.845	5.23		0.2198	9.017

As shown in the table above, the calculated values for tuning has been acquired with the use of the Ziegler-Nichols tuning equations. The next step is to input these values into the controller and start tweaking to achieve the utmost results as shown in Figure 15 & Figure 16. The final PID gain that will be used for this system are shown in the table below.

Table 3 - Ziegler-Nichols Tuning Values after Tweaking

PID Tuning Values	Tank 1	Tank 2
K_P	4.5	4.5
K_I	0.2179	0.2198
K_D	10	10

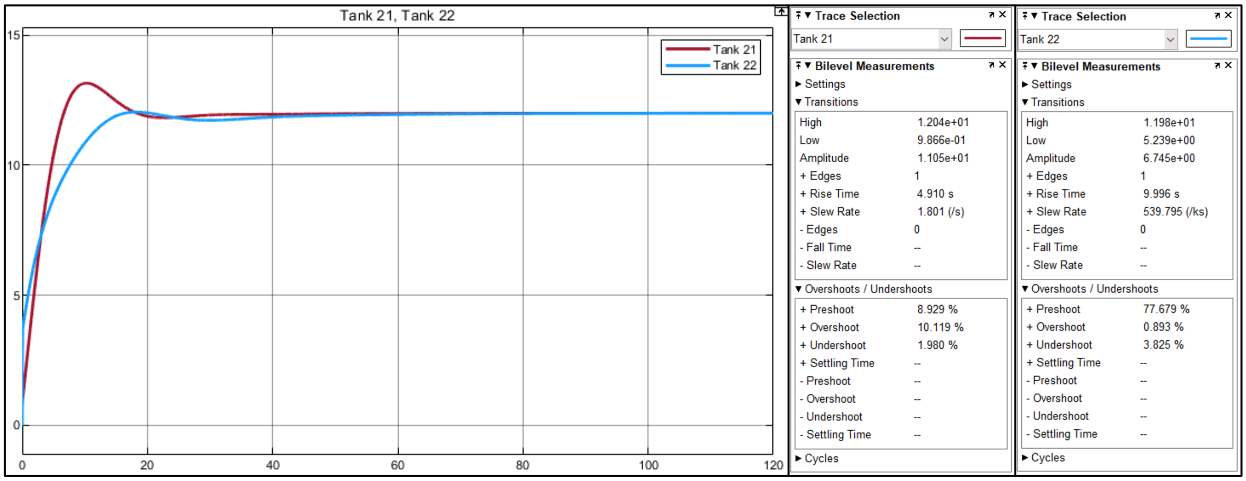

Figure 15 – Model Height (cm) vs. Time (s) with PID Controller. Pump 1: 12 volts, Pump 2: 12 volts. Yellow: Tank 21 Output, Blue: Tank 22 Output

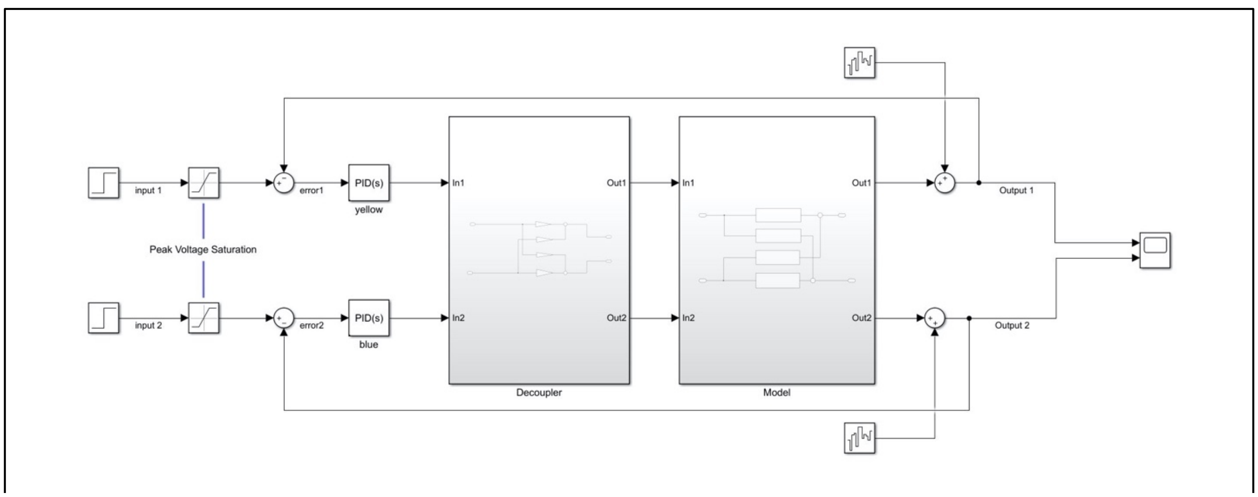

Figure 16 - Model with Noise Inputs on Height of the Tanks

After achieving the PID controller values, the nest step was to test the system and check how robust it is. This was attained by adding a noise source to each of the tanks. This noise is caused by water splashes or leaks. When the noise was added the results of the system where steady and the system reacted well as shown in Figure 17. This shows that the system is indeed robust and can react well to any type of noise.

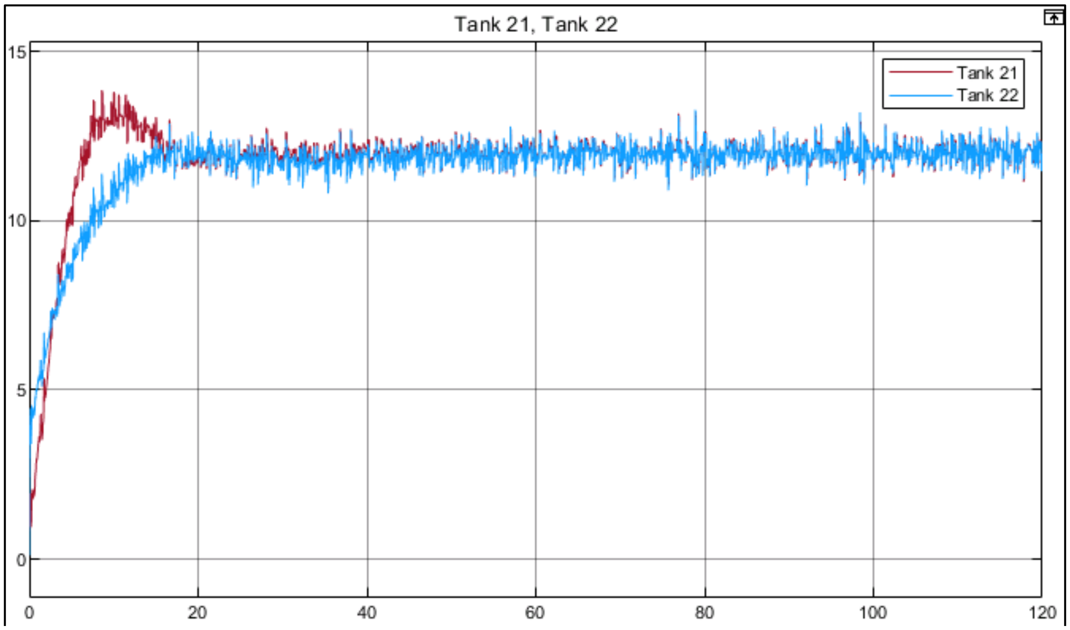

Figure 17 – Height (cm) vs. Time (s) Response with Noise. Yellow: Tank 21 Output, Blue: Tank 22 Output, Pump 1: 12 volts, Pump 2: 12 volts.

Another robustness test has been conducted by removing/adding water to the tanks. At 20 seconds, 5 cm of water is added to Tank 21, and 5 cm of water is removed from Tank 22 (Figure 18). It is evident that the PID controller returns the water level to the desired setpoint of 12 cm, rather than Figure 32, as shown with no controller.

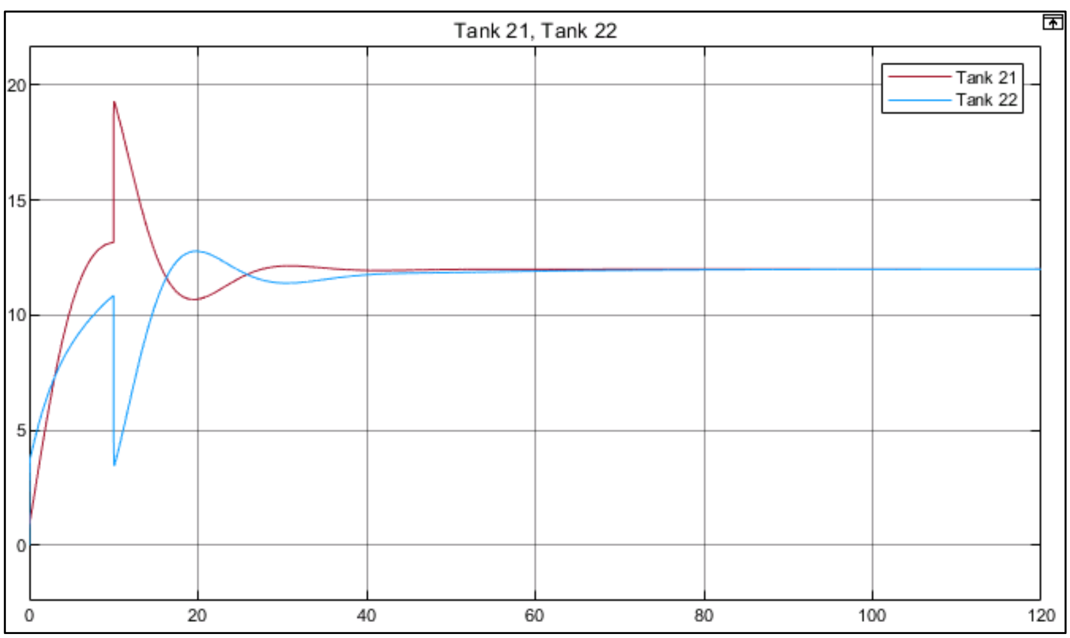

Figure 18 - Height (cm) vs. Time (s) with 5cm of water added/removed to Tank 21 and Tank 22 at 10 seconds

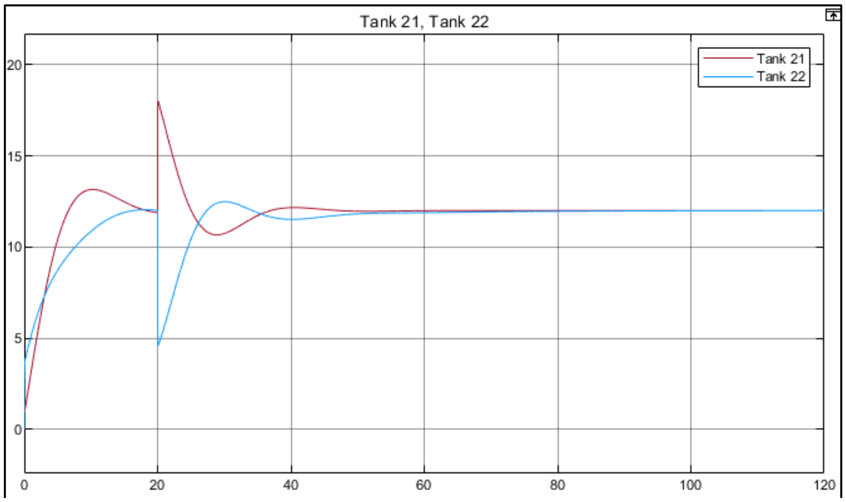

Figure 19 - Height vs. Time with no controller, adding/removing 5cm of water at 20 seconds

6.0 IMC CONTROLLER DESIGN

6.1 IMC CONTROLLER DESIGN METHODOLOGY

The final choice for the type of controller to use is an Internal Model Control (IMC) based approach (Figure 20). IMC operates on the basis of comparing the physical plant to the modelled system, then use a controller to control the error of this comparison. IMC controllers are extremely robust, and only require a single tuning parameter, making them easy to tune. The downside of the IMC method is that it does not provide a large improvement for settling time. Since the Plant used in the system has been decoupled, a separate SISO IMC can be designed for each of the bottom tanks.

