# CSE 317 *Design and Analysis of Algorithms*

## Problem Set 4

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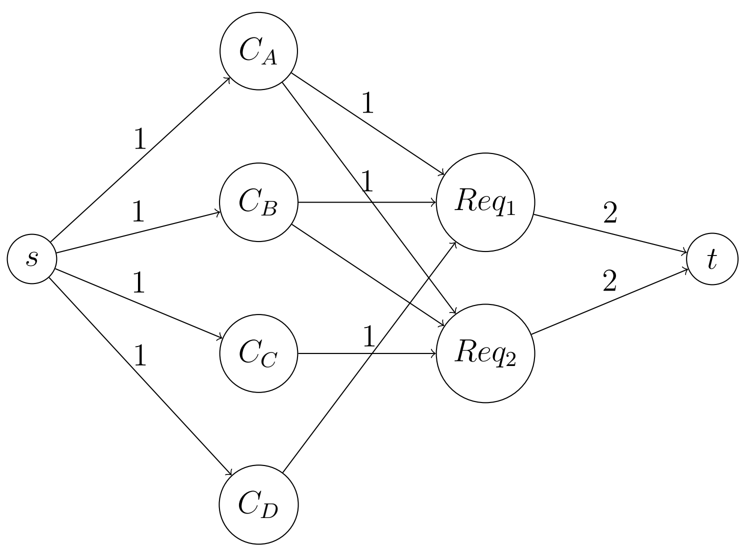
# Question 1

Now, we want to have courses. We will then have a list of courses that a student can choose from. The path from source node $S$ to the courses is of cost 1. Then, we have another list of requirements that a student needs to fulfill to be eligible to graduate. The cost from courses to requirement is again 1. And the cost from requirement to sink node t is of cost $k_i$.

We will now apply the Edmonds-Karp Algorithm on this graph, and then find the max flow. After some trivial calculations we find that the max flow is 4. Now to see if it holds for use cases. If a student has a requirement, and the courses are overlapping, the path to requirement node will never reach as the capacity for the overlapped course will be full. Hence the reduction holds.




- $S$ is the source node.
- $C_1, C_2, \ldots, C_n$ are the courses taken by the student.
- $Req_1, Req_2, \ldots, Req_m$ are the nodes representing requirements.
- $T$ is the sink node.

I have made an example graph above, where there are two requirements, $req_1$ and $req_2$, and four courses, $c_A$, $c_B$, $c_C$, and $c_D$.

Requirement $req_1$ requires $req_1$ courses to be taken from the set $\{c_A, c_B, c_D\}$, and requirement $req_2$ requires $req_2$ courses to be taken from the set $\{c_A, c_B, c_C\}$.

## Question 2

# Question 3

So for this question i will follow the approach done in class.
Original Linear Program:

$$\text{Max } = x_1 + 3x_2 - 2x_3$$

We will now number these 1,2,3

$$\begin{aligned} x_1 + x_2 + 2x_3 &\leq 2 && \text{(Eq. 1)} \\ 7x_1 + 2x_2 + 5x_3 &\leq 6 && \text{(Eq. 2)} \\ 2x_1 + x_2 - x_3 &\leq 1 && \text{(Eq. 3)} \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Now we will have $y_1$, $y_2$, $y_3$. We will now multiply them by the corresponding equation number: $y_1$ multiplied by $x_1 + x_2 + 2x_3 \leq 2$.

These $y_i$ must be greater than 0 to ensure the sign of the inequality doesn't flip.

From this we get $(y_1 + 7y_2 + 2y_3)x_1 + (y_1 + 2y_2 + y_3)x_2 + (2y_1 + 5y_2 - y_3)x_3$ ( we want this side to be equal to the original LP objective function )

Dual Linear Program:

$$\text{Minimize } = 2y_1 + 6y_2 + y_3$$

This will give us :

$$\begin{aligned} y_1 + 7y_2 + 2y_3 &\geq 1 \\ y_1 + 2y_2 + y_3 &\geq 3 \\ 2y_1 + 5y_2 - y_3 &\geq -2 \\ y_1, y_2, y_3 &\geq 0 \end{aligned}$$

Similarly we will solve the second Linear program in the same manner.
Original Linear Program:

$$\text{Max } = x_1 - 3x_2 + 2x_3$$

$$\begin{aligned}
-3x_1 - 2x_3 &\leq -2 && \text{(Eq. 1)} \\
2x_2 - x_3 &\leq 5 && \text{(Eq. 2)} \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

Now we will have $y_1$, $y_2$. We will now multiply them by the corresponding equation number: $y_1$ multiplied by $-3x_1 - 2x_3 \leq -2$.

These $y_i$ must be greater than 0 to ensure the sign of the inequality doesn't flip.

From this we get $(-3y_1)x_1 + (2y_2)x_2 + (-2y_1 - y_2)x_3$

( we want this side to be equal to the original LP objective function )

Dual Linear Program:

$$\text{Minimize } = -2y_1 + 5y_2$$

This will give us :

$$\begin{aligned}
-3y_1 &\geq 1 \\
2y_2 &\leq -3 \\
-2y_1 - y_2 &\geq 2 \\
y_1, y_2 &\geq 0
\end{aligned}$$

## Question 4

# Acknowledgments

I discussed the problem set questions with the following students:

- Yahya Ahmed -24442
- Fatima Mahmood -24314