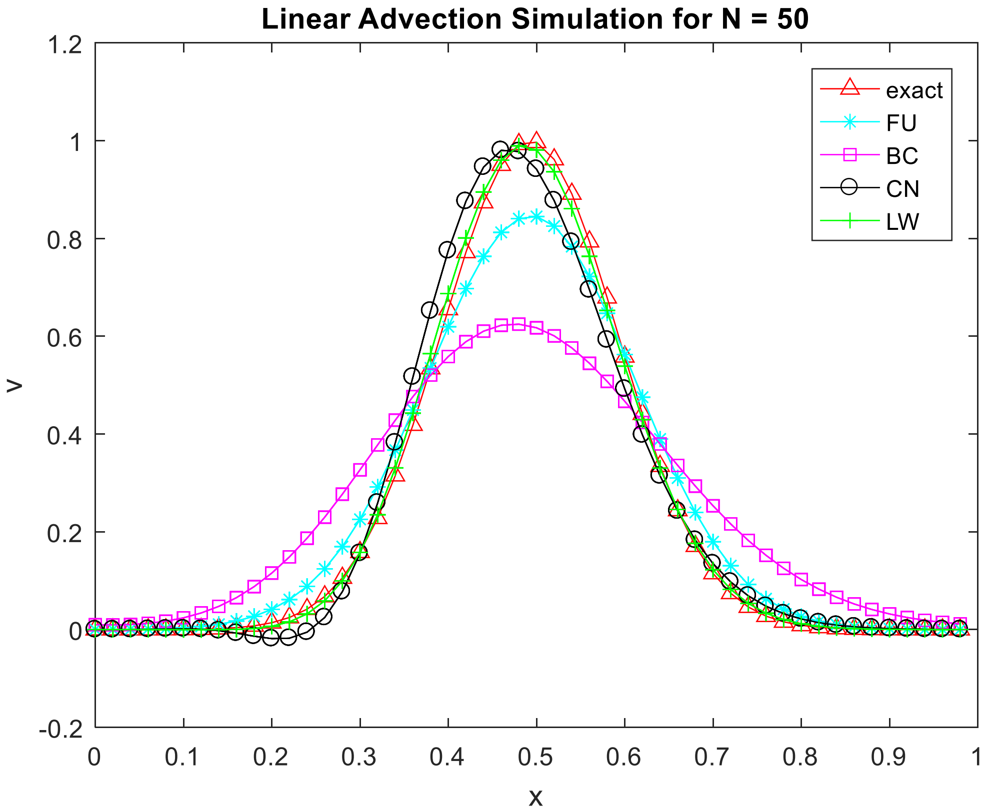
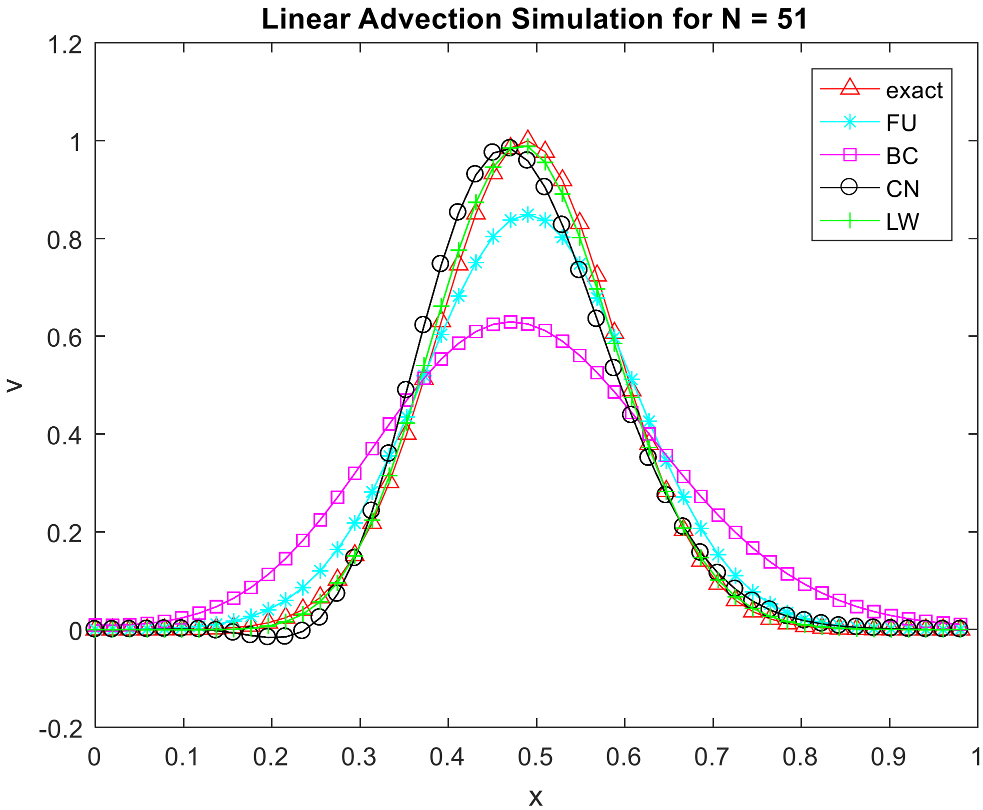
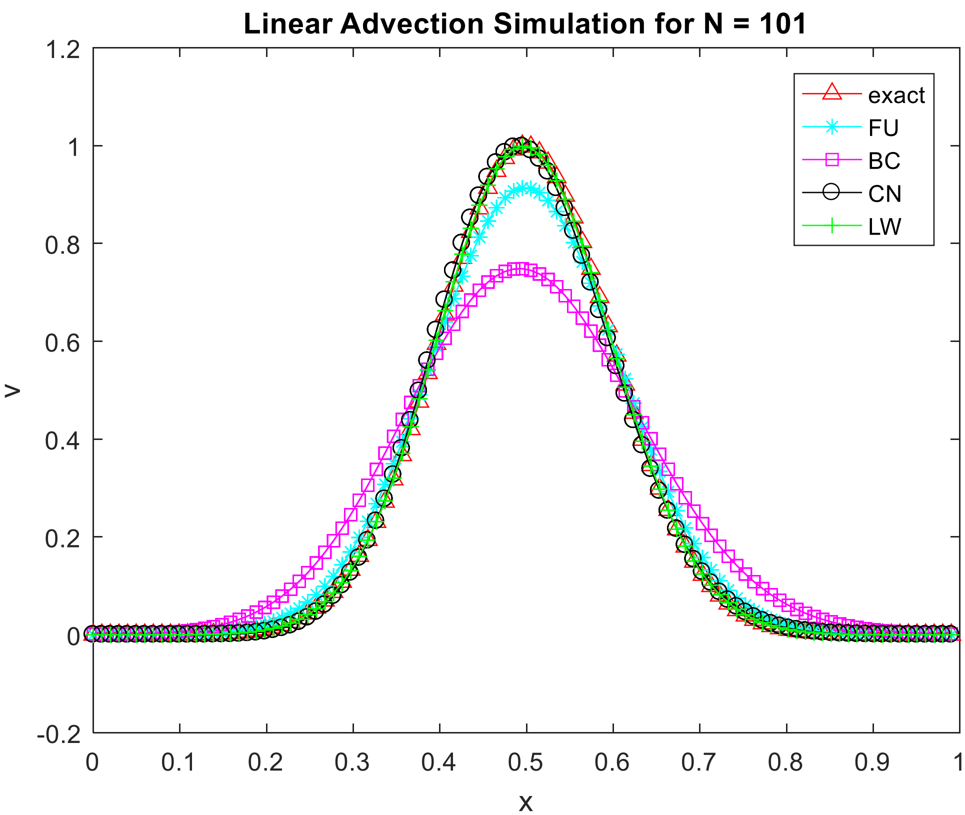


