COLLATE

Digital Communcation

Notes

Unit 3

Syllabus

UNIT – III

Analysis of digital receiver, Prediction Filter, Design and Property of Matched filter, Correlator Receiver,
Orthogonal Signal, Gram-Schmidt Orthogonalization Procedure, Maximum likelihood receiver, Coherent receiver
design, Inter Symbol Interference, Eye Pattern.

of all
sampling

(1)

UNIT III

Analysis of Digital Receivers

1→ Earlier :

(1) analog w/fs ⟶ digital data ⟶ electrical w/fs or symbols ⟶ (2)

(2) detection of symbols and recovery of digital data from received w/f.

⇓ Problem in recovering.

NOISE

2→ NOISE (unwanted electrical sig.)

Man-made
- → Switching Transients
- → Other radiating EM signals

Natural
- → Electrical signals in electrical ckt and component noise.
- → atmospheric disturbances

↓

can be eliminated through filtering and shielding.

3→ Noise that cannot be eliminated
→ Thermal or Johnson noise :

↓

caused by random motion of $e^{\ominus}$s in all components eg: resistors, wires, etc

↓

Thermal noise = zero mean Gaussian random process

→ in a digital tx^n sys, one of M possible wfts are txed.

$$s_i(t) \in \{ s_1(t), s_2(t), \ldots, s_M(t) \} \text{ for } 0 \le t \le T$$

At the Rx:

$r(t) = s_i(t) + \underline{n(t)}$ — causes a corruption in the original signal.

$r(t)$ → received sig.
$s_i(t)$ → original txed sig
$n(t)$ → noise (AWGN)

A ⇒ Additive (adds 'noise' onto the signal)
W ⇒ White (the noise has a flat spectrum $\bar{c}$ PSD. = $\frac{N_0}{2}$ (W/Hz) from $-\infty$ to ∞ Hz)
G ⇒ Gaussian (amplitude of the noise voltage fluctuations follows a Gaussian distribution)

→ Now, Noise ⇒ Random process

∴ it can be described in terms of :-
→ Mean (or avg.) value
→ Variance (or std. deviation)
→ pdf (prob. density fun^n)

→ Now, noise is assumed to be AWGN, so its pdf is :

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2}$$

$n(t)$ has mean $= 0$
& variance $= \sigma^2$

→ The expression for pdf shows that the noise voltage amplitudes are distributed according to Gaussian distribution.

→ The most probable amplitudes of noise are those $\bar{c}$ small +ve or -ve values.

→ Since, $n(t)$ → randomly varying quantity,
So, $r(t)$ → will also be randomly varying quantity.

∴ there will be uncertainity in finding the value of $r(t)$ ⇒ lead to error in recovery of digital data.

→ The fun' of a digital Rx is to find ~~which~~ which (and not "what") of the signalling w/fs (or symbols) was txed in any given signalling interval.

* Analog Rx → what signal was txed
* Digital Rx → which " "

→ In a digital Rx,
set of txed signals $\{s_1(t), s_2(t), \ldots s_M(t)\}$ is known a priori.

→ The Rx knows what to expect & it has to find out $\underline{c}$ sig. ~~was~~ is being txed in any given time interval.

→ Digi. Rx has superior noise performance over analog Rx.

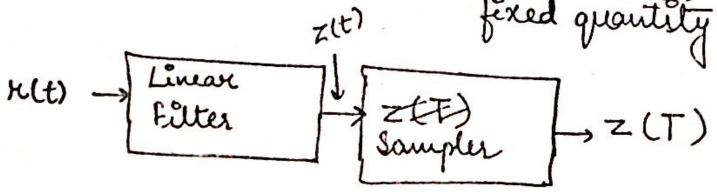
⇒ **How to detect a Digital Signal ?**

2 basic steps :

Step 1 :

reduce $r(t) \xrightarrow{\text{to}} z(T)$

(Test statistic)
fixed quantity



Step 2 : Compare $z(T)$ $\bar{c}$ a reference value or threshold γ

(This will find out which sig. was txed)

$$z(T) \underset{s_2}{\overset{s_1}{\gtrless}} \gamma$$

if $z(T) > \gamma$, ⇒ $s_1(t)$ is considered to have been txed

& if $z(T) < \gamma$, ⇒ $s_2(t)$ - - - - - " - - -

* Sometimes, due to noise, we can have errors i.e wrong estimation of signal.

Since, $n(t)$ = rand var ⇒ $r(t)$ & hence $z(t)$ or $z(T)$ is also random

∴ $z(T)$ can be considered by its mean or variance or pdf's.

→ The analysis of digit. Rx begins $\bar{c}$ the concept of matched filter.

⇒ MATCHED FILTER (Design and Property)

→ It is a linear time-invariant (LTI) filter that leads to the optimum detection of a signal w/f that is immersed in AWGN.

* optimum = min. prob. of an error occuring

→ Here, we match the impulse response of the filter (H(f) or h(t)) to the sig. w/f ($s_i(t)$).

→ The filter is designed to detect the presence of signal $s_i(t)$ (∵ is buried in the noisy rxed signal $r(t)$).

→ * More precisely; Matched filter is designed to maximise the SNR at the filter o/p, for a given w/f $s_i(t)$ at sampling instant $t = T$.

→ $r(t)$ is filter i/p

$$r(t) = \underbrace{s_i(t)}_{\text{(original signal)}} + \underbrace{n(t)}_{\text{(noise)}}$$

→ Since, the filter is linear

⇒ the o/p at $t = T$ is :-

$$z(T) = a_i + n_0$$

a_i ↓ (signal component) ↓ Deterministic

n_0 ↓ (noise component) ↓ random

→ Now, the o/p Noise power (variance or avg. power)

$= \sigma^2$.

→ SNR of matched filter at sampling time instant t=T is :-

$$\left(\frac{S}{N}\right)_{t=T} = \frac{a_i^2}{\sigma^2}$$

Aim : we have to find the optimum filter transfer funⁿ $H_0(f)$ that maximises $(S/N)_{t=T}$. This will minimise the probability of making an incorrect decision i.e to minimise the prob. of error.

→ Expⁿ for SNR can be re-written as :-

$$\left(\frac{S}{N}\right)_{t=T} \leq \frac{2E}{N_0}$$

$\left(\frac{S}{N}\right)$ dep. on → sig. energy 'E'

and

→ Noise power spectral Density ($N_0/2$)

→ Now, max. value of ~~A~~ (S/N) is at :

$$\boxed{\frac{S}{N} = \frac{2E}{N_0}}$$

Correspondingly, the max. value of $H(f)$, denoted $H_{opt}(f)$ occurs when :

impulse response in freq. domain

$$\boxed{H_{opt}(f) = k\, S_i^*(f)\, e^{-j2\pi fT}}$$

k → an arbitrary constt.

* → denotes complex conjugate

$S_i(f)$ → Fourier Transform of sig. w/f $s_i(t)$.

→ In time domain, the impulse response of the optimum filter $h_{opt}(t)$ is given by inverse FT of $H_{opt}(f)$ i.e

$$h_{opt}(t) = \mathcal{F}^{-1}\{H(f)\} \quad (\text{IFT or } \mathcal{F}^{-1})$$

$$= \mathcal{F}^{-1}\{k S_i^*(f) e^{-j2\pi fT}\}$$

$$= k s_i(T-t)$$

→ So, the impulse response of the optimum filter is a time-inversed and delayed version of the i/p sig $s_i(t)$.

→ In other words, $h_{opt}(t)$ is "matched" to the i/p signal $s_i(t)$.

→ An LTI sys defined in this way is termed as a matched filter.

