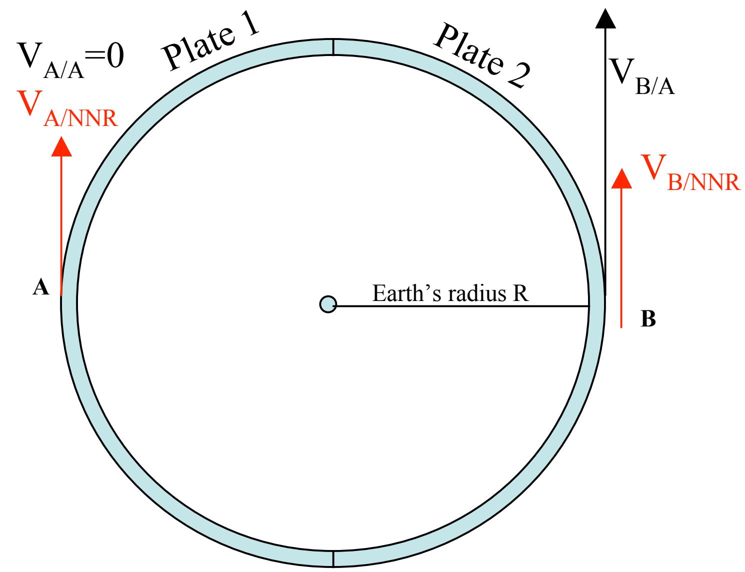
The international Terrestrial Reference System: ITRS

- Definition adopted by the IUGG and IAG: see http://tai.bipm.org/iers/conv2003/conv2003.html
- Tri-dimensional orthogonal (X,Y,Z), equatorial (Z-axis coincides with Earth's rotation axis)
- Non-rotating (actually, rotates with the Earth)
- Geocentric: origin = Earth's center of mass, including oceans and atmosphere.
- Units = meter and second S.I.
- Orientation given by BIH at 1984.0.
- Time evolution of the orientation ensured by imposing a **no-net-rotation** condition for horizontal motions.

The no-net-rotation (NNR) condition

- Objective:
  - Representing velocities without referring to a particular plate.
  - Solve a datum defect problem: ex. of 2 plates $\Rightarrow$ 1 relative velocity to solve for 2 "absolute" velocities… (what about 3 plates?)
- The no-net-rotation condition states that the total angular momentum of all tectonic plates should be zero.
- See figure for the simple (and theoretical) case of 2 plates on a circle.
- The NNR condition has no impact on **relative** plate velocities.
- It is an additional condition used to define a reference for plate motions that is not attached to any particular plate.



$$M_A = R \times V_{B/NNR}$$
$$M_B = R \times V_{A/NNR}$$
$$\Sigma M = 0$$
$$\Rightarrow V_{B/NNR} = -V_{A/NNR} = V_{B/A} / 2$$

