Q1

ANSWER:

Minimum scalar multiplications = 43

Optimal paranthesisation = $[(M_1 M_2) M_3] M_4$

WORKING:

Input:

M_1	M_2	M_3	M_4
1×5	5×2	2×3	3×9
$m_0 \quad m_1$	$m_1 \quad m_2$	$m_2 \quad m_3$	$m_3 \quad m_4$

$M_i = m_{i-1} \times m_i$

m_0	m_1	m_2	m_3	m_4
1	5	2	3	9

Cases:

$C(i,i) = 0$

$$C(i,j) = \min \left\{ C(i,k) + C(k+1,j) + (m_{i-1} \times m_k \times m_j) \right. \quad \forall k \text{ s.t } i \leq k < j$$

Run-through (done diagonally):

$C(1,1) = C(2,2) = C(3,3) = C(4,4) = 0$

$\forall C(i,j)$ s.t $i > j = X$

$$C(1,2) = \min \left\{ C(1,1) + C(2,2) + (\overset{m_0 \times m_1 \times m_2}{1 \times 5 \times 2}) \right. = 10$$

($C(1,1) = 0$, $C(2,2) = 0$)

$$C(2,3) = \min \left\{ C(2,2) + C(3,3) + (m_1 \times m_2 \times m_3) \right. = 30$$

($C(2,2) = 0$, $C(3,3) = 0$, $m_1 = 5$, $m_2 = 2$, $m_3 = 3$)

$$C(3,4) = \min \left\{ C(3,3) + C(4,4) + (m_2 \times m_3 \times m_4) \right. = 54$$

($C(3,3) = 0$, $C(4,4) = 0$, $m_2 = 2$, $m_3 = 3$, $m_4 = 9$)

	1	2	3	4
1	0			
2	X	0		
3	X	X	0	
4	X	X	X	0

	1	2	3	4
1	0	10		
2	X	0	30	
3	X	X	0	54
4	X	X	X	0

Marks _____ Signature of Instructor _____

$C(1,3) = \min \begin{cases} C(1,1) + C(2,3) + (m_0 \times m_1 \times m_3) = 45 \\ \quad 0 \quad\quad 30 \quad\quad 1 \;\; 5 \;\; 3 \\ C(1,2) + C(3,3) + (m_0 \times m_2 \times m_3) = 16 \checkmark \\ \quad 10 \quad\quad 0 \quad\quad 1 \;\; 2 \;\; 3 \end{cases}$

0	10	96	
X	0	30	121
X	X	0	54
X	X	X	0

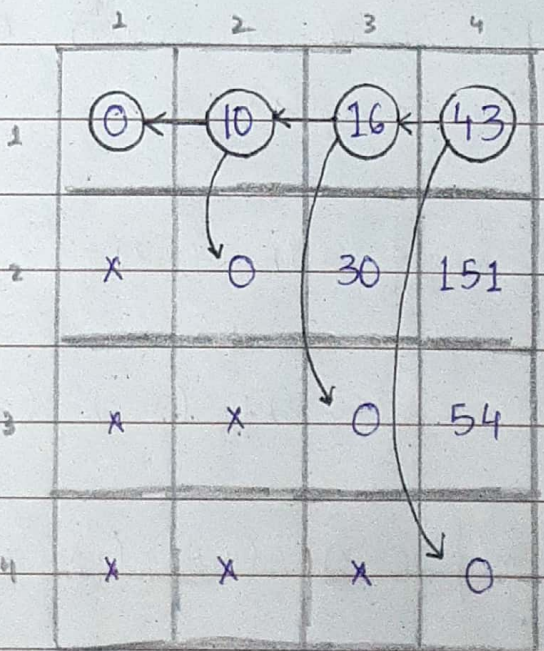
$C(2,4) = \min \begin{cases} C(1,2) + C(3,4) + (m_1 \times m_2 \times m_4) = 154 \\ \quad 10 \quad\quad 54 \quad\quad 5 \;\; 2 \;\; 9 \\ C(1,3) + C(4,4) + (m_1 \times m_3 \times m_4) = 151 \checkmark \\ \quad 16 \quad\quad 0 \quad\quad 5 \;\; 3 \;\; 9 \end{cases}$

0	16	16	
X	0	30	151
X	X	0	54
X	X	X	0

$C(1,4) = \min \begin{cases} C(1,1) + C(2,4) + (m_0 \times m_1 \times m_4) = 196 \\ \quad 0 \quad\quad 151 \quad\quad 1 \;\; 5 \;\; 9 \\ C(1,2) + C(3,4) + (m_0 \times m_2 \times m_4) = 82 \\ \quad 10 \quad\quad 54 \quad\quad 1 \;\; 2 \;\; 9 \\ C(1,3) + C(4,4) + (m_0 \times m_3 \times m_4) = 43 \checkmark \\ \quad 16 \quad\quad 0 \quad\quad 1 \times 3 \times 9 \end{cases}$

