Segmentation of Liver Tumors in Ultrasound Images based on Scale-Space Analysis of the Continuous Wavelet Transform

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Abstract—

We have developed a simple, yet robust method for segmentation of low-contrast objects embedded in noisy images. Our technique has been applied to segmenting of liver tumors in B-scan ultrasound images with hypoechoic rims. In our method, first a B-scan image is processed by a median filter for removal of speckle noise. Then several one-dimensional profiles are obtained along multiple radial directions which pass through the manually identified center of the region of a tumor. After smoothing by a Gaussian kernel smoother, these profiles are processed by Sombrero's continuous wavelets to yield scalograms over a range of scales. The modulus maxima lines, which represent the degree of regularity at individual points on the profiles, are then utilized for identifying candidate points on the boundary of the tumor. These detected boundary points are fitted by an ellipse and are used as an initial configuration of a wavelet snake. The wavelet snake is then deformed so that the accurate boundary of the tumor is found. A preliminary result for several metastases with various sizes of hypoechoic rims showed that our method could extract boundaries of the tumors which were close to the contours drawn by expert radiologists. Therefore, our new method can segment the regions of focal liver disease in sonograms with accuracy, and it can be useful as a preprocessing step in our scheme for automated classification of focal liver disease in sonography.

I. Introduction

Segmentation is often a first step in a computer vision system for diagnosis of medical images [1]. Segmentation is performed in an attempt to reduce the large amount of information present in an image to a point where an automated process can recognize the objects of interest in the images. For example, computer-aided diagnosis (CAD) for classification of focal liver tumors in sonography [2] requires segmentation as a preprocessing step for successive texture analyses of the tumors. However, effective segmentation is a difficult task for noisy images such as B-scan ultrasound images, because the boundaries of the tumors of interest can be fuzzy and have low contrast. Although some texture features such as the entropy [3], [4] can be used for segmentation of an object, the gray value of the pixels and the profile analysis of images can be a powerful means for segmentation. In this study, we have developed a simple, yet robust method based on the continuous wavelet transform for segmentation of low-contrast objects embedded in noisy images. This technique is applied to segmenting of focal liver tumors such as malignant progressive metastases, which often show a hypoechoic rim, or halo, in B-scan ultrasound images.

II. Detection of Singularities of Profile with Continuous Wavelets

In our method, the continuous wavelet transform is used for detection of the abrupt changes (or *singularities*) due to the halo of a focal liver tumor, from the profile that is obtained along a radial direction and passes through the center of the tumor. More precisely, we attempt to detect the singularities resulting from a discontinuity of the α^{th} derivative of the profile, where α is a real number. Let $f(x)$ be the profile of the tumor. We assume that the profile has the following additive structure:

$$f(x) = \gamma(x - x_0)_+^\alpha + n(x)\,, \qquad (1)$$

where $n(x)$ represents white Gaussian noise with a zero mean and variance σ^2 present in the image. The distribution [5]

$$(x - x_0)_+^\alpha = \begin{cases} 0 & x \le x_0 \\ (x - x_0)^\alpha & x > x_0 \end{cases} \qquad (2)$$

represents an abrupt change at x_0 with a regularity α, and γ represents the strength of the distribution.

The continuous wavelet transform can be used to characterize the above singularity in the following manner [6]. Let $\psi(x)$ be a mother wavelet that is defined as the n^{th} derivative of a localized function ϕ, which is positive everywhere, in the class of l-continuously differentiable functions C^l $(l > n)$. Then the continuous wavelet transform W is defined by the following convolution operation:

$$W[f](x,a) \equiv f \star \psi_a(x) = \int f(t)\psi_a(x - t)dt\,, \qquad (3)$$

where $\psi_a(x) \equiv a^{-1}\psi(x/a)$, and $a > 0$ is a *scale* parameter. Because of the linearity of W, the wavelet transform of the profile in Eq. (1) is given by

$$W[f](x,a) = \gamma W\left[(t - t_0)_+^\alpha\right](x,a) + W[n](x,a)\,. \qquad (4)$$