The Tisserand reference system

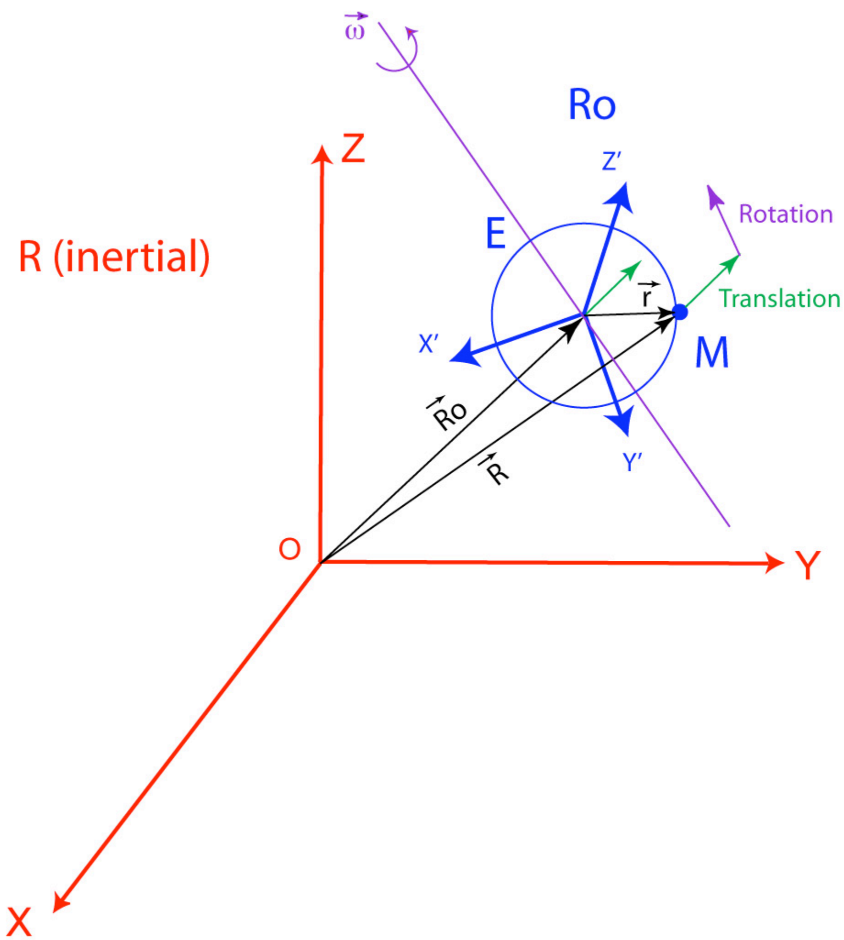
- “Mean” coordinate system in which deformations of the Earth do not contribute to the global kinetic moment (important in Earth rotation theory)

- Let us assume two systems R (inertial) and Ro (translates and rotates w.r.t. R). Body E is attached to Ro. At point M, one can write:

$$\begin{cases} \vec{R} = \vec{R}_o + \vec{r} \\ \vec{V} = \vec{V}_o + \vec{v} + \vec{\omega} \times \vec{r} \end{cases}$$

- One can show that the Tisserand condition is equivalent to:

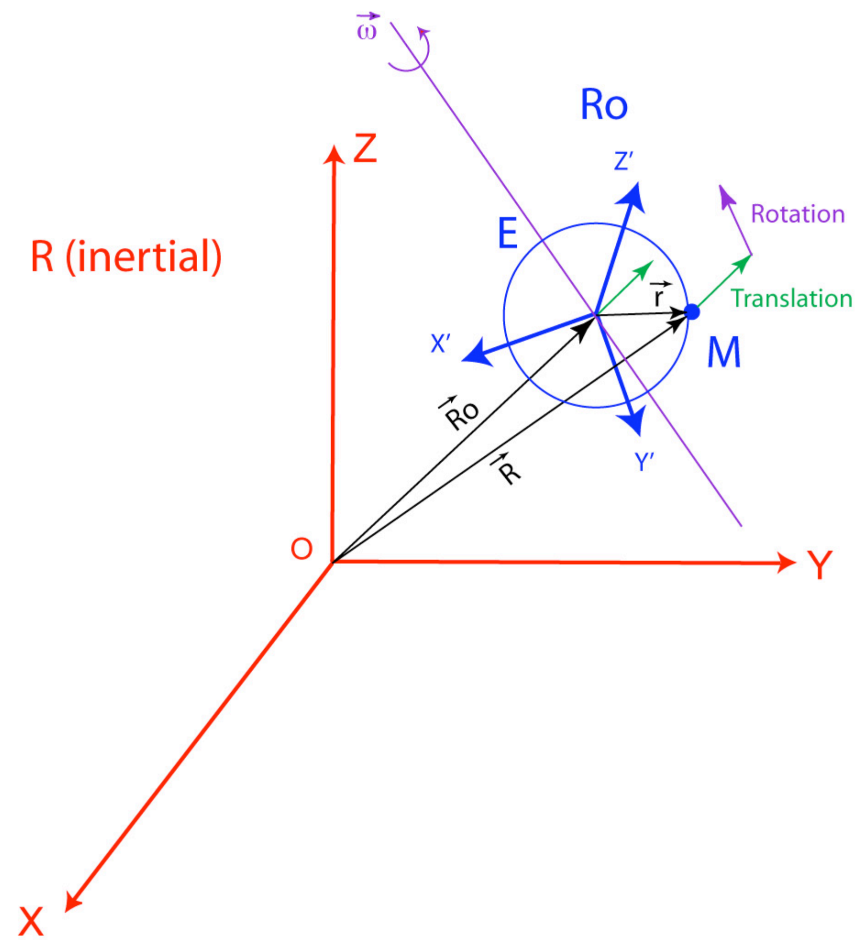
$$\begin{cases} \int_E \vec{v} \; dm = \vec{0} & \text{No translation condition} \\ \int_E \vec{v} \times \vec{r} \; dm = \vec{0} & \text{No rotation condition} \end{cases}$$



