Spring 2022

CSE 317: Design and Analysis of Algorithms, Quiz - 1 [Section 4]

Monday, February 14, 2022. Total marks: 10 point, Duration: 15 minutes.

Name: _____, Student ID: _____

1. (10 points) Analyze the following algorithm and determine how many times the Algorithm MULTIPLY performs halving (line o. 5). Use O-notations.

Algorithm: MULTIPLY

Input: y and z such that $y, z \in \mathbb{N}$
Output: Returns yz

1. $x = 0$
2. **while** $(z > 0)$
3. **if** $z \mod 2 = 1$ **then** $x = x + y$
4. $y = 2y$
5. $z = \lfloor z/2 \rfloor$
6. **return** x

Solution: We see that every time **while** loop is executed, line no. 5 is executed. The number of times **while** loop is executed is depended on how quickly z becomes 0. Given that $z \in \mathbb{N}$ such that $2^{k-1} \leq z \leq 2^k$ for some $k \in \mathbb{N}$ and $k > 0$.

In each iteration of the **while** loop, the value of z is updated (decreased) to $\lfloor z/2 \rfloor$. So after the first iteration $\lfloor z/2 \rfloor$, after the second iteration $\lfloor z/2^2 \rfloor$, and so on. We can generalize that after j iteration we have $\lfloor z/2^j \rfloor = 1$ so in $(j+1)$-st iteration line no. 5 is executed last time and algorithm comes out of **while** loop.

We can conclude that line no. 5 is executed at least $j + 1$ times such that $\lfloor z/2^j \rfloor = 1$, i.e.,

$$\begin{aligned} 0 < \quad \frac{z}{2^j} &\leq 1 \\ z &\leq 2^j. \end{aligned}$$

Since j is an integer we can conclude is at most $\log_2 z$. Therefore the number of times the line no. 5 is executed is $\Theta(\log z)$.