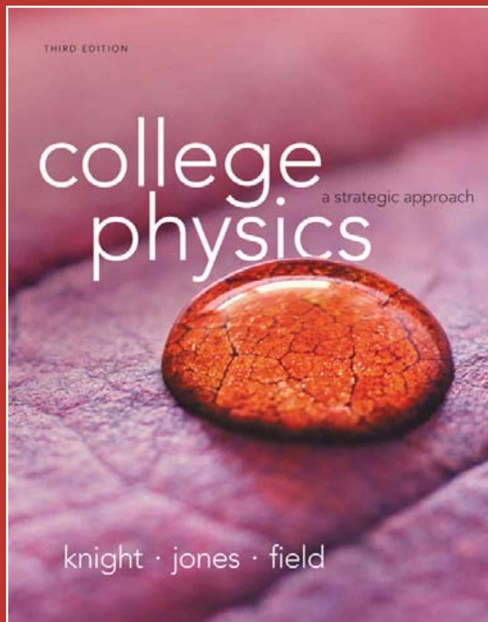


Lecture Presentation

Chapter 3

Vectors and Motion in Two Dimensions

Suggested Simulations for Chapter 3

- **PhETs**
  - *Vector Addition*
  - *Ladybug Motion 2D*
  - *Maze Game*
  - *Motion in 2D*
  - *Projectile Motion*
  - *Ladybug Revolution*

Chapter 3 Vectors and Motion in Two Dimensions

Chapter Goal: To learn more about vectors and to use vectors as a tool to analyze motion in two dimensions.

 Slide 3-3

Chapter 3 Preview
Looking Ahead: Vectors and Components

- The dark green vector is the ball's initial velocity. The light green component vectors show initial horizontal and vertical velocity.

- You'll learn to describe motion in terms of quantities such as distance and velocity, an important first step in analyzing motion.

 Slide 3-4

Chapter 3 Preview
Looking Ahead: Projectile Motion

- A leaping fish's parabolic arc is an example of projectile motion. The details are the same for a fish or a basketball.

- You'll see how to solve projectile motion problems, determining how long an object is in the air and how far it travels.

Chapter 3 Preview
Looking Ahead: Circular Motion

- The riders move in a circle at a constant speed, but they have an acceleration because the direction is constantly changing.

- You'll learn how to determine the magnitude and the direction of the acceleration for an object in circular motion.

Chapter 3 Preview
Looking Ahead

Vectors and Components

The dark green vector is the ball's initial velocity. The light green **component vectors** show initial horizontal and vertical velocity.

You'll learn how to find components of vectors and how to use these components to solve problems.

Projectile Motion

A leaping fish's parabolic arc is an example of **projectile motion.** The details are the same for a fish or a basketball.

You'll see how to solve projectile motion problems, determining how long an object is in the air and how far it travels.

Circular Motion

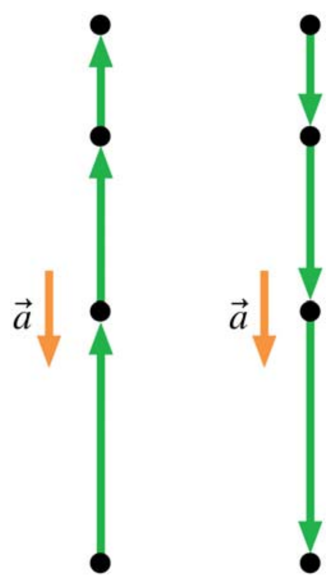
The riders move in a circle at a constant speed, but they have an acceleration because the direction is constantly changing.

You'll learn how to determine the magnitude and the direction of the acceleration for an object in circular motion.

Text: p. 64

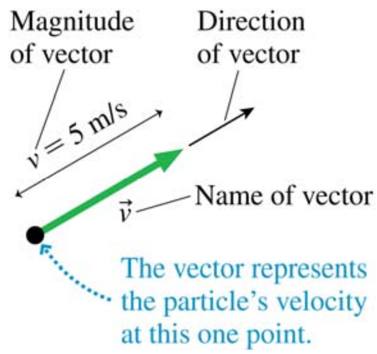
Chapter 3 Preview
Looking Back: Free Fall

- You learned in Section 2.7 that an object tossed straight up is in free fall. The acceleration is the same whether the object is going up or coming back down.
- For an object in projectile motion, the vertical component of the motion is also free fall. You'll use your knowledge of free fall to solve projectile motion problems.



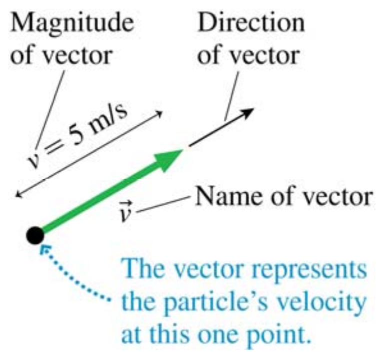
Using Vectors

- A vector is a quantity with both a size (magnitude) and a direction.
- Figure 3.1 shows how to represent a particle's velocity as a vector $\vec{v}$.
- The particle's speed at this point is 5 m/s and it is moving in the direction indicated by the arrow.