→ Use of this filter will result in the reception and detection of digital signals $\bar{c}$ smallest prob. of error.

Property / Summary:

1) A filter matched to an i/p sig $s_i(t)$ of duration T is char. by an impulse resp that is time reversed and time delayed

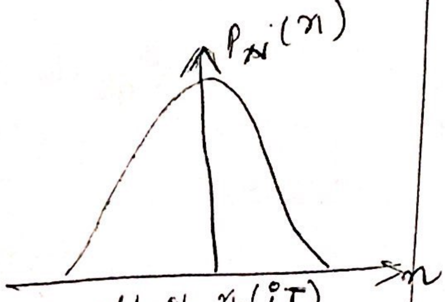
$$\boxed{h_{opt}(t) = k s_i(T-t)}$$

2) In freq domain, the matched filt. is char. by a freq resp. i.e a complex conjugate of the FT of i/p sig $s_i(t)$

$$\boxed{H_{opt}(f) = k S_i^*(f) e^{-j2\pi fT}}$$

3 → Max. SNR deps. upon → E [sig. energy] and → $\left(\frac{N_0}{2}\right)$ [PSD of white Noise]

$$\left(\frac{S}{N}\right)_{t=T} = \frac{2E}{N_0}$$



pdf of $x(iT)$

⇒ **Derivation for Prob. of error (P_e) for the Matched filter**

P_e for optimum filter is :

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right] \quad \text{—— (1)}$$

$x_{01}(T)$, $x_{02}(T)$ } o/p of the Rx in the absence of noise $n(t)$.

Here, we have

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\,df \quad \text{—— (2)}$$

$S_{ni}(f)$ → i/p PSD (of noise)

Here, ($S_{ni}(f)$ is considered as AWGN)

∴ psd of this noise = $S_{ni}(f) = \frac{N_0}{2}$ —— (3)

Hence,

$$\left[\frac{x_{o1}(T) - x_{o2}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\left(\frac{N_0}{2}\right)} df$$

$$= \frac{2}{N_0}\int_{-\infty}^{\infty} |X(f)|^2 df \quad \text{—— (4)}$$

Also, according to Parseval's theorem:

$$\int_{-\infty}^{\infty}|X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)dt = \int_0^T x^2(t)\,dt \quad \text{—— (5)}$$

limits 0 to T (∵ $x(t)$ exists from 0 to T only)

Now,
we know that:

$$x(t) = x_1(t) - x_2(t)$$

∴ the above eqⁿ (5) becomes:

$$\int_{-\infty}^{\infty}|X(f)|^2 df = \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$= \int_0^T [x_1^2(t) + x_2^2(t) - 2x_1(t)x_2(t)]\,dt$$

$$= \int_0^T x_1^2(t)dt + \int_0^T x_2^2(t)dt - 2\int_0^T x_1(t)x_2(t)\,dt \quad \text{—— (6)}$$

Now, $\int_0^T x_1^2(t)\,dt = E_1$ i.e energy of $x_1(t)$

$\sim$ly, $\int_0^T x_2^2(t)\,dt = E_2$

and $\int_0^T x_1(t)\,x_2(t) = E_{12}$ (energy due to autocorrelation b/w $x_1(t)$ & $x_2(t)$)

Now, if we choose $x_1(t) = -x_2(t)$, then:

$$E_1 = E_2 = -E_{12} = E$$

<u>Put in eq. 6</u>

$$\therefore \int_{-\infty}^{\infty} |X(f)|^2 df = [E + E - 2(-E)] = 4E \quad \text{--- (7)}$$

<u>Put (7) in (4)</u>

$$\Rightarrow \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0} \cdot 4E = \frac{8E}{N_0}$$

<u>Taking $\sqrt{\ }$ on b/s</u>

$$\left.\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} = 2\sqrt{2}\sqrt{\frac{E}{N_0}} \quad \text{--- (8)}$$

<u>Put eq^n (8) in (1)</u>

$\therefore$ prob. of error P_e for matched filter is :-

$$\boxed{P_e = \frac{1}{2}\,\text{erfc}\left[\sqrt{\frac{E}{N_0}}\right]}$$

⇒ Theorem → Orthog. sets are linearly independent.
(if $S = \{v_1, v_2, \dots, v_n\}$ is an orthog set of non-zero vectors in an inner prod. space V, then S is linearly indep)

Corollary → If V is an inner prod. space of dimension n, then any orthog set of 'n' non-zero vectors is a basis for V

$\boxed{S \text{ is a basis of } V \text{ of dimension } n.}$

This means that we have to check that all the pairs are orthog. or not

Eg:
(2) Show that the following set is a basis for $\mathbb{R}^4$

dimension $= 4$ & S also has 4 vectors

$S = \{(2,3,2,-2), (1,0,0,1), (-1,0,2,1), (-1,2,-1,1)\}$
with $v_1 = (2,3,2,-2)$, $v_2 = (1,0,0,1)$, $v_3 = (-1,0,2,1)$, $v_4 = (-1,2,-1,1)$

So,

(2)
$$\left.\begin{aligned} v_1 \cdot v_2 &= 0 \\ v_1 \cdot v_3 &= 0 \\ v_1 \cdot v_4 &= 0 \\ v_2 \cdot v_3 &= 0 \\ v_2 \cdot v_4 &= 0 \\ v_3 \cdot v_4 &= 0 \end{aligned}\right\}$$

1^{st} you have to verify this (check pair-wise orthogonality)

So, S is a basis for $\mathbb{R}^4$

⇒ Co-ordinates relative to an orthonormal basis.
if $B = \{v_1, v_2, \dots v_n\}$ is an orthon. basis for V, then the coord. rep^n of a vector 'w' relative to B is :-

$$w = \underbrace{\langle w, v_1\rangle}_{\text{dot prod. of } w \,\&\, v_1} v_1 + \langle w, v_2\rangle v_2 + \dots + \langle w, v_n\rangle v_n \quad \text{—— (1)}$$

① Find the coord. matrix of $w = (5, -5, 2)$ relative to the following orthon. basis for R^3

$$B = \{ \underbrace{(\tfrac{3}{5}, \tfrac{4}{5}, 0)}_{v_1}, \underbrace{(-\tfrac{4}{5}, \tfrac{3}{5}, 0)}_{v_2}, \underbrace{(0, 0, 1)}_{v_3} \}$$

① in short: find matrix 'w' for orthon. basis B.

→ These coordinates are called Fourier coeff. of 'w' relative to B.

Sol^n: ∵ B is orthon. (as $v_1 \cdot v_2 = 0$, $v_1 \cdot v_3 = 0$, $v_2 \cdot v_3 = 0$)

So, to find coord. of 'w', we use eq^n ①

$$w \cdot v_1 = (5, -5, 2) \cdot (\tfrac{3}{5}, \tfrac{4}{5}, 0) =$$

$$= 3 - 4 + 0 = -1$$

$$w \cdot v_2 = (5, -5, 2) \cdot (-\tfrac{4}{5}, \tfrac{3}{5}, 0) = -4 - 3 = -7$$

$$w \cdot v_3 = (5, -5, 2) \cdot (0, 0, 1) = 2$$

So, the coord. matrix relative to B is :-

$$[w]_B = [-1 \;\; -7 \;\; 2]^T \leftarrow \text{Transpose} \quad \text{or} \quad \begin{bmatrix} -1 \\ -7 \\ 2 \end{bmatrix}$$

⇒ Major ques^n

How to generate an orthonormal Basis?

Sol^n: If we are given a basis $B = \{v_1, v_2, \ldots, v_n\}$ for inner space prod V.
then, how to generate an orthonormal basis B'?

