Chapter 5

Experiments

Sommaire

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In this chapter, we evaluate the efficiency of our proposed algorithm M^{2}TML on public datasets for classification problems of univariate time series. First, we describe the datasets. Secondly, we detail the experimental protocol. Finally, we present and discuss the obtained results.

5.1 Description

The efficiency of the learned multi-modal and multi-scale dissimilarities D and $D_{\mathscr{H}}$ is evaluated through a 1−NN classification on 30 public datasets [1] [Keo+11]. The considered data encompass time series that involve global or local temporal differences, require or not time warping, with linearly or non linearly separable neighborhoods. Table 5.1 gives a description of the datasets considered for our experiments and Fig. 5.1 gives the temporal representation of some of the datasets. The learnt metrics D and $D_{\mathscr{H}}$ are compared to three *a priori* combined metrics $D_{Lin}, D_{Geom}, D_{Sig}$ (Eqs. 2.16, 2.17, 2.18) and five alternative uni-modal metrics covering:

1. The standard Euclidean distance and dynamic time warping referenced as d_A (Eq. 2.1) and DTW
2. The behavior-based measures d_B (Eq. 2.6) and $d_{B-\text{DTW}}$ its counterpart for asynchronous time series, that is d_B is evaluated once time series synchronized by dynamic programing

[1] PowerCons: `https://archive.ics.uci.edu/ml/datasets/Individual+household+electric+power+consumption`, BME and UMD: `http://ama.liglab.fr/~douzal/tools.html`.

3. The frequential-based metric d_F (Eq. 2.4).

The *a priori* combined metrics $(D_{Lin}, D_{Geom}, D_{Sig})$ relies, for synchronous (resp. asynchronous), on 2 log-normalized dissimilarities d_A, d_B (resp. DTW, $d_{B-\text{DTW}}$). The alternative metrics and the *a priori* combined metrics are evaluated as usual by involving the all time series elements (*i.e.* at the global scale). For D and $D_{\mathscr{H}}$, we consider a 21-dimensional embedding space $\mathscr{E}$ that relies, for synchronous (resp. asynchronous) data, on 3 log-normalized dissimilarities d_A^s, d_B^s (resp. DTW^s, $d_{B-\text{DTW}}^s$), and d_F^s, at 7 temporal granularities $s \in \{0, ..., 6\}$ obtained by binary segmentation as described in Section 4.1.

Dataset	Nb. Class	Nb. Train	Nb. Test	TS length
1 CC	6	300	300	60
2 GunPoint	2	50	150	150
3 CBF	3	30	900	128
4 OSULeaf	6	200	242	427
5 SwedishLeaf	15	500	625	128
6 Trace	4	100	100	275
7 FaceFour	4	24	88	350
8 Lighting2	2	60	61	637
9 Lighting7	7	70	73	319
10 ECG200	2	100	100	96
11 Adiac	37	390	391	176
12 FISH	7	175	175	463
13 Beef	5	30	30	470
14 Coffee	2	28	28	286
15 OliveOil	4	30	30	570
16 CinCECGtorso	4	40	1380	1639
17 DiatomSizeReduc	4	16	306	345
18 ECG5Days	2	23	861	136
19 FacesUCR	14	200	2050	131
20 InlineSkate	7	100	550	1882
21 ItalyPowerD	2	67	1029	24
22 MedicalImages	10	381	760	99
23 MoteStrain	2	20	1252	84
24 SonyAIBOII	2	27	953	65
25 SonyAIBO	2	20	601	70
26 Symbols	6	25	995	398
27 TwoLeadECG	2	23	1139	82
28 PowerCons	2	73	292	144
29 BMEsmooth	3	300	1500	128
30 UMDsmooth	3	360	1440	150

Table 5.1: Dataset description giving the number of classes (Nb. Class), the number of time series for the training (Nb. Train) and the testing (Nb. Test) sets, and the length of each time series (TS length).