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The first term of this equation can be rewritten as

$$\begin{aligned} W\left[(x-x_0)_+^\alpha\right](x,a) &= \frac{1}{a}\frac{d^n}{d(x/a)^n}\phi\left(\frac{x}{a}\right) \star (x-x_0)_+^\alpha \\ &= \Gamma(\alpha+1)a^\alpha \phi^{(n-\alpha-1)}\left(\frac{x-x_0}{a}\right) , \end{aligned}$$

where $\phi^{(n)}$ represents the n^{th} derivative of ϕ. Let us assume that $|\phi^{(n)}(x)|$ has N maxima $\hat{\phi}_j$ at x_j $(1 \le j \le N)$. Then the modulus of the above wavelet transform of the distribution has N *maxima lines*

$$\{ax_j + x_0 \, : \, 1 \le j \le N\} , \tag{5}$$

which converge to the singularity point x_0 when $a \to 0$. We can then define the *ridge function* r_j as the modulus of the wavelet transform along the j^{th} maxima line as follows:

$$\begin{aligned} r_j(x_0,a) &\equiv \left|W\left[(t-t_0)_+^\alpha\right](ax_j+x_0,a)\right| & (6) \\ &= \Gamma(\alpha+1)a^\alpha \hat{\phi}_j(x_0) . & (7) \end{aligned}$$

This shows that, when the ridge function is plotted in a log-log plot, it becomes a straight line whose slope is equal to the regularity α. One can use this fact to estimate the regularity α at the singularity point x_0 [6].

The second term in Eq. (4) represents the wavelet transform of noise. The linearity of the wavelet transform ensures that $W[n](x,a)$ is also a Gaussian process with a variance

$$\begin{aligned} \mathrm{Var}\{W[n]\} &= \sigma^2 \int \psi_n^2(x)dx & (8) \\ &= \frac{\sigma^2}{a}\int \psi^2(x)dx \propto \frac{\sigma^2}{a} . & (9) \end{aligned}$$

This expression indicates that the variance of $W[n](x,a)$ is inversely proportional to the scale parameter a. Combined with the expression for $W\left[(x-x_0)_+^\alpha\right](x,a)$, the signal-to-noise ratio of $W[f]$ is shown to increase proportionally to $a^{\alpha+1/2}$. Therefore, for a sufficiently large scale, $a \gg 1$, the wavelet transform of the singularity with $\alpha > 1/2$ dominates $W[f]$:

$$W[f] \approx W\left[(t-t_0)_+^\alpha\right](x,a) , \quad a \gg 1 . \tag{10}$$

On the other hand, the noise is dominant only when the scale is small. In other words, most of the maxima lines due to the singularities continue from a small to a large scale, whereas the maxima lines due to noise tend to fail to reach the large scales.

Therefore, we can detect the singularities by extracting the maxima lines in the following manner. First, for a given profile f shown in Fig. 1, a two-dimensional plot of $W[f]$, called a *scalogram* as shown in Fig. 2, is obtained over a range of scales. Then the modulus maxima of $W[f]$ are identified at each scale. The modulus maxima corresponding to the same point on the profile are connected to form a maxima line corresponding to that point, as shown in Fig. 3. Maxima lines obtained from all of the points on the profile are thresholded by their lengths and average intensities, and those points corresponding to the remaining maxima lines are identified as the "true" singularities.

