

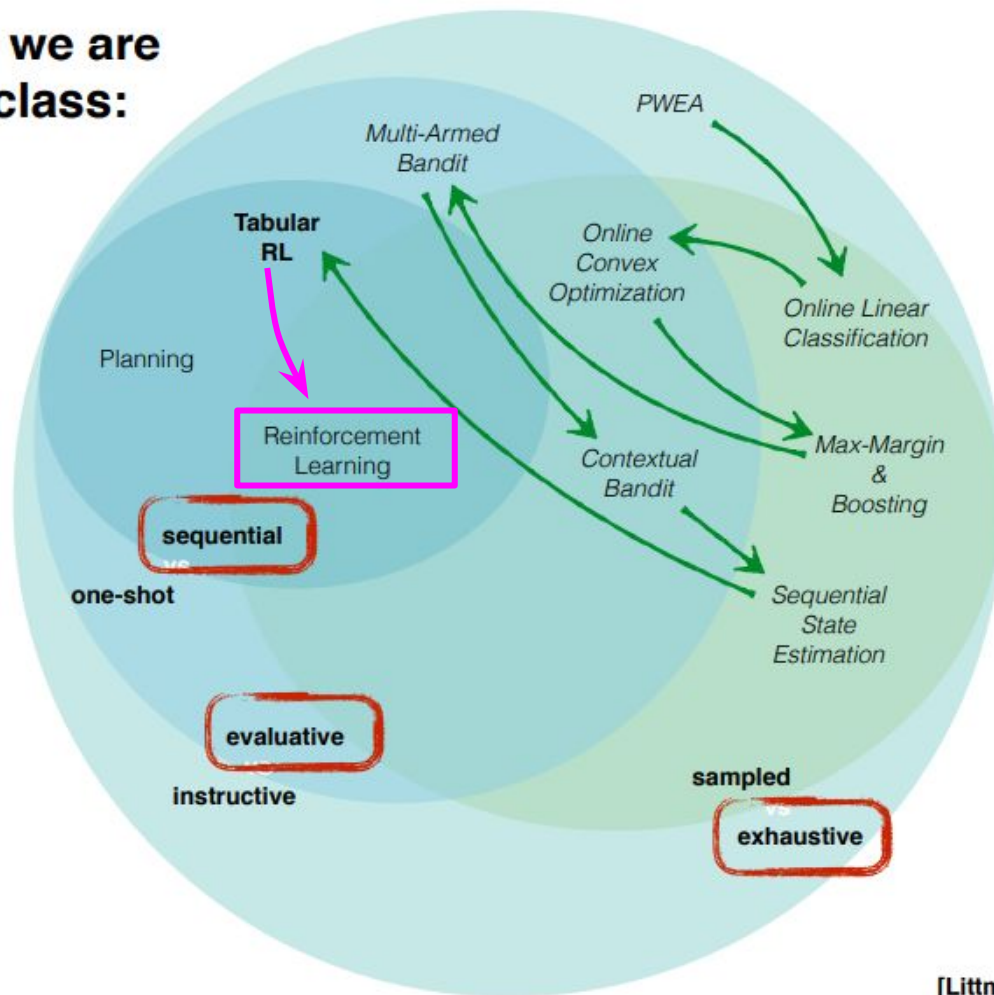


# A3C: Asynchronous Advantage Actor Critic

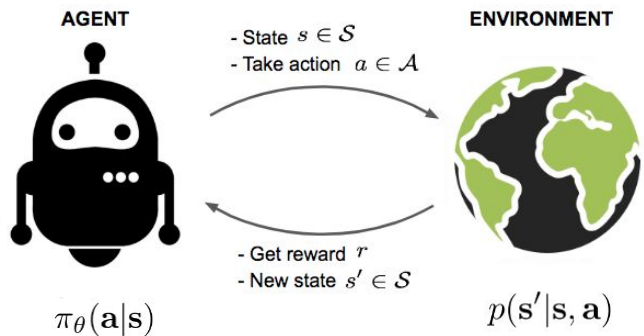
16-831 Statistical Techniques in Robotics

**Carnegie Mellon University (Malhar Bhoite, Vinay Varma, Ashish Roongta)**

## Where we are in the class:



# Reinforcement Learning (Recap)



Markov decision process

$\mathcal{S}$  – state space

$\mathcal{A}$  – action space

$r$  – reward function

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

states  $s \in \mathcal{S}$  (discrete or continuous)

actions  $a \in \mathcal{A}$  (discrete or continuous)

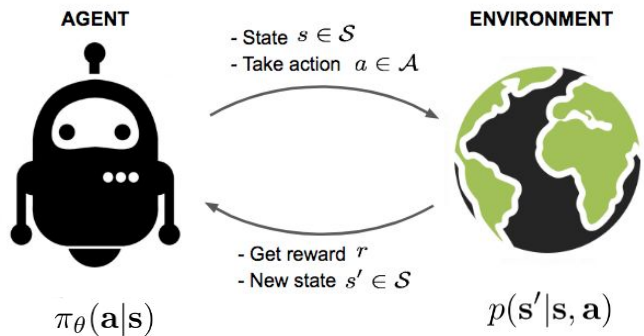
$$r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

$r(s_t, a_t)$  – reward

**Objective:**

*Learn to take actions that give the most reward*

# Reinforcement Learning (Recap)



Markov decision process

$\mathcal{S}$  – state space

$\mathcal{A}$  – action space

$r$  – reward function

$$\mathcal{M} = \{\mathcal{S}, \mathcal{A}, \mathcal{T}, r\}$$

states  $s \in \mathcal{S}$  (discrete or continuous)

actions  $a \in \mathcal{A}$  (discrete or continuous)

$$r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$$

$r(s_t, a_t)$  – reward

**Objective:**

*Learn to take actions that give the most reward*

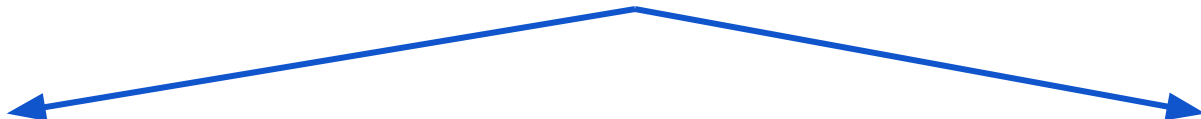
*What are the various techniques to solve RL we have learnt so far?*

# Reinforcement Learning (Recap)

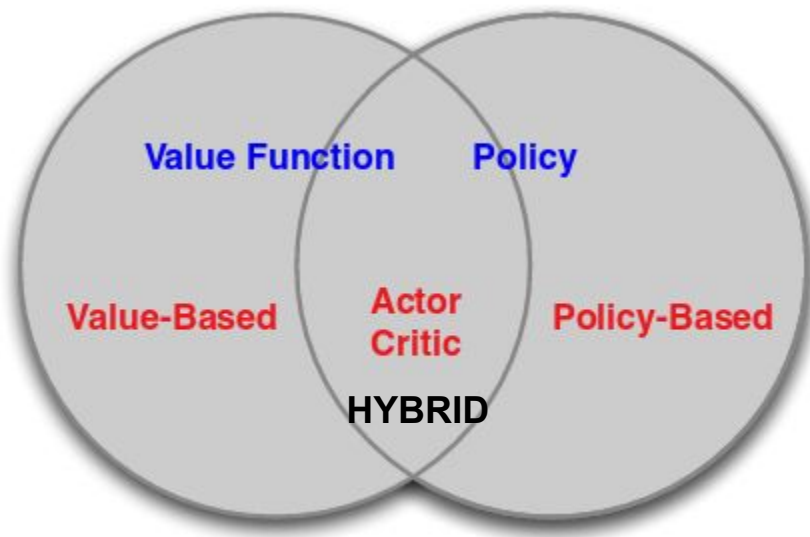
Methods for Solving Reinforcement Learning (Learning the Policy)

Policy Gradient Methods

Temporal Difference Learning  
(Value-Based Methods)

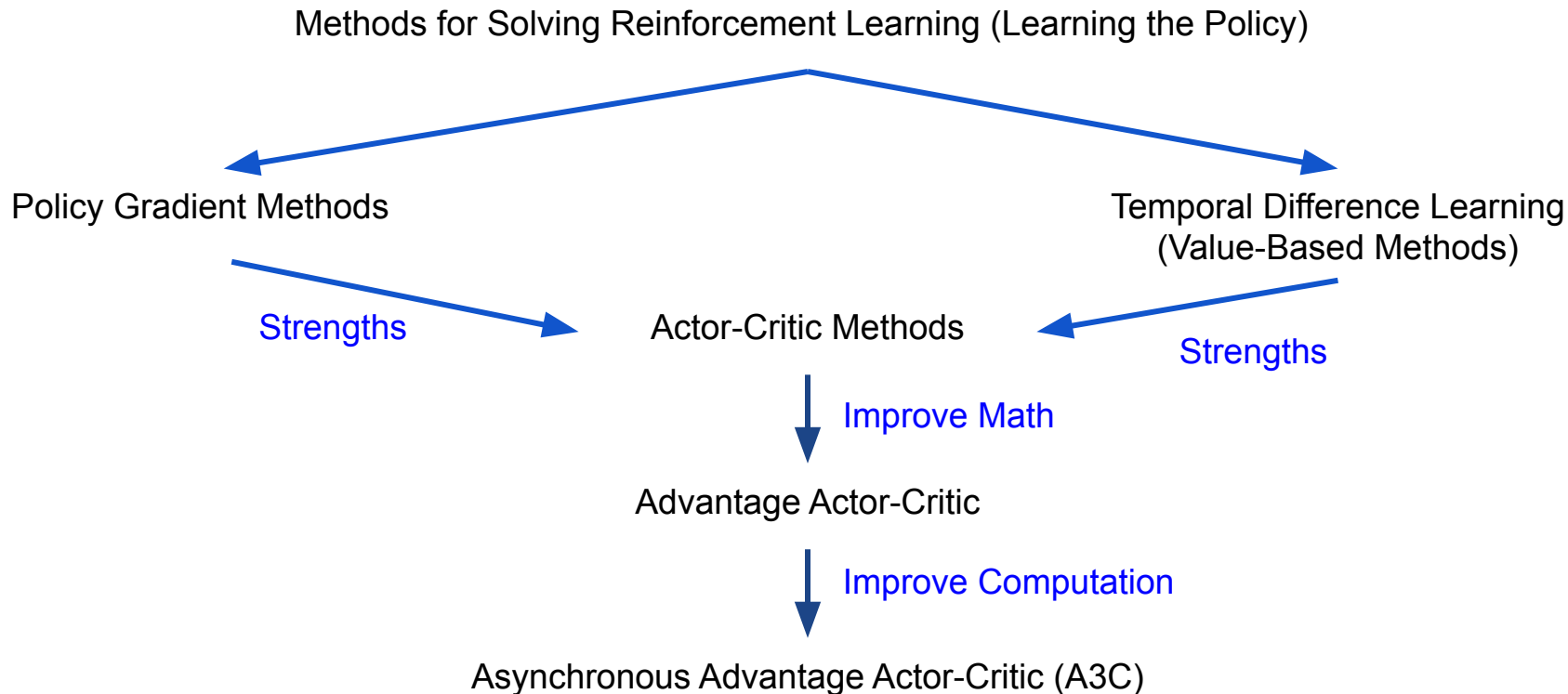


# Actor-Critic Methods



*Can we combine the strengths of both methods?*

# Roadmap



---

## Asynchronous Methods for Deep Reinforcement Learning

---

Volodymyr Mnih<sup>1</sup>  
Adria Puigdomènech Badia<sup>1</sup>  
Mehdi Mirza<sup>1,2</sup>  
Alex Graves<sup>1</sup>  
Tim Harley<sup>1</sup>  
Timothy P. Lillicrap<sup>1</sup>  
David Silver<sup>1</sup>  
Koray Kavukcuoglu<sup>1</sup>

<sup>1</sup> Google DeepMind

<sup>2</sup> Montreal Institute for Learning Algorithms (MILA), University of Montreal

VMNIH@GOOGLE.COM  
ADRIAP@GOOGLE.COM  
MIRZAMOM@IRO.UMONTREAL.CA  
GRAVESA@GOOGLE.COM  
THARLEY@GOOGLE.COM  
COUNTZERO@GOOGLE.COM  
DAVIDSILVER@GOOGLE.COM  
KORAYK@GOOGLE.COM