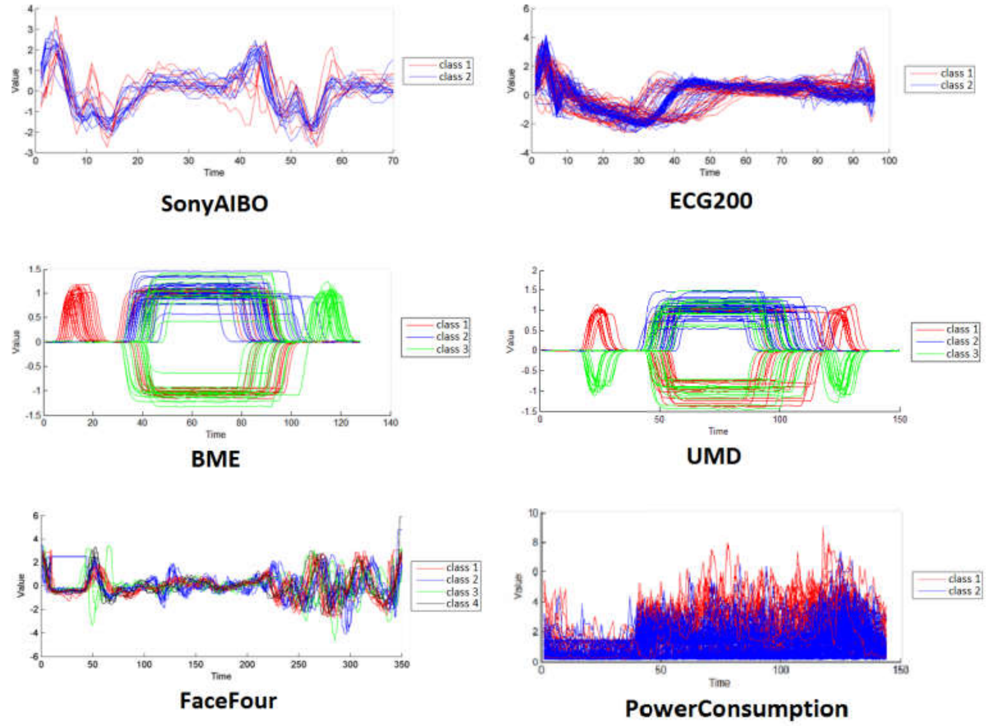


Figure 5.1: Temporal representation of some datasets (SonyAIBO, ECG200, BME, UMD, FaceFour, PowerConsumption) considered in our experiments.

5.2 Experimental protocol

The combined metrics D and $D_{\mathscr{H}}$ (K as the Gaussian kernel) are learned under L_1 and L_2 regularization, respectively. The parameters are estimated on a validation set by line/grid search. A cross-validation and stratified sampling for unbalanced datasets are used (Section 1.1.2). Particularly, for each couple (r, λ) $r \in \{1, 4, 10\}$ and $\lambda \in \{0, 10, 30\}$, the pairwise SVM parameters (C, α, γ) are learned by grid search as indicated in Table 5.2. The temporal order r is noise-dependent, typically 1 is retained for noise-free data. The parameter λ corresponds to the strength of the '*push*' term; precisely, if no, moderate or strong '*push*' is required during the training process, a λ value of 0, 10 and 30 is learned, respectively. For the *a priori* combined metrics D_{Lin}, D_{Geom} and D_{Sig}, the same protocol (grid search, cross-validation) is applied to learn the best parameter α. The parameters retained are those that:

- **First**, minimize the average classification error Err on the validation set (Section 1.1.2 & Section 1.1.3.a).
- **Secondly**, in the case of multiple solutions (C, α, γ) leading to equal performances, the most discriminant one is retained (i.e. making closer positive pairs and far a way negative pairs). Precisely, it minimizes the ratio $\frac{d_{intra}}{d_{inter}}$ where d_{intra} and d_{inter} stands respectively to the mean of all intraclass and interclass distances according to the considered metric D, $D_{\mathscr{H}}$, D_{Lin}, D_{Geom} or D_{Sig} .

	Parameter	Ranges
d_B	r	$\{1, 2, 3, , \dots, T-1\}$
$D_{Lin}, D_{Geom}, D_{Sig}$	α	$\{0, 0.1, \dots, 1\}$
D, $D_{\mathscr{H}}$	λ	$\{0, 10, 30\}$
D, $D_{\mathscr{H}}$	C	$\{10^{-3}, 0.5, 1, 5, 10, 20, 30, ..., 150\}$
D, $D_{\mathscr{H}}$	α	$\{1, 2, 3\}$
$D_{\mathscr{H}}$	γ	$\{10^{-3}, 10^{-2}, \dots, 10^{3}\}$

Table 5.2: Parameter ranges

5.3 Results

Table 5.3 reports the 1-NN classification test errors for uni-modal and M²TML metrics; the results that are statistically and significantly better than the rest are indicated in bold (z-test at 5% risk detailed in Section 1.1.3.a). In particular, we compare the 1-NN error rates when based on uni-modal metrics (first 5 columns), three *a priori* combined metrics ($D_{Lin}, D_{Geom}, D_{Sig}$) and on D and $D_{\mathscr{H}}$. The last column 'WARP' indicates the synchronous (✓) or asynchronous (×) data type.

