

Answer all questions. Use separate answer sheet. Be to the point. Show your work.

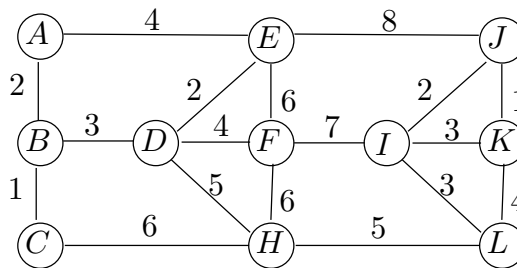
Please give clear and rigorous answers.

Name: \_\_\_\_\_

ERP: \_\_\_\_\_

**Question 1: Greedy Algorithms** ..... 6 marks

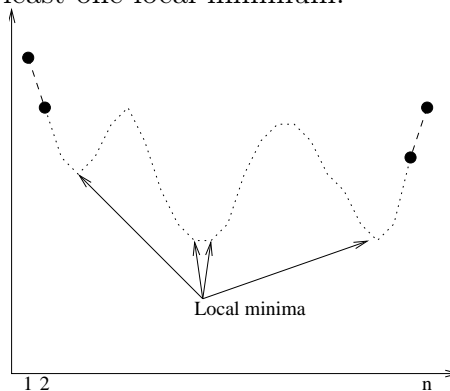
(a) [4 marks] Consider the weighted graph below.



- Run Prim's algorithm starting from vertex A. Write the edges in the order which they are added to the minimum spanning tree.
  - Run Kruskal's algorithm. Write the edges in the order which they are added to the minimum spanning tree.
  - What is the total weight of minimum spanning tree in this graph?
- (b) [2 marks] Prove that if the weights of the edges of a connected graph  $G$  are distinct, then  $G$  has a unique minimum spanning tree.

**Question 2: Divide & Conquer** ..... 5 marks

- (a) [3 marks] In this problem we consider an array  $A$  of length  $n$  for which we know that  $A[1] \geq A[2]$  and  $A[n-1] \leq A[n]$ . We say that  $A[x]$  is a *local minimum* if  $A[x-1] \geq A[x]$  and  $A[x] \leq A[x+1]$ . Note that  $A$  must have at least one local minimum.



We can obviously find a local minimum in  $O(n)$  time by scanning through  $A$ . Describe an  $O(\log n)$  algorithm for finding a local minimum.

- (b) [2 marks] Solve the recurrence:  $T(n) = 3T(n/4) + n$  using master theorem.

**Question 3: Dynamic Programming** ..... 4 marks

In this question we will compute edit-distance between the strings  $X = \textit{your-firstname}$  and  $Y = \textit{lastname}$ .

- (a) [1 mark] Write the recurrence for computing the optimal cost of a problem given the optimal solution of relevant subproblems. How many subproblems we get?
- (b) [3 marks] Fill in the appropriate table using the recurrence from previous part. (If your first-name or last-name have more than 5 letters then consider the first 5 letters only).