CSE317 Design & Analysis of Algorithms

Final Examination – Spring'18

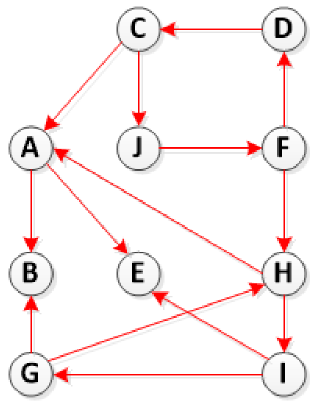
Max Marks: 40 Duration: 3 hours

Answer all questions. Use seperate answer sheet. Be to the point. Show your work.
Please give clear and rigorous answers.

Name: ____________________ **ERP:** __________

Question 1: Graphs .... *8 marks*

(a) [4 marks] Run the strongly connected components algorithm on the following directed graph G. When doing DFS on G^R: whenever there is a choice of vertices to explore, always pick the one that is alphabetically first.

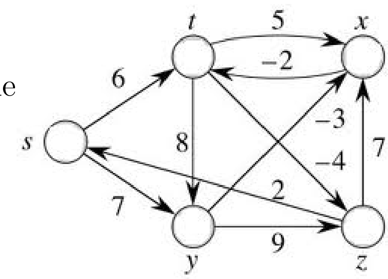


i. In what order are the strongly connected components (SCCs) found?

ii. Which are source SCCs and which are sink SCCs?

iii. Draw the "metagraph" (each meta-node is an SCC of G).

(b) [4 marks] Suppose Bellman-Ford's algorithm is run on the following graph, starting at node s.



i. Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

ii. Show the final shortest-path tree.

Question 2: Asymptotic Notations .... *6 marks*

(a) [2 marks] Prove that $T(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3 = O(n^3)$ using the formal definition of the Big-O notation. Hint: Find a constant c and threshold n_0 such that $cn^3 \geq T(n)$ for $n \geq n_0$.

(b) [4 marks] Determine whether each statement is TRUE or FALSE and correct the formula (inside $O()$, $\Theta()$, or $\Omega()$) in the latter case.

i. $3n^2 + 10n \log n = O(n \log n)$

ii. $3n^2 + 10n \log n = \Omega(n^2)$

iii. $10\sqrt{n} + \log n = O(n)$

iv. $2\sqrt{n} + \log n = \Theta(\sqrt{n})$

Question 3: Minimum Spanning Trees .... *8 marks*

(a) [4 marks] Consider a set of 5 towns. The cost of constraction of a road between towns i and j is a_{ij}. Find the minimum cost road network connecting the towns with each other.

$$\begin{bmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & \infty & 10 \\ 11 & 9 & \infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{bmatrix}$$

(b) [4 marks] Your colleague has provided you with a function called `mst` that takes as input a weighted undirected graph G, and returns a minimm spanning tree for G. Unfortunately, her function throws an exception if any of the edge weights are negative, yet the graphs you are working with model neural networks, and allow for both positive and negative synapse weights. Describe in a few sentences how you could implement a wrapper function called `my_mst` that allows for an input graph G with positive or negative weights, and returns a minimum spanning tree for G. Moreover, it accomplishes this, in part, by making a call to `mst`. However `mst` is located in a pre-compiled library, and so you may not change any of its code.

Question 4: Divide & Conquer .... *9 marks*

(a) [4 marks] Given an array `A[0..N-1]`, find a peak element in it in $O(\log n)$ time. An element `A[i]` is a peak element if it is not smaller than its neighbor(s). I.e, `A[i] >= A[i-1]` and `A[i] >= A[i+1]` for $0 < i < N - 1$. (Boundary cases: If `A` contains at least two elements, `A[0]` is peak if `A[0] >= A[1]` and `A[N-1]` is peak element if `A[N-1] >= A[N-2]`. If `A` contains single element then that element is peak.)

For example, peak elements in $[8, 9, 10, 2, 5, 6]$ are 10 and 6. In $[8, 9, 10, 12, 15]$ the peak element is 15, and peak element of $[10, 8, 6, 5, 3, 2]$ is 10.

There might be multiple peak element in a array, your algorithm need to find *only one* peak element.

(b) [1 mark] Give formal statement of Master theorem.

(c) [4 marks] Solve following recurrences using Master theorem.

i. $T(n) = 4T(n/2) + n$

ii. $T(n) = 2T(n/4) + \sqrt{n}$

Question 5: Dynamic Programming .... *9 marks*

(a) [4 marks] In this question we will compute edit-distance between the strings X = *your-firstname* and Y = *lastname.*

i. Write the recurrence for computing the optimal cost of a problem given the optimal solution of relevant subproblems. How many subproblems we get?

ii. Fill in the appropriate table using the recurrence from previous part. (If your first-name or last-name have more than 5 letters then consider the first 5 letters only).

(b) [5 marks] A pharmacist has W pills and n empty bottles. Let $\{p_1, p_2, \ldots, p_n\}$ denote the number of pills that each bottle can hold.

i. Describe a greedy algorithm, which, given W and $\{p_1, p_2, \ldots, p_n\}$, determines the fewest number of bottles needed to store the pills. Prove that your algorithm is correct (that is, prove that the first bottle chosen by your algorithm will be in some optimal solution).

ii. Consider a variation when each bottle also has an associated cost c_i, and you want to minimize the total cost of the bottles used to store all the pills.
Let $MinPill(i, j)$ be the minimum cost obtainable when storing j pills using bottles among 1 through i. Give a recurrence expression for $MinPill(i, j)$. We shall do the base case for you:

- $MinPill(i, 0) = 0$ for all i
- $MinPill(0, j) = \infty$ for $j > 0$

iii. Describe briefly how you would design an algorithm for it using dynamic programming and analyse its running time.