	Alternative uni-modal metrics					A priori combinations			M²TML		WARP
Dataset	d_A	d_B	d_F	DTW	$d_{B-\text{DTW}}$	$D_{Lin}(\alpha)$	$D_{Geom}(\alpha)$	$D_{Sig}(\alpha)$	$D(\lambda^*)$	$D_{\mathscr{H}}(\lambda^*)$	WARP
1 CC	0.120	0.113	0.383	**0.007**	0.027	**0.007**	**0.007**	**0.007**	**0.003** (0)	**0.007** (0)	✓
2 GunPoint	0.087	0.113	**0.027**	0.093	**0.027**	**0.027**	**0.027**	**0.040**	**0.020** (10)	**0.040** (10)	✓
3 CBF	0.148	0.140	0.382	**0.003**	**0.000**	**0.000**	**0.000**	**0.000**	0.031 (30)	**0.003** (0)	✓
4 OSULeaf	0.484	0.475	0.426	0.409	**0.265**	**0.264**	**0.264**	**0.322**	0.380 (0)	0.376 (0)	✓
5 SwedishLeaf	0.211	0.186	0.146	0.208	**0.109**	**0.115**	**0.110**	**0.125**	**0.110** (0)	**0.114** (0)	✓
6 Trace	0.240	0.240	0.140	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000** (0)	**0.010** (0)	✓
7 FaceFour	0.216	0.216	0.239	0.170	0.136	0.170	0.170	0.170	**0.000** (0)	0.034 (0)	✓
8 Lighting2	**0.246**	**0.246**	**0.148**	**0.131**	**0.213**	**0.131**	**0.131**	**0.131**	**0.148** (0)	**0.131** (0)	✓
9 Lighting7	**0.425**	**0.411**	**0.316**	**0.274**	**0.288**	**0.342**	**0.356**	**0.342**	**0.397** (0)	**0.233** (0)	✓
10 ECG200	**0.120**	**0.070**	0.160	0.230	0.190	**0.070**	**0.070**	**0.070**	**0.080** (0)	**0.080** (0)	×
11 Adiac	0.389	**0.297**	**0.261**	0.396	0.338	0.373	0.363	0.402	0.358 (0)	0.361 (0)	×
12 FISH	0.217	**0.149**	0.229	**0.166**	**0.137**	**0.109**	**0.137**	**0.126**	**0.149** (0)	0.240 (0)	✓
13 Beef	0.467	0.300	0.500	0.500	0.500	0.367	0.267	0.467	**0.033** (0)	0.267 (0)	×
14 Coffee	0.250	**0.000**	0.357	0.179	0.143	**0.000**	**0.000**	**0.071**	**0.000** (0)	**0.000** (10)	×
15 OliveOil	**0.133**	**0.133**	**0.167**	**0.200**	**0.100**	**0.133**	**0.133**	**0.133**	**0.167** (0)	**0.100** (10)	✓
16 CinCECGtorso	0.103	0.367	0.167	0.349	0.367	**0.094**	**0.094**	**0.093**	**0.092** (0)	**0.079** (0)	×
17 DiatomSizeR	0.065	0.076	0.069	**0.033**	**0.029**	**0.033**	**0.033**	**0.042**	**0.026** (0)	**0.029** (0)	✓
18 ECG5Days	0.203	0.153	**0.006**	0.232	0.236	0.203	0.203	0.203	**0.007** (10)	0.024 (0)	×
19 FacesUCR	0.231	0.227	0.175	0.095	0.102	0.098	0.098	0.099	**0.068** (10)	**0.059** (0)	✓
20 InlineSkate	**0.658**	**0.658**	0.675	**0.616**	**0.623**	**0.605**	**0.605**	**0.602**	0.733 (10)	**0.625** (0)	✓
21 ItalyPowerD	0.045	**0.028**	0.078	0.050	0.055	**0.028**	**0.028**	**0.030**	**0.028** (30)	**0.037** (10)	×
22 MedicalImages	0.316	0.313	0.345	**0.263**	0.290	**0.263**	**0.263**	**0.263**	**0.237** (0)	**0.236** (10)	✓
23 MoteStrain	**0.121**	0.263	0.278	0.165	0.171	0.260	0.248	0.188	0.185 (0)	0.153 (10)	✓
24 SonyAIBOII	**0.141**	**0.142**	**0.128**	0.169	0.194	**0.142**	**0.142**	**0.144**	**0.155** (0)	**0.131** (0)	×
25 SonyAIBO	0.305	0.308	0.258	0.275	0.343	0.308	0.308	0.293	**0.188** (0)	**0.165** (30)	×
26 Symbols	0.101	0.111	0.080	**0.050**	**0.043**	**0.051**	**0.050**	**0.052**	**0.034** (30)	**0.046** (30)	✓
27 TwoLeadECG	0.253	0.153	0.103	0.096	**0.008**	**0.005**	**0.005**	0.018	**0.006** (0)	0.016 (10)	✓
28 PowerCons	**0.366**	0.445	**0.315**	0.397	0.401	0.401	0.401	0.401	**0.318** (0)	**0.308** (0)	✓
29 BME	0.173	0.180	0.373	0.107	0.120	0.107	0.107	0.107	**0.040** (30)	**0.000** (10)	✓
30 UMD	0.194	0.222	0.299	0.118	**0.090**	0.111	0.111	0.118	0.104 (0)	**0.042** (0)	✓

