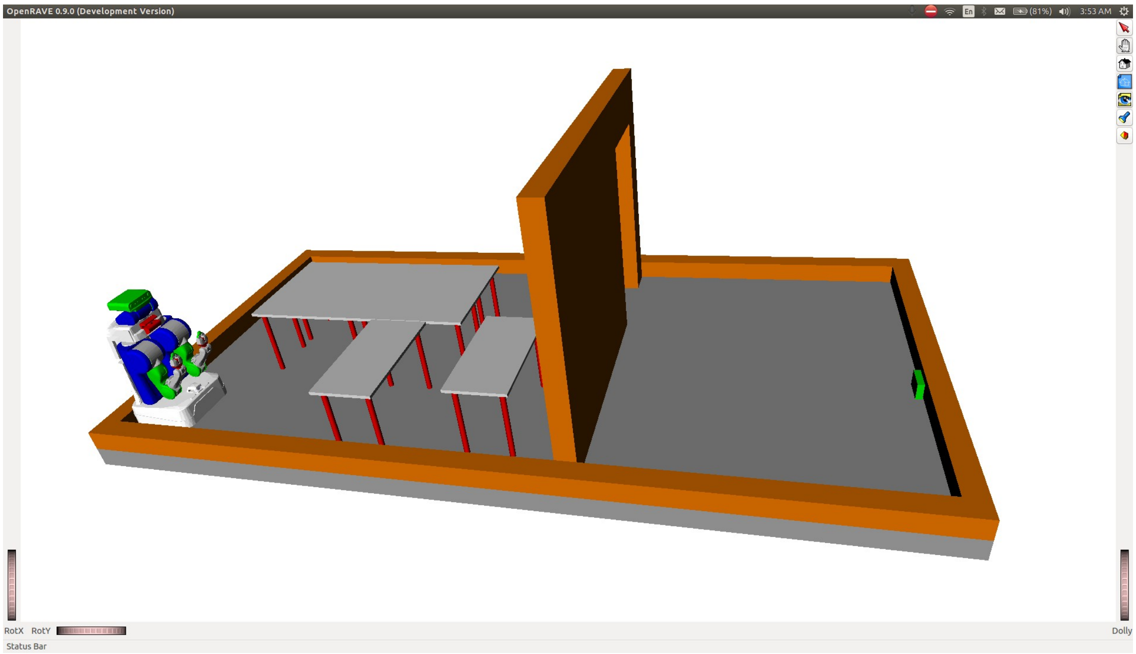
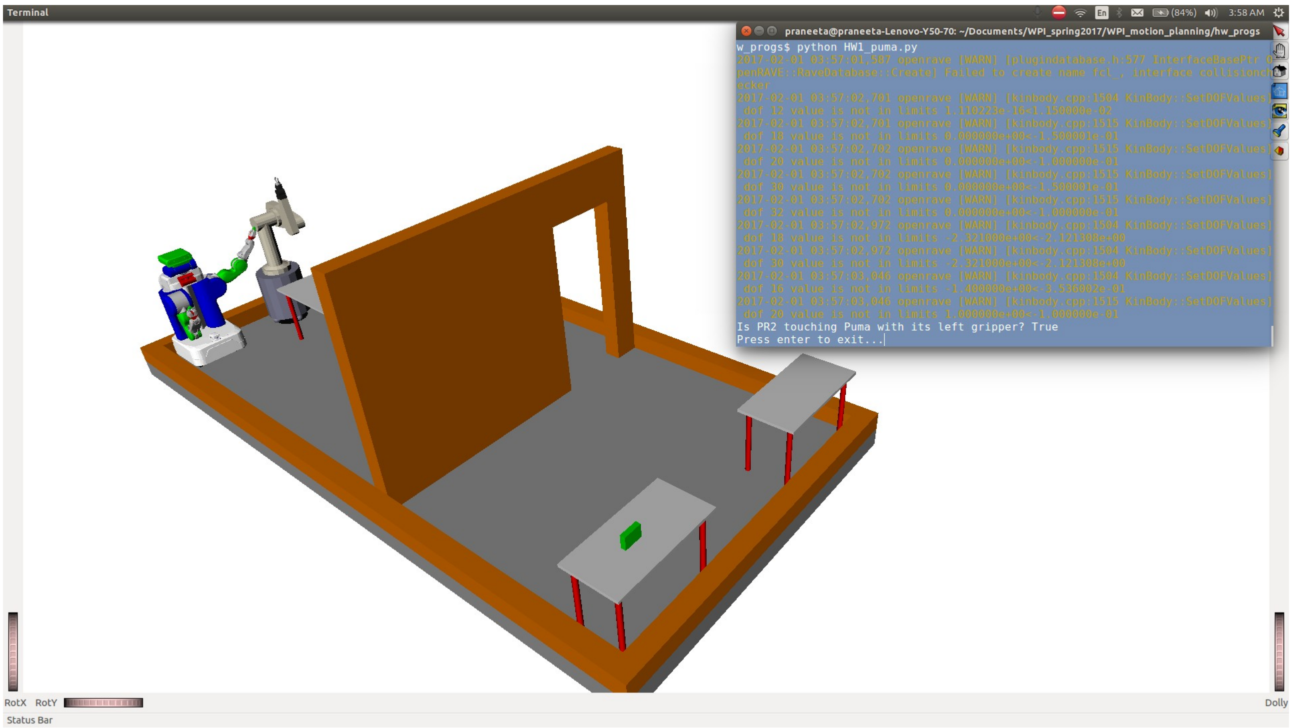
Motion planning Spring 2017
Assignment 1

