

**Part A: Knowledge and Understanding****Name:** \_\_\_\_\_

1. Complete the following identities:

a)  $\cos 2x = \cos^2 x - \sin^2 x$

b)  $\cos(x - y) = \cos x \cos y + \sin x \sin y$

c)  $\sin(x + y) = \sin x \cos y + \cos x \sin y$

d)  $\tan(-x) = -\tan x$

e)  $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

f)  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

*Please hand-in this portion and receive Part 2 (no calculator).*

**Part B: Knowledge and Understanding (NO CALCULATOR) Name: \_\_\_\_\_**

1. Determine the **exact** value for:  $\sin \frac{7\pi}{12}$

$$\begin{aligned}
 & \sin \left( \frac{\pi}{3} + \frac{\pi}{4} \right) \\
 [3] \quad &= \sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\
 &= \left( \frac{\sqrt{3}}{2} \right) \left( \frac{1}{\sqrt{2}} \right) + \left( \frac{1}{2} \right) \left( \frac{1}{\sqrt{2}} \right) \\
 &= \frac{\sqrt{3} + 1}{2\sqrt{2}}
 \end{aligned}$$

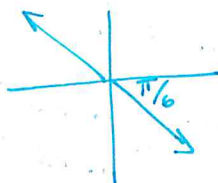
2. Simplify to a single term:  $\sin(x - y)\cos y + \cos(x - y)\sin y$

$$\begin{aligned}
 [1] \quad &= \sin [x - y + y] \\
 &= \sin x \\
 &= \sin x \cos^2 y - \cancel{\sin y \cos x \cos y} + \cancel{\sin y \cos x \cos y} + \sin x \sin^2 y \\
 &= \sin x \cos^2 y + \sin x \sin^2 y \\
 &= \sin x (\cos^2 y + \sin^2 y) \\
 &= \sin x
 \end{aligned}$$

3. For  $0 \leq \theta \leq 2\pi$ , find the exact value(s) of  $\theta$  where:

a)  $\tan \theta = -\frac{1}{\sqrt{3}}$

b)  $\sin \theta = 0$



no opp

$$\therefore \theta = 0, \pi \text{ or } 2\pi$$

[4]

$$\therefore \theta = \frac{5\pi}{6} \text{ or } \frac{11\pi}{6}$$

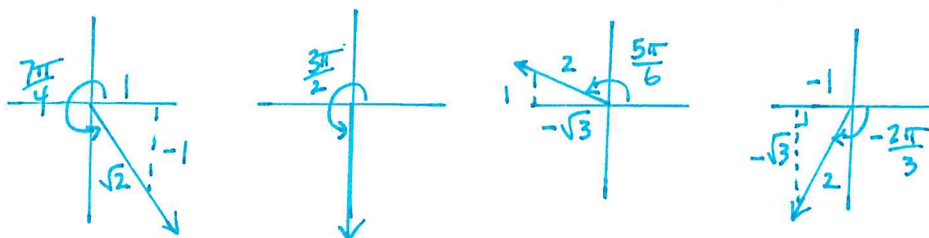
4. Given a circle that has a radius of 7cm. What is the length of the arc that is created by an angle of 3 radians?

[1]

$$\begin{aligned}
 a &= 7 \times 3 \\
 &= 21 \text{ cm}
 \end{aligned}$$

5. Determine the exact value for:  $\cos\left(\frac{7\pi}{4}\right)\sin\left(\frac{3\pi}{2}\right) - \sec\left(\frac{17\pi}{6}\right)\cot\left(-\frac{2\pi}{3}\right)$

[4]



$$\begin{aligned}
 &= \left(\frac{1}{\sqrt{2}}\right)(-1) - \left(\frac{2}{-\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) \\
 &= -\frac{1}{\sqrt{2}} + \frac{2}{3} \\
 &= \frac{-3 + 2\sqrt{2}}{3\sqrt{2}}
 \end{aligned}$$

$= \frac{4 - 3\sqrt{2}}{6}$

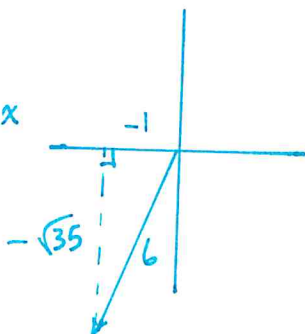
6. Given  $\cos x = -\frac{1}{6}$  where  $\pi \leq x \leq \frac{3\pi}{2}$ . Find the exact value of:

a)  $\sin\left(\frac{5\pi}{3} - x\right)$

$$= \sin\frac{5\pi}{3}\cos x - \cos\frac{5\pi}{3}\sin x$$

[4]

$$\begin{aligned}
 &= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{6}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sqrt{35}}{6}\right) \\
 &= \frac{\sqrt{3} + \sqrt{35}}{12}
 \end{aligned}$$



b)  $\cos 2x$

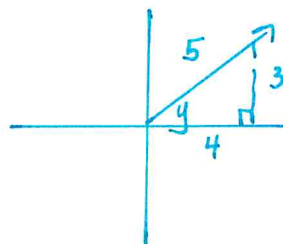
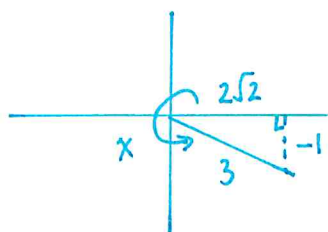
$$\begin{aligned}
 &= 2\cos^2 x - 1 \\
 &= 2\left(-\frac{1}{6}\right)^2 - 1 \\
 &= 2\left(\frac{1}{36}\right) - 1 \\
 &= \frac{1}{18} - 1 \\
 &= -\frac{17}{18}
 \end{aligned}$$

or

$$\begin{aligned}
 &\cos^2 x - \sin^2 x \\
 &= \left(-\frac{1}{6}\right)^2 - \left(\frac{\sqrt{35}}{6}\right)^2 \\
 &= \frac{1}{36} - \frac{35}{36} \\
 &= -\frac{34}{36} = -\frac{17}{18}
 \end{aligned}$$