Table 5.3: 1-NN test error rates for standard, *a priori* combined and M²TML measures.

5.4 Discussion

From Table 5.3, we can see first that the 1-NN classification reaches the best results in:

1. Less than one-third of the data when based on d_A, d_B or d_F
2. Slightly more than one-third for DTW and $d_{B-\text{DTW}}$
3. More than two-thirds (23 times on 30) when based on D or $D_{\mathscr{H}}$.

Particularly, note that for nearly all datasets for which an uni-modal metric succeeds, the M^2TML metrics succeed similarly or lead to equivalent results. However, for several challenging datasets (*e.g.* FaceFour, Beef, FaceUCR, SonyAIBO, BME) M^2TML realizes drastic improvements, to the best of our knowledge never achieved before for these challenging public data. For instance, the impressive scores of 3% obtained for Beef against an error rate varying from 30% to 50% for alternative metrics, and of 0% obtained for FaceFour v.s. 13% to 23% for alternative metrics. Finally, D and $D_{\mathscr{H}}$ are all the more outperforming if only compared to the standard metrics d_A (the Euclidean distance) and DTW.
Secondly, if we compare *a priori* combined metrics (D_{Lin}, D_{Geom}, D_{Sig}) based on only the unimodal metrics involved in the combination (either d_A and d_B or DTW and $d_{B-\text{DTW}}$), we observe that *a priori* combined metrics achieved on two-third of the data with an equivalent or better score. Compared to the learnt metrics (D, $D_{\mathscr{H}}$), the results are globally similar results except for 8 datasets where the learnt metrics perform better (FaceFour, Beef, ECG5Days, FaceUCR, SonyAIBO, PowerCons, BME, UMD) and one where the *a priori* combined metrics perform better (OSULeaf). Note that the combined metric D_{Sig} is limited to two components and can't be easily extend to other metrics in its combination. D_{Lin} and D_{Geom} could be easily extended. One proposition could be:

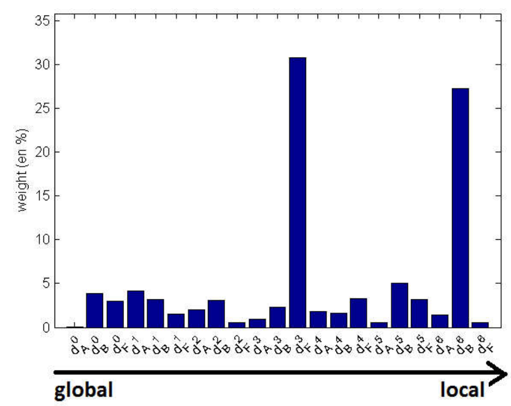
$$D_{Lin}(\mathbf{x}_i, \mathbf{x}_j) = \sum_{h=1}^{p} \alpha_h d_h(\mathbf{x}_i, \mathbf{x}_j) \tag{5.1}$$

$$D_{Geom}(\mathbf{x}_i, \mathbf{x}_j) = \prod_{h=1}^{p} \alpha_h d_h(\mathbf{x}_i, \mathbf{x}_j) \tag{5.2}$$

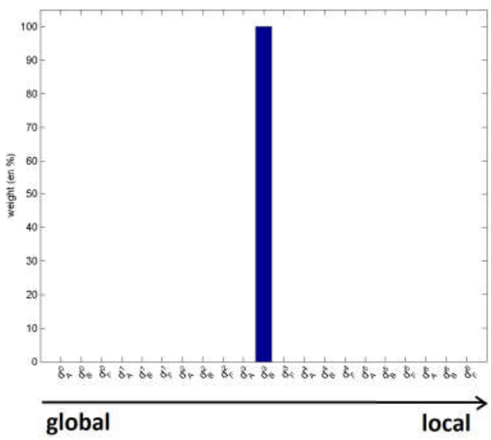
However, by considering p metrics d_h the resulting models requires to optimize p parameters. The grid search to find the best parameters α_h can become time consuming.

