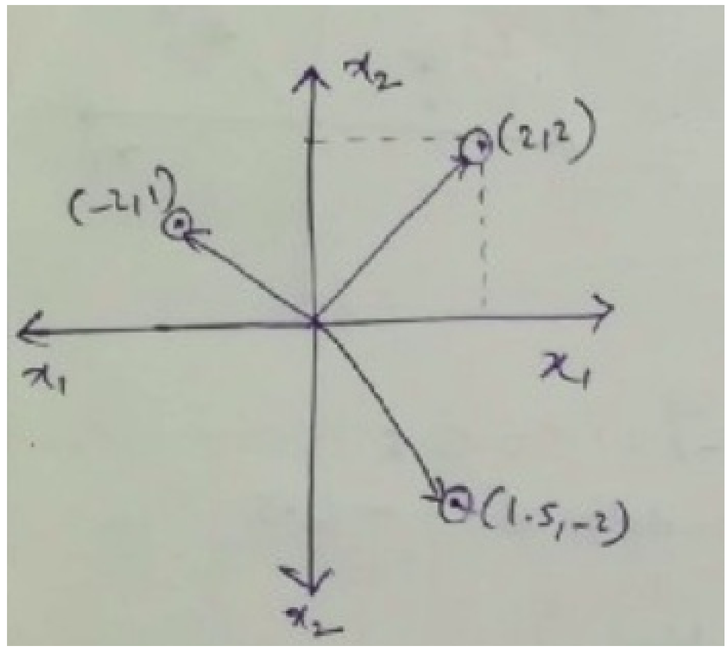
Vectors and Matrices

medium.com/@manveetdn/notes-on-vectors-and-matrices-padhai-onefourthlabs-course-a-first-course-on-deep-learning-ae38b208c42b

> **Disclaimer: This is notes on "Notes on Vectors and Matrices" Lesson (PadhAI onefourthlabs course "A First Course on Deep Learning".)**

The Graph of x1.x2 and y

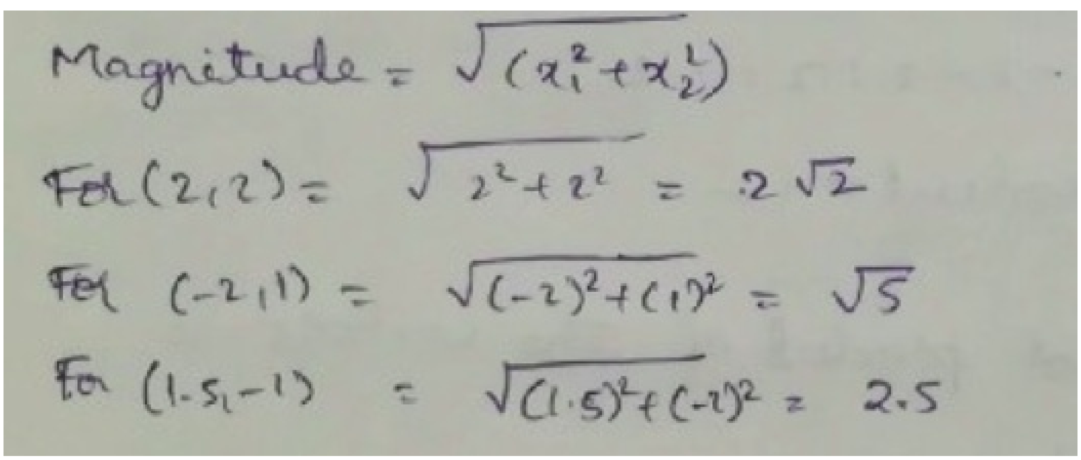
The beside graph shows the plots between **x1 , x2(the input data)**and the (**output) y .**



Here in the beside graph Y has a special phase because it is the output we will use only x1 and x2 to plot the data.

Vectors are quantified by:

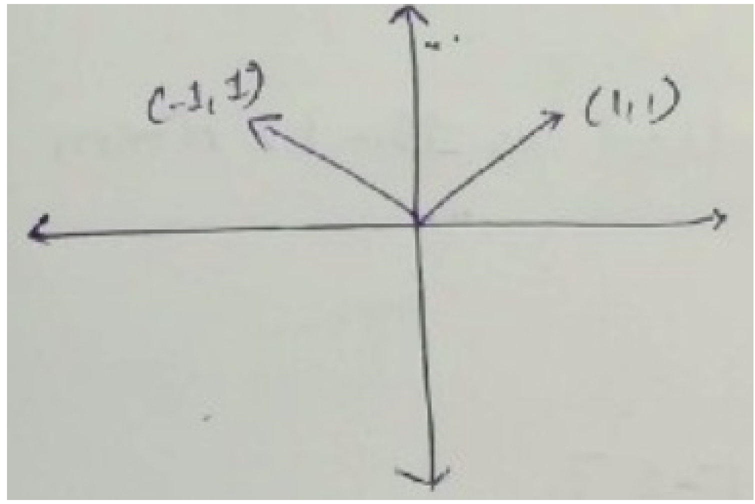
1. Magnitude of the vector
2. Direction of the vector.



