For the following Multiple Choice and True/False problems, write the UPPERCASE letter in the space provided.

1. Given the polynomial: $f(x) = x^4 - 2x^3 + 5x^2 - 15$, the factor theorem suggests test values for x when discovering factors should include: **[2]** A
   A) $\pm 1, \pm 3, \pm 5, \pm 15$
   B) $\pm 1, \pm 2, \pm 3, ... \pm 15$
   C) $\pm 1, \ \pm 2, \ \pm 5, \ \pm 15$
   D) $1, -2, 5, \ -15$
   E) None of the above

2. A polynomial $f(x)$ is divided by $(2x + 7)$, the remainder can be determined by evaluating: **[2]** D
   A) $f(-7)$
   B) $f(7)$
   C) $f(\frac{7}{2})$
   D) $f(-\frac{7}{2})$
   E) $f(-\frac{2}{7})$
   F) Not enough information

3. True/False: **[2]**
   a) All polynomials of degree 4 have zeroes. F
   b) All polynomials of degree 5 will have at least one change in direction. F
   c) Some polynomials of degree 6 may have only 1 zero. T
   d) Polynomials of degree 7 can have at most 6 changes in direction. T

4. Perform the following long division: $\left(x^3 - 2x - 4\right) \div (x - 2)$ **[2]**

$$
\begin{array}{r}
x^2 + 2x + 2 \\
x-2 \overline{\big) \, x^3 + 0x^2 - 2x - 4} \\
\underline{x^3 - 2x^2 \qquad\qquad} \\
2x^2 - 2x \qquad \\
\underline{2x^2 - 4x \qquad} \\
2x - 4 \\
\underline{2x - 4} \\
0
\end{array}
$$

5. Factor: **[4]**

a) $6x^3 + 19x^2 + 11x - 6$ $= f(x)$

$f(-2) = -48 + 76 - 22 - 6$
$= 0$

$\therefore$ $x+2$ is a factor

$\therefore$ $(x+2)(6x^2 + 7x - 3)$ $\quad \begin{matrix} 3 & -1 \\ 2 & +3 \end{matrix}$

$= (x+2)(3x-1)(2x+3)$

b) $(x + g)^3 + 125w^3$

$= (x+g+5w)\left[(x+g)^2 - 5w(x+g) + 25w^2\right]$

6. Solve: $x^4 - 4x^3 - 4x^2 + 11x + 10 = 0$ $= f(x)$ **[3]**

$f(-1) = 0$

$\therefore$ $x+1$ is a factor

$\therefore$ $(x+1)(x^3 - 5x^2 + x + 10) = 0$

$f(2) = 0$

$\therefore$ $x-2$ is a factor

$\therefore (x+1)(x-2)(x^2 - 3x - 5) = 0$

$\therefore x = -1$ or 2 or $x = \dfrac{3 \pm \sqrt{9+20}}{2}$

$= \dfrac{3 \pm \sqrt{29}}{2}$

7. Solve: **[3]** $\quad \dfrac{6x^2 - 3x + 2}{2x+1} \geq 1$

$\dfrac{6x^2 - 3x + 2 - 2x - 1}{2x+1} \geq 0$

$\begin{matrix} 3 & -1 \\ 2 & -1 \end{matrix}$ $\quad \dfrac{6x^2 - 5x + 1}{2x+1} \geq 0$

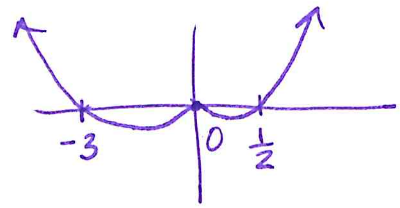
$\dfrac{(3x-1)(2x-1)}{2x+1} \geq 0$

$\therefore$ Key #'s $\frac{1}{3}, \frac{1}{2}, \frac{-1}{2}$

$(-\infty, -\frac{1}{2})$	$(-\frac{1}{2}, \frac{1}{3}]$	$[\frac{1}{3}, \frac{1}{2}]$	$[\frac{1}{2}, \infty)$
-1	0	0.4	1
$\frac{--}{-}$	$\frac{--}{+}$	$\frac{+-}{+}$	$\frac{++}{+}$
$= -$	$= +$ ✓	$= -$	$= +$ ✓