For the learnt metric D, thanks to the L_1 regularization, the learned SVM reveals the features that most differentiate positive from negative pairs. Table 5.4 shows the sparse, mutimodal and multi-scale potential of M^2TML approach. It gives for each dataset, the weights of the top five 'discriminative' features that contribute to the definition of D. For instance, for FaceFour D reaches the 0% by combining, in the order of importance, the behavior, frequential and amplitude modalities, at the global (I^0) and local (I^4, I^5, I^2) scales. For Beef, besides the impressive error rate of 3%, the learned model is very sparse as D involves only the behavior

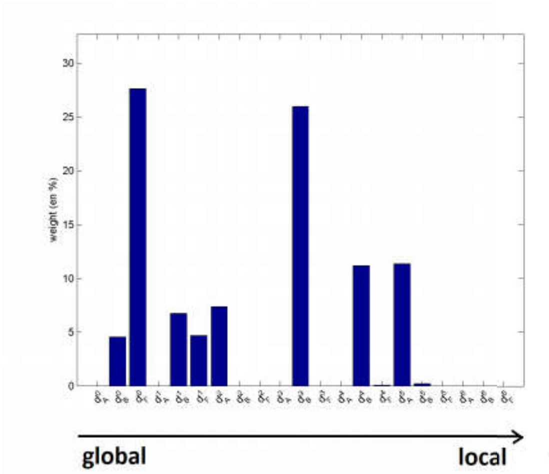
modality based on the segment I^3 (d_B^3). Similarly for Coffee, the obtained 0% involves only the behavior and frequential modalities at several scales.
In Fig. 5.2 we plot the the weights of all features for SonyAIBO, Beef, CincECGtorso and FaceFour cases as an example. It illustrates both the sparsity of the M^{2}TML approach (Beef, CincECGtorso and FaceFour) and the ability of the algorithm to combine all the features into the metric D when it is necessary (SonyAIBO). Note in particular that the algorithm is able to either select one single feature (Beef) or combine a number of selected features (CinCECGTorso, FaceFour). Fig. 5.3 illustrates the temporal locations of the most discriminative features for these datasets. In summary, we can emphasize that for almost all datasets, the definition of D involves no more than five features (the most contributive ones), that assesses not only the model's sparsity but also the representativeness of the revealed features.



