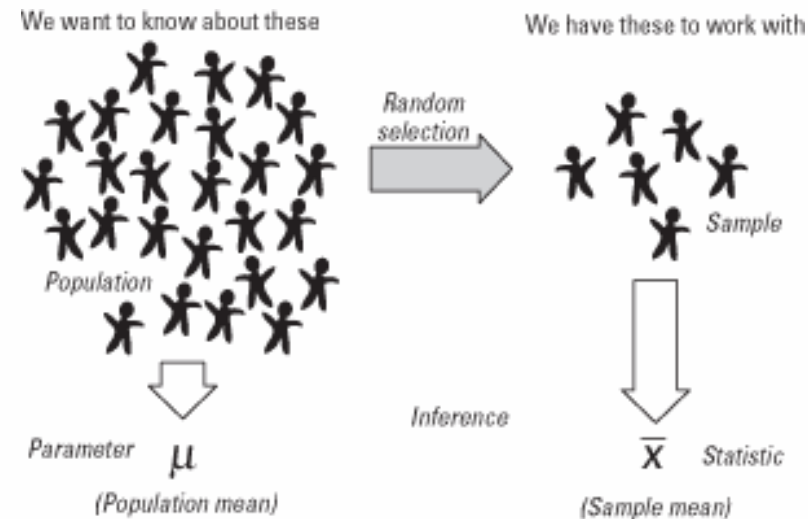# Hypothesis Testing

Statistical Hypothesis

- A hypothesis is a statement or assertion about the state of nature. e.g. The campaign was effective.

- Every hypothesis implies its contradiction or alternative. e.g. The campaign was not effective.

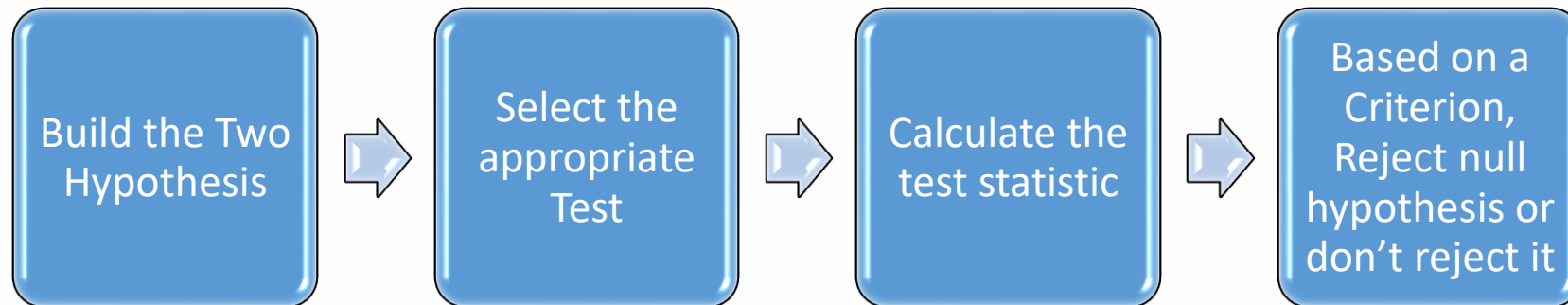- Either of the hypothesis statement can be true or false.

Testing the Hypothesis

- For testing any hypothesis we make use of the sample data.
- Based on the sample data, we reject either of the hypothesis statements.
- The decision to reject or failing to reject any of the hypothesis can be either correct or wrong.

We want to know about these

We have these to work with

Random selection

Population

Sample

Parameter μ

(Population mean)

Inference

$\overline{X}$ Statistic

(Sample mean)

Courtesy: www.clifffnotes.com

Flow for Hypothesis Testing

```
┌─────────────┐      ┌─────────────┐      ┌─────────────┐      ┌─────────────┐
│ Build the   │ ──▶  │ Select the  │ ──▶  │ Calculate   │ ──▶  │ Based on a  │
│ Two         │      │ appropriate │      │ the test    │      │ Criterion,  │
│ Hypothesis  │      │ Test        │      │ statistic   │      │ Reject null │
│             │      │             │      │             │      │ hypothesis  │
│             │      │             │      │             │      │ or don't    │
│             │      │             │      │             │      │ reject it   │
└─────────────┘      └─────────────┘      └─────────────┘      └─────────────┘
```

Ground Truth and Findings

- Found fact would be based on sample whereas ground truth would be based on population.

