Countability

- Sets
  - Finite
  - Infinite
    - Countable
    - Uncountable

Countable: A set 'S' is said countable if all the elements of the set can be put in one to one correspond-ence with the set of natural numbers.

Uncountable: A set is uncountable, if it is infinite and not countable.

- set of Even ~~a~~number, $E = \{0, 2, 4, 6, 8, 10, 12, \ldots\}$ $2i$

  Natural no, $N = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$ $i+1$

  Index of 0   Index of 7   Index of $(2i)$.

  Set of even number is countable.

Index: If every element in the given set is going to have a index.

- Set of all odd number, $O = \cancel{\emptyset}\{1, 3, 5, 7, 9, 11 \ldots\}$ $2i+1$

  $N = \{1, 2, 3, 4, 5, 6 \ldots\}$ $i+1$

  Index of $(2i+1)$

  set of odd no is countable.

→ Set of all ~~a~~ Real Number is uncountable.

A set is said to be countable if there exists an enumera-tion methode using which all the elements of set can be generated and for any particular element, it takes only finite number of steps to generate it. The finite number of sets taken to generate an element can be used as its index and hence a mapping into natural numbers.

[Ex-1] • set of all even numbers –

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for (i = 0 to ∞)
  { generate (2i)
  }
```

0, 2, 4, 6, 8 ...... — It is countable.
1 2 3 4 5 .....
Index

• set of all odd number –