The Tisserand reference system

$$\begin{cases} \int_E \vec{v} \quad dm = \vec{0} & \text{No translation condition} \\ \int_E \vec{v} \times \vec{r} \quad dm = \vec{0} & \text{No rotation condition} \end{cases}$$

- The system of axis defined by the above conditions is called “Tisserand system”.
- Integration domain:
  - Should be entire Earth volume
  - But velocities at surface only => integration over surface only
- With hypothesis of spherical Earth + uniform density, volume integral becomes a surface integral



The NNR reference system

- The Tisserand no-rotation conditions is also called “no-net-rotation” condition (NNR)

- For a spherical Earth of unit radius and uniform density, the NNR conditions writes:

$$\int_S \vec{r} \times \vec{v} \; dA = \vec{0}$$

- The integral can be broken into a sum to account for discrete plates:

$$\int_S \vec{r} \times \vec{v} \; dA = \sum_P \int_P \vec{r} \times \vec{v} \; dA$$

- With, for a given plate:

$$L_P = \int_P \vec{r} \times \vec{v} \; dA$$

The NNR reference system

- Assuming rigid plates, velocity at point *M* (position vector *r* in NNR) on plate *P* is given by:

$$\vec{v}(\vec{r}) = \vec{\omega}_P \times \vec{r} \qquad \Rightarrow L_P = \int_P \vec{r} \times (\vec{\omega}_P \times \vec{r}) \; dA$$

- Developing the vector product with the triple product expansion gives:

$$L_P = \int_P ((\vec{r}.\vec{r})\vec{\omega}_P - (\vec{r}.\vec{\omega}_P)\vec{r}) \; dA = \int_P (\vec{r}.\vec{r})\vec{\omega}_P \; dA - \int_P (\vec{r}.\vec{\omega}_P)\vec{r}\, dA$$

- Assuming a spherical Earth of unit radius (*r* = *1*), the first term introduces the plate area A_P:

$$\int_P (\vec{r}.\vec{r})\vec{\omega}_P \; dA = r^2\vec{\omega}_P \int_P dA = \vec{\omega}_P A_P$$

- Dealing with the second term is a bit more involved, see next.

The NNR reference system

$$(\vec{r}.\vec{\omega}_P)\vec{r} = (x_1\omega_1 + x_2\omega_2 + x_3\omega_3)\vec{r}$$

$$= \begin{bmatrix} x_1^2\omega_1 + x_1x_2\omega_2 + x_1x_3\omega_3 \\ x_1x_2\omega_1 + x_2^2\omega_2 + x_2x_3\omega_3 \\ x_1x_3\omega_1 + x_2x_3\omega_2 + x_3^2\omega_3 \end{bmatrix}$$

$$= \begin{bmatrix} x_1^2 & x_1x_2 & x_1x_3 \\ x_1x_2 & x_2^2 & x_2x_3 \\ x_1x_3 & x_2x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Therefore: $$\int_P (\vec{r}.\vec{\omega}_P)\vec{r}\, dA =$$

$$\begin{bmatrix} \int x_1^2 & \int x_1x_2 & \int x_1x_3 \\ \int x_1x_2 & \int x_2^2 & \int x_2x_3 \\ \int x_1x_3 & \int x_2x_3 & \int x_3^2 \end{bmatrix} \vec{\omega}_P .dA$$

