## Recap

- vectors, tensors
- calculus (derivatives & integrals)
- kinematics. ($F$, $\nabla u$, $\varepsilon$, $\omega$)

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## Quick exercise (vectors)

# You are given a fault plane's orientation in spherical coordinates (strike $= \phi^\circ$, dip $= \delta^\circ$), calculate the unit vectors $\hat{s}, \hat{d}, \hat{n}$

$\hat{s}$ – unit vector in the strike direction

$\hat{d}$ – unit vector in the dip direction

$\hat{n}$ – unit vector normal to the plane

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# You have made observations of $(\phi, \delta)$ from bedding planes in a foldbelt. Calculate the average $(\hat{\phi}, \hat{\delta})$

$[\phi] = [12, 8, 25, 290, 352, 1, 355]$

$[\delta] = [30, 24, 41, 18, 25, 34, 27]$

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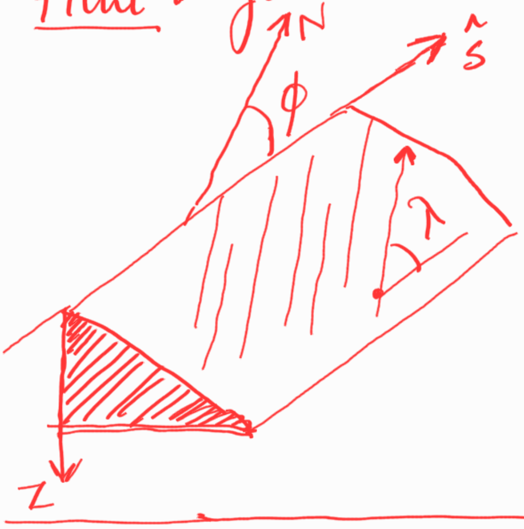
# Along with the $(\phi, \delta)$ observations, you also found

the fault planes. These are oriented

striations on the fault p... at an angle $\lambda$ to the horizontal. Compute the unit vector $\hat{r}$ in the rake direction.

Hint: you can express $\hat{r}$ using $\hat{s}$ & $\hat{d}$.




## Dynamics

**Forces**

→ Body forces

$$F_b = \iiint_V \rho \, \underline{f}(\underline{r}) \, dV$$

$\rho$ - mass density

$\underline{f}(\underline{r})$ - force density

$V$ - volume

→ Surface forces.

$$F_s = \int_S \underline{P}(\underline{r}) \, dS$$

$\underline{P}(\underline{r})$ - pressure/traction

$S$ - surface area

## Conservative & non-conservative forces

- for a conservative vector field $\underline{F}$,

① $\oint \underline{F} \cdot d\underline{r} = 0$ (path-independant work)

(2) $\underline{F} = \nabla \phi$

(3) $\nabla \times \underline{F} = 0$

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**Stress** — Action of a force over an infinitesimal area

$\sigma = \frac{F}{A}$ ; $\underline{F} \longrightarrow$ vector

$[\sigma] \rightarrow$ tensor why?

[think about orientations]

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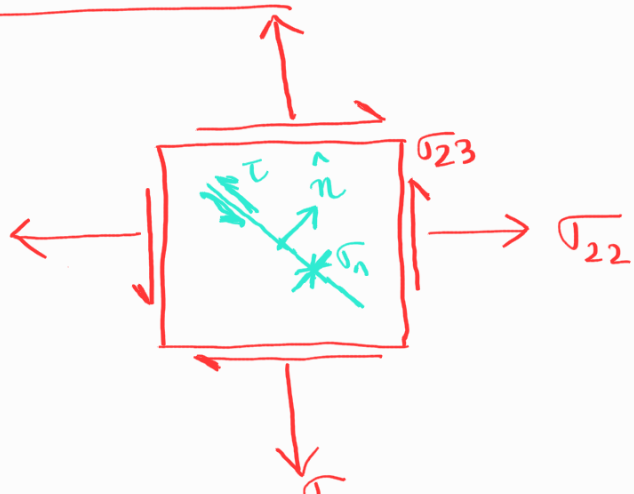
**Surface forces & traction vectors**

$\underline{t} = [\sigma] \cdot \hat{n}$




- stress tensor

& an orientation

$\sigma_{33}$

— vector quantity

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1) Given a 2-d stress tensor

$$\sigma = \begin{bmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{bmatrix}$$ and a boundary

$\hat{n} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, compute the traction vector acting on this boundary.