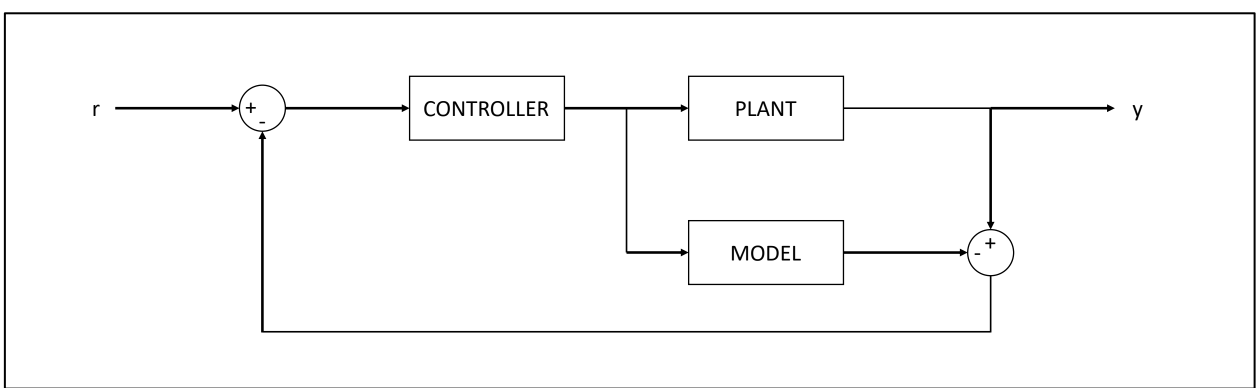

Figure 20 - IMC Controller Schematic

The model for the IMC is created by analyzing the closed-loop system response of the decoupled plant (Figure 13). By observation, it is decided to simplify Tank 21 as a second-order system, and Tank 22 as a first-order system. The theory behind creating this simplified model is that the poles and zeros of the simplified model can be used for the controller. This creates the opportunity to design two separate SISO IMC systems rather than using MIMO IMC.

To model Tank 21, a second-order system, the natural frequency and peak overshoot are required. These characteristics are both determined from observing the plot. The open-loop transfer function is then determined (Equation 5, Equation 6, Equation 7, Equation 8, Equation 9).

Equation 5 - Natural Frequency of Tank 21

$$\omega_n = \frac{2\pi}{Frequency\ Period} = 0.17285\, rad/s$$

Equation 6 - Percent Overshoot of Tank 21

$$\%OS = \frac{9.414 - 8.388}{8.388} * 100\% = 12.23\%$$

Equation 7 - Gain of Second-Order System

$$k = \frac{Steady\ State}{Setpoint} = \frac{8.388}{12}$$

Equation 8 - Damping Ratio Formula

$$\zeta = \frac{\left|\ln \frac{\&OS}{100}\right|}{\sqrt{\pi^2 + \ln^2\left(\frac{\%OS}{100}\right)}}$$

Equation 9 - Second-Order Transfer Function

$$G(s) = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The final open-loop transfer function is then converted into its closed-loop equivalent (Equation 10).

Equation 10 - Tank 21 Model Open-Loop to Closed-Loop Transfer Function

$$G_{CL}(s) = \frac{-G}{G-1}$$

$$G_{21,CL}(s) = \frac{\mathbf{0.02088}\boldsymbol{s^2} + \mathbf{0.004041} + \mathbf{0.000624}}{\boldsymbol{s^4} + \mathbf{0.3844}\boldsymbol{s^2} + \mathbf{0.07581}\boldsymbol{s^2} + \mathbf{0.00747}\boldsymbol{s} + \mathbf{0.0002687}}$$

To model Tank 22, a first-order system, Tao and the gain of Tank 22 are required. These values can also be determined by analyzing the plot. This can then be used to determine the closed-loop transfer function (Equation 11, Equation 12, Equation 13, Equation 14).

Equation 11 – Time Constant (Tao) of Tank 22

$$\tau = 5.411$$

Equation 12 - Gain of First-Order System

$$k = \frac{Steady\ State}{Setpoint} = \frac{3.254}{5.5}$$

Equation 13 - First-Order Transfer Function

$$G(s) = \frac{k}{\tau s + 1}$$

Equation 14 – Tank 22 Model Closed-Loop Transfer Function

$$G_{22,OL}(s) = \frac{\left(\frac{3.254}{5.5}\right)}{5.411s + 1}$$

$$G_{22,CL}(s) = \frac{\mathbf{3.201}\boldsymbol{s} + \mathbf{0.5916}}{\mathbf{29.28}\boldsymbol{s}^{\mathbf{2}} + \mathbf{7.621}\boldsymbol{s} + \mathbf{0.4084}}$$

The closed-loop decoupled plant and the calculated models are compared to each other using Simulink (Figure 21). The result of this simplification is very accurate to the decoupled model (Figure 22). The simplified model of Tank 21 is extremely similar to the decoupled plant. For Tank 22, the first-order simplification is slightly different during the transient part of the response.

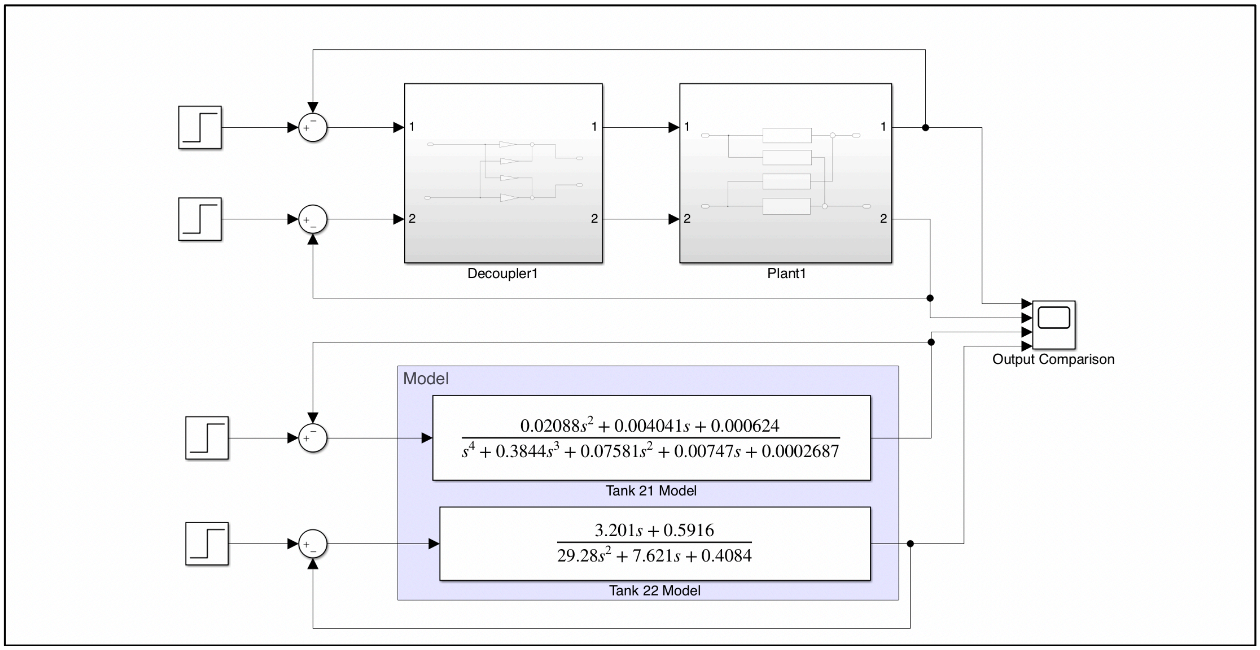

Figure 21 - Simulink Model for Comparing Decoupled Plant and Simplified Model

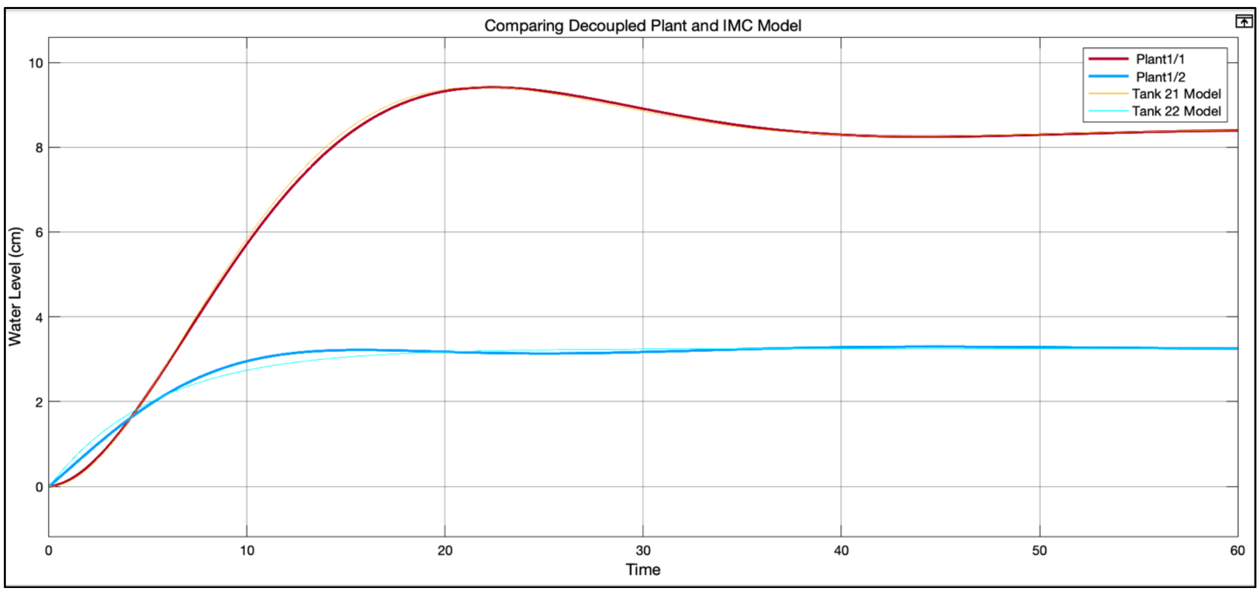

Figure 22 - Comparison of Decoupled Plant and Simplified Model

6.2 TUNING

The IMC tuning method offers stable and robust alternatives to other techniques. This method is simple to use, as it uses only one tuning parameter which is lambda. The n is only used to make the system stable, it is used when the system has a positive pole in which we can account for using the IMC controller. The IMC offers the following advantages to the plant: 1) No overshoot in the system. Which will make it easier not to overflow the tanks. 2) The tuning rules are less sensitive to errors. 3) The IMC is very robust, which means that the control loop will stay stable even if the process characteristics change. 4) Absorbs disturbances and passes less of them to the plant.

Equation 15 - IMC Tuning Variables

$$F(s) = \frac{1}{(\lambda s + 1)^n}$$

To create an IMC controller equation the following steps must be followed for Tank 21 & Tank 22. The first step to modelling an IMC is to get the plant transfer function:

$$Tank\ 21 = G_p = \frac{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)}{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)(s + 0.1116)(s + 0.0806)}$$

Step 2: Equate the plant transfer function to the model transfer function

$$G_p = \hat{G}_p = \frac{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)}{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)(s + 0.1116)(s + 0.0806)}$$

Step 3: Split the positive and negative zeros

$$G^-{}_p = \frac{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)}{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)(s + 0.1116)(s + 0.0806)}$$

$$G^+{}_p = 0$$

Step 4: Multiply IMC equation by the reciprocal of Gp^-

$$q = F(s) * \frac{1}{G_P{}^-}$$

$$q = \frac{1}{(\lambda s + 1)^n} * \frac{1}{G_P{}^-}$$

Step 5: Choose filter n that is proper for this system, in this case for Tank 21 n = 2

$$q_{tank\ 21} = \frac{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)(s + 0.1116)(s + 0.0806)}{(s + 0.0961 + 0.1437i)(s + 0.0961 - 0.1437i)(\lambda s + 1)^2}$$

This shows the final controller transfer function q for Tank 21. Similarly, for Tank 22 the equation q will be:

$$q_{tank\ 22} = \frac{(s + 0.1848)(s + 0.0755)}{(s + 0.1848)(\lambda s + 1)^1}$$

The final step is to tune the IMC by changing lambda. It follows a simple trial and error method until the system reaches the desired output as shown in Figure 23. Table 4 below shows the lambda and n used for this system.

