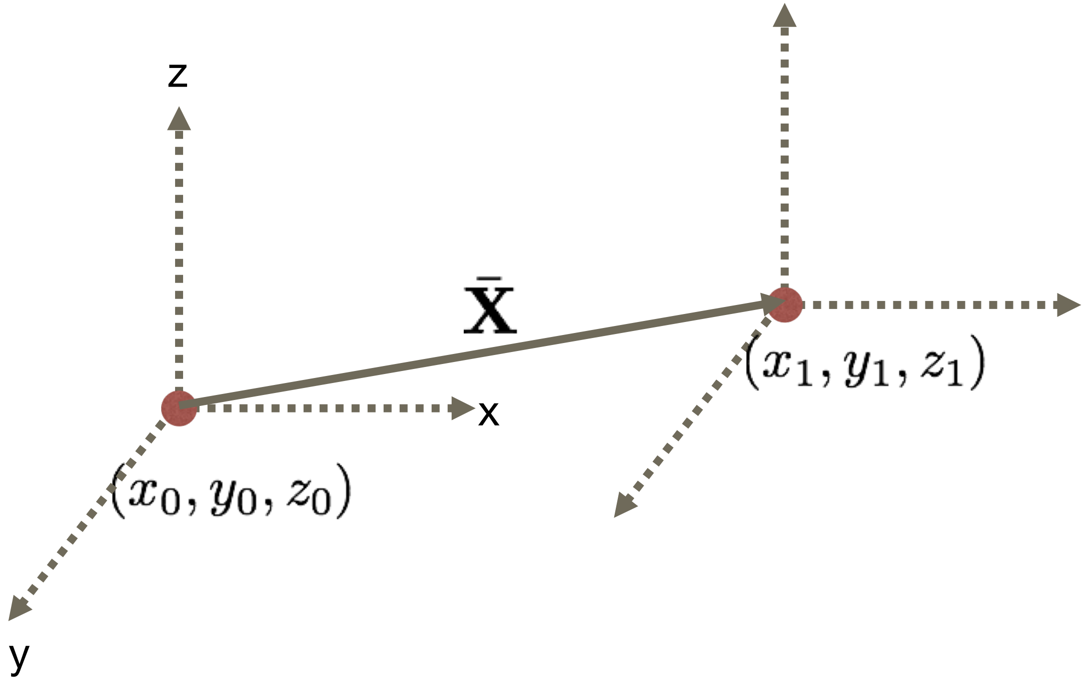
Week 2

1. Rigid block motion
2. Plate motion on a sphere
3. Continuum kinematics (intro)

Motion and coordinate systems

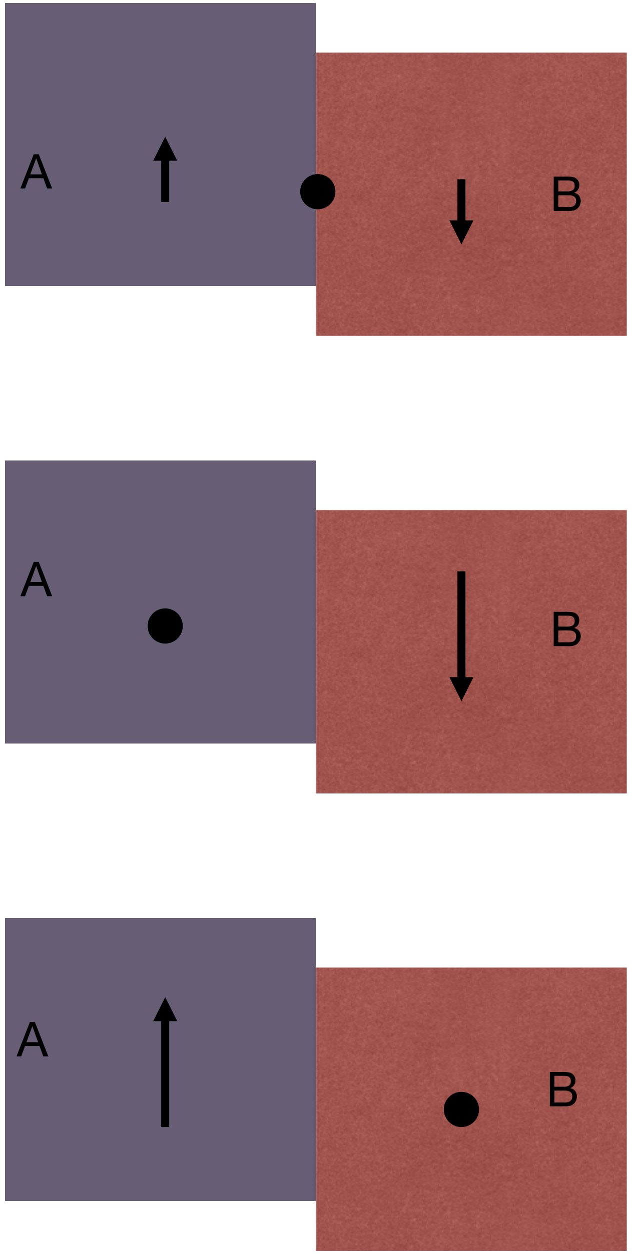
- Motion is a vector quantity
- All motion is with respect to an origin
- Velocity is the time derivative of the displacement vector



$$\bar{\mathbf{U}} = \frac{\partial \bar{\mathbf{X}}}{\partial t}$$

$$\bar{\mathbf{X}} = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

$$\bar{\mathbf{U}} = \left(\frac{x_1 - x_0}{t_1 - t_0}, \frac{y_1 - y_0}{t_1 - t_0}, \frac{z_1 - z_0}{t_1 - t_0} \right)$$

