

1. Complete the following identities:

a) $\sin(x - y) = \sin x \cos y - \cos x \sin y$

b) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

c) $\cos(-x) = \cos x$

d) $\sin 2x = 2 \sin x \cos x$

e) $\cos(x + y) = \cos x \cos y - \sin x \sin y$

f) $\tan(\frac{\pi}{2} - x) = \cot x$

[3]

Please hand-in this portion and receive Part 2 of 3 parts.

No Calculator!

Name: _____

1. Determine an exact value for each expression below.

a) $\sin \frac{11\pi}{12}$

$$\sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right)$$

$$= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6}$$

[6]

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

b) $\cos \frac{4\pi}{3} \cot \frac{5\pi}{6} - \sin \frac{7\pi}{4} \csc \left(-\frac{3\pi}{2}\right)$

$$= \left(-\frac{1}{2}\right)\left(-\sqrt{3}\right) - \left(-\frac{1}{\sqrt{2}}\right)(1)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}$$

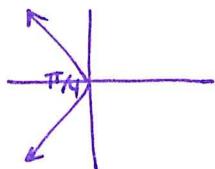
$$= \frac{\sqrt{3}+\sqrt{2}}{2}$$

2. For $0 \leq \theta \leq 2\pi$, find the value(s) of θ :

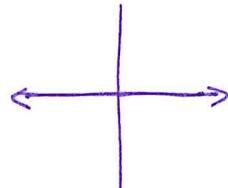
a) $\cos \theta = -\frac{1}{\sqrt{2}}$

b) $\tan \theta = 0$

[4]



$$\therefore \theta = \frac{3\pi}{4} \text{ or } \frac{5\pi}{4}$$



$$\therefore \theta = 0, \pi, 2\pi$$

$$\text{opp} = 0$$

3. Convert 36° to exact radians.

$$36^\circ \times \frac{\pi}{180^\circ}$$

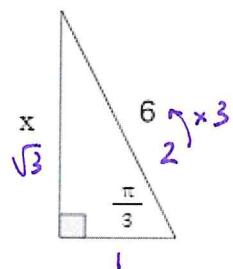
[1]

$$= \frac{\pi}{5}$$

[1]

4. Determine the missing side length:

$$\therefore x = 3\sqrt{3}$$



Please hand-in this portion and receive Part 3 of 3 parts. You may use your calculator on Part 3.

Calculator Permitted

Name: _____

1. Find the length of the arc which subtends an angle of 5.2 radians at the center of a circle of radius 4 cm.

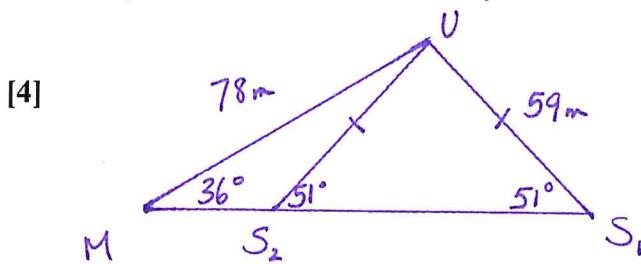
$$\begin{aligned} \text{arc length} &= 4 \times 5.2 \\ [2] \quad &= 20.8 \text{ cm} \end{aligned}$$

2. If $\sin x = -\frac{12}{18}$ with $\frac{3\pi}{2} \leq x \leq 2\pi$, then find the exact value of: $\cos\left(\frac{5\pi}{4} - x\right)$.

[3]

$$\begin{aligned} &\cos \frac{5\pi}{4} \cos x + \sin \frac{5\pi}{4} \sin x \\ &= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{6\sqrt{5}}{18}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{12}{18}\right) \\ &= \frac{-6\sqrt{5} + 12}{18\sqrt{2}} \\ &= \frac{-6\sqrt{5} + 12}{18\sqrt{2}} = \frac{2\sqrt{2} - \sqrt{10}}{6} \end{aligned}$$

3. From different locations on the ground, Mulder and Scully both spot a UFO hovering above Area 51. Mulder estimates that the UFO is about 78 m away from him. If Mulder looks at Scully, then looks up at the UFO, the angle of elevation is 36° . Scully estimates that the UFO is about 59 m away from her. How far apart could Mulder and Scully be from each other? Provide a clearly labelled diagram.



