1. Uniform Distribution
2. Geometric Distribution
3. Exponential Distribution

The **exponential distribution** is a continuous distribution that models the time between randomly occurring events. Thus, it is often used in such applications as modeling the time between customer arrivals to a service system or the time to or between failures of machines, lightbulbs, hard drives, and other mechanical or electrical components.

Similar to the Poisson distribution, the exponential distribution has one parameter, λ. In fact, the exponential distribution is closely related to the Poisson distribution; if the number of events occurring *during* an interval of time has a Poisson distribution, then the time *between* events is exponentially distributed. For instance, if the number of arrivals at a bank is Poisson-distributed, say with mean $\lambda = 12$/hour, then the time between arrivals is exponential, with mean $\mu = 1/12$ hour, or 5 minutes.

The exponential distribution has the density function

$$f(x) = \lambda e^{-\lambda x}, \quad \text{for } x \geq 0 \tag{5.23}$$

and its cumulative distribution function is

$$F(x) = 1 - e^{-\lambda x}, \quad \text{for } x \geq 0 \tag{5.24}$$

$$S(x) = e^{-\lambda x}$$

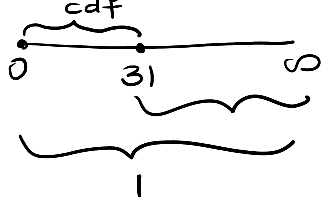
The expected value of the exponential distribution is $1/\lambda$ and the variance is $(1/\lambda)^2$.

43. The actual delivery time from a pizza delivery company is exponentially distributed with a mean of 28 minutes.

a. What is the probability that the delivery time will exceed 31 minutes?

b. What proportion of deliveries will be completed within 25 minutes?

$0 \leq X < \infty$



$$E(X) = 28 = \frac{1}{\lambda} \qquad \therefore \lambda = \frac{1}{28}$$

$$\text{(a)} \quad P(X > 31) = e^{-\left(\frac{1}{28} \times 31\right)}$$

$$= 0.3305$$

$$\text{(b)} \quad P(X \leq 25) = 1 - e^{-\left(\frac{1}{28}\right)25} = 0.59051$$

42. According to WHO, around 1.35 million people die each year due to road accidents. Suppose that on a specific highway, an average of four accidents occur within every hour. Find the probability that the duration between accidents is less than or equal to 30 minutes.

Duration betn accidents : $\frac{1}{4}$ hours $= \frac{1}{4} \times 60 = 15$ minutes

$\lambda = \frac{1}{15}$; $P[X \leq 30] = 1 - e^{-\left(\frac{1}{15}\right)30}$

$= 0.86466$

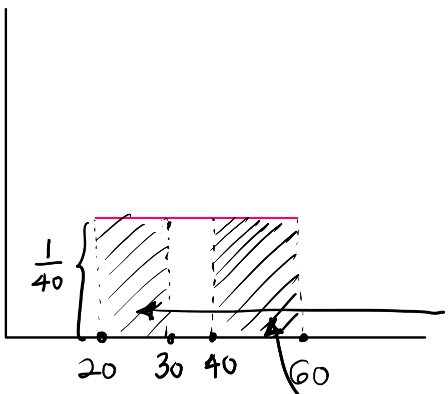
Distribution	Type	Expression	Stats
Binomial	Discrete	$P(X=k) = {}^nC_k \, p^k q^{n-k}$ $k=0,1,\ldots n$ Binomial (n, p)	$E(X) = np$ $Var(X) = npq$
Poisson	Discrete	$P[X=k] = \frac{e^{-\lambda}\lambda^k}{k!}$; $k=0,1,\ldots\infty$ Poisson(λ)	$E(X) = \lambda$ $Var(X) = \lambda$
Geometric	Discrete	$P[X=k] = (1-p)^{k-1}p$ Geometric (p) $k=0,1\ldots\infty$	$E(X) = \frac{1}{p}$ $Var(X) = (1-p)/p^2$
Uniform	Continuous	$f(x) = \frac{1}{b-a}$ $a \leq x \leq b$ Uniform (a,b)	$E(X) = (a+b)/2$ $Var(X) = (b-a)^2/12$
Uniform	Discrete	$P[X=k] = \frac{1}{n}$ for n items	$E(X) = (n+1)/2$ $Var(X) = (n^2-1)/12$
Exponential	Continuous	$f(x) = \lambda e^{-\lambda x}$ $x \geq 0$ Exponential (λ)	$E(X) = \frac{1}{\lambda}$ $Var(X) = 1/\lambda^2$
Normal	Continuous	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $\sigma > 0$ (μ, σ^2) $-\infty < x < \infty$ $-\infty < \mu < \infty$ Normal (μ, σ^2)	$E(X) = \mu$ $Var(X) = \sigma^2$

35. The time required to play a game of Battleship™ is uniformly distributed between 20 and 60 minutes.

a. Find the expected value and variance of the time to complete the game.

b. What is the probability of finishing within 30 minutes?

c. What is the probability that the game would take longer than 40 minutes?



$$f(x) = \frac{1}{b-a} = \frac{1}{40}$$

$$E(X) = \frac{a+b}{2} = 40, \quad Var(X) = \frac{(b-a)^2}{12}$$

$$= 133.33$$

$$P(X \leq 30) = \frac{10}{40} = 0.25$$

$$P(X > 40) = \frac{1}{40} \times 20 = 0.5$$