Part A: Knowledge and Understanding

1. Solve for x, for the given domain. Give all exact solution(s) in radians. Round approximate solution(s) to the nearest hundredth of a radian. **Exact answers must be given when possible.**

a) $2\sin(2x) - \sqrt{3} = 0, \quad 0 \leq x \leq 2\pi$

$\sin 2x = \frac{\sqrt{3}}{2}$

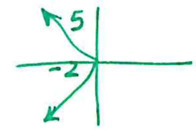
$\therefore 2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

b) $3\sec x - 5 = 5\sec x, \quad 0 \leq x \leq 2\pi$

$-5 = 2\sec x$

$\sec x = \frac{-5}{2}$



$\alpha = \cos^{-1}\left(\frac{2}{5}\right)$

$\alpha \doteq 1.16$

$\therefore x = \pi - 1.16$

or

$\pi + 1.16$

$\therefore x \doteq 1.98$ or 4.3

12

c) $5\cos^2 x + 14\sin x - 13 = 0, \quad x \in R$

$5(1 - \sin^2 x) + 14\sin x - 13 = 0$

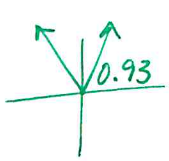
$-5\sin^2 x + 14\sin x - 8 = 0$

$5\sin^2 x - 14\sin x + 8 = 0 \qquad \begin{matrix} 5 & -4 \\ 1 & -2 \end{matrix}$

$(5\sin x - 4)(\sin x - 2) = 0$

$\therefore \sin x = \frac{4}{5}$ or $\sin x = 2$

no solution



$\therefore x = 0.93$ or $\pi - 0.93$

$\therefore x \doteq 0.93 + 2k\pi$

or

$2.21 + 2k\pi$

d) $\cot x \sin x = \cot x, \quad x \in R$

$\cot x (\sin x - 1) = 0$

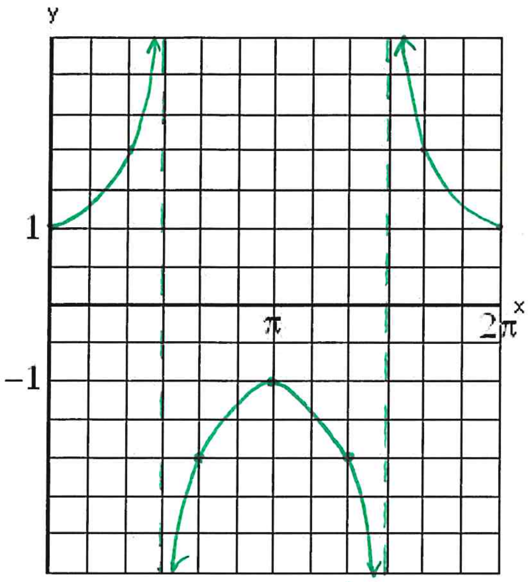
$\therefore \cot x = 0$ or $\sin x = 1$

$\therefore x = \frac{\pi}{2} + k\pi \qquad \therefore x = \frac{\pi}{2} + 2k\pi$

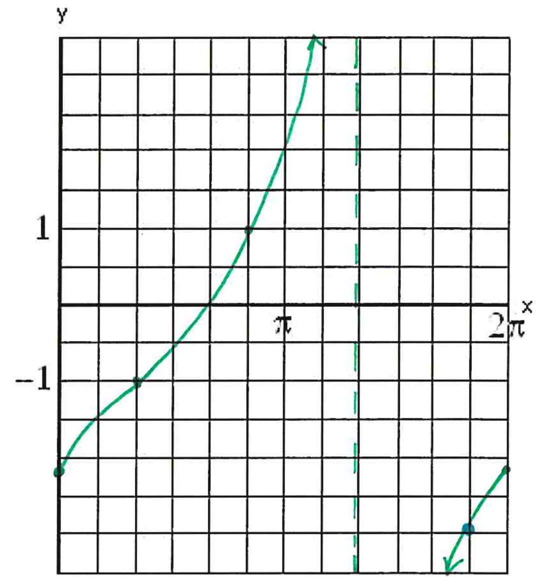
$\therefore x = \frac{\pi}{2} + k\pi$

2. Graph each function for the interval $0 \le x \le 2\pi$.

a) $y = sec\, x$



b) $y = 2\tan\left(\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right) - 1$



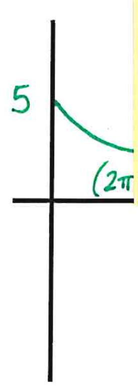
5

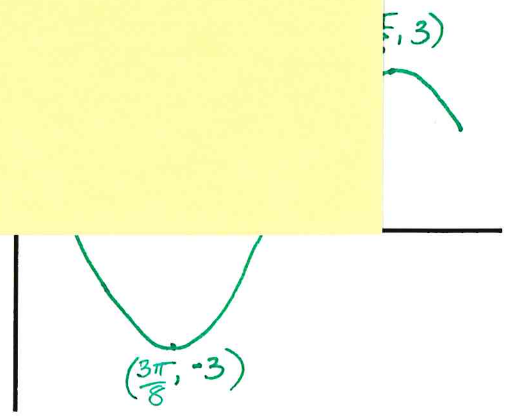
y-int ✓
trans root ✓
trans $\frac{\pi}{4}$ ✓

− pd .5 match
−1 x
− .5 for → $\frac{\pi}{6}$

3. Sketch the f[...] one maximum p[...]

a) $y =$ −[...]





