

1. Complete the following identities:

a)  $\sin(x - y) = \sin x \cos y - \cos x \sin y$       b)  $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

c)  $\cos(-x) = \cos x$

d)  $\sin 2x = 2 \sin x \cos x$

e)  $\cos(x + y) = \cos x \cos y - \sin x \sin y$

f)  $\tan\left(\frac{\pi}{2} - x\right) = \cot x$

[3]

*Please hand-in this portion and receive Part 2 of 3 parts.*

# No Calculator!

Name: \_\_\_\_\_

1. Determine an **exact** value for each expression below.

a)  $\sin \frac{11\pi}{12}$

[6]

$$\begin{aligned} & \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) \\ &= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \cos \frac{3\pi}{4} \sin \frac{\pi}{6} \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}} \\ &= \frac{\sqrt{6}-\sqrt{2}}{4} \end{aligned}$$

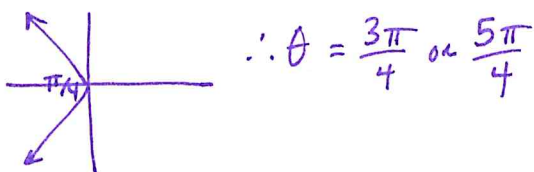
b)  $\cos \frac{4\pi}{3} \cot \frac{5\pi}{6} - \sin \frac{7\pi}{4} \csc \left(-\frac{3\pi}{2}\right)$

$$\begin{aligned} &= \left(-\frac{1}{2}\right)\left(-\sqrt{3}\right) - \left(-\frac{1}{\sqrt{2}}\right)(1) \\ &= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{3}+\sqrt{2}}{2} \end{aligned}$$

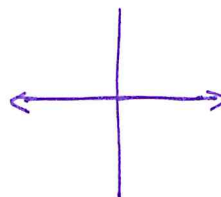
2. For  $0 \leq \theta \leq 2\pi$ , find the value(s) of  $\theta$ :

a)  $\cos \theta = -\frac{1}{\sqrt{2}}$

[4]



b)  $\tan \theta = 0$



$\therefore \theta = 0, \pi, 2\pi$

opp = 0

3. Convert  $36^\circ$  to exact radians.

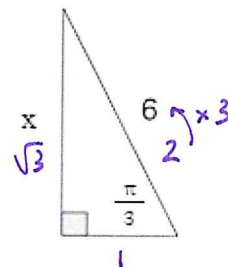
[1]

$$\begin{aligned} & 36^\circ \times \frac{\pi}{180^\circ} \\ &= \frac{\pi}{5} \end{aligned}$$

[1]

4. Determine the missing side length:

$\therefore x = 3\sqrt{3}$



# Calculator Permitted

Name: \_\_\_\_\_

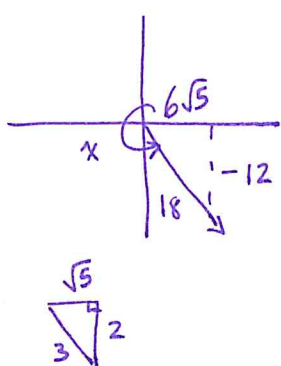
1. Find the length of the arc which subtends an angle of 5.2 radians at the center of a circle of radius 4 cm.

$$\begin{aligned} \text{arc length} &= 4 \times 5.2 \\ &= 20.8 \text{ cm} \end{aligned}$$

[2]

2. If  $\sin x = -\frac{12}{18}$  with  $\frac{3\pi}{2} \leq x \leq 2\pi$ , then find the exact value of:  $\cos\left(\frac{5\pi}{4} - x\right)$ .

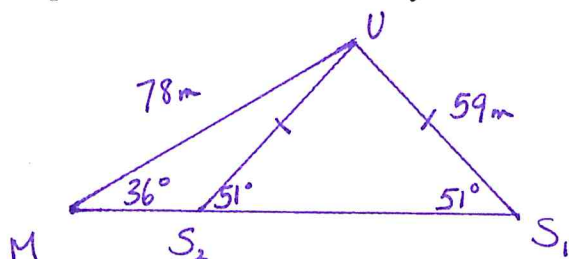
[3]



$$\begin{aligned} &\cos \frac{5\pi}{4} \cos x + \sin \frac{5\pi}{4} \sin x \\ &= \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{6\sqrt{5}}{18}\right) + \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{12}{18}\right) \\ &= \frac{-6\sqrt{5} + 12}{18\sqrt{2}} = \frac{-6\sqrt{10} + 12\sqrt{2}}{36} = \frac{2\sqrt{2} - \sqrt{10}}{6} \end{aligned}$$

3. From different locations on the ground, Mulder and Scully both spot a UFO hovering above Area 51. Mulder estimates that the UFO is about 78 m away from him. If Mulder looks at Scully, then looks up at the UFO, the angle of elevation is  $36^\circ$ . Scully estimates that the UFO is about 59 m away from her. How far apart could Mulder and Scully be from each other? Provide a clearly labelled diagram.

