Week 6 Topics

(1) Unicycle library
- Reading fault geometry
- Computing displacements & stresses in 2-d & 3-d
- Stress transfer b/w 2 faults

(2) Friction on faults
- Coulomb friction (static)
- Dynamic friction coefficient (velocity dependance)
  - linear
  - power-law
  - logarithmic
- Rate-state dependant friction

discuss FD integration

(3) Governing equations for fault-slip in an elastic medium
- $\nabla \cdot \sigma + \underline{f} = 0$ ← static equilibrium

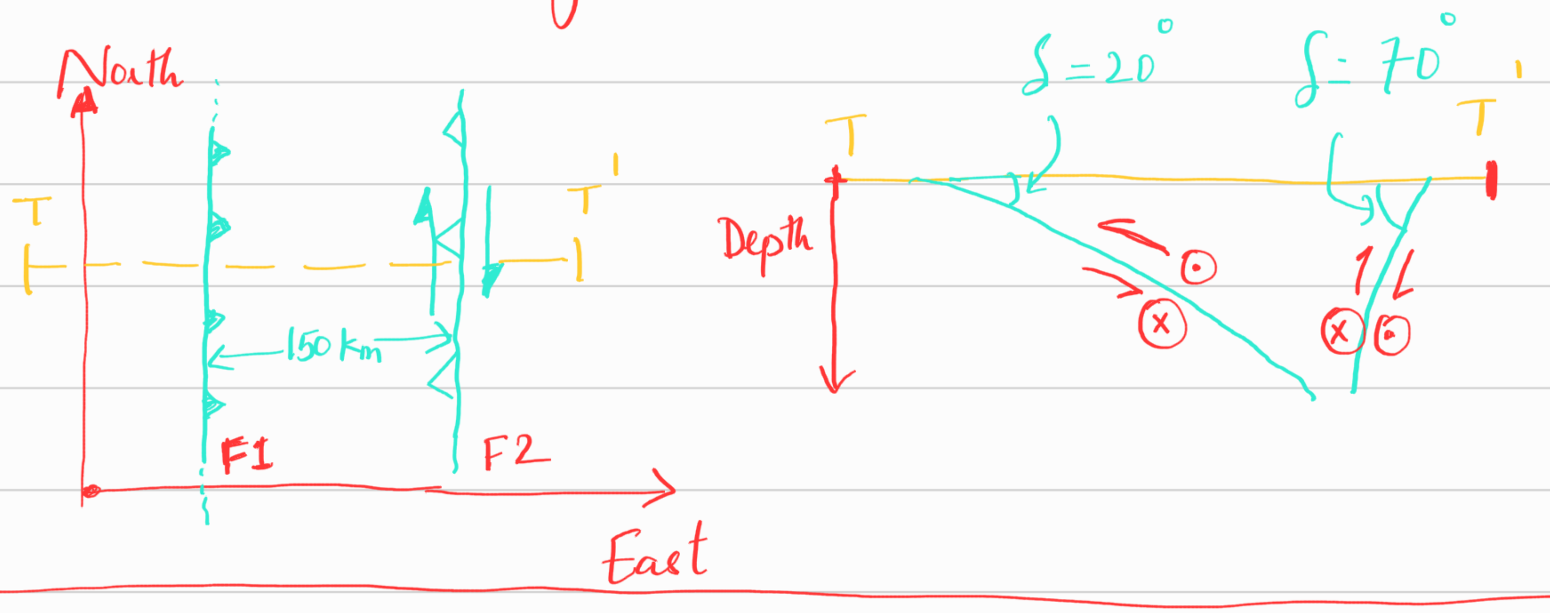
subject to frictional BCs.

taking $\frac{\partial}{\partial t}(\,)$, $\nabla \cdot \dot{\sigma} + \dot{\underline{f}} = 0$ ← quasi-static equilibrium

- solutions using ODE solvers
  - Implicit / Explicit Euler
  - Adams-Bashforth / Adams-Moulton
  - Crank-Nicholson
  - Runge-Kutta

In-class exercise,

You know there are 2 parallel faults, both of whose surface trace are aligned with the longitudes that pass through them.



