Chap - Sets

Set - It is a collection of unordered elements. eg-
$A = \{1, 2, 3\}$, $B = \{1, 3, 2\}$, $C = \{1, 2, 2, 1, 3, 1, 2, 3, 3\}$
$A = B = C$

Roster method : $V = \{a, e, i, o, u\}$
Set builder notation: $V = \{x \mid x \text{ is a vowel}\}$

Types of sets

1. Universal set (U) - It is a set which contains all the elements of a particular domain.

2. Empty/Null set : $\{\}$ or ϕ. It is a set containing zero elements.

3. Singleton set - A set containing only one element.
$\{\phi\}$ is a singleton set.

4. Power Set of a set S is the set containing all the subsets of S.

5. Subset - Set A is said to be a subset of set B if and only if every element of A is also an element of set B. i.e. $\forall x, x \in A \xrightarrow{\text{implication}} x \in B \Rightarrow A \subseteq B$.

$p \to q$ if p then q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

6. Proper subset - A is said to be proper subset of B, if A is a subset of B but A is not equal to B. A = {1,2,3}, B = {1,2,3,4,5}, A ⊂ B

S = {1,2,3}

Power Set = {ϕ, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3}}

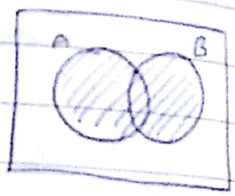
Q Proof Prove that $\phi \subseteq P(S)$, $S \subseteq P(S)$

Set Cardinality - It is number of elements in a set represented by |S|. No of elements in a set. Finite sets and infinite sets.

Set Operations

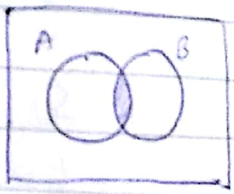
1. Union

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



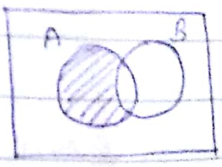
2 Intersection

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



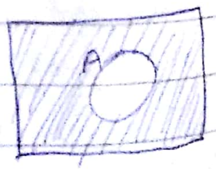
3 Difference

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$



4. Complement

$\bar{A} = \{x \mid x \notin A\} = U - A$

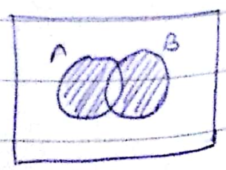


5. Symmetric difference.

$A \oplus B$ or $A \Delta B$ = either in A or in B but not in both.

$= (A-B) \cup (B-A)$

$= (A \cup B) - (A \cap B)$



Q $A = \{1,2,3\}$, $B = \{2,4,6,8\}$

ans $A \cup B = \{1,2,3,4,6,8\}$

$A \cap B = \{2\}$

$A - B = \{1,3\}$

$\bar{A} = \{4,5,6,7,8,9,10\}$

$A \Delta B = \{1,3,6,8\}$

6 Cartesian product

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$ $\{(1,a),(1,b),(2,a),(2,b)\}$

$A = \{1,2\}$, $B = \{a,b\}$; $A \times B =$ ~~$\{(1,a),(1,b),(2,b)$~~

$A \times B = B \times A$

if $A = \phi$ or $B = \phi$ such that $A \times B = \phi$

if $A = B$.

Representation of discrete structures.

using Venn diagrams, Bit representation

$U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{2,4,5\}$, $B = \{1,3,4,5\}$, $C = \{5,7,8\}$

$A = \{0,1,0,1,1,0,0,0,0,0\}$ $B = \{1,0,1,1,1,0,0,0,0,0\}$

$C = \{0000101011$

De Morgan's law

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Set identities (Table 1 of sec 2.2 Rosen)

$\overline{A \cap B} = \bar{A} \cup \bar{B}$ [using $\neg(x \in A \wedge x \in B) = \neg(x \in A) \vee \neg(x \in B)$]

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I By subset method

To prove:

$\overline{A \cap B} \subseteq \bar{A} \cup \bar{B}$ and $\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$

Let $x \in \overline{A \cap B}$

$x \notin A \cap B$

$\neg(x \in A \cap B)$

$\neg(x \in A \cap x \in B)$

$\neg(x \in A) \cup \neg(x \in B)$

$x \notin A \cup x \notin B$

$x \in \bar{A} \cup x \in \bar{B}$

$x \in \bar{A} \cup \bar{B}$

$\therefore \overline{A \cap B} \subseteq \bar{A} \cup \bar{B}$

Let $x \in \bar{A} \cup \bar{B}$

$\cancel{\neg(x}$ $x \in \bar{A} \vee x \in \bar{B}$

$x \notin A \vee x \notin B$

$\neg(x \in A) \vee \neg(x \in B)$

$\neg(x \in A \cap x \in B)$

$\neg(x \in A \cap B)$

$x \in A \cap B$

$x \in \overline{A \cap B}$

$\Rightarrow \bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$

$\Rightarrow \overline{A \cap B} = \bar{A} \cup \bar{B}$

II Using set builder notation

$\overline{A \cap B} = \{x \mid x \in \overline{A \cap B}\}$

$= \{x \mid x \notin A \cap B\}$

$= \{x \mid \neg(x \in A \cap B)\}$

$= \{x \mid \neg(x \in A \cap x \in B)\}$

$= \{x \mid \neg(x \in A) \cup \neg(x \in B)\}$

$= \{x \mid x \in A \vee x \in B\}$
$= \{x \mid x \in \bar{A} \cup x \in \bar{B}\}$
$= \{x \mid x \in \bar{A} \cup \bar{B}\}$

III Membership Table

A	B	$A \cap B$	~~$\overline{AB}$~~ $\overline{A \cap B}$	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Multisets

$S = \{m_1 . a_1, m_2 . a_2, \ldots m_r . a_r\}$
m_i = multiplicity of element a_i
$i = 1$ to r.

eg $A = \{3a, 2b, 1c\}$
$A = \{a, a, a, b, b, c\}$

Operations on ~~set~~ multisets

Union - Union of two multisets A and B is the multiset where the multiplicity of an element is the maximum of its multiplicities in A and B.

Intersection - Intersection of two ~~set~~ multisets A and B is the multiset where multiplicity of each element is the minimum of its multiplicities in A or B.

Difference - Difference of two multisets A and B is the multiset of where the multiplicity of the an element ~~tm~~ is its multiplicity in A minus its multiplicity in B unless it comes out to be negative ~~the~~ then its is taken as zero.

Addition - The addition of two multiplicities w.r.t sum of ~~an~~ the multiplicity of an element in A added with multiplicity of its in B.