$$\begin{aligned} \Delta MUS_1 \\ \frac{\sin S}{78} &= \frac{\sin 36^\circ}{59} \\ \sin S &= \frac{78 \sin 36^\circ}{59} \\ S &= 51^\circ \end{aligned}$$

$$\begin{aligned} \Rightarrow U &= 180 - (36 + 51) \\ &= 180 - 87 \\ &= 93^\circ \\ \frac{u}{\sin 93^\circ} &= \frac{59}{\sin 36^\circ} \\ u &= \frac{59 \sin 93^\circ}{\sin 36^\circ} \end{aligned}$$

$$u = 100.24 \text{ m}$$

$$\begin{aligned} \Delta MUS_2 \\ S &= 180 - 51 \\ &= 129^\circ \\ U &= 180 - (36 + 129) \\ &= 180 - 165 \\ &= 15^\circ \end{aligned}$$

$$\begin{aligned} \frac{u}{\sin 15^\circ} &= \frac{59}{\sin 36^\circ} \\ u &= \frac{59 \sin 15^\circ}{\sin 36^\circ} \end{aligned}$$

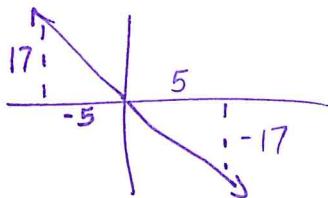
$$u = 25.98 \text{ m}$$

$$\therefore 100.24 \text{ m or } 25.98 \text{ m}$$

2. For $0 \leq \theta \leq 2\pi$, find the value(s) of θ when $\tan(\frac{\pi}{2} - \theta) = -\frac{5}{17}$ (nearest hundredth)..

$$\therefore \cot \theta = -\frac{5}{17} \quad \therefore \alpha \approx 1.28 \text{ rads}$$

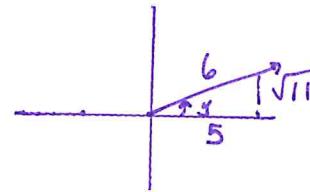
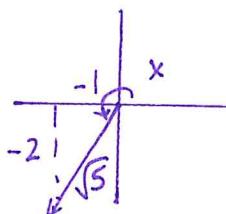
[2]



$$\therefore \theta = \pi - 1.28 \text{ or } 2\pi - 1.28 \\ \approx 1.86 \text{ or } 5 \text{ rads}$$

3. If $\tan x = 2$ with $\pi \leq x \leq \frac{3\pi}{2}$ and $\cos y = \frac{5}{6}$ with $0 \leq y \leq \frac{\pi}{2}$,

DIAGRAMS (label, include x & y):



Find the exact values of:

a) Find $\tan(x - y)$

b) Find $\sin(x - y)$

$$\begin{aligned} &= \frac{\tan x - \tan y}{1 + \tan x \tan y} \\ &= \frac{2 - \frac{\sqrt{11}}{5}}{1 + 2(\frac{\sqrt{11}}{5})} \\ &= \frac{\frac{10 - \sqrt{11}}{5}}{\frac{5 + 2\sqrt{11}}{5}} \\ &= \frac{10 - \sqrt{11}}{5 + 2\sqrt{11}} \times \frac{5 - 2\sqrt{11}}{5 - 2\sqrt{11}} \\ &= \frac{50 - 25\sqrt{11} + 2(\sqrt{11})}{25 - 4(\sqrt{11})} \\ &= \frac{72 - 25\sqrt{11}}{-19} \\ &= \frac{25\sqrt{11} - 72}{19} \end{aligned} \quad \left. \begin{aligned} &= \sin x \cos y - \cos x \sin y \\ &= \left(\frac{2}{\sqrt{5}} \right) \left(\frac{5}{6} \right) - \left(\frac{1}{\sqrt{5}} \right) \left(\frac{\sqrt{11}}{6} \right) \\ &= \frac{-10 + \sqrt{11}}{6\sqrt{5}} \\ &= \frac{-10\sqrt{5} + \sqrt{55}}{30} \\ &= \frac{\sqrt{55} - 10\sqrt{5}}{30} \end{aligned} \right\}$$

c) Without using a calculator to support your answer, in what quadrant does angle $x - y$ lie in? (Justify your reasoning).