Each fault is 120 km along-strike & extends to a depth of 20 km. Mesh the surfaces using 10 km × 10 km square patches.

- The fault F1 slipped in a recent earthquake

The slip patch was at the centre of the fault and extended uniformly radially with a radius of 30 km.

$$[S] = \begin{cases} \text{dip-slip} - 2\text{m}, \ \text{strike-slip} - 1\text{m} & r \leq 30 \text{ km} \\ 0 & r > 30 \text{ km} \end{cases}$$

- compute the stress change $\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ & \sigma_{22} & \sigma_{23} \\ & & \sigma_{33} \end{bmatrix}$ at all patch centers for both faults F1 & F2.
- Use the 6 stresses to compute the traction vector $[\underline{t}]$ at each patch.
- compute & plot each traction scalar $(\tau_s, \tau_d, \sigma_n)$ for each patch.
- It appears that fault F2 has a slip-averaged rake angle of 140° (mostly right-lateral strike-slip). Compute the Coulomb stress change on F2 due to an earthquake on F1.

Friction on faults

$\tau_s = f\sigma_n$

If fault-plane is not at failure

σ_3 σ_m σ_1

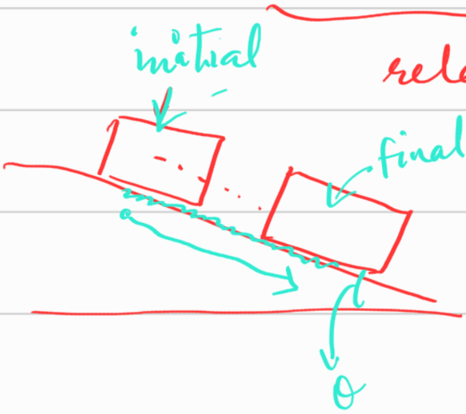
for sliding to occur,

Frictional Stress/strength = Internal stress

- if $\tau_f = f\sigma_n$, & we know earthquakes or frictional slip causes $\Delta\tau \downarrow$, what causes τ_f to change?

→ when sliding started/nucleated,

$\underbrace{\text{Internal elastic stress}}_{\text{released}}$ = Frictional strength



↓

→ to continue sliding, τ_f decreases faster than elastic stress release.

Coulomb friction is only 'static' → insufficient to describe sliding effects.

Dynamic friction

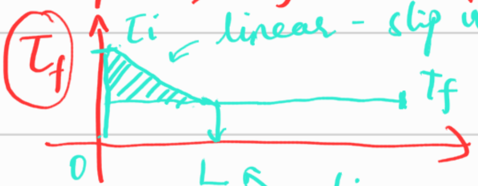
↓ slip-dependant

↳ slip-weakening

$\Delta\tau_f \propto -S$

as slip ↑, τ_f ↓ → positive feedback

linear-slip weakening friction



slip weakening distance

→ $\Delta\tau_f$ due to Δf, minimal $\Delta\sigma_n$ effects

Rate-dependant

↳ velocity-dep friction.

$\boxed{f \propto v \quad \text{linear}}$

$f \propto v^m$ power-law

$f \propto \log v$ log-form

Rate-state dependant friction

$$f = f_0 + \underbrace{a \log\left(\frac{v}{v_0}\right)}_{\substack{\text{velocity-dep} \\ \text{rate-effect}}} + \underbrace{b \log\left(\frac{v_0 \theta}{d_c}\right)}_{\text{related to contact-time}}$$

related to contact-time — another way of slip-dependence → state-effect

$$\frac{d\theta}{dt} = f(v, \theta)$$

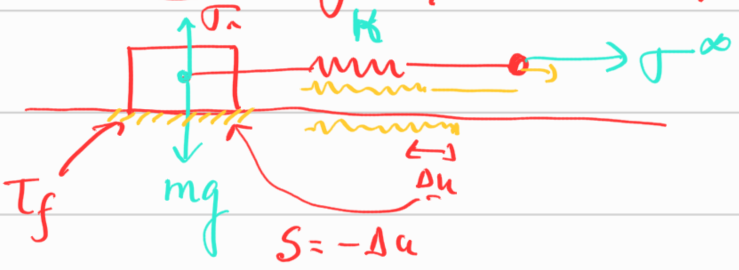
When we did GFs derivations, we solved

$\nabla \cdot \sigma + f = 0$, come from imposed slip on a fault patch.