1.



The image above is a screen shot of the opuput of HW1_tables.py. As seen above, all tables have been moved to be on the PR2's side of the room.

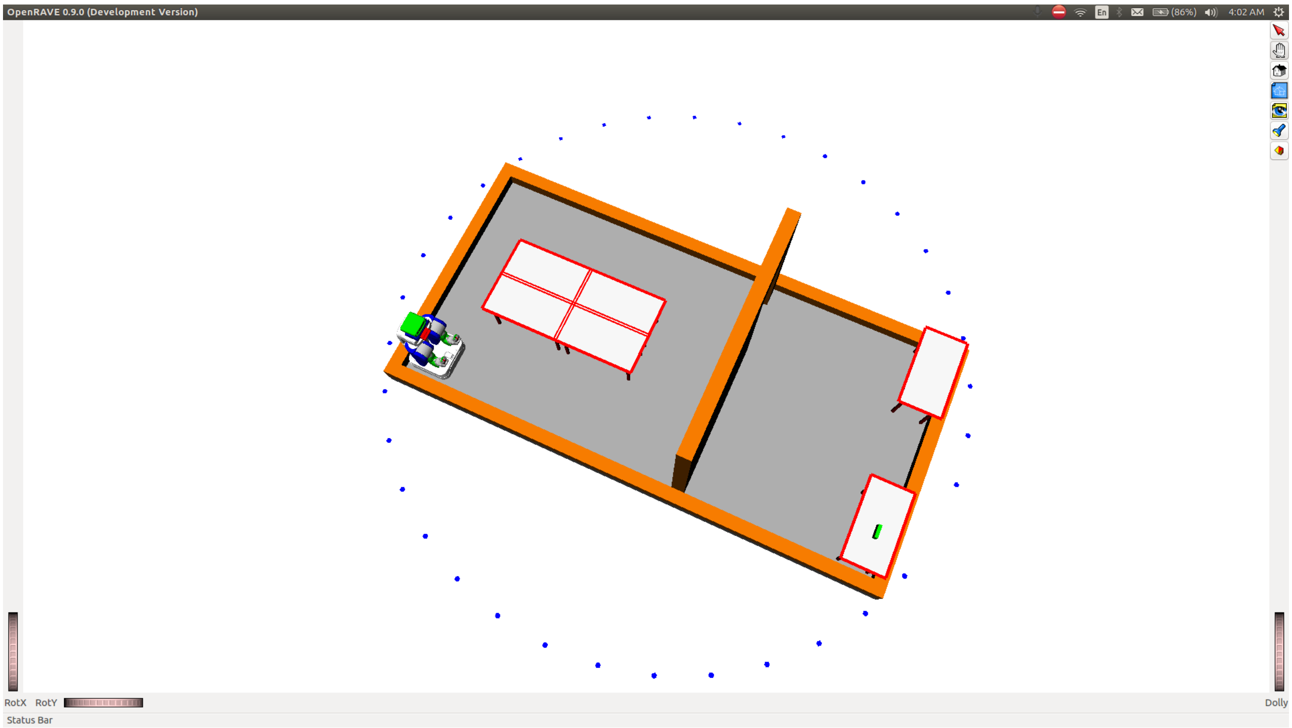
2.



