[Spring 2023] CSE 317: Design and Analysis of Algorithms [All sections]
Homework 3 | Due: April 21, 2023

Instructions: You need to submit it on `Gradescope`. Details for submission will shared soon on `LMS`. Direct your all queries to the course staff (refer to syllabus for contact information and office hours).

---

1. (10 points) This year IBA plans to conduct orientation sessions for the new students. Imagine you just joined IBA and plan to attend the orientation sessions. The orientation consists of n sessions. Each session i is described by a triplet of integers $\langle S_i, D_i, F_i \rangle$, where S_i indicates the session start time, D_i represents the duration of the session and F_i indicates the session finish time. You would like to attend as many sessions as possible. Note that sessions may be overlapping and you can't attend a session after it is already started and you can't leave a session before it finishes. Design an $O(n \log n)$ greedy algorithm that will allow you to do so and prove that your algorithm is correct.

2. (10 points) Let $G = (V, E)$ be a connected undirected graph with nonnegative costs of edges and $e \in E$ be an edge with the minimum cost. Prove that *each minimum spanning tree T of G contains the edge e*.

3. (10 points) We are given n ropes of different lengths $l_1, l_2, \ldots l_n$. We want to tie these ropes into a single rope. The cost to connect two ropes is equal to the sum of their lengths. We want to connect all the ropes at minimum cost.

   For example, if we have four ropes of lengths $4, 3, 2$, and 6. We can connect the ropes in the following ways: first connect ropes of length 2 and 3 (cost is 5). Now we have three ropes of lengths $4, 5$, and 6. Then connect the ropes of length 4 and 5 (cost is 9). Now we have two ropes of lengths 9 and 6. Finally, connect the two ropes (cost is 15). The total cost is $29 = 5 + 9 + 15$.

   Find an efficient *greedy algorithm* for this problem and prove that your solution is correct. Also work-out the worst case running time of your algorithm.

4. (10 points) Given an alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{d}, \texttt{k}, \texttt{r}\}$. Construct a *prefix-free code* for the following string: "`abra ka dabra`" (ignore all whitespace and quotes). [Refer to the textbook for the definition of a prefix-free code.]

5. (10 points) Find the *shortest path* to all vertices in the following directed graph from the source vertex a. Show the length of the shortest path and predecessor information on the vertices as well.