We introduce a 3x3 symmetric matrix S_p with elements defined by: $$S_{Pij} = \int_P (x_i.x_j)\; dA$$

Therefore the integral becomes: $$\int_P (\vec{r}.\vec{\omega}_P)\vec{r}\, dA = S_P\; \vec{\omega}_P$$

The NNR reference system

- Finally: $L_P = \int_P (\vec{r}.\vec{r})\vec{\omega}_P \, dA - \int_P (\vec{r}.\vec{\omega}_P)\vec{r} \, dA$
- Reduces to: $$\begin{aligned} L_P &= \vec{\omega}_P A_P - S_P \vec{\omega}_P \\ &= \left(A_p I - S_p\right)\vec{\omega}_P \\ &= Q_P \vec{\omega}_P \end{aligned}$$
- With: $Q_P = A_P \ I - S_P$
- Q_p is a 3x3 matrix that only depends on the plate geometry, with its components defined by:

$$Q_{Pij} = \int_P \left(\delta_{ij} - x_i x_j\right) dA$$

Kronecker delta: $\delta_{ij} = \begin{cases} 1 & if \ i = j \\ 0 & if \ i \neq j \end{cases}$

The NNR reference system

- The non-rotation condition: $\int_S \vec{r} \times \vec{v} \; dA = \sum_P \int_P \vec{r} \times \vec{v} \; dA = \vec{0}$
- Becomes: $\sum_P Q_P \vec{\omega}_P = \vec{0}$
- Now, observations are relative plate motions, for instance plate *P* w.r.t. Pacific plate. Angular velocities are additive, one can then write:

$$\vec{\omega}_{P/NNR} = \vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR}$$

- Therefore: $\sum_P Q_P \left(\vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR} \right) = \vec{0}$

$$\Rightarrow \sum_P Q_P \; \vec{\omega}_{P/Pacific} + \sum_P Q_P \; \vec{\omega}_{Pacific/NNR} = \vec{0}$$

$$\Rightarrow \sum_P Q_P \; \vec{\omega}_{P/Pacific} + \frac{8\pi}{3} I \; \vec{\omega}_{Pacific/NNR} = \vec{0}$$

(because on a unit radius sphere: $\sum_P Q_P = \frac{8\pi}{3} I$)

The NNR reference system

- Finally, the angular velocity of the Pacific plate w.r.t. NNR can be calculated using:

$$\vec{\omega}_{Pacific/NNR} = -\frac{3}{8\pi}\sum_{P} Q_P \; \vec{\omega}_{P/Pacific}$$

($\omega_{p/\mathrm{Pacific}}$ are known from a relative plate model, Q_p are computed for each plate from its geometry)

- Once the angular velocity of the Pacific plate in NNR is found, the angular velocity of any plate P can be computed using:

$$\vec{\omega}_{P/NNR} = \vec{\omega}_{P/Pacific} + \vec{\omega}_{Pacific/NNR}$$

- This method is the one used to compute the NNR-NUVEL1A model (Argus and Gordon, 1991).

Summary

- Geodetic observations face datum defect problem => need for a reference frame.
- Reference frame in modern space geodesy best implemented using minimal constraints after combination with global solutions (unless regional solution sought).
- Once global position/velocity solution is obtained, question remains of how to express it in a frame independent from any plate = no-net-rotation frame, derived from Tisserand reference system.

