$$\underline{\sigma_s} = \hat{n}\cdot\sigma - (\hat{n}\cdot\sigma\cdot\hat{n})\,\hat{n}$$

$$\begin{bmatrix} \sigma_{22} & \sigma_{23} \\ \sigma_{23} & \sigma_{33} \end{bmatrix} = [\sigma] \qquad \hat{n} = \begin{bmatrix} n_2 \\ n_3 \end{bmatrix}$$

$$\hat{n}\cdot\sigma = \begin{bmatrix} \sigma_{22} n_2 + \sigma_{23} n_3 \\ \sigma_{23} n_2 + \sigma_{33} n_3 \end{bmatrix}$$

$$\hat{n}\cdot\sigma\cdot\hat{n} = \left(\sigma_{22} n_2^2 + 2\sigma_{23} n_2 n_3 + \sigma_{33} n_3^2\right)$$

$$(\hat{n}\cdot\sigma\cdot\hat{n})\,\hat{n} = \begin{bmatrix} \sigma_{22} n_2^3 + 2\sigma_{23} n_2^2 n_3 + \sigma_{33} n_2 n_3^2 \\ \sigma_{22} n_2^2 n_3 + 2\sigma_{23} n_2 n_3^2 + \sigma_{33} n_3^3 \end{bmatrix}$$

$$\underline{\sigma_s} = \begin{bmatrix} \sigma_{22}(n_2 - n_2^3) + \sigma_{23}(n_3 - 2n_2^2 n_3) - \sigma_{33} n_2 n_3^2 \\ -\sigma_{22} n_2^2 n_3 + \sigma_{23}(n_2 - 2n_2 n_3^2) + \sigma_{33}(n_3 - n_3^3) \end{bmatrix}$$

$$\begin{bmatrix} s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} n_2 - n_2^3 & n_3 - 2n_2^2 n_3 & -n_2 n_3^2 \\ -n_2^2 n_3 & n_2 - 2n_2 n_3^2 & n_3 - n_3^3 \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \\ \sigma_{33} \end{bmatrix}$$

subject to $\underline{\underline{\sigma_{22} = -\sigma_{33}}}$ i.e $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \\ \sigma_{33} \end{bmatrix} = 0$

$$\Big[s \quad \Big] \Big[n_2 - n_2^3 + n_2 n_3^2 \qquad n_3 - 2n_2^2 n_3 \quad \Big] \Big[\sigma_{22} \quad \Big]$$

$$\boxed{\begin{bmatrix} s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} n_2 \cdots & \cdots \\ n_3^3 - n_3 - n_2^2 n_3 & n_2 - 2n_2 n_3^2 \end{bmatrix} \begin{bmatrix} \cdots \\ \sigma_{23} \end{bmatrix}}$$

linear equation b/w deviatoric stress state & slip vectors

recall $\begin{bmatrix} n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}$, $\underline{\cos^2\delta - \sin^2\delta = \cos 2\delta}$

$$\begin{bmatrix} s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \sin\delta(1 - \sin^2\delta + \cos^2\delta) & \cos\delta(1 - 2\sin^2\delta) \\ \cos\delta(\cos^2\delta - 1 - \sin^2\delta) & \sin\delta(1 - 2\cos^2\delta) \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \end{bmatrix}$$

$$= \begin{bmatrix} 2\sin\delta\cos^2\delta & \cos\delta(\cos^2\delta - \sin^2\delta) \\ -2\cos\delta\sin^2\delta & \sin\delta(\sin^2\delta - \cos^2\delta) \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 2\sin\delta\cos^2\delta & \cos\delta(\cos^2\delta - \sin^2\delta) \\ -2\sin^2\delta\cos\delta & -\sin\delta(\cos^2\delta - \sin^2\delta) \end{bmatrix} \begin{bmatrix} \sigma_{22} \\ \sigma_{23} \end{bmatrix}}$$

if the slip vector is only along the fault plane,

$$\begin{bmatrix} s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \cos\delta \\ -\sin\delta \end{bmatrix}$$

i.e. $\begin{cases} \cos\delta = 2\sin\delta\cos^2\delta\,\sigma_{22} + \cos\delta(\cos^2\delta - \sin^2\delta)\,\sigma_{23} \\ -\sin\delta = -2\sin^2\delta\cos\delta\,\sigma_{22} - \sin\delta(\cos^2\delta - \sin^2\delta)\,\sigma_{23} \end{cases}$

add

$$(\cos\delta - \sin\delta) = \sigma_{22}\left(\overbrace{2\sin\delta\cos^2\delta - 2\sin^2\delta\cos\delta}^{2\sin\delta\cos\delta(\cos\delta - \sin\delta)}\right)$$

$$\cdots (\cos\delta - \sin\delta)$$

$$+ \sigma_{23} \left(\cos^2\delta - \sin^2\delta\right) \left(\cos\delta \ldots\right)$$

divide by

$$1 = \sigma_{22} \left(2 \sin\delta \cos\delta\right) + \sigma_{23} \left(\cos^2\delta - \sin^2\delta\right)$$

$$1 = \sigma_{22} \sin 2\delta + \sigma_{23} \cos 2\delta$$

$$\Rightarrow \boxed{\sigma_{23} = \sec 2\delta - \sigma_{22} \tan 2\delta}$$

$$\cos\delta = 2 \sin\delta \cos^2\delta\, \sigma_{22} + \cos\delta \cos 2\delta \left[\frac{1}{\cos 2\delta} - \sigma_{22} \tan 2\delta\right]$$

$$\begin{cases} \cancel{\cos\delta} = \cancel{\sin 2\delta \cos\delta\, \sigma_{22}} + \cancel{\cos\delta} - \cancel{\sigma_{22} \sin 2\delta \cos\delta} \\ \cancel{\sin\delta} = -\cancel{\sin 2\delta \sin\delta\, \sigma_{22}} - \sin\delta \cos 2\delta \left(\frac{1}{\cos 2\delta} - \sigma_{22} \frac{\sin 2\delta}{\cos 2\delta}\right) \\ \qquad\qquad - \cancel{\sin\delta} + \cancel{\sigma_{22} \sin 2\delta \sin\delta} \end{cases}$$

$\rightarrow$ satisfied trivially

$\Rightarrow$ only <u>1 independent equation</u>