Q $A = \{3a, 2b, 1c\}$, $B = \{2a, 3b, 4d\}$

Ans- $A \cup B = \{3a, 3b, 1c, 4d\}$
$A \cap B = \{2a, 2b\}$
$A - B = \{1a, 0.b, 1c\}$
$B - A = \{1.b, 4.d\}$
$A + B = \{5a, 5b, 1c, 4d\}$

Q Suppose that A is the multiset has its elements the types of computer equipment needed by 1 department of university where the multiplicities are the no. of pieces of each type needed and B is the analogous multiset for a second department of university. let A = {107. Personal Computers, 44. routers, 6. servers} and B = {14. Personal computers, 6. routers, 2 mainframes} What combination of A and B represent the equipment i) the university should buy assuming both departments use the same equipments ii) that will be used by both departments if both departments use the same equipment iii) ~~that~~ the second department uses but the first department does not if both use same equipment iv) if they do not share equipments

Ans- 1) $A \cup B$ = { 107 PC, 44 routers, 6 servers, 2 mainframes }
$A \cap B$ = { 14 PC, 6 routers }
~~A - B =~~ { B - A = { 2 mainframes }

Fuzzy Set

A fuzzy set S is the set where each element in the universal set U has a degree of membership in the fuzzy set S. degree of membership is denoted by μ and it is a real number between 0 and 1 (both inclusive).
For eg, we have a set F representing the set of famous people and F = { 0.6 Alice, 0.8 Brian, 0.9 Oscar }
degree of membership element

~~Operations on Fuzzy~~ Sets

Complement - The complement of fuzzy set 'S' is '$\bar{S}$' in which the degree of membership of an element is 1 - degree of membership in set S.

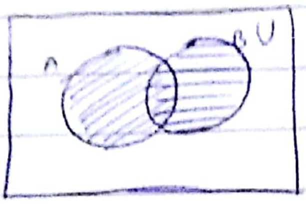
Union - Union of 2 fuzzy sets S and T is the fuzzy set in which each element has its degree of membership as the maximum of its degree of membership in S and T.

Intersection - Intersection of 2 fuzzy sets .. .. is minimum

Q F = { 0.6 Alice, 0.9 Brian, 0.4 Fred, 0.1 Oscar, 0.5 Rita }
R = { 0.4 Alice, 0.8 Brian, 0.2 Fred, 0.9 Oscar, 0.7 Rita }

Ans- $F \cup R$ = { 0.6 Alice, 0.9 Brian, 0.4 Fred, 0.9 Oscar, 0.7 Rita }
$\bar{F}$ = { 0.4 Alice, 0.1 Brian, 0.6 Fred, 0.9 Oscar, 0.5 Rita }
$F \cap R$ = { 0.4 Alice, 0.8 Brian, 0.2 Fred, 0.1 Oscar, 0.5 Rita }

Inclusion Exclusion principle

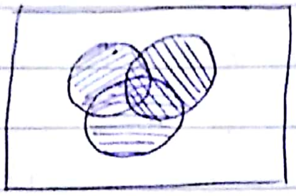


$|A \cup B| = |A| + |B| - |A \cap B|$

Subtraction rule

n_1 ways and n_2 ways

Common



$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| ...$

$2^n - 1$

$n = 2$, terms $2^2 - 1 = 3$

$n = 3$, terms $2^3 - 1 = 7$

Q How many bit strings of length 8 either start with a 1 bit or end with 0 & two bits 00.

Ans -

$\underline{1}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

$\underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{1 \text{ or } 0}\ \underline{0}\ \underline{0}$

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

$\underline{1}\ \_\ \_\ \_\ \_\ \_\ \underline{0}\ \underline{0} = 2^5$

$= 2^7 + 2^6 - 2^5 = 2^5(4 + 2 - 1)$

Q A computer company receives 350 applications from computer graduates for a job opening. 220 of these people majored in computer science 147 major in business and 51 major both in computer science and business. How many of these applicants major neither in computer science nor in business.

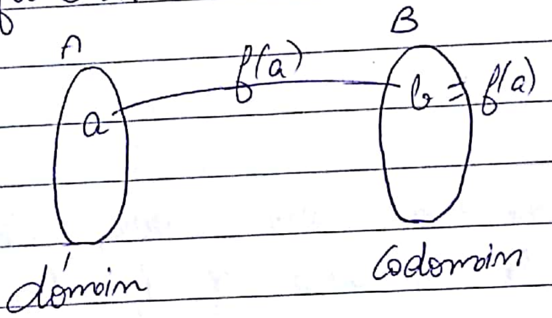
Ans- $A \cup B = 220 + 147 - 51$
$= 316$
$\overline{A \cup B} = 350 - 316 = 34$

Functions:

Let A and B be non empty sets. A function f ~~ff~~ from A to B is an assignment of exactly one element of B to each element of A. We say that ~~f(A ff~~ $f(a) = b$ if b is the unique element of B assigned by the function ~~f~~ F to the element a of A.

$f : A \to B$.

A function is also known as a mapping



One to one / injective functions

A function f is said to be one to one or injective iff ~~[illegible]~~ $f(a) = f(b) \Rightarrow a = b$ for all a and b in the domain of f. In other words an injective function never assigns the same value to two different domain elements.

Q Determine whether $f(x) = x^2$ from the set of integers to set of integers ~~[illegible]~~ is one to one or not.

Ans- No, $f(-2) = f(2)$ but $-2 \neq 2$.

Q Determine $f(x) = x+1$ is one to one or not.

Ans. Yes, let $x+1 = y+1 \Rightarrow x = y$.

Onto function or surjective function

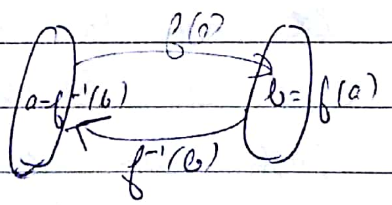
A function f from A to B is called onto or surjective iff for every element $b \in B$ $\exists$ $a \in A$ with $f(a) = b$.

Bijective functions or one to one correspondence

A function f is a one to one correspondence or a bijection if it is both one to one and onto.

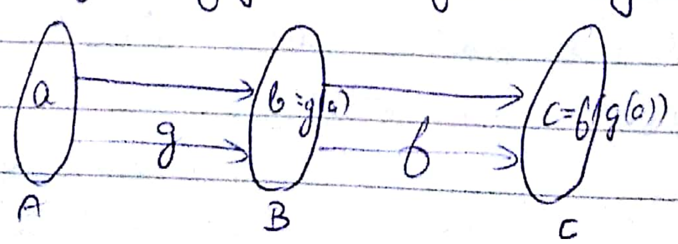
Inverse of a function

Let f be a one to one correspondence from set A to set B, the inverse of f denoted by f^{-1} is a function that to an element b of set B the unique element $a \in A$ such that $f(a) = b$.



Composition of functions

Let g be a function from A to B and f be a function from B to C. then composition of function f and g denoted by $f \circ g$ is defined by $f(g(a))$



For $f \circ g$ to exist, range of g should be equal to domain of f.

Q Let $g: \{a, b, c\} \rightarrow \{a, b, c\}$: $g(a) = b$, $g(b) = c$, $g(c) = a$
let $f: \{a, b, c\} \rightarrow \{1, 2, 3\}$; $f(a) = 3$, $f(b) = 2$, $f(c) = 1$. Find composition of f and g and composition of g and f.

