Value Iteration Algorithm

The Bellman equation is the basis of the value iteration algorithm for solving MDPs.

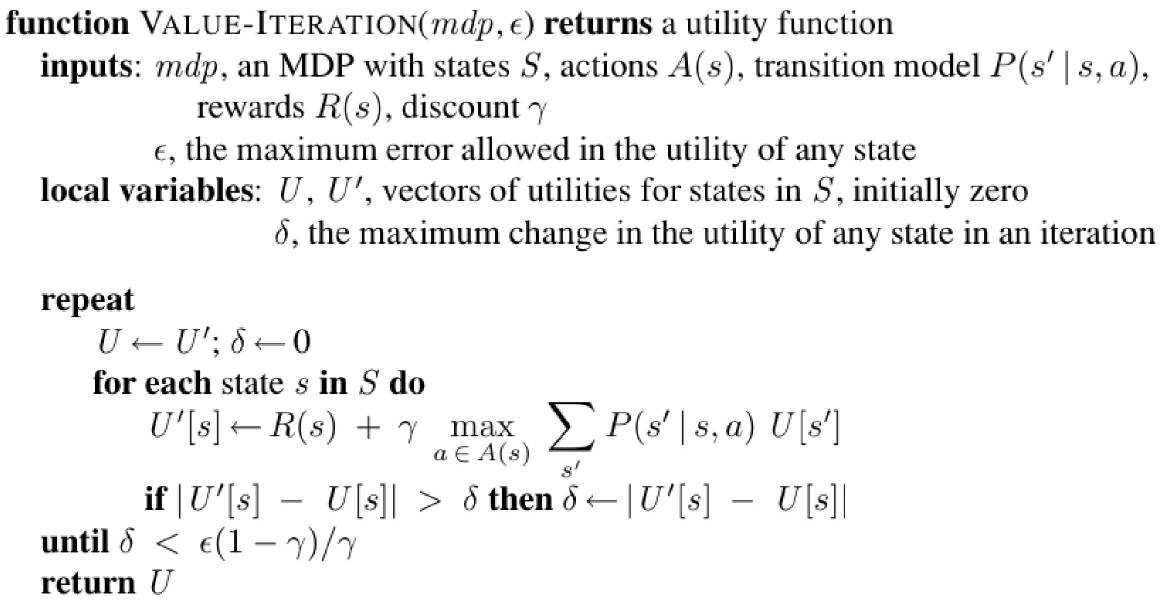
Let Ut(I) be the utility value for state s at the t'th iteration. The iteration step, called a Bellman update, looks like this:

$$U_{t+1}(I) = \max_{A} \left[R(I, A) + \sum_{J} P(J \mid I, A) \cdot U_t(J) \right]$$

Where the Rewards R(I,A) for each state is the expected reward of taking action A in State I. That is :-

$$R(I, A) = \sum_{J} P(J \mid I, A) \cdot R(J, A, I)$$

The value iteration algorithm is as follows :-



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function VALUE-ITERATION($mdp, \epsilon$) returns a utility function
  inputs: $mdp$, an MDP with states $S$, actions $A(s)$, transition model $P(s' \mid s, a)$,
              rewards $R(s)$, discount $\gamma$
          $\epsilon$, the maximum error allowed in the utility of any state
  local variables: $U$, $U'$, vectors of utilities for states in $S$, initially zero
                   $\delta$, the maximum change in the utility of any state in an iteration

  repeat
     $U \leftarrow U'$; $\delta \leftarrow 0$
     for each state $s$ in $S$ do
        $U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\ U[s']$
        if $|U'[s] - U[s]| > \delta$ then $\delta \leftarrow |U'[s] - U[s]|$
  until $\delta < \epsilon(1 - \gamma)/\gamma$
  return $U$
```

TASK 1

> Step Costs : [-20,-20,-20]

> Gamma : 0.99

> Delta : 0.001

> Iterations : 126

From the task_1_trace.txt file obtained, we can make the following inferences about the policy :

- Whenever Lero has 0 stamina, the only optimal policy for that state is to RECHARGE
- Whenever Lero has 0 arrows, he can RECHARGE or DODGE but cannot SHOOT
- Most of the times, Lero is Risk Averse. That is he prefers to RECHARGE than to SHOOT in cases when his stamina = 50 instead of losing an arrow.
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TASK 2

PART 1

> Step Costs : [-0.25,-2.5,-2.5]

> Gamma : 0.99

> Delta : 0.001

> Iterations : 99

From the task_2_part_1_trace.txt file obtained we can make the following inferences about the policy :

- Since the step cose for the SHOOT action is less negative compared to the other actions, Lero now becomes Risk Seeking in situations where the Mighty Dragon's health is beatable.
- The SHOOT action is taken more number of times as compared to the case when all the 3 actions had the same PENALTY
- Convergence is faster. In only 99 iterations, the value iteration algorithm converges when compared to the 126 iterations in TASK 1

PART 2

> Step Costs : [-2.5,-2.5,-2.5]

> Gamma : 0.1

> Delta : 0.001

> Iterations : 4

From the task_2_part_2_trace.txt file obtained we can make the following inferences about the policy :

- All states converge with a policy in the final iteration with more or less same utilities.
- The RECHARGE policy is preferred over the others hence indicating a Risk Averse behaviour.
- Due to a very small discount factor (Gamma), the bellman iterations converge fast, within 4 iterations.

PART 3

> Step Costs : [-2.5,-2.5,-2.5]

> Gamma : 0.99

> Delta : 0.0000000001

> Iterations : 235

From the task_2_part_2_trace.txt file obtained we can make the following inferences about the policy :

- All states converge with a policy in the final iteration with more or less same utilities.
- The RECHARGE policy is preferred over the others hence indicating a Risk Averse behaviour.
- Due to a very small discount factor (Gamma), the bellman iterations converge fast, within 4 iterations.