Table 4 - IMC Controller Parameters

IMC	lambda	n
Tank 21	6.918	2
Tank 22	9.143	1

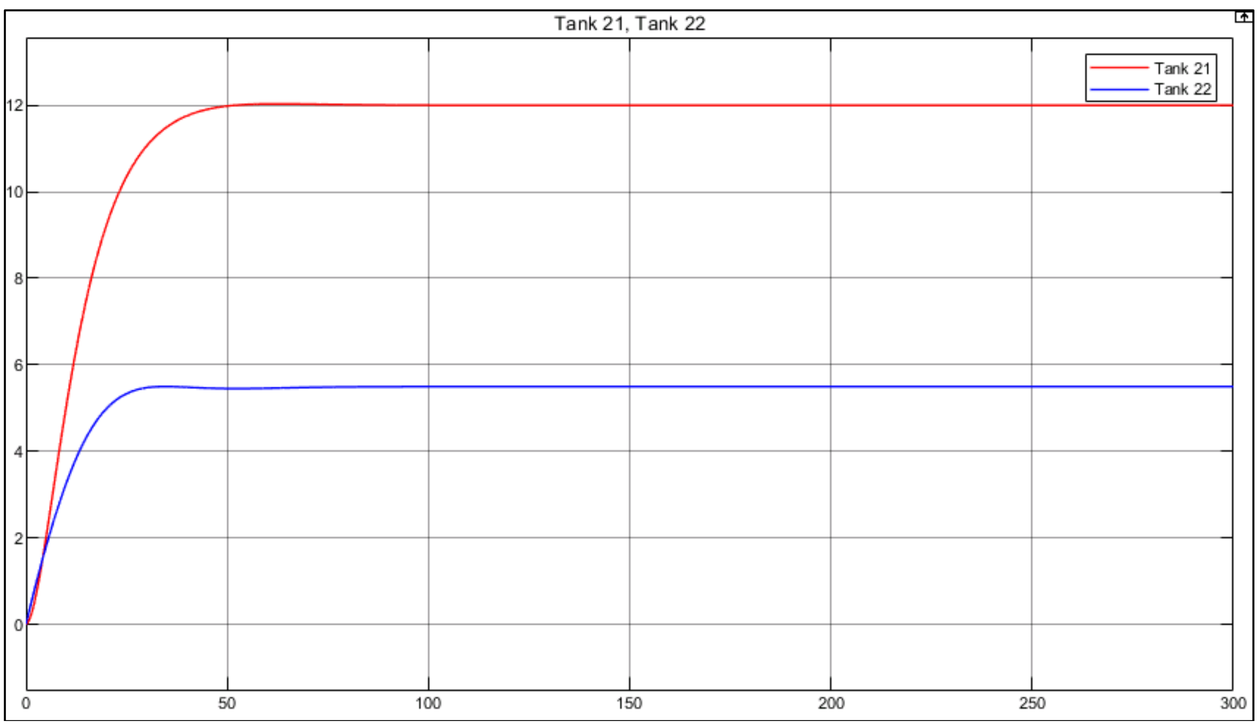

Figure 23 - System Response of the System with IMC. Red: Tank 21, Blue: Tank 22

7.0 SYSTEM RESPONSE

7.1 RESULTS

The first responses we were concerned with were the ones that the Taylor Series linearization was centered around:

```
ho11 = 5.0;
ho12 = 3.0;
ho21 = 12.0;
ho22 = 5.5;
```

Since we are controlling for the bottom two tanks (h21 and h22) those are the primary setpoint inputs. If those work within our constraints, further height setpoints would be tested. h21 is in Red, h22 is in blue. Figure 24 shows that our overshoot is below 1% for both heights, and the settling time is less than 60 seconds, shown by measuring the height at 60 seconds and confirming it is within 2% of the desired setpoint. With this within desired parameters, further tests can be done.

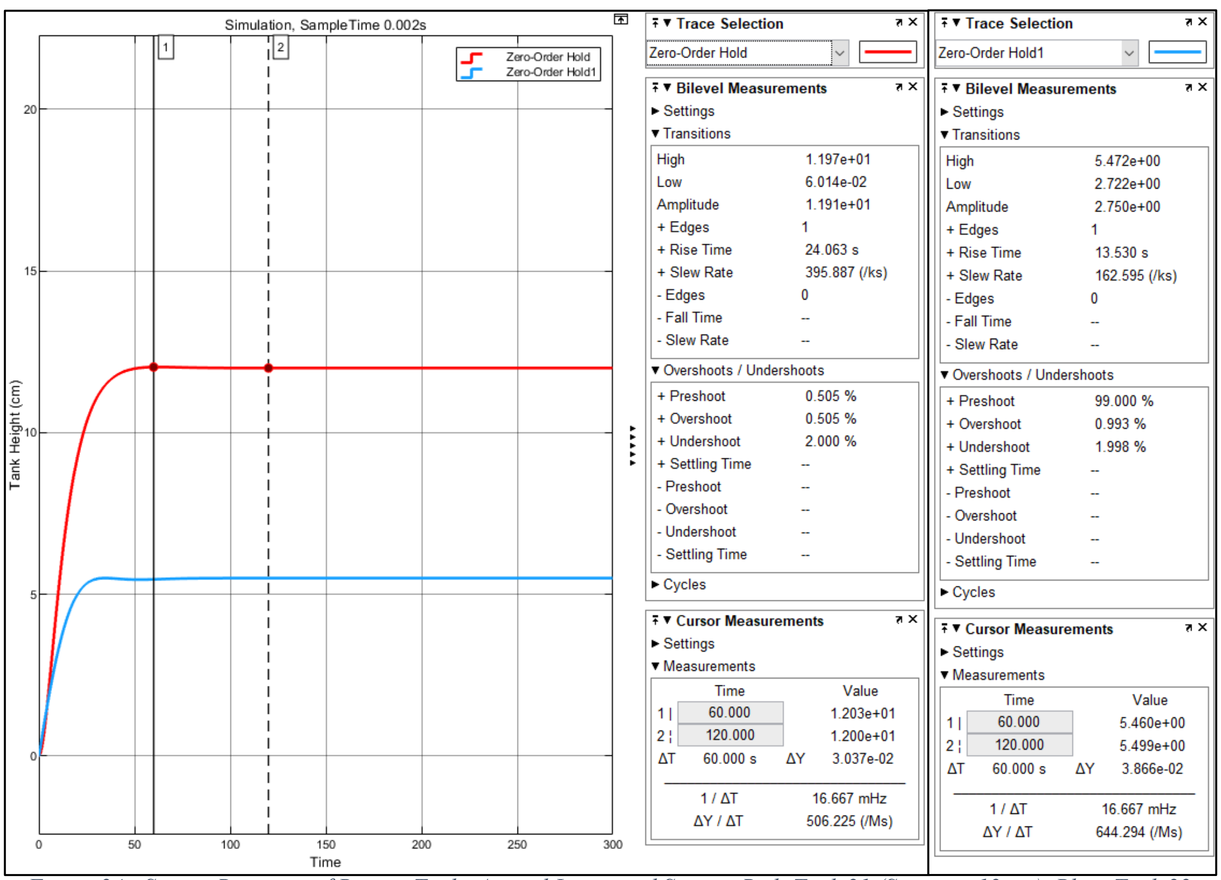

Figure 24 - System Response of Bottom Tanks Around Linearized System. Red: Tank 21 (Setpoint: 12 cm), Blue: Tank 22 (Setpoint: 5.5 cm)

Figure 25 shows the results for h22 (blue) = 11cm and h21 (red) = 3cm. This was to check what would happen if blue was substantially higher than red. Unfortunately, this shows that the red measurement is 2.904, which is less than 2.94 (which is 98% of 3), which breaks our criteria slightly. However, it is within 5% of the final value which is an alternative standard measure of settling time, and it is not for our primary measured height region so it is acceptable.

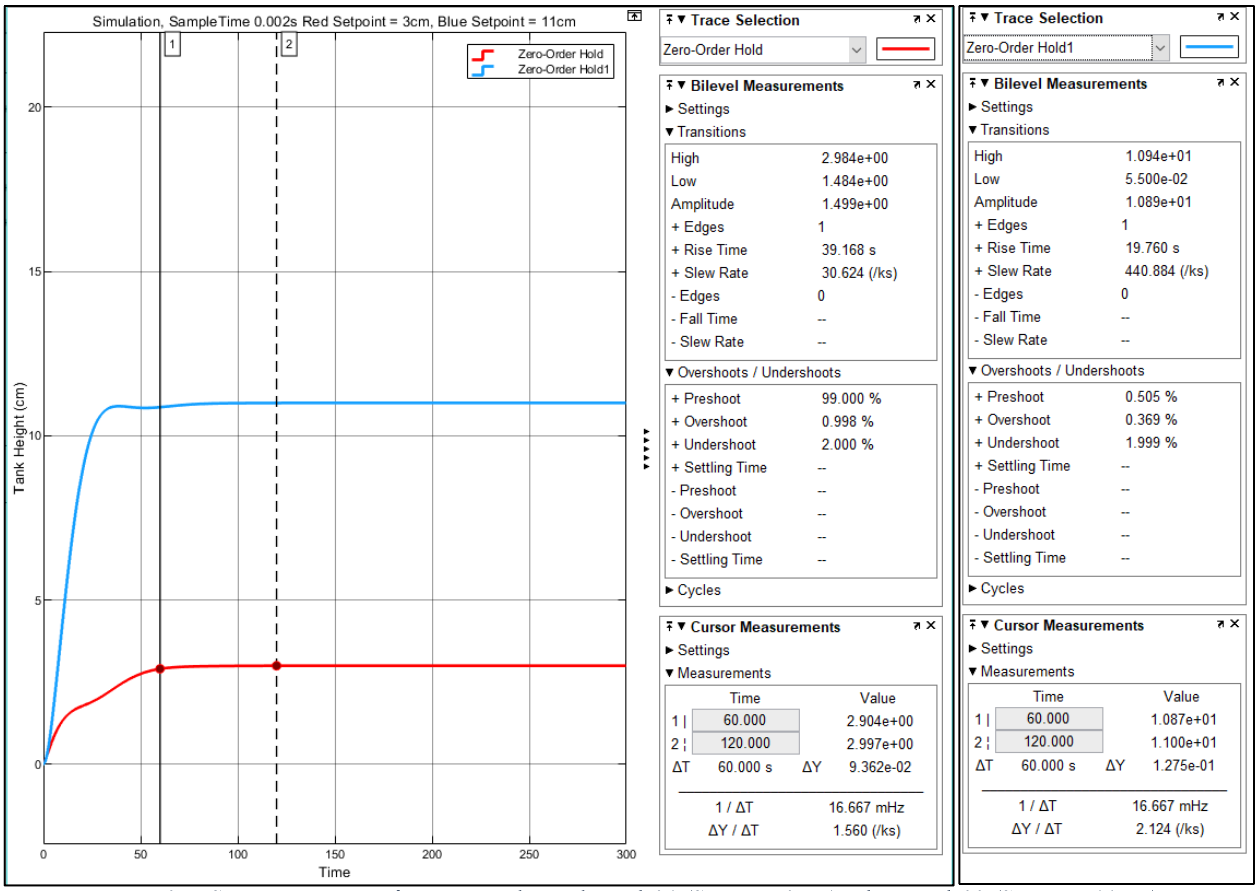

Figure 25 - System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 3 cm), Blue: Tank 22 (Setpoint: 11 cm)

Figure 26 shows the results for h22 (blue) = 2cm and h21 (red) = 1cm. This test was for very small values, and is more successful than the previous one.

