1. Determine the Horizontal asymptote for $f(x) = \frac{3x + 7}{2x^2 + 5x}$. Be sure to show the nature of the curve.

$$\lim_{x \to -\infty} \frac{\frac{3}{x} + \frac{7}{x^2}}{2 + \frac{5}{x}}$$

2

$$= \frac{0^- + 0}{2 - 0} = \frac{0^-}{2} = 0^-$$

$\therefore$ below

$$\lim_{x \to \infty} f(x) = \frac{0^+ + 0}{2 + 0} = \frac{0^+}{2} = 0^+$$

$\therefore$ above .5

$\therefore y = 0$ .5

2. A cubic function $f(x)$ with integral coefficients has the following properties: $f(2) = 0$, $-\frac{3}{5}$ is a root, $x + 4$ is a factor and $f(-2) = -27$. Determine $f(x)$.

3

$f(x) = a(x-2)(5x+3)(x+4)$ 1.5

$\because f(-2) = -27$

$\therefore -27 = a(-4)(-7)(2)$

$-27 = 56a$

$a = \frac{-27}{56}$

$\therefore f(x) = \frac{-27}{56}(x-2)(5x+3)(x+4)$

3. Determine the degree of the polynomial function passing through the points:

(−2,11), (−1,0), (0,−5), (1,−4), (2,3), (3,16)

-11 -5 1 7 13

2

6 6 6 6 $\therefore$ degree 2

4. Sketch $f(x) = -2x^3 - 2x^{3} + 3x + 1$

$f(1) = 0$ .5

$\therefore (x-1)$ is a factor

4 $\therefore f(x) = (x-1)(-2x^2-4x-1)$

$= -(x-1)(2x^2+4x+1)$

$\therefore x = 1$ $\therefore x = \frac{-4 \pm \sqrt{16-8}}{4}$

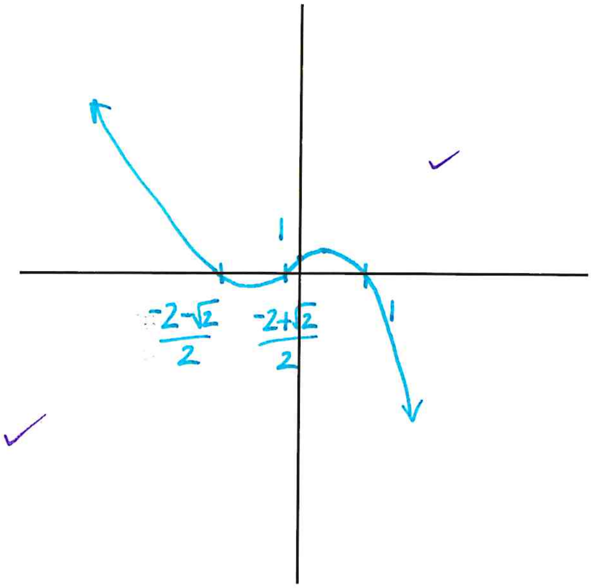
.5

$= \frac{-4 \pm \sqrt{8}}{4}$

$= \frac{-4 \pm 2\sqrt{2}}{4}$

$= \frac{-2 \pm \sqrt{2}}{2}$

(roots)



5. Sketch the function, $f(x) = \frac{2x^2 - 7x + 5}{x-3}$, using asymptote(s) and intercept(s).

INTS.

y-int, $x = 0$

$y = \frac{-5}{3}$ .5

x-int, $y = 0$

$0 = 2x^2 - 7x + 5$ $\begin{matrix} 2 & -5 \\ 1 & -1 \end{matrix}$

$0 = (2x-5)(x-1)$

$\therefore x = \frac{5}{2}$ or 1 .5

VA: @ $x = 3$

$\lim_{x \to 3^-} \frac{(2x-5)(x-1)}{x-3}$

$= \frac{1(2)}{0^-}$

$= \frac{2}{0^-}$

$= -\infty$

$\lim_{x \to 3^+} f(x)$

$= \frac{2}{0^+}$

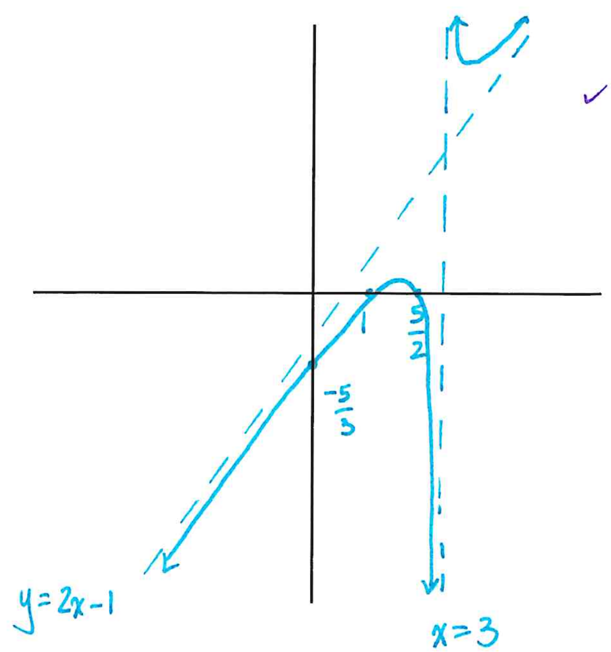
$= \infty$

5

OA.

