

IIT-JEE-Mathematics-Paper2-2008

1. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{(-1)}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1 . The particle moves $\sqrt{2}$ units in the direction of the unit vector $i + j$ and then it moves through an angle $\pi/2$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2 . The point z_2 is given by

- (A) $6 + 7i$
- (B) $-7 + 6i$
- (C) $7 + 6i$
- (D) $-6 + 7i$

2. Let the function $g : (-\infty, \infty) \rightarrow (-\pi/2, \pi/2)$ be given by $g(u) = 2\tan^{-1}(e^u) - \pi/2$. Then, g is

- (A) even and is strictly increasing in $(0, \infty)$
- (B) odd and is strictly decreasing in $(-\infty, \infty)$
- (C) odd and is strictly increasing in $(-\infty, \infty)$
- (D) neither even or odd, but is strictly increasing in $(-\infty, \infty)$

3. Consider a branch of the hyperbola

$$x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$$

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

- (A) $1 - \sqrt{2/3}$
- (B) $\sqrt{3/2} - 1$
- (C) $1 + \sqrt{2/3}$

(D) $\sqrt{(3/2)} + 1$

4. The area of the region between the curves $y = \sqrt{((1 + \sin x)/\cos x)}$ and $y = \sqrt{((1 - \sin x)/\cos x)}$ bounded by the lines $x = 0$ and $x = \pi/4$ is

- (A) $\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$
(B) $\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
(C) $\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$
(D) $\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}} dt$

5. Consider three points $P = (-\sin(\beta-\alpha), -\cos \beta)$, $Q = (\cos(\beta-\alpha), \sin \beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \pi/4$. Then,

- (A) P lies on the line segment RQ
(B) Q lies on the line segment PR
(C) R lies on the line segment QP
(D) P, Q, R are non-collinear

6. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

- (A) 2, 4 or 8
(B) 3, 6 or 9
(C) 4 or 8
(D) 5 or 10

7. Let two non-collinear unit vectors a and b form an acute angle. A point P moves so that at any time t the position vector OP (where O is the origin) is given by $a \cos t + b \sin t$.

When P is farthest from origin O, let M be the length of vector OP and u be the unit vector along vector OP. Then,

- (A) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$
- (B) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + \hat{a} \cdot \hat{b})^{\frac{1}{2}}$
- (C) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$
- (D) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + 2\hat{a} \cdot \hat{b})^{\frac{1}{2}}$

8. Let

$$I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx, J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx.$$

Then, for an arbitrary constant C, the value of $J - I$ equals.

- (A) $\frac{1}{2} \log \left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + C$
- (B) $\frac{1}{2} \log \left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C$
- (C) $\frac{1}{2} \log \left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + C$
- (D) $\frac{1}{2} \log \left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1} \right) + C$

9. Let $g(x)$ & $g(x) = \log f(x)$ where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then, for $N = 1, 2, 3, \dots$,

- $$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$
- (A) $-4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2} \right\}$
 - (B) $4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N-1)^2} \right\}$
 - (C) $-4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2} \right\}$
 - (D) $4 \left\{ 1 + \frac{1}{9} + \frac{1}{25} + \dots + \frac{1}{(2N+1)^2} \right\}$

10. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1: The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P.

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

11. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, 1/\beta$ are the roots of the equation $ax^2 + 2bx + c = 0$, where β^2 is not belongs to $\{-1, 0, 1\}$.

STATEMENT-1: $(p^2 - q)(b^2 - ac) > 0$

and

STATEMENT-2: $b \neq pa$ or $c \neq qa$

- (A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.
- (B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

12. Consider

$$L_1 : 2x + 3y + p - 3 = 0$$

$$L_2 : 2x + 3y + p + 3 = 0$$

$$\text{where } p \text{ is a real number, and } C : x^2 + y^2 + 6x + 10y + 30 = 0$$

STATEMENT-1: If line L_1 is a chord of circle C , then line L_2 is not always a diameter of circle C .

and

STATEMENT-2: If line L_1 is a diameter of circle C , then line L_2 is not a chord of circle C .

