

Spring 2022

## CSE 317: Design and Analysis of Algorithms, Quiz - 1 [Section 3]

Monday, February 14, 2022. Total marks: 10 point, Duration: 15 minutes.

Name: \_\_\_\_\_, Student ID: \_\_\_\_\_

1. (10 points) Analyze the following algorithm and determine how many times the Algorithm POWER performs squaring (line no. 5). Use  $O$ -notations.

**Algorithm:** POWER

**Input:**  $y$  and  $z$  such that  $y \in \mathbb{R}$  and  $z \in \mathbb{N}$

**Output:** Returns  $y^z$

1.  $x = 1$
2. **while** ( $z > 0$ )
3.     **if**  $z$  is odd **then**  $x = x \cdot y$
4.      $z = \lfloor z/2 \rfloor$
5.      $y = y^2$
6. **return**  $x$

**Solution:** We see that every time **while** loop is executed, line no. 5 is executed. The number of times **while** loop is executed is depended on how quickly  $z$  becomes 0. Given that  $z \in \mathbb{N}$  such that  $2^{k-1} \leq z \leq 2^k$  for some  $k \in \mathbb{N}$  and  $k > 0$ .

In each iteration of the **while** loop, the value of  $z$  is updated (decreased) to  $\lfloor z/2 \rfloor$ . So after the first iteration  $\lfloor z/2 \rfloor$ , after the second iteration  $\lfloor z/2^2 \rfloor$ , and so on. We can generalize that after  $j$  iteration we have  $\lfloor z/2^j \rfloor = 1$  so in  $(j+1)$ -st iteration line no. 5 is executed last time and algorithm comes out of **while** loop.

We can conclude that line no. 5 is executed at least  $j+1$  times such that  $\lfloor z/2^j \rfloor = 1$ , i.e.,

$$0 < \frac{z}{2^j} \leq 1 \\ z \leq 2^j.$$

Since  $j$  is an integer we can conclude is at most  $\log_2 z$ . Therefore the number of times the line no. 5 is executed is  $\Theta(\log z)$ .