```
for (i = 0 to ∞)
{
   generate(2i+1)
}
```

→ 1, 3, 5, 7, 9, 11 ...... — It is countable.
Index – 1 2 3 4 5 6 ...

• set of all real numbers.. — Uncountable.

quotients • Set of "P/q", $P, q \in Z_+$ — Countable set.

$S = \{1/1, 1/2, 2/1, 1/3, 1/4, 3/4, \ldots\}$

(2) $1/1$, (3) $1/2, 2/1$ (4) $1/3, 2/2, 3/1$ (5) $1/4, 2/3, 4/1, 3/2$

→ Index (1) (2) (3) (4) (5) (6) (7) (8) (9) (10)

- Set of all string over any finite alphabet countable.

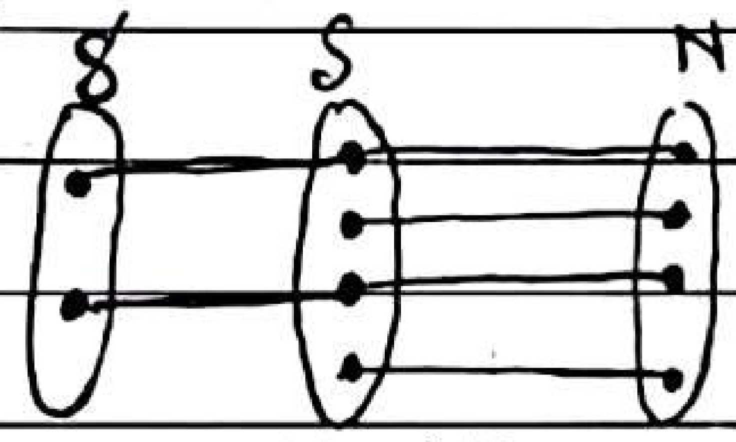
→ $\Sigma = \{a, b\}$

$\Sigma^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa \ldots\}$ countable

① ② ③ ④ ⑤ ⑥ ⑦ ⑧

(Proper Orders)

→ Every subset of countable set is either finite or countable.



countable set

$L \subseteq \Sigma^*$ → every language countable.

- Set of all turing machine is Countable.

→ $\Sigma = \{0, 1\}$

$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$ = coutable

→ every TM can be encoded as a string of 0's and 1's

→ set of all TM (Turing machine), $S \subseteq \Sigma^*$

→ every subset of countable set is either finite or countable.

↳ $\boxed{\text{set of all TM are countable.}}$

- set of all turing machine is Countable.

→ TM are c
REL are c
RcEL are c
CSL are c → LBA are c

CFL are c → PDA (c → countable)
RL are c → FA are countable

** Diagnolization method:

→ To prove that set of all language are uncountable.

$\Sigma = \{a, b\}$, Σ^* is countable, 2^{Σ^*} ~~t~~ is uncountable

→ If 's' is countably infinite, 2^S is uncountable.

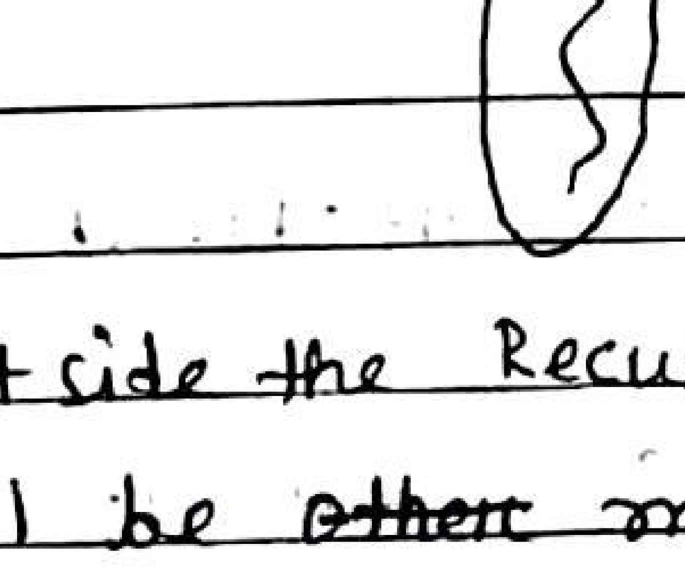
$S = \{a, b\}$, $2^S = \{\{\}, \{a\}, \{b\}, \{ab\}\}$.

		$\Sigma^* =$ ε	a	b	aa	ab	ba	bb	aaa ......
$L=\{a, b, ba\}$	1)	0	1	1	0	0	1	0	0 ......
$L=\{\epsilon\}$	2)	1	0	0	0	0	0	0	0 .....
$L=\{b, aa\}$	3)	0	0	1	1	0	0	0	0 .....
$L=\{b, ab, bb\}$	4)	0	0	1	0	1	0	1	0 ....
$L=\{a, b, ba, bb\}$	5)	0	1	1	0		1	1	0 ....

0 0 1 0 1

1's - 1 1 0 1 0 → This new languag are not present in Index table, so, 2^{Σ^*} is not Countable.

→ assume 2^{Σ^*} is countable.



→ out side the Recursively enumarable languages, there will be ~~other~~ more language.

atlantis

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Some problems on countability –

→ If S_1 and S_2 are countable sets, then $S_1 \cup S_2$ is countable and $S_1 \times S_2$ is countable.

$$\begin{cases} S_1: \overset{1}{a_1}\ \overset{2}{a_2}\ \overset{3}{a_3}\ \overset{4}{a_4}\ \overset{5}{a_5}\ \overset{6}{a_6} \cdots \\ S_2: \overset{1}{b_1}\ \overset{2}{b_2}\ \overset{3}{b_3}\ \overset{4}{b_4}\ \overset{5}{b_5}\ \overset{6}{b_6} \cdots \end{cases}$$

$S_1 \cup S_2$: $a_1 b_1\ a_2 b_2\ a_3 b_3$

1 2 3 4

$S_1 \times S_2$:

addition of index = 2: $\mathrm{Ind}(1) + \mathrm{Ind}(1)$ (a_1, b_1)

3: 1 2, 2 1 $(a_1, b_2)\ (a_2, b_1)$

4: 1 3, 2 2, 3 1 $(a_1, b_3)(a_2 b_2)(a_3 b_1)$.

→ The ~~cros~~ cartesion product of finite number of countable sets ~~of~~ is countable.

→ The set of all languages that are not recursively enumerable is uncountable.