Binomial

Thursday, November 23, 2023 12:49 PM

$\binom{n}{k}$ or ${}^{n}C_{k}$: No. of ways of choosing k objects among n objects

$$f(k;n,p) = \Pr(X=k) = \binom{n}{k} p^k (1-p)^{n-k} = {}^{n}C_{k}\, p^{k} q^{n-k}$$

$p+q=1$

$(k+1)^{th}$ term in the expansion of $(a+b)^n$ is ${}^{n}C_{k}\, a^{k} b^{n-k}$

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= {}^{2}C_{0}\, a^{2} b^{2-2} + {}^{2}C_{1}\, a^{1} b^{2-1} + {}^{2}C_{2}\, a^{0} b^{2-0}$$

$X: 0, 1, 2, \dots n$

$$(p+q)^n = \sum_{k=0}^{n} {}^{n}C_{k}\, p^{k} q^{n-k} = 1$$

Bernoulli Trail: Experiment with 2 possible outcomes

In a typical Month, an Insurance agent presents life insurance plans to 40 potential customers. Historically, one in four such customers chooses to buy Life Insurance from this agent. Based on the relevant binomial distribution , answer the following questions :

Buys or Won't Buy

$P(B) = p = \frac{1}{4} = 0.25$ $\quad n=40$; X: no. of customers buying

$P(NB) = q = 1 - \frac{1}{4} = 0.75$ $\quad P[X=k] = {}^{40}C_{k}\, 0.25^{k}\, 0.75^{40-k}$

1. What is the probability that exactly 5 customers will buy life Insurance from this agent in the coming month ? $P[X=5] = {}^{40}C_{5}\, 0.25^{5}\, 0.75^{35}$
$$= \frac{40 \times 39 \times 38 \times 37 \times 36}{5 \times 4 \times 3 \times 2 \times 1} \times 0.25^{5}\, 0.75^{35} = 0.02723$$
2. What is the probability that not more than 10 customers will buy life insurance from this agent in the coming month ? $P[X \le 10] = \sum_{k=0}^{10} P[X=k]$

```
In [10]: print(binom.cdf(10,40,0.25))
0.5839040780287896
```

3. What is the probability that at least 20 customers will buy life insurance from this agent in the coming month ? $P[X \geq 20] = P[X > 19]$

–

```
In [11]: print(binom.sf(19,40,0.25))
0.0005724311071761386
```

4. Determine the mean and variance of the number of customers who will buy life insurance from this agent in the coming month.

```
In [12]: print(binom.stats(40,0.25))
(10.0, 7.5)
```

Cumulative Distribution Function : $P[X \leq k]$

$x=0,1,\dots$

$P[X \leq 5] = P[X=0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5]$

$= P[X=0] + P[X=1] + \dots + P[X=5]$

binom.cdf(k,n,p,...)

$P[X \leq 10] = \sum_{k=0}^{10} P[X=k]$

E-notation

1.00565852e-05 $= 1.0056585 \times 10^{-5}$

$= 0.000010056585$

The incidence of a certain disease is such that on an average, 15% of workers suffer from it. If some 20 workers are chosen at random, find the probability that $n=20$, $p=0.15$

- Exactly 5 workers suffer from the disease : X: workers suffering $q=0.85$

$P[X=5]$

```
binom.pmf(5, 20, 0.15)
0.10284517954557217
```

- More than 12 workers suffer from the disease

$P[X>12]$

```
binom.sf(12, 20, 0.15)
5.295123107511796e-07
```

$= 5.295123107 \times 10^{-7}$

$= 0.0000005295123$

- At most 10 workers suffer from the disease

$P[X \leq 10]$

```
binom.cdf(10, 20, 0.15)
0.9999613672517919
```