The no-net-rotation (NNR) condition

- The NNR condition actually has a “dynamic” origin.
- First proposed by Lliboutry (1977) as an approximation of a reference frame where moment of forces acting on lower mantle is zero.
- In its original definition, this implies:
  - Rigid lower mantle
  - Uniform thickness lithosphere
  - No lateral viscosity variations in upper mantle

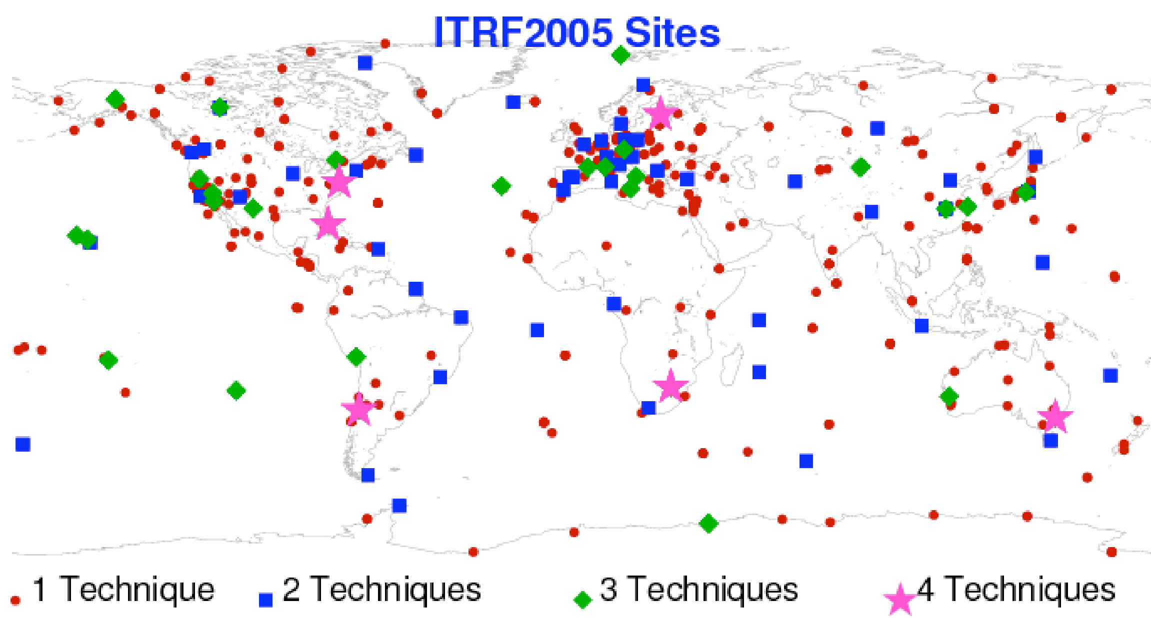
  $\Rightarrow$ NNR is a frame in which the internal dynamics of the mantle is null.
- These conditions are not realistic geophysically, in particular because of slabs in upper and lower mantle, that contribute greatly to driving plate motions (Lithgow-Bertelloni and Richards, 1995)
- But that’s ok, as long as NNR is simply used as a **<u>conventional</u>** reference.

The international Terrestrial Reference Frame: ITRF

- Positions (at a given epoch) and velocities of a set of geodetic sites (+ associated covariance information) = <u>dynamic</u> datum

- Positions and velocities estimated by combining independent geodetic solutions and techniques.