### Abstract

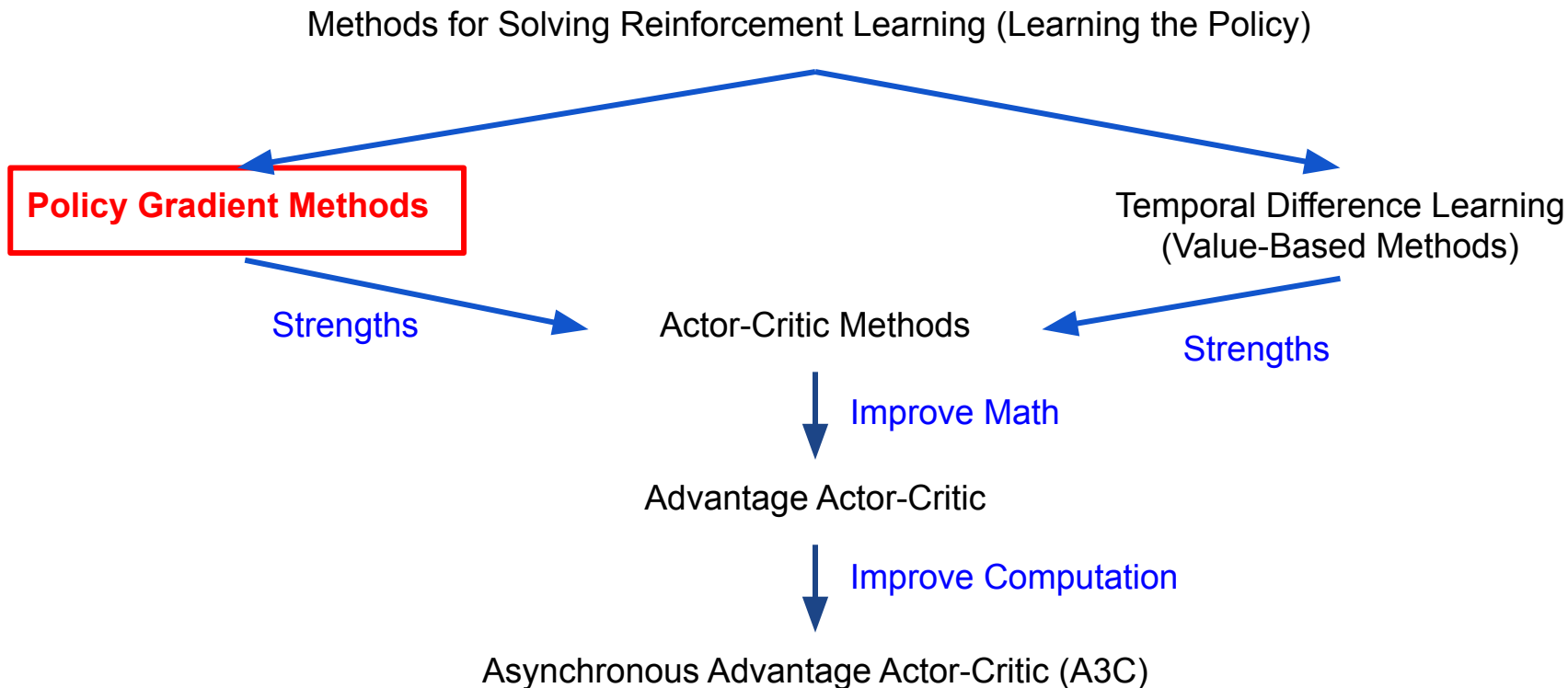
We propose a conceptually simple and lightweight framework for deep reinforcement learning that uses asynchronous gradient descent for optimization of deep neural network controllers. We present asynchronous variants of four standard reinforcement learning algorithms and show that parallel actor-learners have a stabilizing effect on training allowing all four methods to successfully train neural network controllers. The best performing method, an asynchronous variant of actor-critic, surpasses the current state-of-the-art on the Atari domain while training for half the time on a single

line RL updates are strongly correlated. By storing the agent's data in an experience replay memory, the data can be batched (Riedmiller, 2005; Schulman et al., 2015a) or randomly sampled (Mnih et al., 2013; 2015; Van Hasselt et al., 2015) from different time-steps. Aggregating over memory in this way reduces non-stationarity and decorrelates updates, but at the same time limits the methods to off-policy reinforcement learning algorithms.

Deep RL algorithms based on experience replay have achieved unprecedented success in challenging domains such as Atari 2600. However, experience replay has several drawbacks: it uses more memory and computation per real interaction; and it requires off-policy learning algorithms



# Roadmap



# Policy Gradient Theorem (Recap)

1. Start by Defining a Parameterized Policy  $\pi_\theta$

**Objective: Maximize Expected Rewards**

2. Define an Objective Function  $J(\theta)$  
$$J(\theta) = \sum_{s \in S} d^\pi(s) V^\pi(s) = \sum_{s \in S} d^\pi(s) \sum_{a \in A} \pi_\theta(a|s) Q^\pi(s, a)$$

3. Calculate the Gradient of the Objective Function  $\nabla_\theta J(\theta)$

$$\nabla_\theta J(\theta) = E_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

4. Plug values in the update rule  $\theta \leftarrow \alpha \nabla_\theta J(\theta)$

5. Repeat till converged

Recall:

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t|\mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

# Policy Gradient Theorem (Recap)

1. Start by Defining a Parameterized Policy  $\pi_\theta$

**Objective: Maximize Expected Rewards**

2. Define an Objective Function  $J(\theta)$  
$$J(\theta) = \sum_{s \in S} d^\pi(s) V^\pi(s) = \sum_{s \in S} d^\pi(s) \sum_{a \in A} \pi_\theta(a|s) Q^\pi(s, a)$$

3. Calculate the Gradient of the Objective Function  $\nabla_\theta J(\theta)$

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) Q^{\pi_\theta}(s, a)]$$

4. Plug values in the update rule  $\theta \leftarrow \alpha \nabla_\theta J(\theta)$

**What are these terms?**

5. Repeat till converged

# Policy and the Gain

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) Q^{\pi_{\theta}}(s, a)]$$

State-Action Value Function

Policy

# Policy and the Gain

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

## ‘Policy’

Function Approximator

(Eg. Logistic Regressor, Neural Network, etc.)

State  $\rightarrow$  Action

## ‘Gain’ Term (General)

Value Function - Estimates expected return  
Scalar value

Intuitively, this **evaluates** the policy

# Policy and the Gain

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

## ‘Policy’

Function Approximator

(Eg. Logistic Regressor, Neural Network, etc.)

State  $\rightarrow$  Action

## ‘Gain’ Term (General)

Value Function - Estimates expected return  
Scalar value

Intuitively, this **evaluates** the policy

# Policy and the Gain

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

**‘Policy’**

Function Approximator

(Eg. Logistic Regressor, Neural Network, etc.)

State  $\rightarrow$  Action

**‘Gain’ Term** (General)

Value Function - Estimates expected return  
Scalar value

Intuitively, this **evaluates** the policy

*What are the Different ways of estimating this gain term?*

# Policy Evaluation Methods (Estimating Gain)

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

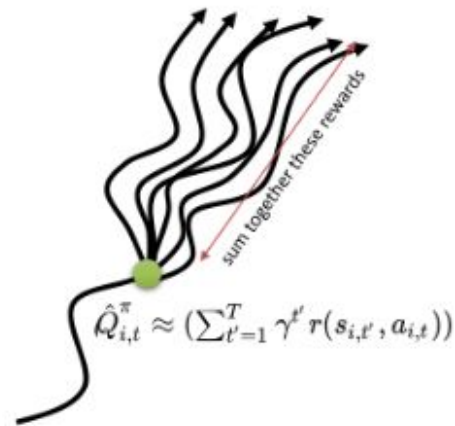
- $\sum_{t'=t}^T \gamma^{t'-t} r(s_{i,t'}, a_{i,t'})$  Total Reward of Simulated Trajectories (Monte Carlo Rollout)
- $\sum_{t'=t}^T \gamma^{t'-t} r(s_{i,t'}, a_{i,t'}) - b(s_t)$  Monte Carlo Rollout with Baseline
- $Q^{\pi}(s_t, a_t)$  State-Action Value Function
- $A^{\pi}(s_t, a_t)$  Advantage Function
- $r_t + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t)$  TD Residual



# Empirically Calculate Gain (REINFORCE)

Simulate episodes and calculate **empirical mean return** instead of **expected return**:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}^{\pi}$$



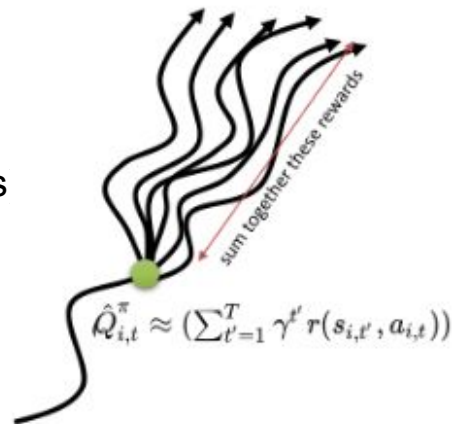
# Empirically Calculate Gain (REINFORCE)

Simulate episodes and calculate **empirical mean return** instead of **expected return**:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}^{\pi}$$

## Problems with Monte Carlo Rollout/ REINFORCE:

- Calculating the Gain term is **slow** (Wait till every episode terminates)
- Empirical mean is unbiased, but has **high variance**



# Empirically Calculate Gain (REINFORCE)

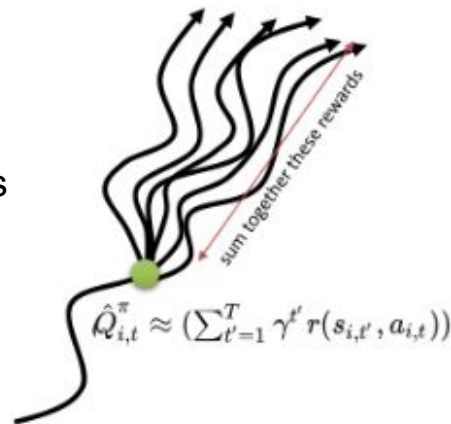
Simulate episodes and calculate **empirical mean return** instead of **expected return**:

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}^{\pi}$$

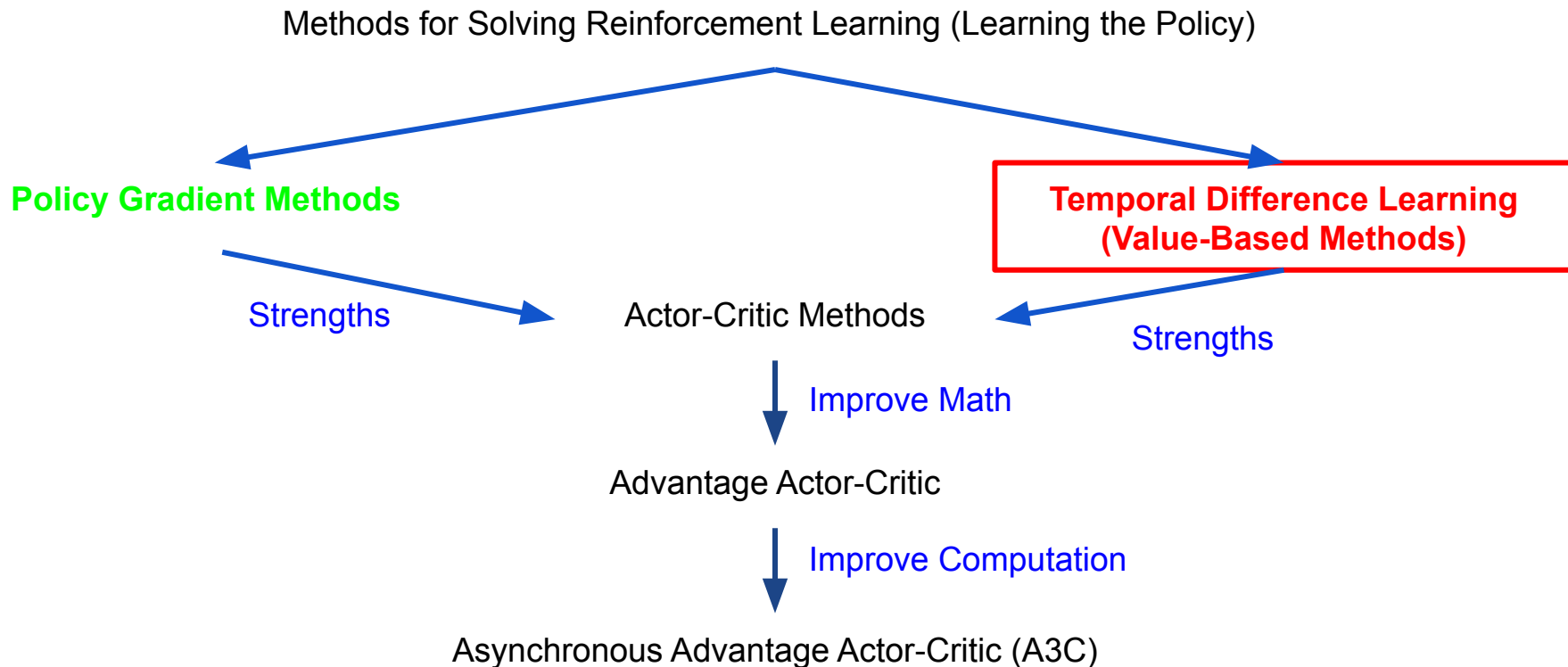
## Problems with Monte Carlo Rollout/ REINFORCE:

- Calculating the Gain term is **slow** (Wait till every episode terminates)
- Empirical mean is unbiased, but has **high variance**

**Any faster way of estimating the Gain term?**



# Roadmap



# Temporal Difference Methods (Recap)

TD Methods give **another way to estimate** the **value function**

$$\delta = \underline{R_{t+1} + \gamma V^\pi s_{t+1}} - V^\pi s_t \quad \bigg| \quad \delta = \underline{R_{t+1} + \gamma Q^\pi s_{t+1}} - Q^\pi s_t$$

*Value Estimate after one step, TD(0)*

# Temporal Difference Methods (Recap)

TD Methods give **another way to estimate the value function**

$$\delta = \underline{R_{t+1} + \gamma V^\pi s_{t+1}} - V^\pi s_t$$

$$\delta = \underline{R_{t+1} + \gamma Q^\pi s_{t+1}} - Q^\pi s_t$$

*Value Estimate after one step, TD(0)*

- Computationally much faster : One-Step vs Complete Episode (Monte Carlo)
- **Low variance**
- Can learn in environments without a final outcome

# Temporal Difference Methods (Recap)

TD Methods give **another way to estimate the value function**

$$\delta = \underline{R_{t+1} + \gamma V^\pi s_{t+1}} - V^\pi s_t \quad \bigg| \quad \delta = \underline{R_{t+1} + \gamma Q^\pi s_{t+1}} - Q^\pi s_t$$

***Value Estimate after one step, TD(0)***

- Computationally much faster : One-Step vs Complete Episode (Monte Carlo)
- **Low variance**
- Can learn in environments without a final outcome

***But we're dealing with very large state and action spaces - what frameworks can we use?***

# Value Function Approximation

$$\delta = R_{t+1} + \gamma V^\pi s_{t+1} - V^\pi s_t$$

$$\delta = R_{t+1} + \gamma Q^\pi s_{t+1} - Q^\pi s_t$$

**New Function Approximator to Estimate the Value Function/ Evaluate Policy**

*Parameters  $w$*





# Value Function Approximation (General Method)

$$C(w) = \mathbb{E}_{\pi} [(V_{\pi}(s) - \hat{V}(s, \mathbf{w}))^2]$$

Target/True Value Function      Approximate Value Function

*Notation Alert: ^ for Value Function Approx.*

# Value Function Approximation (General Method)

$$C(\mathbf{w}) = \mathbb{E}_{\pi} [\underbrace{V_{\pi}(s)}_{\text{Target/True Value Function}} - \underbrace{\hat{V}(s, \mathbf{w})}_{\text{Approximate Value Function}}]^2]$$

*Notation Alert: ^ for Value Function Approx.*

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} C(\mathbf{w}) = \alpha \mathbb{E}_{\pi} [(V_{\pi}(s) - \hat{V}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})]$$

Solve using **Gradient Descent**

# Value Function Approximation (General Method)

$$C(w) = \mathbb{E}_\pi [\underbrace{(V_\pi(s))}_{\text{Target/True Value Function}} - \underbrace{\hat{V}(s, \mathbf{w})}_{\text{Approximate Value Function}})^2]$$

*Notation Alert: ^ for Value Function Approx.*

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} C(\mathbf{w}) = \alpha \mathbb{E}_\pi [(V_\pi(s) - \hat{V}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})] \quad \text{Solve using **Gradient Descent**}$$

$$\Delta \mathbf{w} = \alpha [(V_\pi(s) - \hat{V}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})] \quad \text{Update parameters}$$

# Value Function Approximation (TD(0) Update)

$$C(\mathbf{w}) = \mathbb{E}_{\pi}[(\underbrace{V_{\pi}(s)}_{\text{Target/True Value Function}} - \underbrace{\hat{V}(s, \mathbf{w})}_{\text{Approximate Value Function}})^2]$$

*Notation Alert: ^ for Value Function Approx.*

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} C(\mathbf{w}) = \alpha \mathbb{E}_{\pi}[(V_{\pi}(s) - \hat{V}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})] \quad \text{Solve using **Gradient Descent**}$$

$$\Delta \mathbf{w} = \alpha [(V_{\pi}(s) - \hat{V}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})] \quad \text{Update parameters}$$

**Target Value  $\rightarrow$  Value Function using TD Estimation**

$$\Delta \mathbf{w} = \alpha [R_{t+1} + \gamma \hat{V}(s_{t+1}, \mathbf{w}) - \hat{V}(s, \mathbf{w})] \nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})$$

**Same logic applicable to Q Value Function**

# To Summarize...

## **Value-Based Methods** (e.g. DQN)

### **Strength:**

Fast Estimators of Action  
Quality  
(given state, action)

### **Weaknesses:**

Cannot deal with very  
large/continuous action  
spaces (*argmax over  
actions!*)

## **Policy Gradient Methods** (e.g. REINFORCE)

### **Strength:**

Tackle Large Action  
Spaces

### **Weaknesses:**

Action quality estimation is  
slow (Monte Carlo rollout)

# An Idea - Combine Strengths

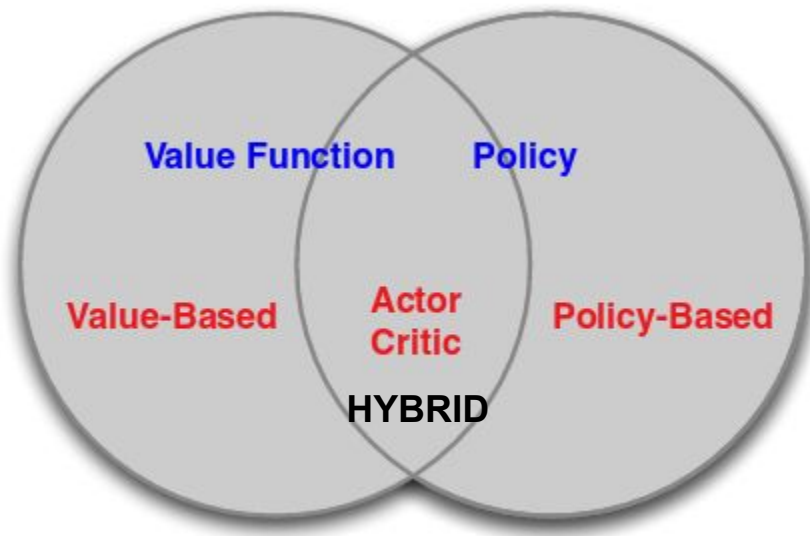
## Value-Based Methods (e.g. DQN)

### Strength:

Fast Estimators of Action  
Quality  
(given state, action)

### Weaknesses:

Cannot deal with very  
large/continuous action  
spaces (*argmax* over  
actions!)



## Policy Gradient Methods (e.g. REINFORCE)

### Strength:

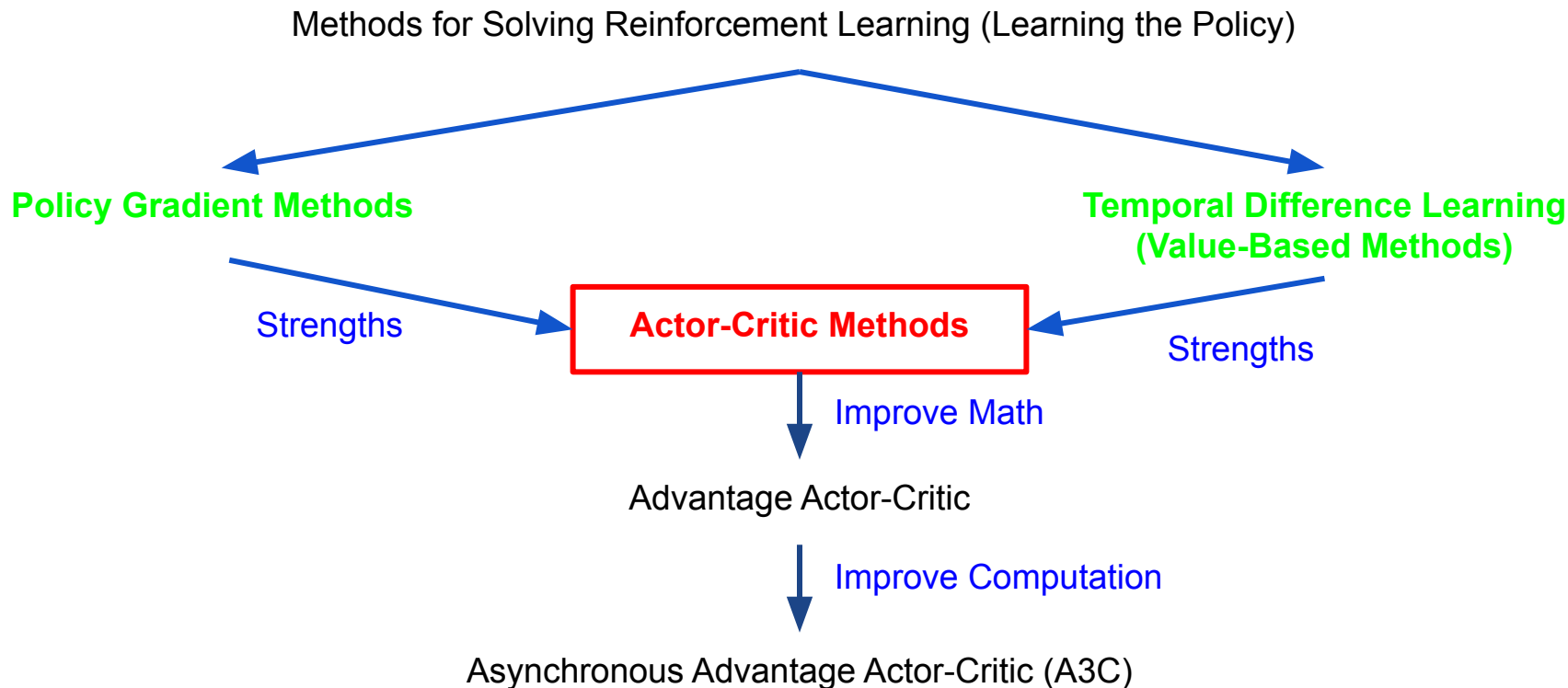
Tackle Large Action  
Spaces

### Weaknesses:

Action quality estimation is  
slow (Monte Carlo rollout)

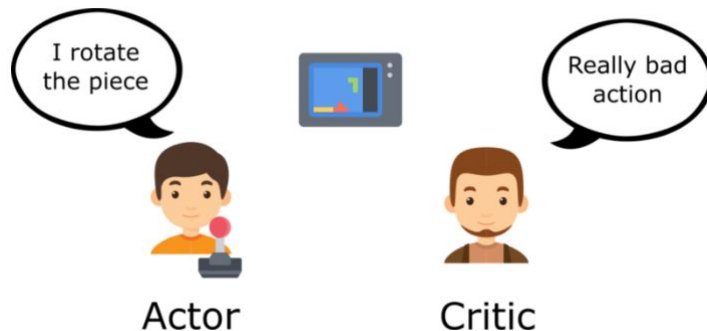
*Can we combine the strengths of both methods?*

# Roadmap



# Actor-Critic Methods: Intuition

**Take Actions Using  
Policy Gradient Methods**  
*State*  $\rightarrow$  *Action*



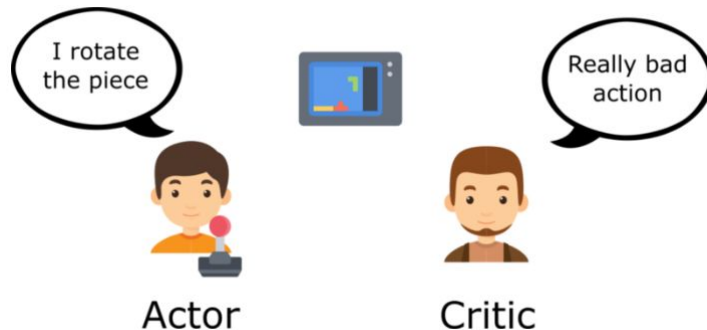
**Evaluate Policy Using TD  
Learning**  
*State*  $\rightarrow$  *Value Function*