Ans- $f \circ g$ = $f(g(a)) = f(b) = 2$
$f(g(b)) = f(c) = 1$
$f(g(c)) = f(a) = 3$

$g \circ f$ = $g(f(a)) = g(1)$ = Not defined
$g(f(b)) = g(2)$ = Not defined
$g(f(c)) = g(3)$ = Not defined
{ Range of f is not domain of g }

Q Let $f(x) = 2x+3$ and $g(x) = \cancel{g}\ 3x+2$. Find $f \circ g$ and $g \circ f$

Ans- $f(g(x)) = \cancel{f(2x+} \ f(3x+2) = 2(3x+2)+3 = 6x+7$
$g(f(x)) = g(2x+3) = 3(2x+3)+2 = 6x+11$

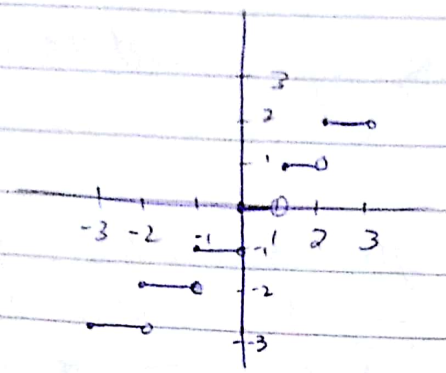
* When the composition of a function and its inverse is formed, we obtain an identity function.

$f(a) = b$, $f^{-1}(b) = a$

$f^{-1} \circ f(a) = f^{-1}(f(a)) = f^{-1}(b) = a$
$(f \circ f^{-1})(b) = f(f^{-1}(b)) = f(a) = b$

~~Floo~~ Floor function $\lfloor \ \rfloor$

$\lfloor 7.2 \rfloor = 7$
$\lfloor -7.2 \rfloor = -8$
$\lfloor 7 \rfloor = 7$



Ceil function: $\lceil \ \rceil$

$\lceil 7.2 \rceil = 8$

$\lceil -7.2 \rceil = -7$

$\lceil 7 \rceil = 7$

Q Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes each byte is made up of 8 bits. How many bytes are required to infer 100 bits of data.

Ans- $\frac{100}{8} = 12.5$

$\lceil 12.5 \rceil = 13$

Q In asynchronous transfer mode, data are organised into cells of 53 bytes. How many cells can be transmitted in one minute over a connection that transmits data at the rate of 500 ~~kbt~~ kilobits/sec.

Ans- $= \frac{500 \times 1000}{8 \times 53} \times 60$

$= \lfloor 70754.7 \rfloor$

$= 70754$

Relations

A relation is a a subset of the cartesian product. Let A and A be two sets then a relation R from A to B is a subset of $A \times B$.

Representing relations:

$A = \{1, 2, 3\}$, $B = \{a, b, c\}$ $R = \{(1,a), (2,a), (3,b), (3,c)\}$

It can be represented by table, matrix, digraph/directed graph.

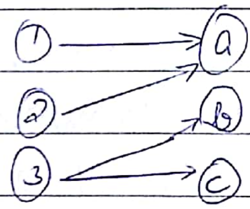
Table

R	a	b	c
1	X		
2	X		
3		X	X

Matrix

$$\begin{array}{c|ccc} R & a & b & c \\ \hline 1 & & & \\ 2 & & & \\ 3 & & & \end{array} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

digraph



Q How many relations are there on a set with n elements.

Ans- 2^{n^2}

Q How many reflexive relations are possible on a set with n elements?

Ans-

Properties of relations

1. Reflexive – A relation R on set A is called reflexive if $(a,a) \in R$, $\forall a \in A$.

2. Irreflexive – A relation R on set is said to be irreflexive if none of the elements of set A is related to itself.

$A = \{1, 2, 3\}$

$R = \{(1,1), (1,2), (2,2), (2,3), (3,1)\}$

Not reflexive. Not irreflexive because two elements have, but ...

3. Symmetric - A relation R on set A is said to be symmetric if $(b,a) \in R$ whenever $(a,b) \in R$

4. Assymetric - A relation R on set A is said to be assymetric if $(a,b) \in R$ then $(b,a) \notin R$.

5. Anti symmetric - A relation R on set A is said to be anti symmetric if $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$.

6. Transitive - A relation R on set A is said to be transitive if whenever $(a,b) \in R$ and (b,c) then $(a,c) \in R \; \forall \; a, b, c \in A$.

Q Consider $A = \{1, 2, 3, 4\}$ $R = \{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$
Check.

Ans- Not reflexive, Not irreflexive
Not symmetric, Not assymetric, Not anti symmetric
Transitive

Q $R = \{(2,4), ~~and~~ (4,2)\}$

Ans- Irreflexive, Symmetric

Equivalence relation and partial order

A relation which is reflexive, symmetric and transitive is known as equivalence relation.

A relation which is reflexive, antisymmetric ~~assym~~ and transitive is known as partial order.

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Q Let m be a positive integer. Show that relation $R = \{(a,b) \mid a \equiv b \pmod m\}$ is an equivalence relation on the set of integers, where $\equiv$ is congruence modulo m
$\Rightarrow (a-b)$ is divisible by m.

Ans- $(a,a) \in R$ as $a-a = 0$ divisible by m. $\Rightarrow R$ is reflexive

Let $(a,b) \in R$ to prove $(b,a) \in R$.

$a-b =$ divisible by m.

$b-a = -(a-b)$ is also divisible by m.

So $(b,a) \in R$. $\Rightarrow$ It is symmetric

Let $(a,b) \in R$ and $(b,c) \in R$ To prove $(a,c) \in R$

$a-b =$ divisible by m -(1) $\Rightarrow a-b = km$

$b-c =$ divisible by m -(2) $\Rightarrow b-c = lm$

$a-c =$ divisible by m. $\Rightarrow a-c = m(k+l)$

$\therefore$ It is transitive.

Since it is reflexive, symmetric and transitive.

So it is ~~eqre~~ equivalence relation.

Equivalence classes

Let R be an equivalence relation on set A. The set of all elements that are related to an element a of A is called the equivalence class of a.

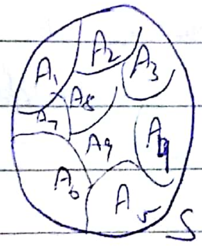
Partition of a set S

It is a collection of disjoint non empty subsets of S that have S as their union. The collection of subsets A_i forms a partition of S iff

i) $A_i \neq 0$

ii) $A_i \cap A_j = \phi$ where $i \neq j$.

iii) $\cup A_i = S$



Q eg. Suppose that $S = \{1,2,3,4,5,6\}$. The collection of sets $A_1 = \{1,2,3\}$, $A_2 = \{4,5\}$, $A_3 = \{6\}$ is taken under consideration. Determine whether A_1, A_2, A_3 form a partition or not.

Ans- $A_1 \cup A_2 \cup A_3 = S$

So it they are partition.

<u>Partial ordering</u>

A relation R on set S is called partial order if it is reflexive, antisymmetric and transitive. A set S together with its partial ordering are is called partially ordered set or poset and is denoted by (S, R).

Q For example, show that the relation $\geq$ is a partial ordering on the set of integers.

Ans- $a \geq a$. So it is reflexive.