0	10	16	43
X	0	30	151
X	X	0	54
X	X	X	0

Backtracking:



ROUGH WORK & PROOF OF CONCEPT : (Shown on next pages)

$M_1 \cdot M_2 \cdot M_3 \cdot M_4$

$1 \times 5 \quad 5 \times 2 \quad 2 \times 3 \quad 3 \times 9$

$m_0 \; m_1 \quad m_1 \; m_2 \quad m_2 \; m_3 \quad m_3 \; m_4$

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Rules:

$$c(i,i) = 0$$

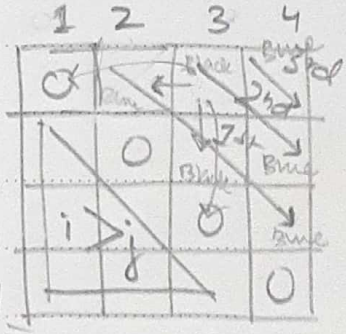
$$c(i,j) = \min_{\forall k \text{ s.t } i \leq k < j} \left\{ c(i,k) + c(k+1,j) + (m_{i-1} \times m_k \times m_j) \right.$$

dimensions:

$$M_i = m_{i-1} \times m_i$$

m_0	m_1	m_2	m_3	m_4
1	5	2	3	9

First diagonal:



$k \neq j$ $\quad c(i,k) \quad c(k+1,j)$

i	j	k	$k+1$	m_{i-1}	m_k	m_j
(1, 2)		1	2	m_0	m_1	m_2

$1 \times 5 \times 2 = 10$

(2,3)		2	3	(2,2)	(3,3)	m_1 m_2 m_3
(3,4)		3	4	(3,3)	(4,4)	m_2 m_3 m_4
(1,3)	MIN 1		2	(1,1)	(2,3)	$m_0 \times m_1 \times m_3$
(1,3)	2		3	(1,2)	(3,3)	$m_0 \times m_2 \times m_3$

m_0	m_1	m_2	m_3	m_4
1	5	2	3	9

UEP

All $i \le k < j$: $k \neq j$

i	j	i-1	k	k+1	$m_{i-1} \times m_k \times m_j$	c(i,k)	c(k+1,j)	x_k
1	2	0	1	2	$m_0 \times m_1 \times m_2$ $1 \times 5 \times 2$ $= 10$	c(1,1) 0	c(2,2) 0	10
2	3	1	2	3	$m_1 \times m_2 \times m_3$ $5 \times 2 \times 3$ $= 30$	c(2,2) 0	c(3,3) 0	30
3	4	2	3	4	$m_2 \times m_3 \times m_4$ $2 \times 3 \times 9$ $= 54$	c(3,3) 0	c(4,4) 0	54
1	3	0	1	2	$m_0 \times m_1 \times m_3$ $1 \times 5 \times 3$ $= 15+30 = 45$	c(1,1) 0	c(2,3) 30	16
1	3	0	2	3	$m_0 \times m_2 \times m_3$ $1 \times 2 \times 3$ $= 6+10 = (16)$ ✓	c(1,2) 10	c(3,3) 0	
2	4	1	2	3	$m_1 \times m_2 \times m_4$ $5 \times 2 \times 9$ $= 90+10+54 = 154$	c(1,2) 10	c(3,4) 54	151
2	4	1	3	4	$m_1 \times m_3 \times m_4$ $5 \times 3 \times 9$ $= 135+16 = (151)$ ✓✓	c(1,3) 16	c(4,4) 0	

i	j	i-1	k	k+1	$m_{i-1} \times m_k \times m_j$	c(i,k)	c(k+1,j)	Minimum
1	4	0	1	2	$m_0 \times m_1 \times m_4$ $1 \times 5 \times 9$ $= 45 + 151$	c(1,1) 0	c(2,4) 151	
1	4	0	2	3	$m_0 \times m_2 \times m_4$ $1 \times 2 \times 9$ $= 18 + 10 + 54 =$	c(1,2) 10	c(3,4) 54	43
1	4	0	3	4	$m_0 \times m_3 \times m_4$ $1 \times 3 \times 9$ $= 27 + 16 =$ (43)	c(1,3) 16	c(4,4) 0	

	1	2	3	4
1	(0)	(10)	(16)	(43)
2	x	(0)	30	151
3	x	x	(0)	54
4	x	x	x	(0)

Answer is 10