6

min $(\frac{\pi}{2}, 1) \to (2\pi, 3)$

max $(\frac{3\pi}{2}, -1) \to (6\pi, 7)$

y-int , $x=0$

$y = -2\sin 0 + 5$

$= 5$

max $(2\pi, 1) \to (\frac{7\pi}{8}, 3)$

min $(\pi, -1) \to (\frac{3\pi}{8}, -3)$

$\frac{\pi}{2} - \frac{\pi}{8}$

$= \frac{3\pi}{8}$

y-int , $x=0$

$y = 3\cos\frac{\pi}{4}$

$= \frac{3\sqrt{2}}{2} \doteq 2.12$

4. The height, $h(t)$, of a basket on a water wheel at time t can be modelled by
$h(t) = 2\cos[\frac{\pi}{4}(t-1)] + 1.5$, where t is in seconds and $h(t)$ is in metres above the water.

a) How long does it take for the wheel to make one complete revolution?

$$pd = \frac{2\pi}{K} \rightarrow pd = \frac{2\pi}{\frac{\pi}{4}} \rightarrow pd = 8_s$$

b) What is the height above the water after 10 sec?

$$h(10) = 2\cos\left(\frac{9\pi}{4}\right) + 1.5$$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + 1.5$$

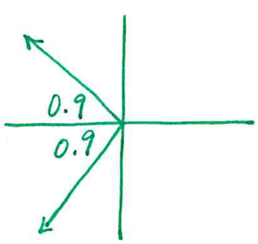
5

$$\doteq 2.91 \text{ m}$$

c) From the previous problem, within the first rotation, for how many seconds is the basket higher than 0.25 m above the water?

$$0.25 = 2\cos\left[\frac{\pi}{4}(t-1)\right] + 1.5$$

$$-\frac{5}{8} = \cos\left[\frac{\pi}{4}(t-1)\right]$$



$\therefore \frac{\pi}{4}(t-1) = \pi - 0.9$ ✓ or $\frac{\pi}{4}(t-1) = \pi + 0.9$ ✓

$t = \frac{4(\pi - 0.9)}{\pi} + 1$ $\qquad$ $t = \frac{4(\pi + 0.9)}{\pi} + 1$

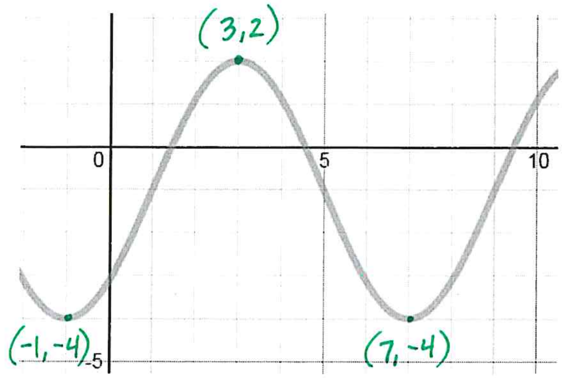
$\doteq 3.85_s$ $\qquad$ $\doteq 6.15_s$

$$\therefore 6.15 - 3.85$$

$$= 2.3_s \checkmark$$

$\therefore$ 5.7 seconds

5. Determine an equation for the function whose graph is identical to the given graph.



$a = 3$
$d = -1$
$pd = 8$
$\therefore k = \frac{\pi}{4}$

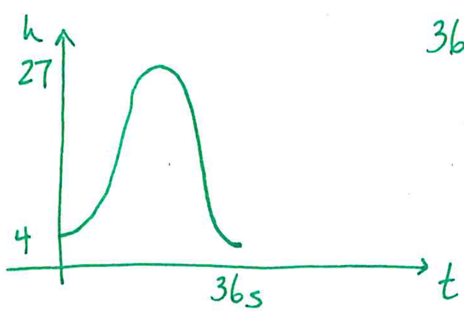
$y = 3\cos\left[\frac{\pi}{4}(x-3)\right] - 1$

2

$y = 3\sin\left[\frac{\pi}{4}(x-1)\right] - 1$

$y = -3\cos\left[\frac{\pi}{4}(x+1)\right] - 1$

6. A Ferris wheel with a diameter of 23 m makes one complete revolution every 36 s. The bottom of the wheel is 4 m above the ground. Determine an equation to model this situation. Include a sketch.



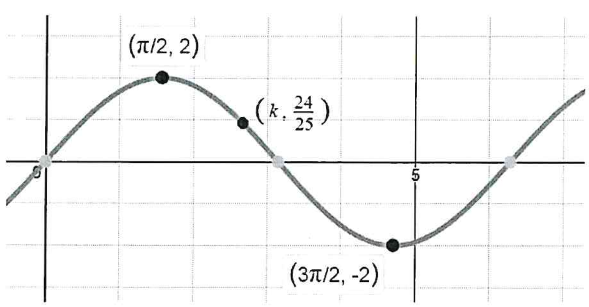
$36s = \frac{2\pi}{k}$

$k = \frac{\pi}{18}$

$h = -11.5\cos\left(\frac{\pi}{18}t\right) + 15.5$

3

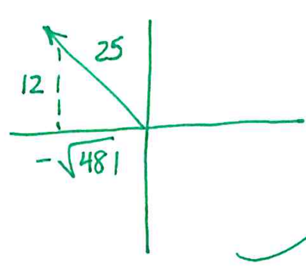
7. The graph represents the function $y = a\sin(bx)$. Find $\cos k$.
Find an approximate value (2 marks)
Find an exact value (4 marks)



$a = 2$
$b = 1$

$\therefore y = 2\sin x$
sub $\left(k, \frac{24}{25}\right)$
$\therefore \frac{24}{25} = 2\sin k$
$\frac{12}{25} = \sin k$

4



$\therefore \cos k = \frac{-\sqrt{481}}{25}$