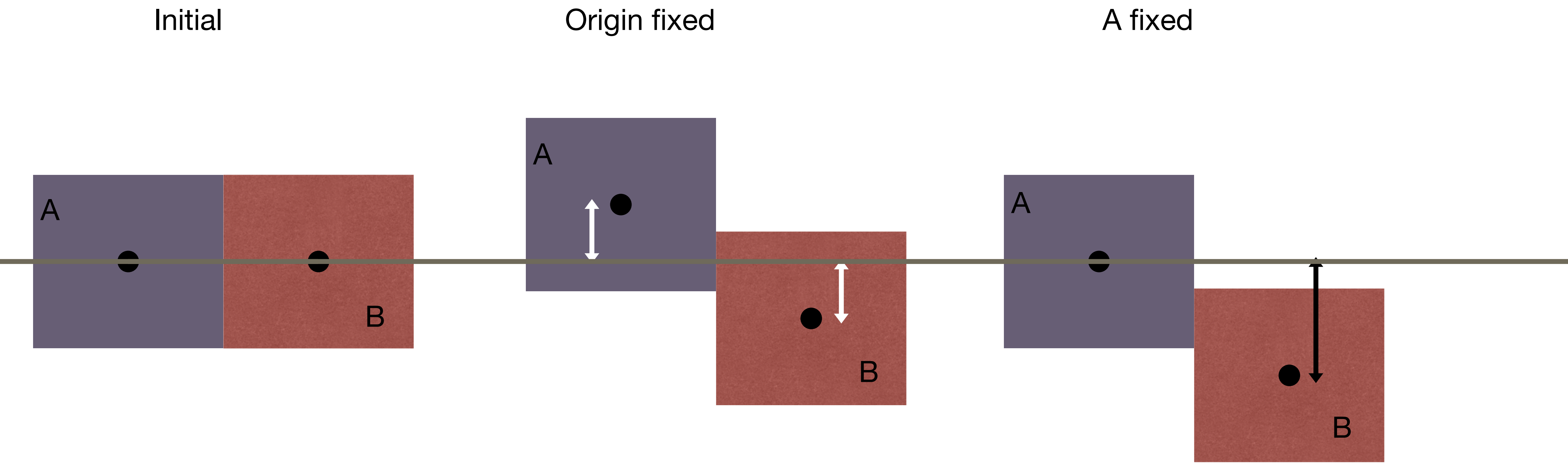


Reference frames and relative motion

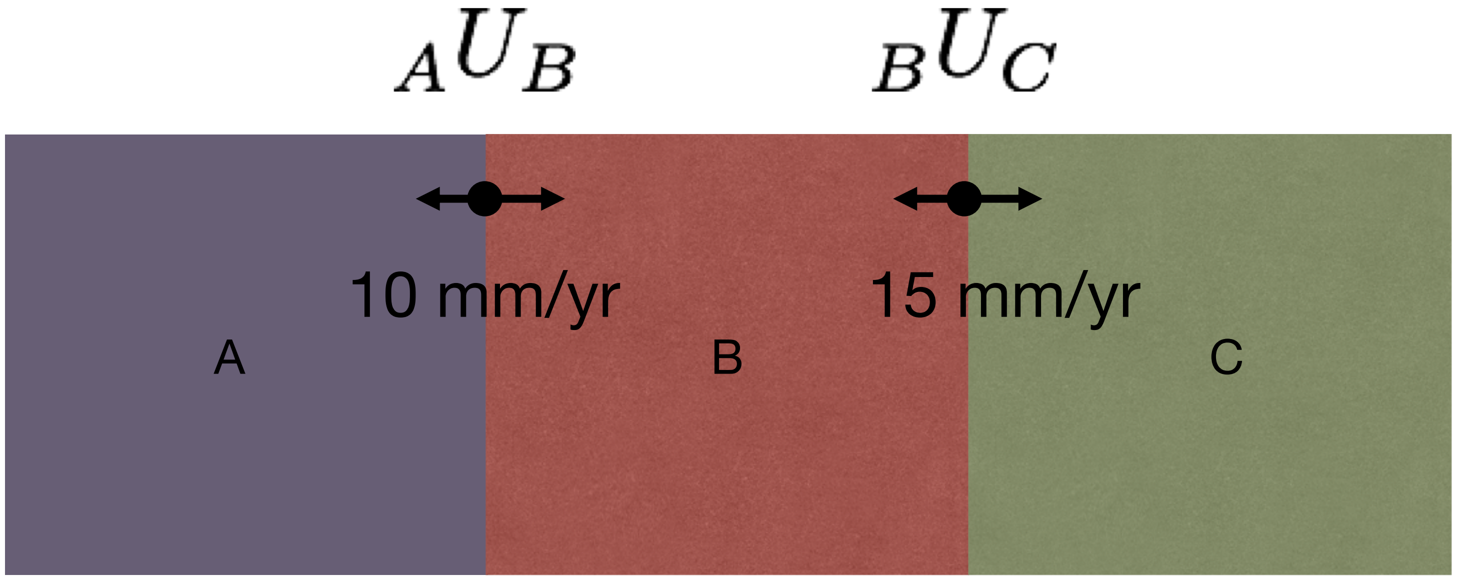
$$ {}_{A}U_{B} = -{}_{B}U_{A} $$

- Blocks move relative to an origin/reference frame
- You can decide where that origin is