Answer: We use Gram-Schmidt Orthogonalization Process (to calc. an orthon basis B' from B)

1. Let $B = \{v_1, v_2, \dots, v_n\}$ be a basis for an inner prod space V.

2. Let $B' = \{w_1, w_2, \dots, w_n\}$ when ~~w_i~~ w_i is given by:-

$$w_1 = v_1$$

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$$

$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$$

$$\vdots$$

$$w_n = v_n - \frac{\langle v_n, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_n, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2 - \dots - \frac{\langle v_n, w_{n-1} \rangle}{\langle w_{n-1}, w_{n-1} \rangle} w_{n-1}$$

Here, $B' \rightarrow$ Orthogonal basis for V
it is not yet orthonormal. (it is orthog. right now)

The orthonormal basis is $B'' = \{u_1, u_2, \dots, u_n\}$

$u_1, u_2, \dots \rightarrow$ all unit vectors ($\because$ orthonormal)

So, $\boxed{u_i = \frac{w_i}{\|w_i\|}}$

So, B'' is an orthon basis for V.

Moreover,

$$\text{span}\{v_1, v_2, \dots, v_k\} = \text{span}\{u_1, u_2, \dots, u_k\}$$

for $k = 1, 2, 3, \dots, n$.

Lets do an example.

(1) Apply Gram-Schmidt Orthog' Process for the following basis for R^3

$$B = \{\overbrace{(1,1,0)}^{v_1}, \overbrace{(1,2,0)}^{v_2}, \overbrace{(0,1,2)}^{v_3}\}$$

(1) <u>Step 1</u>: Find $B' = \{w_1, w_2, w_3\}$

So, $w_1 = v_1 = (1, 1, 0)$ ——— (1)

$$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$$

$$= (1,2,0) - \frac{\{(1,2,0)\cdot(1,1,0)\}}{\{(1,1,0)\cdot(1,1,0)\}}(1,1,0)$$

$$= (1,2,0) - \left[\frac{(1+2)}{(1+1)}\right](1,1,0)$$

$$= (1,2,0) - \left(\frac{3}{2}, \frac{3}{2}, 0\right)$$

$$= \left(-\frac{1}{2}, \frac{1}{2}, 0\right) = w_2$$ ——— (2)

$$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$$

$$= (0,1,2) - \frac{\{(0,1,2)\cdot(-\frac{1}{2}, \frac{1}{2}, 0)\}}{\{(-\frac{1}{2}, +\frac{1}{2}, 0)\cdot(-\frac{1}{2}, \frac{1}{2}, 0)\}}\left(-\frac{1}{2}, \frac{1}{2}, 0\right)$$

$$- \frac{\{(0,1,2)\cdot(1,1,0)\}}{\{(1,1,0)\cdot(1,1,0)\}}(1,1,0)$$

$\frac{1}{4}+\frac{1}{4}$

$$= (0,1,2) - \frac{\{\cancel{(\frac{1}{2})}\}}{\{\cancel{\frac{1}{2}}\}}\left(-\frac{1}{2}, \frac{1}{2}, 0\right) - \frac{\{(1)\}}{\{2\}}(1,1,0)$$

$$= (0,1,2) - \left(-\frac{1}{2}, \frac{1}{2}, 0\right) - \left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

$$=(0,1,2)-\left\{\left(\frac{-1}{2},\frac{1}{2},0\right)-\left(\frac{1}{2},\frac{1}{2},0\right)\right\}$$

$$=(0,1,2)-\left(-\frac{1}{4},0,0\right)$$

$$=\left(\frac{1}{4},1,2\right)=w_3 \quad \text{③}$$

Step 2 : Calc. $B'' = \{u_1, u_2, u_3\}$

So, $u_1 = \dfrac{w_1}{||w_1||}$

$$= \frac{(1,1,0)}{||(1,1,0)||} = \frac{(1,1,0)}{\sqrt{(1^2+1^2+0^2)}} = \frac{(1,1,0)}{\sqrt{2}}$$

$$= \left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right) \quad \text{④}$$

$$u_2 = \frac{w_2}{||w_2||}$$

$$= \frac{\left(-\frac{1}{2},\frac{1}{2},0\right)}{\sqrt{\left(\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+0^2\right)}} = \frac{\left(-\frac{1}{2},\frac{1}{2},0\right)}{\sqrt{\left(\frac{1}{4}+\frac{1}{4}+0\right)}} = \frac{\left(-\frac{1}{2},\frac{1}{2},0\right)}{\sqrt{(1/2)}}$$

$$= \left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right) \quad \text{⑤}$$

$$u_3 = \frac{w_3}{||w_3||} = \frac{(1,1,2)}{\sqrt{6}} = \left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{\sqrt{2}}{3}\right).$$

$$= \frac{\left(\frac{1}{4},1,2\right)}{\sqrt{\left(\frac{1}{16}+1+4\right)}} = \cancel{\frac{\left(\frac{1}{4},1,2\right)}{\sqrt{(17/90)}}} = \cancel{\left(\frac{17}{360},\frac{17}{90},\frac{34}{90}\right)} \quad \text{⑥}$$

$\frac{1+16}{90}$

$\frac{17}{90}$

we say that B'' is an orthonormal basis for R^3.

$u_1, u_2, u_3 \rightarrow$ all are unit vectors $\bar{c}$ length 1

'w_j' → prediction filter coeff.

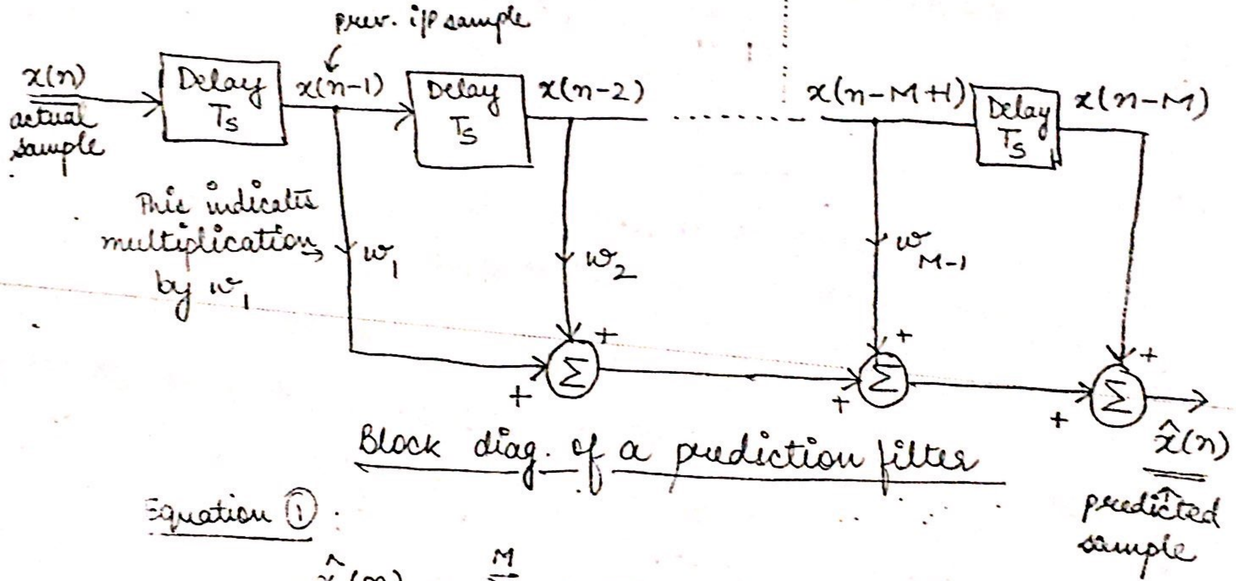
$R(k-j) = \overline{x(n-k) \cdot x(n-j)}$, is autocorrelation of ∴

and

$R(k) = \overline{x(n) \cdot x(n-k)}$, " " " $x(n)$

→ <u>Prediction Filter</u> :-

→ implementation of eqⁿ ① is called a prediction filter.



