Last 3 weeks

- Describing motion $\longrightarrow$ $\underline{x}, \underline{u}, \dfrac{d\underline{u}}{dt}$ spatial ref; $\underline{X}, \underline{U}, \dfrac{d\underline{U}}{dt}$ material ref — same?

- Describing deformation $(F, \nabla \underline{u})$ $\longrightarrow$ Rotation; Strain $\longrightarrow$ Isotropic ($\nabla \cdot \underline{u} \neq 0$), Deviatoric ($\nabla \cdot \underline{u} = 0$)

- Forces — source of deformation — sign conventions

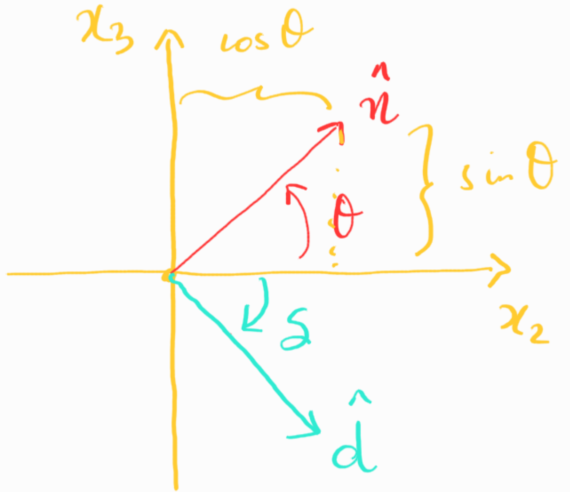
$\boxed{\text{Mohr's Circle}}$ — Graphical representation of the stress tensor

- very intuitive in 2-d.

$$[\sigma] = \begin{bmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{bmatrix} ; \quad \hat{n} = \begin{bmatrix} n_2 \\ n_3 \end{bmatrix}$$

traction vector

$$\underline{t} = \begin{bmatrix} \underline{\sigma_{22} n_2 + \sigma_{23} n_3} \\ \underline{\sigma_{23} n_2 + \sigma_{33} n_3} \end{bmatrix}$$



$$\boxed{\sigma_n} \begin{cases} \sigma_{22}' = \sigma_{22}\left[\cos^2\theta\right] + \sigma_{33}\left[\sin^2\theta\right] + 2\,\sigma_{23}\left[\cos\theta \sin\theta\right] \\ \sigma' = -(\sigma_{22} - \sigma_{33})\left[\sin\theta \cos\theta\right] + \sigma_{23}\left[\cos^2\theta - \sin^2\theta\right] \end{cases}$$

$\boxed{\sigma_s}$ (σ_{23}

You now know $\underline{\sigma_n}$ & $\underline{\sigma_s}$ as scalars, but if you wanted to describe them as vectors,

$$\underline{\sigma_s} = |\sigma_s|\,\hat{d}, \qquad \underline{\sigma_n} = (\overbrace{\underbrace{\hat{n}\cdot\sigma}_{\text{traction vector}}\cdot\hat{n}}^{\text{scalar}})\,\hat{n}$$

$$\underline{\sigma_s} = \hat{n}\cdot\sigma - \underline{\sigma_n}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

$$\sigma_n \begin{cases} \sigma_{22}' = \left(\dfrac{\sigma_{22}+\sigma_{33}}{2}\right) + \left(\dfrac{\sigma_{22}-\sigma_{33}}{2}\right)\cos 2\theta + \sigma_{23}\sin 2\theta \end{cases}$$

$$\sigma_s \begin{cases} \sigma_{23}' = -\left(\dfrac{\sigma_{22}-\sigma_{33}}{2}\right)\sin 2\theta + \sigma_{23}\cos 2\theta \end{cases}$$

