CSE317 Design & and Analysis of Algorithms

Second Term Examination – Fall'15

Max Marks: 33 Time Allowed: 90 minutes

Attempt all questions. Be to the point. Show your work.

1. [9 marks] For each of the following statements, decide whether it is true or false. Give brief justification of your answer.

   (a) If a DFS of a directed graph contains a back edge, any other DFS of the same graph will also contain at least one back edge.

   (b) A DFS of a directed graph always produces the same number of tree edges, i.e., independent of the order in which vertices are considered for DFS.

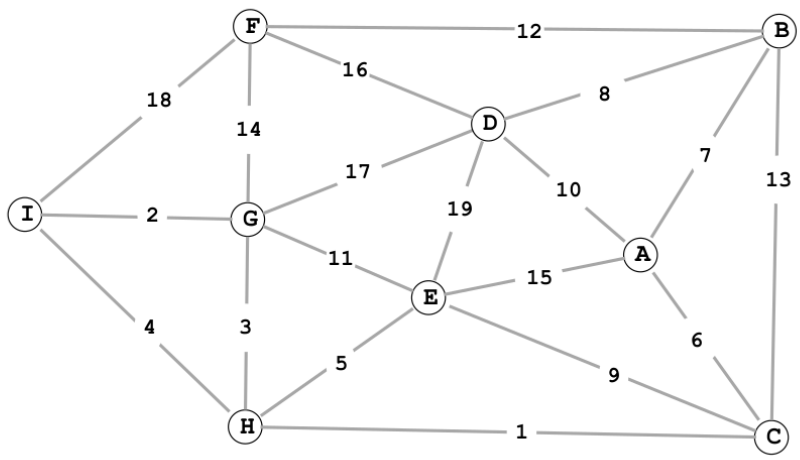
   (c) If the DFS finishing time $f[u] > f[v]$ for two vertices u and v in a directed graph G, and u and v are in the same DFS tree in the DFS forest, then u is an ancestor of v in the depth first tree.

   (d) An edge with maximum weight in a graph cannot be part of its MST.

   (e) A shortest path in a graph remains shortest after incrementing weight of each edge by 1.

   (f) Given a connected graph $G = (V, E)$, if a vertex v is visited during level k of a breadth-first search from source vertex s, then every path from s to v has length at most k.

2. Consider the following weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.

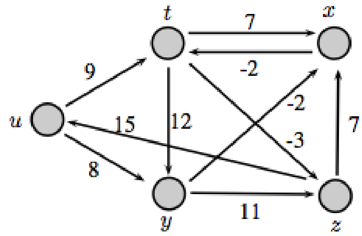


   (a) [2 marks] Give the sequence of edges in the MST in the order that Kruskal's algorithm includes them.

   (b) [2 marks] Suppose that the edge D-I of weight w is added to the graph. For which values of w is the edge D-I in a MST?

   (c) [2 marks] Explain why the cut property implies the correctness of Kruskal's algorithm.

3. Suppose Bellman-Ford's algorithm is run on the following graph, starting at node u.



   (a) [4 marks] Draw a table showing the intermediate distance values of all the nodes at each iteration of the algorithm.

   (b) [2 marks] Show the final shortest-path tree.

4. Your job is to arrange n ill-behaved children in a straight line, facing front. You are given a list of m statements of the form "i hates j". If i hates j, then you do not want put i somewhere behind j, because then i is capable of throwing something at j.

   (a) [3 marks] Give an algorithm that orders the line, (or says that it is not possible) in $O(m + n)$ time.

   (b) [3 marks] Suppose instead you want to arrange the children in rows such that if i hates j, then i must be in a lower numbered row than j. Give an efficient algorithm to find the minimum number of rows needed, if it is possible.

5. For a list of intervals representing start and finish time of n lectures, recall the problem of finding a schedule of lectures into minimum number of classrooms.

   (a) [2 marks] *Lower bound.* Describe a good lower bound on the minimum number of class rooms needed by any schedule.

   (b) [2 marks] *Optimal greedy algorithm.* Give an optimal greedy algorithm, i.e., your algorithm should return a schedule that use minimum possible number of classrooms. Argue why your algorithm always return an optimal solution.

   (c) [2 marks] *Non-optimal greedy algorithm.* Give another greedy strategy that is not optimal and prove its non-optimality by giving a counter-example.