[4]



$$\frac{\sin S}{78} = \frac{\sin 36^\circ}{59}$$

$$\sin S = \frac{78 \sin 36^\circ}{59}$$

$$S \approx 51^\circ$$

$$\begin{aligned} U &= 180 - (36 + 51) \\ &= 180 - 87 \\ &= 93^\circ \end{aligned}$$

$$\frac{u}{\sin 93^\circ} = \frac{59}{\sin 36^\circ}$$

$$u = \frac{59 \sin 93^\circ}{\sin 36^\circ}$$

$$u \approx 100.24 \text{ m}$$

$$\Delta MVS_2$$

$$\begin{aligned} S &= 180 - 51 \\ &= 129^\circ \end{aligned}$$

$$\begin{aligned} U &= 180 - (36 + 129) \\ &= 180 - 165 \\ &= 15^\circ \end{aligned}$$

$$\begin{aligned} \frac{u}{\sin 15^\circ} &= \frac{59}{\sin 36^\circ} \\ u &= \frac{59 \sin 15^\circ}{\sin 36^\circ} \end{aligned}$$

$$u \approx 25.98 \text{ m}$$

$$\therefore 100.24 \text{ m or } 25.98 \text{ m}$$

2. For  $0 \leq \theta \leq 2\pi$ , find the value(s) of  $\theta$  when  $\tan(\frac{\pi}{2} - \theta) = -\frac{5}{17}$  (nearest hundredth)..

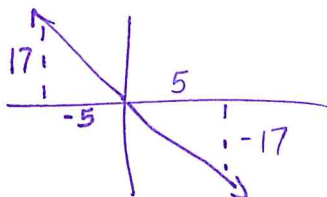
$$\therefore \cot \theta = -\frac{5}{17}$$

$$\therefore \alpha \doteq 1.28 \text{ rads}$$

$$\therefore \theta = \pi - 1.28 \text{ or } 2\pi - 1.28$$

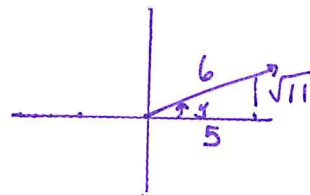
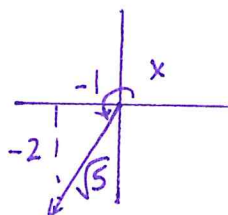
$$\doteq 1.86 \text{ or } 5 \text{ rads}$$

[2]



3. If  $\tan x = 2$  with  $\pi \leq x \leq \frac{3\pi}{2}$  and  $\cos y = \frac{5}{6}$  with  $0 \leq y \leq \frac{\pi}{2}$ ,

DIAGRAMS (label, include  $x$  &  $y$ ):



Find the exact values of:

a) Find  $\tan(x - y)$

b) Find  $\sin(x - y)$

[5]

$$\begin{aligned} &= \frac{\tan x - \tan y}{1 + \tan x \tan y} \\ &= \frac{2 - \frac{\sqrt{11}}{5}}{1 + 2\left(\frac{\sqrt{11}}{5}\right)} \\ &= \frac{10 - \sqrt{11}}{5} \cdot \frac{5 - 2\sqrt{11}}{5 - 2\sqrt{11}} \\ &= \frac{50 - 25\sqrt{11} + 2(11)}{25 - 4(11)} \\ &= \frac{72 - 25\sqrt{11}}{-19} \\ &= \frac{25\sqrt{11} - 72}{19} \end{aligned}$$

$$\begin{aligned} &= \sin x \cos y - \cos x \sin y \\ &= \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{5}{6}\right) - \left(-\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{11}}{6}\right) \\ &= \frac{-10 + \sqrt{11}}{6\sqrt{5}} \\ &= \frac{-10\sqrt{5} + \sqrt{55}}{30} \\ &= \frac{\sqrt{55} - 10\sqrt{5}}{30} \end{aligned}$$

c) Without using a calculator to support your answer, in what quadrant does angle  $x - y$  lie in? (Justify your reasoning).

