Spring 2022

CSE 317: Design and Analysis of Algorithms, Quiz - 2 [Section 4]

Monday, February 28, 2022. Total marks: 10 point, Duration: 15 minutes.

Name: ______________________________, Student ID: ______________________________

1. (10 points) Let $A = \langle a_1, \dots a_n \rangle$ a be a sequence of n distinct positive integers. An element a_i, $1 \le i \le n$, is called a *peak element* if none of its neighbors are larger than it. Two elements a_i and a_j are neighbors if $|i - j| = 1$. Design a *divide and conquer* $O(\log n)$-algorithm to find **one** peak element.

   For example, peak elements in different cases of A are underlined next: $A = \langle 1, 2, \underline{3} \rangle$, $A = \langle \underline{6}, 5, 4 \rangle$ and $A = \langle 1, \underline{5}, 2, \underline{4}, 3 \rangle$. [**Note:** It is possible that a given sequence contains more than one peak elements (see the last example) but your algorithm is supposed to return just one of them.]

Solution:

Algorithm: FIND-PEAK

Input: A sequence $A = \langle a_l, \dots a_h \rangle$ of $h - l + 1$ distinct positive integers.

Output: A peak element a_j such that $a_j > a_{j+1}$ and $a_j > a_{j-1}$.

1. $m = \lfloor (h + l)/2 \rfloor$
2. **if** $a_m > a_{m-1}$ **and** $a_m > a_{m+1}$ **then return** a_m
3. **if** $a_m < a_{m-1}$ **then return** FIND-PEAK($\langle a_l, \dots, a_{m-1} \rangle$)
4. **if** $a_m < a_{m+1}$ **then return** FIND-PEAK($\langle a_{m+1}, \dots, a_h \rangle$)

Time complexity: Let $h - l + 1 = n$ initially and $T(n)$ be time required to compute FIND-PEAK, then $T(n) = T(n/2) + 1$, with $T(1) = 1$ so, $T(n) = O(\log n)$.