# Actor-Critic Methods: Intuition

Take Actions Using  
Policy Gradient Methods  
 $State \rightarrow Action$

Use 'Policy' Function  
Approximator with  
Parameter  $\theta$



Evaluate Policy Using TD  
Learning  
 $State \rightarrow Value Function$

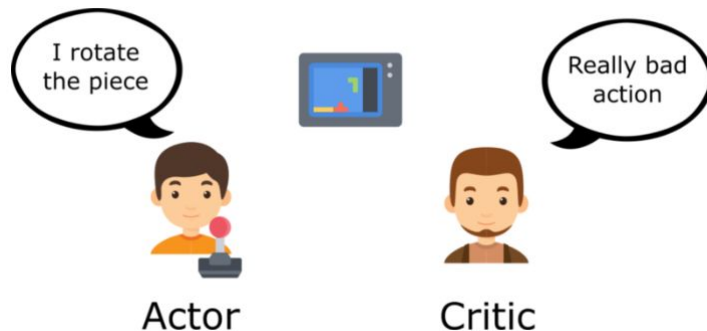
Use 'Value' Function  
Approximator with  
parameters  $w$  to calculate  
Value Function (Q or V)

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

# Actor-Critic Methods: Intuition

Take Actions Using  
Policy Gradient Methods  
 $State \rightarrow Action$

Use 'Policy' Function  
Approximator with  
Parameter  $\theta$



Evaluate Policy Using TD  
Learning  
 $State \rightarrow Value Function$

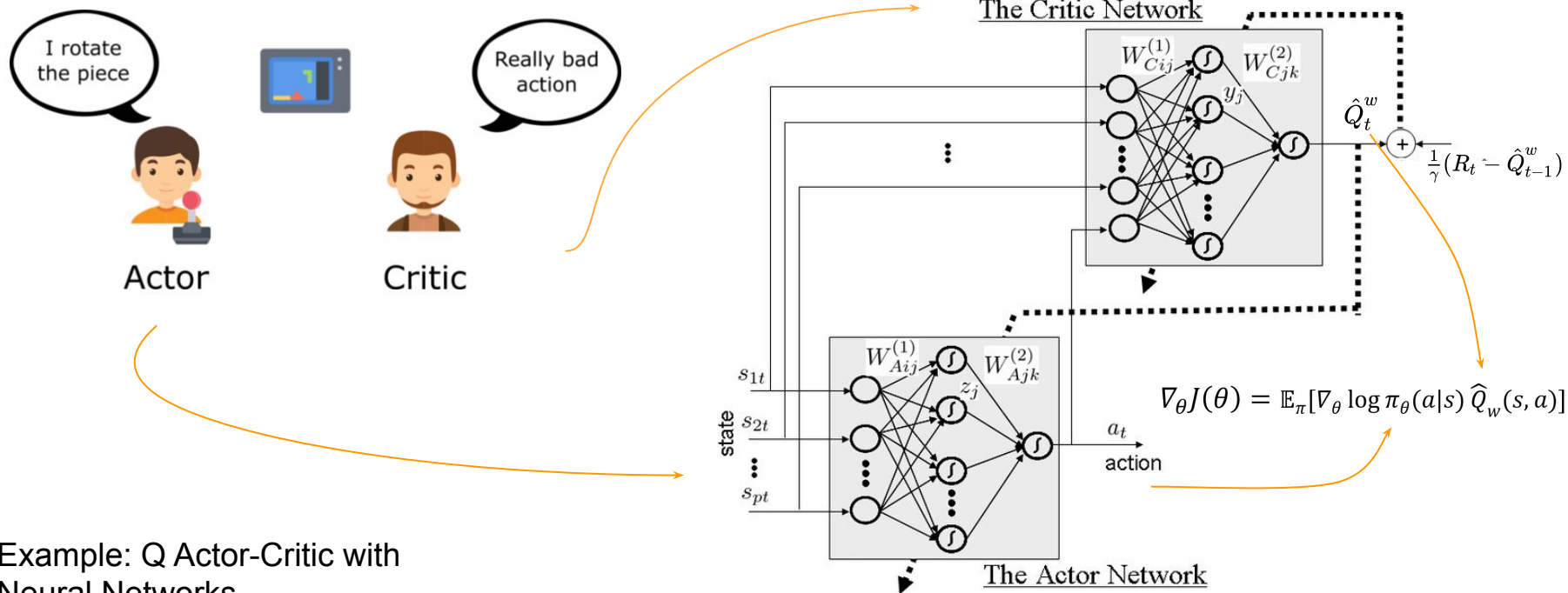
Use 'Value' Function  
Approximator with  
parameters  $w$  to calculate  
Value Function (Q or V)

$$\nabla_{\theta} J(\theta) = E_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

Q Actor-Critic

Advantage Actor-Critic

# Actor-Critic Methods: Visualization



Example: Q Actor-Critic with Neural Networks

# Q Actor Critic

## Actor Update (Gradient Ascent)

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) Q^{\pi_{\theta}}(s, a)]$$

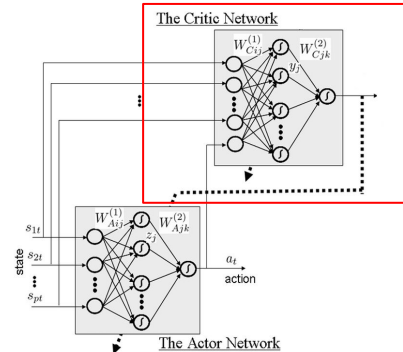
$$Q^{\pi_{\theta}} \approx \hat{Q}_w(s, a)$$

$$\nabla_{\theta} J(\theta) \approx [\nabla_{\theta} \log \pi_{\theta}(s, a) \hat{Q}_w(s, a)]$$

$$\Delta \theta = \alpha \nabla_{\theta} \log \pi_{\theta}(s, a) \hat{Q}_w(s, a)$$

**Replace Gain Term**

**Use Value calculated from Value function approximator**



# Q Actor Critic

## Actor Update (Gradient Ascent)

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) Q^{\pi_{\theta}}(s, a)]$$

$$Q^{\pi_{\theta}} \approx \hat{Q}_w(s, a)$$

$$\nabla_{\theta} J(\theta) \approx [\nabla_{\theta} \log \pi_{\theta}(s, a) \hat{Q}_w(s, a)]$$

$$\Delta \theta = \alpha \nabla_{\theta} \log \pi_{\theta}(s, a) \hat{Q}_w(s, a)$$

## Critic Update (Gradient Descent)

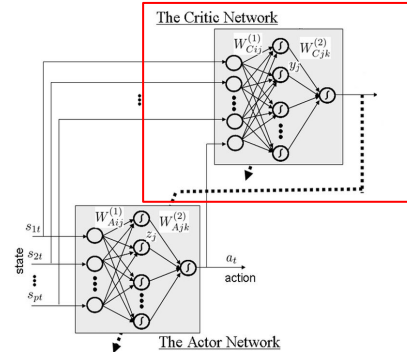
$$\Delta \mathbf{w} = \alpha [\underbrace{R_{t+1} + \gamma \hat{Q}(s', a', \mathbf{w})}_{\text{One step TD error}} - \hat{Q}(s, a, \mathbf{w})] \nabla_{\mathbf{w}} \hat{Q}(s, a, \mathbf{w})$$

One step TD error

Gradient Of Value Function

**Replace Gain Term**

**Use Value calculated from Value function approximator**



# Advantage Actor Critic

## Actor Update (Gradient Ascent)

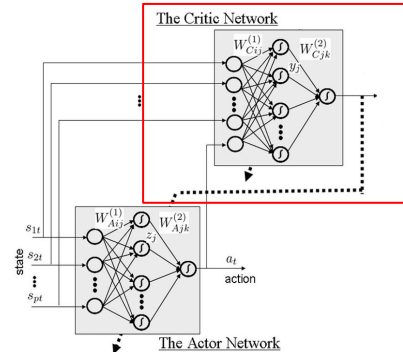
$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) \hat{A}_w(s, a)]$$

## Critic Update (Gradient Descent)

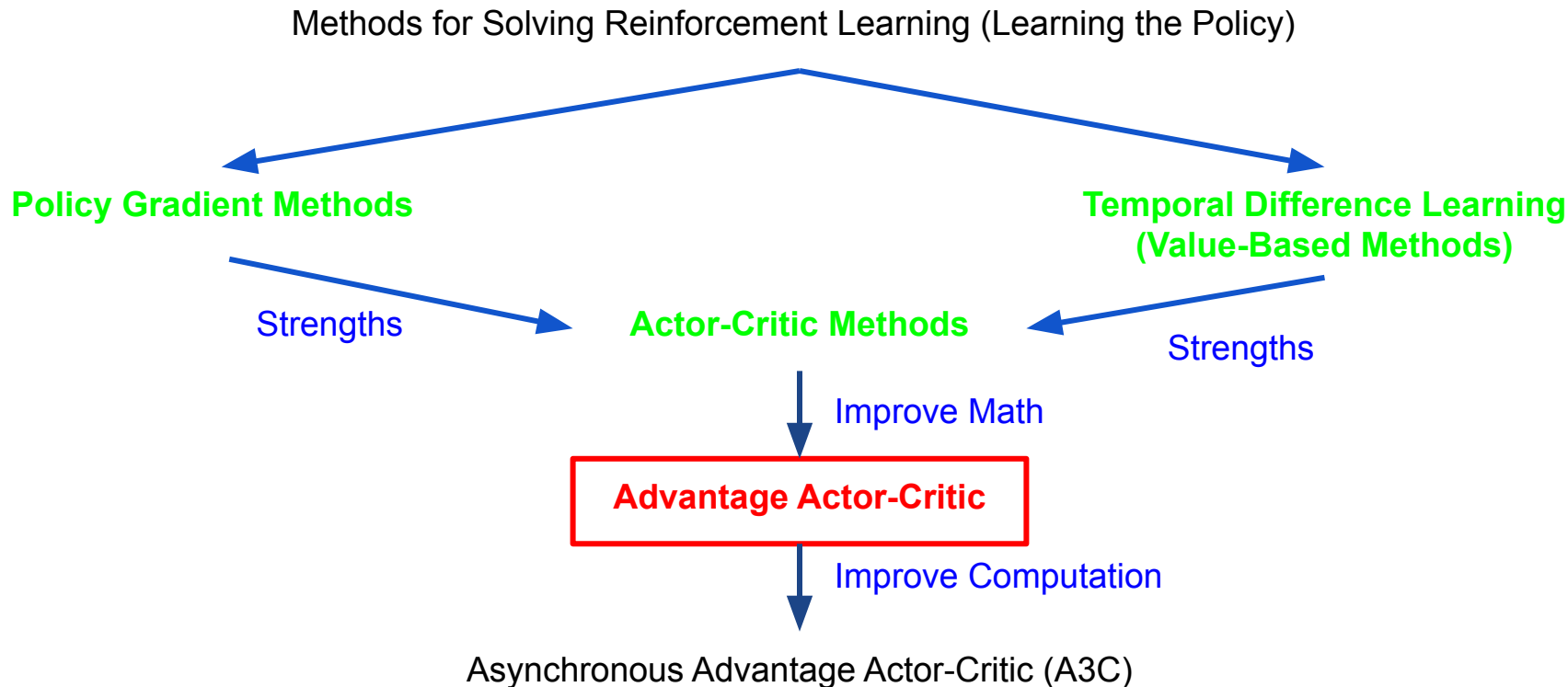
$$\Delta \mathbf{w} = \alpha [\underbrace{R_{t+1} + \gamma \hat{V}(s_{t+1}, \mathbf{w}) - \hat{V}(s, \mathbf{w})}_{\text{One step TD error}} \underbrace{\nabla_{\mathbf{w}} \hat{V}(s, \mathbf{w})}_{\text{Gradient Of Value Function}}]$$

One step TD error

Gradient Of Value Function



# Roadmap



Advantage


$$A(s, a) = Q(s, a) - V(s)$$



# Advantage

$$A(s, a) = Q(s, a) - V(s)$$

Intuitively, this means how much better it is to take a specific action compared to the average, general action at the given state


$$\sum_a \pi(a|s)Q(s, a)$$

# Advantage Actor Critic

Policy Gradient:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Advantage Actor Critic

## Policy Gradient:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) \overset{A_{\pi_{\theta}}(s, a)}{\cancel{Q_{\pi_{\theta}}(s, a)}}]$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) A_{\pi_{\theta}}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Advantage Actor Critic

Why??

Is this a different Policy Gradient??

Policy Gradient:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) \cancel{Q_{\pi_{\theta}}(s, a)}^{A_{\pi_{\theta}}(s, a)}]$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) A_{\pi_{\theta}}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Baseline: Variance reduction

$$\text{Var}(f - \phi) = \text{Var}(f) - 2\underline{\text{Cov}(f, \phi)} + \text{Var}(\phi)$$

Large when  $f, \phi$  strongly correlated  $\longrightarrow \text{Var}(f - \phi) < \text{Var}(f)$

If  $\phi$  and  $f$  are strongly correlated, so that the covariance term on the right hand side is greater than the variance of  $\phi$ , then a variance improvement has been made over the original estimation problem.