Calculating the magnitude of vectors.

As shown in the image, the calculated the magnitude the formula is like that magnitude of the vectors from points (2,2),(-2,1),(1.5,1) is calculated. and the points.

vectors through (-1,1) and(1,1) are opposite in directions but equal in magnitude.

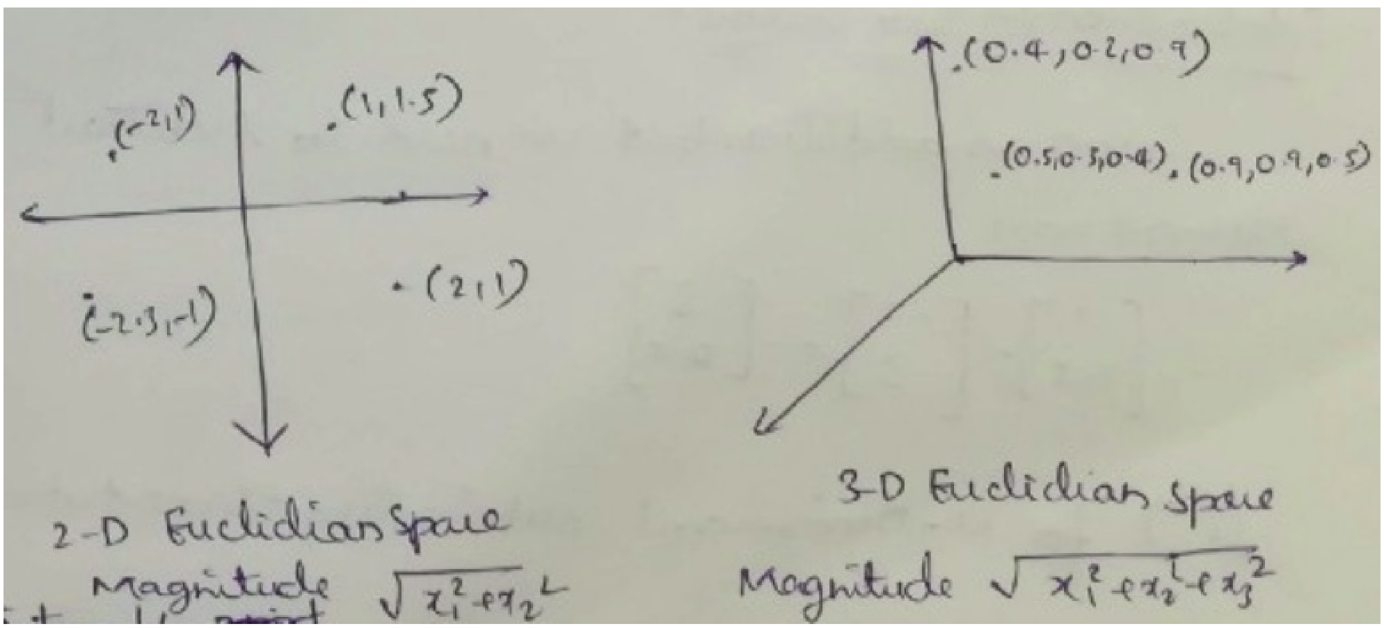


Note: Two vectors points in different direction can have same magnitude like the vectors of points points (-1,1)and (1,1).

Vectors:

In two dimensional space we will have two coordinates x1,x2 and in the same way three dimensional plane we have x1,x2,x3 three coordinates and

Similarly, 'N' dimensional space we will have x1,x2,x3,.............,xn coordinates.



2-D and 3-D Euclidean space.

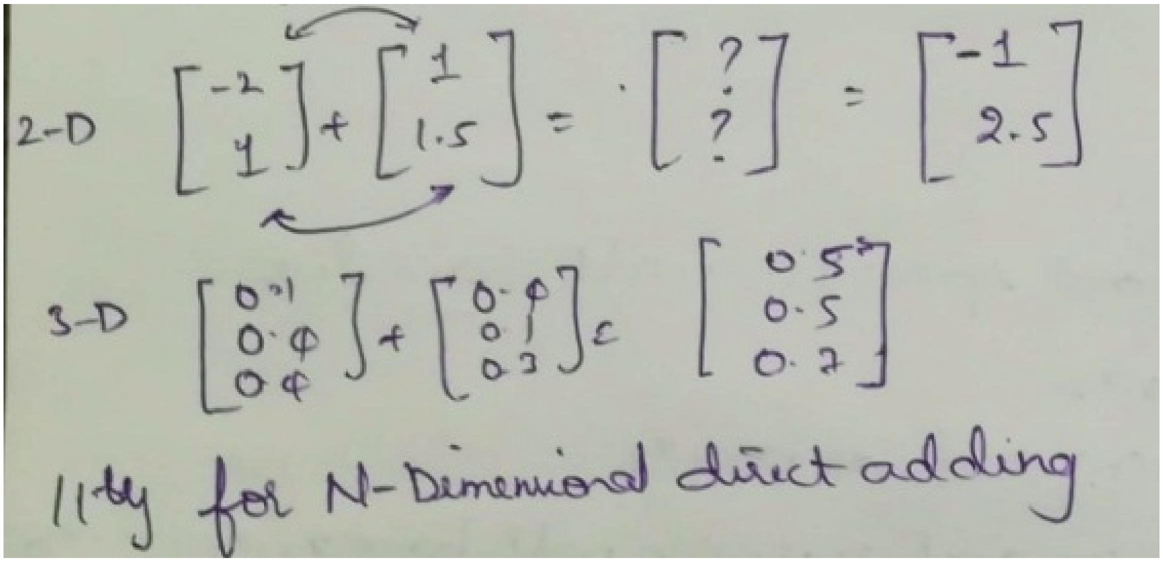
Euclidean Norm

Magnitude for the N dimensional Euclidean space is square root of summation of squares of the coordinates from x1 to xn.

