1. For the following functions, list all the function(s) that satisfy the following conditions.

A. $A(x) = -\frac{1}{2}(3)^{2x+4} + 4$

B. $B(x) = -6log_{\frac{1}{2}}(x + 3) + 4$

C. $C(x) = 3log_4[3(x + 4)] - 2$

D. $D(x) = 5\left(\frac{1}{4}\right)^{x+2} - 3$

Write the UPPERCASE letter of the function(s) in the space provided

i) Is an increasing function: B, C

ii) Has a horizontal asymptote at $y = 4$: A

4

iii) Has a shift left four units: C

iv) Has more than one stretch that will make the function steeper: C

2. Complete the following table with exact, fully simplified values.

Function	$y = -4log_5\left[\frac{1}{2}(x + 5)\right] + 8$	$y = \frac{1}{3}e^{2(x-5)} - 4$
Increasing / Decreasing	DEC	✓ INC
Equation of the Asymptote	$x = -5$	✓ $y = -4$
Intercept (not the transformed-intercept)	x-int: 45 ✓	y-int: ✓ $\frac{1-12e^{10}}{3e^{10}}$
Domain and Range	D: $x > -5$ R: $y \in R$ ✓	D: $x \in R$ R: $y > -4$
For only: $y = \frac{1}{3}e^{2(x-5)} - 4$ **state the transformations in order to be performed**	• Vertical stretch by a factor of $\frac{1}{3}$ ∴ flatter • horizontal stretch by a factor of $\frac{1}{2}$ ∴ steeper • shifted right 5 units, down 4 units	

Rough work for above:

x-int, $y = 0$

$2 = \log_5\left(\frac{1}{2}(x+5)\right)$

$\therefore 25 = \frac{1}{2}(x+5)$

$50 = x+5$

$x = 45$

y-int, $x = 0$

$y = \frac{1}{3}e^{-10} - 4$

$= \frac{1}{3e^{10}} - 4$

$= \frac{1-12e^{10}}{3e^{10}}$

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3. Sketch the following functions. Label the asymptote, transformed intercept and applicable intercept.