This is a generic approach to reducing variance of Monte carlo estimates for integrals

**How does variance reduction help here?**

Better estimates of policy gradient

# Advantage Actor Critic

Why??

Variance reduction

Is this a different Policy Gradient??

Policy Gradient:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) \cancel{Q_{\pi_{\theta}}(s, a)}^{A_{\pi_{\theta}}(s, a)}]$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) A_{\pi_{\theta}}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Reward shaping

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) (Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s))] \\ &= \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)] - \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)]\end{aligned}$$

# Baseline

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) (Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s))] \\ &= \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)] - \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)]\end{aligned}$$

Policy gradient with Q function



# Baseline: Unbiased estimate

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) (Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s))] \\ &= \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)] - \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)]\end{aligned}$$

Policy gradient with Q function


$$\mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)] = 0$$

Unbiased estimate of the policy gradient for any baseline shift  $b(s)$  in reward. Here,  
 $b(s) = V(s)$

# Baseline: Unbiased estimate

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) (Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s))] \\ &= \underbrace{\mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)]}_{\text{Policy gradient with Q function}} - \underbrace{\mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)]}\end{aligned}$$


$$\mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)] = 0$$

Unbiased estimate of the policy gradient for any baseline shift  $b(s)$  in reward. Here,

$$b(s) = V(s)$$

**Proof**

# Baseline: Unbiased estimate

$$\begin{aligned}\nabla_{\theta} J(\theta) &= \mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) (Q_{\pi_{\theta}}(s, a) - V_{\pi_{\theta}}(s))] \\ &= \underbrace{\mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) Q_{\pi_{\theta}}(s, a)]}_{\text{Policy gradient with Q function}} - \underbrace{\mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)]}\end{aligned}$$


$$\mathbb{E}_{\pi}[\nabla_{\theta} \log \pi_{\theta}(a|s) V_{\pi_{\theta}}(s)] = 0$$

Unbiased estimate of the policy gradient for any baseline shift  $b(s)$  in reward. Here,  
 $b(s) = V(s)$

**Proof**

Solve HW4!

# Advantage Actor Critic

Why??

Variance reduction

Is this a different Policy Gradient??

Unbiased estimate of policy gradient

## Policy Gradient:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) \cancel{Q_{\pi_{\theta}}(s, a)}^{A_{\pi_{\theta}}(s, a)}]$$

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi} [\nabla_{\theta} \log \pi_{\theta}(a|s) A_{\pi_{\theta}}(s, a)]$$

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Estimating the Advantage function

For the true **value function**  $V^{\pi_\theta}(s)$  the TD error:

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

is an **unbiased estimate** of the advantage function:

$$\begin{aligned}\mathbb{E}_{\pi_\theta} [\delta^{\pi_\theta} | s, a] &= \mathbb{E}_{\pi_\theta} [r + \gamma V^{\pi_\theta}(s') | s, a] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s) \\ &= A^{\pi_\theta}(s, a)\end{aligned}$$

# Estimating the Advantage function

For the true **value function**  $V^{\pi_\theta}(s)$  the TD error:

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

is an **unbiased estimate** of the advantage function:

$$\begin{aligned}\mathbb{E}_{\pi_\theta} [\delta^{\pi_\theta} | s, a] &= \mathbb{E}_{\pi_\theta} [r + \gamma V^{\pi_\theta}(s') | s, a] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s) \\ &= A^{\pi_\theta}(s, a)\end{aligned}$$

So we can use the TD error to compute the **policy gradient**

$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \delta^{\pi_\theta}]$$

# Estimating the Advantage function

For the true **value function**  $V^{\pi_\theta}(s)$  the TD error:

$$\delta^{\pi_\theta} = r + \gamma V^{\pi_\theta}(s') - V^{\pi_\theta}(s)$$

is an **unbiased estimate** of the advantage function:

$$\begin{aligned}\mathbb{E}_{\pi_\theta} [\delta^{\pi_\theta} | s, a] &= \mathbb{E}_{\pi_\theta} [r + \gamma V^{\pi_\theta}(s') | s, a] - V^{\pi_\theta}(s) \\ &= Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s) \\ &= A^{\pi_\theta}(s, a)\end{aligned}$$

So we can use the TD error to compute the **policy gradient**


$$\nabla_\theta J(\theta) = \mathbb{E}_{\pi_\theta} [\nabla_\theta \log \pi_\theta(s, a) \delta^{\pi_\theta}]$$

In practice we can use an **approximate TD error**


$$\delta^{\pi_\theta} \approx r + \gamma \hat{V}_\phi(s') - \hat{V}_\phi(s)$$

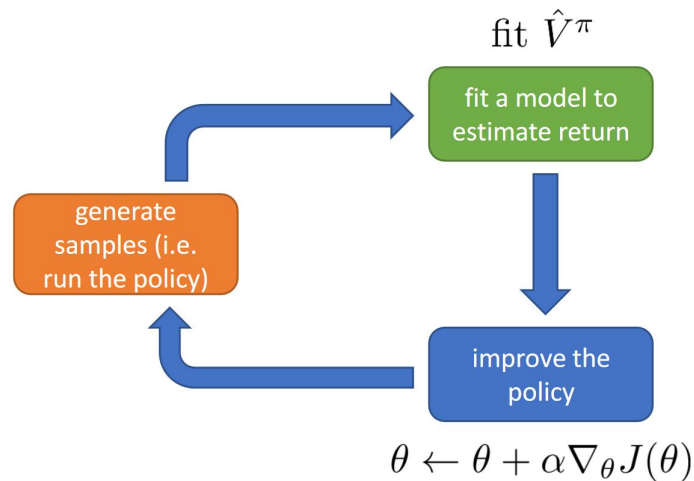
# Advantage Actor-Critic algorithm

batch actor-critic algorithm:

- 
1. sample  $\{\mathbf{s}_i, \mathbf{a}_i\}$  from  $\pi_\theta(\mathbf{a}|\mathbf{s})$  (run it on the robot)
  2. fit  $\hat{V}_\phi^\pi(\mathbf{s})$  to sampled reward sums
  3. evaluate  $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}_\phi^\pi(\mathbf{s}'_i) - \hat{V}_\phi^\pi(\mathbf{s}_i)$
  4.  $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
  5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

online actor-critic algorithm:

- 
1. take action  $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$ , get  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
  2. update  $\hat{V}_\phi^\pi$  using target  $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
  3. evaluate  $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
  4.  $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
  5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$



$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

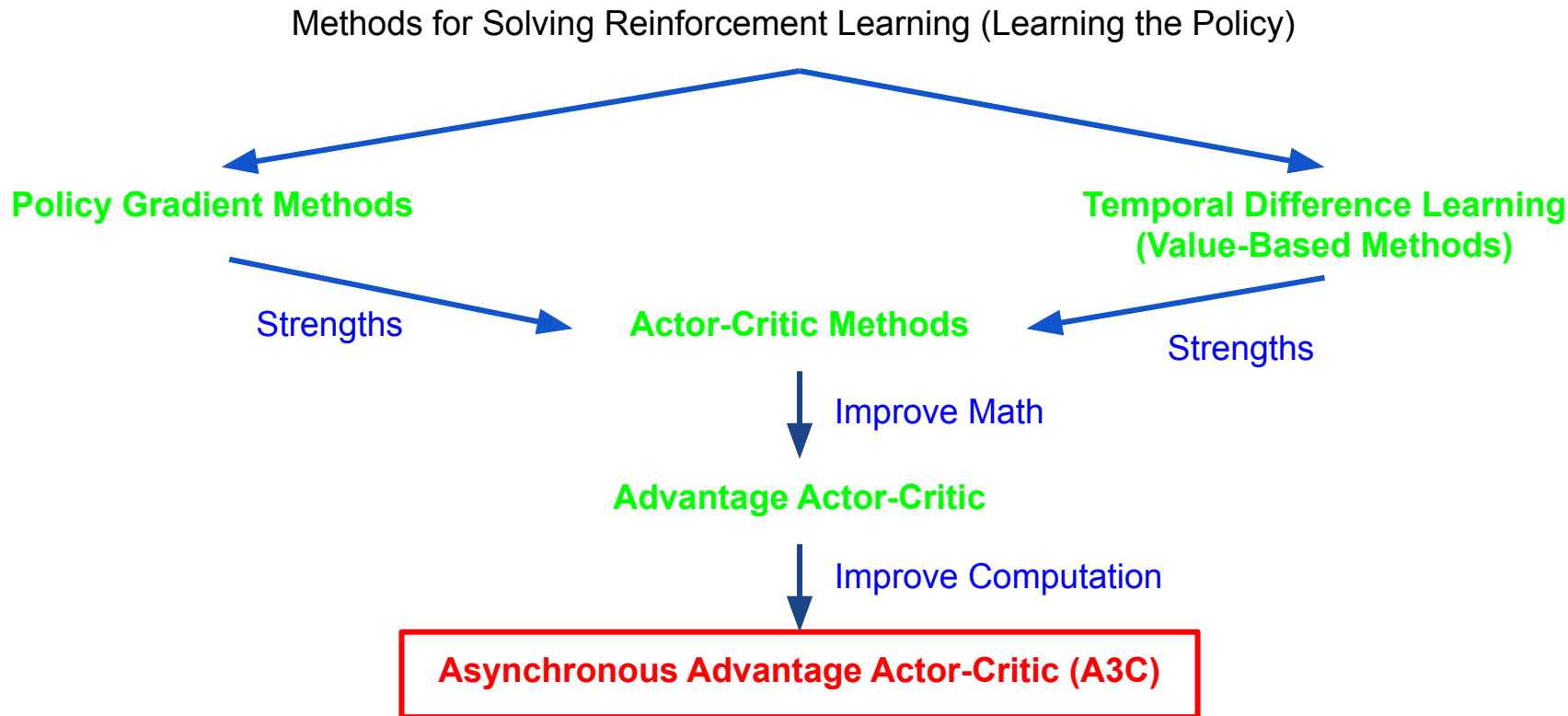


# Recap

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{\pi_{\theta}} [\nabla_{\theta} \log \pi_{\theta}(s, a) R(s, a)]$$

- $\sum_{t'=t}^T r(s_{i,t'}, a_{i,t'})$  Total Reward of Simulated Trajectories (Monte Carlo Rollout)
- $\sum_{t'=t}^T r(s_{i,t'}, a_{i,t'}) - b(s_t)$  Monte Carlo Rollout with Baseline
- $Q^{\pi}(s_t, a_t)$  State-Action Value Function
- $A^{\pi}(s_t, a_t)$  Advantage Function
- $r_t + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t)$  TD Residual

