Probability

We will be covering…

- Concept of Probability
- Types of Events
  - Collectively Exhaustive
  - Mutually Exclusive
  - Independent
- Rules
  - Complement
  - Addition
  - Multiplicative

Probability

- A probability in statistical theory is a number between 0 and 1 that measures the likelihood that some event will occur.
- A zero probability of an event signifies event not occuring and probability one signifies event being almost sure

Collectively Exhaustive Events

- Events A1, A2, … An are said to be collectively exhaustive events if at least one of the events must occur
- Example:
  - A1: Rain
  - A2: Sunny Weather
  - A3: Cloudy
  - A4: Snowfall

Rules

$P(A) = 0.7$

$P(A^c) = 1 - 0.7 = 0.3$

- If A and B are events then:
  - Rule of Complement : P(Non-occurrence of A) = 1 – P(A)
  - Addition Rule: P(Occurrence of at least one of the events) = P(A) + P(B) – P(Occurrence of both A and B)
  - Multiplication Rule: P(Occurrence of A given that B has occurred) = P(Occurrence of both A and B) / P(B)

Addition Rule

- P(Occurrence of at least one of the events) = P(A) + P(B) – P(Occurrence of both A and B)
- By Notation:

  $P(A \text{ or } B \text{ or } A \text{ and } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Example:
  - A: Ad Campaign will fail
  - B: Competitor launches a new product
  - $A \cap B$: A and B both occur

  Say $P(A) = 0.23$, $P(B) = 0.39$, $P(A \cap B) = 0.18$ then

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.23 + 0.39 - 0.18$$
  $$= 0.44$$

Mutually Exclusive Events

- The two events A and B are said to be mutually exclusive (disjoint) if they both cannot occur simultaneously

Not being independent

- In case of mutually exclusive events A and B,
  - $P(A \cap B) = 0$
  - $P(A \cup B) = P(A) + P(B)$

Example: A: Snowfall, B: Temperature > 50 degrees Celsius

$P(A) = 0.25$, $P(B) = 0.20$ then $P(A \text{ or } B) = 0.45$

Multiplication Rule

- P(Occurrence of both A and B) = P(Occurrence of A given that B has occurred)*P(B)

Or

- P(Occurrence of A given that B has occurred) = P(Occurrence of both A and B) / P(B)
- By Notations, $P(A \mid B) = P(A \cap B) / P(B)$, $P(A \mid B)$ is read as A given B
- Similarly, $P(B \mid A) = P(A \cap B) / P(A)$
- $P(A|B)$ and $P(B|A)$ are called conditional probabilities
- Example:
  - A: Ad Campaign will fail
  - B: Competitor launches a new product
  - $A \cap B$: A and B both occur

  $P(A) = 0.25$, $P(B) = 0.32$, $P(A \cap B) = 0.17$ then

  $P(A|B) = P(A \cap B) / P(B) = 0.17 / 0.32 = 0.5312$,
  $P(B|A) = P(A \cap B) / P(A) = 0.17 / 0.25 = 0.68$

$$P(A \cap B) = P(A)\,P(B|A)$$
$$= P(B)\,P(A|B)$$

Independence of Events

- If A and B are independent events then: $\Rightarrow P(B|A) = P(B)$ $\Rightarrow P(A|B) = P(A)$
  - P(Occurrence of both A and B) = P(A) P(B)
- By Notations, $P(A \cap B) = P(A) * P(B)$
- Example:
  - A: Ad Campaign will fail
  - B: Sensex goes up
  - $A \cap B$: A and B both occur

  Say, P(A) = 0.32, P(B) = 0.64, $P(A \cap B) = P(A) * P(B) = 0.2048$