The above image is a screen shot of the output of HW1_puma.py. As seen above, PR2 is touching Puma with its left gripper. The command line terminal indicates the collision of the two robots to be

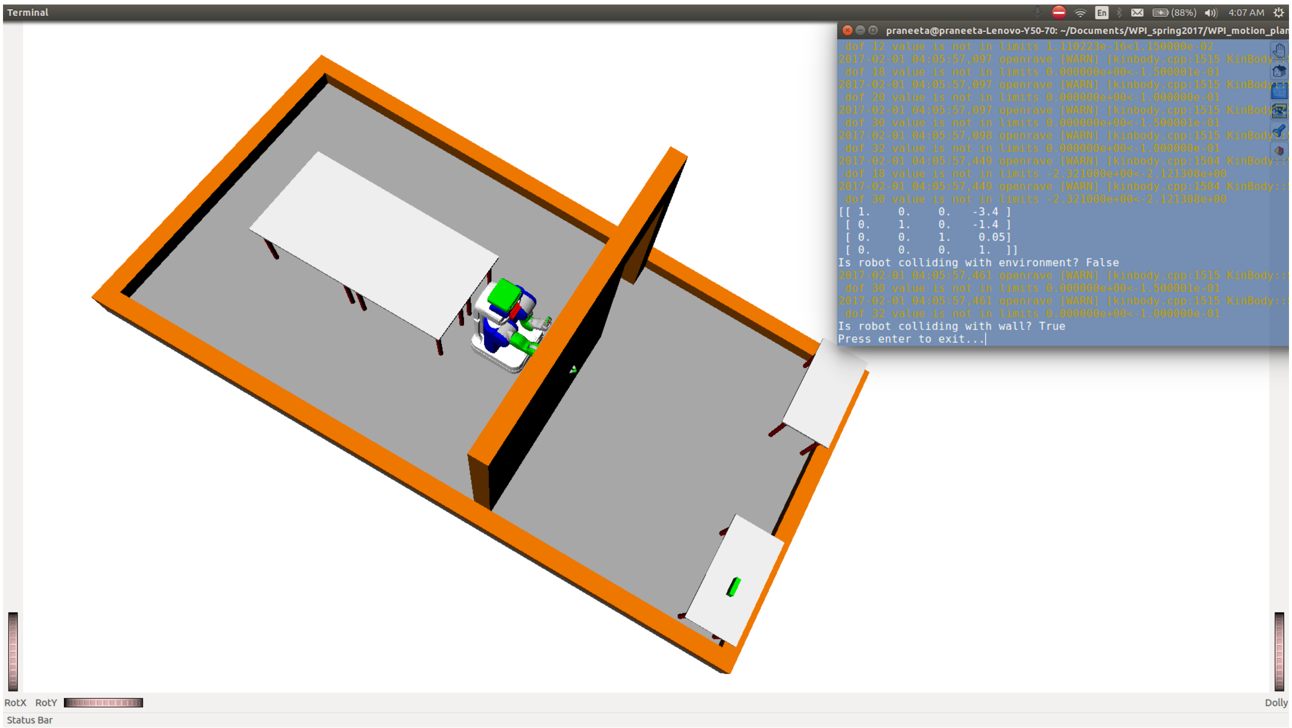
True.

3.



The above image is a screen shot of the output of HW1_drawing.py. 35 blue points have been drawn around the entire environment and red rectangles have been drawn around the boundaries of the table.

4.



The above image is a screen shot of the output of HW1_collision.py. As seen above, The robot was placed at the center of the environment, facing the wall. The command line terminal indicates its collision with the environment/wall as *False*. The joint values were set to the point the right arm pointing straight forward. The collision with the robot is is found (and displayed) to be *True* before the controller is called.

Explain why you placed waitrobot() inside/outside the "with env:". What would happen if it were placed in the other location and why?