<u>Block diag. of a prediction filter</u>

<u>Equation ①</u> :

$$\hat{x}(n) = \sum_{k=1}^{M} w_k \, x(n-k)$$

→ eqⁿ ① is called 'filter' ∵ it represents a linear convolutⁿ.

* <u>Convolution</u> : it is a mathematical way of combining 2 signals to form a third signal.
<u>Linear convolution</u> is the basic operation to calculate the o/p for any linear time invariant sys given its i/p and its impulse resp.
convolution <u>calculates</u> the resp of an LTI sys.
whereas,
cross-correlation is used for pattern matching.

(2.)

Hence, $x(n-1), x(n-2), \ldots, x(n-m)$ → past i/p's

$x(n)$ → present i/p.

→ These i/p samples are used for filtering c̄ the help of coeff. $w_1, w_2, \ldots, w_M$. Such filter is of the type of finite impulse response digital filters.

→ Filtering operation can be modified by changing the value of filter coeff.

→ Prediction filter is used earlier in DPCM.

→ Prediction error can also be calculated using prediction filter.

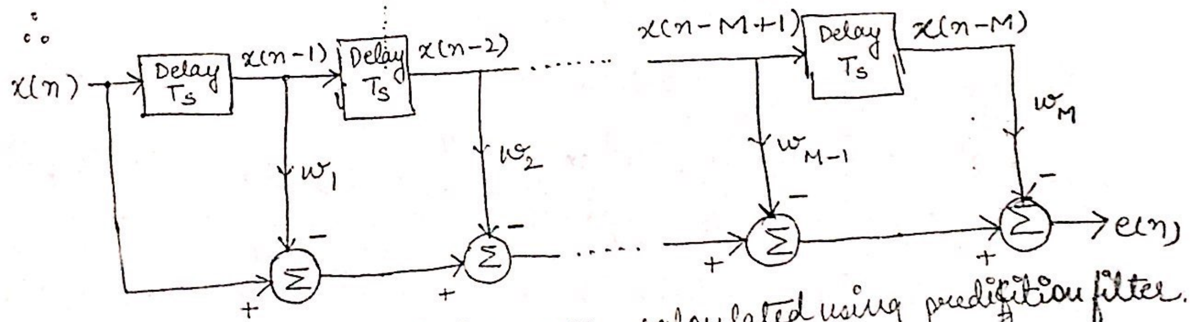
$$e(n) = x(n) - \hat{x}(n)$$

Put ① in above eq^n

$$e(n) = x(n) - \sum_{k=1}^{M} w_k \, x(n-k)$$

$$= x(n) - [w_1 x(n-1) + w_2 x(n-2) + \cdots + w_M x(n-M)]$$

$$= x(n) - w_1 x(n-1) - w_2 x(n-2) - \cdots - w_M x(n-M)$$

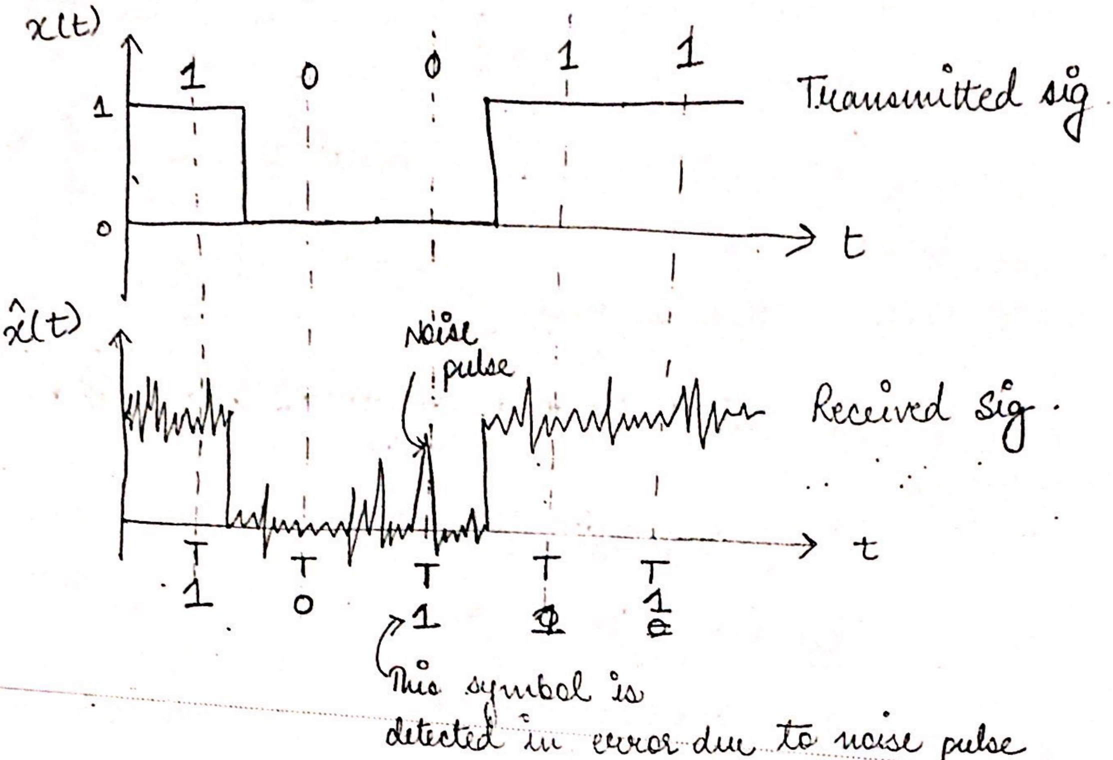


Prediction error calculated using prediction filter.

→ in the above diag., $w_1 x(n-1), w_2 x(n-2), \ldots w_M x(n-M)$ are subtracted from $x(n)$. Hence o/p of prediction filter is error $e(n)$.

⇒ MATCHED FILTER

→ used for detection of signals in baseband and passband tx^n.



→ The above fig. shows txed ~~sig~~ digital sig. and received noisy sig. The txed sig sequence is 1 0 0 1 1. The pulse is checked at the point 'T' of every bit period. Because ~~the~~ of noise pulse present in the third bit at the instant 'T' of checking, it is detected in error

→ Requirements of detection error

(i) SNR of Rx must be improved.

(ii) Signal must be checked at the instant in bit period, when SNR is max.

(iii) The error probability should be min.

→ Matched filter

(i) It satisfies all the above requirements.

(ii) It is called matched filter since its impulse response is matched to the shape of i/p signal.

and

where we have used equations (13.15), (13.13) and (13.9). Recognizing that $|\cos\theta| \leq 1$, the Schwarz inequality of equation (13.16) immediately follows from equation (13.17). Moreover, from the first line of equation (13.17), it may be noted that $|\cos\theta| = 1$ if and only if $s_2 = cs_1$, *i.e.*, $s_2(t) = cs_1(t)$, where c is an arbitrary constant.