Reference frames and relative motion

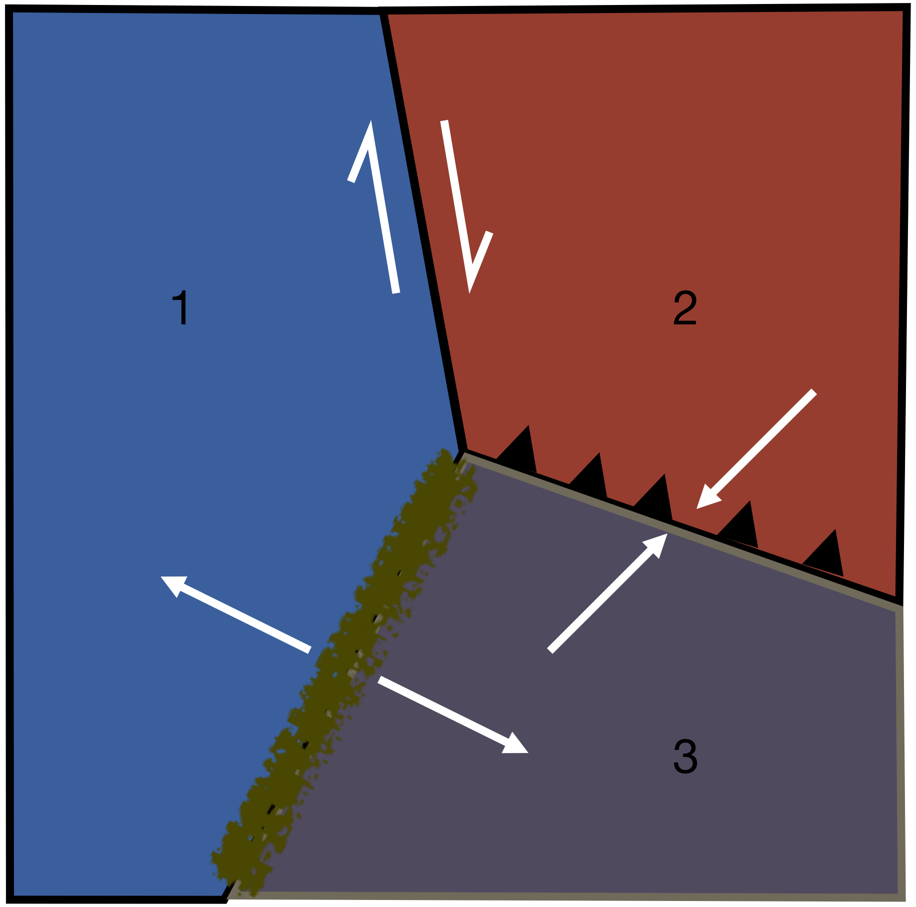


Relative motion



Find $_AU_C$

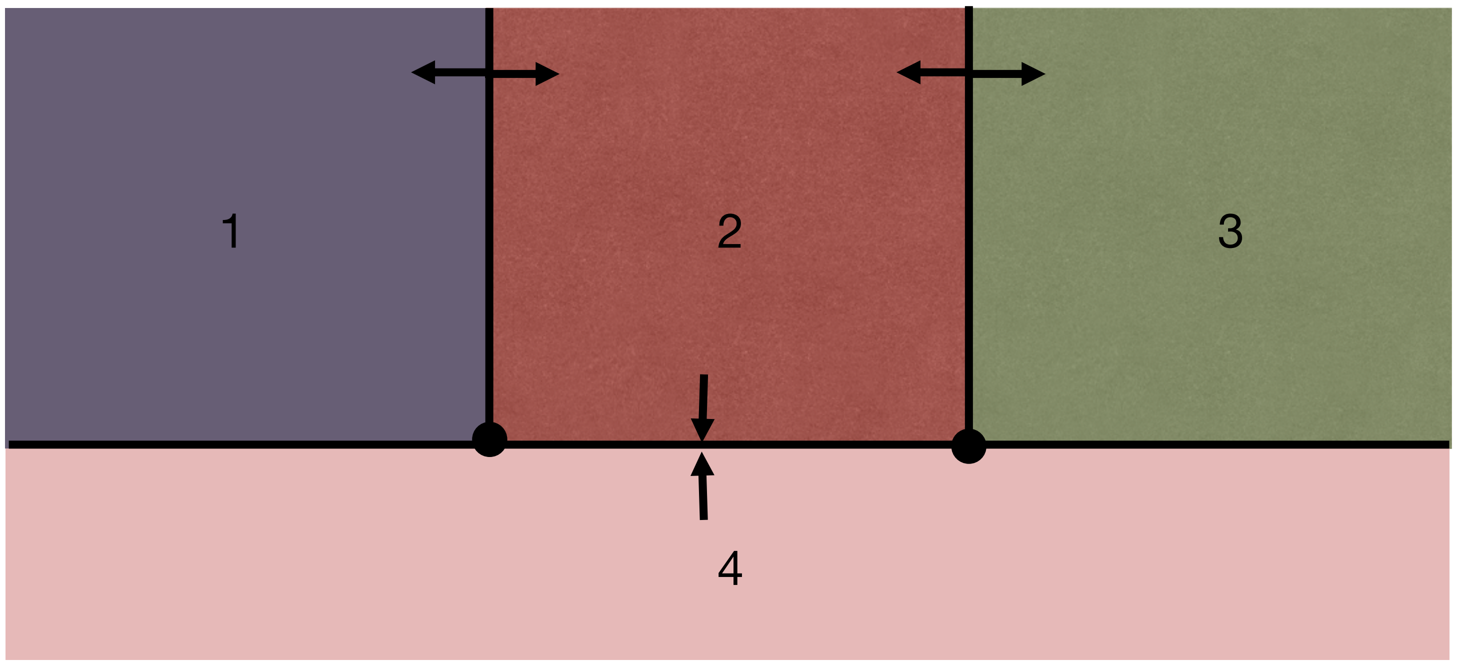
Circuit closure



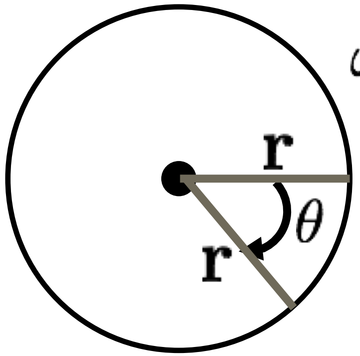
$$ {}_1U_2 + {}_2U_3 + {}_3U_1 = 0 $$

Circuit closure

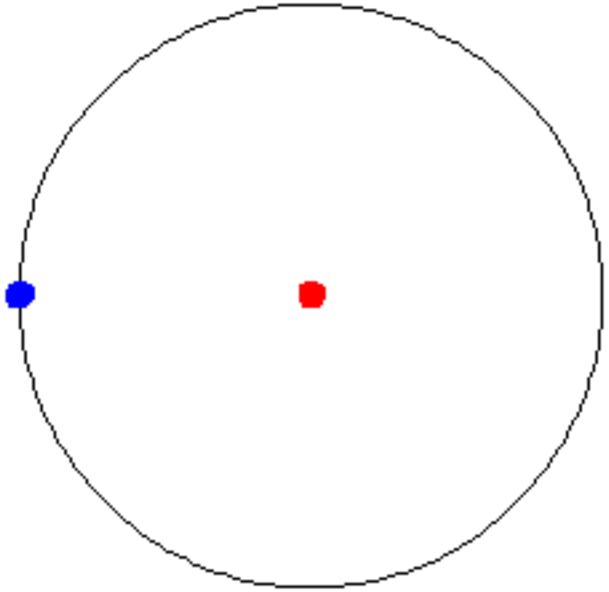
Find ${}_1U_4$ and ${}_3U_4$



Rotations in Cartesian coordinates

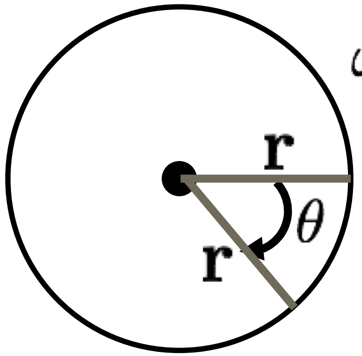


$$\omega = \frac{\partial \theta}{\partial t}$$



- Blocks motion can be described as a rotation around an origin (Euler pole)

