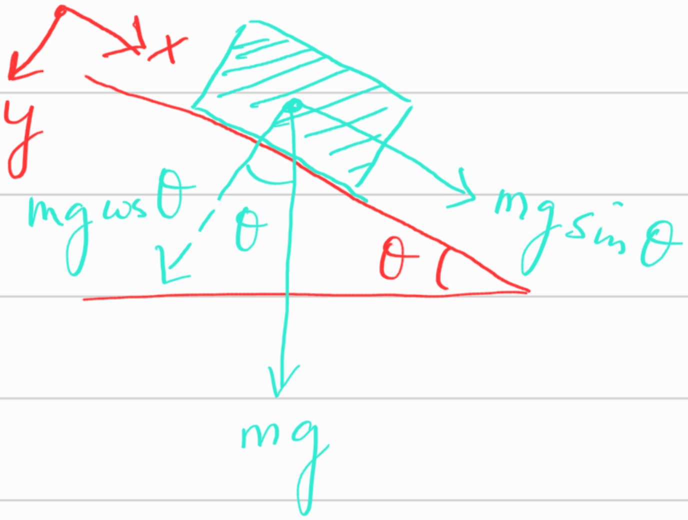
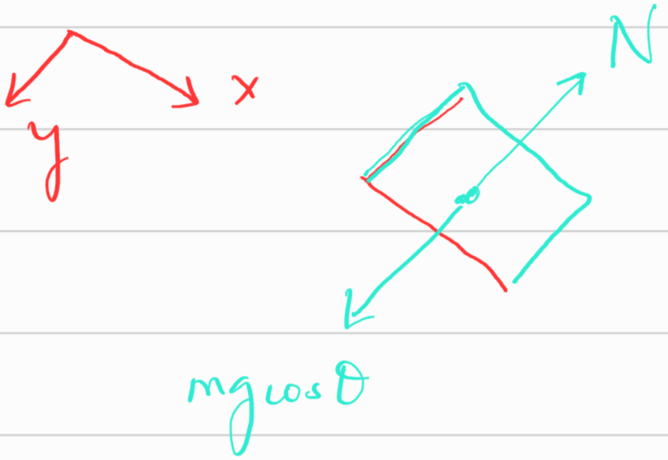
Friction co-efficients & angles



Force balance on a block sliding down an inclined ramp.

