Poisson

Friday, November 24, 2023 9:00 AM

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \qquad X = 0, 1, 2, \ldots \infty$$

where

- e is Euler's number ($e = 2.71828\ldots$)
- $k!$ is the factorial of k.

$$Mean = E(X) = \lambda$$

$$Variance(X) = \lambda$$

Scenarios for Poisson Distribution:

1. Number of Customers arriving at a counter in a certain period of time
2. Number of Customers calling a customer care division in a certain period of time
3. Number of Customers getting served at the counter in a certain period of time
4. Number of accidents on a specific road in a certain period of time
5. Number of accidents at a work place in a certain period of time

The annual number of industrial accidents occurring in a particular manufacturing plant is known to follow Poisson distribution with mean 12.

X: no. of accidents

a) What is the probability of observing exactly 5 accidents at this plant during the coming year ? $P[X=5]$

```
poisson.pmf(5, 12)
0.012740638735861376
```

b) What is the probability of observing not more than 12 accidents at this plant the coming year ? $P[X \leq 12]$

```
poisson.cdf(12,12)
0.5759652485730645
```

c) What is the probability of observing at least 15 accidents at this plant during the coming year ? $P[X \geq 15] = P[X > 14]$

c) What is the probability of observing at least 15 accidents at this plant during the coming year ? $P[X \geq 15] = P[X > 14]$

```
poisson.sf(14, 12)
0.2279754676964551
```

d) What is the probability of observing between 10 and 15 accidents (inclusive) at this plant during the coming year ?

$$P[10 \leq X \leq 15] = \sum_{k=10}^{15} P[X = k]$$

$$X : \underbrace{\underbrace{0, 1, 2, 3 \ldots 9}_{cdf(9)}, 10, 11, \ldots, 15}_{cdf(15)}, 16 \ldots \infty$$

$$cdf(15) - cdf(9)$$

The number of customers served at a counter per hour are 4. Find the following:

$\lambda = 4$

a. Probability that more than 5 customers will be served in an hour

$P[X > 5]$

```
poisson.sf(5,4)
0.2148696129695948
```

Probability that less than 3 customers will be served in an hour

$P[X < 3] = P[X \leq 2]$

```
poisson.cdf(2,4)
0.23810330555354436
```