$GF(x_1, x_2, x_3; y_1, y_2, y_3, \underbrace{\phi, \delta, \lambda}_{\hat{s}, \hat{d}}, L, W)$ ← linear b/w $u_i \propto S_i$

but how much should a fault slip?

Governing equations for fault slip



if block is stationary, and σ^∞ is applied then spring will extend
⇒ elastic/spring stress ↑

→ if the block slides, then spring will contract ⇒ elastic stress ↓

$$\sigma^\infty + \underbrace{K(\Delta u)}_{(-KS)} = f(\sigma_n)$$

this is time-invariant

$\frac{\partial}{\partial t}(\sigma^\infty) = c$ elastic stress

f is v-dependent $\Big\}$ $\frac{\partial}{\partial t}(\;)$

$$\dot{\sigma}^\infty - K v = \frac{d}{dt}(f)\,\sigma_n \quad \text{—— (1)}$$

$v \to$ unknown, 1 differential equation.

$\to$ we need an initial condition $\Rightarrow$ IVP

$f = Av \longrightarrow$ apply in 1,

$\varepsilon_{ij} = \dfrac{\partial u_i}{\partial x_j} \to$ given

you wanted u_i

$$\dot{\sigma}^{\infty} - Kv = A\sigma_n \frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} + \frac{K}{A\sigma_n} v = \dot{\sigma}^{\infty}}$$

$\longrightarrow$ subject to $v = v_i$ at $t = 0$.

$\underbrace{v = v_i \text{ at } t=0}_{IC}$

$$\frac{dv}{dt} = \left(\dot{\sigma}^{\infty} - \frac{K'v'}{A\sigma_n}\right)$$

$$\frac{dv}{\left[\dot{\sigma}^{\infty} - \frac{K}{A\sigma_n} v\right]} = dt.$$

$$\int \frac{dx}{(a-x)} = \int dt$$

$$-\ln(a-x) = t + c.$$

$$\frac{A\sigma_n}{K} \log\left(\dot{\sigma}^{\infty} - \frac{K}{A\sigma_n} v\right) = t + c.$$

$\longrightarrow$ insert IC to calculate C.

$\Rightarrow$ solution for $v(t)$.

Numerical solution

$$\frac{dv}{dt} + \frac{K}{A\sigma_n} v = \dot{\sigma}^{\infty}, \qquad \underbrace{v(t=0) = v_i}_{IC}$$

$v_0 \quad v_1 \quad v_2 \quad v_3 \quad v_4 \quad \ldots$

$t_0 \quad t_1 \quad t_2 \quad t_3$

Δt

$$\frac{dv}{dt} = \dot{\sigma}^{\infty} - \frac{K}{A\sigma_n} v$$

$$\frac{v_{i+1} - v_i}{\Delta t} = \dot{\sigma}^{\infty} - \frac{K}{A\sigma_n} v_i$$

$$v_{i+1} = \Delta t \left(\dot{\sigma}^{\infty} - \frac{K}{A\sigma_n} v_i\right) + v_i \leftarrow \text{Finite difference approximation.}$$

Known

Cheryl $\to$ Adams-Bashforth / Adams-Moulton methods

Grace $\longrightarrow$ Implicit & Explicit Euler method

Iatan $\longrightarrow$ $\underbrace{\text{Runge-Kutta methods}}_{\text{Methods to solve ODEs}}$ $\rightarrow \dfrac{d^n y}{dt^n} \leftarrow$ vector