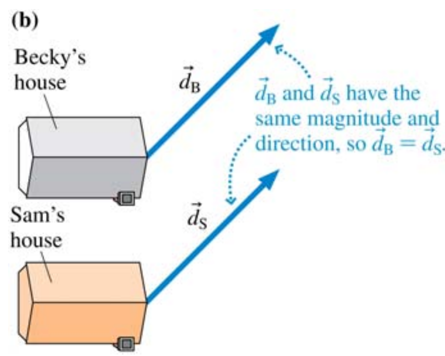
Using Vectors

- The magnitude of a vector is represented by the letter without an arrow.
- In this case, the particle's speed—the magnitude of the velocity vector $\vec{v}$—is $v = 5$ m/s.
- The magnitude of a vector, a *scalar* quantity, cannot be a negative number.



Using Vectors

- The displacement vector is a straight-line connection from the initial position to the final position, regardless of the actual path.
- **Two vectors are equal if they have the same magnitude and direction.** This is regardless of the individual starting points of the vectors.



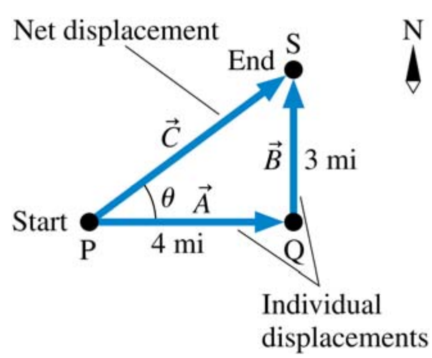
Vector Addition

- $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's having first displacement $\vec{A}$, then displacement $\vec{B}$.
- The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$:

$$\vec{C} = \vec{A} + \vec{B}$$

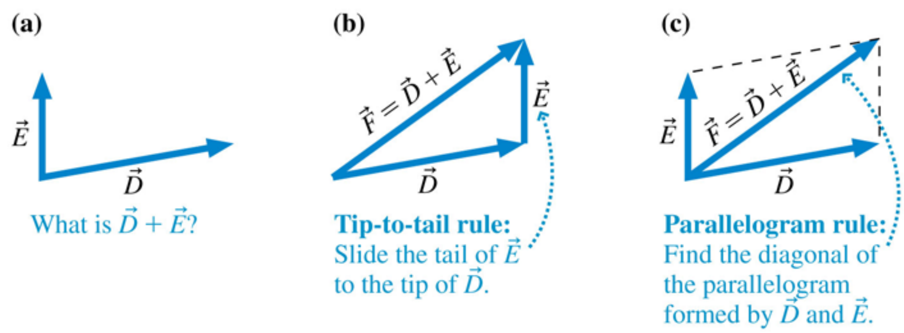
- The sum of the two vectors is called the resultant vector. Vector addition is commutative:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



Vector Addition

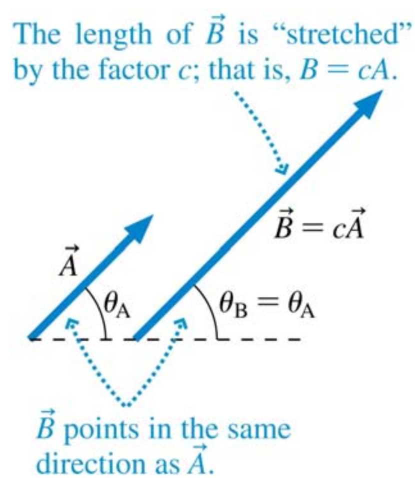
- The figure shows the *tip-to-tail rule* of vector addition and the *parallelogram rule* of vector addition.



Slide 3-13

Multiplication by a Scalar

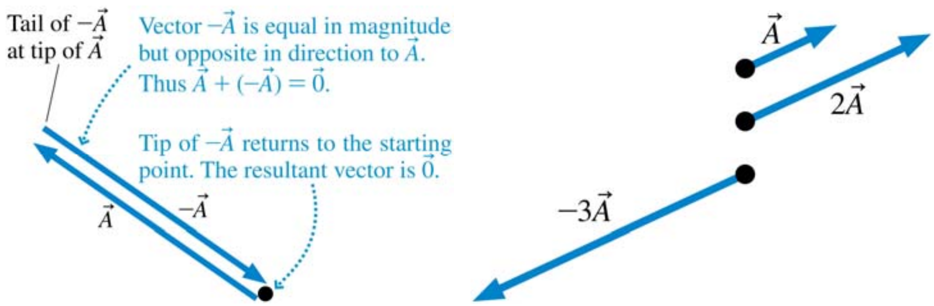
- **Multiplying a vector by a positive scalar gives another vector of *different magnitude* but pointing in the *same direction*.**
- If we multiply a vector by zero the product is a vector having zero length. The vector is known as the **zero vector**.



