Spring 2022

CSE 317: Design and Analysis of Algorithms, Quiz - 3 [Section 3]

Wednesday, March 30, 2022. Total marks: 10 point, Duration: 15 minutes.

Name: _____, Student ID: _____

1. (10 points) Given a positive integer n, count all n-digit numbers without any consecutive 1's using dynamic programming algorithm. What is the time complexity of your algorithm?

   For example, for $n = 5$, there are 13 binary numbers that satisfy the given constraints:

   00000, 00001, 00010, 00100, 00101, 01000, 01001, 01010, 10000, 10001, 10010, 10100, and 10101.

Solution: Let a_i be the number of binary strings of length i which do not contain any two consecutive 1's and which end in a 0. Similarly, let b_i be the number of such strings which end in a 1. We can append either a 0 or a 1 to a string ending in 0, but we can only append 0 to a string ending in 1. This yields the recurrence relation:

$a_i = a_{i-1} + b_{i-1}$

$b_i = a_{i-1}$

The base cases of above recurrences are $a_1 = b_1 = 1$.

The total number of strings of length i is thus just $a_i + b_i$.

Time complexity: $O(n)$