Rotations in Cartesian coordinates



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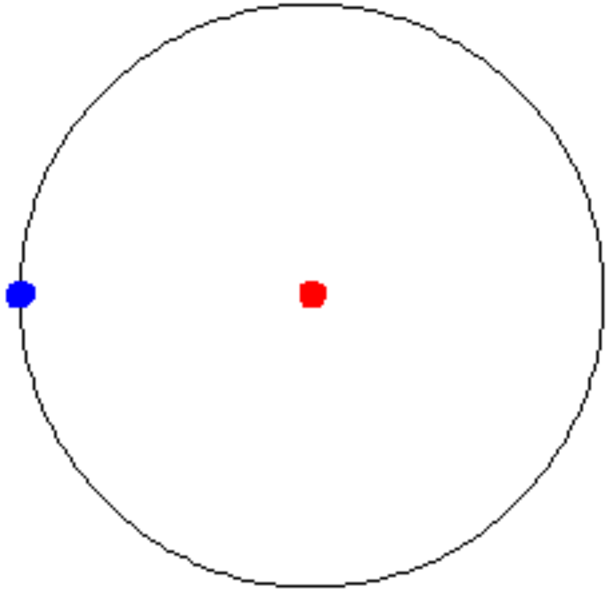
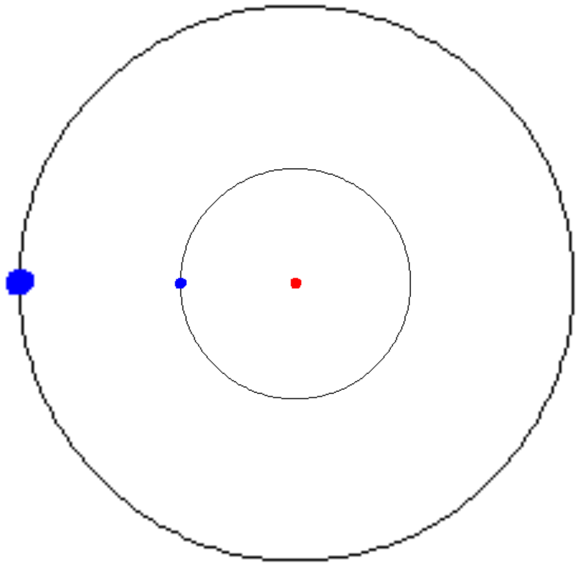
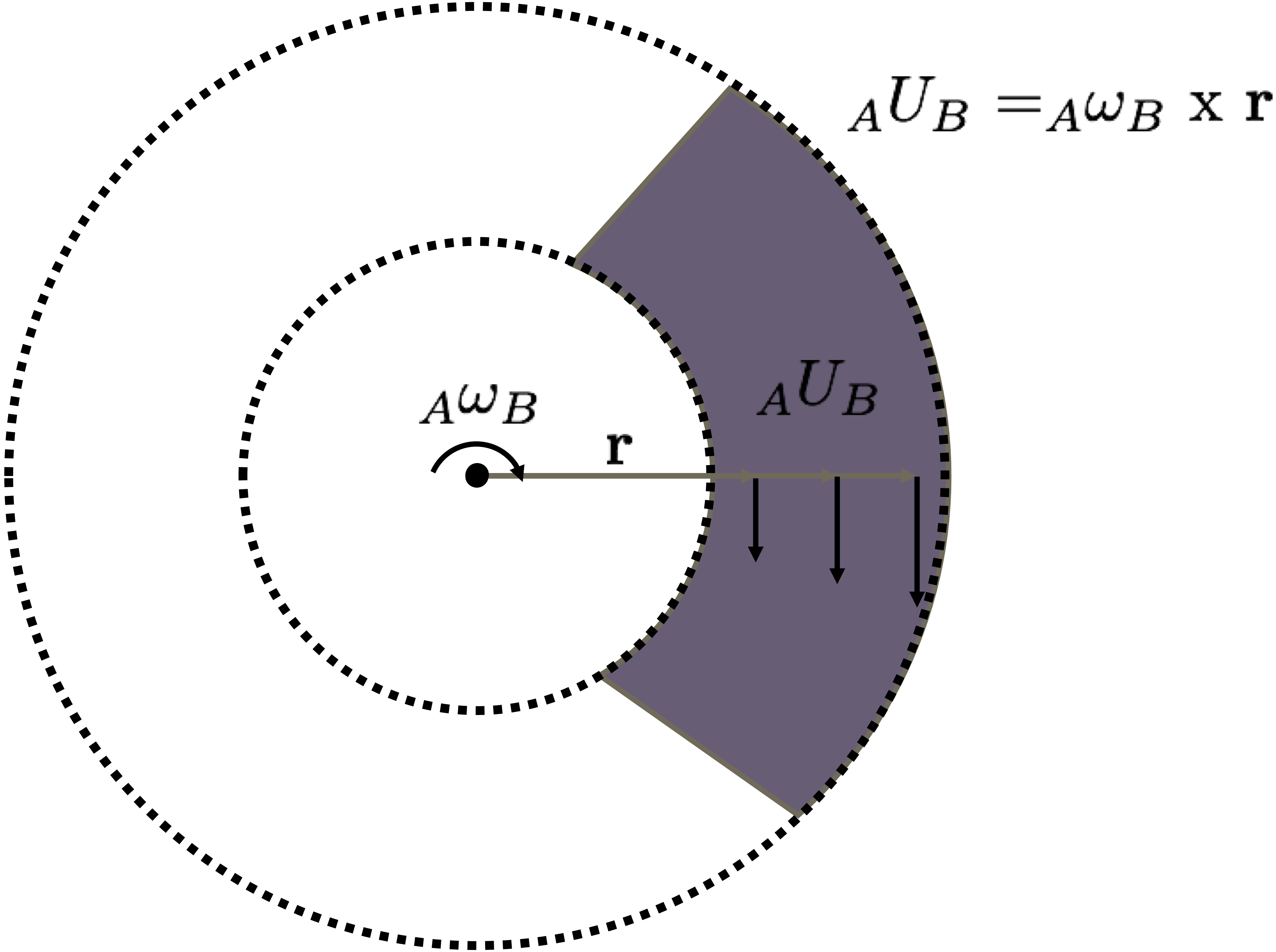




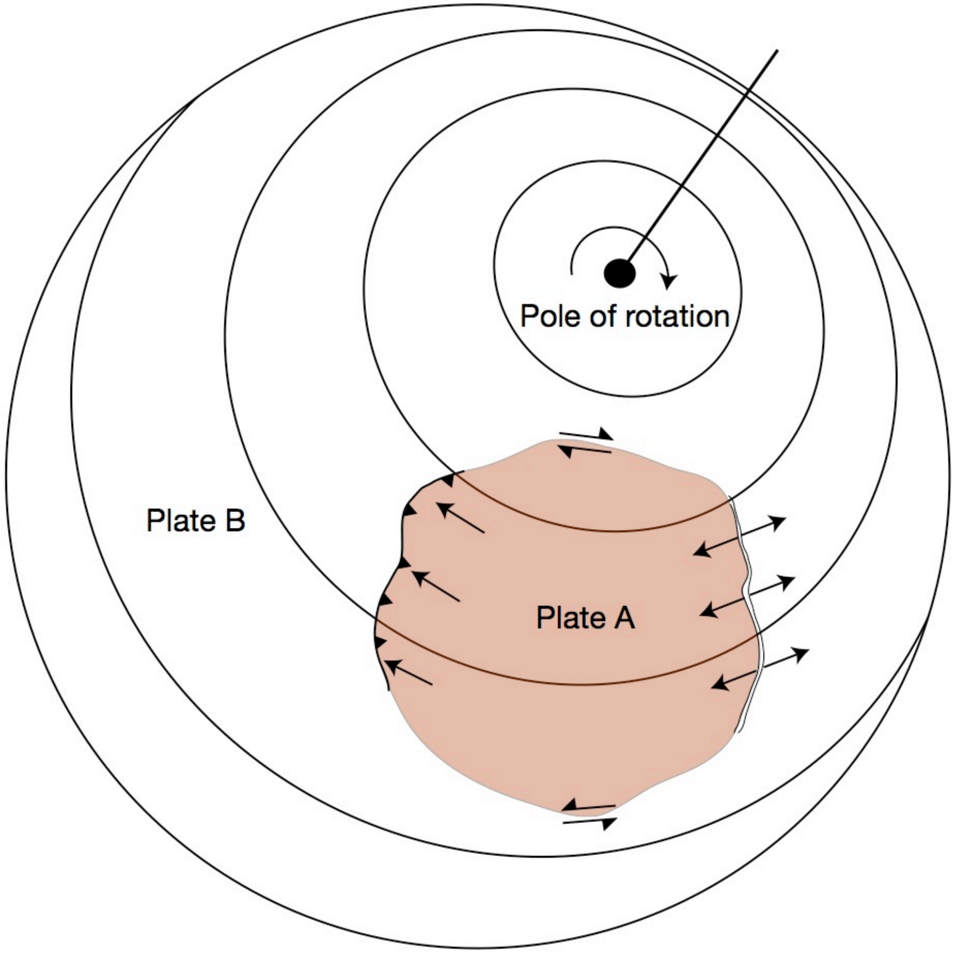
- Blocks motion can be described as a rotation around an origin (Euler pole)
- Velocity varies with distance, but angular velocity is constant



