Introduction to Solid Mechanics

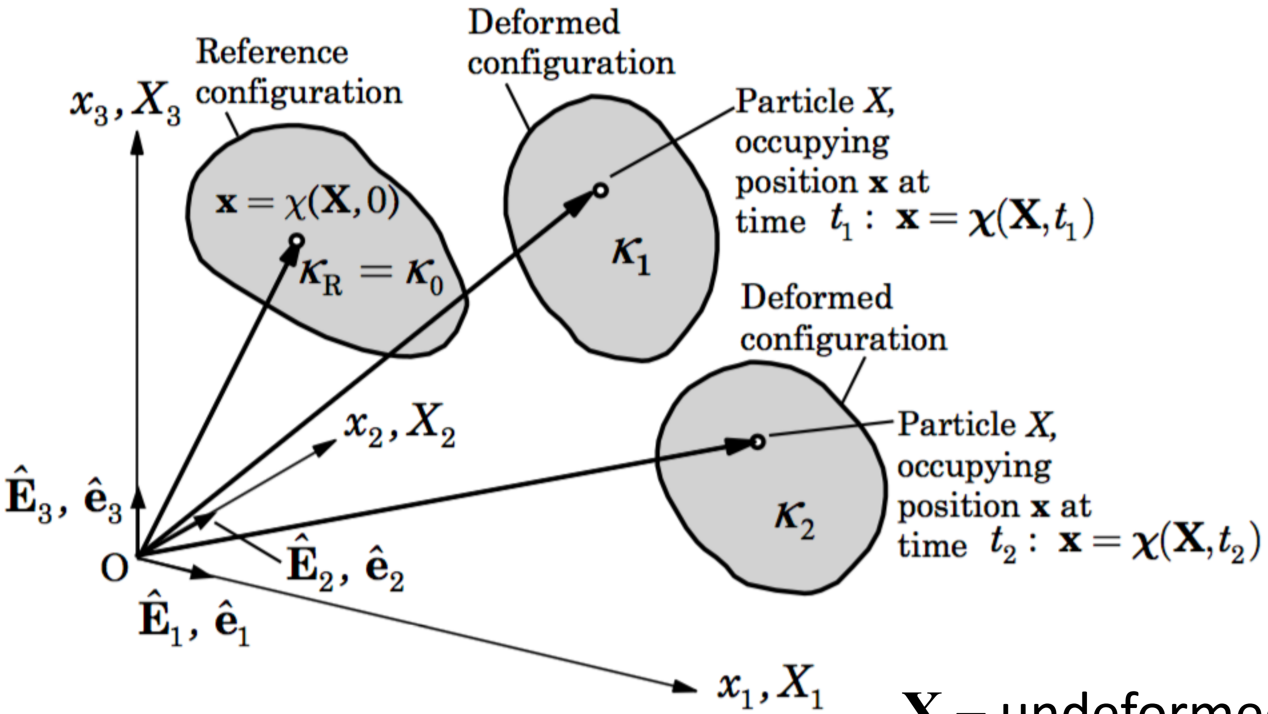
1. Displacements and deformation
2. Strains and rotations