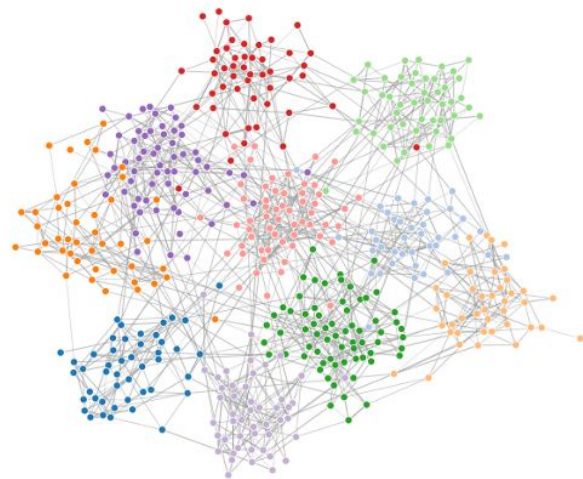
# Roadmap



# **Asynchronous** Advantage Actor-Critic

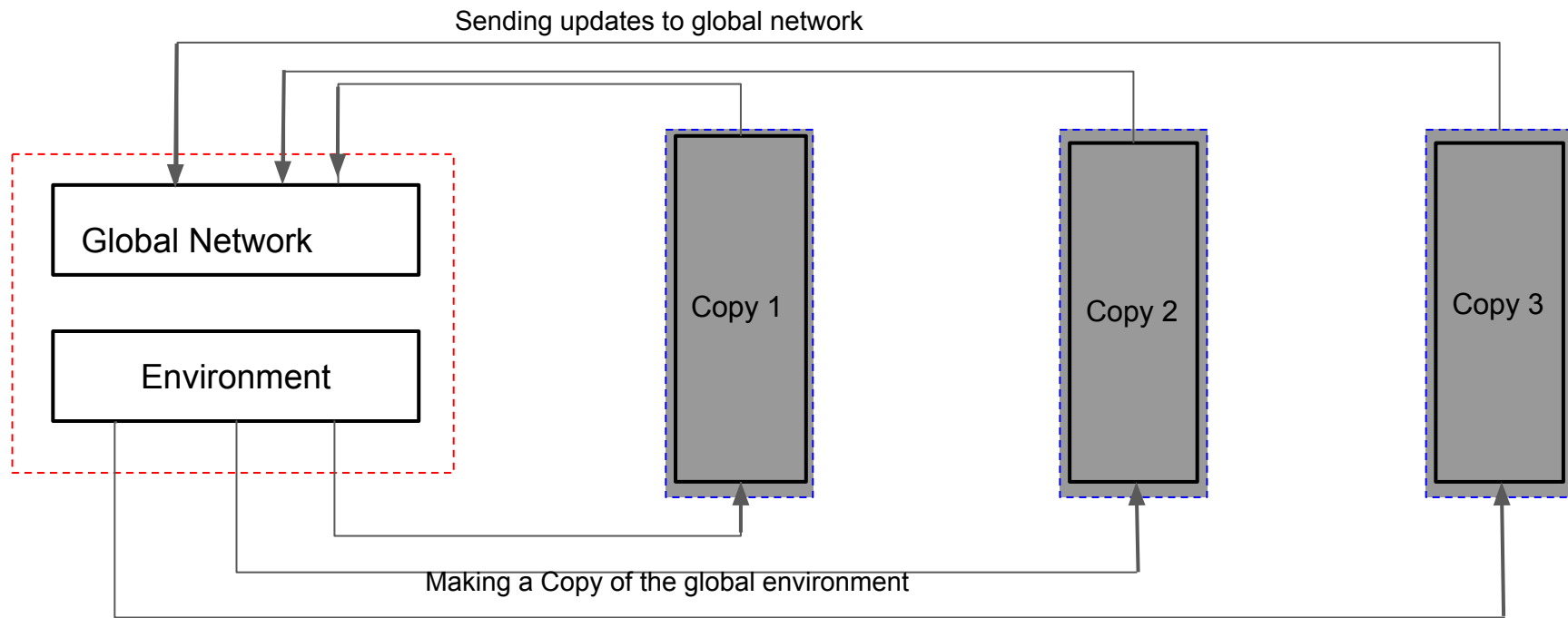
# Why do we need Asynchronous Actor-Critic Methods - Breaking Sequential Correlation

- Learning from strongly correlated sequences (MDP) is inefficient.
- Heavily correlated sequences lead to strong bias in trained parameters.
- Sequential correlation makes learning process unstable



# Asynchronous

- Parallel multi-agents with no coordination between them.



# Asynchronous (No Coordination)

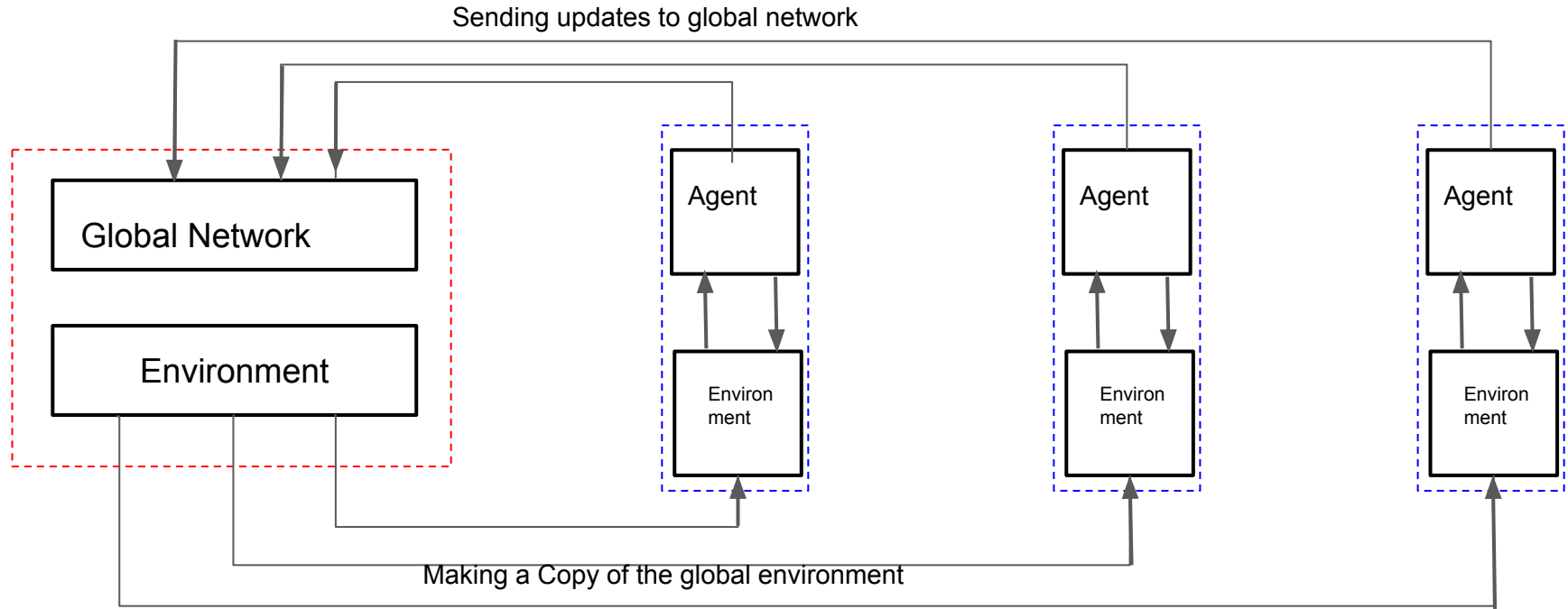
- Motivation:

Find RL algorithms that can train deep neural network policies reliably (breaking sequential correlation) without large resources.

- Asynchronously execute parallel agents, each with its own copy of the environment.
- The gradients are asynchronously sent to global network (central copy of the model).

# Asynchronous

- Parallel multi-agents with no coordination between them.



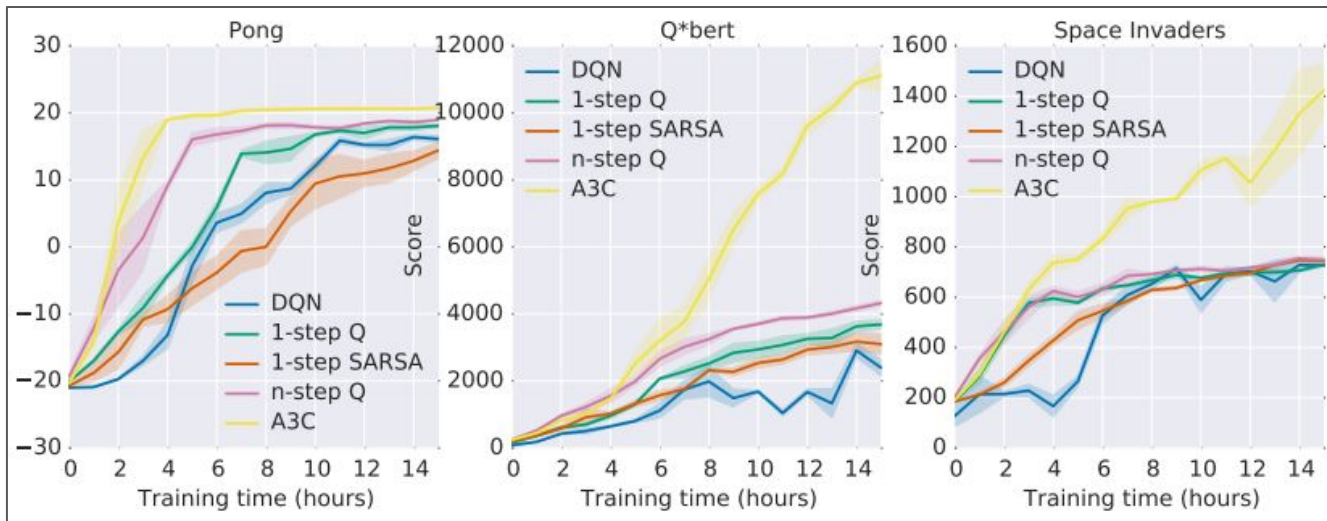
# Advantages of Asynchronous

- Multiple actors applying asynchronous updates is less likely to be correlated than single agent.
- Can be applied to most standard reinforcement algorithms.
- They are able to train variety of networks stably.
- Works for both discrete as well as continuous methods.



# Advantages of Asynchronous

- Multiple actors applying asynchronous updates is less likely to be correlated than single agent.
- Can be applied to most standard reinforcement learning algorithms.
- They are able to train variety of networks stably.
- Works for both discrete as well as continuous methods.



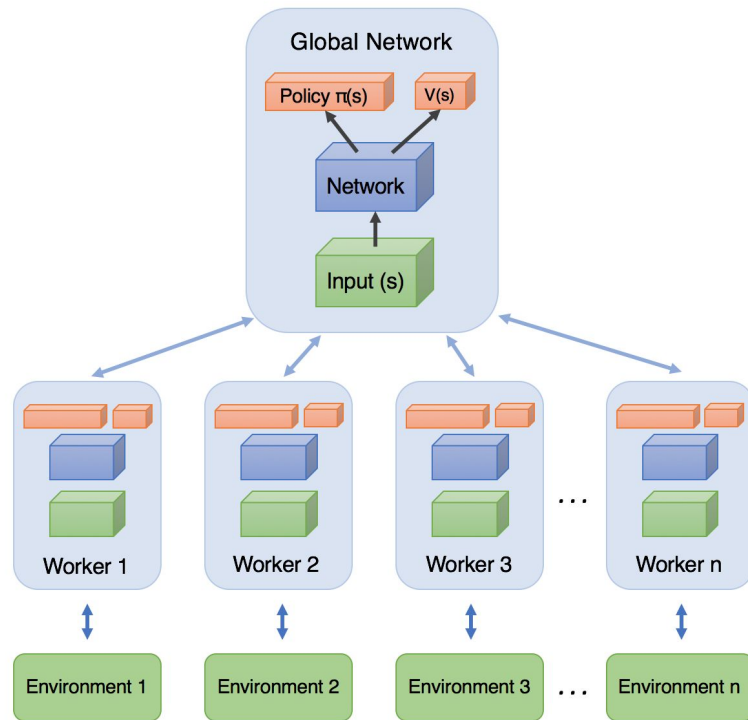
# **Asynchronous Advantage Actor-Critic**

# A3C (Asynchronous Advantage Actor-Critic)

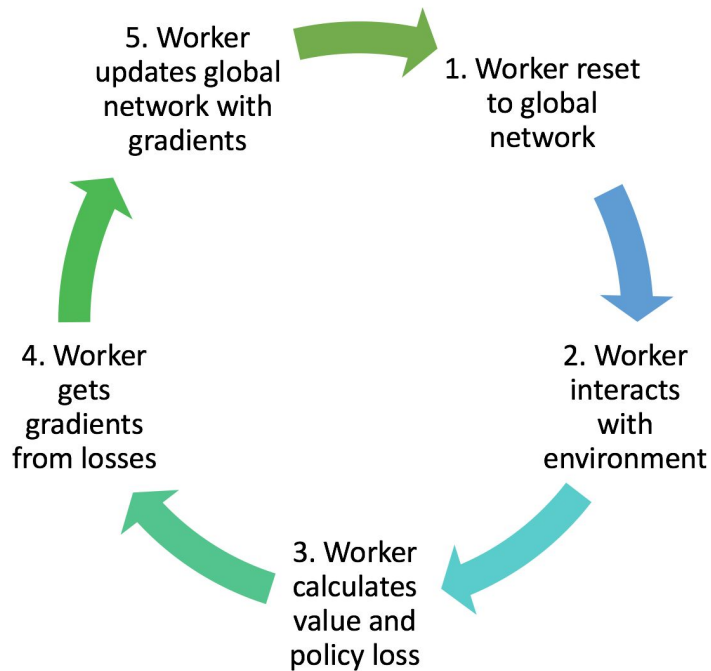
- Multiple actors are trained in **parallel**
- Each agent asynchronously updates the global network.
- Multiple CPU threads for separate actor copies.

# A3C (Asynchronous Advantage Actor-Critic)

- Multiple actors are trained in **parallel**
- Each agent asynchronously updates the global network.
- Multiple CPU threads for separate actor copies.

