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Quiz-3

Max. Time: 20 min **Max. Points: 20**

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [8] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting is allowed.

i. Every linear transformation is a matrix transformation.

(A) True (B) False

ii. A transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ with A as its standard matrix, is onto if and only if the columns of A span $\mathbb{R}^m$.

(A) True (B) False

iii. The columns of the standard matrix for a linear transformation A from $\mathbb{R}^n$ to $\mathbb{R}^m$ are not the images of the of the columns of $n \times n$ identity matrix $\boldsymbol{I}$.

(A) True (B) False

iv. When two linear transformations are performed one after another, the combined effect will always be linear.

(A) True (B) False

v. If A is a 2×3 matrix representing a linear transformation T, then T cannot map $\mathbb{R}^3$ onto $\mathbb{R}^2$.

(A) True (B) False

vi. The codomain of the transformation $\mathbf{x} \mapsto \boldsymbol{A}\mathbf{x}$ is the set of all linear combinations of the columns of $\boldsymbol{A}$.

(A) True (B) False

vii. The transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

(A) True (B) False

viii. If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and $\mathbf{c}$ is in $\mathbb{R}^m$, then whether $\mathbf{c}$ is in the range of T is a uniqueness question.

(A) True (B) False

Q.2. [7+5]

a) For the following transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$, check if the given vector $\mathbf{b}$ is in its range.

$$\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 7 & -5 & -1 \\ 0 & 1 & -4 & 3 & 1 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & -1 \\ 0 & 1 & -4 & 3 & 1 \\ 0 & 2 & -8 & 6 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & -1 \\ 0 & 1 & -4 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system is consistent, so $\mathbf{b}$ is in the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

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b) Consider $T: \mathbb{R}^2 \to \mathbb{R}^2$. Give the standard matrix for the linear transformation A that first reflects points through the vertical axis x_2 and then rotates points $\frac{\pi}{2}$ radians.

$$\mathbf{e}_1 \to \mathbf{e}_1 \to -\mathbf{e}_2 \text{ and } \mathbf{e}_2 \to \mathbf{e}_2 - 2\mathbf{e}_1 \to -\mathbf{e}_1 + 2\mathbf{e}_2, \text{ so } A = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$$

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