"with env:" is used to lock the environment to be thread safe. The waitrobot() must be placed outside the "with env:" block. Waitrobot() ensures that the true configuration of the robot that was assigned to it in the "with env:" construct takes effect on the OpenRave GUI. It is a function that waits while the robot is busy.

If waitrobot() was placed inside the "with env:" block, then the changes being made inside the block's body will be interrupted and this is anti intuitive to the usage of "with env:" which locks the environment from external threads making changes.

1. For a general rigid body in $\mathbb{R}^3$, the dimension of the C-space is :

$x, y, z \in \mathbb{R}^3$

roll, pitch, yaw $\in \begin{cases} S^3 & \text{(without twist) and} \\ RP^3 & \text{(with twist).} \end{cases}$

$\therefore$ The C-space is given by.

$$C = C_1 \times C_2 = \mathbb{R}^3 \cdot \mathbb{R}P^3$$
$$\text{dimension} = 6.$$

Since the rod does not exhibit any detectable changes in position / orientation if rotated about it's central axis, its C_2 reduces to

$$C_2 = \begin{cases} \mathbb{R}P^2 & \text{(with twist)} \\ S^2 & \text{(without twist).} \end{cases}$$

Thus, the C-space is given by :

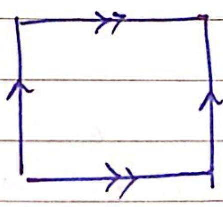
$$C = \begin{cases} \mathbb{R}^3 \times \mathbb{R}P^2 & \text{(with twist) or} \\ \mathbb{R}^3 \times S^2 & \text{(without twist)} \end{cases}$$

The dimension of the c-space is = $\underline{\underline{5}}$

★ Since the question does not talk about twist, we can assume that the C-space is given by :

$$\boxed{C = \mathbb{R}^3 \times S^2 \text{ with a dimension of } 5}$$

2. <u>Astroids - style game</u> :



The screen is a Torus (T^2)

A space craft in 2D has 3 properties that tell us its location on the screen :

$$x, y \in \mathbb{R}^2 \quad \& \quad \phi \text{ (orientation of spacecraft)} \in S^1$$

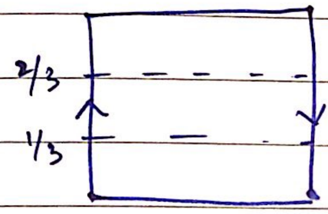
Thus $C_1 = \mathbb{R}^2$ & $C_2 = S^1$

$$C = C_1 \times C_2$$

$$\boxed{C = R^2 \times S^1}$$

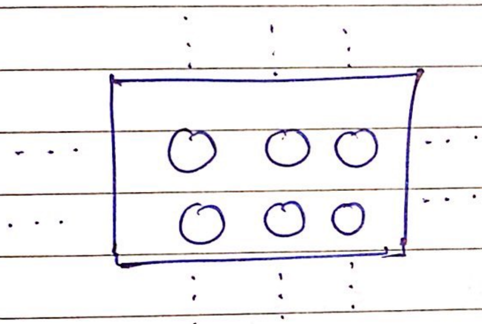
Considering the property of the screen, the C-space could be affected by ~~its~~ this since the space craft can enter and exit the 2 identified side pairs. This makes the C-space $T^2 \times S^1$

(a)



After slicing the Mobius band as shown, the resultant is 2 interlinked mobius bands, where One is longer (and thinner) than the second.

(b)



The resulting plane after tearing holes out of the plane is not a Manifold since the points around the periphery of the holes have lost the points in their ~~vicinity.~~ vicinity.

3 A. Let

$$h_1 = a + bi + cj + dk.$$

then

$a = \cos\left(\frac{\theta}{2}\right)$

$b = v_1 \cdot \sin\left(\frac{\theta}{2}\right)$

$c = v_2 \sin\left(\frac{\theta}{2}\right)$

$d = v_3 \sin\left(\frac{\theta}{2}\right)$

when θ = rotation = $\frac{\pi}{2}$ (given)

& $v = [v_1, v_2, v_3] = \left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ (given)

$\therefore \quad a_1 = \frac{1}{\sqrt{2}}$

$b_1 = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{6}}$

$c_1 = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{6}}$

$d_1 = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{6}}$

$\therefore \; h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}(i + j + k)$

B. Similar to part A, now we have

$v = [0, 1, 0]$ and rotation $\theta = \pi$.

