CSE317 Design & Analysis of Algorithms

First Term Examination

Max Marks: 20 Time Allowed: 1½ hours

Answer all questions. Use seperate answer sheet. Please give clear and rigorous answers. Be to the point. Show your work.

Name: ____________________ **ERP:** __________

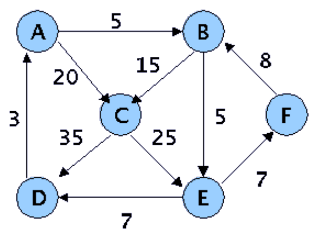
Question 1: .... *5 marks*

(a) [3 marks] Clearly indicate the following trees in the weighted graph pictured below. Some of these subproblems have more than one correct answer. (You do not need to show the intermediate steps, just give the required trees)

i. A DFS tree rooted at A

ii. A BFS tree rooted at A

iii. A shortest-path tree rooted at A



(b) [1 mark] Consider the two standard representations of directed graphs: the adjacency-list representation and the adjacency-matrix representation. Find a problem that can be solved more efficiently in the adjacency-list representation than in the adjacency-matrix representation, and another problem that can be solved more efficiently in the adjacency-matrix representation than in the adjacency-list representation.

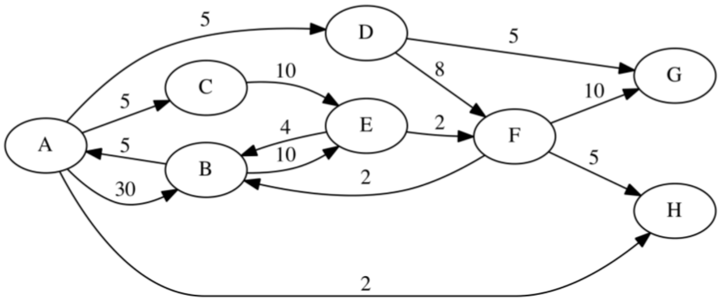
(c) [1 mark] Prove or disprove (by giving a counter-example) the following claim: If a directed graph G contains a path from a vertex u to a vertex v, then any depth-first search must result in $pre(v) < post(u)$.

Question 2: .... *3 marks*

Suppose a CS curriculum consists of n courses, all of them mandatory. The pre-requisite graph $G(V, E)$ with $|V| = n$ has a vertex for each course, and a directed edge from course v to course w if and only if v is a pre-requisite for w. Give an algorithm that computes the minimum number of semesters necessary to complete the curriculum. You may assume that a student can take any number of courses in one semester. The running time of your algorithm should be $O(|V| + |E|)$. Assume adjacency list representation of the graph. Assume that $G(V, E)$ does not have a directed cycle.

Question 3: .... *4 marks*

Consider the directed weighted graph below.



(a) [1½ marks] Complete the table of `prev[]` and `dist[]` values immediately after the first five vertices (`A, H, C, D, G`) have been relaxed during the execution of Dijkstra's algorithm. Some values have already been provided for you.

v	A	B	C	D	E	F	G	H
prev[v]	nil		A	A			D	A
dist[v]	0		5	5			10	2

(b) [½ mark] What vertex will be relaxed next by Dijkstra's algorithm?

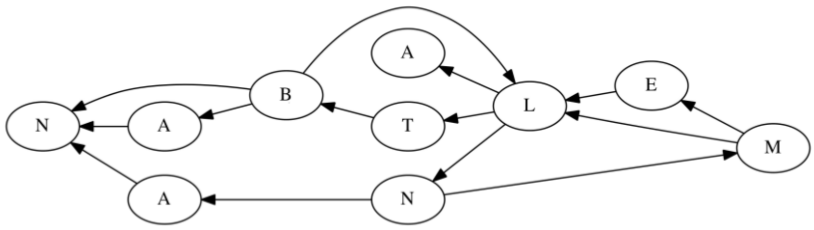
(c) [1 mark] Fill in the table of `prev[]` and `dist[]` values after the 6th vertex you listed in part b is relaxed – you are only required to list any values that have changed since part a (i.e. leave rows blank which did not change).

(d) [1 mark] A modified version of Dijkstra's algorithm with two additional lines of code is shown below (last two lines). Given a graph G for which Dijkstra's algorithm returns a correct result, will this version of Dijkstra's algorithm always return the correct result G? Give an intuitive reason for your answer (you do not need to provide a full proof).

function DIJKSTRA(G, s)
  for all $u \in V$ **do**
    $dist[u] \leftarrow \infty$, $prev[u] \leftarrow nil$
  $dist[s] \leftarrow 0$
  $H \leftarrow MakeQueue(V)$ dist-values as keys
  while H is not empty **do**
    $u \leftarrow ExtractMin(H)$
    for all $(u, v) \in E$ **do** // relax outgoing edges of u
      $relax((u, v))$
    for all $(w, v) \in E$ **do** // relax all edges of G
      $relax((w, v))$

Question 4: .... *4 marks*

(a) [2 marks] Run depth-first search on the digraph below, starting at vertex M. Assume the adjacency lists are in sorted order: for example, when exploring vertex L, consider the edge $L \to A$ before $L \to N$.



List the vertices in preorder and postorder.

(b) [2 marks] Consider two vertices x and y that are simultaneously on the function-call stack at some point during the execution of depth-first search from vertex s in a digraph. Which of the following must be true? Justify your answer.

A. There is both a directed path from s to x and a directed path from s to y.

B. If there is no directed path from x to y, then there is a directed path from y to x.

C. There is both a directed path from x to y and a directed path from y to x.

Question 5: .... *4 marks*

Consider the problem of converting currencies modelled by a directed graph $G = (V, E)$ with $|V|$ vertices representing currencies and $|E|$ directed edges (u, v) each of which has a strictly positive weight $w(u, v) > 0$ representing the exchange rate. For instance, for any real number x, we have $\$x = w(usd, pkr) \cdot x$ Rupees. Our goal is, given a pair of currencies $s, t \in V$, to find the least expensive way of exchanging from s to t, possibly by using more than one exchange.

(a) [2 marks] How could you transform the graph by reweighting the edges so that the problem could be solved with a shortest path algorithm? Indicate which shortest path algorithm is used.

(b) [2 marks] How would you deal with negative-weight cycles if they occurred in the transformed graph? Give the perspective of the currency trader as well as that of a computer scientist.