UNIVERSITY OF LONDON

# Probability and Statistics: To $p$, or not to $p$?

## Module Leader: Dr James Abdey

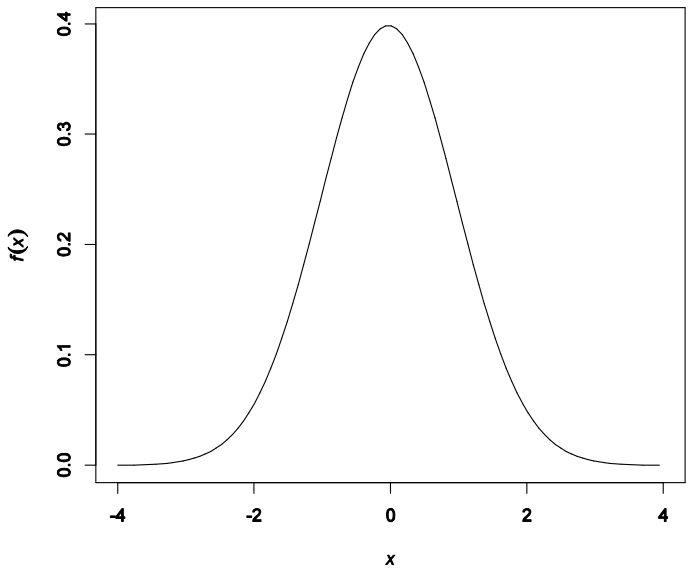
## 1.5 Safe to assume? Beware, when model assumptions go wrong!

We have defined a model to be a deliberate simplification of reality. To assist with the process of model building, we often make **assumptions** – usually *simplifying* assumptions.

Returning to the (geographically-accurate) London Underground map, the Circle line (in yellow) is not a perfect geometric circle, but here it is reasonable to assume the line behaves like a circle as it does go round in a loop. So adopting the name 'Circle line' assumes its path closely approximates a circle. I do not think anyone would seriously suggest the name 'Circle line' is inappropriate!

Moving to statistical models, we often make **distributional assumptions**, i.e. we assume a particular probability distribution (a concept introduced next week) for a particular variable.

In due course we will meet the **normal distribution**, the familiar bell-shaped curve:



The normal distribution is frequently-used in models. One example is that financial returns on assets are often assumed to be normally distributed.

Under this assumption of normality, the probability of returns being within three standard deviations of the mean (mean and standard deviation will be introduced in weeks 2 and 3) is approximately 99.7%. This means that the probability of returns being *more* than three standard deviations from the mean is approximately 0.3%. (In the graph above, the mean is 0 and the standard deviation is 1, so 'mean $\pm$ 3 standard deviations' equates to the interval $[-3, 3]$. This means that 99.7% of the total area under the curve is between $-3$ and 3.)

Assuming market returns follow a normal distribution is fundamental to many models in finance, for example Markowitz's modern portfolio theory and the Black–Scholes–Merton option pricing model. However, this assumption does *not* typically reflect actual observed market returns and 'tail events', i.e. **black swan events** (which recall are low-probability, high-impact events), tend to occur more frequently than a normal distribution would predict!

For now, the moral of the story is to beware assumptions – if you make a wrong or invalid assumption, then decisions you make in good faith may lead to outcomes far from what you expected. As an example, the subprime mortgage market in the United States assumed house prices would only ever increase... but what goes up usually comes down at some point!