${}_A U_B = {}_A\omega_B \times \mathbf{r}$
${}_A\omega_B$
$\mathbf{r}$
${}_A U_B$

Rotations on a spherical Earth

- The same rules as for flat system
- The Euler pole does not have to be on any plate - it is imaginary
- We need the Euler pole to describe relative rotation

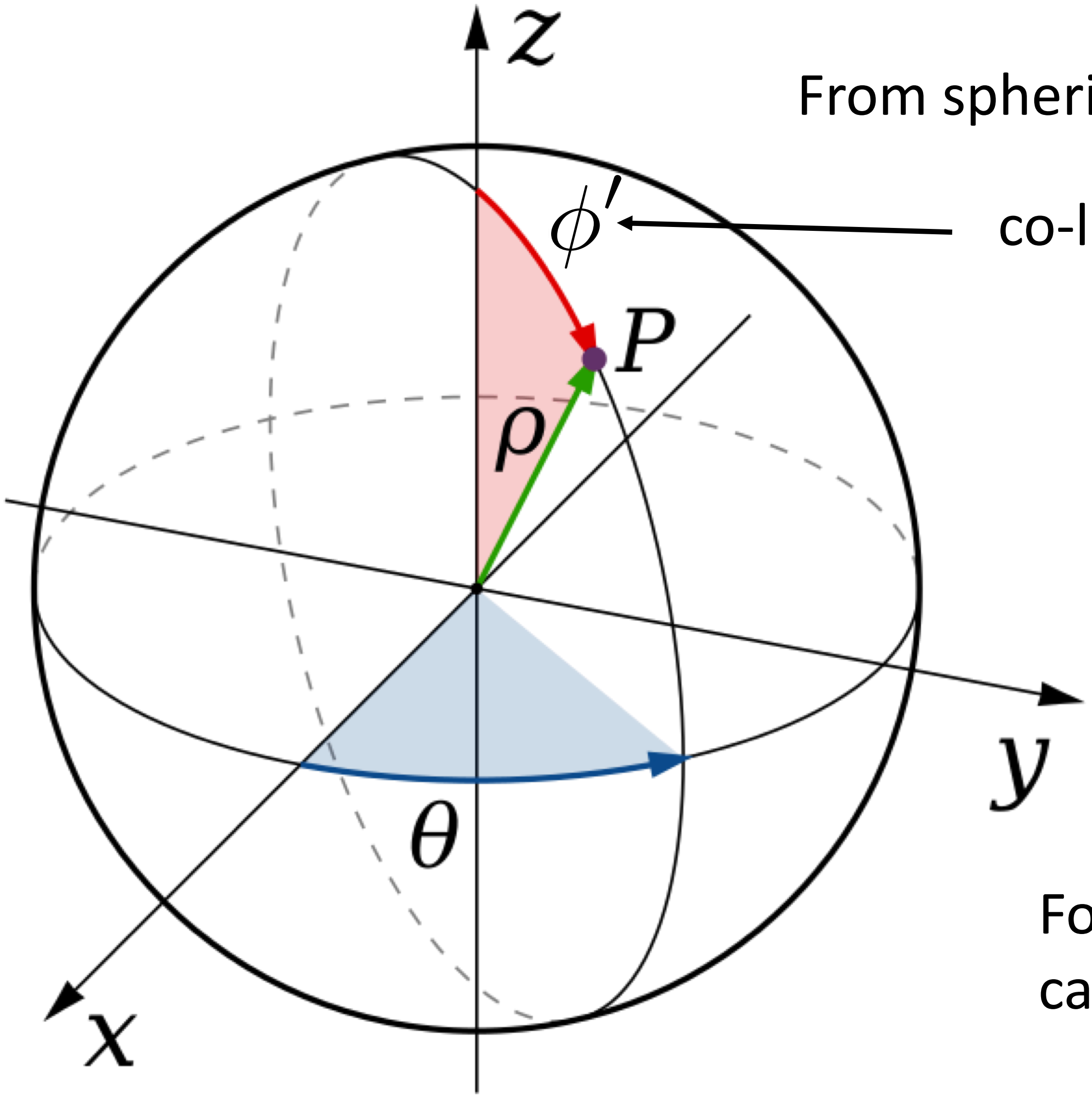


$$_{i}U_{i+1} = {_{i}\omega_{i+1}} \times \mathbf{r}$$

Linear velocities are calculated from angular velocities

Coordinate transformations

From spherical <-> 3-d cartesian



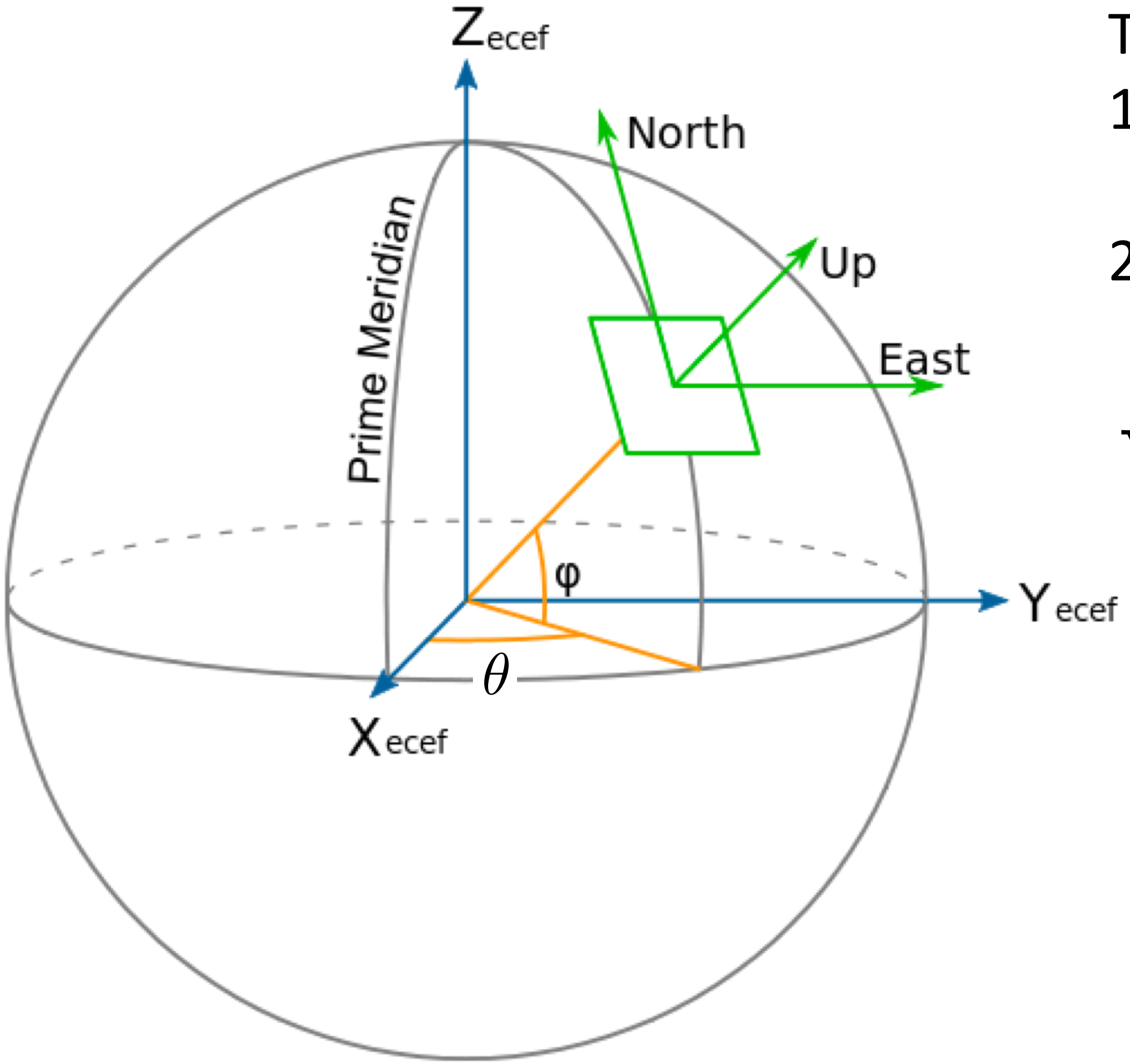
co-latitude

$$x = R\cos\theta\cos\phi$$
$$y = R\sin\theta\cos\phi$$
$$z = R\sin\phi$$