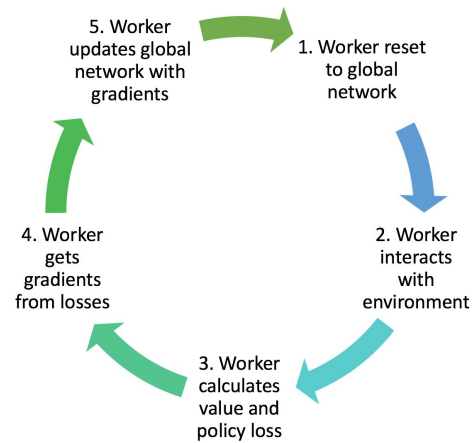


# Algorithm outline



**For Each Agent**

# Algorithm outline



**For Each Agent**

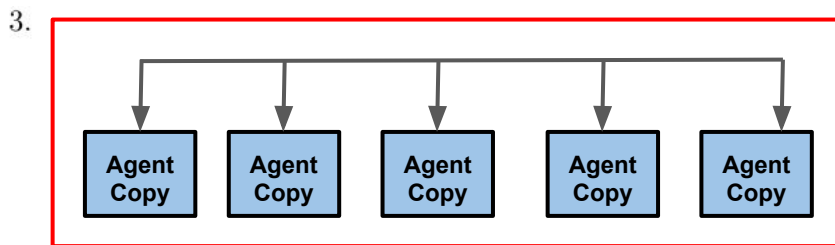
# Algorithm outline

#Assume global shared parameter vectors  $\theta$ .

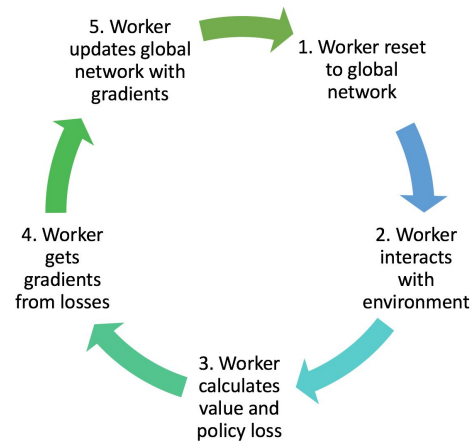
#Assume thread-specific parameter vectors  $\theta'_k$  for each worker agent k.

1. Reset gradients:  $d\theta \leftarrow 0$  and  $d\theta'_k \leftarrow 0$

2. Duplicate global parameters to thread-specific parameters  $\theta'_k = \theta$



4.  $\theta \leftarrow \theta + \alpha \nabla_{\theta'_k} J(\theta'_k)$



**For Each Agent**

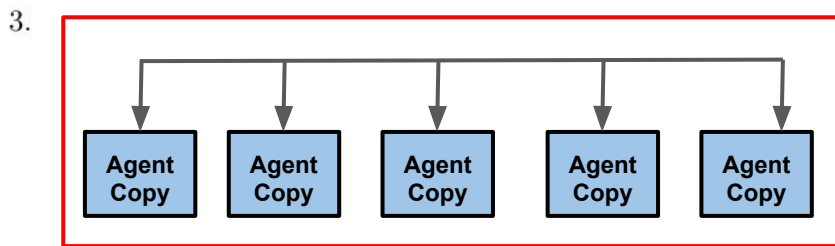
# Algorithm outline

#Assume global shared parameter vectors  $\theta$ .

#Assume thread-specific parameter vectors  $\theta'_k$  for each worker agent k.

1. Reset gradients:  $d\theta \leftarrow 0$  and  $d\theta'_k \leftarrow 0$

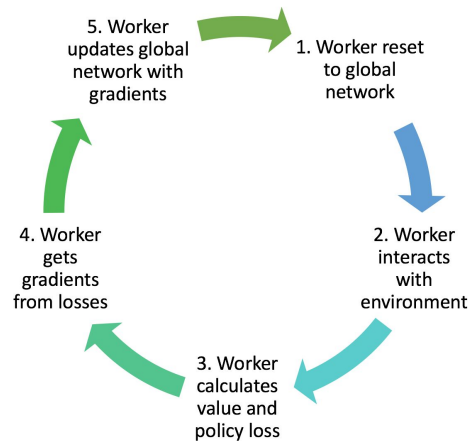
2. Duplicate global parameters to thread-specific parameters  $\theta'_k = \theta$



4.  $\theta \leftarrow \theta + \alpha \nabla_{\theta'_k} J(\theta'_k)$

## Agent Copy

3. (a) sample  $\{s_i, a_i\}$  from  $\pi_{\theta'_k}(a|s)$  (run it on each agent)
- (b) fit  $\hat{V}_{\Phi}^{\pi}(s)$  to sampled reward sums.
- (c) evaluate  $\hat{A}^{\pi}(s_i, a_i) = r(s_i, a_i) + \gamma \hat{V}_{\Phi}^{\pi}(s'_i) - \hat{V}_{\Phi}^{\pi}(s_i)$
- (d)  $\nabla_{\theta'_k} J(\theta'_k) \approx \Sigma_i \nabla_{\theta' + k} \log \pi_{\theta'_k}(a_i | s_i) \hat{A}^{\pi}(s_i, a_i)$



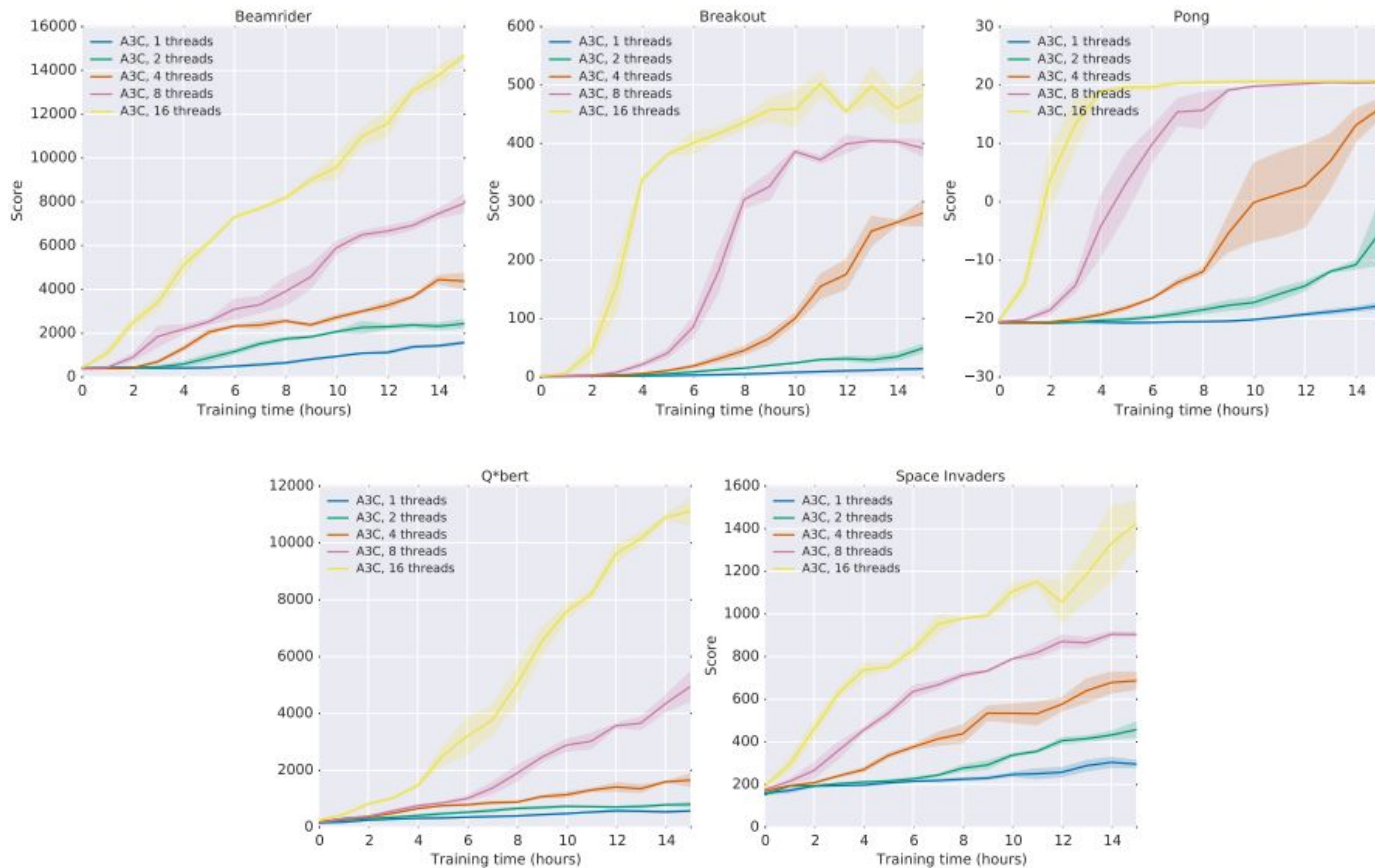
For Each Agent



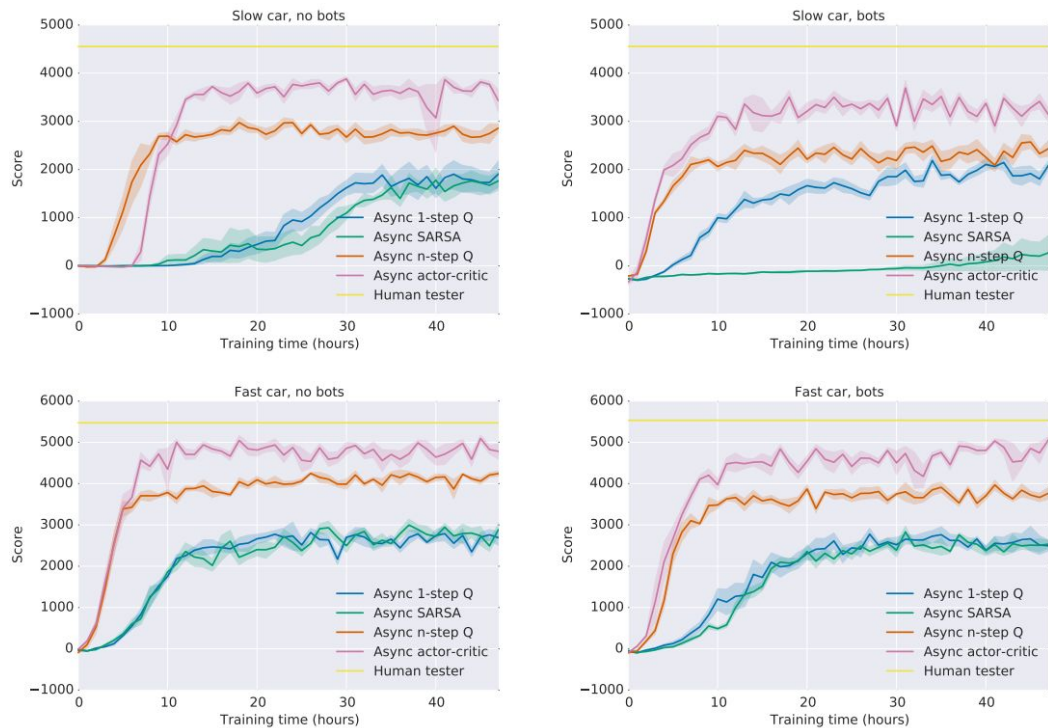
# Advantages

- Stabilizes training (by breaking correlation).
- Using parallel actor-learners to update a shared model has a stabilizing effect.
- Multiple actors running in parallel explore different parts of the environment.
- Different exploration policies in each actor-learner can be explicitly implemented to maximize this diversity.
- Reduced training time (roughly linear in the number of parallel actor-learners).
- No GPU required. Multi-core CPU works.
- Eliminate the need of experience replay.

# Reduced training time-linear in number of parallel actors



# Performance of A3C



## Asynchronous Methods for Deep Reinforcement Learning

Volodymyr Mnih<sup>1</sup>  
 Adrià Puigdomènech Badia<sup>1</sup>  
 Mehdi Mirza<sup>1,2</sup>  
 Alex Graves<sup>1</sup>  
 Tim Harley<sup>1</sup>  
 Timothy P. Lillicrap<sup>1</sup>  
 David Silver<sup>1</sup>

Koray Kavukcuoglu<sup>1</sup>

<sup>1</sup> Google DeepMind

<sup>2</sup> Montreal Institute for Learning Algorithms (MILA), University of Montreal

VMNIH@GOOGLE.COM  
 ADRIAP@GOOGLE.COM  
 MIRZAMOM@IRO.UMONTREAL.CA  
 GRAVESA@GOOGLE.COM  
 THARLEY@GOOGLE.COM  
 COUNTZERO@GOOGLE.COM  
 DAVIDSILVER@GOOGLE.COM  
 KORAYK@GOOGLE.COM

## Abstract

We propose a conceptually simple and lightweight framework for deep reinforcement learning that uses asynchronous gradient descent for optimization of deep neural network controllers. We present asynchronous variants of four standard reinforcement learning algorithms and show that parallel actor-learners have a stabilizing effect on training allowing all four methods to successfully train neural network controllers. The best performing method, an asynchronous variant of actor-critic, surpasses the current state-of-the-art on the Atari domain while training for half the time on a single multi-core CPU instead of a GPU. Furthermore, we show that asynchronous actor-critic succeeds on a wide variety of continuous motor control problems as well as on a new task of navigating random 3D mazes using a visual input.

