CSE317 Design & Analysis of Algorithms

Institute of Business Administration Karachi
Leadership and Ideas for Tomorrow

First Term Examination – Spring'18

Max Marks: 20 Duration: 1½ hours

IBA FCS
Faculty of Computer Science

Answer all questions. Use seperate answer sheet. Be to the point. Show your work.
Please give clear and rigorous answers.

Name: ________________ **ERP:** __________

Question 1: .... *5 marks*

(a) [2 marks] Draw a directed graph, having as its nine vertices the strings 'ape', 'ate', 'eat', 'era', 'pea', 'rap', 'rat', 'ear' and 'tea', and including an edge from word x to word y whenever the last two letters of x are the same as the first two letters of y; for instance, you should include an edge from 'ape' to 'pea'.

(b) [2 marks] Write down a sequence in which the vertices of this graph could be visited by breadth first search, starting from 'era'. Also draw resulting BFS tree.

(c) [1 mark] Does this graph have a topological ordering? Explain why or why not.

Question 2: .... *5 marks*

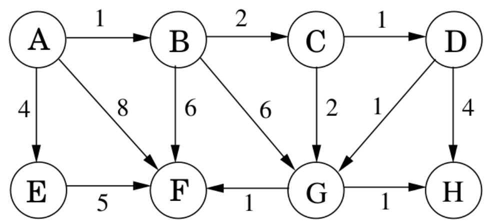
Suppose you are implementing a spreadsheet program, in which you must maintain a grid of cells. Some cells of the spreadsheet contain numbers, but other cells contain expressions that depend on other cells for their value. However, the expressions are not allowed to form a cycle of dependencies: for example, if the expression in cell E1 depends on the value of cell A5, and the expression in cell A5 depends on the value of cell C2, then C2 must not depend on E1.

(a) [2½ marks] Describe an algorithm for making sure that no cycle of dependencies exists (or finding one and complaining to the spreadsheet user if it does exist).

(b) [2½ marks] If the spreadsheet changes, all its expressions may need to be recalculated. Describe an efficient method for sorting the expression evaluations, so that each cell is recalculated only after the cells it depends on have been recalculated.

Question 3: .... *7 marks*

Consider the following graph.



(a) [4 marks] Use Dijkstra's algorithm to calculate the single-source shortest paths from vertex A to every other vertex. Show your steps in the table below. If during your algorithm two unvisited vertices have the same distance, use alphabetical order to determine which one is selected first.

Iteration	**Node removed**	**`dist[.]` / `prev[.]` value**							
		A	B	C	D	E	F	G	H
	-	0 / nill	∞ / nill	∞ / nill	∞ /nill	∞ /nill	∞ /nill	∞ /nill	∞ /nill
1									
$\vdots$									

(b) [1 mark] Draw shortest-path tree found by Dijkstra's algorithm in the previous part.

(c) [1 mark] Let $G = (V, E)$ be a weighted graph. Under what assumption does Dijkstra's algorithm correctly compute shortest paths?

(d) [1 mark] Is it true that the shortest path between two vertices u, v in a directed graph with all edge weights equal to 1 can be computed in $O(|V| + |E|)$ time?

Question 4: .... *3 marks*

Assume that you have an algorithm NEG-CYCLE such that given any directed, connected graph G with weight function $w : E \to R$, NEG-CYCLE terminates by giving you a negative weight cycle (a cycle such that the sum of the edge weights on the cycle is less than 0) or FALSE if no negative weight cycle exists in G.

Consider a currency trader who deals in the currency of n countries and makes money through arbitrage. For example, if 1 dollar is selling for 0.98 euros, and 1 euro is selling for 110 yen and 1 yen is selling for 0.0098 dollars, then the trader can convert 1 dollar into 0.98 euros, convert that into $0.98 \times 110 = 107.8$ yen, and convert it back to $107.8 \times 0.0098 = 1.05644$ dollars for a profit of 5.644 cents.

The currency trader has at any time an $n \times n$ matrix A where $A[i, j]$ represents the conversion rate from i to j. How can he use NEG-CYCLE to make money?