$$\sqrt{\sum_{i=1}^{n} x_i^2}$$

The magnitude is also called the L2 Norm or Euclidean Norm.

Addition of Vectors:

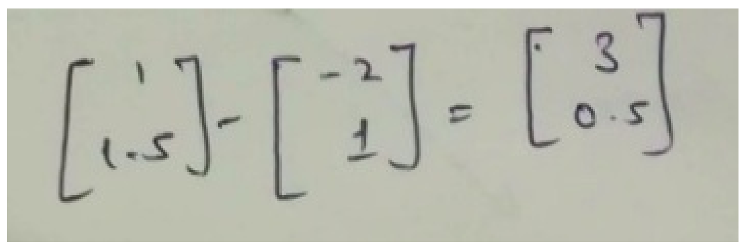


Addition of vectors.

In the case of addition of matrices, we will add row wise and write the final matrix as shown. Similar is the case with the N-dimensional Matrix.

Subtraction of matrix:

Substration of vectors.



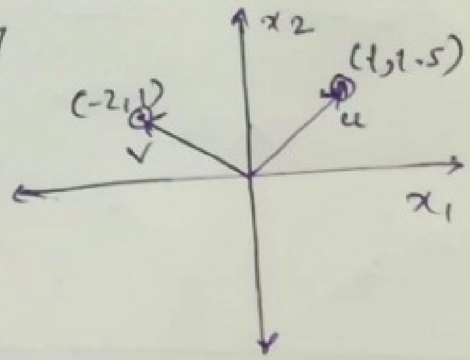
Same is the procedure as we follow which we have done in the addition of matrix, we will perform the subtraction operation row wise and write the final matrix.Same is the case with subtraction of N-dimensional matrix.

Dot product of vectors:

The below is process we follow for the dot-product of two vectors. the dot product is also called the multiplication product of vectors. Will perform multiplication operation to each row and we will take summation all the all final values.

Dot product of Vectors (or) multiplying two vectors:-

let $u = [u_1, u_2]$. $v = [v_1, v_2]$



$$\operatorname{dot}(u,v) = u \cdot v = u^T v$$

$$= \sum_{i=1}^{n} u_i v_i$$

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1.5 \end{bmatrix} = -2\times 1 + 1\times 1.5 = -2 + 1.5 = 0.5$$

$\therefore$ dot product $= 0.5$

let another be

$$\overset{u}{\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}} \overset{v}{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}} =$$

u_1	u_2	u_3
-2	1	3
v_1	v_2	v_3
1	2	4

$$= (-2\times 1) + (1\times 2) + (3\times 4)$$

$$= -2 + 2 + 12 = 12$$

$\therefore$ dot product $= 12$

Like this dot product of the vectors is calculated.

Do product of two vectors.

Unit Vector:

Ex:- $(0,1)$ $(1,0)$. $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ $(1,0,0)$ $(0,1,0)$ $(0,0,1)$, $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Magnitude $(0,1) = \sqrt{0+1^2} = 1$. $(1,0) = \sqrt{1^2+0} = 1$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{1} = 1$$

Unit vector of different dimesions.

Any vector of any dimension having its magnitude as one is called the unit vector.

Ex of unit vectors: (0,1), (0,1), (1/root(2),1/root(2)), (1,0,0),(0,1,0) , (0,0,1). (1/root(3),1/root(3),1/root(3)).

Unit vector in the direction of any given vector:

The unit vector in the direction of anyvector is

If vector is $u = [2, 1.5]$

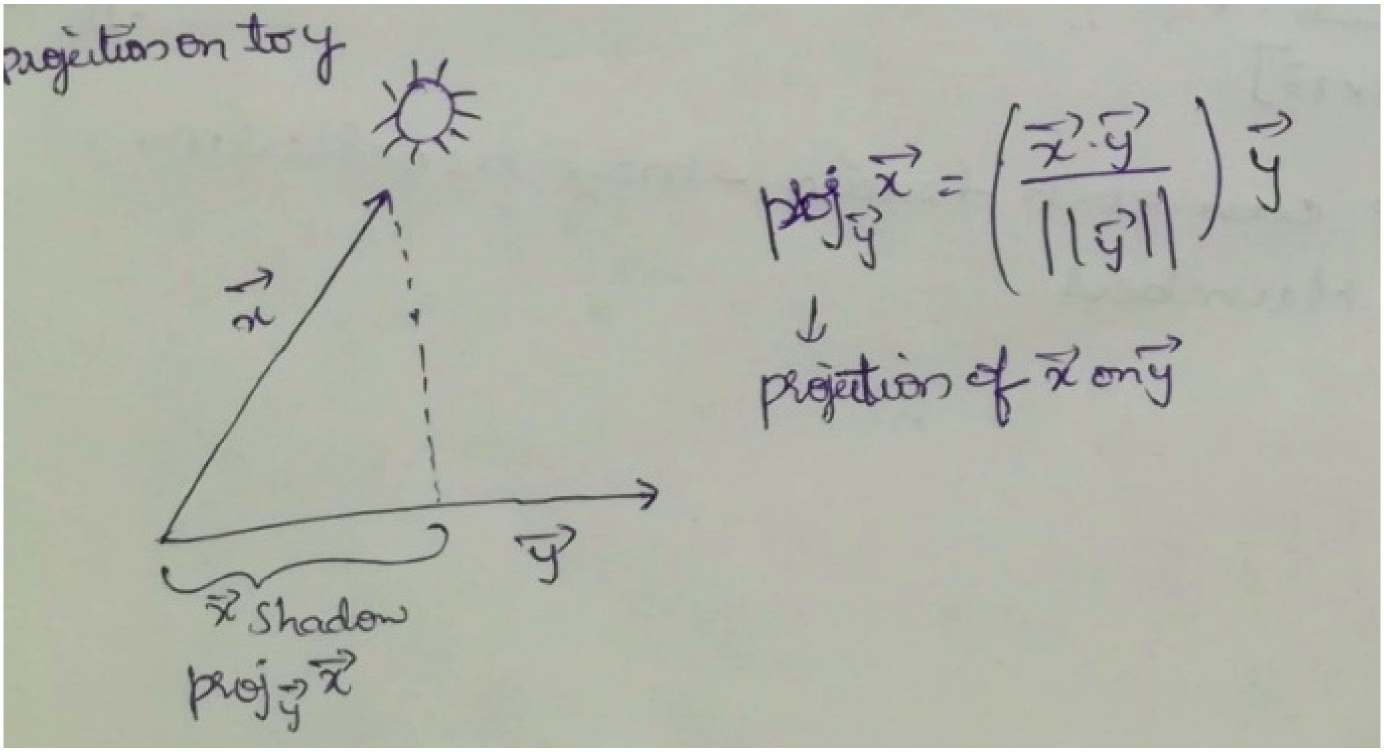
The unit vector in direction of u is $= \left[\frac{2}{\sqrt{(2)^2+(1.5)^2}}, \frac{1.5}{\sqrt{(2)^2+(1.5)^2}}\right]$

$$= (0.8, 0.6)$$

Formula.

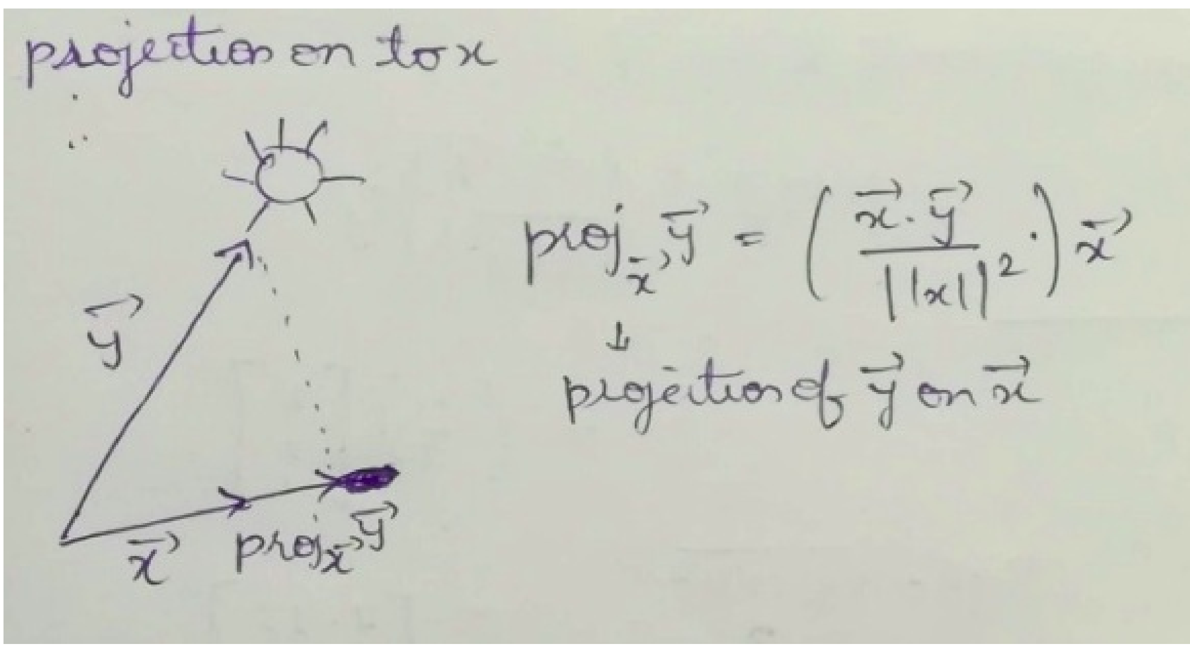
Projection of one vectors on to the another:

1. **Projection of vector x on vector y:**



Projection of vector x on vector y.

