

PadhAI: The Convolution Operation

One Fourth Labs

The 2D convolution operation

What about 2D inputs? What are the neighbors that we consider?

1. In a nutshell, the convolution operation boils down to taking a given input and re-estimating it as a weighted average of all the inputs around it.
2. The above definition is easy to visualise in 1D, but what about 2D?
3. In 2D, we would consider neighbors along the rows and columns, using the following formula

$$s_{ij} = (I * K)_{ij} = \sum_{a=0}^{m-1} \sum_{b=0}^{n-1} I_{i+a, j+b} * K_{a,b}$$

- a. K refers to kernel or weights and I refers to the input. And * refers to the convolution operation

a = rows

K	K_{00}	K_{01}
	K_{10}	K_{11}

b = cols

- b. Let a be the number of rows and b be the number of columns
- c. m & n specify the size of the matrix, in this case we consider them to be 2 each. So it's a 2x2 matrix. Therefore a & b range from 0-1 each.
- d. Now, to calculate the new value at a particular pixel I_{ij} , we simply need to fill in the values into the formula.
- e. $s_{ij} = I_{i+0, j+0}K_{0,0} + I_{i+0, j+1}K_{0,1} + I_{i+1, j+0}K_{1,0} + I_{i+1, j+1}K_{1,1}$
- f. Here is a pictorial representation

a	b	c	d	Convolution ->	aw+bx+ey+fz	bw+cx+fy+gz	cw+dx+gy+hz
e	f	g	h		ew+fx+iy+jz	fw+gx+jy+kz	gw+hx+ky+lz
i	j	k	l				
Input				Kernel	Output		

- g. This is how the convolutional operation looks like in 2D
- h. Instead of only choosing successive points, we must also consider previous points, on both

sides of the reference pixel. $s_{ij} = (I * K)_{ij} = \sum_{a=-\frac{m}{2}}^{\frac{m}{2}} \sum_{b=-\frac{n}{2}}^{\frac{n}{2}} I_{i-a, j-b} * K_{\frac{m}{2}+a, \frac{n}{2}+b}$