$$\sigma_m = \frac{\sigma_{22}+\sigma_{33}}{2}$$

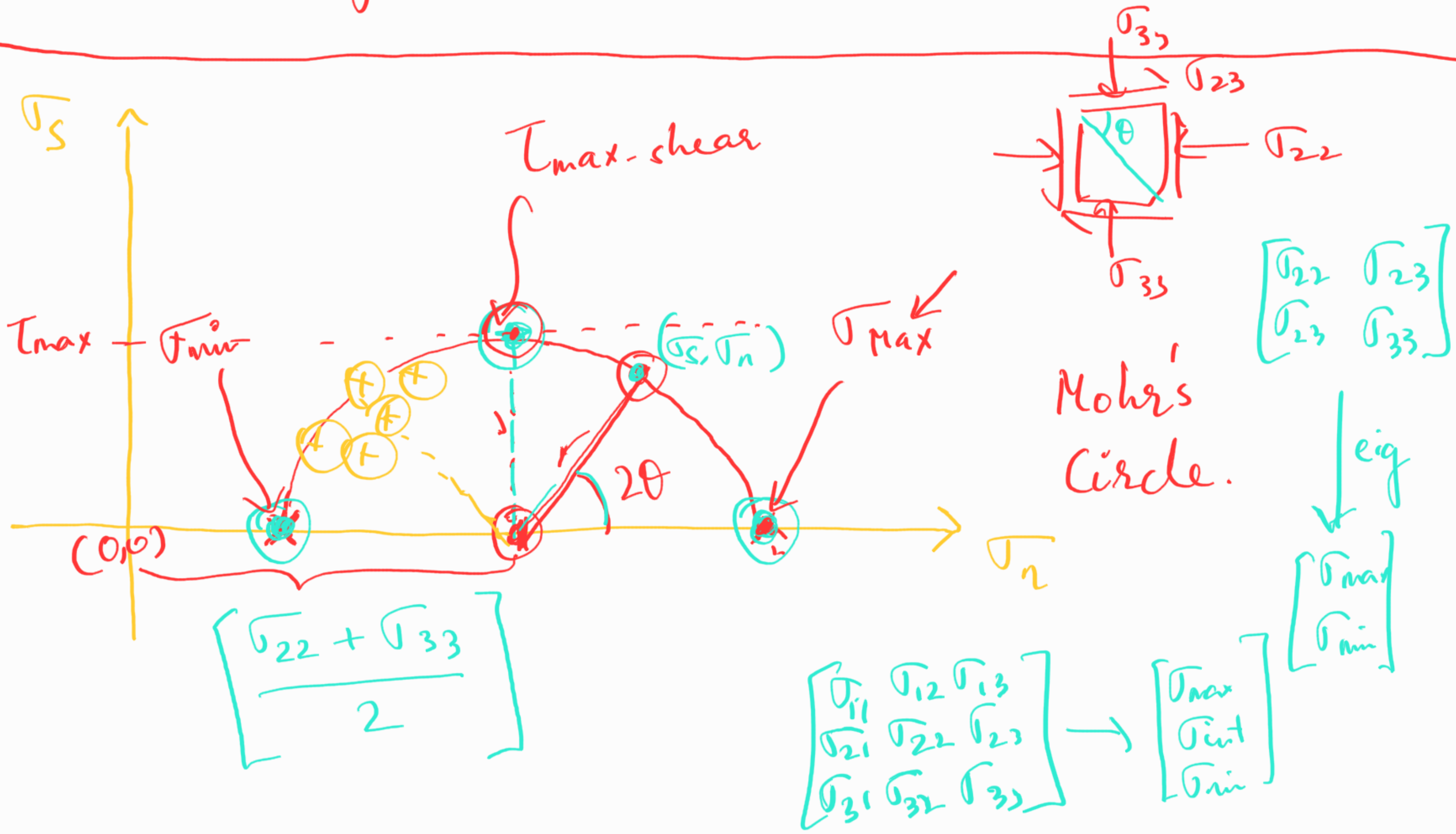
$$(\sigma_{22}' - \sigma_m) = \frac{\sigma_{22}-\sigma_{33}}{2}\cos 2\theta + \sigma_{23}\sin 2\theta$$

$$\sigma_{23}' = -\left(\frac{\sigma_{22}-\sigma_{33}}{2}\right)\sin 2\theta + \sigma_{23}\cos 2\theta$$

$$\boxed{x^2 + y^2 = r^2} \leftarrow \text{equation of a circle}$$

$$\overbrace{\left[\underline{\sigma_n} - \left(\frac{\sigma_{22}+\sigma_{33}}{2}\right)\right]^2}^{\sigma_{22}' \quad x^2} + \overbrace{\underline{\sigma_s}^2}^{\sigma_{23}'} = \overbrace{\left[\left(\frac{\sigma_{22}-\sigma_{33}}{2}\right)^2 + \sigma_{23}^2\right]}^{r^2}$$

origin $\quad y^2 \quad$ radius



Homework Assignment

- We can use fault slip vectors (observations) to infer the stress orientations at a given location & a given time interval.

