$$e = (d - Gm)$$

$$(G^T G m)^T = m^T G^T G$$

$$\phi = (d - Gm)^T (d - Gm)$$
$$= (d^T - m^T G^T)(d - Gm)$$
$$\phi = d^T d - m^T G^T d - d^T G m + m^T G^T G m$$
$$\frac{\partial \phi}{\partial m} = 0 - d^T G - d^T G + m^T G^T G + m^T G^T G$$
$$-2 d^T G + 2 m^T G^T G = 0$$
$$m^T G^T G = d^T G$$
$$G^T G m = G^T d$$
$$m = (G^T G)^{-1} G^T d$$

$$\frac{\partial (m^T G^T d)}{\partial m}$$

$$m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}, \quad d = [d_1 \ d_2 \ d_3 \ d_4]^T$$

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & \cdots & \\ G_{41} & & G_{43} \end{bmatrix}$$

$$m^T G^T d = [m_1 \ m_2 \ m_3] \begin{bmatrix} G_{11} & G_{21} & G_{31} & G_{41} \\ G_{12} & G_{22} & G_{32} & G_{42} \\ G_{13} & G_{23} & G_{33} & G_{43} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

$$= [m_1 \ m_2 \ m_3] \begin{bmatrix} G_{11} d_1 + G_{21} d_2 + G_{31} d_3 + G_{41} d_4 \\ G_{12} d_1 + G_{22} d_2 + G_{32} d_3 + G_{42} d_4 \\ G_{13} d_1 + G_{23} d_2 + G_{33} d_3 + G_{43} d_4 \end{bmatrix}$$

$$\frac{\partial (m^T G^T d)}{\partial m_k} = \sum G_{jk} d_j$$

$$d^T G m$$
$$= d_i G_{ij} m_j$$

$$\partial (d^T G m) = \sum G_{ij} d_i$$

for $m^T G^T d = m_i G_{ij} d_i$

$$\frac{\partial (\ldots)}{\partial m_K} = \ldots G_{jK} d_j \qquad \frac{\partial \phi}{\partial m_K} = G_{iK} d_K$$

$m_1 a_1 + m_2 a_2$

$$\phi_i = m^T G^T = m_j G_{ij} \rightarrow [\ i \longrightarrow\ ]$$

$$\phi_i = Gm = G_{ij} m_j \rightarrow [\ ]\ i \downarrow \qquad \frac{\partial \phi_i}{\partial m_j} = G_{ij}$$

$$Am = \begin{bmatrix} A_{11}m_1 + A_{12}m_2 + A_{13}m_3 \ldots \\ A_{21}m_1 + A_{22}m_2 + A_{23}m_3 \ldots \\ A_{31}m_1 + A_{32}m_2 + A_{33}m_3 \ldots \\ \vdots \end{bmatrix}$$

$$\frac{\partial Am}{\partial m_1} = \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \\ \vdots \end{bmatrix} \qquad \frac{\partial Am}{\partial m_2} = \begin{bmatrix} A_{12} \\ A_{22} \\ A_{32} \end{bmatrix} \ldots \ldots$$

$$\frac{\partial (Am)}{\partial m} = \begin{bmatrix} \left(\frac{\partial Am}{\partial m_1}\right) & \left(\frac{\partial Am}{\partial m_2}\right) & \left(\frac{\partial Am}{\partial m_3}\right) & \ldots \\ \vdots & \vdots & \vdots & \ldots \end{bmatrix} = A$$

$$m^T A = [m_1 \quad m_2 \quad m_3] \begin{bmatrix} A_{11} & A_{12} & \ldots \\ A_{21} & A_{22} & \ldots \\ A_{31} & A_{32} & \ldots \end{bmatrix}$$

$$\frac{\partial (m^T A)}{\partial m_1} = [A_{11} \quad A_{12} \quad \ldots]^T$$

$$\frac{\partial m^T A}{\partial m_2} = [A_{21} \quad A_{22} \quad \ldots]^T$$

$$\frac{\partial(m^T A)}{\partial m} = \begin{bmatrix} \left[\frac{\partial m^T A}{\partial m_1}\right] & \left[\frac{\partial m^T A}{\partial m_2}\right] & \left[\frac{\partial m^T A}{\partial m_3}\right] & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} & \cdots \\ A_{12} & & & \\ A_{13} & & & \end{bmatrix} = A^T$$

---

## Kalman Filter

$x = \begin{bmatrix} s \\ v \end{bmatrix}, \quad u = [a]$

$P_k$

$$\hat{\underline{x}}_k = F \hat{\underline{x}}_{k-1} + B \hat{\underline{u}}$$

$Q \leftarrow$ input

$P_{k-1}$

$$\hat{\underline{x}}_k = \begin{bmatrix} \hat{s}_k \\ \hat{v}_k \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{s}_{k-1} \\ \hat{v}_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix} \hat{a}_{k-1}$$