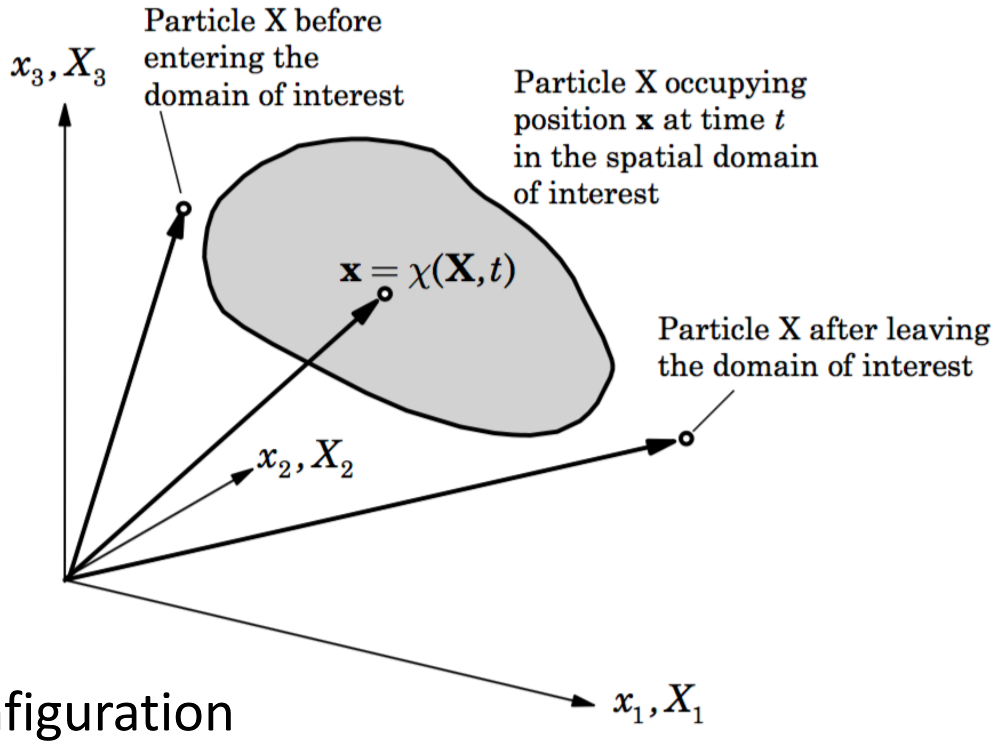
Displacements and deformations

Material and Spatial descriptions

$\mathbf{x} = \chi(\mathbf{X},t)$

$\mathbf{X} = \mathbf{X}(\mathbf{x},t) = \chi^{-1}(\mathbf{x},t)$





Motion described in terms of $\mathbf{X}$

$\mathbf{X}$ – undeformed configuration

$\mathbf{x}$ – deformed configuration

Motion described in terms of $\mathbf{x}$

Displacements and deformations

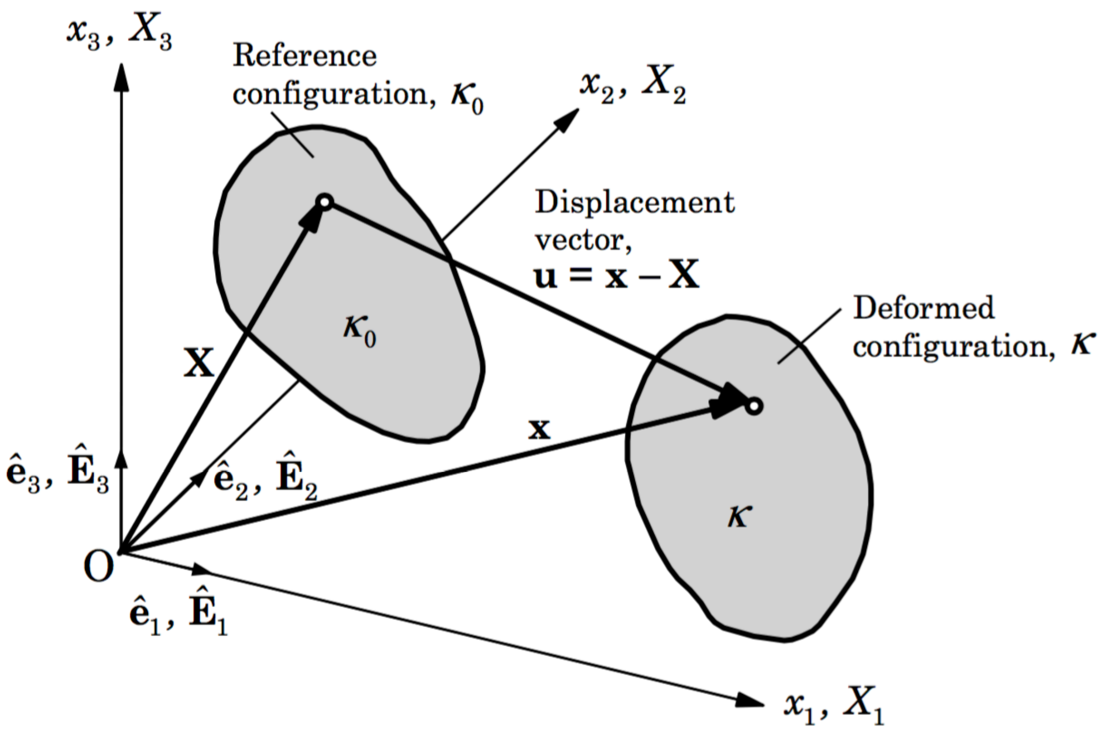
Displacement field

$$\mathbf{u}(\mathbf{X},t) = \mathbf{x}(\mathbf{X},t) - \mathbf{X}$$

Lagrangian – GPS, levelling, weather balloons, device moves with the medium

$$\mathbf{u}(\mathbf{x},t) = \mathbf{x} - \mathbf{X}(\mathbf{x},t)$$

Eulerian – pitot tubes, air temp, OBP, common in fluid mechanics (maybe InSAR)



Displacements and deformations

In typical geophysical studies, displacements are small such that the material and spatial descriptions are nearly identical. But in geodynamics, where we simulate large deformations and changing configurations, this distinction is important.

