

Warwick School: SnapPy part 2 Tuesday, September 12, 2017.

For links for SnapPy, SageMath, and other needed topics see: <http://dunfield.info/warwick>.

1. Here's how you get the exterior of a randomly chosen 14-crossing prime knot:

```
knots = HTLinkExteriors(cusps=1, crossings=14)
M = knots.random()
```

- (a) Python uses the `len` function to access the length of any list-like object, so do `len(knots)` to see how many such knots there are.
- (b) Try creating the Dirichlet domain for M at the command line. Most of the time you will get an error message saying that this failed! (If not, pick a different example which does fail for the rest of this problem ;-).
- (c) By default, the hyperbolic structure on M is computed using standard double-precision floating-point numbers (about 15 decimal digits). It turns out this isn't enough to find the Dirichlet domain for a manifold this complicated. Use the `high_precision` method of M to upgrade it to a `ManifoldHP` called H .
- (d) Compute the volumes of M and H . Are the answers consistent with the hyperbolic structure on H being computed to quad-double precision?
- (e) Try computing the Dirichlet domain D using H , which will most likely succeed though it typically takes a few seconds.
- (f) Interrogate D to get a pretty picture and find out how many faces and vertices D has.

2. Programming and searching.

- (a) Find all manifolds in the `OrientableClosedCensus` which have a 2- or 3-fold covering space with $b_1 > 0$.
- (b) The smallest volume manifold in the `OrientableClosedCensus` with $b_1 > 0$ is $m160(3,1)$ which has volume ≈ 3.1663333212 . Find a closed manifold with $b_1 > 0$ with smaller volume by searching through 0 surgeries on the knots in `HTLinkExteriors`.
- (c) Express the first manifold M that you found in (b) as a Dehn filling on a 1-cusped manifold in the `OrientableCuspedCensus`.
- (d) The reason that the closed manifold M in part (c) is not in the `OrientableClosedCensus` is that its shortest geodesic has length smaller than the cutoff that was chosen by Hodgson and Weeks when they created this census. Determine this cutoff by finding length of the shortest geodesic for the all the manifolds in the `OrientableClosedCensus`.

3. Recall that two 3-manifolds are commensurable if they have a common finite cover.

- (a) In the `OrientableClosedCensus`, find several pairs of 1-cusped manifolds that appear to have the same volume. (You don't have to find all such pairs, though that can certainly be done.)
- (b) For each pair, increase your confidence that the volumes are the same by using the `ManifoldHP` type, which works with quad-double precision.

- (c) For each pair, try to either (a) find a common finite cover, or (b) show they are incommensurable by looking at the cusp density.
4. SageMath and SnapPy are friends.
- (a) Find the first nontrivial knot in `HTLinkExteriors` which has the same Alexander polynomial as the unknot.
 - (b) Let M be the knot exterior you found in part (a). Via Matthias's new `verify` module in SnapPy, use interval arithmetic to rigorously prove that this manifold is indeed hyperbolic. Consequently, the corresponding knot is not the unknot.
 - (c) Find exact expressions for the tetrahedra shapes of M , which live in some number field.
 - (d) Check that the number field found in (c) is the same as the trace field of M .
5. SageMath and SnapPy are friends, part II.
- (a) Find many cusped hyperbolic 3-manifolds in the various censuses whose trace fields are $\mathbb{Q}(i)$. Hint: Computing trace fields is really expensive, so either (a) try with very small parameters or (b) numerically recognize elements of $\mathbb{Q}(i)$ that have small denominators.
 - (b) Amongst the your examples in (a), find one with a non-integral trace. This manifold will not be arithmetic and will also contain a closed incompressible surface.