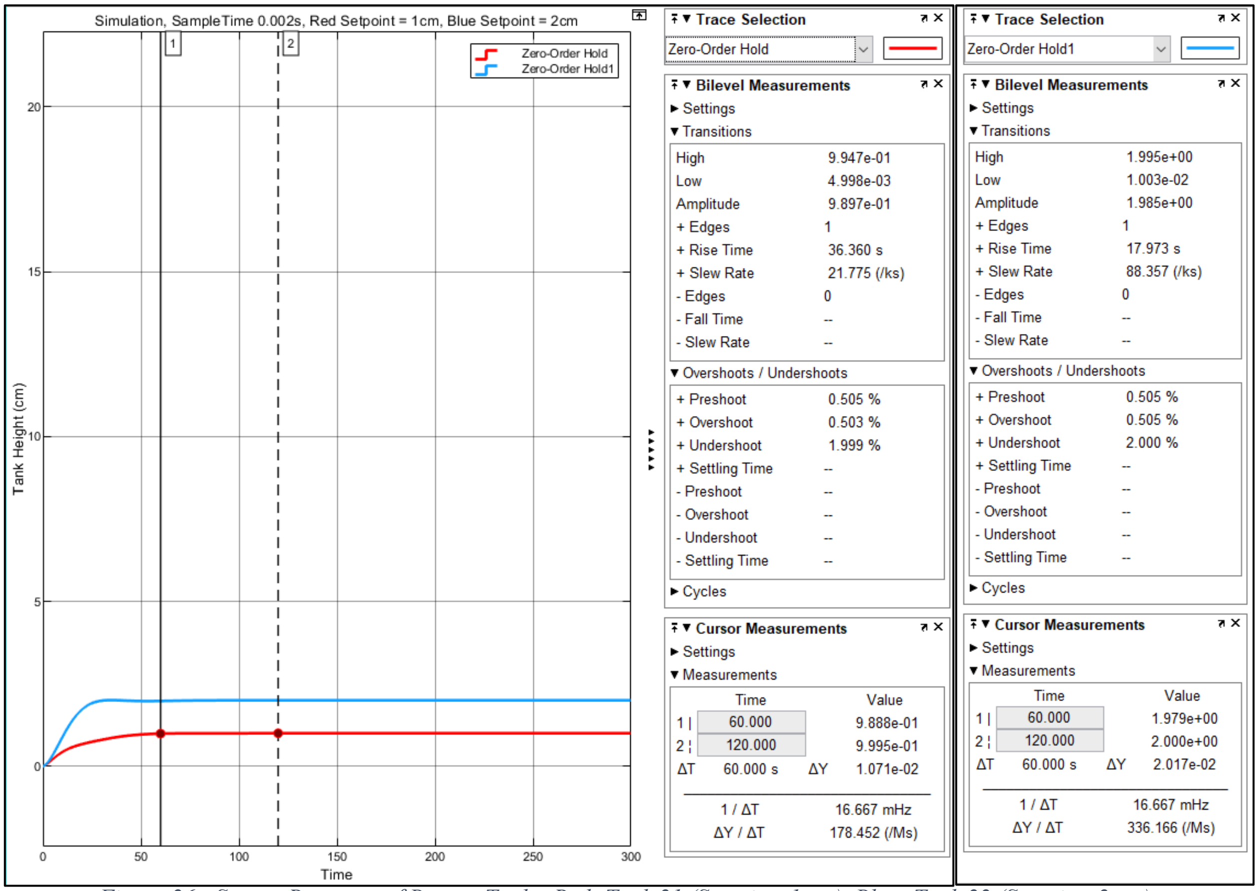

Figure 26 - System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 1 cm), Blue: Tank 22 (Setpoint: 2 cm)

Figure 27 shows the results for h22 (blue) = 20cm and h21 (red) = 22cm. This test was for very large heights (close to spilling over if the overshoot was too large), and is acceptable for tank h21 (red), but the results for blue aren't in the acceptable range at all. Looking at the output after the voltage approximation block from our IMC controller shows that the voltage is peaked at 12 volts and never decreases (the same situation happens in Figure 29), meaning this height is the maximum that h22 (blue) can reach in general. This may be improved if we adjust the IMC model simplification so idealized blue is made to be a 2nd order transfer function. This would be a way to improve our design given more time.

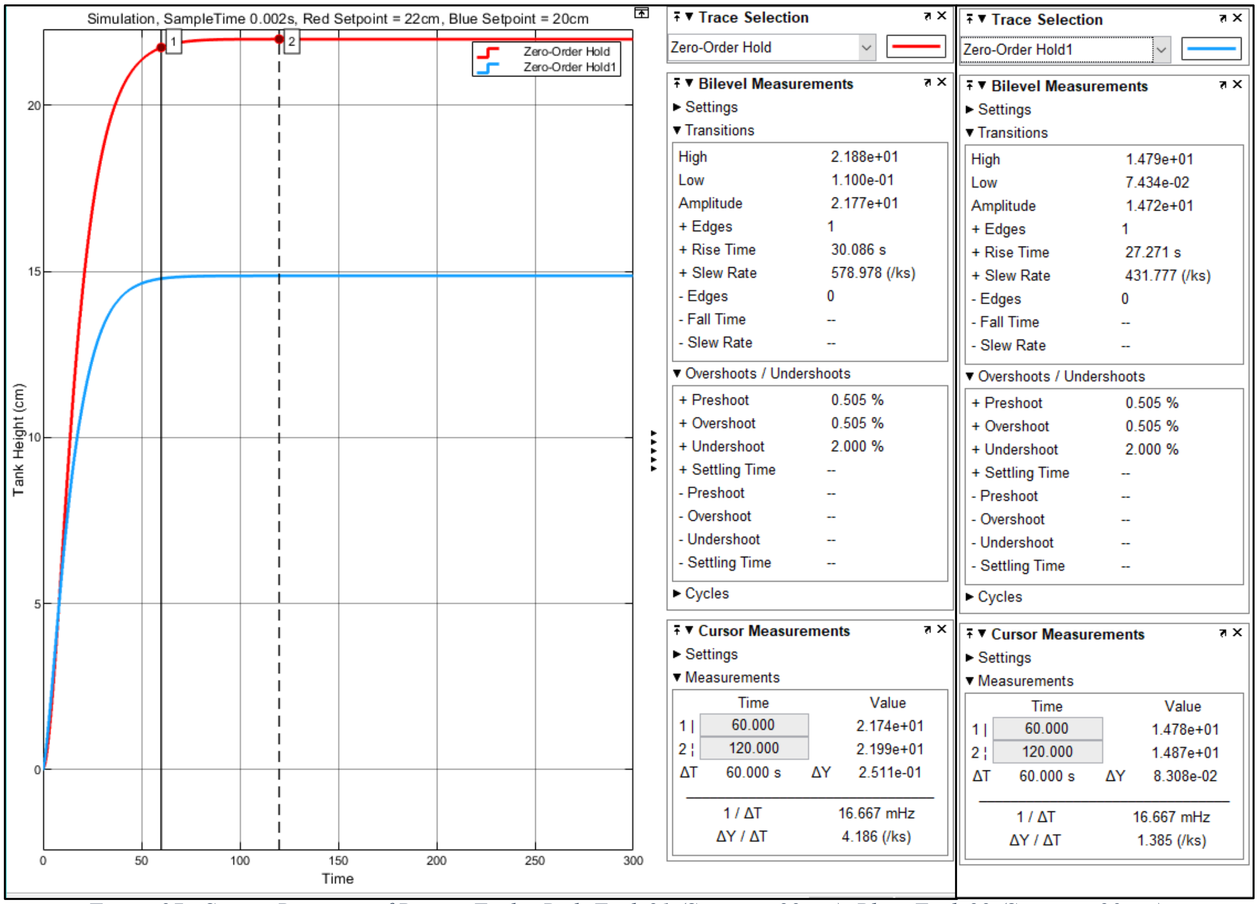

Figure 27 - System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 22 cm), Blue: Tank 22 (Setpoint: 20 cm)

Figure 28 is an additional test, blue setpoint is 14cm, red setpoint is 1cm. This extreme difference appears to still saturate the blue voltage (Figure 29). The set saturation point was chosen as 12V because that is the rated voltage for our system's DC pumps.

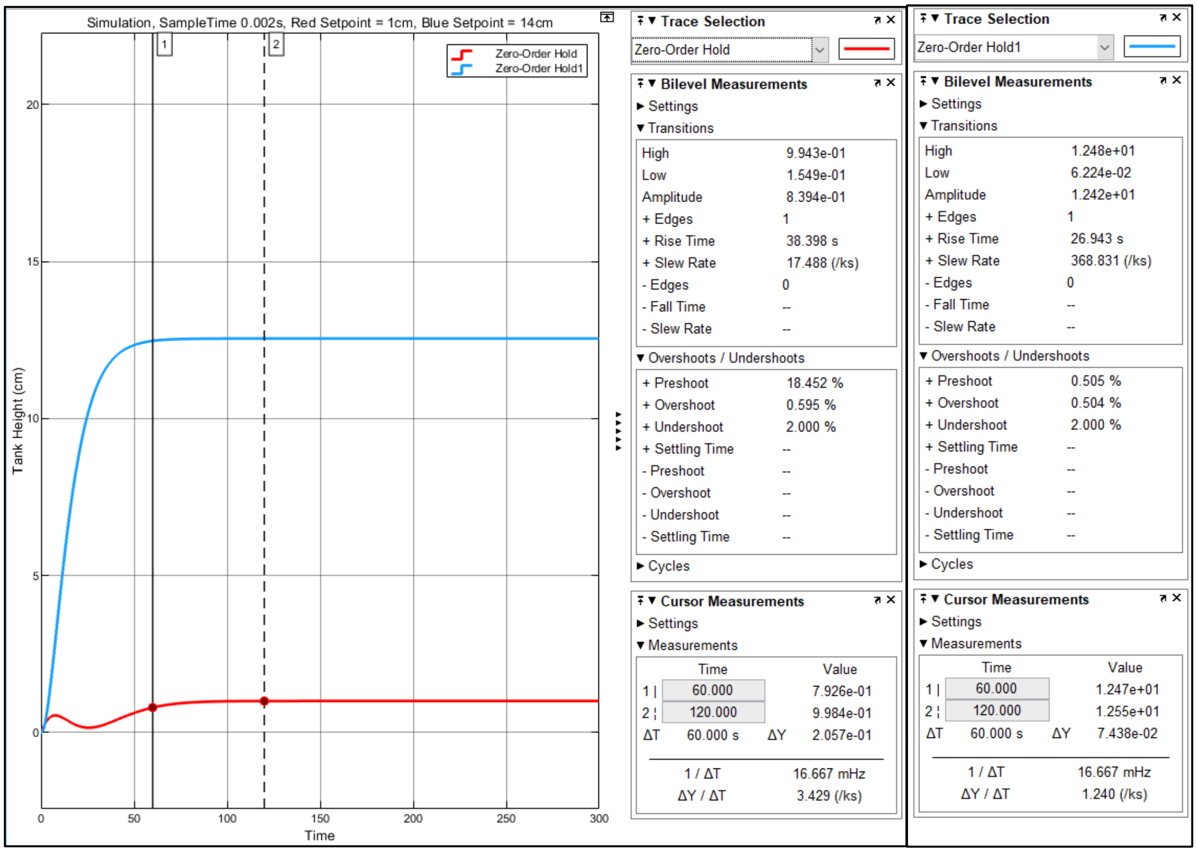

Figure 28 - System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 1 cm), Blue: Tank 22 (Setpoint: 14 cm)