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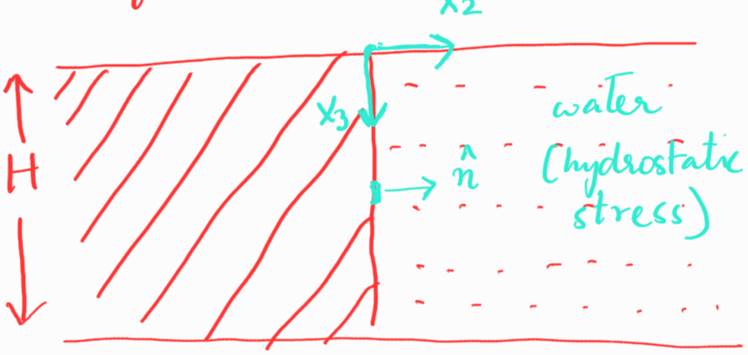
2) Now assuming the plane $\hat{n}$ describes the wall of a water reservoir of height $H$, calculate the total force acting on the wall when the dam is full. Hint: $\underline{F} = \int_0^H \underline{t}\, dA$

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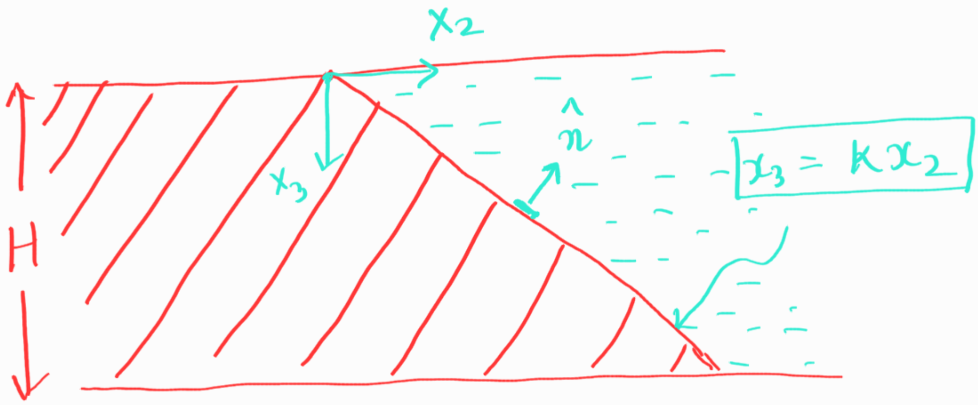
3) If the dam wall was not flat, but instead sloping such that equation of the line describing the boundary is $\boxed{x_3 = K x_2}$, compute the total

the boundary is $\boxed{x_3 = \quad x_2}$, compute the
force on this wall. Why does the direction of the force change compared to the previous problem?
**Hint**: first compute $\hat{n}$ in terms of $k$ & then calculate $\underline{t} = \sigma \cdot \hat{n}$.




[• Reading fault geometry & calculating stress change]

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## Mohr's Circle — Graphical representation of the stress tensor

- very intuitive in 2-d.

$$[\sigma] = \begin{bmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{bmatrix} ; \quad \hat{n} = \begin{bmatrix} n_2 \\ n_3 \end{bmatrix}$$

$$\underline{t} = \begin{bmatrix} \sigma_{22} n_2 + \sigma_{23} n_3 \\ \sigma_{23} n_2 + \sigma_{33} n_3 \end{bmatrix}$$

$$(1)\begin{cases} \sigma_{22}' = \sigma_{22}\cos^2\theta + \sigma_{33}\sin^2\theta + 2\sigma_{23}\sin\theta\cos\theta \\ \sigma_{23}' = -(\sigma_{22}-\sigma_{33})\sin\theta\cos\theta + \sigma_{23}(\cos^2\theta - \sin^2\theta) \end{cases}$$

recall that $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$, $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

& $\sin 2\theta = 2\sin\theta\cos\theta$ —— (2)

substituting (2) in (1), we get

$$\boxed{\begin{aligned} \sigma_{22}' &= \tfrac{1}{2}(\sigma_{22}+\sigma_{33}) + \tfrac{1}{2}(\sigma_{22}-\sigma_{33})\cos 2\theta + \sigma_{23}\sin 2\theta \\ \sigma_{23}' &= -\tfrac{1}{2}(\sigma_{22}-\sigma_{33}\ \sin 2\theta + \sigma_{23}\cos 2\theta \end{aligned}}$$