- We won't be able to directly find the ground truth due to the large size of population.

- Hence we analyze the sample and try to find the fact.

- It may happen that:
    - Found fact from the sample is same as ground truth
    - Found fact from the sample is exactly opposite of the ground truth.

Actual and Findings

- When found fact from sample is not same as the ground truth from the population then we would be wrong.

- We are interested in knowing the probability of us getting wrong.

Statistical Hypothesis

- Null Hypothesis: A **null hypothesis**, denoted by H_0, is an assertive statement about one or more population parameters. This is the statement we hold to be true until we have sufficient statistical evidence to reject it.

- The **alternative hypothesis**, denoted by H_1, is the assertive statement of all situations *not* covered by the null hypothesis.

Example

- H0: The mean sales are 1400.
- H1: The mean sales are not 1400.

- OR

- H0: The mean sales are less than or equal to 1400.
- H1: The mean sales are greater than 1400.

Note: Null Hypothesis should contain equality

Example

- Consider the example:
  - H0: The mean sales are less than or equal to 1400.
  - H1: The mean sales are greater than 1400.
- Suppose we draw a random sample for testing the above hypotheses.
- We can be wrong in the either of the cases:
  - The mean sales for population are less than or equal to 1400 and our sample suggests it as greater than 1400.(Rejecting true H0)
  - The mean sales for population are greater than 1400 and our sample suggests it as less than or equal to 1400.(Failing to reject the false H0)

Errors

- Type I: Rejecting true H0
- Type II: Failing to reject the false H0
- P(Type I Error) = α
- P(Type II Error) = β
- If we try to reduce α, then β increases and if we try to reduce β, then α increases.
- More serious is the error α. Hence a level for α is maintained. This level is called level of significance. It is denoted by α. Usually its is maintained as 0.1 or 0.05 or 0.01.
- More often maintained as 0.05.

Types of Errors

Decision	H0 True	H0 False
Reject H0	Type I Error (α) Producer's Risk	Correct
Fail to reject H0	Correct	Type II Error (β) Consumer's Risk

P - Value

- The *p-value* is the probability of getting a value of the test statistic as extreme as, or more extreme than, the actual value obtained, when the null hypothesis is true.

- The p-value is the smallest level of significance, α, at which the null hypothesis may be rejected using the obtained value of the test statistic.

- **Policy to be followed: When the *p*-value is less than α , reject H_0, otherwise we do not reject H_0.**

Power of a Test

- The **power** of a statistical hypothesis test is the probability of rejecting the null hypothesis when the null hypothesis is false.