If $a \geq b$, $b \geq c \Rightarrow a \geq c$. So transitive.

If $(a,b) \in R$ and $(b,a) \in R$ and $a = b$.

$a \geq b$, $b \geq a \Rightarrow a = b$. So it is antisymmetric.

So it is partial ordering.

Q Is the relation a divides b, a partial ordering on the set of integers?

Ans- $\frac{a}{a}$. So it is reflexive.

If a divides b and b divides c $\Rightarrow$ a divides c. So transitive.

a divides b and b divides a. $\Rightarrow a = b$. So antisymmetric

$\therefore$ It is partial ordering

~~Comparable and uncomparable sets~~

The elements a and b of a poset $(S, \leq)$ are called comparable, if either $a \leq b$ or $b \leq a$. When a and b are elements of S such that neither $a \leq b$ nor $b \leq a$.

Q In the poset $(Z^+, \text{divisibility} (|))$. Are 3 and 9 comparable. What can be said about ~~5~~ 9 and 7.

Ans- 3 and 9 are comparable because $3|9$.
5 and 7 are uncomparable.

Totally ordered set or Linearly ordered set

If (S, ~~≤~~ partial ordering) is a poset and every two elements of S are comparable then S is called a totally ordered or linearly ordered set. It is also known as a chain.

Q ~~If~~ Is poset $(Z, \leq)$ a chain?
Ans. First check poset ~~is~~ if not given.
Every two elements of integers are comparable. So it is a chain.

Inverse relation ~~and complimentary relation~~

An inverse relation are inverse of relation R can be defined by $R^{-1} = (b, a)$ such that $(a, b) \in R$

Q $R = \{(a, b) \mid a \leq b\}$. Find R^{-1}.
Ans- $R^{-1} = \{(b, a) \mid a \leq b\}$ or $R^{-1} = \{(a, b) \mid b \leq a\}$

Complimentary relation
The relation $\bar{R}$ of a relation R ~~is equal to~~
$\bar{R} = \{(a, b) \mid (a, b) \notin R\}$
Q $R = \{(a, b) \mid a \leq b\}$. Find $\bar{R}$.
Ans- $\bar{R} = \{(a, b) \mid a > b\}$

1. For reflexive, matrix representation should have its diagonal elements as 1.
2. For symmetric, $M_R = M_R^t$

Q $A = \{1,2,3\}$, $B = \{1,2,3,4\}$
$R_1 = \{(1,1),(2,2),(3,3)\}$
$R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$
Find $R_1 \cup R_2$ and $R_1 \cap R_2$? Find $R_1 - R_2$ and $R_2 - R_1$

Ans- $R_1 \cup R_2 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,4)\}$
$R_1 \cap R_2 = \{(1,1)\}$
$R_1 - R_2 = \{(2,2),(3,3)\}$
$R_2 - R_1 = \{(1,2)(1,3),(1,4)\}$

$$M_{R_1} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & & & \\ 2 & & 1 & & \\ 3 & & & 1 & \end{array} \qquad M_{R_2} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & & & & \\ 3 & & & & \end{array}$$

$$M_{R_1 \cap R_2} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & & & \\ 2 & & & & \\ 3 & & & & \end{array} = \{(1,1)\}$$

Composition of relations

$M_{S \circ R} = M_R \odot M_S$

Let R be a relation from A to B and S be a relation from B to C. The composite of R and S is a relation consisting of ordered pairs (a, c) where $a \in A$ and $c \in C$ and for which there exists an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$. The composite of R and S is denoted by $S \circ R$.

$R^2 = R \circ R$

$R^3 = R^2 \circ R$

$M_{R^3} = M_{R^2} \circ M_R$

Q. Find the matrix representing the relation $S \circ R$ where the matrices representing R and S are $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

Ans- $M_{S \circ R} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Q. Closures of relations

1. Reflexive closure
2. Symmetric closure
3. transitive closure.

Reflexive closure- The reflexive closure of R can be formed by adding to R all pairs of the form (a,a) with $a \in A$.

$\Delta = \{(a,a) \mid a \in A \rightarrow$ diagonal relation $\}$

$Ref(R) = R \cup \Delta$

Q. $R = \{(a,b) \mid a < b\}$ on set of integers. Find $Ref(R)$.

Ans- $Ref(R) = R \cup \Delta$

$= \{(a,b) \mid a < b\} \cup \{(a,a) \mid a \in A\}$

$= \{(a,b) \mid a \leq b\}$

Q. Find $Ref(R)$, $R = \{(1,1),(1,2),(2,1),(3,2)\}$, $A = \{1,2,3\}$

Ans- $Ref(R) = \{(1,1),(1,2),(2,1),(3,2),(2,2),(3,3)\}$

Symmetric closure - The symmetric closure of a relation R can be formed by adding all the ordered pairs of the form (b, a) whenever $(a, b) \in R$.

$R = \{(a, b) \mid a < b\}$

$R^{-1} = \{(b, a) \mid a < b\}$

$Sym(R) = R \cup R^{-1}$

$= \{(a, b) \mid a < b\} \cup \{(b, a) \mid a < b\}$

$= \{(a, b) \mid a \neq b\}$

Q $R = \{(1,1), (1,2), (2,1), (3,2)\}$ Find $Sym(R)$

Ans- $Sym(R) = \{(1,1), (1,2), (2,1), (3,2), (2,3)\}$

Transitive Closure

Q $R = \{(1,1), (1,2), (2,1), (3,2)\}$, Find $Tran(R)$.

Ans- $R: \{(1,1), (1,2), (2,1), (3,2), (3,1), (2,2)\}$

Two methods to find transitive closure

1. Using Connectivity relation R^* - Let R be a relation on a set A. The connectivity relation R^* consist of the pairs (a, b) such that there is a path of length atleast 1 from A to B in R.

$R' = \{(a, b) \mid \text{path length} = 1\}$

$R^2 = \{(a, b) \mid \text{path length} = 2\}$

$R^n = \{(a, b) \mid \text{path length} = n\}$

$R^* = \bigcup_{n=1}^{\infty} R^n$

Let M_R be the 0-1 matrix of relation R on set with n elements then the 0-1 matrix of the transitive closure of or the connectivity relation R^* is $M_{R^*} = M_R$

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \dots \vee M_R^{[n]}$$

Q Find the 0 1 matrix of the transitive closure of relation R where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]}$$

$$R^2 = M_R \odot M_R$$

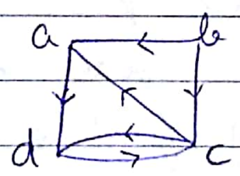
$$M_R^{[2]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$ – The place where element is other than 0, it becomes 1.

$$M_R^{[3]} = M_R^{[2]} \odot M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

2. Warshall ~~algo~~ algorithm



$$W_0 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

[Using only a]

$$W_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & \boxed{1} \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

using [a,b]

$$W_2 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} = W_1$$

$$W_3(a,b,c) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ \boxed{1} & 0 & 1 & \boxed{1} \end{bmatrix}$$

$$W_4(a,b,c,d) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} \boxed{1} & 0 & \boxed{1} & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & \boxed{1} & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Lexicographic ordering

It is same as ordering done in dictionary. Lexicographic ordering on $A_1 X A_2$ is defined by specifying that one pair is less than the second pair if the first entry of first pair is less than the first entry of second pair. or in case if the first ~~two~~ entries are equal ~~two~~ in both pairs then the second entry of first pair should be less than the second entry of second pair.