Slide 3-14

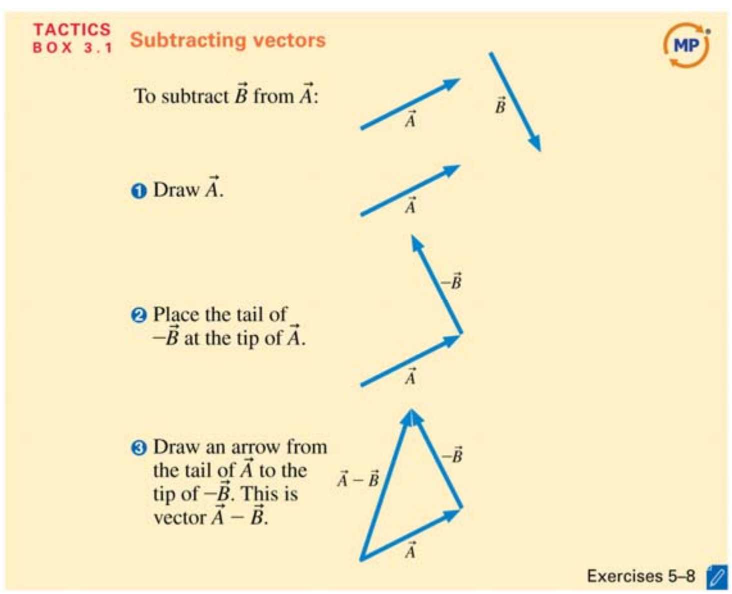
Multiplication by a Scalar

- **A vector cannot have a negative magnitude.**
- If we multiply a vector by a negative number we reverse its direction.
- Multiplying a vector by –1 reverses its direction without changing its length (magnitude).



 Slide 3-15

Vector Subtraction



Text: p. 67

 Slide 3-16

Using Vectors on Motion Diagrams

- In two dimensions, an object's displacement is a vector:

$$\vec{v} = \frac{\vec{d}}{\Delta t} = \left(\frac{d}{\Delta t}, \text{same direction as } \vec{d}\right)$$

Definition of velocity in two or more dimensions

- The velocity vector is simply the displacement vector multiplied by the scalar $1/\Delta t$.
- Consequently **the velocity vector points in the direction of the displacement.**

Slide 3-17

Using Vectors on Motion Diagrams

- The vector definition of acceleration is a straightforward extension of the one-dimensional version:

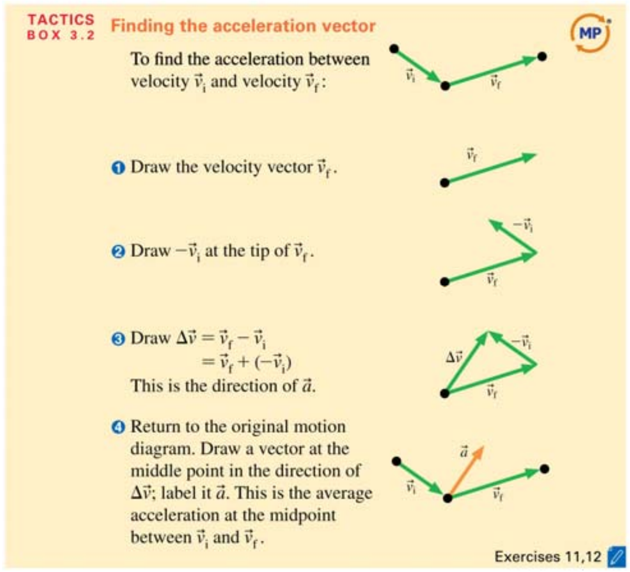
$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

Definition of acceleration in two or more dimensions

- There is an acceleration whenever there is a change in velocity. Velocity can change in either or both of two possible ways:
  1. The magnitude can change, indicating a change in speed.
  2. The direction of motion can change.