The proof of the Schwarz inequality applies to real-valued signals. It may be readily extended to complex-valued signals, in which case equation (13.16) is reformulated as under:

$$\left|\int_{-\infty}^{\infty} s_1(t)s_2^*(t)\,dt\right|^2 \leq \left(\int_{-\infty}^{\infty} |s_1(t)|^2\,dt\right)\left(\int_{-\infty}^{\infty} |s_2(t)|^2\,dt\right) \qquad ...(13.18)$$

where the equality holds if and only if $s_2(t) = cs_1(t)$, where c is a constant.

13.6 GRAM-SCHMIDT ORTHOGONALIZATION PROCEDURE *(Expected)*

After discussing geometric representation of energy signals, let us now discuss the Gram-Schmidt orthogonalization procedure for which we require a complete orthonormal set of basis functions. To start, let us assume that we have a set of M energy signals represented by $s_1(t), s_2(t), ..., s_M(t)$.

If, we start with $s_1(t)$, which chosen from this set arbitrarily, the first basis function is defined as

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} \qquad ...(13.19)$$

where E_1 = energy of the signal $s_1(t)$

From equation (13.19), we write

$$s_1(t) = \sqrt{E_1}\,.\,\phi_1(t)$$

or

$$s_1(t) = s_{11}\,\phi_1(t) \qquad ...(13.20)$$

where coefficient $s_{11} = \sqrt{E_1}$, and

$\phi_1(t)$ has unit energy, as required.

Next, using the signal $s_2(t)$, we can define the coefficient s_{21} as

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt \qquad ...(13.21)$$

Therefore, we can introduce a new intermediate function

$$g_2(t) = s_2(t) - s_{21}\phi_1(t) \qquad ...(13.22)$$

which is orthogonal to $\phi_1(t)$ over the interval $0 \leq t \leq T$ by equation (13.21) and by the fact that the basis function $\phi_1(t)$ has unit energy.

Now, we can define the second basis function as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} \qquad ...(13.23)$$

Now, substituting equation (13.22) into equation (13.23) and simplifying, we get the desired result as under:

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \qquad ...(13.24)$$

where E_2 is the energy of the signal $s_2(t)$.

From equation (13.23), it is obvious that

$$\int_0^T \phi_2^2(t)\,dt = 1$$

and, from equation (13.24), it is obvious that

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0$$

This means that $\phi_1(t)$ and $\phi_2(t)$ form an orthonormal pair, as required.

Depending upon the above discussion, we may write the general form as under:

$$g_i(t) = s_i(t) - \sum_{j=1}^{i=1} s_{ij}\phi_j(t) \qquad \text{...(13.25}$$

where the coefficients s_{ij} are defined as

$$s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt\,, \quad j = 1, 2, ..., i-1 \qquad \text{...(13.26}$$

It may be noted that equation (13.22) is a special case of equation (13.25) with $i = 2$. Furthe for $i = 1$, the function $g_i(t)$ reduces to $s_i(t)$.

Given the $g_i(t)$, now, we can define the set of basis functions as under:

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}, \quad i = 1, 2, ... N \qquad \text{...(13.27}$$

which forms on orthonormal set.

Further, the dimension N is less than or equal to the number of given signals, M dependin on one of two possibilities:

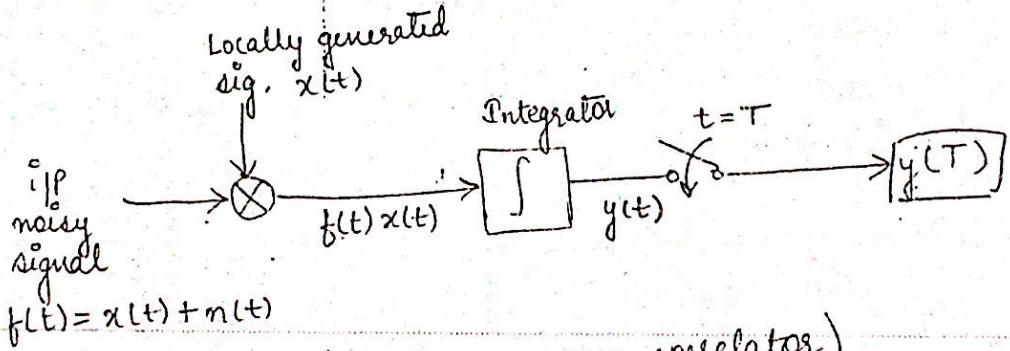
(*i*) The signals $s_1(t)$, $s_2(t)$, ..., $s_M(t)$ form a linearly independent set, in which case $N = M$.

(*ii*) The signals $s_1(t)$, $s_2(t)$, ..., $s_M(t)$ are not linearly independent, in which case, $N < M$, an the intermediate function $g_i(t)$ is zero for $i > N$.

⇒ CORRELATOR RECEIVER

→ Correlation is a pattern matching process where ~~where~~ we find out similarities b/w 2 signals.

→ In practice, a linear matched filter is more often implemented as a correlator. This allows a cheaper implementation of filters.



(Block diag. of a correlator)

Steps:-

working of correlator:

- i/p noisy sig. $f(t)$ is multiplied to a replica of i/p sig $x(t)$

↓

- Result $(f(t)x(t))$ is fed to an integrator

↓

- o/p of integrator is sampled at $t = T$

→ Why it is called a correlator?

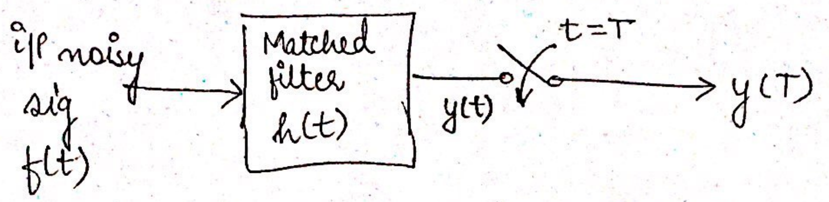
→ ∵ it correlates received sig. $f(t)$ c̄ i/p signal $x(t)$.

So, o/p $y(t) = \int_0^T f(t)\,x(t)\,dt$ ——— ①

Now, at $t = T$

O/P of correlator is :

$$y(T) = \int_0^T f(t)\, x(t)\, dt \quad \text{——— (2)}$$



Now, consider the matched filter ~~acc~~ in ~~to~~ the above diagram.

→ The o/p y(t) is obtained by convolution of i/p. f(t) & its impulse response h(t).

* here, we donot need a locally generated replica of the i/p signal x(t).

∴

$$y(t) = f(t) \otimes h(t)$$

$$= \int_{-\infty}^{\infty} f(\tau) \cdot h(t-\tau)\, d\tau \quad \text{——— (3)}$$

Now, we know that :

impulse resp. of matched filter, h(t) is :-

$$h(t) = \frac{2k}{N_0} x(T-t) \quad \text{——— (4)}$$

$$\therefore h(t-\tau) = \frac{2k}{N_0} x(T-t+\tau)$$

Now, integration is performed over one bit period.

∴ limits change to 0 to T

So,

$$y(t) = \frac{2k}{N_0}\int_0^T f(\tau)\, x(T-t+\tau)\, d\tau$$

at $t = T$

$$y(T) = \frac{2k}{N_0}\int_0^T f(\tau)\, x(\cancel{T}-\cancel{T}+\tau)\, d\tau$$

$$= \frac{2k}{N_0}\int_0^T f(\tau)\, x(\tau)\, d\tau$$

let $\tau = t$ (only for convenience)

$$\therefore\ y(T) = \frac{2k}{N_0}\int_0^T f(t)\, x(t)\, dt \quad \text{——— (5)}$$

from eqn (2) and (5)

eqn (5) gives the o/p of matched filter

eqn (2) " " " " correlator.

we may say that both are identical
as eqn (2) = (5).