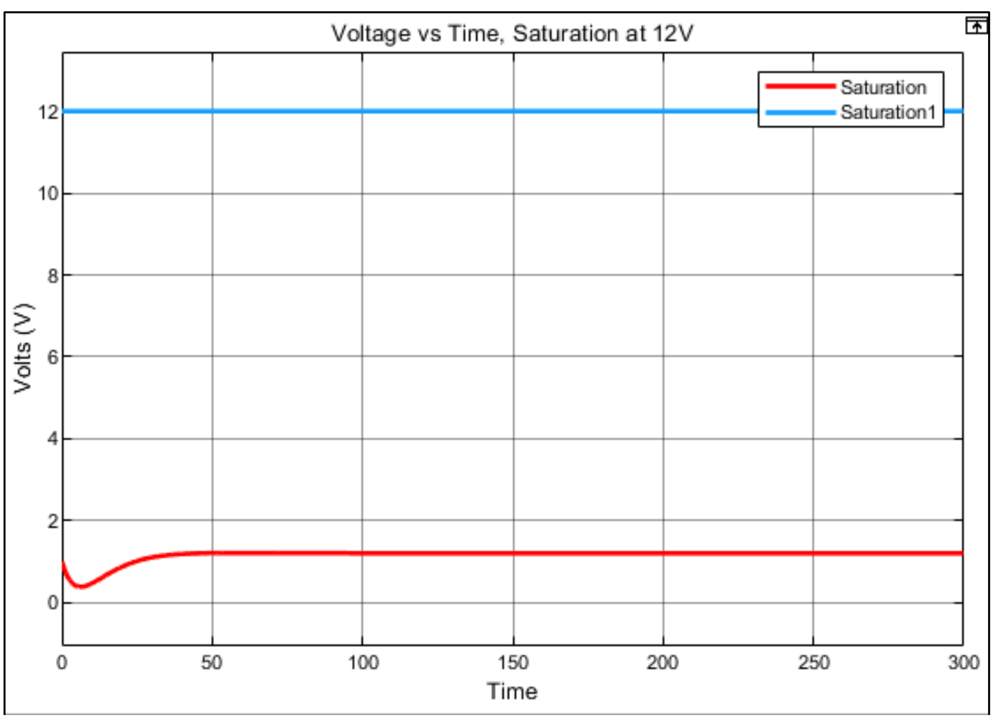

Figure 29 - Voltage Input for System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 12 cm), Blue: Tank 22 (Setpoint: 5.5 cm) (Saturation at 12V)

For testing's sake, Figure 30 shows the result if I raised the saturation to 20V (which is still within the range for the pump, its maximum value is 22V, but it's not something I'd want to test on the real system for safety reasons). Figure 30 shows that finally settling times and steady state error are much better but the red tank completely empties and is not within the desired settling time.

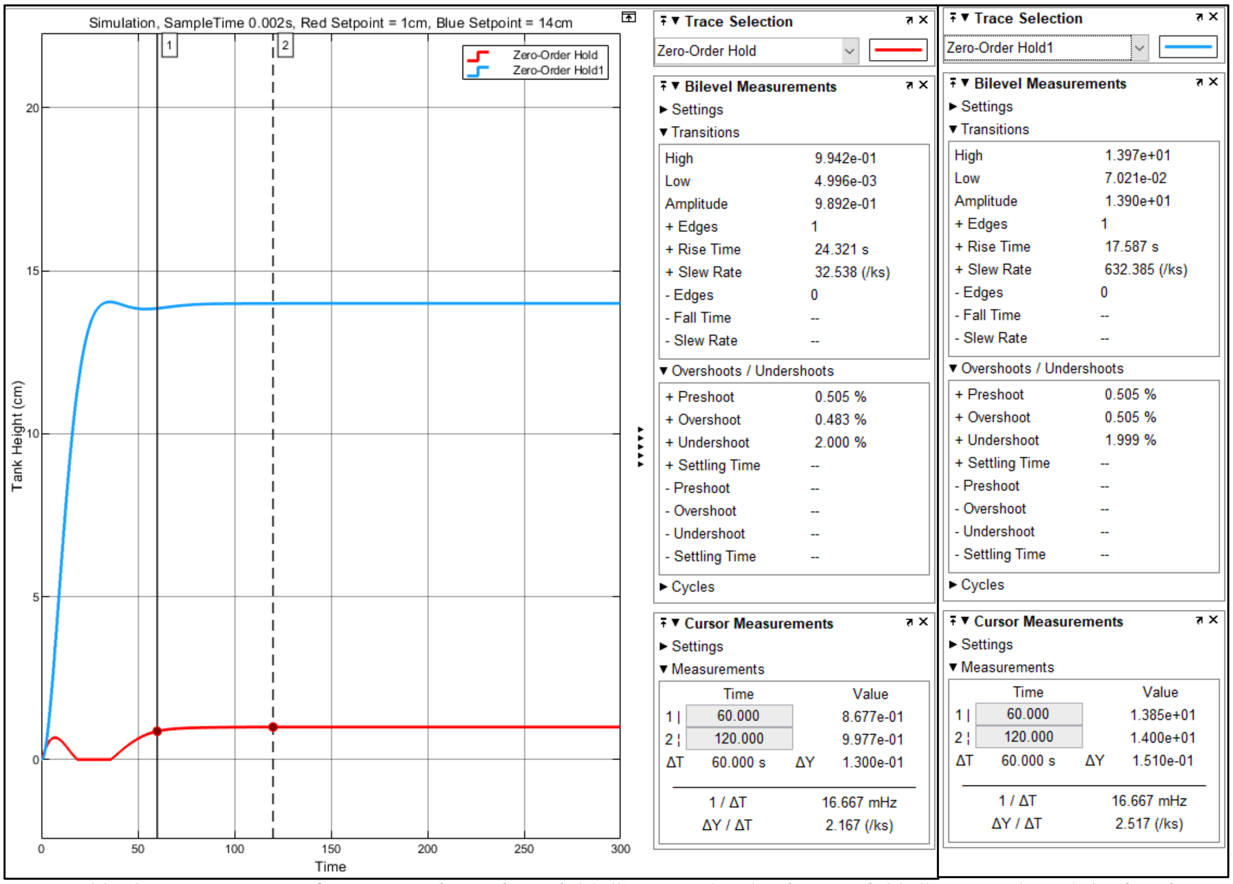

Figure 30 - System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 1 cm), Blue: Tank 22 (Setpoint: 14 cm) (with Voltage Saturation of 20V)

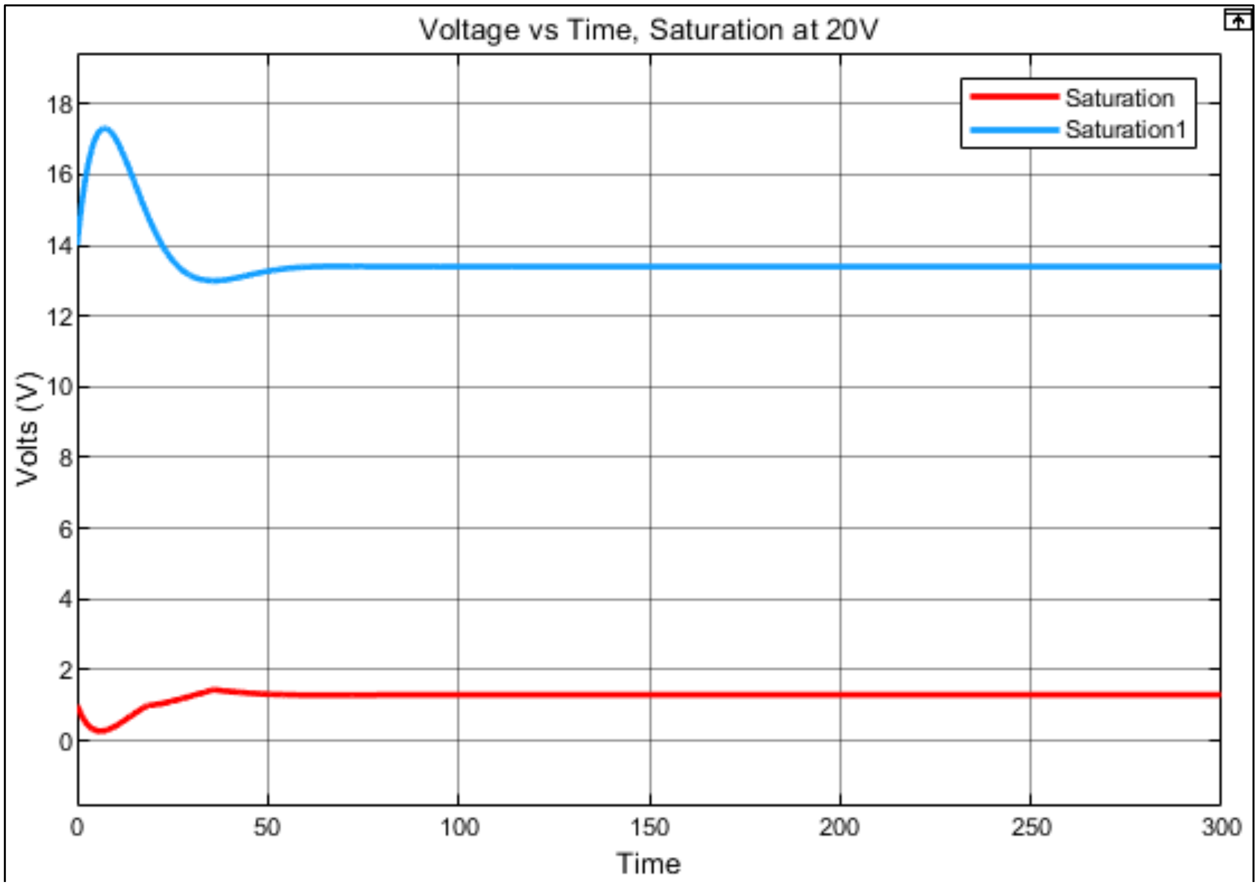

Figure 31 - Voltage Input for System Response of Bottom Tanks. Red: Tank 21 (Setpoint: 12cm), Blue: Tank 22 (Setpoint: 5.5cm) (Saturation at 20V)

These figures have shown that when the parameters used for (Figure 24) and below are used, the system responds acceptably, but the system should be retuned if higher heights are needed.

7.2 SOURCES OF ERROR

This project has many errors that can affect the plant functionality. These errors vary from systematic to random errors. Looking at the systematic errors there are multiple to consider such as Instrumental errors, where the tank system can leak due to random cracks in the tank or gaps where the tubing is attached. This type of leakage can affect the measurements of how much water is in the tank. Likewise, this can also occur within the water pump where random leakages can happen in the pump itself or the tubing that is connected to it. Furthermore, there can be error due to the voltage input produced by the Matlab Simulink simulation which is a steady voltage. However, when running the plant in real time the voltage input was fluctuating which causes the pump to under/over perform. In addition, the changing of atmospheric pressure means that without constant calibration the pressure sensor detecting the height of the water in each tank will have some amount of error. Finally, there can be theoretical errors, such as approximation done while calculating the values. All these numbers can affect the modeling of the system but in a very small-scale. Lastly, another possible error can occur due to human errors such as calculation mistakes, wrong inputted voltage values etc.

8.0 CONTINUOUS TO DISCRETE

Figure 40, is the response from the heights our Taylor Series was linearized around, plus white noise that looked similar to the noise observed from the real system and a sampling time of 0.002s, which is 500Hz which is practically analog for MATLAB simulations.

The figures in Appendix D – Analog-Digital Conversion Emulation, shows that past Figure 42's sampling times of 5.58s for "red" (output1) and 4.27s for "blue" (output2), the system's overshoot and settling time would get worse and worse quickly, becoming undesirable by 30s sampling time. Note: the values of 5.58s and 4.27s were derived by finding the poles of the simplifed Model transfer functions for output1 and output2 (Figure 38) using the Matlab "pole()" command, and following the equation below [3]:

$$Maximum\ Sample\ Time \leq \left(\frac{1}{5}\right) \times \left(\frac{2\pi}{\omega_{max}}\right)\ \ where\ \omega_{max} = \max(|pole\ of\ TF|)$$

$$\omega_{max}(red) = 0.2253\ , \omega_{max}(blue) = 0.2941$$

$$Maximum\ Sample\ Time\ (red) = 5.58s, Maximum\ Sample\ Time(blue) = 4.27s$$

Luckily, the physical ADC on the real system's pressure-height sensors operate between 0.1 and 0.002 sampling periods [1], so the system should theoretically work if COVID-19 ended soon and our group could test our controller on the physical system using heights below 12cm (as discussed in 7.0 System Response).