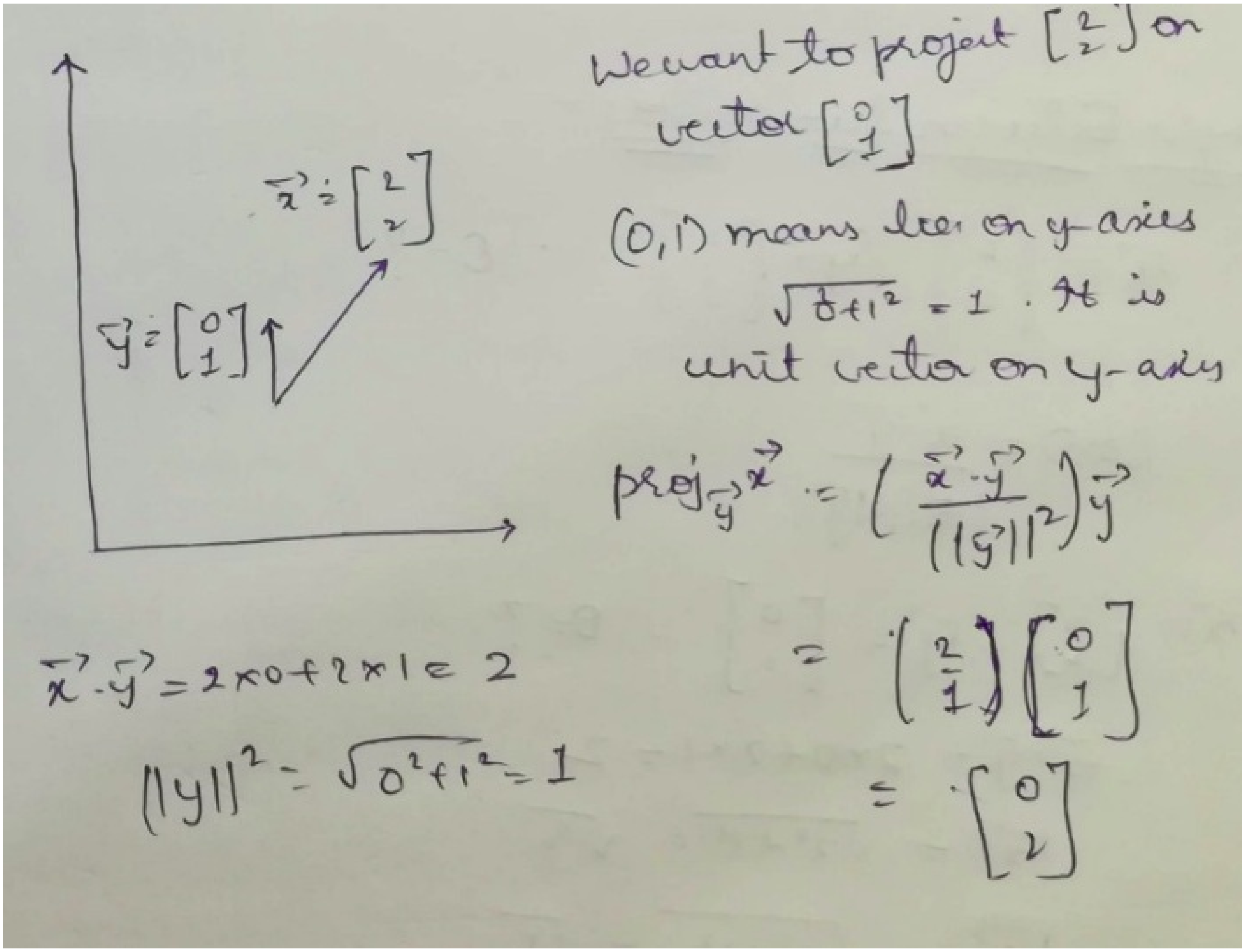
2. **Projection of vector y on vector x:**