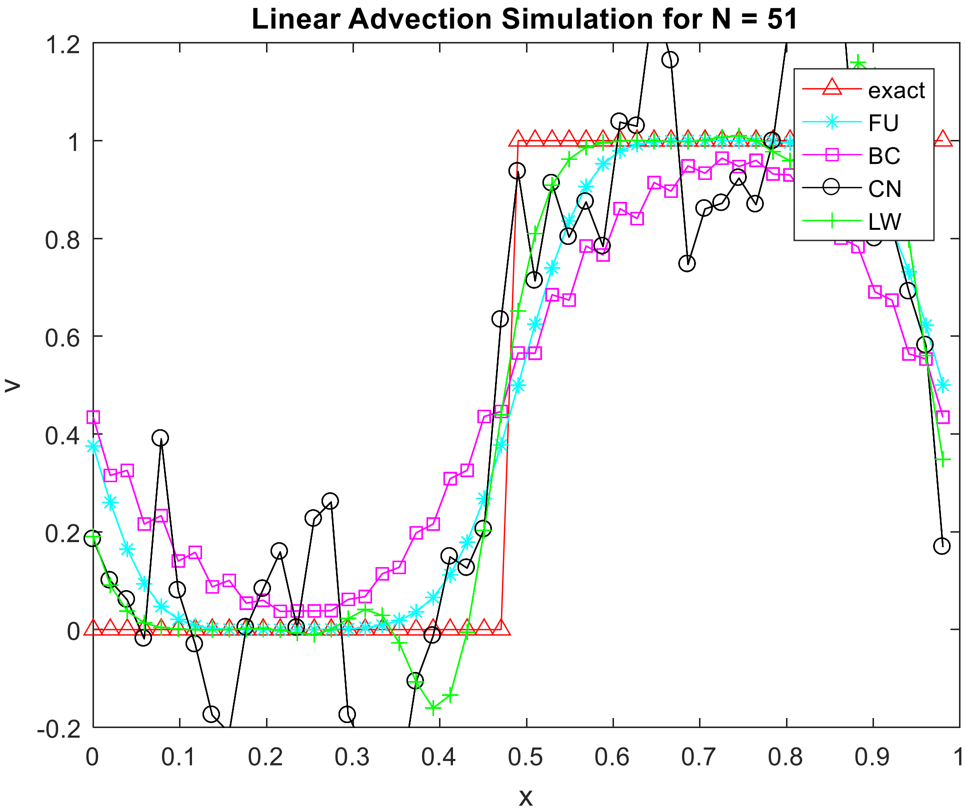
Question 5 (b):