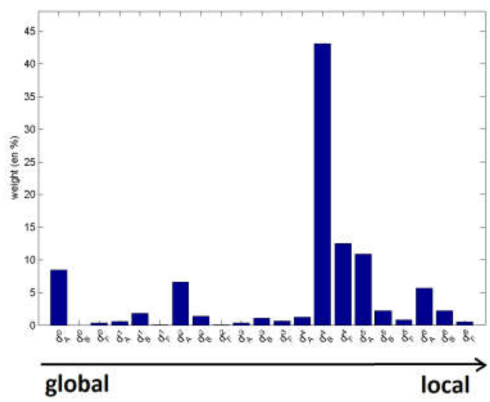
(a) SonyAIBO



(b) Beef



(c) CinC ECG torso



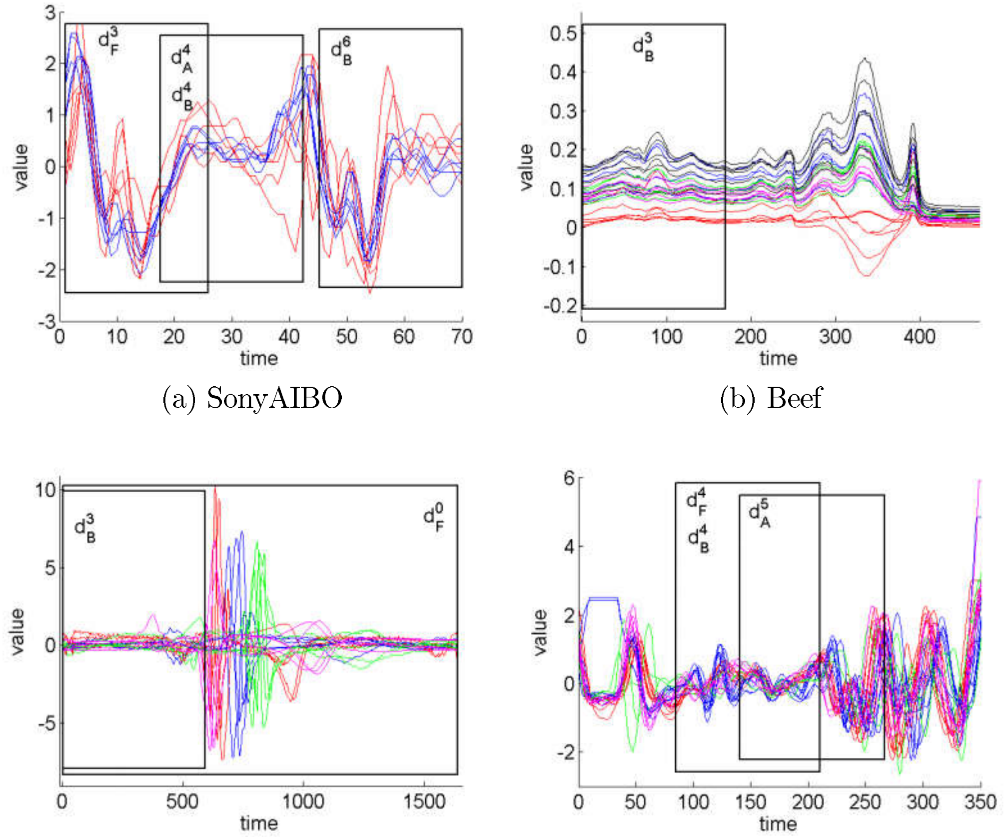
(d) FaceFour

Figure 5.2: M^{2}TML feature weights for 4 datasets.