$(a_1, a_2) < (b_1, b_2)$ if $a_1 < b_1$ or $a_2 < b_2$ if $a_1 = b_1$

e.g.- $(1,2) < (2,1)$

$(1,2,3) < (1,2,4)$

discrete < discretion

discreet < discrete

Hasse diagrams

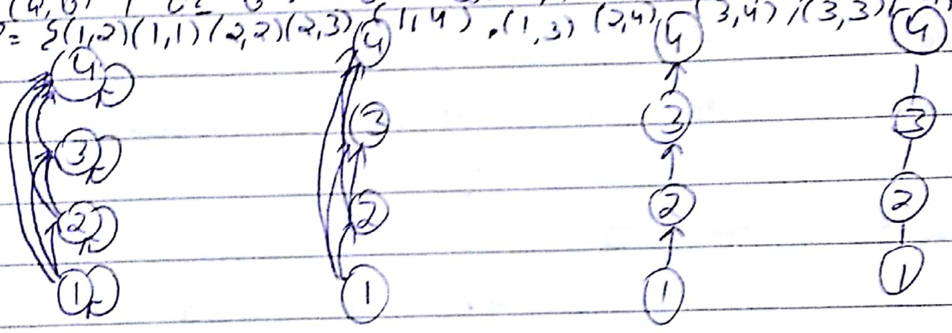
These are created for directed graphs of a finite posets. Steps to draw:

1. Create a directed graph for a poset.
2. Remove all the loops from the graph.
3. Remove all transitive edges
4. Redraw graph such that all the edges point upwards
5. Remove the arrows from the edges.

Q Draw Hasse diagram representing the partial ordering $\{(a,b) \mid a \le b\}$ on set $A = \{1,2,3,4\}$.

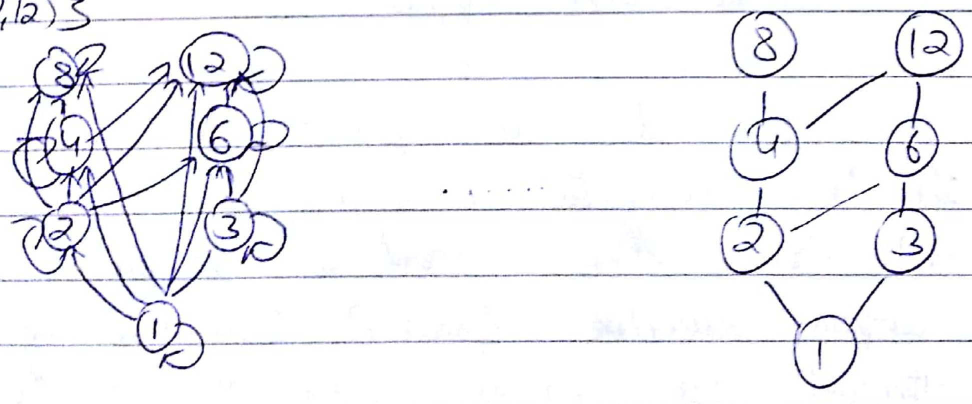
$R = \{(1,2)(1,1)(2,2)(2,3),(1,4),(1,3)(2,4),(3,4),(3,3),(4,4)\}$

Ans-



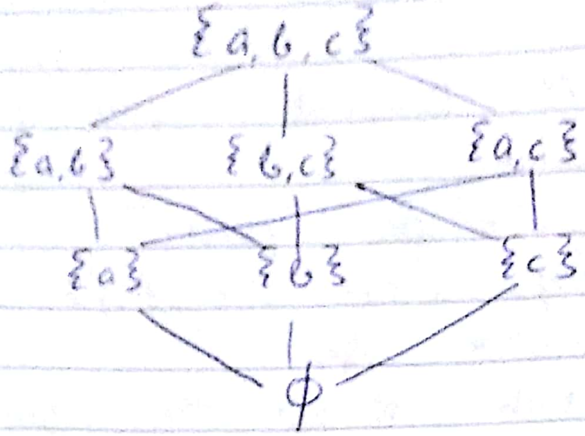
Q Draw Hasse diagram representing partial ordering $\{(a,b) \mid a \text{ divides } b\}$ on the set $\{1,2,3,4,6,8,12\}$

Ans- $R = \{(1,2),(1,3),(1,4),(1,6),(1,8),(1,12),(2,4),(2,6),(2,8),(2,12),$
$(3,6),(3,12),(4,8),(4,12),(6,12),(1,1),(2,2),(3,3),(4,4),(6,6),(8,8),$
$(12,12)\}$



Q Draw Hasse diagram for partial ordering $\{(a,b) \mid A \subseteq B\}$ on the power set $P(S)$ where $S = \{a,b,c\}$.

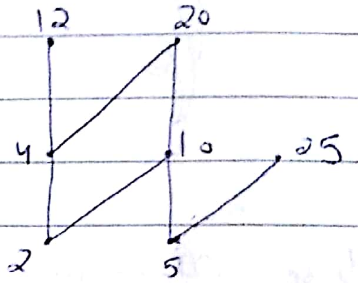
Ans-



Q Which elements of the poset are maximal and minimal { (2, 4, 5, 10, 12, 20, 25), | a divides b }

Ans- * Maximal - When no element is above it.
* Minimal - When no element is below it.

~~Maximal - 2 and 5~~



Maximal - 12, 20, 25

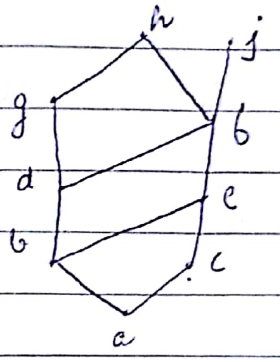
Minimal - 2, 5

<u>Lower bounds and upper bounds</u>

If an element is greater than or equal to all the elements in a subset A of poset (S, ≤) then this element is known as the upper bound of subset A. Similarly if there is an element less than or equal to all the elements in A then such an element is known as the lower bound of subset A. Upper bounds or lower bounds can be more than 1 in num

Least upper bound (LUB) is the smallest of all the upper bounds of A and Greatest lower Bound (GLB) is the greatest of all the lower bounds.

Q Find lower and upper bounds of subset {a, b, c}, {j, h} and {a, c, d, f} and {g, e}



Ans- {a, b, c} - LB - a
UB - e, f, h, j
LUB = e
GLB = a

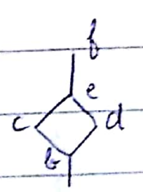
{j, h} - UB - No
LB - f, d, b, e, a, c
GLB - f
LUB - No

{a, c, d, f} - LB - a
UB - f, j, h
GLB - a
LUB - f

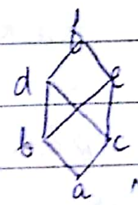
{g, e} - LB - a, b, UB - h, GLB - b LUB - h

Lattices

A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a lattice.