9.0 CONCLUSION AND FURTHER RECOMMENDATIONS

The current control of the modelled system uses decoupling and a PID controller. This control type allows for two controllers to accurately control each of the bottom tanks without affecting each other. The final tuning of the controller meets the overshoot and settling time specifications that were decided to control the system. By adding additional controller methods, the systems control can be greatly improved.

Feedforward control will be used to greatly enhance the performance of a system. The two key characteristics that a feedforward will be useful for: 1) Is an identifiable disturbance that is affecting the measured variable, adding a feedforward will help control and regulate these affects. 2) The second key aspect is if the disturbance can be measured or from a known source, this way the system will be able to control the source of disturbance before it affects the main process. Unlike a cascade control which rejects the disturbance, feedforward control just allows the system to measure the disturbances and make the main controller compensate for these values. A feedforward controller will be implemented in this system to account for all the disturbances that may be introduced.

An Internal Model Control (IMC) will also be introduced in the plant model. The IMC tuning method offers stable and robust alternatives to other techniques. This method is simple to use, as it uses only one tuning parameter which is λ as shown in the equation below. The 'n' value stabilizes the system, and is used when the system has a positive pole in which we can account for using the IMC controller. The IMC offers the following advantages to the plant: 1) No overshoot in the system, reducing risk of overflowing the tanks. 2) The tuning rules are less sensitive to errors. 3) The IMC is very robust, which means that the control loop will stay stable even if the process characteristics change. 4) Absorbs disturbances and passes less of them to the plant [2].

WORKS CITED

[1] Quanser Inc., "User Manual Couple Tank Experiment Set up and Configuration," Markham, Ontario, 2014.

[2] Dataforth, "Tuning Control Loops with the IMC Tuning Method," [Online]. Available: https://www.dataforth.com/tuning-control-loops-with-imc-tuning-method.aspx.

[3] M. Cooper-Stachowsky, "Sheets 82 back to 88," [Online]. Available: https://courselink.uoguelph.ca/d2l/le/content/593709/viewContent/2364546/View.

[4] M. Cooper-Stachowsky and Julie Vale, "ENGG4280 Four-tank system model.," University of Guelph, Guelph, 2019.

APPENDICES

APPENDIX A – SYSTEM SPECIFICATIONS

Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	$cm^3/s/V$
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

Figure 32 - Properties of the Quanser Four Tank System [1]