latitude

For a given euler pole (spherical coordinates), we can calculate the rotation vector in cartesian coordinates

Coordinate transformations



To calculate velocities in a local N,E,U basis

1. we need 1 rotation along the co-latitude (from Z to U)
2. and a second rotation to align X with E.

$$\mathbf{V}_{[n,e,u]} = R_x\left(\frac{\pi}{2} - \phi\right) R_z\left(\frac{\pi}{2} + \theta\right) (\boldsymbol{\omega} \times \mathbf{r})_{[x,y,z]}$$

2 rotations to convert from x,y,z to e,n,u

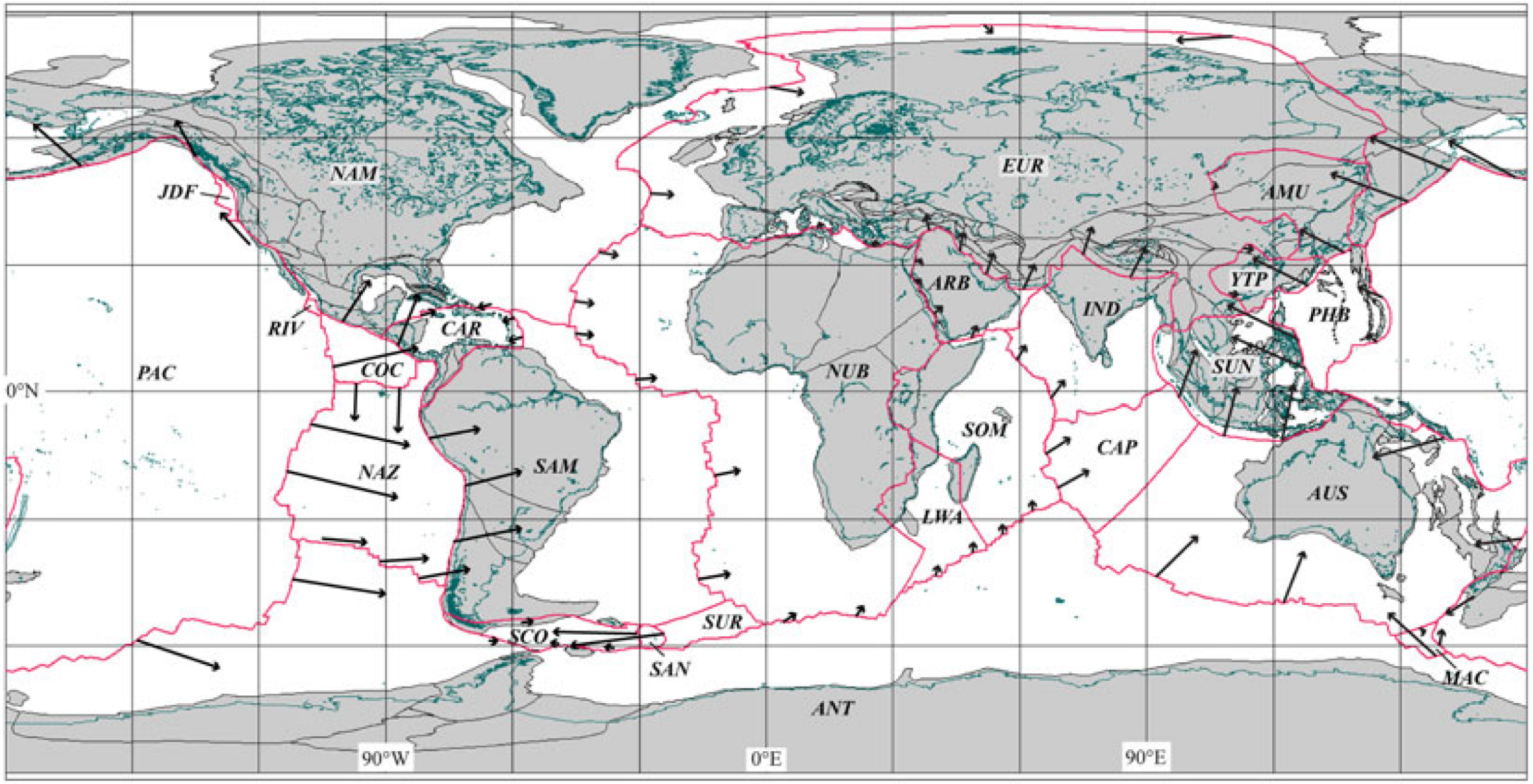
$$\begin{bmatrix} -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \end{bmatrix}$$