- The main assumption is called the Wallace-Bott hypothesis i.e the fault slip vector is parallel to the shear traction vector.

$$\hat{s} = \frac{\sigma_s}{|\sigma_s|} \longleftarrow \left(\underbrace{\hat{n}\cdot\sigma}_{\text{traction}} - \underbrace{(\hat{n}\cdot\sigma\cdot\hat{n})}_{\text{normal traction vector}}^{\sigma_n}\,\hat{n}\right)$$

$$\underline{\sigma_s} + \underline{\sigma_n} = \underline{t}$$

– 2-d case

- Derive a linear relationship b/w $\hat{r} = \begin{bmatrix} r_x \\ r_z \end{bmatrix}$ & a deviatoric stress tensor

$$\sigma^{dev} = \begin{bmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & -\sigma_{22} \end{bmatrix}$$

$$\begin{bmatrix} r_x \\ r_z \end{bmatrix} = \begin{bmatrix} f(n_x, n_z) \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \end{bmatrix} \qquad d = G\,m$$

- Using this linear relationship, and the fault slip vector observations provided to you, estimate the average stress tensor orientation.

  Hint: $[r] = G(\underline{n_x}, \underline{n_z}) \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \end{bmatrix}$ can be solved in MATLAB using $[\underline{\sigma}] = G \backslash [r]$.

  The uncertainties in $\underline{\sigma}$ is captured by the covariance matrix $C_\sigma = inv(G^T G)$

- Use the estimated $\underline{\sigma}$ & covariance C_σ to estimate the uncertainties in the orientation of σ_{max} & σ_{min}. σ_{33} $[pv, ev] = eig(\sigma)$

  Hint: from $(\sigma_{22}, \sigma_{23}, -\sigma_{22})$, you can estimate the principal stresses $\begin{bmatrix} \sigma_{max} \\ \sigma_{min} \end{bmatrix}$ & their orientations. Use 'mvnrnd$(\sigma, C_\sigma, N_{samples})$'

to sample from the posterior distribution.

– 3-d case

- Derive a linear relationship b/w $\hat{r}$ & σ^{dev}, given the fault plane ($\hat{n}$).

$$\overset{(\phi,\delta,\lambda)}{3\left\{\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}\right.} = \begin{bmatrix} f(n_x, n_y, n_z) \\ \uparrow \\ (\phi, \delta) \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix}$$

$\sigma_{11}+\sigma_{22}+\sigma_{33}=0$

$\sigma_{33} \rightarrow -(\sigma_{11}+\sigma_{22})$

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & -(\sigma_{11}+\sigma_{22}) \end{bmatrix} \xrightarrow{eig} [\text{eigenvectors}] \rightarrow \begin{bmatrix} \sigma_{max} \\ \sigma_{int} \\ \sigma_{min} \end{bmatrix}$$

$$[\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{13}, \sigma_{23}] \rightarrow \begin{bmatrix} \sigma_{max} \\ \sigma_{int} \\ \sigma_{min} \end{bmatrix}$$

- Comment on the uniqueness issues in this problem.

- For a given set of (ϕ, δ, λ) from fault slip vectors, estimate the orientation of $[\sigma_{max}, \sigma_{int}, \sigma_{min}]$. (Extract events from your AOI)

- Read the GCMT datafile using the script provided. Use both nodal planes for each earthquake as inputs to constructing $[f(n_x^{(1)}, n_y^{(1)}, n_z^{(1)})]$. You will now have a

$\left[\left(n_x^{(2)}, n_y^{(2)}, n_z^{(2)}\right) \right]$

$[n]$ vector that is $3 \times 2 \times N$earthquakes.

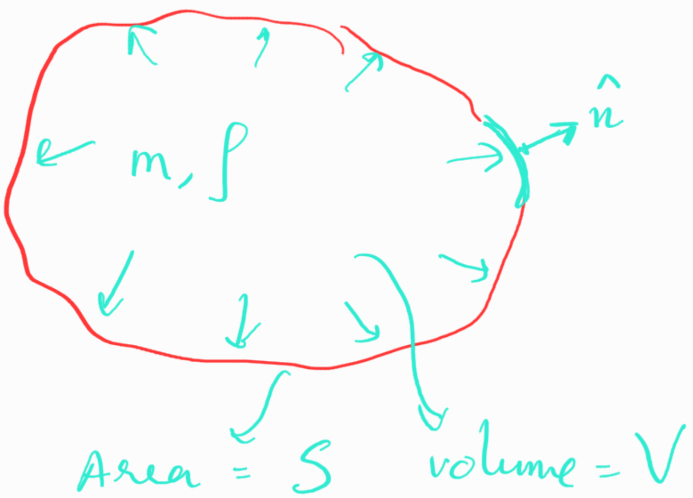
Conservation Laws

(1) Conservation of Mass

(2) Conservation of linear momentum

(3) Conservation of angular momentum

Mass Conservation



— consider control volume

Mass flux : (flow of material)