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

13. Let a solution $y = y(x)$ of the differential equation

$$x\sqrt{x^2-1} dy - y\sqrt{y^2-1} dx = 0$$

satisfy $y(2) = 2/\sqrt{3}$.

STATEMENT-1: $y(x) = \sec(\sec^{-1} x - \pi/6)$

and

STATEMENT-2: $y(x)$ is given by

$$1/y = (2\sqrt{3})/x - \sqrt{(1-1/x^2)}$$

(A) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for statement-1.

(B) Statement-1 is True, Statement-2 is True, Statement-2 is not a correct explanation for statement-1.

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Paragraph

Consider the function $f : (-\infty, \infty) \rightarrow (-\infty, \infty)$ defined by

$$f(x) = (x^2 - ax + 1)/(x^2 + ax + 1), \quad 0 < a < 2.$$

14. Which of the following is true?

- (A) $(2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0$
- (B) $(2 - a)^2 f''(1) - (2 + a)^2 f''(-1) = 0$
- (C) $f'(1)f'(-1) = (2 - a)^2$
- (D) $f'(1)f'(-1) = (2 + a)^2$

15. Which of the following is true?

- (A) $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$
- (B) $f(x)$ is increasing on $(-1, 1)$ and has a local maximum at $x = 1$
- (C) $f(x)$ is increasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$.
- (D) $f(x)$ is decreasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at $x = 1$.

16. Let

$$g(x) = \int_0^{ex} (f'(t))/(1+t^2) dt.$$

Which of the following is true?

- (A) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
- (B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
- (C) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
- (D) $g'(x)$ does not change sign on $(-\infty, \infty)$

Paragraph

Consider the lines

$$L1 : (x+1)/3 = (y+2)/1 = (z+1)/2$$

$$L2 : (x-2)/1 = (y+2)/2 = (z-3)/3$$

17. The unit vector perpendicular to both L₁ and L₂ is

- (A) $(-\hat{i} + 7\hat{j} + 7\hat{k})/\sqrt{99}$
- (B) $(-\hat{i} - 7\hat{j} + 5\hat{k})/(5\sqrt{3})$
- (C) $(-\hat{i} + 7\hat{j} + 5\hat{k})/(5\sqrt{3})$
- (D) $(7\hat{i} - 7\hat{j} - \hat{k})/\sqrt{99}$

18. The shortest distance between L₁ and L₂ is

- (A) 0
- (B) $17/\sqrt{3}$
- (C) $41/(5\sqrt{3})$
- (D) $17/(5\sqrt{3})$

19. The distance of the point (1, 1, 1) from the plane passing through the point (-1, -2, -1) and whose normal is perpendicular to both the lines L₁ and L₂ is

- (A) $2/\sqrt{75}$
- (B) $7/\sqrt{75}$
- (C) $13/\sqrt{75}$
- (D) $23/\sqrt{75}$

20. Consider the lines given by

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

Match the Statements/Expressions in Column I with the Statements/ Expressions in Column II.

Column I		Column II	
(A)	L_1, L_2, L_3 are concurrent if	(p)	$k = -9$
(B)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q)	$k = -6/5$

(C)	L_1, L_2, L_3 form a triangle if	(r)	$k = 5/6$
(D)	L_1, L_2, L_3 do not form a triangle if	(s)	$k = 5$

21. Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		Column II	
(A)	The minimum value of $x^2+2x+4 / x+2$ is	(p)	0
(B)	Let A and B be 3×3 matrices of real numbers, where A is symmetric, b is skew-symmetric, and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q)	1
(C)	Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3-a)} < 2$, must be less than	(r)	2
(D)	If $\sin q = \cos f$, then the possible values of $1/\Pi(\theta \pm \emptyset - \Pi/2)$ are	(s)	3

22. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the Statements/Expressions in Column I and the Statements/ Expressions in Column II.

Column I		Column II	
(A)	The number of permutations containing the word ENDEA is	(p)	$5!$
(B)	The number of permutations in which the letter E occurs in the first and the last positions i	(q)	$2 \times 5!$
(C)	The number of permutations in which one of the letters, D, L, N occurs in the last five positions is	(r)	$7 \times 5!$
(D)	The number of permutations in which the letters A, E, O occur only in odd positions is	(s)	$21 \times 5!$