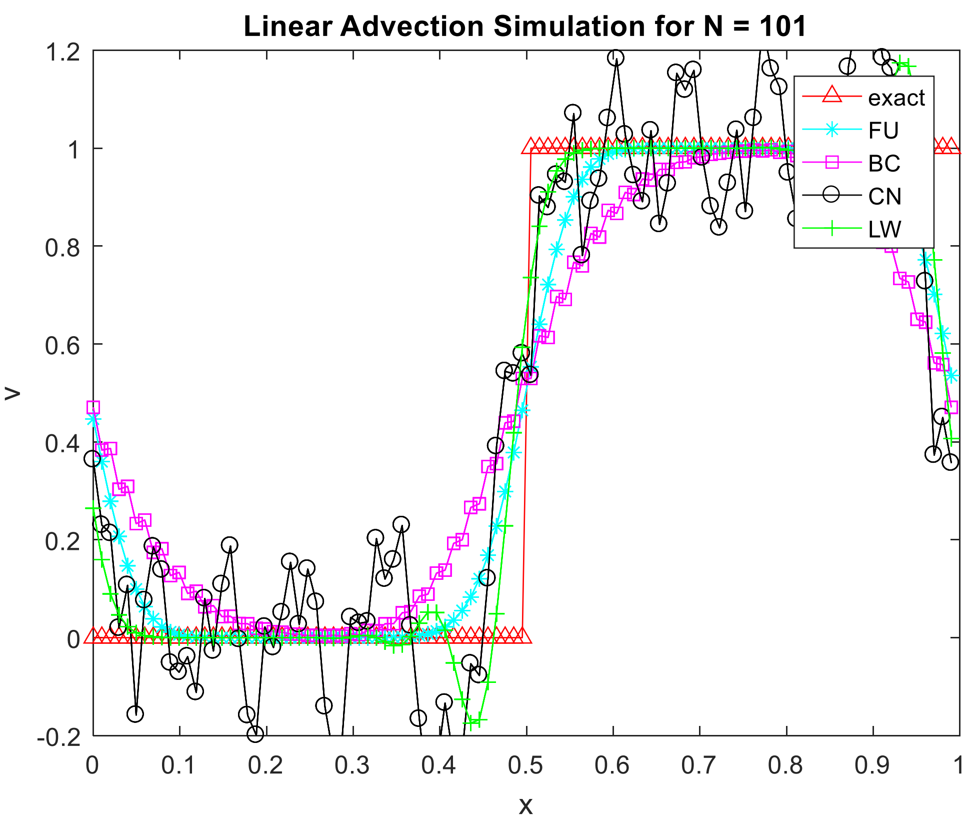
At N=51, Lax-Wendroff method provided the most appropriate numerical solution for problem 1. Considering Crank Nicolson method, dispersion has been discovered therefore the method is less accurate. The plot of Forward Upwind confirms a case of dissipation that has dampened the amplitude of numerical solution. Finally, Backward Central method incurred both dispersion and dissipation; in fact it has the highest dissipation. Backward Central is the least accurate method.
We know LW and CN methods have 2nd order of convergence; at N=101 both methods converged towards exact solution, however minor dispersion is visible in CN method. Further, numerical dissipation still exist in BC and FU methods in fact BC has higher dissipation and still the most inaccurate method.