Projection of vector y on vector x

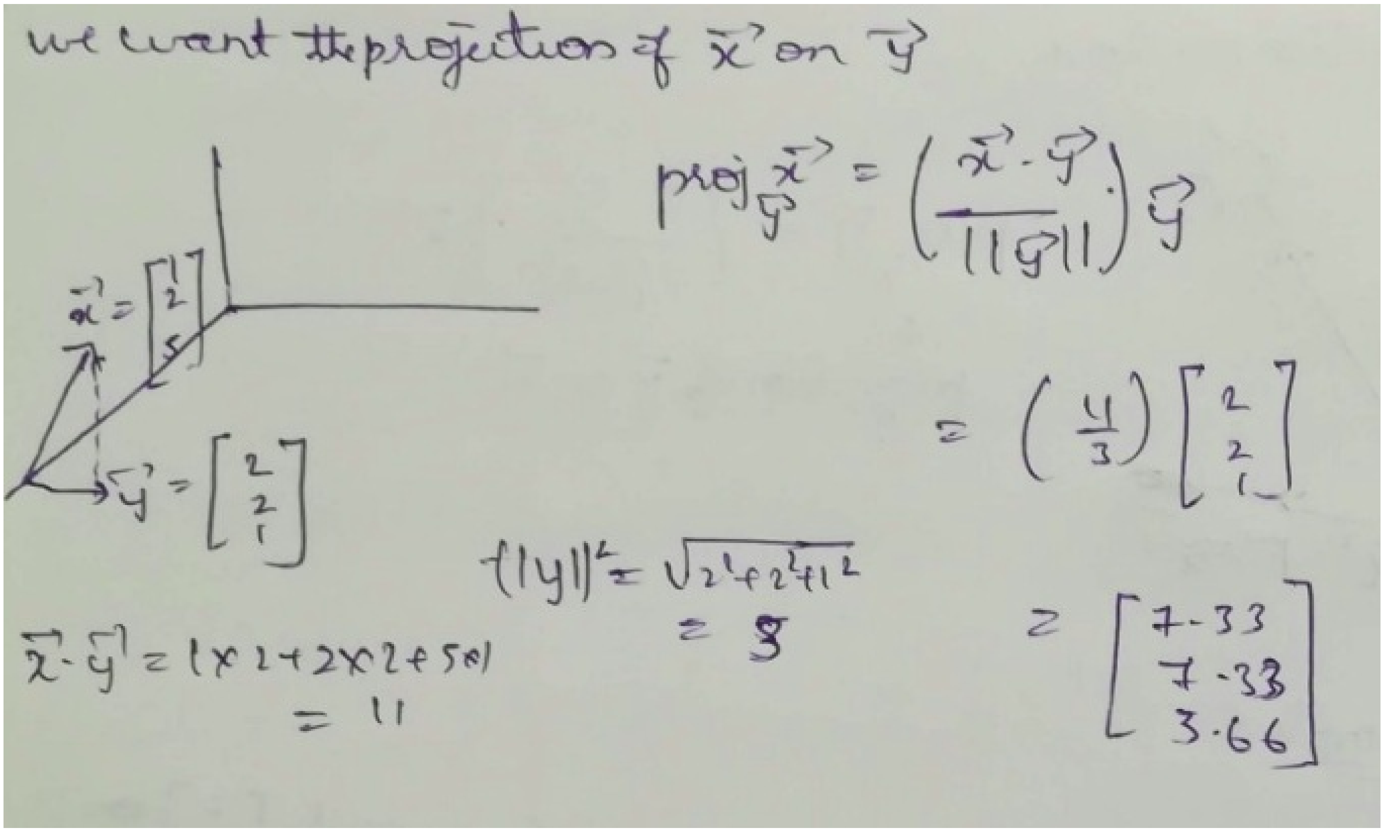
Example of the projection:

1. Vector X on Vector Y:



Example of projection of x on y.

2. Vector Y on Vector X:



Example of projection of y on x

Angle between two vectors:

Angle between two vectors x and y is given by the formula cosine of the angle as shown in the below.

$$\vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \cdot \quad \theta = ?$$

$$\cos\theta = \frac{\vec{x} \cdot \vec{y}}{\|\vec{x}\|\|\vec{y}\|}$$

$$\vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \cdot \quad \theta = ?$$

$$\vec{x} \cdot \vec{y} = 2 \times 0 + 2 \times 1 = 2$$

$$|\vec{x}| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$|\vec{y}| = \sqrt{0^2 + 1^2} = \sqrt{1}$$