When we calculate the material derivative of some quantity that is being advected, we get -

advection

$$\frac{D\phi(\mathbf{x},t)}{Dt} = \frac{\partial\phi(\mathbf{x},t)}{\partial t} + \frac{Dx_i}{Dt}\frac{\partial\phi(\mathbf{x},t)}{\partial x_i}$$

Time derivative for a particle
(using spatial description – **x**, not **X**)

Differing derivatives based on coordinate system

We start with a square block of side 1 units, with its BL edge at (0,0). Motion in the medium is described by the mapping

$$\chi(\mathbf{X}, t) = \mathbf{x} = (X_1 + AtX_2)\hat{\mathbf{e}}_1 + (X_2 - AtX_1)\hat{\mathbf{e}}_2 \tag{1}$$

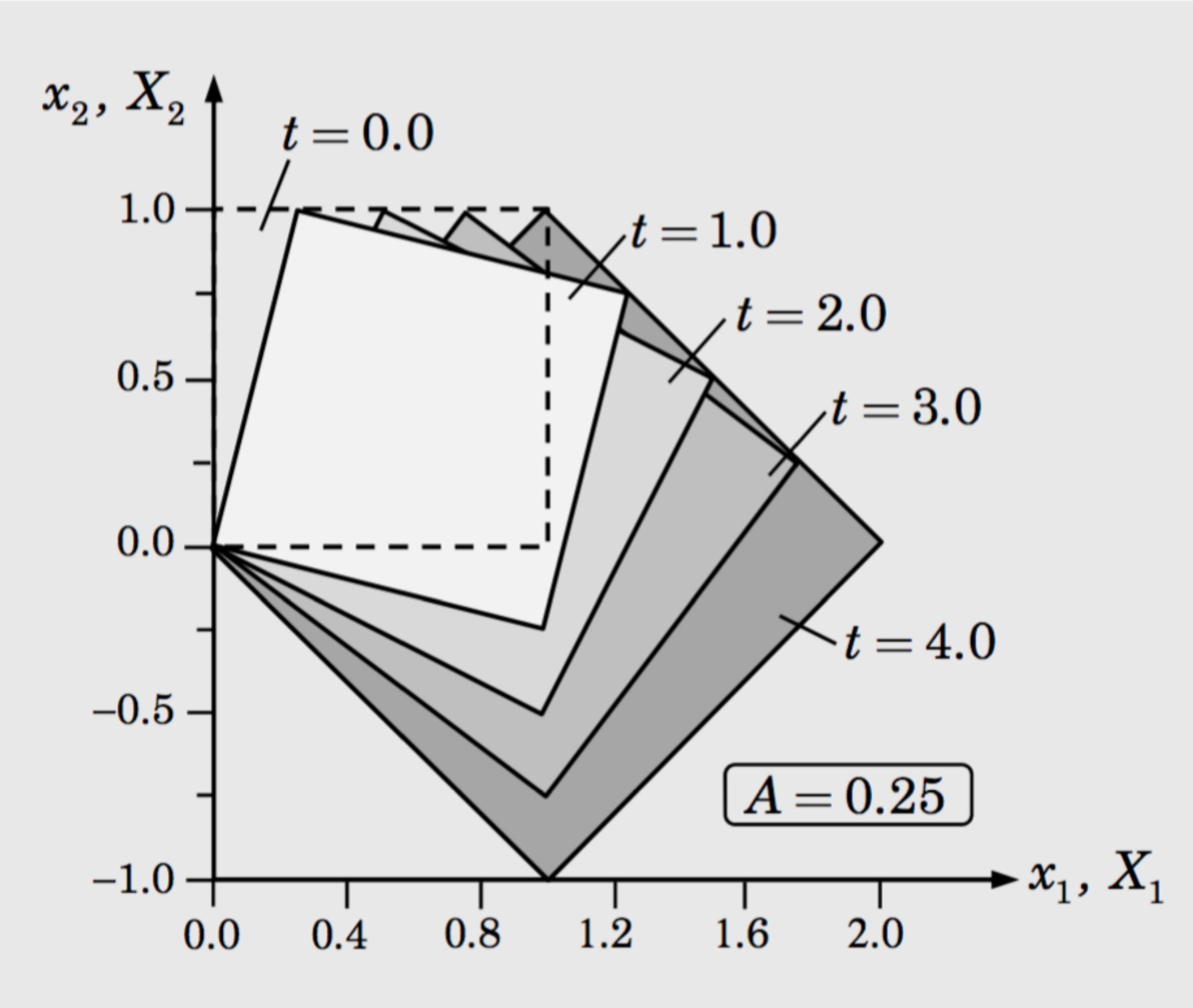
The temperature in the continuum is known in the spatial description as

$$T(\mathbf{x}, t) = x_1 + tx_2 \tag{2}$$

Determine the velocity components $\left(\frac{D\mathbf{x}}{Dt}\right)$, and the total time derivatives of T in the spatial and material description.

Differing derivatives based on coordinate system

(X_1, X_2, X_3)		(x_1, x_2, x_3)
$(0,0,0)$	$\rightarrow$	$(0,0,0)$
$(1,0,0)$	$\rightarrow$	$(1,-At,0)$
$(0,1,0)$	$\rightarrow$	$(At,1,0)$
$(1,1,0)$	$\rightarrow$	$(1+At,1-At,0)$



$$\frac{D}{Dt}\left(T(\mathbf{X},t)\right) = \frac{D}{Dt}\left(X_1 + (A+1)tX_2 - At^2X_1\right) = (A+1)X_2 - 2AtX_1$$

$$\frac{D}{Dt}\left(T(\mathbf{x},t)\right) = x_2 + v_1 + v_2 t = (A+1)X_2 - 2AtX_1$$