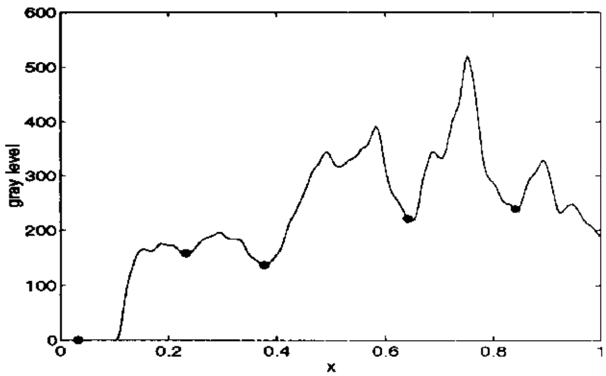

Fig. 1. A profile across a liver tumor in a B-scan image

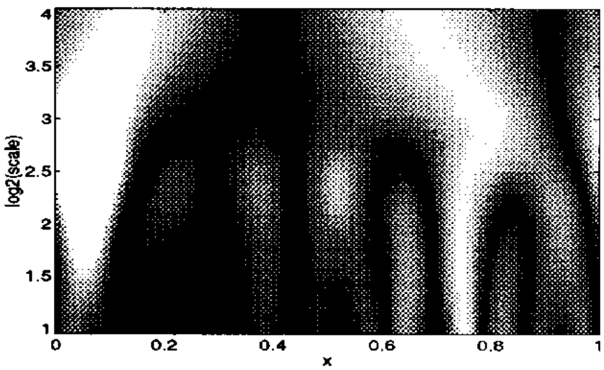

Fig. 2. A scalogram $W[f]$ of the profile in Fig. 1.

III. Detection of Boundary of Tumor

To detect the boundary of the halo of a tumor in a B-scan image such as the one shown at the center of Fig. 4, we first process the image by a median filter for removal of speckle noise. Then a one-dimensional profile is obtained along a radial direction, which passes through the manually identified center of the region of a tumor, as shown by the lines in Fig. 5. After smoothing by a Gaussian kernel smoother, the profile is processed by Sombrero's wavelet

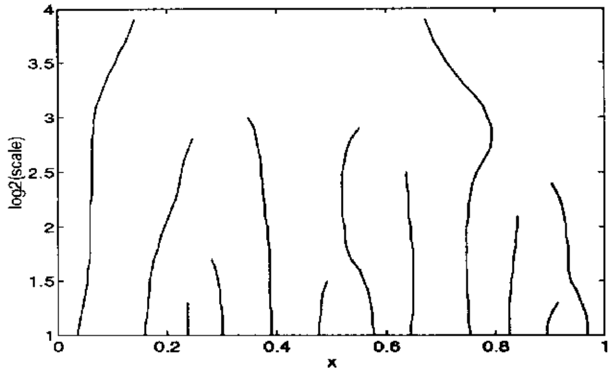

Fig. 3. Maxima lines extracted from the scalogram in Fig. 2.

that is a second derivative of a Gaussian. Then the maxima lines are obtained and the singularities on the profile are identified by use of the method described in the previous section. Among these singularities, those at local minima are selected, as represented by black circles in Fig. 1. Then, two local minima closest to the center, one from the left side and another from the right side of the center are identified as the boundary points of the halo of the tumor. This is because, generally, the halo appears as a low gray-level rim. These procedures are repeated on the radial profiles with multiple angles to produce a set of points, as shown by cross marks in Fig. 5. As observed in these cross marks, some points are incorrectly identified as boundary points. To eliminate these "outliers," we calculated the Mahalanobis distances [1] for these points, and those points with larger distances than the mean + standard deviation of the Mahalanobis distances are eliminated as outliers. Finally, the remaining points are connected to provide a coarse estimate of the boundary of the tumor, as represented by the rugged contour in Fig. 6.

The points representing the estimated boundary of the tumor are then used for fitting of an ellipse. We used a method of direct least-square fitting of ellipses [7], the result of which is shown in Fig. 6 by an ellipse.

We then used the fitted ellipse as an initial configuration of a *wavelet snake* [8] to find an accurate boundary of the halo of the tumor. Briefly, the wavelet snake is a smooth and closed active contour defined by a wavelet basis. The shape of the snake is determined by a set of wavelet coefficients. The deformation of the wavelet snake is driven by a maximum a posteriori (MAP) estimate to find a border of two regions with substantially different gray levels. The MAP estimate is performed by a gradient descent algorithm, which is expressed in a closed form by means of the fast wavelet transform. Therefore, the deformation of the snake can be computationally inexpensive, whereas most of

Fig. 4. A B-scan image with a tumor surrounded by a hypoechoic rims (or hello) at the center

the other snake models involve computationally expensive energy minimization processes. An advantage of using the wavelet snake is that the smoothness of the snake can easily be controlled by selection of the minimum scale of the wavelet coefficients being updated in the gradient descent algorithm. For example, the deformed snake may keep a smooth and round shape when the wavelet coefficients at several high scales are updated and those of the other, low scales are kept intact.