Moving towards problem 2, LW to great extent an accurate numerical solution technique however minor oscillations have taken place at both N=51 and N=101. CN is the least reliable method due significant fluctuations in the solution. At N=51, dissipation is evident in BC method that later reduced in linear advection simulation conducted at N=101. Finally, FU is the most reliable method for problem 2 at both N=51 and N=101 irrespective of its slower convergence rate as compared to LW method at horizontal level of the exact plot. Forward Upwind method provided the most appropriate numerical solution for problem 2.

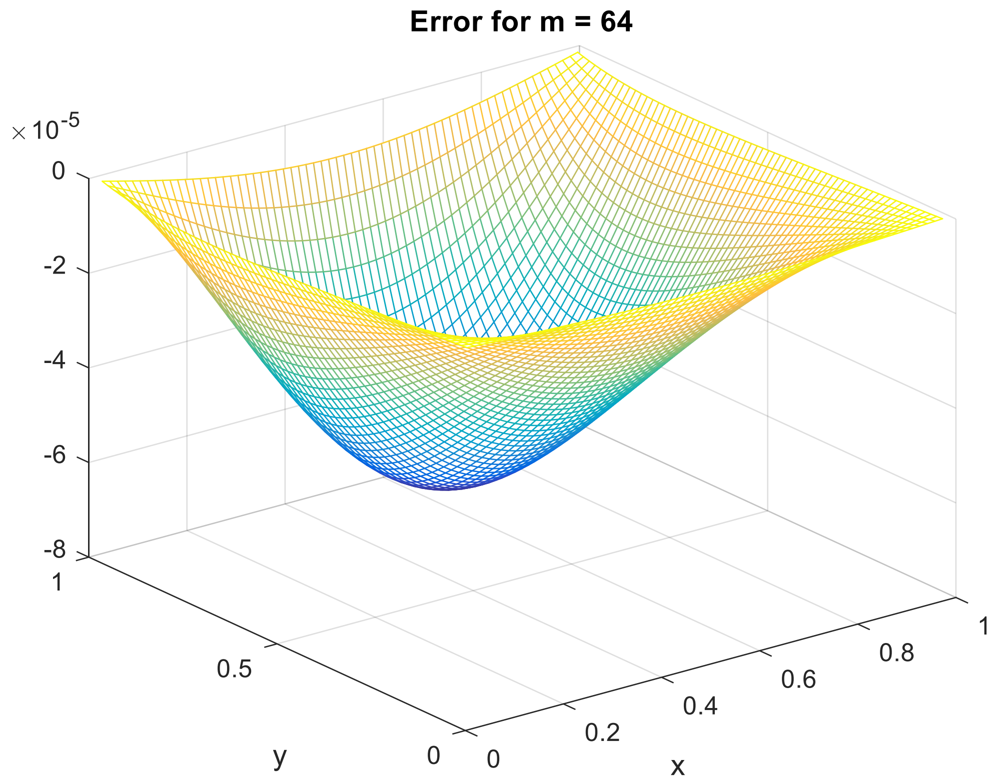

Linear Advection Simulation for N = 51
exact
FU
BC
CN
LW
v
x

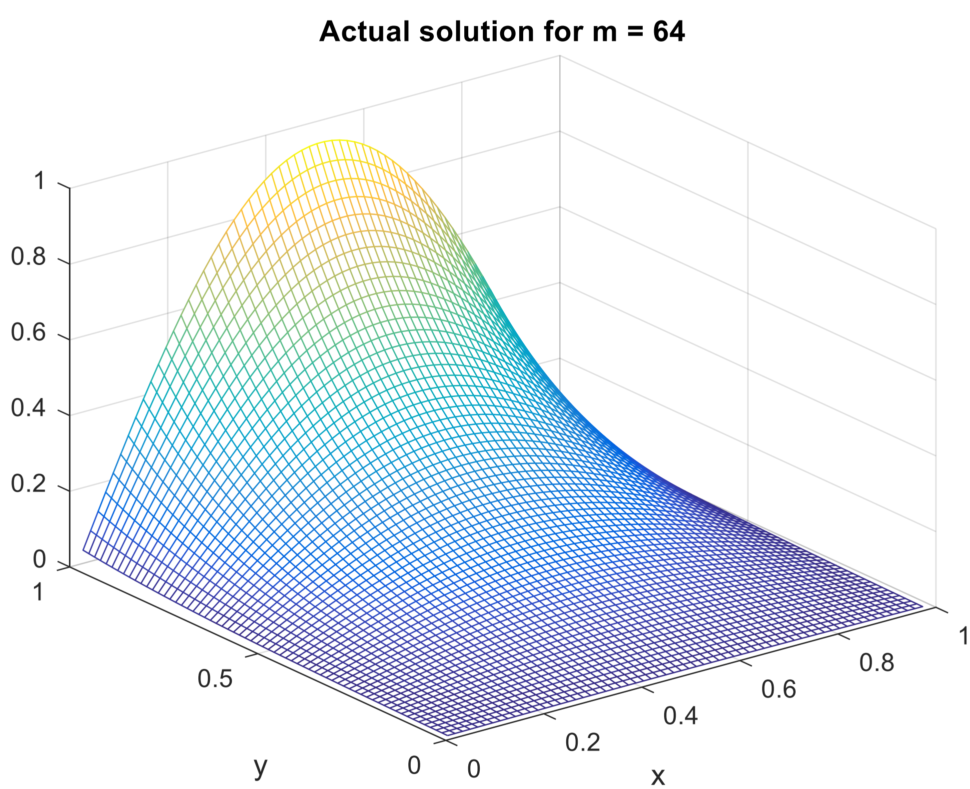

Linear Advection Simulation for N = 101
exact
FU
BC
CN
LW
v
x

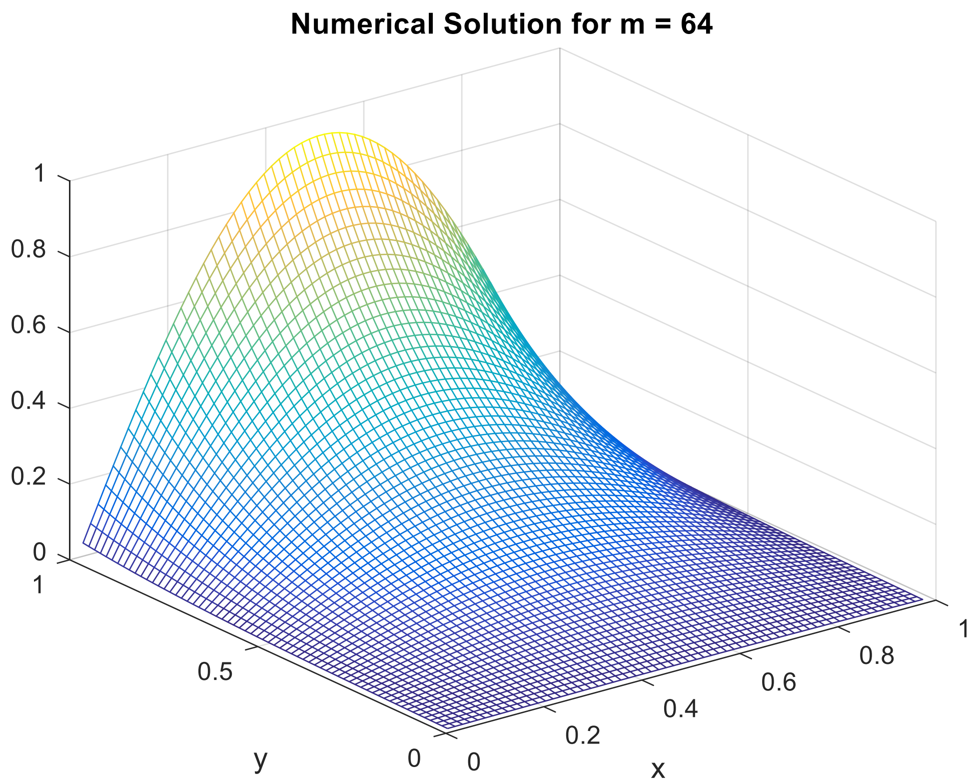

Linear Advection Simulation for N = 51
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
v
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
exact
FU
BC
CN
LW


Linear Advection Simulation for N = 101
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
v
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
exact
FU
BC
CN
LW

Question 6 :


Error for m = 64
×10^-5
0
-2
-4
-6
-8
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Actual solution for m = 64
1
0.8
0.6
0.4
0.2
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x



m	Norm	Ratio
2	0.013599637361553	
4	0.005328604746247	0.39181962
8	0.001691951556060	0.317522435
16	4.781488083797510e-04	0.28260195
32	1.271802416184308e-04	0.265984646
64	3.280062793129024e-05	0.257906633
128	8.329078181054410e-06	0.253930449

Theoretically predicted O(h^2) convergence states the ratio converges to 0.25 asymptotically. After conducting convergence study, with m = 2, 4, 8, 16, 32, 64, 128 the ratio between $\|E^h\|_2$ and successive values of m; the O(h^2) convergence is achieved.

$$\frac{(m+2)^2}{(2m+2)^2} \to \frac{1}{4} \quad as \quad m \to \infty$$