$\tan(x - y)$  is positive  $\therefore Q_1 \text{ or } Q_3$

$\sin(x - y)$  is negative  $\therefore Q_3 \text{ or } Q_4$

$\therefore Q_3$

1. Prove

$$a) \frac{1}{1 + \frac{\sin 2A}{2\cos A}} = \sec^2 A - \frac{\tan A}{\cos A}$$

$$\begin{aligned} LS &= \frac{1}{1 + \frac{2\sin A \cos A}{2\cos A}} \\ &= \frac{1}{1 + \sin A} \end{aligned}$$

$$\begin{aligned} RS &= \frac{1}{\cos^2 A} - \frac{\sin A}{\cos^2 A} \\ &= \frac{1 - \sin A}{\cos^2 A} \\ &= \frac{1 - \sin A}{1 - \sin^2 A} \end{aligned}$$

$$\begin{aligned} &= \frac{1 - \cancel{\sin A}}{(1 + \sin A)(1 - \cancel{\sin A})} \\ &= \frac{1}{1 + \sin A} \end{aligned}$$

$$b) 1 - (\sin x - \cos y)^2 - \cos^2 x + \cos^2 y = \sin(x + y) + \sin(x - y)$$

$$\begin{aligned} LS &= 1 - (\sin^2 x - 2\sin x \cos y + \cos^2 y) - \cos^2 x + \cos^2 y \\ &= 1 - \sin^2 x + 2\sin x \cos y - \cos^2 y - \cos^2 x + \cos^2 y \\ &= \cos^2 x + 2\sin x \cos y - \cos^2 x \\ &= 2\sin x \cos y \end{aligned}$$

$$\begin{aligned} RS &= \sin x \cos y + \cos x \sin y + \sin x \cos y - \cos x \sin y \\ &= 2\sin x \cos y \end{aligned}$$

$$c) \frac{\cos 2A}{1 + \sin 2A} = \frac{\cot A - 1}{\cot A + 1}$$

$$LS = \frac{\cos^2 A - \sin^2 A}{1 + 2 \sin A \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin^2 A + 2 \sin A \cos A + \cos^2 A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{(\sin A + \cos A)^2}$$

$$= \frac{\cos A - \sin A}{\sin A + \cos A}$$

$$= \frac{\cos A - \sin A}{\sin A} \cdot \frac{\sin A}{\sin A + \cos A}$$

$$= \frac{\cot A - 1}{1 + \cot A}$$

2. You Choose. Do either a) **OR** b). Do NOT do both.

$$a) \frac{(2 \cos A + 3)}{24 \sin A \cos A} = (1 - 2 \sin^2 A) \sqrt{\frac{4 \cos^2 A + 12 \cos A + 9}{36 \sin^2 4A}} \quad \frac{2+3}{2+3}$$

$$b) \frac{\sin x - \sin 3x}{\cos x + \cos 3x} = -\tan x$$

$$a) RS = \cos 2A \sqrt{\frac{(2 \cos A + 3)^2}{36 \sin^2 4A}}$$

$$= \cos 2A \left( \frac{2 \cos A + 3}{6 \sin 4A} \right)$$

$$= \cancel{\cos 2A} \left( \frac{2 \cos A + 3}{6 (2 \sin 2A \cos 2A)} \right)$$

$$= \frac{2 \cos A + 3}{12 \sin 2A}$$

$$= \frac{2 \cos A + 3}{12 (2 \sin A \cos A)}$$

$$= \frac{2 \cos A + 3}{24 \sin A \cos A}$$

$$b) \frac{\sin x - \sin(2x+x)}{\cos x + \cos(2x+x)}$$

$$= \frac{\sin x - \sin 2x \cos x - \cos 2x \sin x}{\cos x + \cos 2x \cos x - \sin 2x \sin x}$$

$$= \frac{\sin x - 2\sin x \cos^2 x - (\cos^2 x - \sin^2 x) \sin x}{\cos x + (\cos^2 x - \sin^2 x) \cos x - 2\sin^2 x \cos x}$$

$$= \frac{\sin x - 2\sin x \cos^2 x - \sin x \cos^2 x + \sin^3 x}{\cos x + \cos^3 x - \sin^2 x \cos x - 2\sin^2 x \cos x}$$

$$= \frac{\sin x - 3\sin x \cos^2 x + \sin^3 x}{\cos x - 3\sin^2 x \cos x + \cos^3 x}$$

$$= \frac{\sin x (1 - 3\cos^2 x + \sin^2 x)}{\cos x (1 - 3\sin^2 x + \cos^2 x)}$$

$$= \frac{\sin x (1 - 3(1 - \sin^2 x) + \sin^2 x)}{\cos x (1 - 3\sin^2 x + 1 - \sin^2 x)}$$

$$= \frac{\sin x (-2 + 4\sin^2 x)}{\cos x (2 - 4\sin^2 x)}$$

$$= \frac{-\sin x (2 - 4\cancel{\sin^2 x})}{\cos x (2 - 4\cancel{\sin^2 x})}$$

$$= -\tan x$$