$\hat{\underline{x}}_{k|k-1}$

$$[P_k] = F P_{k-1} F^T + Q$$

observation $\underline{x}_k \leftarrow R_k$

$$H = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\underline{y}_k = \underline{x}_k - H \hat{\underline{x}}_{k|k-1}$$

$$[S_k] = H P_k H^T + R_k$$

$$K = P_k H^T S_k^{-1}$$

$$\hat{\underline{x}}_{k|k} = \hat{\underline{x}}_{k|k-1} + K \underline{y}_k$$

$$[P_{k|k}] = (I - KH) P_{k|k-1}$$

$$\underline{y}_{k|k} = \underline{x}_k - H\hat{x}_{k|k}$$

---

$$u_1(x_2, x_3; y_2, y_3, w)$$

$$= \frac{s}{2\pi}\left\{ \tan^{-1}\frac{x_3-y_3}{x_2-y_2} - \tan^{-1}\frac{x_3+y_3}{x_2-y_2} - \tan^{-1}\frac{x_3-y_3-w}{x_2-y_2} + \tan^{-1}\frac{x_3+y_3+w}{x_2-y_2} \right\}$$

$$u_1(x_2, x_3; y_2, y_3, w, \delta)$$

$$= \frac{s}{2\pi}\left\{ \tan^{-1}\frac{x_3-y_3}{x_2-y_2} - \tan^{-1}\frac{x_3+y_3}{x_2-y_2} - \tan^{-1}\frac{x_3-y_3-w\sin\delta}{x_2-y_2-w\cos\delta} + \tan^{-1}\frac{x_3+y_3+w\sin\delta}{x_2-y_2-w\cos\delta} \right\}$$

$$\frac{\partial u_1}{\partial x_3} = \frac{s}{2\pi}\left\{ \frac{x_2-y_2}{(x_2-y_2)^2+(\cancel{x}_3-y_3)^2} - \frac{x_2-y_2}{(x_2-y_2)^2+(\cancel{x}_3+y_3)^2} \right.$$

$$\left. - \frac{(x_2-y_2-w\cos\delta)}{(x_2-y_2-w\cos\delta)^2+(\cancel{x}_3-y_3-w\sin\delta)^2} + \frac{(x_2-y_2-w\cos\delta)}{(x_2-y_2-w\cos\delta)^2 + \cdots} \right\}$$

$$+ \left(x_3 + y_3 + w\sin\delta\right)\Big]\Big]$$

at $x_3 = 0$,

$$\frac{\partial u_1}{\partial x_3} = \frac{s}{2\pi}\left\{ 0 \right\} \rightarrow \text{boundary condition satisfied.}$$

$$u_1\left(x_2, 0, y_2, y_3, w \to \infty, \delta\right)$$

$$= \frac{s}{2\pi}\left\{ \tan^{-1}\frac{-y_3}{x_2 - y_2} - \tan^{-1}\frac{y_3}{x_2 - y_2} - \tan^{-1}(-\tan\delta) + \tan^{-1}\tan\delta \right\}$$

$$= \frac{s}{2\pi}\left\{ -2\tan^{-1}\frac{y_3}{(x_2 - y_2)} + 2\delta \right\}$$

$$= \frac{s}{\pi}\tan^{-1}\left(\frac{x_2 - y_2}{y_3}\right) + \frac{s\delta}{\pi} \quad \text{—— (1)}$$

$$u_1\left(x_2, 0, y_2, y_3, w \to \infty\right)$$

$$= \frac{s}{2\pi}\left\{ -2\tan^{-1}\frac{y_3}{x_2 - y_2} - \underbrace{\tan^{-1}(-\infty)}_{-\pi} + \underbrace{\tan^{-1}(\infty)}_{\pi} \right\}$$

$$= \frac{s}{2\pi}\left\{2\tan^{-1}\left(\frac{x_2-y_2}{y_3}\right) + \pi\right\}$$

$$= \frac{s}{\pi}\tan^{-1}\left(\frac{x_2-y_2}{y_3}\right) + \frac{s}{2} \longrightarrow (2)$$

when $\delta = \frac{\pi}{2}$ in (1), $\frac{s\delta}{\pi} = \frac{s}{2}$

---

$$K(V_{pl} - v) = \frac{A\sigma_n}{m} v^{\frac{1}{m}-1} \frac{dv}{dt}.$$

for $m=2$,

$$K(V_{pl} - v) = \frac{A\sigma_n}{m\sqrt{v}} \frac{dv}{dt}.$$

$$\frac{mK}{A\sigma_n} = \frac{1}{\sqrt{v}(V_{pl}-v)}\frac{dv}{dt}.$$

$$\boxed{\frac{dy}{dt} = k y^{\frac{1}{2}}(a-y)}$$

$$\frac{mK}{A\sigma_n}dt = \frac{dv}{(V_{pl}-v)\sqrt{v}}$$

$$\xi = \sqrt{\frac{v}{V_{pl}}}$$

$$d\xi = \frac{1}{2\sqrt{v V_{pl}}}dv$$

$$= \frac{2\cancel{\sqrt{v}}\sqrt{V_{pl}}\,d\xi}{\cancel{\sqrt{v}}\,V_{pl}(1-\xi^2)}$$