$$\begin{array}{r} 2x - 1 \\ x-3 \enclose{longdiv}{2x^2 - 7x + 5} \\ \underline{2x^2 - 6x} \\ -x + 5 \\ \underline{-x + 3} \\ 2 \end{array}$$

$\therefore y = 2x - 1$



6. Determine the equation of the **cubic** polynomial function, $f(x)$, passing through the following points:

$$(-4, -378), (-3, -210), (-2, -100), (-1, -36), (0, -6), (1, 2), (2, 0)$$

Sketch the curve and label all intercepts.

$f(x) = (x-2)(ax^2+bx+c)$

$\because f(0) = -6$

$\therefore -6 = -2c$

$c = 3$

$\therefore f(x) = (x-2)(ax^2+bx+3)$ ✓

$\because f(1) = 2$

$\therefore 2 = -(a+b+3)$

$-2 = a+b+3$

$a+b = -5$ ①

$\because f(-1) = -36$

$\therefore -36 = -3(a-b+3)$

$12 = a-b+3$

$9 = a-b$ ②

✓✓

5

① + ②

$2a = 4$

$a = 2$

sub $a = 2$ in ①

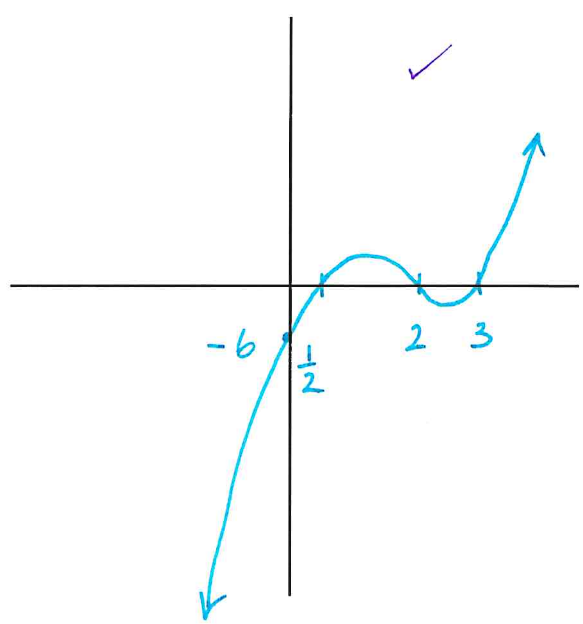
$2+b = -5$

$b = -7$

$\therefore f(x) = (x-2)(2x^2-7x+3)$ $\begin{matrix} 2 & -1 \\ 1 & -3 \end{matrix}$

$= (x-2)(2x-1)(x-3)$ ✓

$\therefore x = 2, \frac{1}{2}, 3$ (roots)



✓

7. Using intercept(s), asymptote(s), and nature of the curve, sketch the function $f(x) = \frac{x^2 - 9}{x^2 + 2x}$.

INTS

y-int, $x = 0$

$y = \frac{-9}{0}$ $\therefore$ undefined

✓

x int, $y = 0$

$0 = (x+3)(x-3)$

$\therefore x = \pm 3$

VA. @ $x = 0$, $x = -2$

$$\lim_{x \to -2^-} \frac{(x+3)(x-3)}{x(x+2)} = \frac{1(-5)}{-2(0^-)} = \frac{-5}{0^+} = -\infty$$

✓

$$\lim_{x \to -2^+} f(x) = \frac{-5}{-2(0^+)} = \frac{-5}{0^-} = \infty$$

$$\lim_{x \to 0^-} f(x) = \frac{3(-3)}{0^-(2)} = \frac{-9}{0^-} = \infty$$

✓

$$\lim_{x \to 0^+} f(x) = \frac{-9}{0^+(2)} = \frac{-9}{0^+} = -\infty$$

HA.

5

$$\lim_{x \to -\infty} \frac{1 - \frac{9}{x^2}}{1 + \frac{2}{x}}$$

$= \frac{1-0}{1-0}$ bigger $\therefore$ above .5

$= 1$

✓

$$\lim_{x \to \infty} f(x)$$

$= \frac{1-0}{1+0}$ smaller $\therefore$ below .5

$= 1$

✓