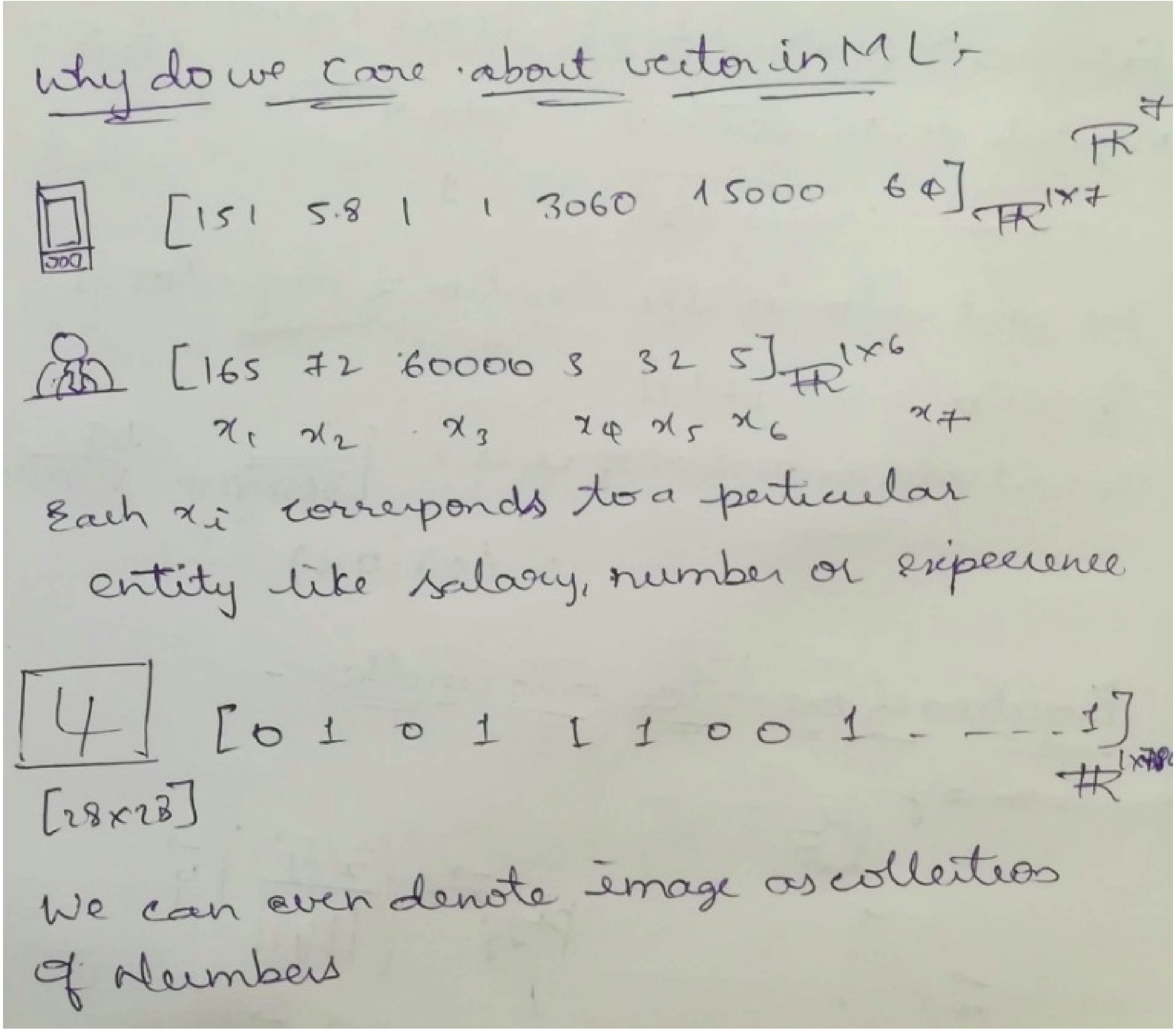
$$\cos\theta = \frac{1}{\sqrt{2}}$$

Angle between two vectors.

Orthogonal or perpendicular vectors:

If the angle between the two vectors is 90 then they are orthogonal vectors and the also the dot product of the vectors will be equal to zero.

Why do we care about vectors in ML:



Here the input in the data set we ill in the form of the vectors and matrics of and each xi corresponds to a particular entity like salary,number or experience.

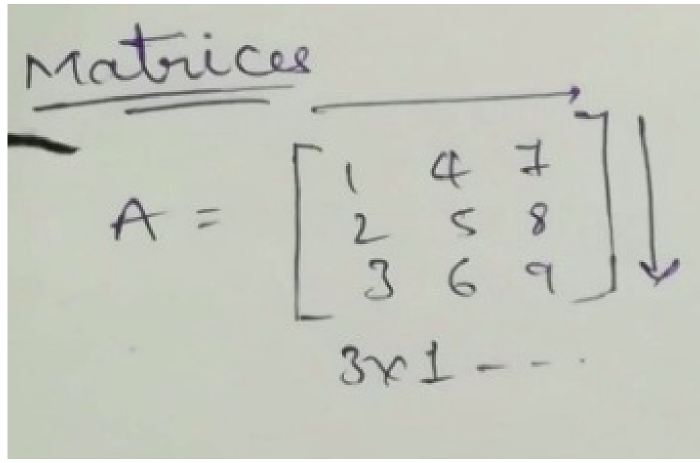
The MNIST dataset each image is a minimum of (28x28) and each pixel has a particular values 0 or 1.Like that we use the vectors and Matrices there.

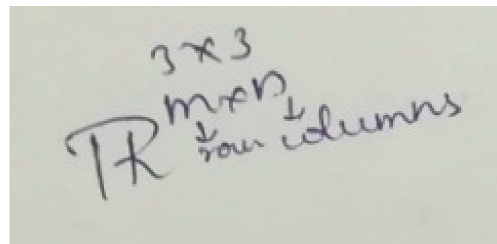
We can we even denote image as the collection of images.

Matrices:

Matrix is a n- dimensional representation of data.

3X3 Matrix and representation of matrix of m-rows and n-columns.