$\therefore$

$a_2 = 0 \; \left(= \cos\frac{\pi}{2}\right)$

$b_2 = 0 \cdot \sin\frac{\pi}{2} = 0$

$c_2 = 1 \cdot \sin\frac{\pi}{2} = 1$

$d_2 = 0 \cdot \sin\frac{\pi}{2} = 0$

$\therefore \; h_2 = 1 \cdot j$

c. Let h_3 represent rotation represented by h_1 followed by rotation represented by h_2

Then

$h_3 = h_2 \cdot h_1 = a_3 + b_3 i + c_3 j + d_3 k$

$\cancel{= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}(i+j+k)\right) \cdot j}$

$\cancel{= \frac{1}{\sqrt{2}} j + \frac{1}{\sqrt{6}}(-1)} *$

Let $h_1 = a_1 + b_1 i + c_1 j + d_1 k$ & $h_2 = a_2 + b_2 i + c_2 j + d_2 k$

$a_3 = a_1 a_2 - b_1 b_2 - c_1 c_2 - d_1 d_2 = \frac{-1}{\sqrt{6}}$

$b_3 = a_1 b_2 + a_2 b_1 + c_1 d_2 - c_2 d_1 = \cancel{\frac{1}{\sqrt{6}}}\; 0 + 0 + 0 - \frac{1}{\sqrt{6}} = \frac{-1}{\sqrt{6}}$

$c_3 = a_1 c_2 + a_2 c_1 + b_2 d_1 - b_1 d_2 = \frac{1}{\sqrt{2}} + 0 + 0 - 0 = \frac{1}{\sqrt{2}}$

$d_3 = a_1 d_2 + a_2 d_1 + b_1 c_2 - b_2 c_1 = 0 + 0 + \frac{1}{\sqrt{6}} - 0 = \frac{1}{\sqrt{6}}$

classmate

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$\therefore h_3 = \frac{-1}{\sqrt{6}} - \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{2}} j + \frac{1}{\sqrt{6}} k$

$\therefore \theta = 2 \cos^{-1}(a_3)$

$= 2 \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right).$

$\theta = 3.9827$ rad.

$\cancel{v_1 = 2 \sin^{-1}(b_3)}$

$\cancel{= 2 \sin^{-1}\left(\frac{-1}{6}\right)}$

$v_1 = \frac{b_3}{\sin\left(\frac{\theta}{2}\right)} = \frac{b_3}{\sqrt{1 - \cos^2\left(\frac{\theta}{2}\right)}} = \frac{b_3}{\sqrt{1 - a_3^2}}$

$= \frac{-1/\sqrt{6}}{\sqrt{\frac{5}{6}}} = -\frac{1}{\sqrt{5}}$

$v_2 = \frac{c_3}{\sqrt{1 - a_3^2}} = \frac{1/\sqrt{2}}{\sqrt{\frac{5}{6}}} = \frac{\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{3}{5}}$

$v_3 = \frac{d_3}{\sqrt{1 - a_3^2}} = \frac{1/\sqrt{6}}{\sqrt{5/6}} = \frac{1}{\sqrt{5}}$

$\therefore v = [v_1, v_2, v_3]$

$v = \left[\frac{-1}{\sqrt{5}}, \sqrt{\frac{3}{5}}, \frac{1}{\sqrt{5}}\right]$ (axis)

with rotation $= 3.9827$ rad. (angle)

4. For a single polyhedral body in 3D :

$$C = \mathbb{R}^3 \cdot \mathbb{RP}^3 \quad \text{——} \quad (1)$$

For n polyhedral bodies, the C space is simply a Cartesian product of eq (1), n-times.

ie $$C\Big|_{n\,\text{bodies}} = \left(\mathbb{R}^3 \cdot \mathbb{RP}^3\right)^n$$

given : 6 polyhedral bodies

$\Rightarrow$ $n = 6$

$$\therefore \quad C = \left(\mathbb{R}^3 \times \mathbb{RP}^3\right)^6$$

$$= \mathbb{R}^{18} \times \left(\mathbb{RP}^3\right)^6 \; //$$

$\therefore$ The dimension of the C-spaces = $\underline{\underline{36}}$