$$\textbf{Power} = \textbf{(1 - β)}$$

- Power is the probability that your test will reject the null hypothesis when the null hypothesis is false, or the probability that you will detect a difference when a difference actually exists.
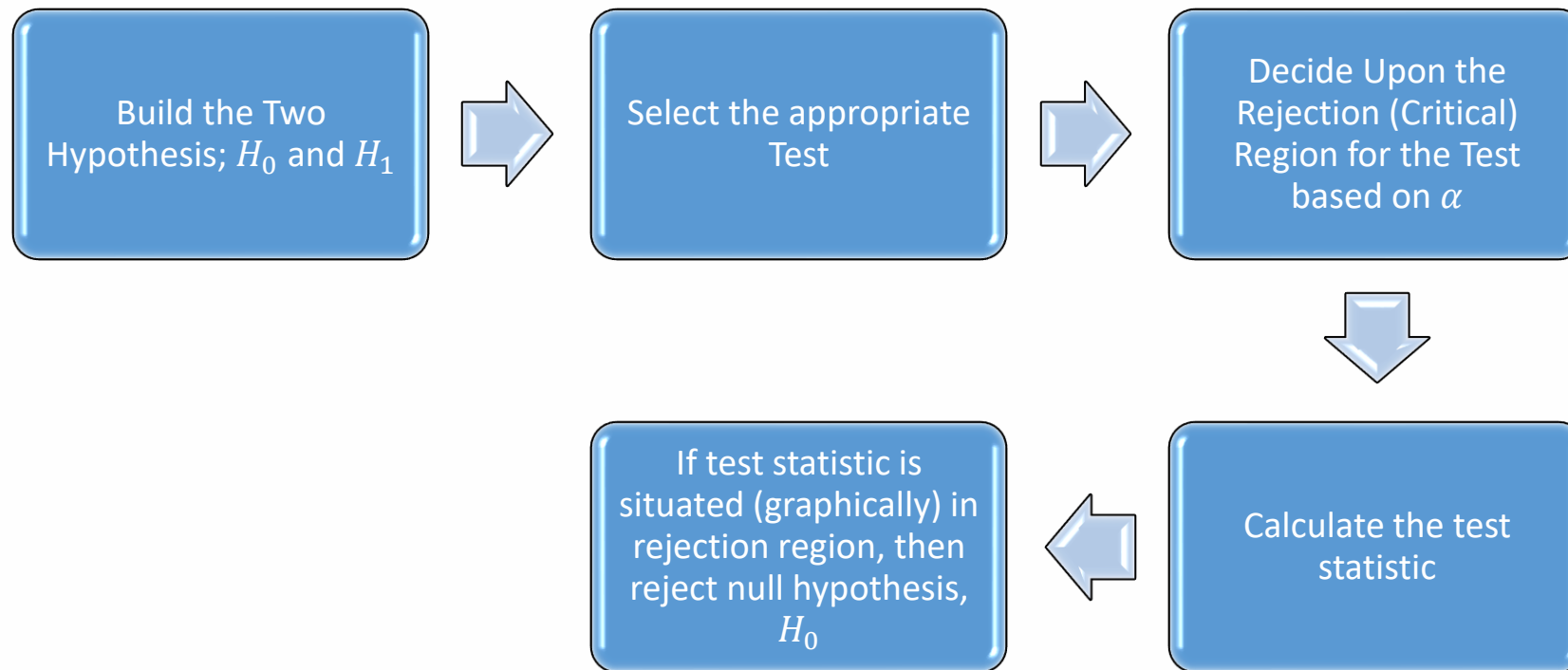
Tail of Test

- H0: The mean sales are 1400. i.e. $\mu = 1400$
- H1: The mean sales are not 1400. i.e. $\mu \neq 1400$
  - This is **two tailed** test as in H1, mean sales can be greater then or less than 1400.

- H0: The mean sales are less than or equal to 1400. i.e. $\mu \leq 1400$
- H1: The mean sales are greater than 1400. i.e. $\mu > 1400$
  - This is **right tailed** or **upper tailed** test as in H1, mean sales are greater than 1400.