Slide 3-18

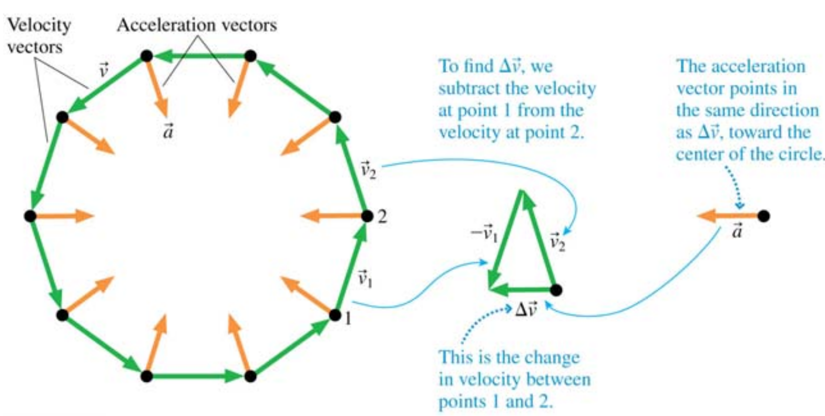
Finding the Acceleration Vector



Text: p. 69

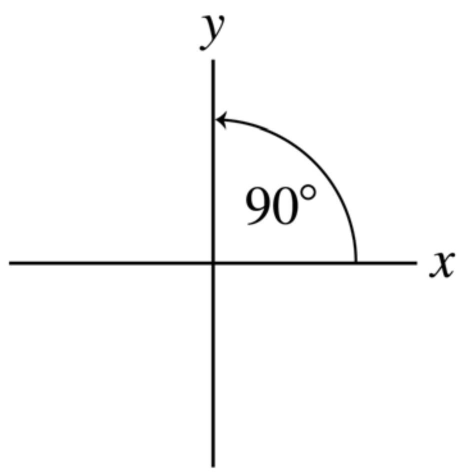
Vectors and Circular Motion

- Cars on a Ferris wheel move at a constant speed but in a continuously changing direction. They are in **uniform circular motion**.



Coordinate Systems

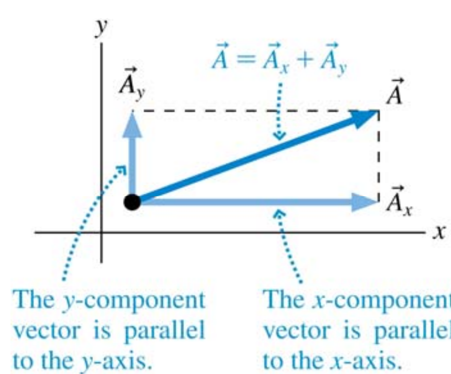
- A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements.
- We will generally use **Cartesian coordinates**.
- Coordinate axes have a positive end and a negative end, separated by a zero at the origin where the two axes cross.



Component Vectors

- For a vector *A* and an *xy*-coordinate system we can define two new vectors parallel to the axes that we call the component vectors of $\vec{A}$.
- You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



Components

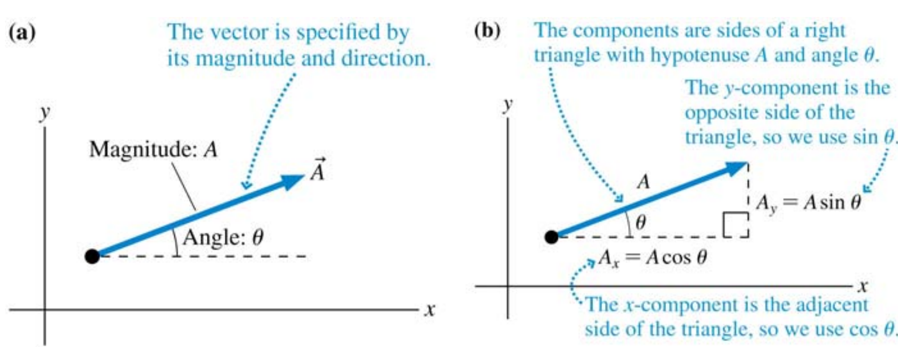
TACTICS BOX 3.3 **Determining the components of a vector**

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

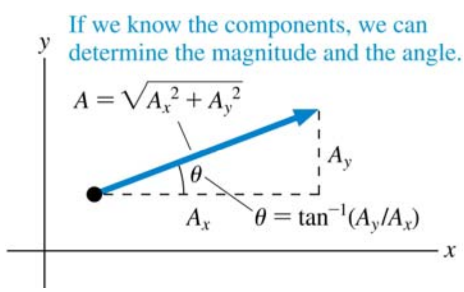
Exercises 16–18

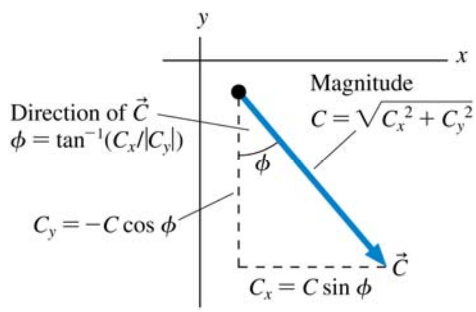
Text: p. 71

Components



Components



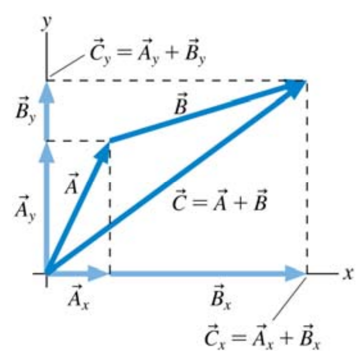


Working with Components

- We can add vectors using components.
- Let's look at the vector sum $\vec{C} = \vec{A} + \vec{B}$ for the vectors shown in FIGURE 3.19. You can see that the component vectors of $\vec{C}$ are the sums of the component vectors of $\vec{A}$ and $\vec{B}$. The same is true of the components: $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

$$D_x = A_x + B_x + C_x + \cdots$$

$$D_y = A_y + B_y + C_y + \cdots$$



Working with Components

$$\vec{F} = \vec{n} + \vec{w} + \vec{f}$$

- Equation 3.18 is really just a shorthand way of writing the two simultaneous equations:

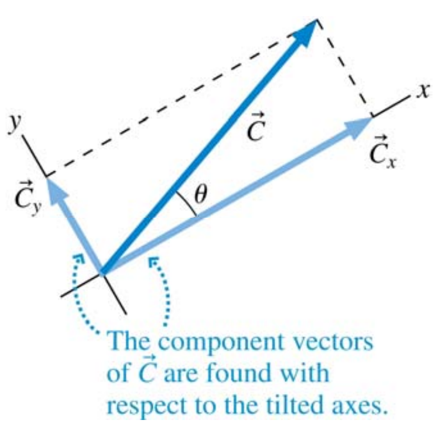
$$F_x = n_x + w_x + f_x$$

$$F_y = n_y + w_y + f_y$$

- In other words, a vector equation is interpreted as meaning: Equate the x-components on both sides of the equals sign, then equate the y-components. Vector notation allows us to write these two equations in a more compact form.

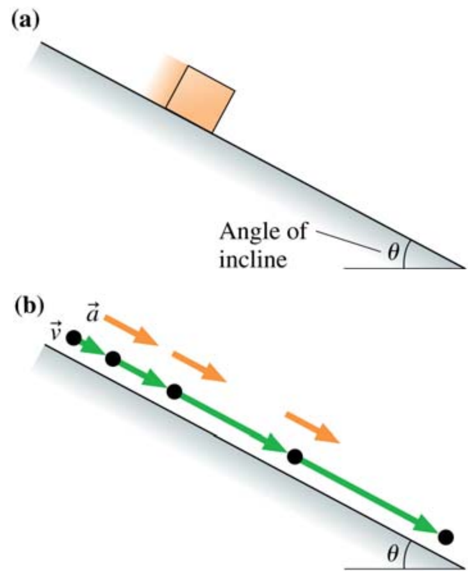
Tilted Axes

- For motion on a slope, it is often most convenient to put the x-axis along the slope.
- When we add the y-axis, this gives us a tilted coordinate system.
- Finding components with tilted axes is done the same way as with horizontal and vertical axes. The components are parallel to the tilted axes and the angles are measured from the tilted axes.



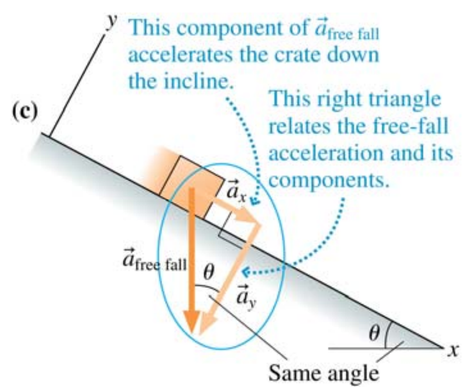
Accelerated Motion on a Ramp

- A crate slides down a frictionless (i.e., smooth) ramp tilted at angle θ.
- The crate is constrained to accelerate parallel to the surface.
- Both the acceleration and velocity vectors are parallel to the ramp.



Accelerated Motion on a Ramp

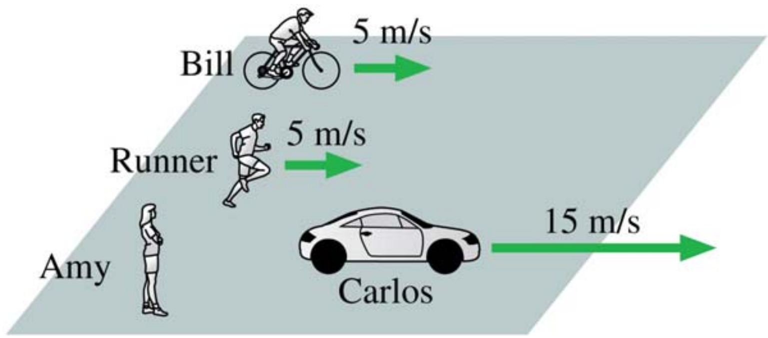
- We choose the coordinate system to have the x-axis along the ramp and the y-axis perpendicular. All motion will be along the x-axis.
- The acceleration parallel to the ramp is a component of the free-fall acceleration the object would have if the ramp vanished:



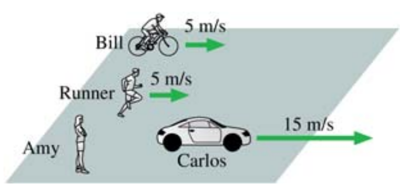
$$a_x = \pm g \sin\theta$$

Relative Motion

- Amy, Bill, and Carlos are watching a runner.
- The runner moves at a different velocity relative to each of them.