$\tan(x-y)$ is positive $\therefore Q_1 \text{ or } Q_3$

$\sin(x-y)$ is negative $\therefore Q_3 \text{ or } Q_4$

$\therefore Q_3$

Trig. Identities Quiz

Total: ____ /12

Name: ANSWER KEY

1. Prove

$$\text{a) } \frac{1}{1 + \frac{\sin 2A}{2\cos A}} = \sec^2 A - \frac{\tan A}{\cos A}$$

$$\begin{aligned} \text{LS} &= \frac{1}{1 + \frac{2\sin A \cos A}{2\cos A}} \\ &= \frac{1}{1 + \sin A} \\ \text{RS} &= \frac{1}{\cos^2 A} - \frac{\sin A}{\cos^2 A} \\ &= \frac{1 - \sin A}{\cos^2 A} \\ &= \frac{1 - \sin A}{1 - \sin^2 A} \\ &= \frac{1 - \sin A}{(1 + \sin A)(1 - \sin A)} \\ &= \frac{1}{1 + \sin A} \end{aligned}$$

$$\text{b) } 1 - (\sin x - \cos y)^2 - \cos^2 x + \cos^2 y = \sin(x + y) + \sin(x - y)$$

$$\begin{aligned} \text{LS} &= 1 - (\sin^2 x - 2\sin x \cos y + \cos^2 y) - \cos^2 x + \cos^2 y \\ &= 1 - \sin^2 x + 2\sin x \cos y - \cos^2 y - \cos^2 x + \cos^2 y \\ &= \cos^2 x + 2\sin x \cos y - \cos^2 x \\ &= 2\sin x \cos y \end{aligned}$$

$$\begin{aligned} \text{RS} &= \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y \\ &= 2\sin x \cos y \end{aligned}$$

$$c) \frac{\cos 2A}{1 + \sin 2A} = \frac{\cot A - 1}{\cot A + 1}$$

$$LS = \frac{\cos^2 A - \sin^2 A}{1 + 2 \sin A \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin^2 A + 2 \sin A \cos A + \cos^2 A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{(\sin A + \cos A)^2}$$

$$= \frac{\cos A - \sin A}{\sin A + \cos A}$$

$$\begin{aligned} &= \frac{\cos A - \sin A}{\sin A} \\ &= \frac{\sin A + \cos A}{\sin A} \\ &= \frac{\cot A - 1}{1 + \cot A} \end{aligned}$$

2. You Choose. Do either a) OR b). Do NOT do both.

$$a) \frac{(2\cos A + 3)}{24 \sin A \cos A} = (1 - 2 \sin^2 A) \sqrt{\frac{4\cos^2 A + 12\cos A + 9}{36 \sin^2 4A}} \quad \stackrel{2+3}{2+3}$$

$$b) \frac{\sin x - \sin 3x}{\cos x + \cos 3x} = -\tan x$$

$$a) RS = \cos 2A$$

$$\sqrt{\frac{(2\cos A + 3)^2}{36 \sin^2 4A}}$$

$$= \cos 2A \left(\frac{2\cos A + 3}{6 \sin 4A} \right)$$

$$= \cos 2A \left(\frac{2\cos A + 3}{6(2\sin 2A \cos 2A)} \right)$$

$$= \frac{2\cos A + 3}{12 \sin 2A}$$

$$= \frac{2\cos A + 3}{12(2\sin A \cos A)}$$

$$= \frac{2\cos A + 3}{24 \sin A \cos A}$$

$$\begin{aligned}
 b) & \frac{\sin x - \sin(2x+x)}{\cos x + \cos(2x+x)} \\
 &= \frac{\sin x - \sin 2x \cos x - \cos 2x \sin x}{\cos x + \cos 2x \cos x - \sin 2x \sin x} \\
 &= \frac{\sin x - 2\sin x \cos^2 x - (\cos^2 x - \sin^2 x) \sin x}{\cos x + (\cos^2 x - \sin^2 x) \cos x - 2\sin^2 x \cos x} \\
 &= \frac{\sin x - 2\sin x \cos^2 x - \sin x \cos^2 x + \sin^3 x}{\cos x + \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x} \\
 &= \frac{\sin x - 3\sin x \cos^2 x + \sin^3 x}{\cos x - 3\sin^2 x \cos x + \cos^3 x} \\
 &= \frac{\sin x (1 - 3\cos^2 x + \sin^2 x)}{\cos x (1 - 3\sin^2 x + \cos^2 x)} \\
 &= \frac{\sin x (1 - 3(1 - \sin^2 x) + \sin^2 x)}{\cos x (1 - 3\sin^2 x + 1 - \sin^2 x)} \\
 &= \frac{\sin x (-2 + 4\sin^2 x)}{\cos x (2 - 4\sin^2 x)} \\
 &= \frac{-\sin x (2 - 4\sin^2 x)}{\cos x (2 - 4\sin^2 x)} \\
 &= -\tan x
 \end{aligned}$$