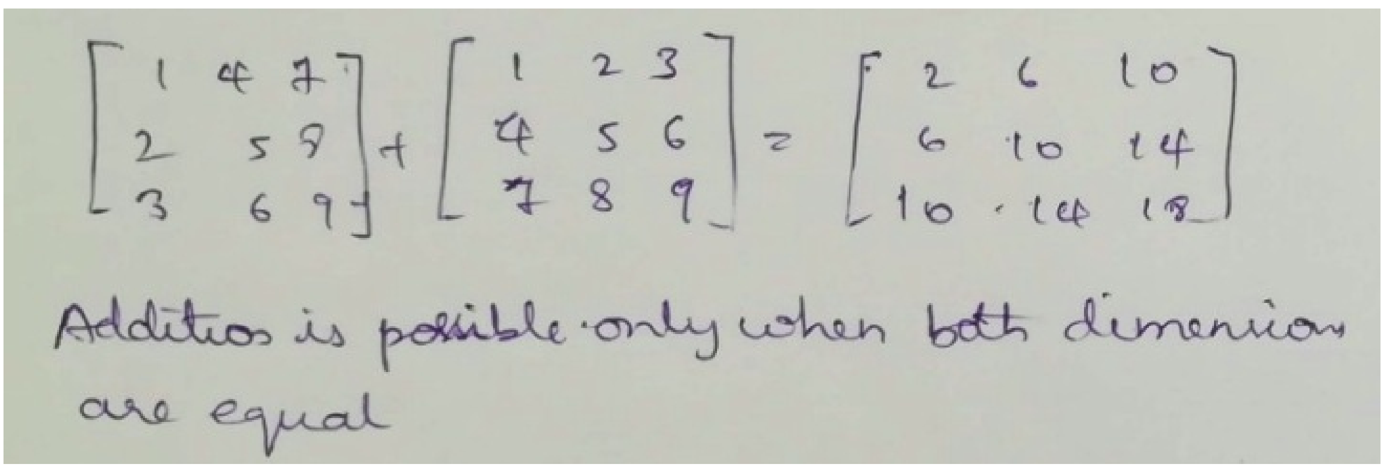




Addition of matrices:

both the matrices much be of same dimensions to perform the addition and substation operation on matrices.

The procedure is same same as the follow adding corresponding position elements and write the final matrix.



Addition of matrices.

Multiplication of two matrix :

To perform the multiplication operation the matrix the number of columns of first matrix should match the number of rows of second matrix.

$$\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix} \times \begin{bmatrix}5\\ 6\end{bmatrix} = \begin{bmatrix}?\\ ?\end{bmatrix}$$

$R^{2\times 2}$ $\quad$ $R^{2\times 1}$

Both m values should match to matrix multiplication

$$\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix} \times \begin{bmatrix}5\\ 6\end{bmatrix} = \begin{bmatrix}1\times5 + 3\times6\\ 2\times5 + 4\times6\end{bmatrix} = \begin{bmatrix}23\\ 34\end{bmatrix}_{2\times 1}$$

$\mathbb{R}^{2\times 2}$ $\quad$ $\mathbb{R}^{2\times 1}$ $\quad\quad$ $\mathbb{R}^{2\times 1}$

Multiplying matrix of 2 dimensions.

Conclusion: Number of columns in the matrix should be same as the number of rows in the vector

Matrix Multiplication:-

$$\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 1 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 1 \\ 0 & 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 15 \\ 4 & 5 & 12 \\ 11 & 10 & 15 \end{bmatrix}$$

Multiply each row with each column of the second matrix

$$R^{m \times n} \times R^{n \times k} = R^{m \times k}$$

Conclusion: Any $m \times n$ can be multiplied with a $n \times k$ matrix to get a $m \times k$ output

Multiplying matrix of 3 dimensions.

Alternative way of multiplying matrix:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} \times \begin{bmatrix} b_{11} \\ b_{21}\end{bmatrix} = \begin{bmatrix} a_{11}\times b_{11} + a_{12}\times b_{21} \\ a_{21}\times b_{11} + a_{22}\times b_{21}\end{bmatrix}$$

$$= b_{11}\times \begin{bmatrix} a_{11}\\ a_{21}\end{bmatrix} + b_{21}\begin{bmatrix} a_{12}\\ a_{22}\end{bmatrix}$$

which of the form

$$y = m x_1 + n x_2$$

this is linear combination where m & n are weights

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{bmatrix} = \begin{bmatrix} b_{11}\times \begin{bmatrix} a_{11}\\ a_{21}\end{bmatrix} + b_{21}\times \begin{bmatrix} a_{12}\\ a_{22}\end{bmatrix} \\ b_{21}\times \begin{bmatrix} a_{11}\\ a_{21}\end{bmatrix} + b_{22}\begin{bmatrix} a_{12}\\ a_{22}\end{bmatrix}\end{bmatrix}$$

Another way of multiplying two matrices.

We see more application of the concept in the course a head:

Vector

$$\underbrace{W}_{\text{matrix}}\overset{\text{Vector}}{x} + \underbrace{b}_{\text{Vector}}$$

W is $\mathbb{R}^{m\times n}$. x is $\mathbb{R}^{n\times 1}$.

$Wx = \mathbb{R}^{m\times 1}$ $\quad$ b is $\mathbb{R}^{m\times 1}$

The basic function y = wx+b.

All the data in the course will be in the form of matrices that's why we use matrices and learn about them.

This is a small try ,uploading the notes . I believe in "**Sharing knowledge is that best way of developing skills**".Comments will be appreciated. Even small edits can be suggested.

> **Each Applause will be a great encouragement.**

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