Dataset	Feature weights (%)				
CC	DTW^2 (56.3%)	d_F^0 (18.8%)	d_F^4 (5.9%)	d_F^1 (4%)	$d_{B-\text{DTW}}^0$ (3.2%)
GunPoint	$d_{B-\text{DTW}}^0$ (42.1%)	DTW^5 (10.9%)	DTW^1 (10.2%)	$d_{B-\text{DTW}}^6$ (8.4%)	d_F^4 (8.1%)
CBF	DTW^4 (56.5%)	d_F^3 (43.4%)	$d_{B-\text{DTW}}^1$ (0.2%)	DTW^0 (0%)	$d_{B-\text{DTW}}^0$ (0%)
OSULeaf	DTW^2 (23.7%)	d_F^0 (19.5%)	$d_{B-\text{DTW}}^0$ (14.6%)	$d_{B-\text{DTW}}^2$ (9.4%)	DTW^1 (9%)
SwedishLeaf	d_F^0 (21.5%)	DTW^0 (15.9%)	$d_{B-\text{DTW}}^0$ (15.2%)	DTW^6 (11.5%)	$d_{B-\text{DTW}}^1$ (6.1%)
Trace	DTW^0 (58.3%)	DTW^6 (6.9%)	$d_{B-\text{DTW}}^0$ (5.8%)	DTW^2 (5.6%)	DTW^5 (5.5%)
FaceFour	$d_{B-\text{DTW}}^4$ (44.5%)	d_F^4 (12.7%)	DTW^5 (11.1%)	DTW^0 (8.3%)	DTW^2 (6.4%)
Lighting2	$d_{B-\text{DTW}}^0$ (30.4%)	DTW^6 (18.7%)	d_F^1 (16.5%)	$d_{B-\text{DTW}}^6$ (13.4%)	DTW^0 (10.7%)
Lighting7	$d_{B-\text{DTW}}^6$ (87.4%)	d_F^6 (8.6%)	$d_{B-\text{DTW}}^5$ (4%)	-	-
ECG200	d_B^0 (89.6%)	d_B^6 (2.4%)	d_A^3 (2.3%)	d_B^1 (2.2%)	d_B^4 (2%)
Adiac	d_F^0 (79.2%)	d_B^4 (13.8%)	d_A^4 (3.5%)	d_F^5 (1.7%)	d_B^5 (1.2%)
FISH	$d_{B-\text{DTW}}^5$ (17.9%)	d_F^0 (10.5%)	$d_{B-\text{DTW}}^6$ (9.9%)	$d_{B-\text{DTW}}^4$ (8.3%)	$d_{B-\text{DTW}}^3$ (7.8%)
Beef	d_B^3 (100%)	-	-	-	-
Coffee	d_B^2 (22.4%)	d_F^4 (20.1%)	d_B^6 (14.6%)	d_B^0 (8.1%)	d_F^5 (7%)
OliveOil	d_F^5 (97%)	$d_{B-\text{DTW}}^2$ (3%)	-	-	-
CinCECGtorso	d_F^0 (38.4%)	d_A^5 (13.1%)	d_B^4 (11.5%)	d_F^1 (11.2%)	d_A^2 (9.8%)
DiatomSizeR	d_F^5 (39.1%)	d_F^0 (36%)	$d_{B-\text{DTW}}^4$ (24.9%)	-	-
ECG5Days	d_B^5 (59.5%)	d_B^6 (32.3%)	d_A^4 (3.9%)	d_B^2 (3.1%)	d_B^4 (1.2%)
FacesUCR	d_F^2 (21.5%)	$d_{B-\text{DTW}}^0$ (19.5%)	d_F^4 (16.7%)	DTW^0 (12.6%)	$d_{B-\text{DTW}}^2$ (8.6%)
InlineSkate	d_F^4 (42.5%)	DTW^5 (22.8%)	DTW^4 (17.6%)	DTW^2 (6.7%)	$d_{B-\text{DTW}}^6$ (5.9%)
ItalyPowerD	d_B^6 (68.7%)	d_B^0 (25.9%)	d_B^3 (5.2%)	d_B^4 (0.2%)	-
MedicalImages	$d_{B-\text{DTW}}^1$ (53.3%)	d_F^3 (12.9%)	$d_{B-\text{DTW}}^2$ (10.7%)	$d_{B-\text{DTW}}^3$ (10.1%)	$d_{B-\text{DTW}}^0$ (3.8%)
MoteStrain	$d_{B-\text{DTW}}^5$ (93.2%)	$d_{B-\text{DTW}}^6$ (6.8%)	-	-	-
SonyAIBOII	d_B^3 (100%)	-	-	-	-
SonyAIBO	d_F^3 (30.8%)	d_B^6 (27.3%)	d_B^5 (5%)	d_A^1 (4.1%)	d_B^0 (3.9%)
Symbols	$d_{B-\text{DTW}}^0$ (45.6%)	$d_{B-\text{DTW}}^6$ (35.3%)	$d_{B-\text{DTW}}^5$ (19%)	DTW^0 (0.1%)	-
TwoLeadECG	$d_{B-\text{DTW}}^4$ (60%)	d_F^1 (12%)	DTW^4 (11.4%)	$d_{B-\text{DTW}}^6$ (7.6%)	$d_{B-\text{DTW}}^1$ (4.2%)
PowerCons	d_F^0 (26.1%)	DTW^0 (20.3%)	d_F^1 (19.3%)	$d_{B-\text{DTW}}^0$ (6.1%)	d_F^2 (5.1%)
BME	$d_{B-\text{DTW}}^0$ (75.2%)	d_F^4 (15.5%)	$d_{B-\text{DTW}}^2$ (5.8%)	$d_{B-\text{DTW}}^1$ (1.9%)	d_F^1 (0.7%)
UMD	$d_{B-\text{DTW}}^0$ (99.8%)	$d_{B-\text{DTW}}^5$ (0.2%)	-	-	-

Table 5.4: Top 5 multi-modal and multi-scale features involved in D

In the second part, we perform a graphical analysis for a global comparison on the whole datasets. In Fig. 5.4 (a), each dataset is projected according to, on the x-axis its best error rate obtained for D and $D_{\mathscr{H}}$, and on y-axis its best performance w.r.t the standard metrics d_A and DTW. In Fig. 5.4 (b), the y-axis is related to the best error rate w.r.t DTW and $d_{B-\text{DTW}}$, the two most performant uni-modal metrics. For both plots we can note that the datasets are principally projected above the first bisector, indicating higher error rates mostly obtained for alternative metrics than for M^2TML. For the less challenging datasets, although almost projected near the bisector denoting equal performances for the compared metrics, M^2TML