It is lattice



No least upper bound

* Maximal, minimal, LB, UB can be more than 1 elements
* Greatest, least, GLB, LUB - unique

Q Is the poset $(Z^+, |)$ divisibility a lattice?

Ans. Yes

Q Determine whether the posets $\{(1,2,3,4,5), |\}$ and $\{(1,2,4,8,16), |\}$ are lattices or not.

Ans - First is not. Second is lattice.

Q Determine whether $(P(S), \subseteq)$ is a lattice or not where S is a set.

Ans - Let A and B be any 2 subsets of S.

$A \cup B \rightarrow LUB$

$A \cap B \rightarrow GLB$

~~Boun~~

Bounded lattice

A lattice is said to be bounded if it has a least element and a greatest element. These bounds are represented by 0 and 1.

Q Is the lattice Z^+ under the partial order of divisibility bounded

Ans - It is not bounded as it has 1 as the least

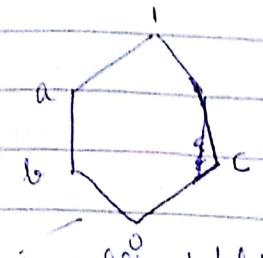
Complimented lattice

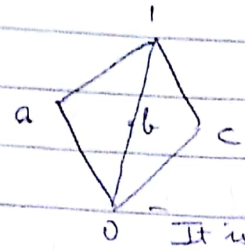
If c is the complement of a then

ca ~~GLA~~ $= 0$ (smallest)

$c + a$ ~~GLA~~ $= 1$ (greatest)

A lattice L is complemented if it is bounded and every element has atleast 1 complement. An element can have multiple compliments and two elements ~~complements~~ can have same compliment ~~element~~ as well.

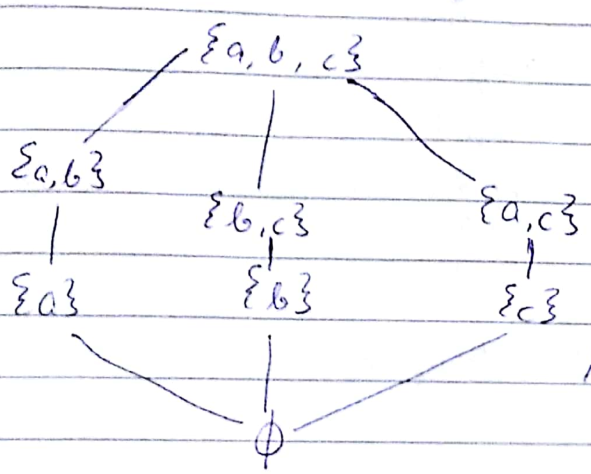




It is complimented lattice
Compliment of $a = c$
compliment of $b = c$
Compliment of $c =$ a and b both

It is complimented lattice
Comp $a =$ b and c
Comp $b =$ a and c
Comp $c =$ a and b

Q



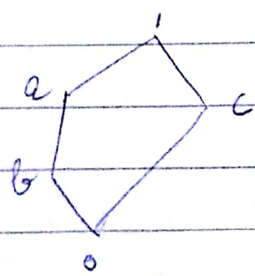
$(P(S), \subseteq)$
A and B be any two subsets.

Ans- $A \cup$ its ~~comp~~ comp $= S$
$A \cap$ its comp $= \phi$

Unique complementary lattice
It is a complimented lattice in which every element has a unique ~~compliment~~ compliment.

Distributive lattice
It ~~is~~ satisfies following two laws, ~~{ x ∧ (y ∨ z)}~~ :
$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Compliments should be unique in a distributive lattice.

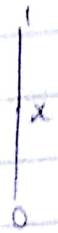


a compliment - c
b compliment - ~~b~~ c
c compliment - a and b both (not unique)
Not a distributive lattice.
No. $a \wedge (b \vee c) = a \wedge 1 = a$ and $(a \wedge b) \vee (a \wedge c) = b \vee 0 = b$ So not distributive

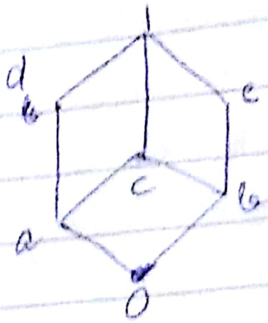
Boolean lattice

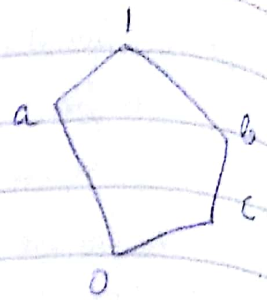
If a lattice is distributive and complimented lattice then it is called a boolean

0 comp 1
1 comp 0



x is not comp 0





a comp e
b comp d
c comp doesn't exist
d comp e, b
e comp d, a

N ary relations and database

S.Name	RollNo	Subject	CGPA
x	23	OS	6.5
y	20	CN	5
z	15	DBMS	6

record

n tuples

field

Student	ID No	Major
Pokemon	231	CS
Adams	888	Phys
Chon	102	CS
good friend	453	maths
Rao	678	maths
Stevens	786	Ph/psych

Operations on N ary relations

1. selection (S_c) 2. Projection (P_c) 3. Join (J_p)

Scanned by CamScanner

Professor	Dept	Course No	Course Name	Course No	Room No	Time
Guz	OS	U301	OS	U301	A101	8AM
Guz	OS	P301	OS	P301	B101	9AM
Forber	CN	U402	CN	U402	A102	8AM
Forber	CN	P402	CN	P402	B102	10AM
Grammer	DS	U406	DS	U406	A103	10AM
Grammer	DS	P406	DS	P406	B103	11AM
Rose	DM	U401	DM	U401	A104	8AM
Rose	DM	P401	DM	P401	B104	10AM

Professor	Course Name	Course No	Room No	Time
Guz	OS	U301	A101	8AM
Guz	OS	P301	B101	9AM

Boolean Algebra.

Sum

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

Product

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

Complement

$\bar{0} = 1$

$\bar{1} = 0$

Q Find value of $1.0 + (\overline{0+1})$

Ans- $0 + \bar{1} = 0 + 0 = 0$

Q $1.0 + (\overline{0+1}) = 0$ Convert into logical expression.

Ans- $(T \wedge F) \vee \neg(F \vee T) \equiv F$

SOP (Sum of Products) - Sum of minterms - product of literals. Output (1) is considered.

variable or its compliment

1 - as it is.

0 - variable complement.

A	B	F	Minterms		
0	0	0	$\bar{A}\bar{B}$	0	m_0
0	1	1	$\bar{A}B$	1	m_1
1	0	0	$A\bar{B}$	2	m_2
1	1	1	AB	3	m_3

$$F = \bar{A}B + AB$$

$$= \Sigma(1,3) \Rightarrow m_1 + m_3$$

POS (Product of Sum) - product of maxterms. output 0 is considered.

0 as it is.

1 variable complement.

