PSET-4

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1 Q1

2 Q2

We will use st-min cut to solve this problem. Lets first define our graph.
We will have two types of nodes (excluding the source and sink node): Concept node (i) and Problem node (P_j).
The capacity of the edges from the source node to the concept node is C_i which represents the cost to learn a concept. So, from the source node we have edges to all the concept nodes.
If a problem P_j requires a concept, there will be an edge from that concept node to that problem node. The capacity of this edge will be infinity. Why? Once a concept is learned (i.e flow has passed through this node), there shouldn't be an additional cost or "flow" to use the learned concept on a problem. As an example, in the question $P_1 = \{2, 3, 5\}$ so we will have edges from concept node 2, 3 and 5 to problem node P_1.

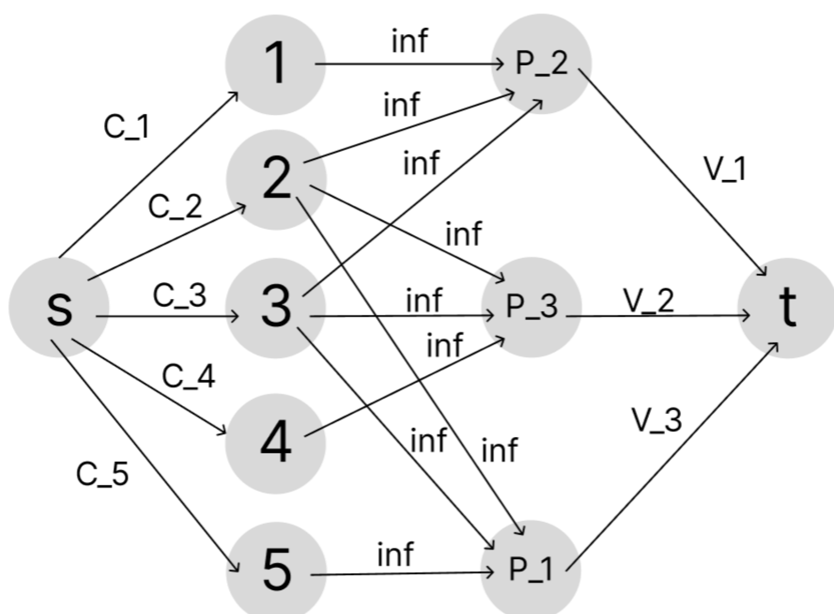

Figure 1: Graph corresponding to example given in question

Each problem nodes P_j will have edges to the sink node. The capacity of these edges is V_j i.e the value of the corresponding problem.
Now we will use st min cut on this graph and our graph will be divided into two disjoint subsets A and B where one subset contains the source node s and the other contains the sink node t. If we have a concept node in the subset which

contains the sink node t, then the capacity of the edge from s to that concept node (C_i)(which crosses the cut) will contribute to the cut value. Cutting an edge here means we are choosing to not learn a concept.
If we have a problem node in the subset which contains the source node s, then the capacity of the edge from P_j to t (V_j) will contribute to the cut value. Cutting an edge here means you are not solving a problem (losing value).
The cut represents the minimum capacity that must be removed from the graph to disconnect it so this cut gives us the minimal cost of learning concepts and the minimal loss of not solving problems.