a) $y = 3log_{\frac{1}{3}}[4(x-1)] + 6$

VA. $x = 1$

TI $(1,0)$ $(\frac{1}{4},0)$ $\rightarrow$ $(\frac{5}{4},6)$

x-int, $y=0$

3

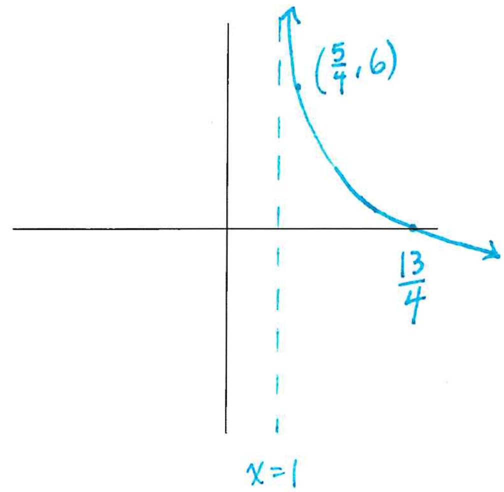
$0 = 3\log_{\frac{1}{3}}[4(x-1)]+6$

$-2 = \log_{\frac{1}{3}}[4(x-1)]$

$\therefore 9 = 4(x-1)$

$\frac{9}{4} = x-1$

$x = \frac{13}{4}$



b) $y = \frac{1}{6}(3)^{x+4} - 1$

HA. $y=-1$

TI $(0,1)$ $(0,\frac{1}{6})$ $\rightarrow$ $(-4,-\frac{5}{6})$

3

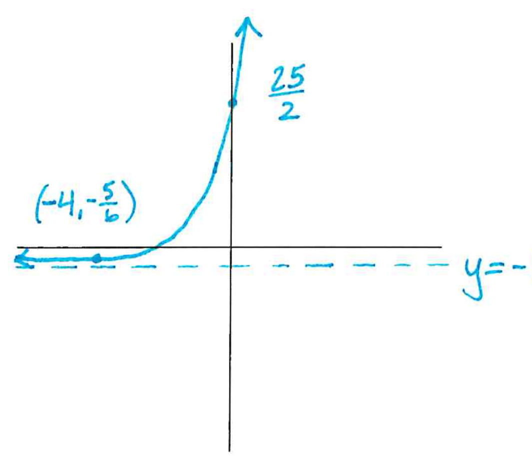
y-int, $x=0$

$y = \frac{1}{6}(3^4)-1$

$= \frac{81}{6}-1$

$= \frac{27}{2}-1$

$= \frac{25}{2}$



c) $y =- 5ln\left[\frac{1}{4}(x-2)\right]^2 + 5$

VA $x=2$

TI $(1,0)$ $(-1,0)$ $(4,0)$ $(-4,0)$ $(6,5)$ $(-2,5)$

3

x-int, $y=0$

$1 = \ln\left[\frac{1}{4}(x-2)\right]^2$

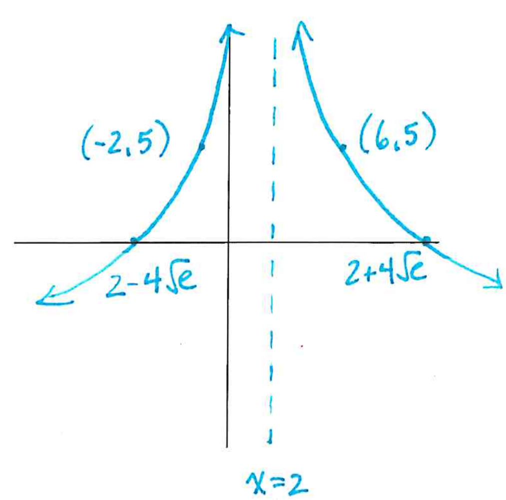
$\therefore e = \left[\frac{1}{4}(x-2)\right]^2$

$\pm\sqrt{e} = \frac{1}{4}(x-2)$

$\pm 4\sqrt{e} = x-2$

$x = 2 \pm 4\sqrt{e}$

$(8.59$ or $-4.59)$



4. For the following problems in #4ab, create the equation, in one variable, to solve the problem.

DO NOT SOLVE THE PROBLEM!!!

a) Jasmine invests \$2500 into an account earning 5% per annum, compounded monthly. How much money will she have in her account after 8 years?

2 $A = 2500\left(1 + \frac{0.05}{12}\right)^{96}$

b) A 80 mg sample of Gtownium decays to 65 mg after 5 mins. Determine the half life of Gtownium.

2 $65 = 80\left(\frac{1}{2}\right)^{5/P}$

For the following problems, SOLVE!

5. You purchased a car 7 years ago. It has been depreciating by 20% each year since it was purchased. If the car has a value of \$10000, that was the original price?

3

$10\,000 = P(0.8)^7$

$P = \frac{10\,000}{0.8^7}$

$P \doteq \$47\,683.72$

6. The half-life of a foul smelling substance is 4 hours. You have 8 oz of this substance today.
a) How much of the substance will be left after 20 hours?

$A = 8\left(\frac{1}{2}\right)^{20/4}$

$= 0.25$ oz

b) How long will it take for the substance to decay to 1.3 oz?

3

$1.3 = 8\left(\frac{1}{2}\right)^{t/4}$

$0.1625 = \left(\frac{1}{2}\right)^{t/4}$

$\log 0.1625 = \log\left(\frac{1}{2}\right)^{t/4}$

$\log 0.1625 = \frac{t}{4}\log\left(\frac{1}{2}\right)$

$t = \frac{4\log 0.1625}{\log\left(\frac{1}{2}\right)}$

$t \doteq 10.486$

$\therefore$ 10.5 hours

7. The population of Logland has increased from 60 000 in 2012 to 410 000 in 2025. Determine the annual growth rate (as a percentage) to 1 decimal place.

$410\,000 = 60\,000 \times G^{13}$

$\frac{41}{6} = G^{13}$

$G = \sqrt[13]{\frac{41}{6}}$

$G \doteq 1.15932$

$\therefore 16\ \%$

3

8. The temperature R (in °C) of a heated metal rod as it cools over time t (in minutes) is modeled by $R(t) = 80 - 15ln(t + 1)$, where $t \geq 0$. In whole minutes (no decimals), how long will it take to **reach** 50°C?

$50 = 80 - 15\ln(t+1)$

$-30 = -15\ln(t+1)$

$2 = \ln(t+1)$

$\therefore e^2 = t+1$

$t = e^2 - 1$

$t \doteq 6.389$

$\therefore 7$ minutes

3

9. The function $f(x) = 5 \bullet 3^x - 4$ is transformed in the following manner:

→ Reflected in the x-axis
→ Vertically stretched by a factor of 2
→ Horizontally stretched by a factor of $log_7 3$
→ Shifted up 6 units

Determine the equation of the resulting function. Show your work to justify your answer.

$f(x) = 5 \cdot 3^x - 4$

reflection : $y = -5 \cdot 3^x + 4$

v. stretch : $y = -10 \cdot 3^x + 8$

h. stretch : $y = -10 \cdot 3^{x/\log_3 7} + 8$

$y = -10 \cdot 7^x + 8$

shift : $y = -10 \cdot 7^x + 14$

3