A	B	F	Maxterms		
0	0	0	$A+B$	0	m_0
0	1	1	$A+\bar{B}$	1	m_1
1	0	0	$\bar{A}+B$	2	m_2
1	1	1	$\bar{A}+\bar{B}$	3	m_3

$$F = \cancel{(A+B).(\bar{A}+\bar{B})}\ (A+B).(\bar{A}+B)$$

$$= \Pi(0,2) \Rightarrow M_0 . M_2$$

Q Find a minterm that equals 1 if $x_1 = x_3 = 0$ and $x_2 = x_4 = x_5 = 1$.

Ans-

x_1	x_2	x_3	x_4	x_5	F
0	1	0	1	1	

$$\text{minterm} = \bar{x}_1 x_2 \bar{x}_3 x_4 x_5$$

* SoP and PoS must contain all the element.

Q Find SOP expansion of the function $F(x,y,z) = (x+y)\bar{z}$

Ans- $F(x,y,z) = (x+y)\bar{z}$

$= x\bar{z} + y\bar{z}$

$= x\bar{z}.1 + y\bar{z}.1$

$= x\bar{z}(y+\bar{y}) + y\bar{z}(x+\bar{x})$

$= x\bar{y}\bar{z} + xy\bar{z} + xy\bar{z} + \bar{x}y\bar{z}$

$= x\bar{y}\bar{z} + xy\bar{z} + \bar{x}y\bar{z}$

Minimization using K-map

x\y	0	1
0		
1		

x\yz	00	01	11	10
0				
1				

wx\yz	00	01	11	10
00				
01				
11				
10				

Q Use K maps to simplify the following expressions.

$wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}\bar{z}$

wx\yz	00	01	11	10
00	1			1
01	1			
11	1			
10	1		1	1

$F = \bar{y}\bar{z} + w\bar{x}y + \bar{x}\bar{z}$

* For corners $\bar{x}\bar{z}$

Q

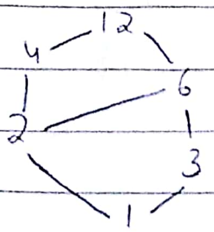
wx\yz	00	01	11	10
00	0			0
01	0			
11	0			
10	0		0	0

$F = (y+z)(\bar{w} + x + \bar{y}).(x + z)$

Covering relation

$\{(1, 2, 3, 4, 6, 12), |\}$

Covering relation = $\{(1,2), (1,3), (2,4), (2,6), (3,6), (4,12), (6,12)\}$



Hashing function

It is used to assign memory locations to records ~~on~~ & files. Most common hashing function is $h(k) = k \bmod m$ where k is the key of the record and m is the number of available memory location.

Properties of hashing function

1. It should be easy to evaluate.
2. It should be onto so that all memory locations can be assigned.
3. It is not one to one.

$$\underbrace{h(k)}_{\text{key of record}} = k \bmod \underbrace{m}_{\text{no of available memory locations}}$$

eg - Roll no - 11, 12, 13, 14, 15, 21, 22, 26

m - 10

0	
1	11
2	12
3	13
4	14
5	15
6	
7	
8	
9	

For 21, it will take location 1 but it is filled already. So collision.

Methods to resolve collision

1. Linear probing - will add +1 till it finds empty space.
2. Quadratic probing - will add $+1^2$ then $+2^2$ ~~then~~ $+3^2$
3. Double hashing - not in detail, data structures.

Recursive functions

A recursive function is a function which is defined in terms of itself.

eg- $factorial(n+1) = (n+1) \cdot factorial(n)$

$factorial(0) = 1$

eg- fibonacci : $F(n) = F(n-1) + F(n-2)$

$F(0) = 0$, $F(1) = 1$

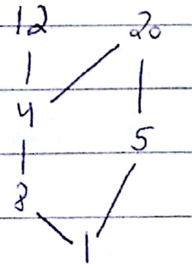
Q Suppose that f is defined recursively by $f(0) = 3$, $f(n+1) = 2f(n) + 3$. Find $f(1), f(2), f(3)$

Ans- $f(1) = 2f(0) + 3 = 9$

$f(2) = 21$

$f(3) = 45$

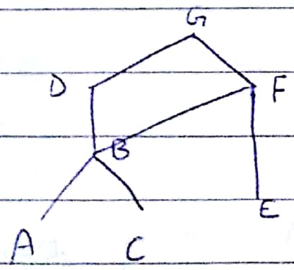
Topological sorting



(Remove all the edges from minimal elements and so on)

Output: 1→5 2→4→12 20

All directions will be from left to right if you want to check

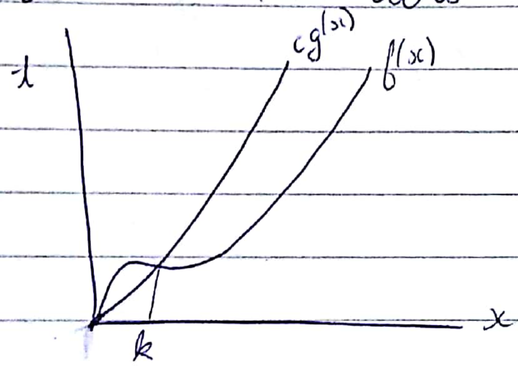


Output A C E B→F D→G

Growth of functions

Big-O notation $\leq$ Little-o $<$
Big-omega notation $\geq$ little omega $>$
Big-theta notation

Let f and g be two functions from the set of integers or the set of real numbers to the set of real numbers, we say that $f(x)$ is $O g(x)$ if there are positive constants c and k such that $f(x) \leq c(g(x))$ whenever $x > k$. c and k are known as witnesses and these need not be unique.
Big O notation gives the upper bound of the function i.e. the worst case.



~~Cannot be a positive~~

Method to find Big O.

Replace all the smaller order terms with the highest order term present in the function.

Q. Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$.

Ans- $x^2 + 2x + 1 \leq c \cdot x^2$

$x^2 + 2x + 1 \leq x^2 + 2x^2 + x^2$

$x^2 + 2x + 1 \leq \underset{c}{4} x^2, \; x \geq \underset{k}{1}$

$\Rightarrow f(x)$ is ~~Big~~ $O(x^2)$ for $c = 4, \; k = 1$

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Q Show that $7x^2$ is Big O of x^3.

Ans- $7x^2 \le c.x^3$

$c=1, \; x\ge 7$

$c=7, \; k=1 \; x\ge 1$

Q Is it also true that x^3 is $O(7x^2)$

Ans- $x^3 \le c(7x^2)$

$x \le 7c$

No values of c and k exist for which these inequations hold

Q Show that n^2 is not $O(n)$

Ans- $n^2 \le c(n)$

$n \le c$

No value of c and k exist for which these inequations hold.