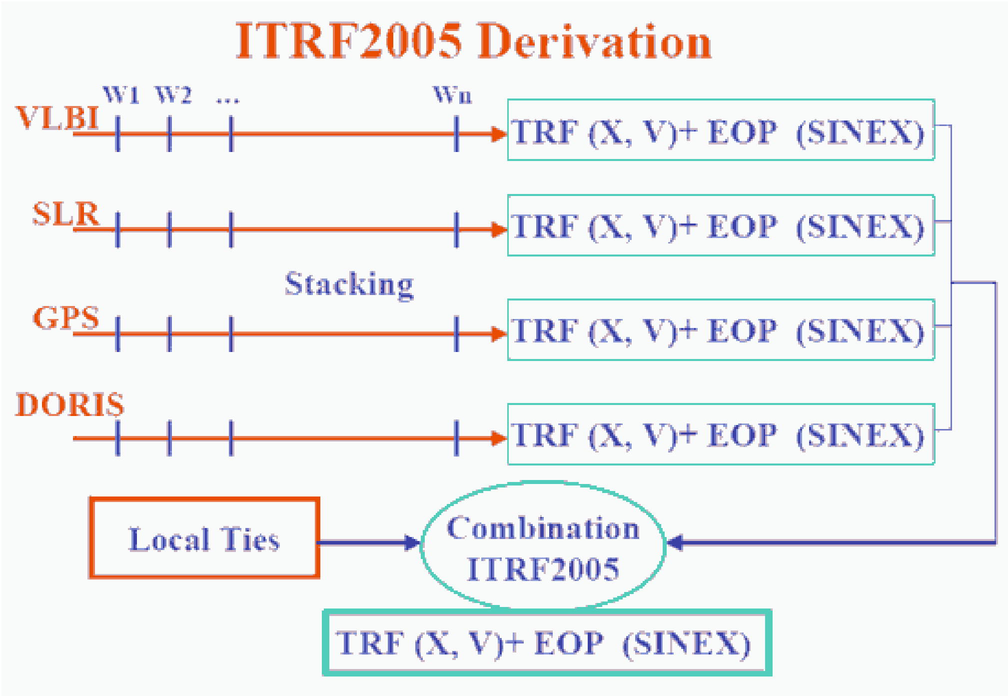
- Combination:
  - “Randomizes” systematic errors associated with each individual solutions
  - Provides a way of detecting blunders in individual solutions
  - Accuracy is equally important as precision



- 1984: VLBI, SLR, LLR, Transit
- 1988: TRF activity becomes part of the IERS => first ITRF = ITRF88
- Since then: ITRF89, ITRF90, ITRF92, ITRF93, ITRF94, ITRF96, ITRF97, ITRF2000
- **Current = ITRF2005:**
  - Up to 25 years of data
  - GPS sites defining the ITRF are all IGS sites
  - Wrms on velocities in the combination: 1 mm/yr VLBI, 1-3 mm/yr SLR and GPS
  - Solutions used: 3 VLBI, 1 LLR, 7 SLR, 6 GPS, 2 DORIS
- ITRF improves as:
  - Number of sites with long time series increases
  - New techniques appear
  - Estimation procedures are improved

The international Terrestrial Reference Frame: ITRF

- Apply minimum constraints equally to all loosely constrained solutions: this is the case of SLR and DORIS solutions
- Apply No-Net-Translation and No-Net-Rotation condition to IVS solutions provided under the form of Normal Equation
- Use as they are minimally constrained solutions: this is the case of IGS weekly solutions
- Form per-technique combinations (TRF + EOP), by rigorously staking the time series, solving for station positions, velocities, EOPs and 7 transformation parameters for each weekly (daily in case of VLBI) solution w.r.t the per-technique cumulative solution.
- Identify and reject/de-weight outliers and properly handle discontinuities using piece-wise approach.
- Combine if necessary cumulative solutions of a given technique into a unique solution: this is the case of the two DORIS solutions.
- Combine the per-technique combinations adding local ties in co-location sites.



The international Terrestrial Reference Frame: ITRF

- <u>Origin</u>: The ITRF2005 origin is defined in such a way that there are null translation parameters at epoch 2000.0 and null translation rates between the ITRF2005 and the ILRS SLR time series.

- <u>Scale</u>: The ITRF2005 scale is defined in such a way that there are null scale factor at epoch 2000.0 and null scale rate between the ITRF2005 and IVS VLBI time series.

