Matrix Computations | Matrices (mathematics) | Vectors (mathematics) | Linear Algebra

Why does repeatedly multiplying a vector by a square matrix cause the vector to converge on or along the matrix's eigenvector?

http://setosa.io/ev/eigenvectors-and-eigenvalues/

2 Answers

Peter Elliott, UCLA Applied Math '13; CMU Statistics PhD Student
Updated Sep 19 2016 · Author has **57** answers and **82k** answer views

First let's assume the matrix A has a full set of eigenvectors $v_1, \ldots, v_m$ with associated eigenvalues $\lambda_1, \ldots, \lambda_m$. Further I'm going to assume that $\lambda_1 = 1$ and for all other k, $|\lambda_k| < 1$.

(Now this might seem like a strong assumption. However, if the largest magnitude eigenvalue were anything else we could simply factor it out to get a matrix $\tilde{A} = \frac{1}{\lambda_1} A$. This matrix has the same eigenvectors as A, and multiplying it by a vector gives only a scaled version of what you get multiplying A by the same vector. As a result, repeated multiplication by $\tilde{A}$ will converge along the same vector as repeated multiplication by A.)

For any vector x we can write it in terms of the eigenvectors, $x = \sum_{k=1}^{m} a_k v_k$. Now let's try repeatedly multiplying x by A.

$$A^n x = \sum_{k=1}^{m} a_k A^n v_k = \sum_{k=1}^{m} a_k \lambda_k^n v_k$$

So the component of x corresponding to each eigenvector is repeatedly multiplied by the associated eigenvalue. For the first component, since the eigenvalue is 1 it just stays the same. For all other components, $\lambda_k^n \to_{n\to\infty} 0$, so they eventually disappear. That means we're left with

$$A^n x \to_{n\to\infty} a_1 v_1 .$$

The intuition here is that when you perform repeated multiplication by the same matrix, you're really repeatedly scaling the coefficients corresponding to the eigenvector basis by the size of the eigenvalue. When you do this a large number of times, the largest eigenvalue/eigenvector will come to dominate.

How can I recover the vector from the matrix (generated from the vector multiplied by its transpose but contaminated by a noise matrix)?

How does multiplying a column vector with a matrix differ from the same with a row vector? How do you know which form a vector is in?

Is it necessary to find the inverse of a given matrix? Should that matrix be a square matrix?

Like a matrix multiplied by a vector can be written as a linear combination of columns of the matrix, what does a matrix multiplied by a matri...

If vector A and vector B are unit vectors, then what is the angle between them so that root 2 A vector - B vector is a unit vector?

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Nadeem Rojenko, Ms. student of Theoretical Physics.
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Pretty simple, eigen vector by definition is a vector that will stay it self (same direction, but may have different length) after performing matrix transformation.

So applying a matrix many times, will keep transforming, rotating, shrinking and expanding the vector till at some stage it will become one of the eigenvectors, then it will not change any more (change in direction).

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