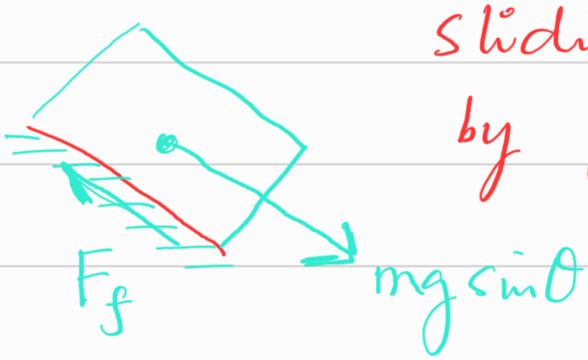
consider force balance in y-direction



Normal force balances the weight of the block

$N = mg\cos\theta$

Force balance in x-direction



Sliding resistance is provided by friction. For the block to remain stationary,

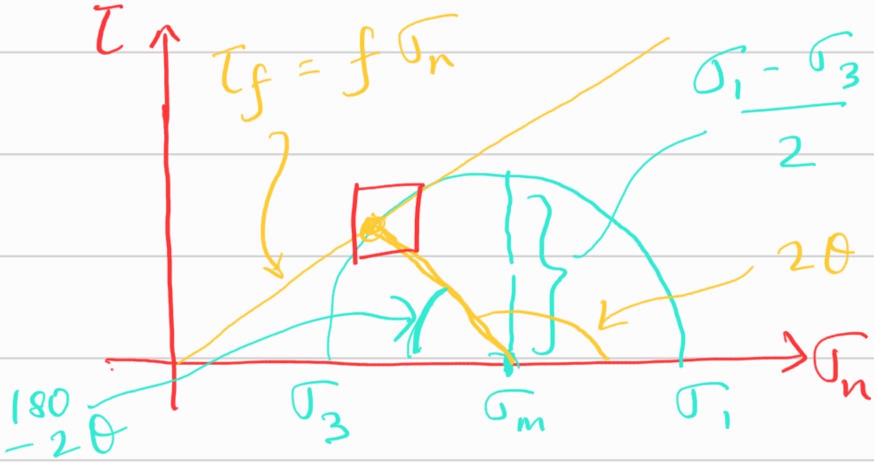
$mg\sin\theta - F_f < 0$.

Sliding initiates when $F_f = mg\sin\theta$

$F_f = f \cdot N$

$\Rightarrow \boxed{f = \tan\theta}$ ← we can estimate the friction co-efficient by finding the angle at which the block starts sliding

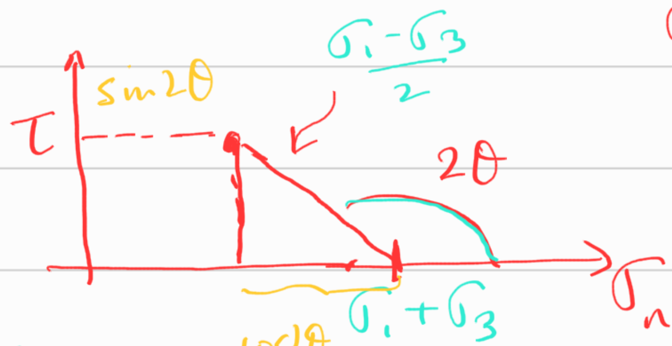
Friction from fault orientations



Failure criterion

$$\tau = f\sigma_n$$

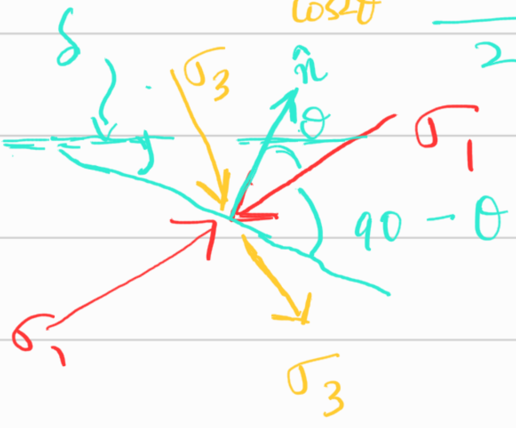
Intersection of the failure criterion & the stress state gives us the optimal orientation of faults.



$$\sigma_n = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta$$

$$\tau = \frac{\sigma_1-\sigma_3}{2}\sin 2\theta$$

stress state for a fault with orientation θ to σ_1



Applying failure criterion,

$$\frac{\sigma_1-\sigma_3}{2}\sin 2\theta = f\left(\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\theta\right)$$

$$\frac{\sigma_1-\sigma_3}{2}\left[\sin 2\theta - f\cos 2\theta\right] = f\sigma_m$$

$$(\sigma_1-\sigma_3) = \frac{2f\sigma_m}{(\sin 2\theta - f\cos 2\theta)}$$

To find optimum $(\sigma_1-\sigma_3)$ for any given θ,
set $\frac{d(\sigma_1-\sigma_3)}{d\theta} = 0$

$$-2f\sigma_m\left(\frac{\overbrace{2\cos 2\theta + 2f\sin 2\theta}^{\text{must be } 0}}{(\sin 2\theta - f\cos 2\theta)^2}\right) = 0$$

$$\cos 2\theta + f\sin 2\theta = 0$$

$$\boxed{f = -\frac{1}{\tan 2\theta}}$$

negative sign because (2θ) is in second quadrant. when $\tan 2\theta < 0$