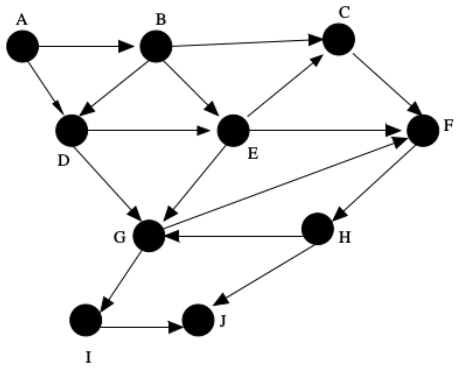
CSE317 Design & and Analysis of Algorithms

Second Term Examination – Spring'15

Max Marks: 50 Time Allowed: 90 minutes

Attempt all questions. Be to the point. Show your work.

1. [10 marks] Do a topological sort of the following graph G using the algorithm discussed in the lecture. Break all ties by picking the vertices in alphabetical order (i.e., A before Z).



2. (a) [4 marks] Consider a weighted graph $G = (V, E)$ with non-negative weights. Suppose you have computed the shortest paths to all nodes from a particular node $s \in V$. Now suppose every weight in the graph is changed: the new weight $l'_e = l_e^2$. Do the shortest paths change? Given an example where it changes or prove it cannot change.

   (b) [3 marks] What is the running time of depth-first search, as a function of $|V|$ and $|E|$, if the input graph is represented by an adjacency matrix instead of an adjacency list?

   (c) [3 marks] How can the number of strongly connected components of a directed graph change if a new edge is added?

3. [10 marks] Give an algorithm to detect whether a given undirected graph contains a cycle. If the graph contains a cycle, then your algorithm should output one. (It should not output all cycles in the graph, just one of them.) The running time of your algorithm should be $O(m + n)$ for a graph with n nodes and m edges.

4. (a) [6 marks] Consider a hash table of size 7 with hash function $h(k) = k \mod 7$. Draw the table that results after inserting, in the given order, the following values: 19, 26, 13, 48, 17 for each of the two scenarios below:
   - When collisions are handled by seperate chaining;
   - When collisions are handled by linear probing.

   (b) [4 marks] If chaining is used to handle collisions in a hash table with load factor α, what is the average number of steps (in terms of α) that will be needed in order to find an object that is in the table? Explain.

5. [10 marks] Your job is to arrange n ill-behaved children in a straight line, facing front. You are given a list of m statements of the form "i hates j". If i hates j, then you do not want put i somewhere behind j, because then i is capable of throwing something at j. Give an algorithm that orders the line, (or says that it is not possible) in $O(m + n)$ time.