CSE317 Design & Analysis of Algorithms

Second Term Examination

Max Marks: 20 | Time Allowed: $1\frac{1}{2}$ hours

Answer all questions. Use seperate answer sheet. Please give clear and rigorous answers. Be to the point. Show your work.

Name: ____________________ **ERP:** __________

Question 1: Divide and Conquer .... *8 marks*

(a) [2 marks] Write the statement of Master theorem (without derivation).

(b) [3 marks] How many lines (as a function of n) does the following program print? Derive a recurrence and solve it. You may assume that n is a power of 4.

```
function f(n)
    if (n > 1) {
        f(n/4);
        f(n/4);
        f(n/4);
        print_line("Hello world!");
    }
```

(c) [3 marks] Given a sorted integer array of length n and number x, find the number of times x occurs in the array. Your algorithm should be $O(\log n)$. Example input/output:

```
count([1,2,2,2,4,6], 2)= 3
```

Hint: find indices of first/last occurence of x in the array.

Question 2: Asymptotics .... *4 marks*

(a) [3 marks] Arrange the following functions according to their increasing order of growth: n, $\sqrt{n}$, $n^{1/n}$, $n \log_2 \log_2 n$, $\sqrt{n} \log_2(n^2)$, $2/n$, 2^n, $2^{(\log_2 n)^2}$.

(b) [1 mark] Compute an appropriately tight O (Big-Oh) bound on the running time of following code fragment, in terms of n. Assume integer arithmetic.

```
for(i = n; i > 0; i = i / 2) {
    for(j = 0; j < n; j++) {
        sum++;
    }
}
```

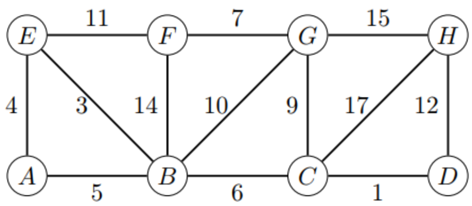
Question 3: Greedy .... *2 marks*

F.Gump is a good runner. He can run upto 100 kilometers uninterrupted, but then he needs to stop and eat a box of chocolates. He wants to run across Pakistan along a certain route. He knows the locations of the stores along the route selling his favourite chocolates. He wants to stop as few time as possible. Design a linear time $O(n)$ algorithm for choosing where to stop, where n is number of chocolate shops along his route. The input to your algorithm is a sorted array $A[1..n]$ of distances of the chocolate shops from the starting point. The last chocolate shop location $A[n]$ is the end point of the route.

Argue why your algorithm is correct.

Question 4: Minimum Spanning Trees .... *6 marks*

(a) [3 marks] Consider the weighted graph below.



i. Run Prim's algorithm starting from vertex A. Write the edges in the order which they are added to the minimum spanning tree.

ii. Run Kruskal's algorithm. Write the edges in the order which they are added to the minimum spanning tree.

(b) [3 marks] You are given an edge-weighted undirected graph, using the adjacency list representation, together with the list of edges in its minimum spanning tree (MST). Describe an efficient algorithm for updating the MST, when each of the following operations is performed on the graph.

i. the weight of an edge that ***was not*** part of the MST is *increased*.

ii. the weight of an edge that ***was*** part of the MST is *increased*.

Give the running time of your algorithm as a function of $|V|$ and/or $|E|$. Assume that common graph operations (e.g., DFS, BFS, finding a cycle, etc.) are available to you, and don't describe how to re-implement them.