APPENDIX B – EXPERIMENTAL RESULTS

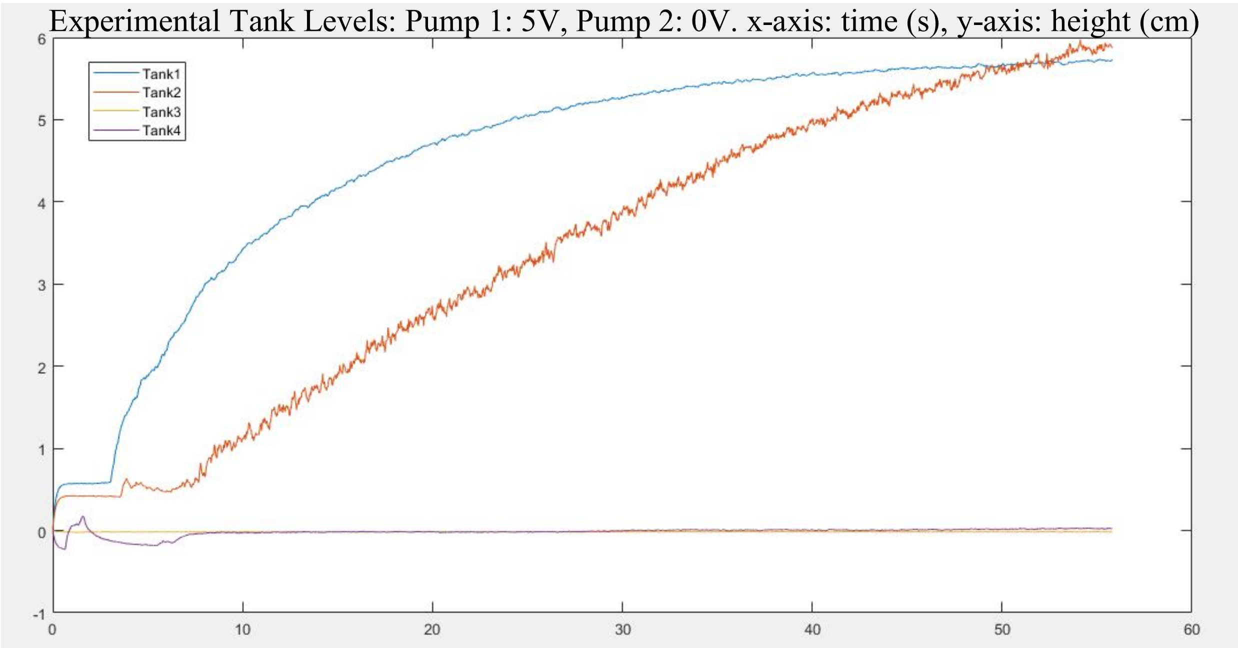

Figure 33 - Experimental Tank Levels. Pump 1: 5V, Pump 2: 0V

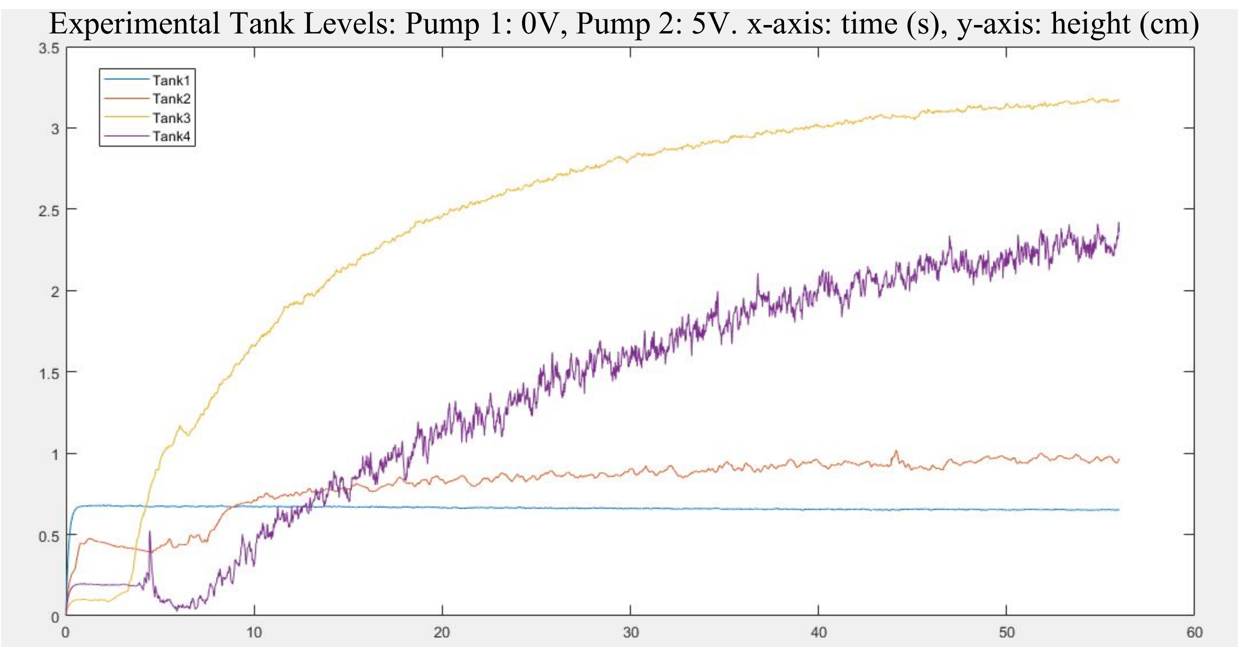

Figure 34 - Experimental Tank Levels. Pump 1: 0V, Pump 2: 5V

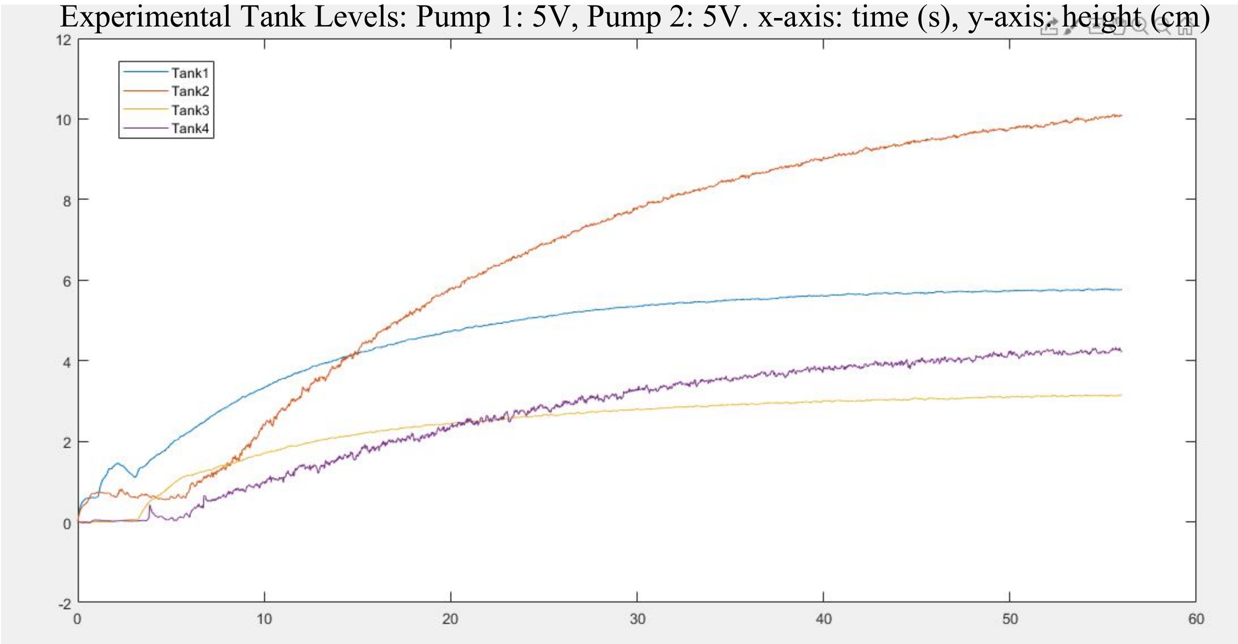

Figure 35 - Experimental Tank Levels. Pump 1: 5V, Pump 2: 5V

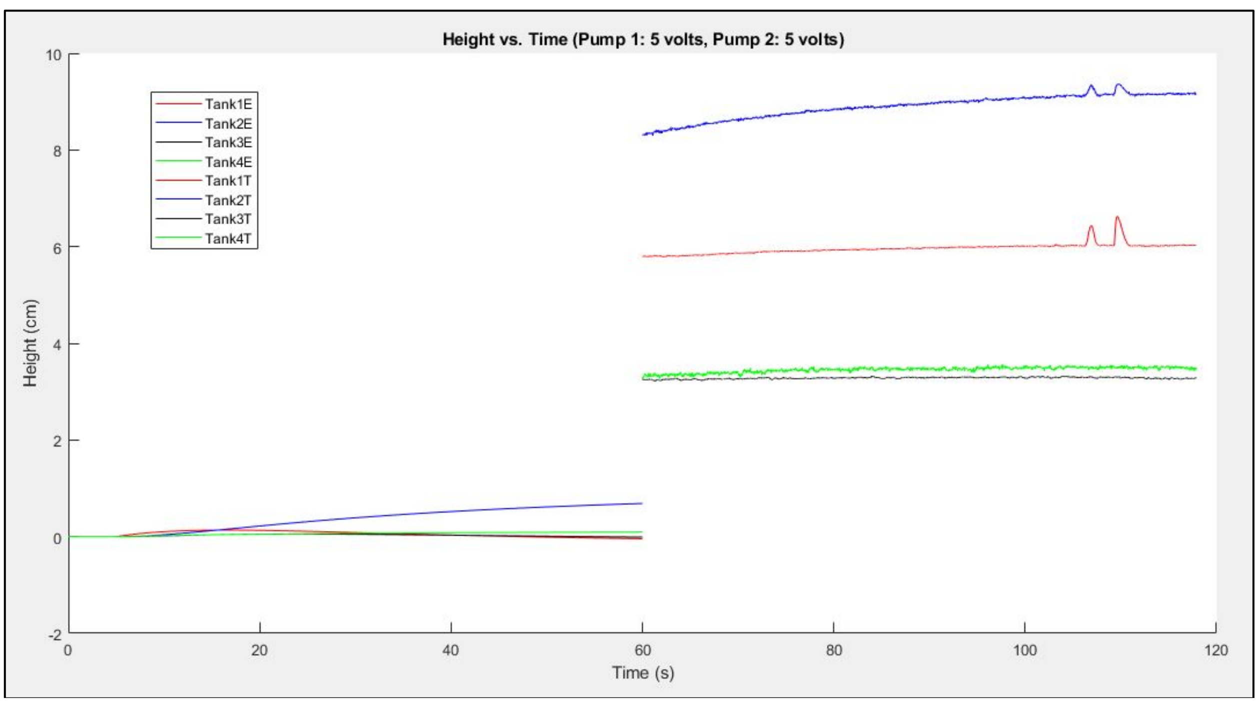

Figure 36 – Second Test for Determining Sensor Gains

APPENDIX C – MATLAB CODE

```
%%%GIVENS
D_out1 = 0.635;
D_out2 = 0.47625;
D_t = 4.445;
D_Md = 0.47625;
g = 981;             %cm/s^2
A = (pi/4)*D_t^2;    %All tank areas are the same (A11 = A12 = A21 = A22)
a = (pi/4)*D_Md^2;   %all tank outlet offices are the same (a11 = a12 = a21 = a22)

%%%EXPERIMENTAL DATA
%pump flow constants
kp11 = A*((25.4597-5.7014)/(27.466-7.842))/5;    %cm^3/(V*s)
kp12 = A*((21.1272-2.4112)/(27.448-5.798))/5;
kp21 = A*((6.1317-0.9284)/(27.464-10.128))/5;
kp22 = A*((2.597-0.3619)/(27.454-11.46))/5;
%sensor gains (for y=sensorgain*height)
sensorGain11 = (6/5.5 + 6.3/4.8)/2;              %h = (Sensor Height)cm/(Observed Height)cm (sensor gain)
sensorGain12 = (3.3/3.2 + 3.5/3.5)/2;
sensorGain21 = (9.2/12 + 9.6/11.9)/2;
sensorGain22 = (3.5/5.2 + 3.9/5)/2;
%desired equilibrium height (current values are steady state values at 1p05 2p05)
ho11 = 5.0;
ho12 = 3.0;
ho21 = 12.0;
ho22 = 5.5;

%%%STATE SPACE
c11 = kp11/A;
c12 = kp12/A;
c21 = kp21/A;
c22 = kp22/A;
b=(a/A)*sqrt(2*g);
el_11 = (1/2)*b/sqrt(ho11);
el_12 = (1/2)*b/sqrt(ho12);
el_21 = (1/2)*b/sqrt(ho21);
el_22 = (1/2)*b/sqrt(ho22);

A = [-el_11     0       0       0  ;
       0     -el_12     0       0  ;
     el_11      0    -el_21     0  ;
       0      el_12     0    -el_22];
B = [c11  0 ;
      0  c12;
      0  c21;
     c22  0 ];
C = [0 0 sensorGain21      0      ;
     0 0      0       sensorGain22];
D = [0 0;
    0 0];
s=tf('s');
sI= [s 0 0 0;
     0 s 0 0;
     0 0 s 0;
     0 0 0 s];
ytf = C*inv(sI-A)*B

%inputs are columns, outputs are rows, so its (rows,cols)
ytf(1,1); %input1 output1 = h21 2nd order
ytf(1,2); %input2 output1 = h11
ytf(2,1); %input1 output2 = h12
ytf(2,2); %input2 output2 = h22 2nd order
```

Figure 37 - MATLAB Code for Determining the Decoupled Model

APPENDIX D – ANALOG-DIGITAL CONVERSION EMULATION

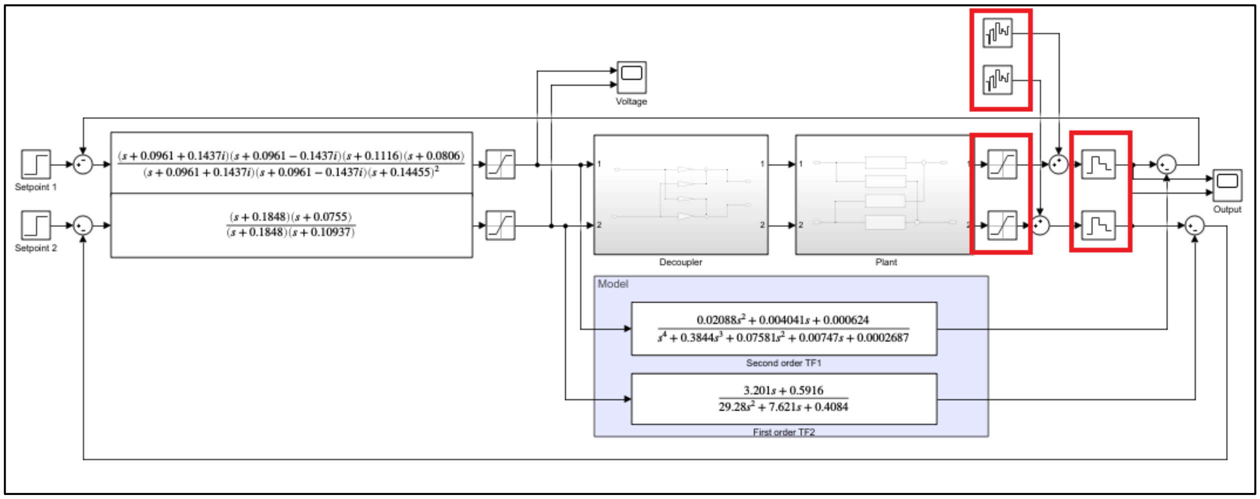

Figure 38 - IMC Controlled System with ADC Conversion

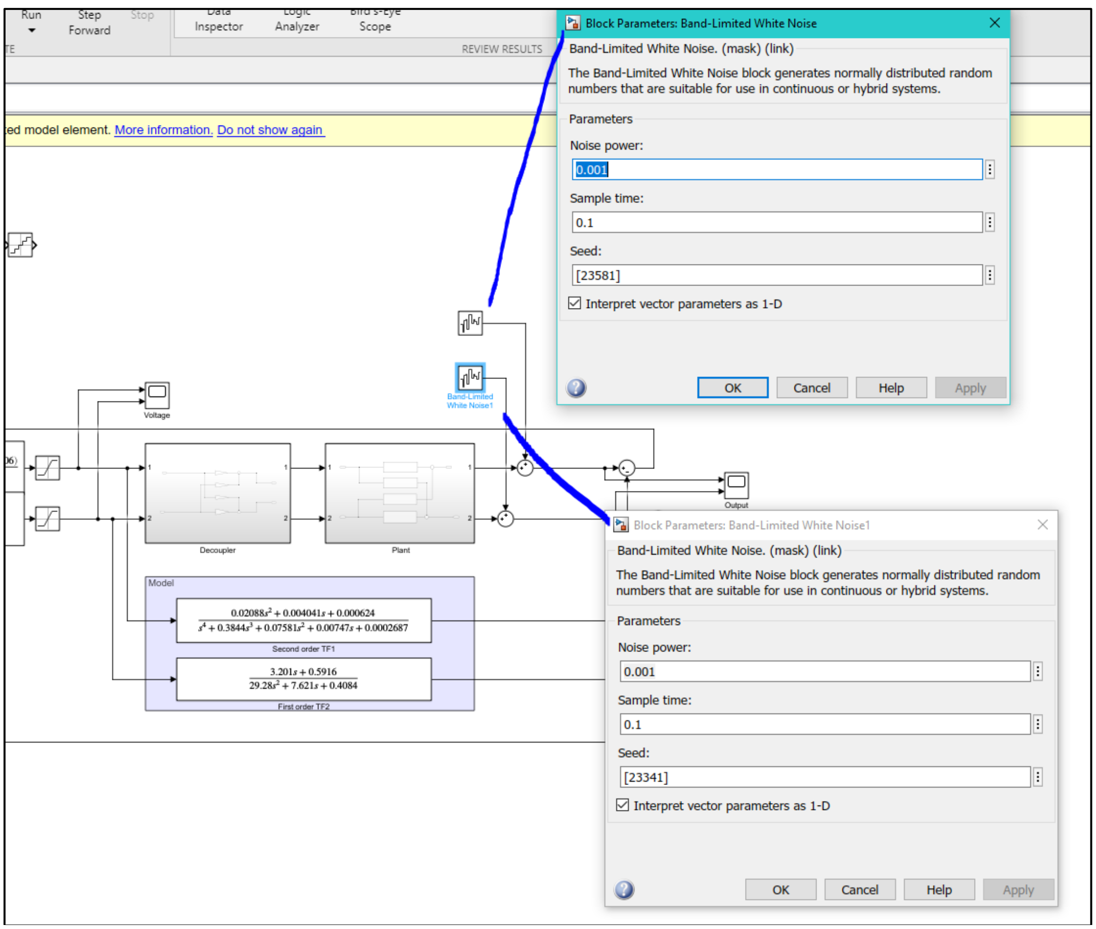

Figure 39 - ADC. Increasing Sampling Period Increases Settling Time

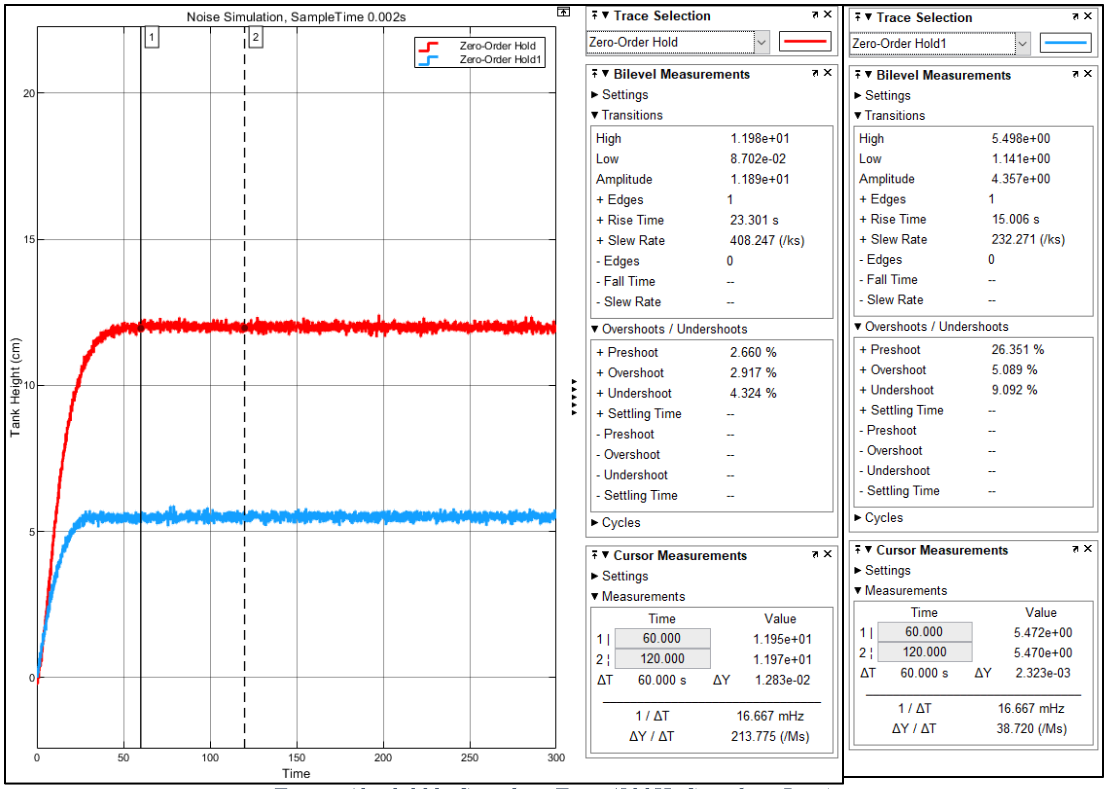

Figure 40 - 0.002s Sampling Time (500Hz Sampling Rate)

