

CSE317 DESIGN & ANALYSIS OF ALGORITHMS

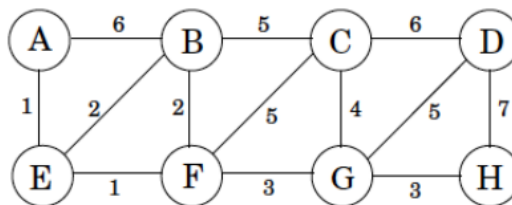
Second Term Examination – Spring'16

Max Marks: 20

Time Allowed: 90 minutes

Attempt all questions. Be to the point. Show your work.

1. (a) [2 marks] Give a counter example that shows that Dijkstra's algorithm may not work for a weighted connected graph with negative weights.
- (b) [3 marks] Explain what adjustments if any need to be made in Dijkstra's algorithm and/or in an underlying graph to solve the following problems.
 - i. Find a shortest path between two given vertices of a weighted graph or digraph. (This variation is called the single-pair shortest-path problem.)
 - ii. Find the shortest paths to a given vertex from each other vertex of a weighted graph or digraph. (This variation is called the single-destination shortest-paths problem.)
2. Consider the following graph.



- (a) [3 marks] Run Kruskal's algorithm on this graph. In what order are the edges added to the MST? For each edge in this sequence, give a cut that justifies its addition.
- (b) [1 mark] How many minimum spanning trees does the graph have altogether?
3. [3 marks] Suppose that a graph has distinct edge weights. Does its lightest edge have to belong to the MST? Can its heaviest edge belong to the MST? Does a min-weight edge on every cycle have to belong to the MST? Prove your answer to each question or give a counterexample.
4. [3 marks] Let $T(n)$ be the running time of an algorithm on input size n defined by the recurrence,

$$T(n) = 3T(n/3) + cn,$$

where c is some constant. Solve the recurrence to find closed form solution for $T(n)$.

5. [5 marks] You are a visitor at a political convention (or perhaps a faculty meeting) with n delegates; each delegate is a member of exactly one political party. It is impossible to tell which political party any delegate belongs to; in particular, you will be summarily ejected from the convention if you ask. However, you can determine whether any pair of delegates belong to the same party or not simply by introducing them to each other – members of the same party always greet each other with smiles and friendly handshakes; members of different parties always greet each other with angry stares and insults.

Suppose more than half of the delegates belong to the same political party. Describe and analyze an $O(n \log n)$ algorithm that identifies all members of this majority party.