Vector calculus

Problem set (solve analytically and numerically)

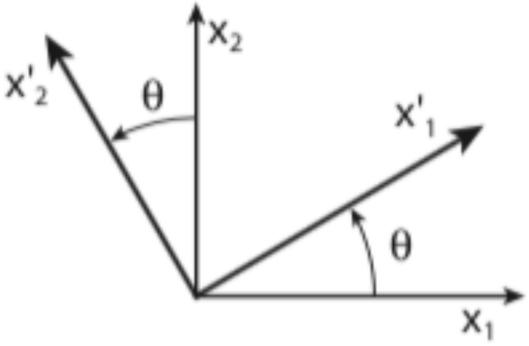
1. The temperature field at any point in a lake is given by $T(x,y,z) = \sin(yz) + \log(1+x^2)$ for all x,y,z. Plot this scalar field at the lake surface ($z=0$) and at 1 unit below the surface (z=1).

2. Compute the gradient of $T(x,y,z)$. Plot this vector field at the lake surface.

3. At the point $(1,1,0)$ on the lake surface, a boat is moving in the $<3,1,0>$ direction. First calculate the unit vector for the boat's motion, and then compute the directional directive of the temperature field as seen by the boat. Hint: directional derivatives are projections of the gradient vector along some unit vector.

Rotating vectors

Coordinate transformations



$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Derive the individual components of the rotation matrix using the dot product of the rotated and original coordinate systems

How do we rotate tensors?

Rotating vectors

Coordinate transformations

1. For a fault with dip = 30º, calculate the unit vector in the dip direction (d) and in the direction normal to the plane (n).
2. This fault exists in a stress field given by a symmetric stress tensor [s22,s23,s33]. Compute the traction vector on this fault plane. Hint: traction vectors are the projection of the stress tensor onto a given plane. And a plane is uniquely described by the vector normal to it i.e. (n)
3. Project this computed traction vector onto the dip direction (d) and the fault-normal direction (n). These are the shear-traction and normal-traction respectively. Compare these expressions with the components you would have obtained simply by rotating the stress tensor by the dip angle. Hint: [S'] = [R][S][R']

Assignment

1. For the provided polygons and velocity vectors in a spherical coordinate system, first project them onto a cartesian coordinate system. Plot the original and transformed data.
2. The data represent the motion of the India-Eurasia collision as seen in the GPS-derived velocity field. Rotate the data and the polygons by 20º to a fault-based coordinate system.
3. How would you estimate the motion of all the sites relative to an India-fixed coordinate system?