Relative Velocity



$$(v_x)_{RC} = (v_x)_{RA} + (v_x)_{AC}$$

The "A" appears on the right of the first expression and on the left of the second; when we combine these velocities, we "cancel" the A to get $(v_x)_{RC}$.

- The runner's velocity relative to Amy is

$$(v_x)_{RA} = 5 \text{ m/s}$$

- The subscript "RA" means "**R**unner relative to **A**my."
- The velocity of Carlos relative to Amy is

$$(v_x)_{CA} = 15 \text{ m/s}$$

- The subscript "CA" means "**C**arlos relative to **A**my."

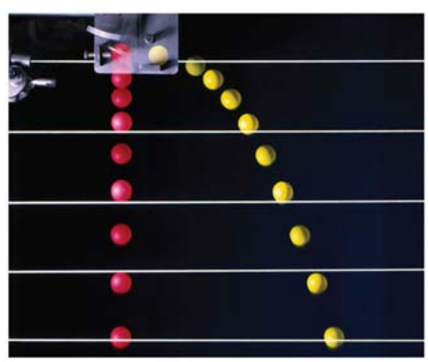
Motion in Two Dimensions: Projectile Motion

- **Projectile motion** is an extension to two dimensions of free-fall motion.
- **A projectile is an object that moves in two dimensions under the influence of gravity and nothing else.**
- As long as we can neglect air resistance, any projectile will follow the same type of path.

Slide 3-33

Motion in Two Dimensions: Projectile Motion

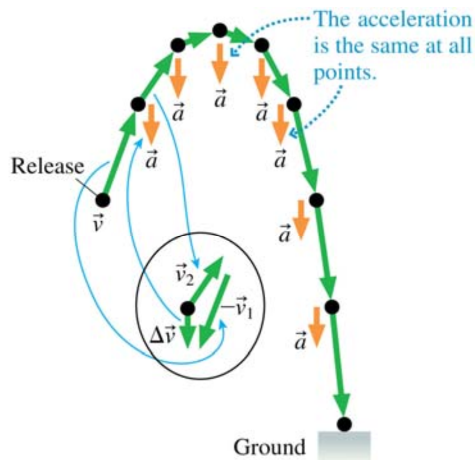
- The vertical motions of the two balls are identical.
- The vertical motion of the yellow ball is not affected by the fact that the ball is moving horizontally.
- **The horizontal and vertical components of an object undergoing projectile motion are independent of each other.**



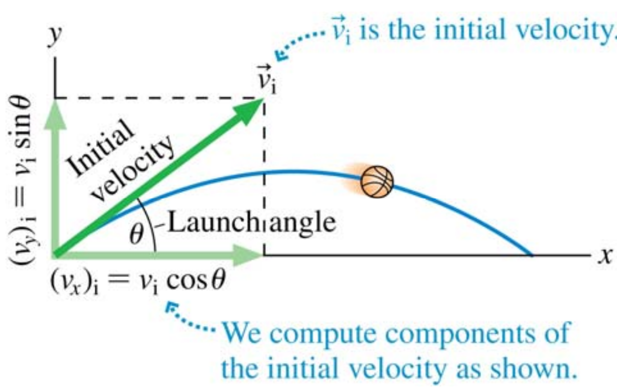
Slide 3-34

Motion in Two Dimensions: Projectile Motion

- **The vertical component of acceleration a_y for all projectile motion is just the familiar $-g$ of free fall, while the horizontal component a_x is zero.**



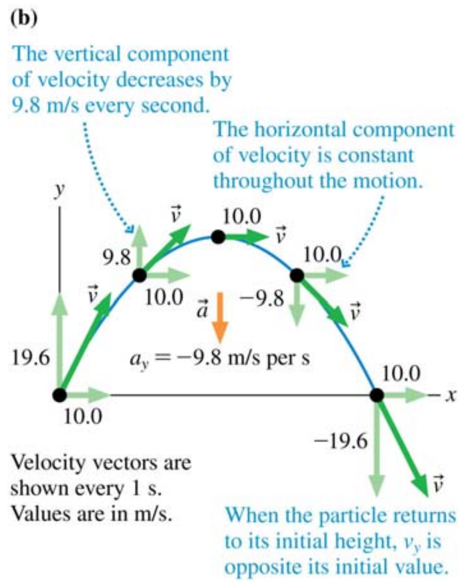
Analyzing Projectile Motion



- The angle of the initial velocity above the horizontal (i.e., above the x-axis) is the **launch angle**.