Rotations on a spherical Earth

$$\sum_{i=1}^{N} ({}_i\omega_{i+1}) = 0$$

$${}_1\omega_2 + {}_2\omega_3 + {}_3\omega_4 + \ldots = 0$$

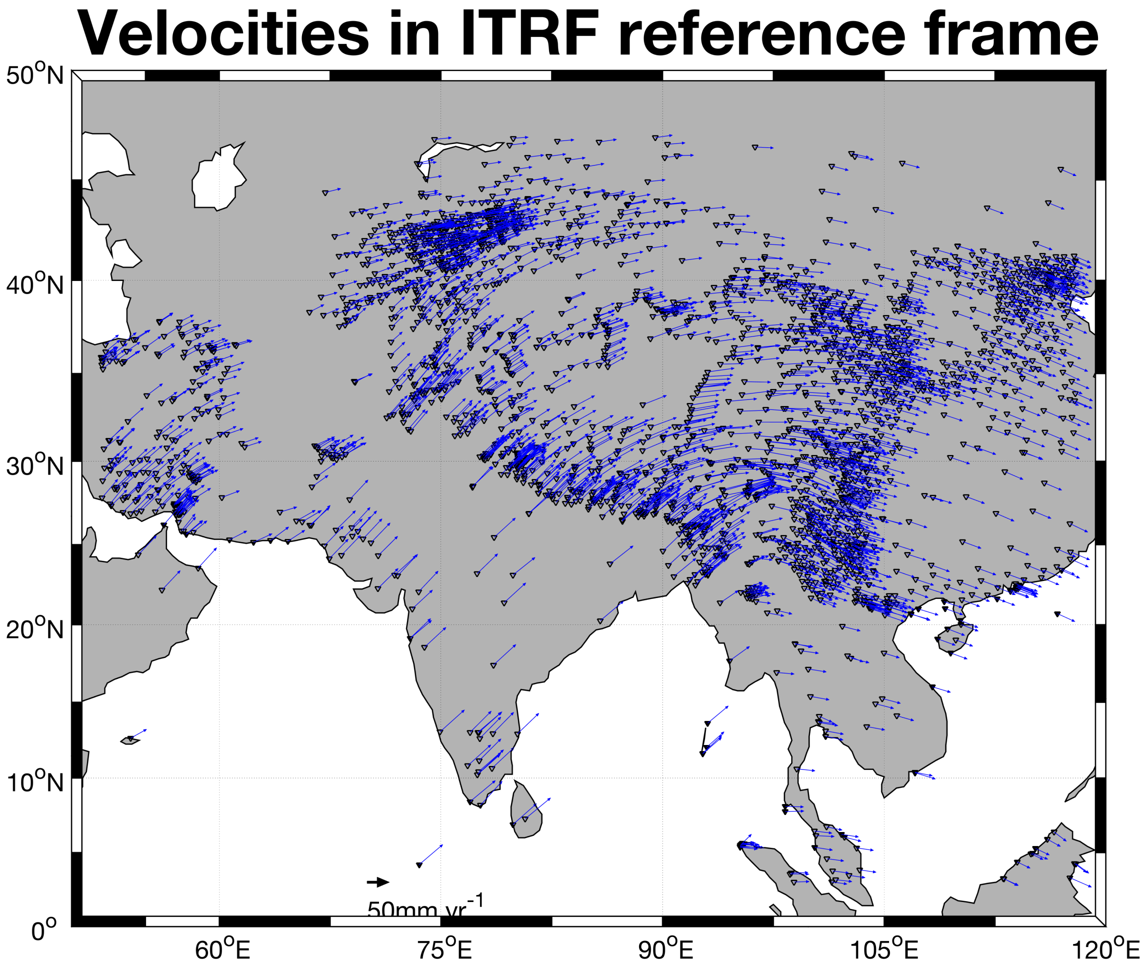
Angular velocities are additive



DeMets et al., (2010)

Reference frames and rotations (again)

Different from rotating vectors into different coordinate systems. Here the vector lengths itself change.



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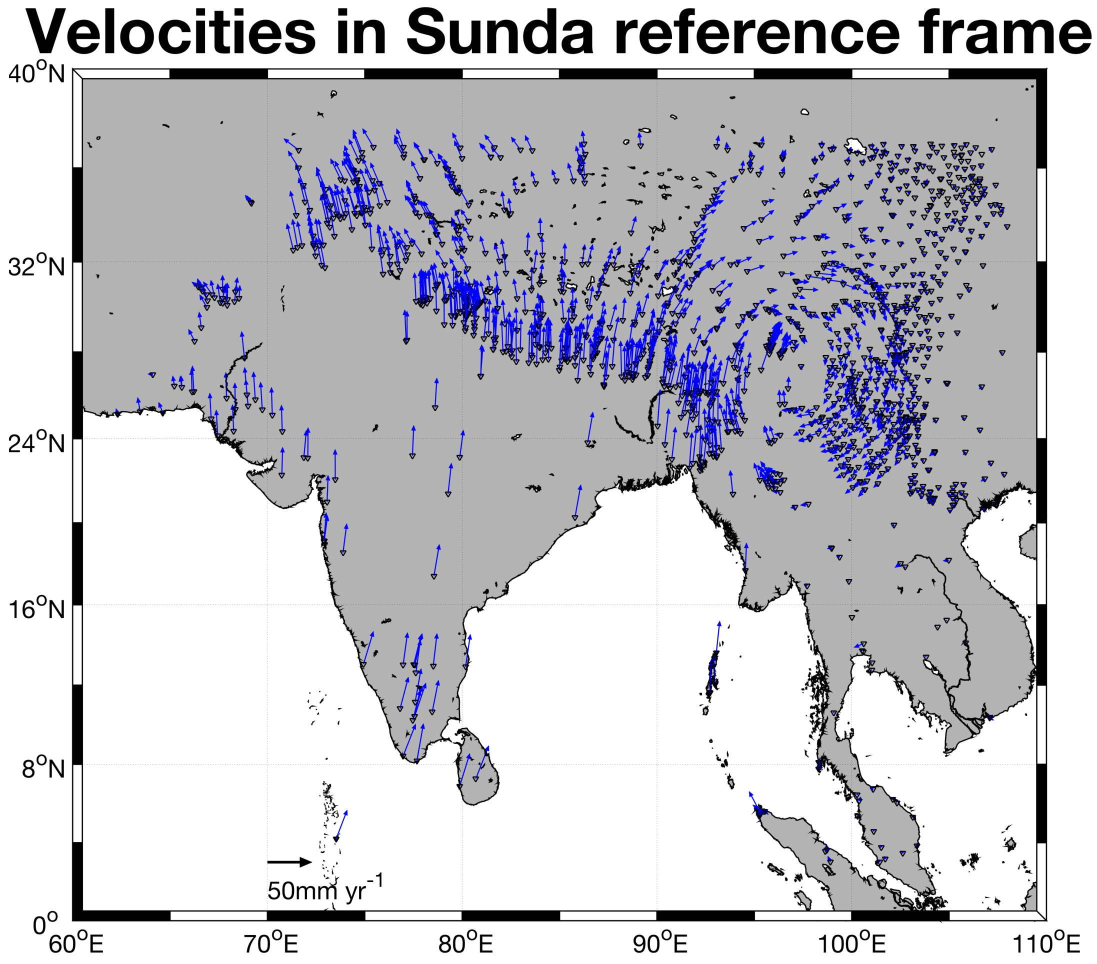