* Leading term of a polynomial dominates its growth so a polynomial of degree n or less is big O of x^n. e.g. $7x^3+2x^2+3$ $O(x^3), O(x^4)$

Q Find O estimate of the sum of first n positive integers

Ans- $\frac{(n)(n+1)}{2} = \frac{n^2+n}{2}$

$\therefore O(n^2)$

$c=1, k=1$

Q Give O estimate for factorial function and log(factorial)

Ans- $f(n) = n!$

$= n(n-1)(n-2)\ldots 1$

$\le n^n$

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$\log(n!) = \log(1 \cdot 2 \cdot 3 \ldots (n-1)(n))$
$= \log 1 + \log 2 + \ldots \log(n-1) + \log n$
$\leq n \log n$
$O(n \log n)$, $c = 1$, $k = 2$

if
$f_1(x) = O(g_1(x))$ and $f_2(x) = O(g_2(x))$,
then $(f_1 + f_2)(x) = O(\max(|g_1(x)|, |g_2(x)|))$
if $f_1(x) = f_2(x) = O(g(x))$,
then $(f_1 + f_2)(x) = O(g(x))$,
if $f_1 = O(g_1(x))$ and $f_2 = O(g_2(x))$
then $(f_1 f_2)(x) = O(g_1(x) g_2(x))$

Q $f(n) = 3n \log(n!) + (n^2 + 3) \log n$
Ans- $3n = O(n)$
$\log n! = O(n \log n)$
$3n \log(n!) = O(n^2 \log n)$
$n^2 + 3 = O(n^2)$
$\log n = O(\log n)$
$(n^2 + 3) \log n = O(n^2 \log n)$
$\therefore 3n \log(n!) + (n^2 + 3) \log n = O(n^2 \log n)$

Q $f(x) = (x+1) \log(x^2+1) + 3x^2$
Ans. $x + 1 = O(x)$
$\log(x^2+1) = O(\log \overset{(x)}{\cancel{(x^2+1)}})$
$3x^2 = O(x^2)$
$(x+1) \log(x^2+1) = O(\cancel{x \log(x^2+1)})$ $O(x \log x)$
$(x+1) \log(x^2+1) + 3x^2 = O(x^2)$

* $1, \log n, n, n \log n, n^2, 2^n, n!$

Big Omega Ω notation

Let f and g be functions from set of +ve integers to set of real numbers, We say that $f(x)$ is $\Omega\, g(x)$ if there are positive constants c and k such that $f(x) \geq c\, g(x)$ ~~where~~ whenever $x > k$.

Ω gives us the lower bound for the function or the best case.

Q Show that $f(x) = 8x^3 + 5x^2 + 7$. is $\Omega(x^3)$.

Ans- $8x^3 + 5x^2 + 7 \geq 8x^3$, $x \geq 1$

$c = 8$, $k = 1$

Q Can we say that x^3 is $O(8x^3 + 5x^2 + 7)$?

Ans- Yes, $x^3 \leq c(8x^3 + 5x^2 + 7)$

Big Theta Θ notation

Let f and g be functions from set of integers or the set of real numbers to the set of real numbers, we say that $f(x)$ is $\Theta(g(x))$. If $f(x)$ is $O(g(x))$ and $f(x)$ is $\Omega(g(x))$ in such a case we say that $f(x)$ is of the order $g(x)$. Θ gives the average case or tight bound of the function.

*order means talking of Θ

Q Show that $3x^2 + 8x\log x$ is $\Theta(x^2)$

Ans. $3x^2 - O(x^2)$

$x\log x - O(x\log x)$

$3x^2 + 8x\log x = O(x^2)$

O . $3x^2 + 8x\log x \leq 3x^2 + 8x^2$

$\leq 11x^2$

c_1

$\Omega \quad 3x^2 + 8x\log x \geq \underset{c_2}{3}x^2 \quad , x \geq 1$

$c_1(g(x)) \leq f(x) \leq c_2(g(x))$

To find Θ find O c_1 and k, Ω c_2 and k, Θ c_1, c_2 and k

$3x^2 \leq 3x^2 + 8x\log x \leq 11x^2$

$c_1 = 3, \; c_2 = 11, \; k =$

1. The leading term of a polynomial determines its order
2. When $f(x)$ is $\Theta(g(x))$, it also means that $g(x)$ is $\Theta(f(x))$.
3. $f(x)$ is $\Theta(g(x))$ iff $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$

<u>Complexity of algorithms</u>

Time complexity — worst case O / avg case Θ / ~~best~~ best case Ω

Space

```
sum of list (A, n)
{
   total = 0;
   for i = 0 to n;
      total = total + A;
   return total;
}
```

cost	No. of times
1	1
2	n+1
2	n
1	1

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Propositional logic - propositions
Predicate logic

Propositions are declarative statements / sentences which are either true or false but not both.
eg p: Chandigarh is the capital of Punjab (✓) True
Chandigarh is the capital of Himachal (X) False
$1+1 = 2$ True
$x+1 = 2$ Not a proposition

Operators / operations on propositions

1. Negation - If we have a proposition p then negation p is denoted by $\neg p$ or $\bar{p}$ and it means 'it is not the case that p'. p $\neg p$, T - F, F - T

2. Conjunction $\wedge$ - $p \wedge q$ eg - p: Today is Friday, q: It is raining
$p \wedge q$: Today is friday and it is raining

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Disjunction $\vee$ - $p \vee q$: Today is friday or it is raining.

4. Exclusive OR, $p \oplus q$ - It is true when exactly one of p and q is true and is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

5. Conditional statement or implication, $p \to q$ if p then

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Also known as p implies q and; p only if q; q if p; q when p; q whenever p; q unless not p; if p, q.

Q Let p be the statement Mary learns discrete maths and q: Mary will find a good job. Express $p \to q$ in english

Ans- If Mary learns discrete maths, then she will find a good job

6. Biconditional statement / bi-implication / double implication $p \leftrightarrow q$

$p \leftrightarrow q \Rightarrow (p \to q) \cap (q \to p)$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	F
F	F	T	T	T

Converse contra position and inverse of proposition

If $p \to q$ is a proposition then its converse is $q \to p$, Contrapositive $= \neg q \to \neg p$.

inverse $\neg p \to \neg q$

Q What are the contrapositive, converse and inverse of statement 'The home team wins whenever it is raining.

Ans- It is q whenever p. p: It is raining q: the home team wins.

Converse - If the home team wins then it is raining

Contrapositive - If home team does not win, then it is not rain

Inverse - If it is not raining then home team does not win

Q **Logical equivalences**

The compound propositions p and q are called logically equivalent if they have same truth value in all the cases or if $p \leftrightarrow q$ is tautology.

Q Show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

Ans-

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$	$P \leftrightarrow Q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Q Show that $\neg(p \rightarrow q)$ and $p \wedge \neg q$ are logically equivalent without using truth table or using a series of logical equivalence.

Ans-
$$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$$
$$\equiv \neg(\neg p) \wedge \neg q$$
$$\equiv p \wedge \neg q$$

Q Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by series of logical equivalencies.

Ans-
$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$$
$$\equiv \neg p \wedge (p \vee \neg q)$$
$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$
$$\equiv F \vee (\neg p \wedge - q)$$
$$\equiv (\neg p \wedge \neg q)$$

$p \wedge T \equiv p$
$p \vee F \equiv p$ } identity laws

$p \vee T \equiv T$
$p \wedge F \equiv F$ } domination laws