CSE317 Design & Analysis of Algorithms

First Term Examination

Max Marks: 25 Time Allowed: 1 hour

Answer all questions. Use seperate answer sheet. Please give clear and rigorous answers. Be to the point. Show your work.

Name: _____ **ERP:** _____

Question 1: Stable Matchings ..... *5 marks*

Suppose preferences are given by the following tables:

Men			
Xavier	Beth	Cara	Amy
Yancey	Amy	Cara	Beth
Zeus	Beth	Amy	Cara

Women			
Amy	Zeus	Yancey	Xavier
Beth	Xavier	Yancey	Zeus
Cara	Xavier	Yancey	Zeus

(a) Find a stable matching using the Gale-Shapley algorithm as discussed in class.

(b) Find a stable matching using a variant of Gale-Shapley algorithm with role of men and women reversed, i.e., women making proposals instead of men.

Question 2: ..... *4 marks*

For each of the following functions f find a simple function g such that $f(n) = \Theta(g(n))$. Briefly justify your answer.

(a) $f_1(n) = n + 1$

(b) $f_2(n) = 1 + 1/n$

(c) $f_3(n) = 2n^3 - 15n^2 + n$

(d) $f_4(n) = n^{100}/2^n$

Question 3: ..... *4 marks*

Use the informal definitions of O, Θ, and Ω to determine whether the following assertions are true or false. Briefly justify your answer.

(a) $n(n+1)/2 \in O(n^3)$

(b) $n(n+1)/2 \in O(n^2)$

(c) $n(n+1)/2 \in \Theta(n^3)$

(d) $n(n+1)/2 \in \Omega(n)$

Question 4: Graphs ..... *4 marks*

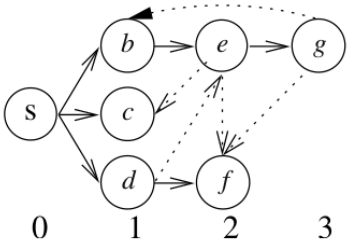
(a) [1 mark] What is the relationship between the sum of the degrees of all vertices and the number of edges of graph $G = (V, E)$?

(b) [1 mark] Show that the number of odd-degree vertices in simple graphs is even.

(c) [2 marks] What is the maximum number of edges in a directed graph with n vertices and no parallel edges? What is the minimum number of edges in a directed graph with n vertices, none of which are isolated?

Question 5: Breadth-First Search ..... *4 marks*

(a) [2 marks] BFS trees are not necessarily unique. Give the BFS tree that is produced when d is removed before b when one performs a BFS from node s in the following graph.

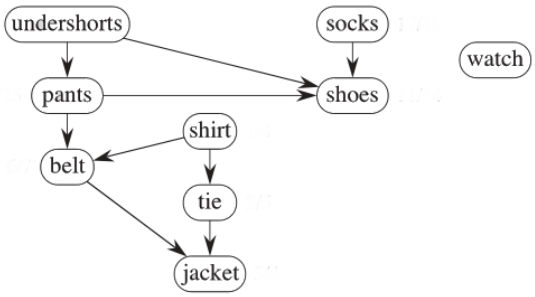


(b) [1 mark] What does the BFS tree tell us about the distance from v to w when neither is at the root?

(c) [1 mark] Suppose you use a stack instead of a queue when running breadth-first search. Does it still compute shortest paths?

Question 6: Topological Sort ..... *4 marks*

(a) [2 marks] Professor Bumstead topologically sorts his clothing when getting dressed. Each directed edge (u, v) in the following graph means that garment u must be put on before garment v. Help the Prof by finding a sequence of garments in which he should wear them while satifying all dependencies.



(b) [2 marks] *Topological sort and BFS.* Explain why the following algorithm does not necessarily produce a topological order: Run BFS, and label the vertices by increasing distance to their respective source.