- H0: The mean sales are greater than or equal to 1400. i.e. $\mu \geq 1400$
- H1: The mean sales are less than 1400. i.e. $\mu < 1400$
  - This is **left tailed** or **lower tailed** test as in H1, mean sales are less than 1400.
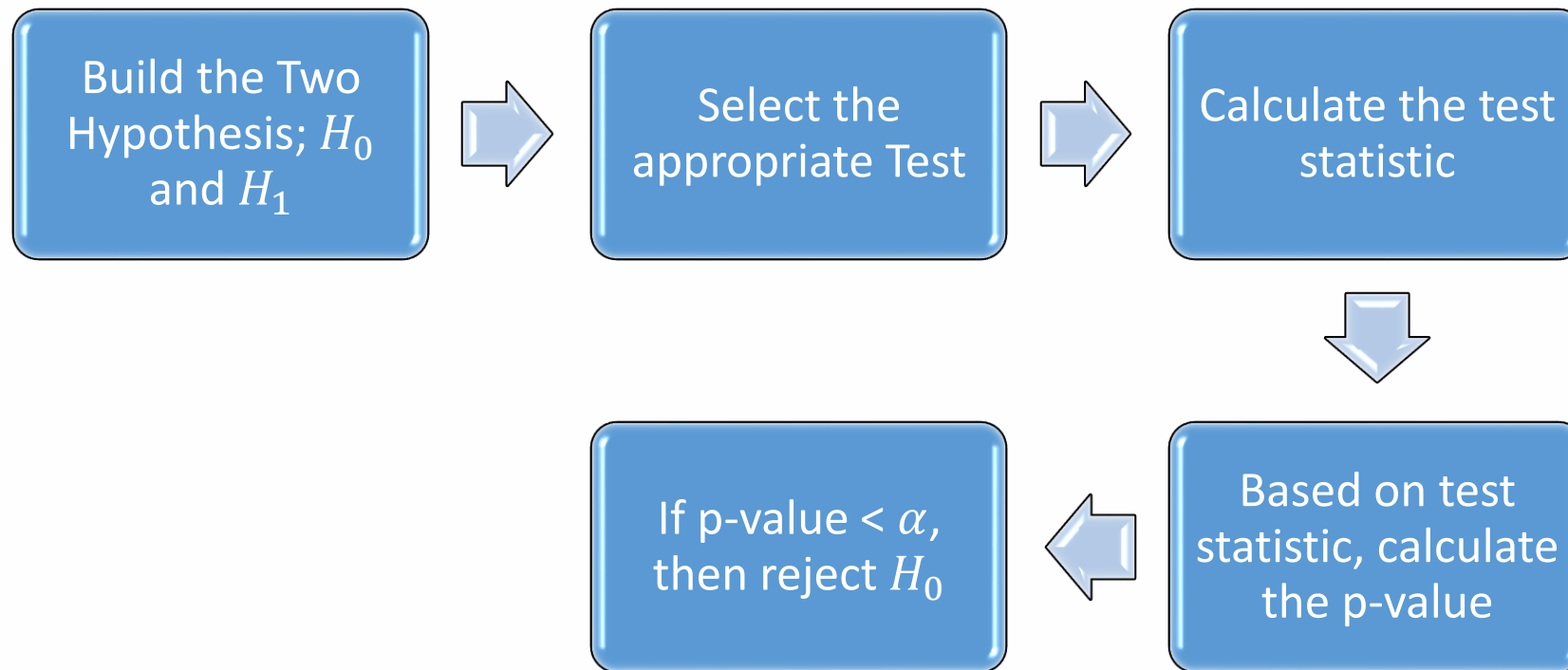
Testing of Hypothesis Approaches

- Critical Value Approach
- P-value Approach

Critical Value Approach

```
┌─────────────────────┐      ┌─────────────────────┐      ┌─────────────────────┐
│   Build the Two     │ ──▶  │ Select the appropriate│ ──▶ │  Decide Upon the    │
│ Hypothesis; $H_0$ and $H_1$ │ │        Test         │      │ Rejection (Critical)│
│                     │      │                     │      │ Region for the Test │
│                     │      │                     │      │   based on $\alpha$ │
└─────────────────────┘      └─────────────────────┘      └─────────────────────┘
                                                                     │
                                                                     ▼
┌─────────────────────┐      ┌─────────────────────┐
│ If test statistic is│ ◀──  │ Calculate the test  │
│ situated (graphically) in │ │     statistic       │
│ rejection region, then │   │                     │
│ reject null hypothesis,│   │                     │
│        $H_0$        │      │                     │
└─────────────────────┘      └─────────────────────┘
```

P-value Approach

Build the Two Hypothesis; H_0 and H_1

Select the appropriate Test

Calculate the test statistic

Based on test statistic, calculate the p-value

If p-value $< \alpha$, then reject H_0

Types of Hypothesis Tests

- Parametric: Tests which assume a particular distribution(Normal) of the population
  - t-test
  - F-test
  - Chi-square test for variance
- Non-Parametric: Tests which assume no particular distribution of the population
  - Median Test
  - Wilcoxon's Rank Sum Test
  - Mann-Whitney Test
  - Kruskal Wallis Test