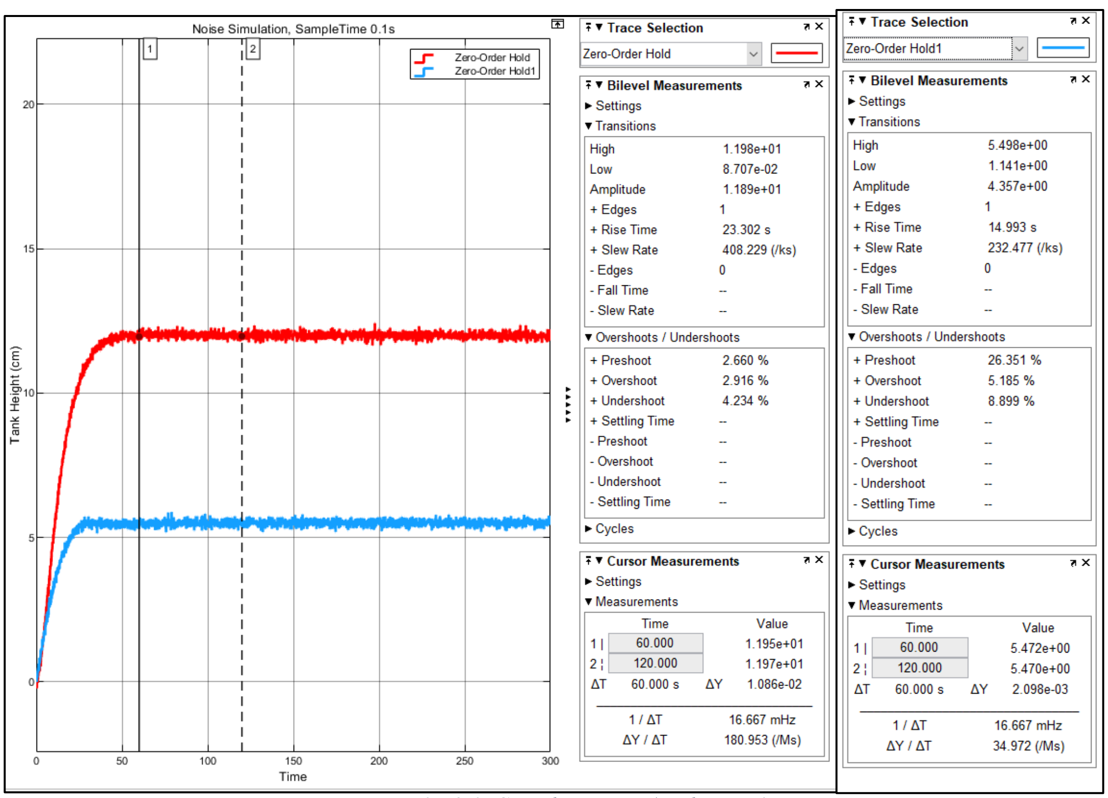

Figure 41 - 0.1s Sampling Time (With Noise)

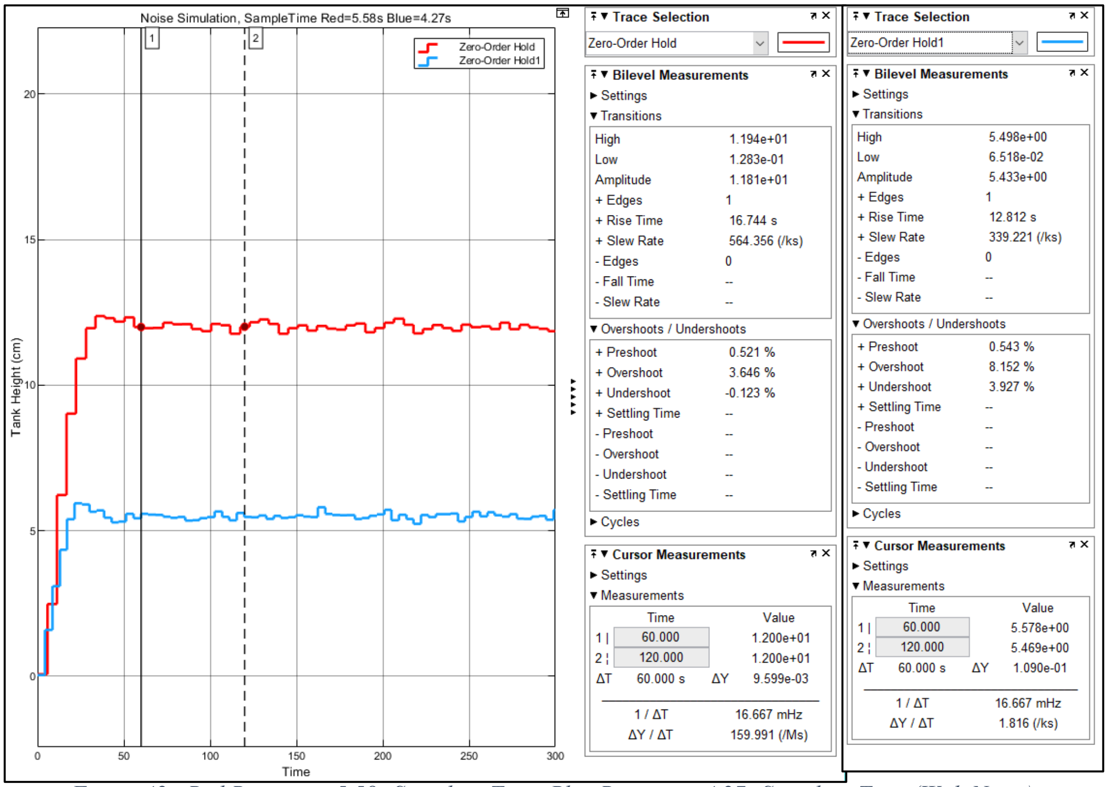

Figure 42 - Red Response: 5.58s Sampling Time. Blue Response: 4.27s Sampling Time (With Noise)

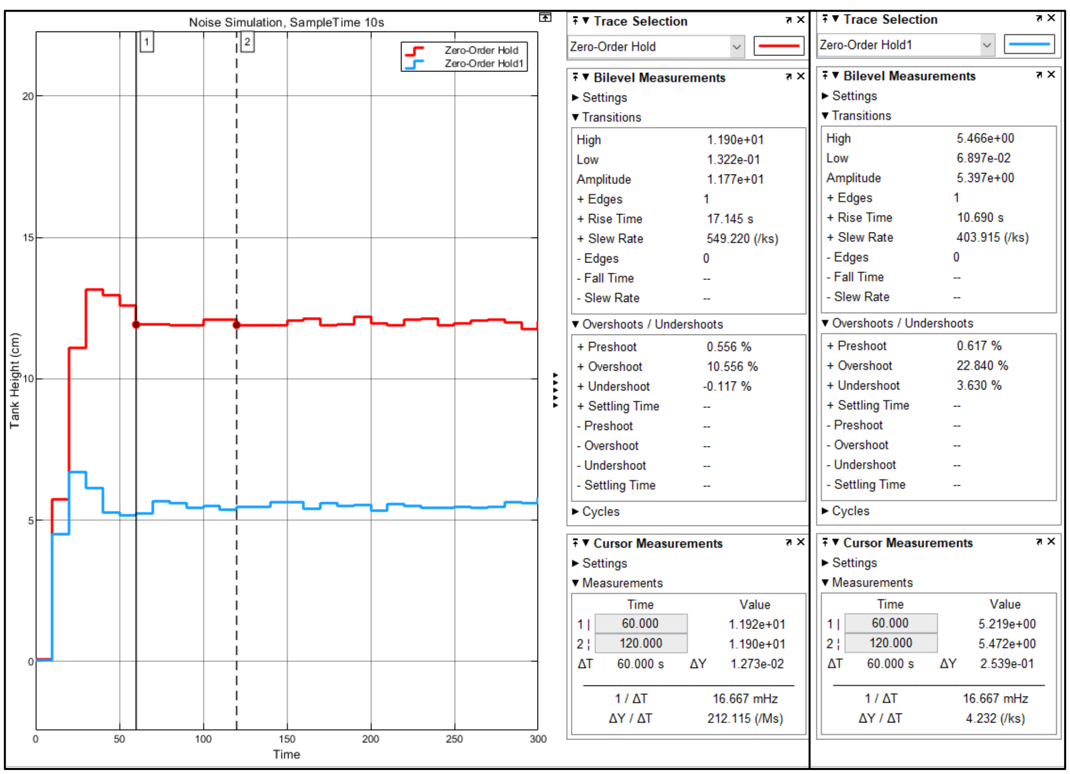

Figure 43 - 10s Sampling Time (With Noise)

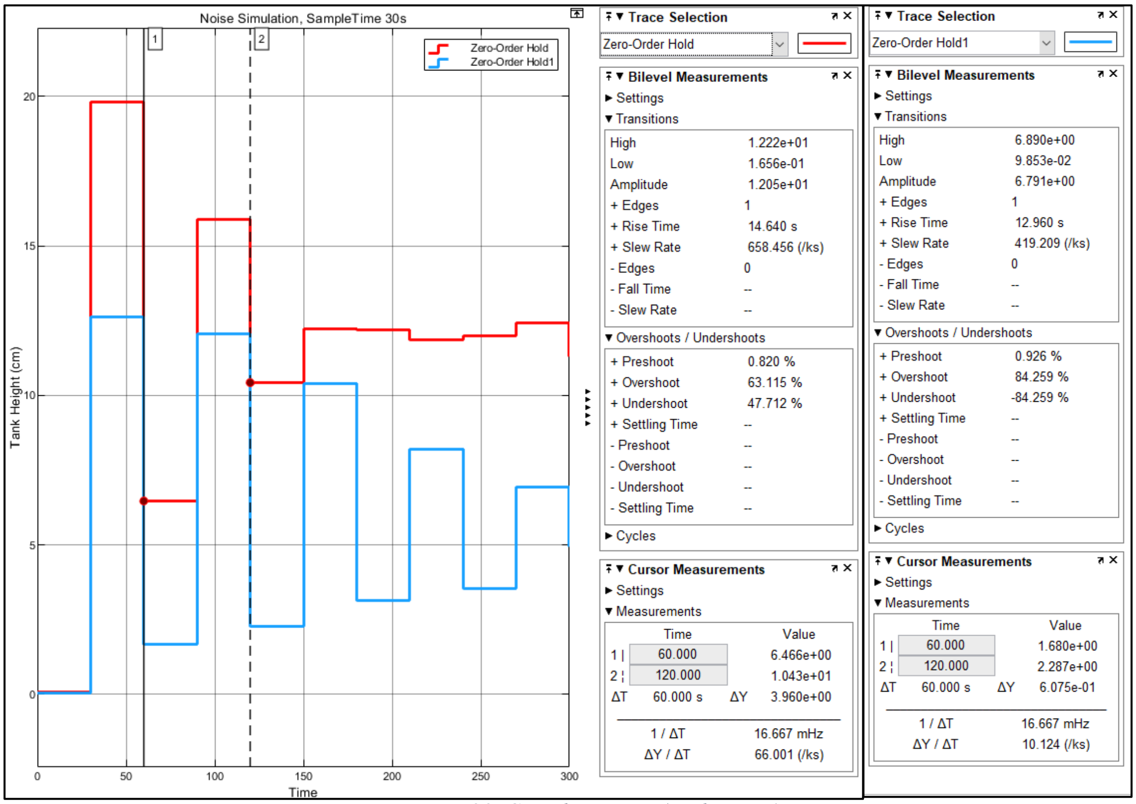

Figure 44 - 30s Sampling Time (With Noise)