- <u>Orientation</u>: The ITRF2005 orientation is defined in such a way that there are null rotation parameters at epoch 2000.0 and null rotation rates between the ITRF2005 and ITRF2000. These two conditions are applied over a core network

ITRF in practice

- Multi-technique combination.
- Origin = SLR, scale = VLBI, orientation = all.
- Position/velocity solution.
- Velocities expressed in no-net-rotation frame:
  - ITRF2000: minimize global rotation w.r.t. NNR-NUVEL1A using 50 high-quality sites far from plate boundaries
  - Subtlety: ITRF does not exactly fulfill a NNR condition because Nuvel1A is biased…
- Provided as tables (position, velocities, uncertainties)
- Full description provided as SINEX file (Solution Indepent Exchange format): ancillary information + vector of unknowns + full variance-covariance matrix (i.e. with correlations).

ITRF in practice

ITRF2005 STATION POSITIONS AT EPOCH 2000.0 AND VELOCITIES
GPS STATIONS

DOMES NB.	SITE NAME	TECH.	ID.	X/Vx	Y/Vy	Z/Vz	Sigmas			SOLN	DATA_START	DATA_END
				m/m/y								
10001S006	PARIS	GPS	OPMT	4202777.434	171367.913	4778660.147	0.005	0.002	0.006			
10001S006				-.0118	0.0170	0.0111	.0011	.0004	.0012			
10002M006	GRASSE	GPS	GRAS	4581690.969	556114.738	4389360.731	0.001	0.000	0.001	1	00:000:00000	03:113:00000
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10002M006	GRASSE	GPS	GRAS	4581690.975	556114.741	4389360.734	0.001	0.000	0.001	2	03:113:00000	04:295:43200
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10002M006	GRASSE	GPS	GRAS	4581690.974	556114.744	4389360.739	0.001	0.001	0.001	3	04:295:43200	00:000:00000
10002M006				-.0139	0.0186	0.0116	.0001	.0001	.0001			
10003M004	TOULOUSE	GPS	TOUL	4627846.086	119629.236	4372999.754	0.001	0.000	0.001			
10003M004				-.0111	0.0191	0.0117	.0003	.0001	.0003			
10003M009	TOULOUSE	GPS	TLSE	4627851.889	119639.921	4372993.492	0.001	0.001	0.001			
10003M009				-.0111	0.0191	0.0117	.0003	.0001	.0003			
10004M004	BREST	GPS	BRST	4231162.638	-332746.764	4745130.859	0.004	0.001	0.004			
10004M004				-.0111	0.0162	0.0134	.0009	.0003	.0009			
10023M001	La Rochelle	GPS	LROC	4424632.623	-94175.321	4577544.022	0.003	0.001	0.003			
10023M001				-.0106	0.0183	0.0123	.0006	.0002	.0006			
10090M001	SAINT JEAN DES	GPS	SJDV	4433469.919	362672.729	4556211.652	0.002	0.001	0.002	1	00:000:00000	99:071:57600
10090M001				-.0118	0.0186	0.0121	.0008	.0002	.0008			
10090M001	SAINT JEAN DES	GPS	SJDV	4433469.921	362672.729	4556211.656	0.001	0.000	0.001	2	99:071:57600	00:000:00000
10090M001				-.0118	0.0186	0.0121	.0008	.0002	.0008			
10202M001	REYKJAVIK	GPS	REYK	2587384.422	-1043033.508	5716563.995	0.001	0.000	0.001	1	00:000:00000	00:169:56460
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M001	REYKJAVIK	GPS	REYK	2587384.410	-1043033.501	5716563.980	0.006	0.003	0.012	2	00:169:56460	00:173:03120
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M001	REYKJAVIK	GPS	REYK	2587384.415	-1043033.509	5716564.003	0.001	0.000	0.001	3	00:173:03120	00:000:00000
10202M001				-.0216	-.0028	0.0059	.0001	.0001	.0002			
10202M003	REYKJAVIK	GPS	REYZ	2587383.736	-1043032.722	5716564.472	0.001	0.001	0.001			
10202M003				-.0216	-.0028	0.0059	.0001	.0001	.0002			

ITRF in practice