The final results obtained by application of the wavelet snake are shown in Fig. 7.

IV. Preliminary Results

We applied our method for detection of the boundaries of tumors in B-scan images to 10 liver metastases with various sizes of halos. These images were acquired by a Diasonics SYNERGY scanner, with a 3.5 MHz convex transducer in a single focal mode (F = 15 cm). The center of the tumors were identified manually by an experienced sonographer. A preliminary result for these metastases showed that our method could extract the boundaries of the tumors which were very close to the contours drawn by the expert sonographer. Therefore, our new method can segment the regions of focal liver disease in sonograms with high accuracy, and the method can be useful as a preprocessing step in our scheme for automated classification of focal liver disease in ultrasound images.

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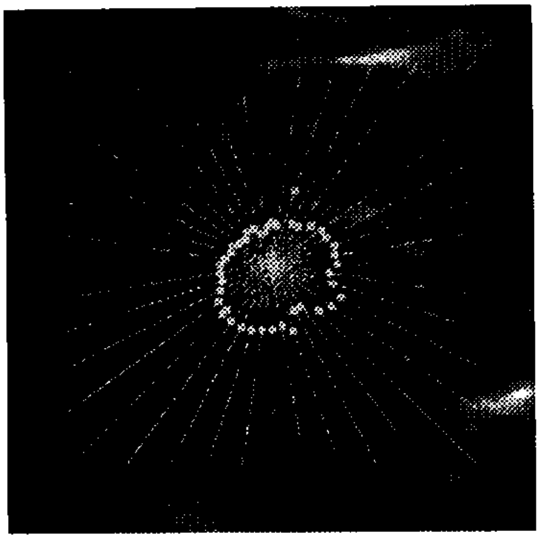

Fig. 5. Example of radial profiles. The detected boundary points are shown by white dots.

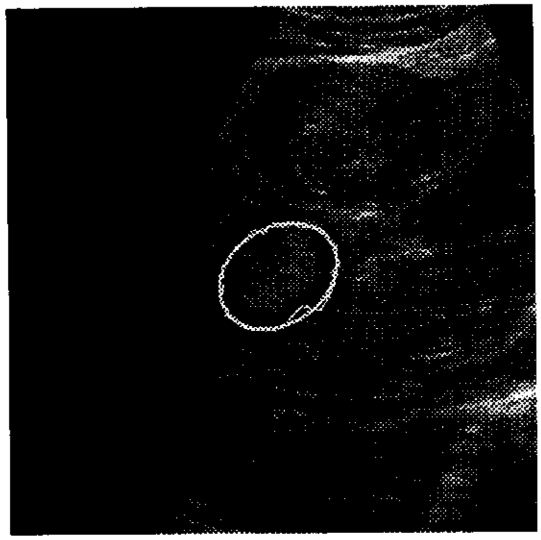

Fig. 6. Ellipse fitted to the detected boundary points. The rugged contour, which are made by connecting the detected boundary points, represents a coarse estimate of the boundary of the tumor.

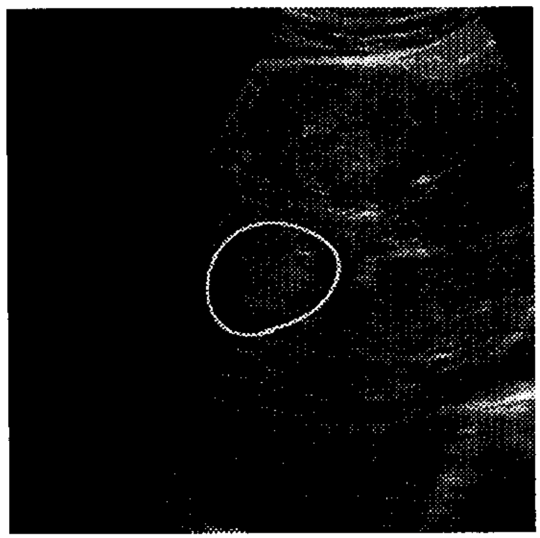

Fig. 7. Final estimate of the boundary of the tumor obtained by the wavelet snake.

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