Spring 2022

CSE 317: Design and Analysis of Algorithms, Quiz - 3 [Section 4]

Wednesday, March 30, 2022. Total marks: 10 point, Duration: 15 minutes.

Name: ____________________________, Student ID: ____________________________

1. (10 points) Let $A = \langle a_1, \ldots, a_n \rangle$ be a sequence of numbers where $n > 0$ and each number $a_i \in \{0, 1\}$. A subsequence $\langle a_{i_1}, \ldots, a_{i_k} \rangle$ of A is a sequence of elements such that $i_1 < \cdots < i_k$ and $1 \leq k \leq n$. A subsequence of A is an alternating sequence if each element is different than the one preceding it. For example, let $A = \langle 0, 1, 0, 0, 0, 1, 0 \rangle$ then $\langle 0 \rangle$, $\langle 1 \rangle$, $\langle 0, 1 \rangle$, $\langle 1, 0 \rangle$ and $\langle 0, 1, 0, 1, 0 \rangle$ are different alternating subsequences while $\langle 1, 1 \rangle$ is not an alternating subsequence. The longest alternating subsequence of A is $\langle 0, 1, 0, 1, 0 \rangle$ and it has length 5.

   Design a dynamic programming algorithm to find the length of the longest alternating subsequence of a given sequence A. What is the time complexity of your algorithm?

Solution:

Let L_i^0 = length of the longest alternating subsequence ending at index i and the last element is 1 and second last element is 0

Let L_i^1 = length of the longest alternating subsequence ending at index i and the last element is 0 and second last element is 1

We can recursively compute for $1 \leq i \leq n$ as following:

$L_i^0 = \max_{j<i}\{L_i^0, L_j^1 + 1\}$ with $A_j = 0$ and $A_i = 1$ and

$L_i^1 = \max_{j<i}\{L_i^1, L_j^0 + 1\}$ with $A_j = 1$ and $A_i = 0$.

Time complexity: $O(n)$

Space complexity: $O(n)$