The constt. in eqn (5) $\left(\frac{2k}{N_0}\right)$ can be normalized to 1.

$\therefore$ we may say that both the matched filter & correlator provides the same o/p.

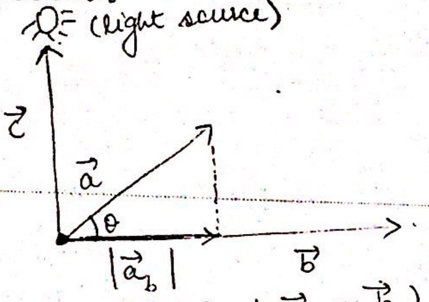
⇒ ORTHOGONAL SIGNALS

Orthogonality: It is the property that allows tx's of more than one signal over a common channel $\bar{c}$ successful detection.

Orthogonal Signals: 2 sig's are orthog. if they are mutually independent.

How to find out whether 2 signals are orthog.?

→ orthog. vectors:



(projection of $\vec{a}$ on $\vec{b}$)
shadow of $\vec{a}$ on $\vec{b}$.
|| → mod rep. mag. of proj^n

$$|\vec{a}_b| = |\vec{a}| \cos\theta$$

$$|\vec{a}_b| . |\vec{b}| \neq 0$$

(mag. of proj^n & mag. of vector $\vec{b}$) ⇒ their X^n $\neq 0$

$$= |\vec{a}| \cos\theta . |\vec{b}| \neq 0$$

$$\underbrace{|\vec{a}||\vec{b}| \cos\theta}_{\text{dot prod. of } \vec{a} \,\&\, \vec{b}} \neq 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} \neq 0$$

∴ $\vec{a}$ & $\vec{b}$ are not independent & we say that these vectors are not orthog. to each other.

Now, if we have another vector (let it be $\vec{c}$)
and we put it under a light source.
we see that, projⁿ of $\vec{c}$ on $\vec{b}$ is NULL.
i.e $\vec{c}$ doesnot have any shadow over $\vec{b}$.

Now, $\theta = 90^\circ$ (b/w $\vec{b}$ & $\vec{c}$)

$\cos 90^\circ = 0$.

$\therefore \vec{b} \cdot \vec{c} = 0$ ($\because bc \cos 90^\circ = 0$)

$\therefore$ we say that $\vec{b}$ & $\vec{c}$ are orthog. (or mutuall indep.)

Signal space $\rightarrow$ inner prod. (Or dot prod.)
(Definite integral)

let there are 2 sigs $x_1(t)$ & $x_2(t)$

~~2 →~~ The 2 sig's are orthog. if

$$\boxed{\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt = 0}$$ [For non-periodic sig's]

and $$\underbrace{\boxed{\int_{0}^{T} x_1(t)\, x_2(t)\, dt = 0}}_{\text{over one time period}}$$ [For periodic sig's]

$\Rightarrow$ Properties of orthog. sigs (4)

(i) 2 harmonics of different frequencies are always $\rightarrow$ orthog.

let $x_1(t) = \sin(n\omega_0 t + \phi_1)$
$x_2(t) = \sin(m\omega_0 t + \phi_2)$] $n \neq m$ & $\phi_1 \neq \phi_2$

So, both phase & frequencies are different
$\Rightarrow$ Hence, $x_1(t)$ & $x_2(t) \longrightarrow 0$

$$\int_0^T \sin(n\omega_0 t + \phi_1)\,\sin(m\omega_0 t + \phi_2)\,dt = 0.$$

(ii) sine & cosine funⁿ $\bar{c}$ same freq. & phase $\rightarrow$ O. (orthog.)

$$\int_0^T \sin(n\omega_0 t + \phi)\,\cos(n\omega_0 t + \phi)\,dt = 0$$

f. & ϕ are same

(iii) dc value & sine funⁿ $\longrightarrow$ O

$$\int_0^T \underset{\substack{\uparrow \\ \text{dc value}}}{a}\,\sin(n\omega_0 t + \phi) = 0$$

Imp (iv) Effects of orthogonality on total energy (E) & avg. power (P) calculations.

if we have

$x_1(t)$ & $x_2(t)$ $\longrightarrow$ O

$$\Rightarrow \int_{-\infty\ 0}^{\infty\ T} x_1(t)\,x_2(t)\,dt = 0$$

& if we have another sig. $y(t)$ $\bar{c}$ is :-

$$y(t) = x_1(t) + x_2(t)$$

$\rightarrow$ Then, the av. power $\underbrace{P_y = P_{x_1} + P_{x_2}}_{①}$ (we can use this directly if $x_1(t)$ & $x_2(t)$ are O)

$\Rightarrow$ Total energy, $\underbrace{E_y = E_{x_1} + E_{x_2}}_{②}$ (")

① Total 'E' is ∞ in case of power sig's
we use eqⁿ ① if $x_1(t)$ & $x_2(t)$ $\rightarrow$ power sig's.

② Avg. power = 0 (zero) in case of energy sig's.
we use eqⁿ ② if $x_1(t)$ & $x_2(t)$ $\rightarrow$ E. sig's.

Example :-

① $y(t) = \overbrace{2\sin(3\omega_0 t + 45^\circ)}^{x_1(t)} + \overbrace{4\sin(4\omega_0 t + 35^\circ)}^{x_2(t)}$.

Cal. avg. power & total energy.

① Solutⁿ:

(i) Find whether $x_1(t)$ & $x_2(t)$ are orthog. or not.

From the above eqⁿ.

$f_1 \neq f_2$ (i.e $3 \neq 4$)

& $\phi_1 \neq \phi_2$ (i.e $45° \neq 35°$)

$\therefore$ ~~b~~ $x_1(t)$ & $x_2(t)$ are $\rightarrow$ O. [from prop. (i)]

(ii) Use property (iv) to calc. Avg. P. & total energy(E)

$P_y = P_{x_1} + P_{x_2}$

$E_y = \infty$ ($\because x_1(t)$ & $x_2(t)$ are periodic.

$\Downarrow$

periodic sig's are power sig's

$\Downarrow$

& in case of power sig's energy, $E = \infty$)

$= \frac{2^2}{2} + \frac{4^2}{2}$

$= \frac{4}{2} + \frac{16}{2}$

$= 2 + 8$

$\boxed{P_y = 10\ W}$ & $\boxed{E_y = \infty}$

$\left[\because P = \frac{A_0^2}{2}\right]$

* How to find whether a sig is E.S. or Power Sig.?

Eg: 1. $x(t) = \underset{x_1(t)}{t}\ \underset{x_2(t)}{u(t)}$

$t \rightarrow$ ramp funⁿ

$x_1(t) = t = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$

Solⁿ: Simply calc. the avg. power (P) & tot. Energy (E) of that sig.

if $\left.\begin{array}{l} E = \text{finite} \\ P = 0 \end{array}\right\}$ Energy Sig

if $\left.\begin{array}{l} P = \text{finite} \\ E = \infty \end{array}\right\}$ Power sig.