## 1. Introduction

Deep neural networks provide rich representations that can enable reinforcement learning (RL) algorithms to perform effectively. However, it was previously thought that the combination of simple online RL algorithms with deep neural networks was fundamentally unstable. Instead, a variety of solutions have been proposed to stabilize the algorithm (Riedmiller, 2005; Mnih et al., 2013; 2015; Van Hasselt et al., 2015; Schulman et al., 2015a). These approaches share a common idea: the sequence of observed data encountered by an online RL agent is non-stationary, and on-

line RL updates are strongly correlated. By storing the agent's data in an experience replay memory, the data can be batched (Riedmiller, 2005; Schulman et al., 2015a) or randomly sampled (Mnih et al., 2013; 2015; Van Hasselt et al., 2015) from different time-steps. Aggregating over memory in this way reduces non-stationarity and decorrelates updates, but at the same time limits the methods to off-policy reinforcement learning algorithms.

Deep RL algorithms based on experience replay have achieved unprecedented success in challenging domains such as Atari 2600. However, experience replay has several drawbacks: it uses more memory and computation per real interaction; and it requires off-policy learning algorithms that can update from data generated by an older policy.

In this paper we provide a very different paradigm for deep reinforcement learning. Instead of experience replay, we asynchronously execute multiple agents in parallel, on multiple instances of the environment. This parallelism also decorrelates the agents' data into a more stationary process, since at any given time-step the parallel agents will be experiencing a variety of different states. This simple idea enables a much larger spectrum of fundamental on-policy RL algorithms, such as Sarsa, n-step methods, and actor-critic methods, as well as off-policy RL algorithms such as Q-learning, to be applied robustly and effectively using deep neural networks.

Our parallel reinforcement learning paradigm also offers practical benefits. Whereas previous approaches to deep reinforcement learning rely heavily on specialized hardware such as GPUs (Mnih et al., 2015; Van Hasselt et al., 2015; Schaul et al., 2015) or massively distributed architectures (Nair et al., 2015), our experiments run on a single machine with a standard multi-core CPU. When applied to a variety of Atari 2600 domains, on many games asynchronous reinforcement learning achieves better results, in far less



Volodymyr Mnih<sup>1</sup>  
 Adrià Puigdomènech Badia<sup>1</sup>  
 Mehdi Mirza<sup>1,2</sup>  
 Alex Graves<sup>1</sup>  
 Tim Harley<sup>1</sup>  
 Timothy P. Lillicrap<sup>1</sup>  
 David Silver<sup>1</sup>

Koray Kavukcuoglu<sup>1</sup>

<sup>1</sup> Google DeepMind

<sup>2</sup> Montreal Institute for Learning Algorithms (MILA), University of Montreal

VMNIH@GOOGLE.COM  
 ADRIAP@GOOGLE.COM  
 MIRZAMOM@IRO.UMONTREAL.CA  
 GRAVES@GOOGLE.COM  
 THARLEY@GOOGLE.COM  
 COUNTZERO@GOOGLE.COM  
 DAVIDSILVER@GOOGLE.COM  
 KORAYK@GOOGLE.COM

### Abstract

We propose a conceptually simple and lightweight framework for deep reinforcement learning that uses asynchronous gradient descent for optimization of deep neural network controllers. We present asynchronous variants of four standard reinforcement learning algorithms and show that parallel actor-learners have a stabilizing effect on training allowing all four methods to successfully train neural network controllers. The best performing method, an asynchronous variant of actor-critic, surpasses the current state-of-the-art on the Atari domain while training for half the time on a single multi-core CPU instead of a GPU. Furthermore, we show that asynchronous actor-critic succeeds on a wide variety of continuous motor control problems as well as on a new task of navigating random 3D mazes using a visual input.

### 1. Introduction

Deep neural networks provide rich representations that can enable reinforcement learning (RL) algorithms to perform effectively. However, it was previously thought that the combination of simple online RL algorithms with deep neural networks was fundamentally unstable. Instead, a variety of solutions have been proposed to stabilize the algorithm (Riedmiller, 2005; Mnih et al., 2013; 2015; Van Hasselt et al., 2015; Schulman et al., 2015a). These approaches share a common idea: the sequence of observed data encountered by an online RL agent is non-stationary, and on-

line RL updates are strongly correlated. By storing the agent's data in an experience replay memory, the data can be batched (Riedmiller, 2005; Schulman et al., 2015a) or randomly sampled (Mnih et al., 2013; 2015; Van Hasselt et al., 2015) from different time-steps. Aggregating over memory in this way reduces non-stationarity and decorrelates updates, but at the same time limits the methods to off-policy reinforcement learning algorithms.

Deep RL algorithms based on experience replay have achieved unprecedented success in challenging domains such as Atari 2600. However, experience replay has several drawbacks: it uses more memory and computation per real interaction; and it requires off-policy learning algorithms that can update from data generated by an older policy.

In this paper we provide a very different paradigm for deep reinforcement learning. Instead of experience replay, we asynchronously execute multiple agents in parallel, on multiple instances of the environment. This parallelism also decorrelates the agents' data into a more stationary process, since at any given time-step the parallel agents will be experiencing a variety of different states. This simple idea enables a much larger spectrum of fundamental on-policy RL algorithms, such as Sarsa, n-step methods, and actor-critic methods, as well as off-policy RL algorithms such as Q-learning, to be applied robustly and effectively using deep neural networks.

Our parallel reinforcement learning paradigm also offers practical benefits. Whereas previous approaches to deep reinforcement learning rely heavily on specialized hardware such as GPUs (Mnih et al., 2015; Van Hasselt et al., 2015; Schaul et al., 2015) or massively distributed architectures (Nair et al., 2015), our experiments run on a single machine with a standard multi-core CPU. When applied to a variety of Atari 2600 domains, on many games asynchronous reinforcement learning achieves better results, in far less

## 6. Conclusions and Discussion

We have presented asynchronous versions of four standard reinforcement learning algorithms and showed that they are able to train neural network controllers on a variety of domains in a stable manner. Our results show that in our proposed framework stable training of neural networks through reinforcement learning is possible with both value-based and policy-based methods, off-policy as well as on-policy methods, and in discrete as well as continuous domains. When trained on the Atari domain using 16 CPU cores, the proposed asynchronous algorithms train faster than DQN trained on an Nvidia K40 GPU, with A3C surpassing the current state-of-the-art in half the training time.

One of our main findings is that using parallel actor-learners to update a shared model had a stabilizing effect on the learning process of the three value-based methods we considered. While this shows that stable online Q-learning is possible without experience replay, which was used for this purpose in DQN, it does not mean that experience replay is not useful. Incorporating experience replay into the asynchronous reinforcement learning framework could

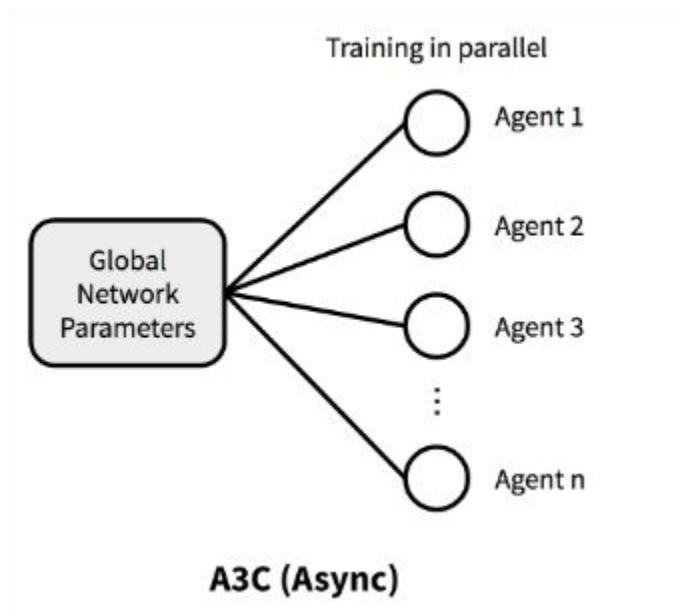
# Limitations of A3C

- Since updates are asynchronous, every update may not be necessarily an optimal update towards convergence.

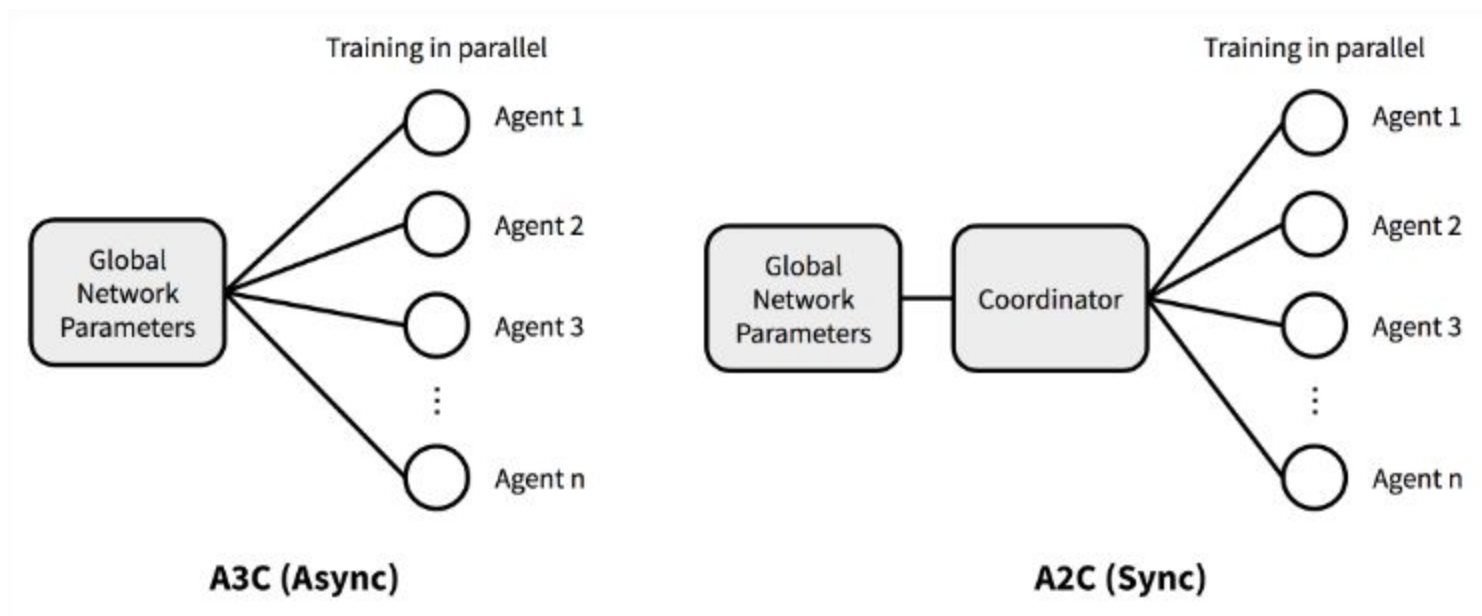
[Each agent updates the global network while other agents are still working with the outdated copy of environment]

- The issue of choosing right features for the actor and critic network remains a challenge.

# A3C vs A2C



# A3C vs A2C





# References

- Mnih, V., Badia, A. P., Mirza, M., Graves, A., Lillicrap, T. P., Harley, T., ... Kavukcuoglu, K. (2016). Asynchronous Methods for Deep Reinforcement Learning. Retrieved from <http://arxiv.org/abs/1602.0178>
- Actor-Critic Algorithms, CS 294-112: Deep Reinforcement Learning Sergey Levine. Retrieve from [http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture\\_5\\_actor\\_critic\\_pdf.pdf](http://rail.eecs.berkeley.edu/deeprlcourse-fa17/f17docs/lecture_5_actor_critic_pdf.pdf)
- Lecture 7: Policy Gradient, David Silver [http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching\\_files/pg.pdf](http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching_files/pg.pdf)
- Lecture 6: Value Function Approximation, David Silver [http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching\\_files/FA.pdf](http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching_files/FA.pdf)
- Policy Gradient Algorithms, Lilian Weng <https://lilianweng.github.io/lil-log/2018/04/08/policy-gradient-algorithms.html>
- Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction; 2nd Edition. 2017.
- An Introduction to Policy Gradients with Doom <https://medium.freecodecamp.org/an-introduction-to-policy-gradients-with-cartpole-and-doom-495b5ef2207f>
- 10703 (Spring 2018): Deep RL and Control <http://www.cs.cmu.edu/~rsalakhu/10703/>