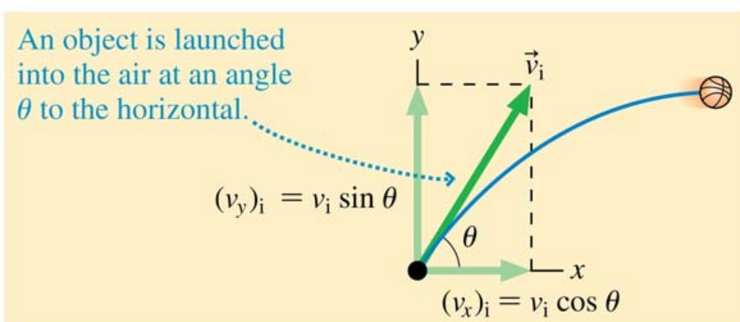
Analyzing Projectile Motion

- The ball finishes its motion moving downward at the same speed as it started moving upward.
- Projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction.



 Slide 3-37

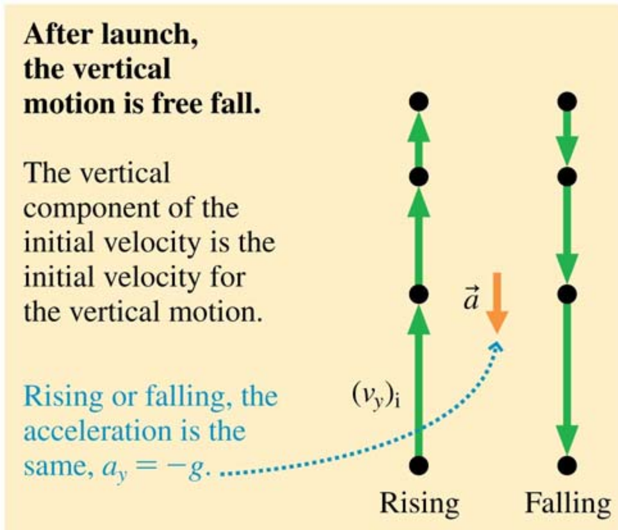
Synthesis 3.1 Projectile Motion



Text: p. 82

 Slide 3-38

Synthesis 3.1 Projectile Motion



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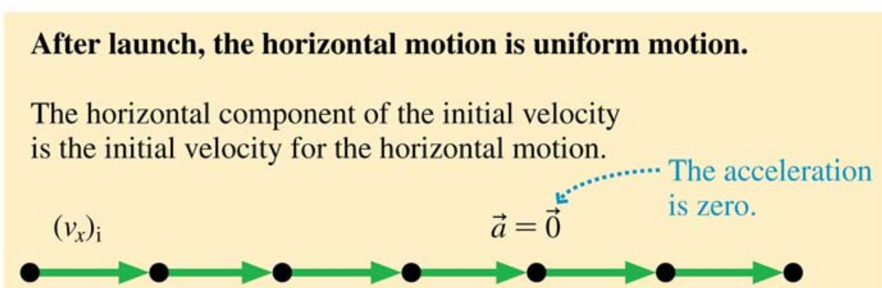
Synthesis 3.1 Projectile Motion



Text: p. 82

Synthesis 3.1 Projectile Motion

The kinematic equations for projectile motion are those for constant-acceleration motion vertically and constant-velocity horizontally:

The vertical motion is free fall.

The free fall acceleration, $g = 9.8\ \text{m/s}^2$.

The horizontal motion is uniform motion.

$$(v_y)_f = (v_y)_i - g\,\Delta t$$
$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$(v_x)_f = (v_x)_i = \text{constant}$$
$$x_f = x_i + (v_x)_i\,\Delta t$$

The two equations are linked by the time interval Δt, which is the same for the horizontal and vertical motion.

Text: p. 82

Projectile Motion Problems

We can solve projectile motion problems by considering the horizontal and vertical motions as separate but related problems.

PREPARE There are a number of steps that you should go through in setting up the solution to a projectile motion problem:

- Make simplifying assumptions. Whether the projectile is a car or a basketball, the motion will be the same.
- Draw a visual overview including a pictorial representation showing the beginning and ending points of the motion.
- Establish a coordinate system with the x-axis horizontal and the y-axis vertical. In this case, you know that the horizontal acceleration will be zero and the vertical acceleration will be free fall: $a_x = 0$ and $a_y = -g$.
- Define symbols and write down a list of known values. Identify what the problem is trying to find.