$x_2(t) \rightarrow$ unit step fun^n

$$x_2(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

av. power, $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$ [std. formula to calc. avg. P]

$$= \lim_{T\to\infty} \frac{1}{2T}\left[\int_{-T}^{0} 0\, dt + \int_{0}^{T} |t \cdot 1|^2 dt\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\left[0 + \left[\frac{t^3}{3}\right]_0^T\right]$$

$$= \lim_{T\to\infty} \frac{1}{2\cancel{T}}\left[\frac{T^{\cancel{3}\,2}}{3}\right]$$

$$= \lim_{T\to\infty} \frac{T^2}{6} = \infty.$$

So, av. Power $= \infty$ for $x(t)$

Tot. energy, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \int_{-\infty}^{0} 0\, dt + \int_{0}^{\infty} t^2 dt$$

$$= \left[\frac{t^3}{3}\right]_0^{\infty}$$

$$= \infty - 0 = \infty$$

tot, $E = \infty$

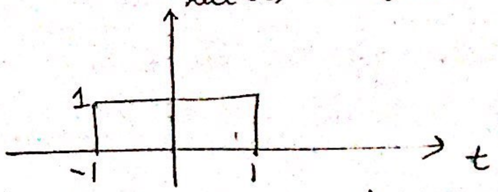
So, sig. $x(t)$ is neither energy sig nor P.S.

(10)

(2) $y(t) = 2\ \text{rect}\underbrace{(t/2)}$

rect(t) ← time scaling (T.S.)

(2)

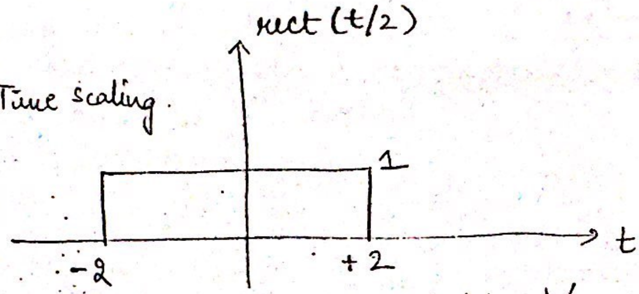


This is a rect. sig.

first, perform time scaling of rect. sig.
2nd, " amplitude " of '2' amplitude

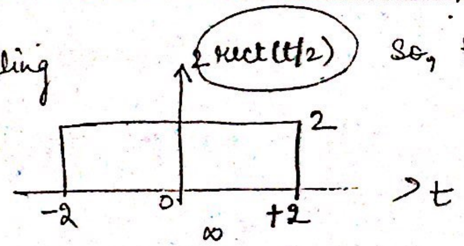
So,

Time scaling.



(divide 1/0.5 on b/s ∵ here we have t/2)
So, time is ÷ed by 0.5

Amp. Scaling



(i) cal. tot. E. $= \int_{-\infty}^{\infty} |y(t)|^2 dt$

$= \int_{-2}^{2} (2)^2 dt = 4[t]_{-2}^{2}$ 2+2

$= 16$ Joules (finite)

(ii) cal. avg. P $= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 4\, dt$

$= \frac{1}{2T} 4 [t]_{-\infty}^{\infty}$ (finite no.)

$= \frac{\infty}{\infty} = 0$ ∴ sig. y(t) = E.S

→ GRAM – SCHMIDT ORTHOGONALIZATION PROCEDURE :-

Goal : To show ① a set of vectors is orthogonal.

$\Downarrow$

forms orthonormal basis

$\Downarrow$ +

rep. a vector relative to an orthonormal basis.

② → apply Gram – Schmidt . O^n procedure.

Definitions:

1. Orthogonal set : A set 'S' of vectors in an inner prod. space V is called orthog. when every pair of vectors in S is orthog.

i.e for every V_i, V_j in 'S' ⇒ $V_i \circ V_j = 0$ [cond^n for orthog.] $i \neq j$.

2. Orthonormal : if, Each vector in the set is a unit vector then, S is called orthonormal.

Eg: ① Show that set 'S' is an orthon basis for R^3.

$$S = \underbrace{\{V_1, V_2, V_3\}}_{\text{these vector lie on diff. planes}} = \left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(-\frac{\sqrt{2}}{6}, +\frac{\sqrt{2}}{6}, \frac{2\sqrt{2}}{3}\right), \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right)\right\}$$

① Solution :

(i) Verify that all vectors are pair-wise orthog. & $\underset{(\text{norm})}{\|\ \|}$ of each vect. = 1.

∴

$v_1 \cdot V_2 = 0$ and $\|V_1\| = 1 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + (0)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

$V_1 \cdot V_3 = 0$ $\qquad \|V_2\| = 1 = \sqrt{\left(\frac{-\sqrt{2}}{6}\right)^2 + \left(\frac{\sqrt{2}}{6}\right)^2 + \left(\frac{2\sqrt{2}}{3}\right)^2} = \frac{2+2}{36} + \frac{8}{9}$

$V_2 \cdot V_3 = 0$ $\qquad = \frac{\frac{4}{36}}{} + \frac{8}{9} = \frac{9}{9} = 1$

$\|V_3\| = 1$

$\hookrightarrow = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$

$= \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = \frac{9}{9} = 1$

∴ we conclude that set 's' is an orthonormal set and it is also a basis for R^3

MAXIMUM LIKELIHOOD DETECTOR

Concept of "Detection of signals in noise"

A Source tx's M diff. signals i.e $S_1(t), S_2(t), \ldots S_M(t)$ etc.

↓

All these are equally likely

↓

∴ each one has a probability of

$$\left(\frac{1}{M}\right)$$

Now, let us consider a time slote of 'T' sec.

↓

if one of these 'M' possible signals be txed in one ('T' sec) duration

↓

Then, for an AWGN channel

$$\boxed{x(t) = s_i(t) + W(t)} \quad \ldots \quad 0 \leq t \leq T$$

where $i = 1, 2, 3, \ldots, M$

$x(t) \rightarrow$ Rxed sig.

$s_i(t) \rightarrow$ ~~the~~ i^{th} msg sig.

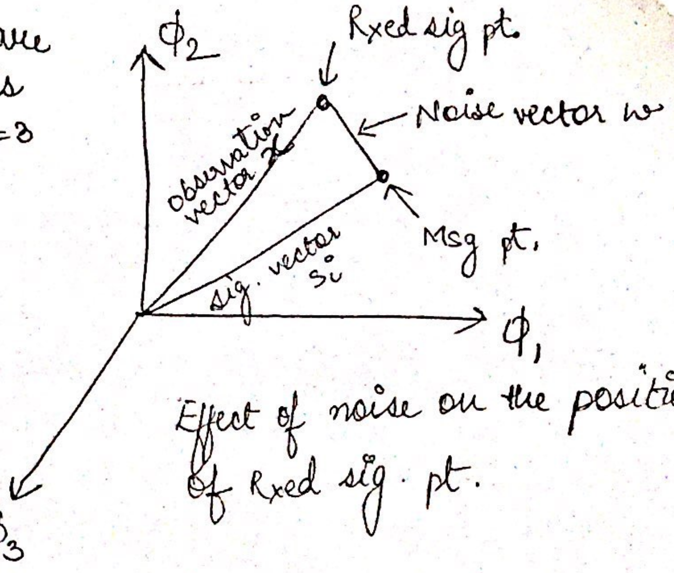
$W(t) \rightarrow$ sample fun. of white noise process

$w(t) \Rightarrow$ zero mean value

$$psd = \frac{N_0}{2}$$

* Note: After Rxing the sig. $x(t)$, the R_x has to make an estimate of the txed signal $s_i(t)$ to decide $\leq$ symbol was txed.

* Here, $\phi_1, \phi_2, \phi_3 \rightarrow$ are N orthonormal basis functions where N=3



Effect of noise on the position of Rxed sig. pt.

In short:

→ ~~Signals~~ The above fig. illustrates relationship b/w the observation vector x, sig vector s_i and noise vector w for $N=3$.

"MLD"

→ Here, we are dealing $\bar{c}$ the detection problem:

→ if observat^n vector 'x' is given then :-
we have to perform a mapping from x to get an estimate $\hat{m}$ of the txed ~~sig~~ symbol m_i, in a way the avg. P_e is minimum
(av. prob. of symbol error)

This is called Max. Likelihood Detection.