$$= 2 \quad d\xi$$

$$\sqrt{V_{pl}} \qquad 1-\xi^2$$

$$\int \frac{2k}{A\sigma_n}\,dt = \frac{2}{\sqrt{V_{pl}}}\int \frac{d\xi}{1-\xi^2}$$

$$\frac{2k}{A\sigma_n}t + C = \frac{2}{\sqrt{V_{pl}}}\tanh^{-1}\xi$$

$$\sqrt{\frac{V}{V_{pl}}}$$

$$= \frac{2}{\sqrt{V_{pl}}}\tanh^{-1}\sqrt{\frac{v}{V_{pl}}}.$$

$$\frac{2kt}{A\sigma_n} + C = \frac{2}{\sqrt{V_{pl}}}\tanh^{-1}\sqrt{\frac{V}{V_{pl}}}.$$

Applying Initial Condition.

$$v(t=0) = V_i^{o}$$

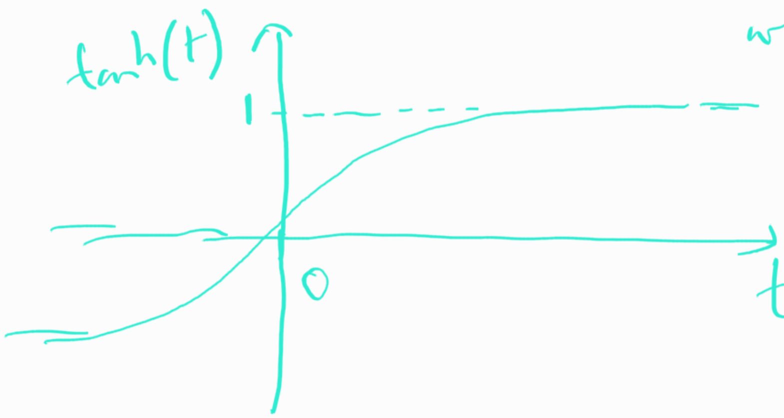
$$C = \frac{2}{\sqrt{V_{pl}}} \tanh^{-1} \sqrt{\frac{V_i}{V_{pl}}}$$

$$\frac{2Kt}{A\sigma_n} + \frac{2}{\sqrt{V_{pl}}} \tanh^{-1} \sqrt{\frac{V_i}{V_{pl}}}$$

$$= \frac{2}{\sqrt{V_{pl}}} \tanh^{-1} \sqrt{\frac{V(t)}{V_{pl}}}.$$

$$\frac{\sqrt{V_{pl}}\, Kt}{A\sigma_n} + \tanh^{-1} \sqrt{\frac{V_i}{V_{pl}}} = \tanh^{-1} \sqrt{\frac{V}{V_{pl}}}.$$

$$\boxed{\sqrt{\frac{V}{V_{pl}}} = \tanh\left\{ \frac{\sqrt{V_{pl}}\, Kt}{A\sigma_n} + \tanh^{-1} \sqrt{\frac{V_i}{V_{pl}}} \right\}}$$




when $t \to \infty$, $\tanh(g(t)) \to 1$

$\Rightarrow v \to V_{pl}.$

$^T (\ldots) + \lambda^2 (b - Am)^T (b - Am)$

$$\phi = (d - Gm)^T(d - Gm) + \lambda^2(b - Am)^T(b - Am)$$

$$= \cancel{d^T d} - \underline{d^T G m - m^T G^T d} + m^T G^T G m$$

$$+ \lambda^2 \left\{ \cancel{b^T b} - \underline{b^T A m - m^T A^T b} + m^T A^T A m \right)$$

$$\frac{\partial \phi}{\partial m} = -2d^T G + (G^T G m)^T + m^T G^T G - 2\lambda^2 b^T A + \lambda^2 (A^T A m)^T + \lambda^2 (m^T A^T A)$$

for optima, $\dfrac{\partial \phi}{\partial m} = 0$,

$$0 = -2d^T G + 2m^T G^T G - 2\lambda^2 b^T A + 2\lambda^2 m^T A^T A$$

$$\cancel{2}\left(d^T G + \lambda^2 b^T A\right) = \cancel{2}\left(m^T G^T G + \lambda^2 m^T A^T A\right)$$

$$\left(G^T d + \lambda^2 A^T b\right) = \left[G^T G + \lambda^2 A^T A\right] m$$

$$\Rightarrow \underline{m = \left(G^T G + \lambda^2 A^T A\right)^{-1}\left(G^T d + \lambda^2 A^T b\right)}$$