7. If  $\sin x = -\frac{1}{3}$  when  $\frac{3\pi}{2} \leq x \leq 2\pi$  and  $\tan y = \frac{3}{4}$  when  $0 \leq y \leq \frac{\pi}{2}$ ,

**Labelled Diagrams (include x & y):**



Find the **EXACT** values for:

a)  $\tan(x - y)$

b)  $\cos(x - y)$

[6]

$$= \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{-\frac{1}{2\sqrt{2}} - \frac{3}{4}}{1 + \left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{3}{4}\right)}$$

$$= \frac{-4 - 6\sqrt{2}}{8\sqrt{2} - 3}$$

$$= \frac{-4 - 6\sqrt{2}}{8\sqrt{2} - 3}$$

$$\begin{aligned} &= \frac{-4 - 6\sqrt{2}}{8\sqrt{2} - 3} \times \frac{8\sqrt{2} + 3}{8\sqrt{2} + 3} \\ &= \frac{-32\sqrt{2} - 12 - 96 - 18\sqrt{2}}{128 - 9} \\ &= \frac{-50\sqrt{2} - 108}{119} \end{aligned}$$

✓.5

$$= \cos x \cos y + \sin x \sin y$$

$$= \left(\frac{2\sqrt{2}}{3}\right)\left(\frac{4}{5}\right) + \left(-\frac{1}{3}\right)\left(\frac{3}{5}\right)$$

$$= \frac{8\sqrt{2} - 3}{15}$$

✓.5

c) What quadrant does the angle  $(x - y)$  lie in? **Justify your reasoning.**

$$\tan(x - y) < 0 \quad \therefore Q_2 \text{ or } Q_4$$

$$\cos(x - y) > 0 \quad \therefore Q_1 \text{ or } Q_4$$

$$\therefore Q_4$$

✓

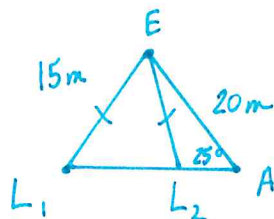
Please hand-in this portion and receive Part 3. You may use your calculator on Part 3.

**Part C: Application (Calculator Permitted)**

**Name:** \_\_\_\_\_

1. Panic is setting in for Blue Jays player, Addison Barger. His lucky baseball glove is in his luggage but the air tag shows that it is not on the conveyor belt at the luggage pickup station. The air tag indicates that his luggage is 15m from the exit and Addison is 20m from the same exit. It also indicates that the angle between Addison's direct path to the exit and the direct path to his luggage is  $25^\circ$ . Determine the approximate distance(s) from Addison to his luggage (nearest tenth).

[4]



$\triangle L_1EA$

$$\frac{\sin L}{20} = \frac{\sin 25}{15}$$

$$\sin L = \frac{20 \sin 25}{15}$$

$$L \approx 34^\circ$$

$$\begin{aligned} E &= 180 - (34 + 25) \\ &= 180 - 59 \\ &= 121^\circ \end{aligned}$$

$$\frac{e}{\sin 121^\circ} = \frac{15}{\sin 25^\circ}$$

$$e = \frac{15 \sin 121^\circ}{\sin 25^\circ}$$

$$e \approx 30.4 \text{ m}$$

$\triangle L_2EA$

$$\begin{aligned} L &= 180 - 34 \\ &= 146 \end{aligned}$$

$$\begin{aligned} E &= 180 - (146 + 25) \\ &= 180 - 171 \\ &= 9^\circ \end{aligned}$$

$$\frac{e}{\sin 9^\circ} = \frac{15}{\sin 25^\circ}$$

$$e = \frac{15 \sin 9^\circ}{\sin 25^\circ}$$

$$e \approx 5.6 \text{ m}$$

2. For  $0 \leq \theta \leq 2\pi$ , find the approximate value(s) of  $\theta$  when  $\sin(\frac{\pi}{2} - \theta) = -\frac{2}{15}$  (nearest hundredth).

$$\therefore \cos \theta = -\frac{2}{15}$$

or

$$\sin \frac{\pi}{2} \cos \theta - \cos \frac{\pi}{2} \sin \theta = -\frac{2}{15}$$

$$(1) \cos \theta - (0) \sin \theta = -\frac{2}{15}$$

$$\cos \theta = -\frac{2}{15}$$

$$\alpha = \cos^{-1}\left(\frac{2}{15}\right)$$

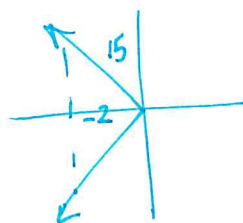
$$\approx 1.44$$

$$\therefore \theta = \pi - 1.44$$

or

$$\pi + 1.44$$

$$\theta \approx 1.7 \text{ or } 4.58$$



[3]