Matrix Computations Matrices (mathematics) Vectors (mathematics) Linear Algebra

Why does repeatedly multiplying a vector by a square matrix cause the vector to converge on or along the matrix's eigenvector?

2 Answers



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First let's assume the matrix A has a full set of eigenvectors v_1, \ldots, v_m with associated eigenvalues $\lambda_1, \ldots, \lambda_m$. Further I'm going to assume that $\lambda_1 = 1$ and for all other $k, |\lambda_k| < 1$.

(Now this might seem like a strong assumption. However, if the largest magnitude eigenvalue were anything else we could simply factor it out to get a matrix $\tilde{A} = \frac{1}{\lambda_1} A$. This matrix has the same eigenvectors as A, and multiplying it by a vector gives only a scaled version of what you get multiplying A by the same vector. As a result, repeated multiplication by \tilde{A} will converge along the same vector as repeated multiplication by A.)

For any vector x we can write it in terms of the eigenvectors, $x = \sum_{k=1}^m a_k v_k$. Now let's try repeatedly multiplying x by A.

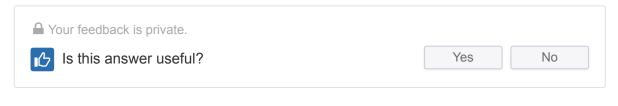
$$A^n x = \sum_{k=1}^m a_k A^n v_k = \sum_{k=1}^m a_k \lambda_k^n v_k$$

So the component of x corresponding to each eigenvector is repeatedly multiplied by the associated eigenvalue. For the first component, since the eigenvalue is 1 it just stays the same. For all other components, $\lambda_k^n \to_{n \to \infty} 0$, so they eventually disappear. That means we're left with

$$A^nx
ightarrow_{n
ightarrow\infty} a_1v_1$$
.

The intuition here is that when you perform repeated multiplication by the same matrix, you're really repeatedly scaling the coefficients corresponding to the eigenvector basis by the size of the eigenvalue. When you do this a large number of times, the largest eigenvalue/eigenvector will come to dominate.

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Nadeem Rojenko, Ms. student of Theoretical Physics.

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Pretty simple, eigen vector by definition is a vector that will stay it self (same direction, but may have different length) after performing matrix transformation.

So applying a matrix many times, will keep transforming, rotating, shrinking and expanding the vector till at some stage it will become one of the eigenvectors, then it will not change any more (change in direction).

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