(a) SonyAIBO (b) Beef

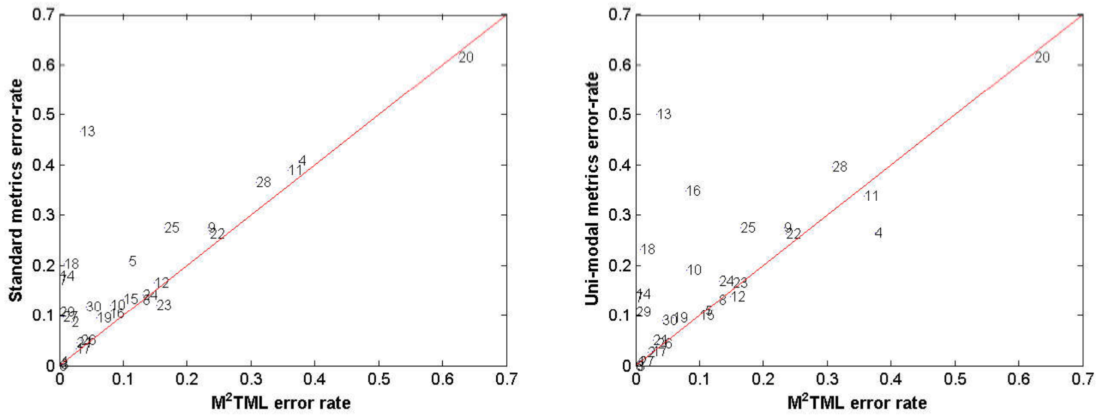
(c) CinC ECG torso (d) FaceFour

Figure 5.3: Temporal representation of the top M^{2}TML feature weights for 4 datasets.

still bring improvements with projections clearly positioned above the bisector. Finally, from Fig. 5.4 (b) we can see that M^{2}TML metrics perform significantly lower than $d_{B-\text{DTW}}$ on OSUleaf, while InlineSkate dataset remains challenging for all studied metrics.

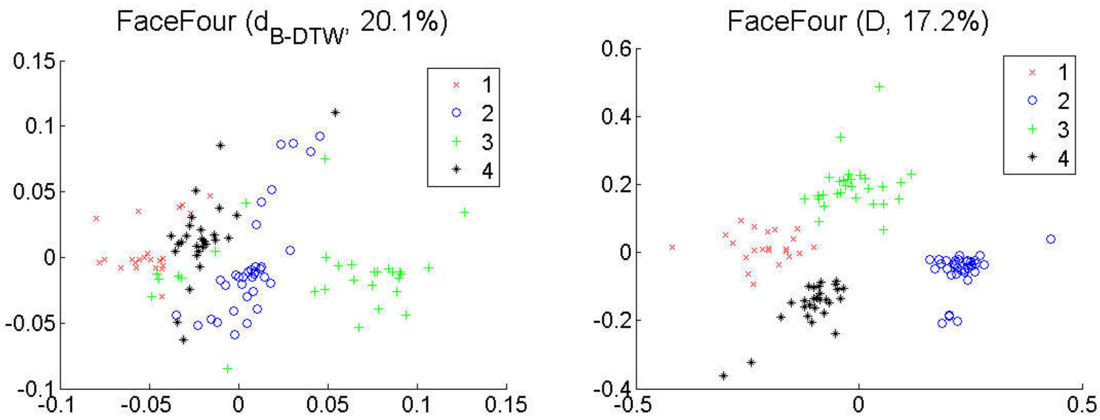
In the last part we compare the global effect of the alternative and M^{2}TML metrics on the 1-NN neighborhood distribution and class discrimination. For that, an MDS[2] is used to vizualize the distribution of samples according to their pairwise dissimilarities. For instance, for FaceFour, Fig. 5.5 shows the first obtained plans and their corresponding stresses, the classes being indicated in different symbols and colors. We can see distinctly the effect of the learned D that leads to more compact and more isolated classes with robust neighborhoods for 1-NN classification (*i.e.* closer positive pairs and far away negative pairs) than the best alternative metric $d_{B-\text{DTW}}$ that shows more overlapping classes and heterogeneous neighborhoods.

[2]matlab function: mdscale for metrics and non metrics



(a) (b)

Figure 5.4: (a) Standard (Euclidean distance d_A and DTW) *vs.* M^{2}TML (D and $D_{\mathscr{H}}$) metrics. (b) Best Uni-modal (DTW and $d_{B-\mathrm{DTW}}$) *vs.* M^{2}TML (D and $D_{\mathscr{H}}$) metrics.



(a) (b)

Figure 5.5: MDS visualization of the $d_{B-\mathrm{DTW}}$ (Fig. a & c) and D (Fig. & d) dissimilarities for FaceFour and CinC ECG torso data

5.5 Conclusion of the chapter

The large conducted experiments and the impressive performances obtained attest the efficiency of the learned M^{2}TML metrics for time series nearest neighbors classification. Finally, let us underline the merit of the M^{2}TML solution, that not only leads to better performances, but also provides a comprehensive and fine-grained information about which modalities are mostly discriminant, how they should be combined and precisely at which temporal granularity.