How, $m_i \rightarrow$ original txed sig.
$\hat{m}_i \rightarrow$ estimated sig.

Assume that x = Observation vector
then,
R_x will make decision if $\hat{m}_i = m_i$

Now, avg. prob. of symbol error in such a decision is:- (2)

$$\boxed{P_e(m_i, x) = P(m_i \text{ not sent}/x) = 1 - P(m_i \text{ sent}/x)} \quad \text{— (1)}$$

$P(m_i \text{ sent}/x) \Rightarrow$ conditional prob. that m_i was sent provided x is rxed.

Now, Optimum Decision Rule

aim $\Rightarrow$ minimize the prob. of error in mapping 'x' into a decision.

Hence, from eqn (1), we have to deduce an optimum decision rule.

The estimate $\hat{m}_i = m_i$ if

$$\boxed{P(m_i \text{ sent}/x) \geq P(m_k \text{ sent}/x)} \text{ for all } k \neq i \quad \text{— (2)}$$

where $K = 1, 2, \ldots, M$.

This rule is called as the maximum a posterior prob.

Now, applying Bayes rule to eqn (2).

we restate the decision rule as:-

The estimate $\hat{m}_i = m_i$ if

$$\left\{ \frac{p_k f_x(x/m_k)}{f_x(x)} \right\} \text{ is max. for } k = i \quad \text{— (3)}$$

$p_k \rightarrow$ prior prob. of occurrence of symbol m_k

$f_x(x/m_k) \rightarrow$ likelihood funn corresponding to t_x of symbol m_k

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$f_X(x) \rightarrow$ unconditional joint pdf of rand. variable X.

Observations from eq^n (3)

(i) denominator term $f_X(x)$ is indep. of txed sig.

(ii) There, a prior prob. $p_K = p_i$

$\because$ all txed sigs are equally likely.

So, simplifying the decision rule :-

The estimate $\hat{m}_i = m_i$, if

$$\boxed{f_X(x/m_k) \text{ is maximum for } k = i} \quad (4)$$

Generally, using natural logarithm of likelihood fun^n is easier than the likeli. fun's itself.

$\Downarrow$

natural logarithm of likelih. fun^n = metric

$\Downarrow$

$f_X(x/m_k) \geq 0$ (always non-negative)

$\Downarrow$

Now, if $X > Y > 0$, then $\log_e X > \log_e Y$

$\therefore$ decision rule can be restated as :-

The estimate $\hat{m}_i = m_i$ if

$$\boxed{\log_e[f_X(x/m_k)] \text{ is max. for } k = i} \quad (5)$$

eq^n (5) is called the max. likeli. rule & device used for its implementat^n is called max. likelihood Detector.

Conclusion :-

From eqⁿ (5) :

The MLD computes the metric of each txed sig m_k $k = 1, 2, \ldots, M$

⇓

compares ~~it c~~ ~~mi~~ these values

⇓

then, makes a decision based on the max. of them.

Scrambling :- Pg - 788

⇒ INTERSYMBOL INTERFERENCE (ISI)

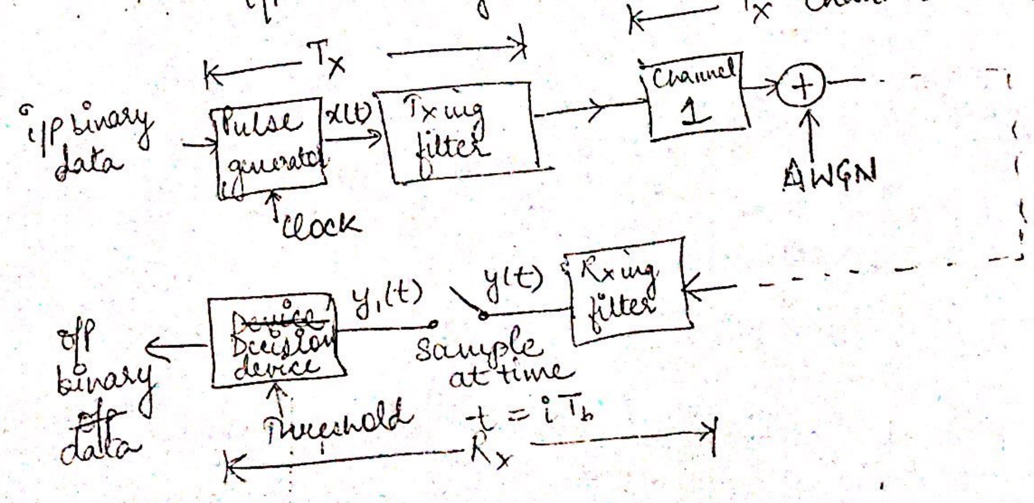
→ ISI defiⁿ

Data is txed in form of bits or pulses, the o/p produced at the R_x due to other bits or symbols interferes $\bar{c}$ the o/p produced by the desired bit. This is ISI.

→ ISI introduces errors in the detected sig.

→ Elements of ~~impulse~~ binary PAM sys.

i/p → binary data seq $= b_k$ $\bar{c}$ bit durat' T_b sec



i/p → binary data seq $\{b_k\}$

The seq. is applied to a pulse generator to produce a discrete PAM signal. $\bar{c}$ is given by :—

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \, v(t - kT_b) \quad \text{——— ①}$$

o/p of pulse generator

$v(t)$ → basic pulse, (normalized such that $v(0) = 1$)

→ The 1st block of the sys. i.e PAM ⇒ converts i/p seq $\{b_k\}$ into polar form i.e

if $b_k = 1 \Rightarrow a_k = 1$

if $b_k = 0 \Rightarrow a_k = -1$

⇓

This PAM sig. $x(t)$ is then fed to txing filter.

⇓

its o/p is fed to the channel. (impulse response of this channel be $h(t)$)

⇓

A random noise is then added to the txed while travelling over the tx channel. So, the Rxed sig. is contaminated $\bar{c}$ noise

⇓

channel o/p is passed through a Rxing filter

⇓

This filter o/p is sampled synchronously $\bar{c}$ the tx

(4)

$\Downarrow$

Seq. of samples obtained at o/p of Rxing is used to reconstruct the original data seq. $\bar{c}$ the help of a decision making device

$\Downarrow$

Each sample is compared to a threshold level in the decision making device.

if amplitude is > threshold level $\Rightarrow$ symbol 1 is Rxed.

if amp. is < threshold level $\Rightarrow$ it is decided that symbol 0 is Rxed.

$\Downarrow$

Rxing filter o/p $\Rightarrow$ $y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b) + n(t)$

$\mu \to$ scaling factor

$n(t) \to$ noise

$p(t-kT_b) \to$ combined impulse response of the Rxing filter.

at time $t = t_i = iT_b$ ($i \to$ any integer)

So,

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(iT_b - kT_b) + n(t_i) \quad \text{——— (2)}$$

Or

$$y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k\, p(iT_b - kT_b) + n(t_i) \quad \text{——— (3)}$$

Eqn (3) shows the Rx o/p $y(t)$ at instant $t = t_i$

Now, from eqn (3) :-

(i) The first term μa_i is produced by i^{th} txed bit. Only this term should be present.

(ii) 2^{nd} term represents the residual effect of all txed bits, obtained at the time of sampling i^{th} bit. This residual effect = ISI.

Pg-767

- Cause of ISI
- Effect of ISI
- Remedy ~~to~~ Reduce ISI

⇒ EYE DIAGRAM

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