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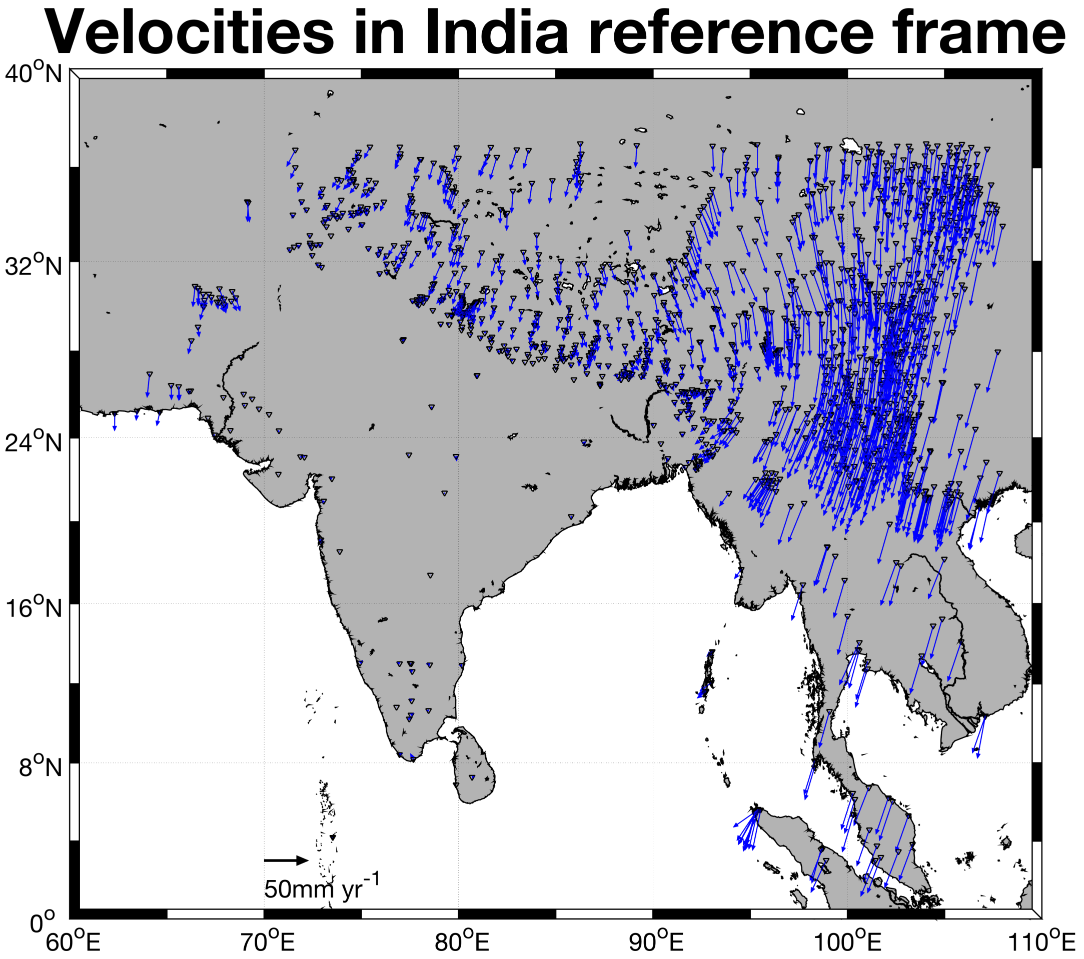

Assignment

1. First derive a linear operation that allows you to rotate the given velocity vectors to any coordinate system, for a known euler vector. Hint: rewrite the cross product as a multiplication of a matrix and a vector, and use the rotation operations we discussed earlier.
2. Now use this linear operation to predict the velocity field in the Indian subcontinent given an Euler pole (N – 51.42º, E – 2.10º, rotation rate – 0.5146 º/Myr). Compare the predicted velocities with the observed velocities.
3. Subtract this predicted velocity field at all sites and visualize the new transformed velocity field. What do you notice?
4. Repeat steps 2 and 3 by choosing the Eurasian Euler pole.

Reading Assignment:

McKenzie and Parker, (1967), The North Pacific - an Example of Tectonics on a Sphere, Nature
Avouac and Tapponnier, (1993), Kinematic Model of Active Deformation in Central Asia, GRL
Kogan et al., (2000), Geodetic Constraints on the Rigidity and Relative Motion of Eurasia and North America, GRL

Plate	Ab	Lat. °N	Lon. °E	w ° Ma^{-1}
Amur	am	63.17	–122.82	0.297 ± 0.020
Antarctica	an	65.42	–118.11	0.250 ± 0.008
Arabia	ar	48.88	–8.49	0.559 ± 0.016
Australia	au	33.86	37.94	0.632 ± 0.017
Capricorn	cp	44.44	23.09	0.608 ± 0.019
Caribbean	ca	35.20	–92.62	0.286 ± 0.023
Cocos	co	26.93	–124.31	1.198 ± 0.045
Eurasia	eu	48.85	–106.50	0.223 ± 0.009
India	in	50.37	–3.29	0.544 ± 0.010
Juan de Fuca	jf	–38.31	60.04	0.951 ± 0.256
Lwandle	lw	51.89	–69.52	0.286 ± 0.026
Macquarie	mq	49.19	11.05	1.144 ± 0.274
Nazca	nz	46.23	–101.06	0.696 ± 0.029
North America	na	–4.85	–80.64	0.209 ± 0.013
Nubia	nb	47.68	–68.44	0.292 ± 0.007
Pacific	pa	–63.58	114.70	0.651 ± 0.011
Philippine Sea	ps	–46.02	–31.36	0.910 ± 0.050
Rivera	ri	20.25	–107.29	4.536 ± 0.630
Sandwich	sw	–29.94	–36.87	1.362 ± 0.744
Scotia	sc	22.52	–106.15	0.146 ± 0.016
Somalia	sm	49.95	–84.52	0.339 ± 0.011
South America	sa	–22.62	–112.83	0.109 ± 0.011
Sunda	su	50.06	–95.02	0.337 ± 0.020
Sur	sr	–32.50	–111.32	0.107 ± 0.028
Yangtze	yz	63.03	–116.62	0.334 ± 0.013