

1) Objective function: $\arg \min \sum_{i,k} \pi_{ik} \|x_i - \mu_k\|^2$

E-step: $\pi_{i,k} = 1$ where $k = \arg \min_k \|x_i - \mu_k\|^2$

let a be k for previous iteration
let b be k for current iteration

$$\|x_i - \mu_a\| \geq \|x_i - \mu_b\|$$

We can say that changing cluster from a to b will only decrease objective function, thus E-step update on membership achieves the minimum objective given the current centroids.

M-step:
$$\mu_k = \frac{\sum_i x_i \pi_{ik}}{\sum_i \pi_{ik}}$$

taking partial derivative of the objective function and

$$\frac{\partial}{\partial \mu_k} \left[\sum_{i,k} \pi_{ik} \|x_i - \mu_k\|^2 \right] = 0$$

$$2 \sum_i \pi_{ik} (x_i - \mu_k) = 0$$

$$\sum_i \pi_{ik} x_i = \sum_i \pi_{ik} \mu_k$$

$$\mu_k = \frac{\sum_i \pi_{ik} x_i}{\sum_i \pi_{ik}}$$

= m step

As we can see, M-step update on centroids achieves the minimum objective given the current membership

No.

Date