Project 2

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https://github.com/malineri/fys3150/tree/main/project_2

PROBLEM 1

In this project we will be looking at a horizontal beam. The beam will be of length L, and a force F will be applied in the endpoint of this beam. This will be described by the second-order differential equation (1).

$$\gamma \frac{d^2u(x)}{dx^2} = -Fu(x). \tag{1}$$

We begin with scaling this formula into a dimensionless equation, by changing the x to a unitless variable $\hat{x} = \frac{x}{L}$. The derivation term then becomes,

$$\frac{d}{dx} = \frac{d\hat{x}}{dx}\frac{d}{d\hat{x}} = \frac{1}{L}\frac{d}{d\hat{x}}.$$

We have a second order differential equation, so we have to do this operation twice. Adding this to equation(1) we get the formula,

$$\frac{\gamma}{L^2}\frac{d^2u(\hat{x})}{d\hat{x}^2} = -Fu(\hat{x}).$$

Which we rearrange slightly,

$$\frac{d^2u(\hat{x})}{d\hat{x}^2} = -\frac{FL^2}{\gamma}u(\hat{x}).$$

Now we introduce a new variable lambda $\lambda = \frac{FL^2}{\gamma}$ then we find that the dimensionless equation of our first equation can be written as,

$$\frac{d^2u(\hat{x})}{d\hat{x}^2} = -\lambda u(\hat{x}). \tag{2}$$

TABLE I. Eigenvalues and corresponding eigenvectors of a tridiagonal matrix A

Eigenvalues λ	Eigenvectors $\vec{v}$
-3.80193774	[-0.23192061, -0.41790651, -0.52112089, -0.23192061, 0.52112089, -0.41790651]
-3.2469796	[0.41790651, 0.52112089, 0.23192061, -0.41790651, 0.23192061, -0.52112089]
-2.44504187	[-0.52112089, -0.23192061, 0.41790651, -0.52112089, -0.41790651, -0.23192061]
-0.19806226	[0.52112089, -0.23192061, -0.41790651, -0.52112089, -0.41790651, 0.23192061]
-1.55495813	[-0.41790651, 0.52112089, -0.23192061, -0.41790651, 0.23192061, 0.52112089]
-0.7530204	[0.23192061, -0.41790651, 0.52112089, -0.23192061, 0.52112089, 0.41790651]

PROBLEM 2

To solve this problem we will be using matrices. We write a short program to set up a tridiagonal NxN matrix **A**, when N = 6. We want this matrix to solve the classic eigenvector, eigenvalue problem $\mathbf{A}\vec{v} = \lambda\vec{v}$ in the same way we formulated our dimensionless equation(2).[**?**]

Algorithm 1 Set up for matrix equation of eigenvalues and eigenvectors in Python

```
from scipy.sparse import coo_matrix
import numpy as np
int N = 6                                          ▷ We begin with setting the length of the matrix N = 6
a, b, c = [1]*(N - 1), [-2]*N, [1]*(N - 1);        ▷ Creating a NxN tridiagonal matrix, with 1, -2 and 1.
A = np.diag(a, -1) + np.diag(b, 0) + np.diag(c, 1)
l, v = np.linalg.eig(A)                            ▷ Finding eigenvalues λ and eigenvektors v⃗
assert np.allclose(v @ np.diag(l), A @ v)
```

Now that we have found the eigenvalues and corresponding eigenvectors we print the results, which we can see in table (1). We can check if these values are correct by comparing the results to the analytical results with the formulas

$$\lambda^i = d + 2a\cos\left(\frac{i\pi}{N+1}\right)$$

$$\vec{v}^i = \left[\sin\left(\frac{i\pi}{N+1}\right), \sin\left(\frac{2i\pi}{N+1}\right), ..., \sin\left(\frac{Ni\pi}{N+1}\right)\right]^T$$

Where i = 1, ... , N.

In this program we used the matrix A, which is just the second derivative, but without the special values of our case.

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

PROBLEM 3

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