Text: p. 83

Projectile Motion Problems

SOLVE There are two sets of kinematic equations for projectile motion, one for the horizontal component and one for the vertical:

Horizontal	Vertical
$x_f = x_i + (v_x)_i \, \Delta t$	$y_f = y_i + (v_y)_i \, \Delta t - \frac{1}{2} g (\Delta t)^2$
$(v_x)_f = (v_x)_i = \text{constant}$	$(v_y)_f = (v_y)_i - g \, \Delta t$

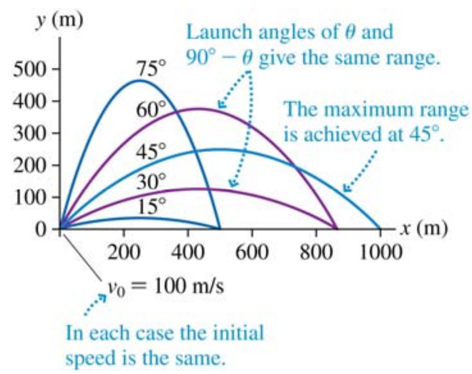
Δt is the same for the horizontal and vertical components of the motion. Find Δt by solving for the vertical or the horizontal component of the motion; then use that value to complete the solution for the other component.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Text: p. 83

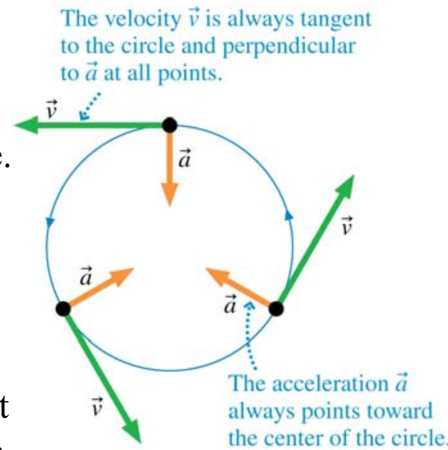
The Range of a Projectile

- The **range** of a projectile is the horizontal distance traveled.
- For smaller objects air resistance is critical, and the maximum range comes at an angle less than 45°.



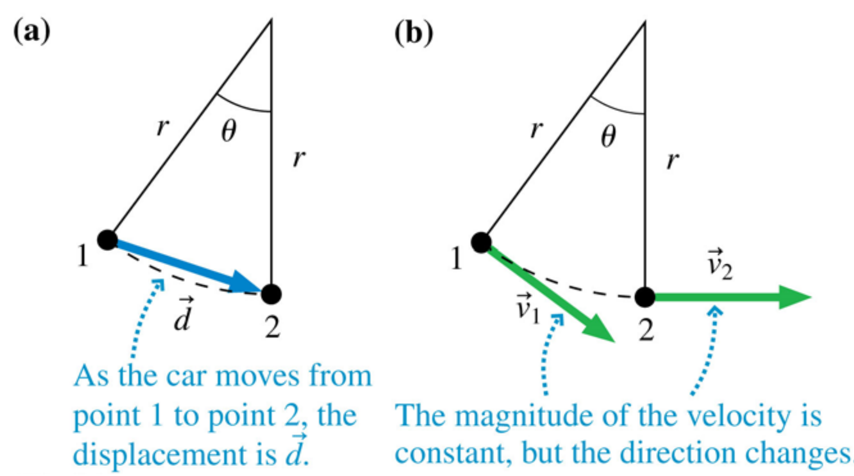
Motion in Two Dimensions: Circular Motion

- For circular motion at a constant speed, the acceleration vector a points toward the center of the circle.
- An acceleration that always points directly toward the center of a circle is called a **centripetal acceleration**.
- Centripetal acceleration is just the **name** for a particular type of motion. It is not a new type of acceleration.



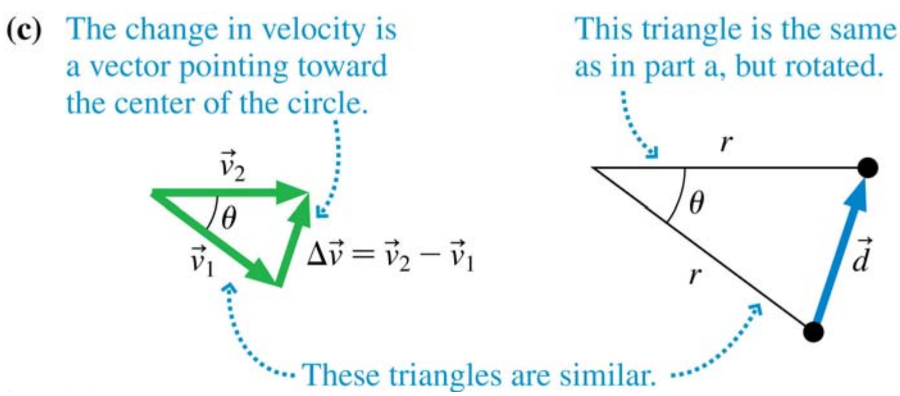
 Slide 3-45

Motion in Two Dimensions: Circular Motion



 Slide 3-46

Motion in Two Dimensions: Circular Motion



$$\frac{\Delta v}{v} = \frac{d}{r}$$

Slide 3-47

Motion in Two Dimensions: Circular Motion

$$d = v\Delta t$$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

$$\vec{a} = \left(\frac{v^2}{r}, \text{toward center of circle}\right)$$

Centripetal acceleration of object moving in a circle of radius r at speed v

Slide 3-48