$\therefore (-\frac{1}{2}, \frac{1}{3}], [\frac{1}{2}, \infty)$

8. Solve: **[3]** $2x^4 + 5x^3 - 3x^2 < 0$



$\begin{matrix} 2 & -1 \\ 1 & +3 \end{matrix}$

$x^2(2x^2+5x-3)<0$

$x^2(2x-1)(x+3)<0$

$\therefore x=0$ or $\frac{1}{2}$ or -3 (roots)

$\therefore (-3,\frac{1}{2}), x\neq 0$

or

$(-3,0)(0,\frac{1}{2})$

9. Determine the value of *k* such that when $2x^3 - 3x^2 + kx - 1$ is divided by $x + 3$, the remainder is -37. **[2]**

$=f(x)$

$f(-3)=-37$

$-54-27-3k-1=-37$

$-3k-82=-37$

$-3k=45$

$k=-15$

10. Determine the x-values that would satisfy both inequalities below: **[4]**

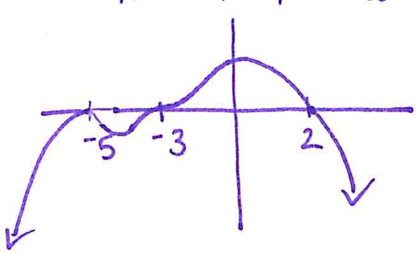
$-(x+5)^2(x+3)^3(x-2)\leq 0$ *AND* $2x^2+5x-7<0$

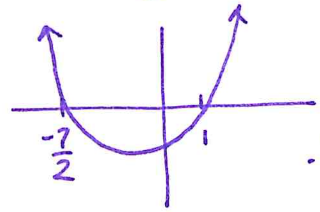
$\begin{matrix} 2 & +7 \\ 1 & -1 \end{matrix}$

$\therefore x=-5,-3,2$ (roots)

$(2x+7)(x-1)<0$

$\therefore x=-\frac{7}{2}$ or 1 (roots)



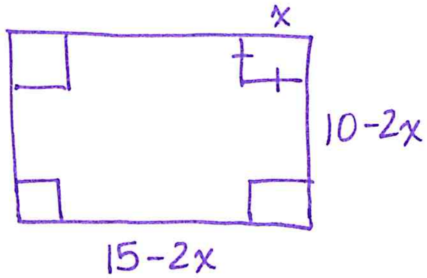


$\therefore(-\infty,-3],[2,\infty)$

$\therefore(-\frac{7}{2},1)$

$\therefore(-\frac{7}{2},-3]$

11. A rectangular piece of cardboard measuring 15 cm by 10 cm is made into an open-topped box by cutting squares from the corners and turning up the sides. If the volume of the box is $104\ cm^3$, what are the dimensions of the box? **[4]**



$V = Lwh$

$104 = x(15-2x)(10-2x)$

$104 = 150x - 50x^2 + 4x^3$

$4x^3 - 50x^2 + 150x - 104 = 0$

$f(x) = 2x^3 - 25x^2 + 75x - 52 = 0$

$f(1) = 0$

$\therefore x-1$ is a factor

$\therefore (x-1)(2x^2 - 23x + 52) = 0$

$\therefore x = 1$ cm or $x = \dfrac{23 \pm \sqrt{529 - 416}}{4}$

$= \dfrac{23 \pm \sqrt{113}}{4}$

$\doteq 8.41$ or 3.09 cm

inadmissible

$\therefore$ The dimensions are

13 cm × 8 cm × 1 cm or 8.82 cm × 3.82 cm × 3.09 cm