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DSC-VI : Practical-08

Demonstration of the Runge-Kutta Method

1 Solve dy/dx = x, where at x=0, y=0

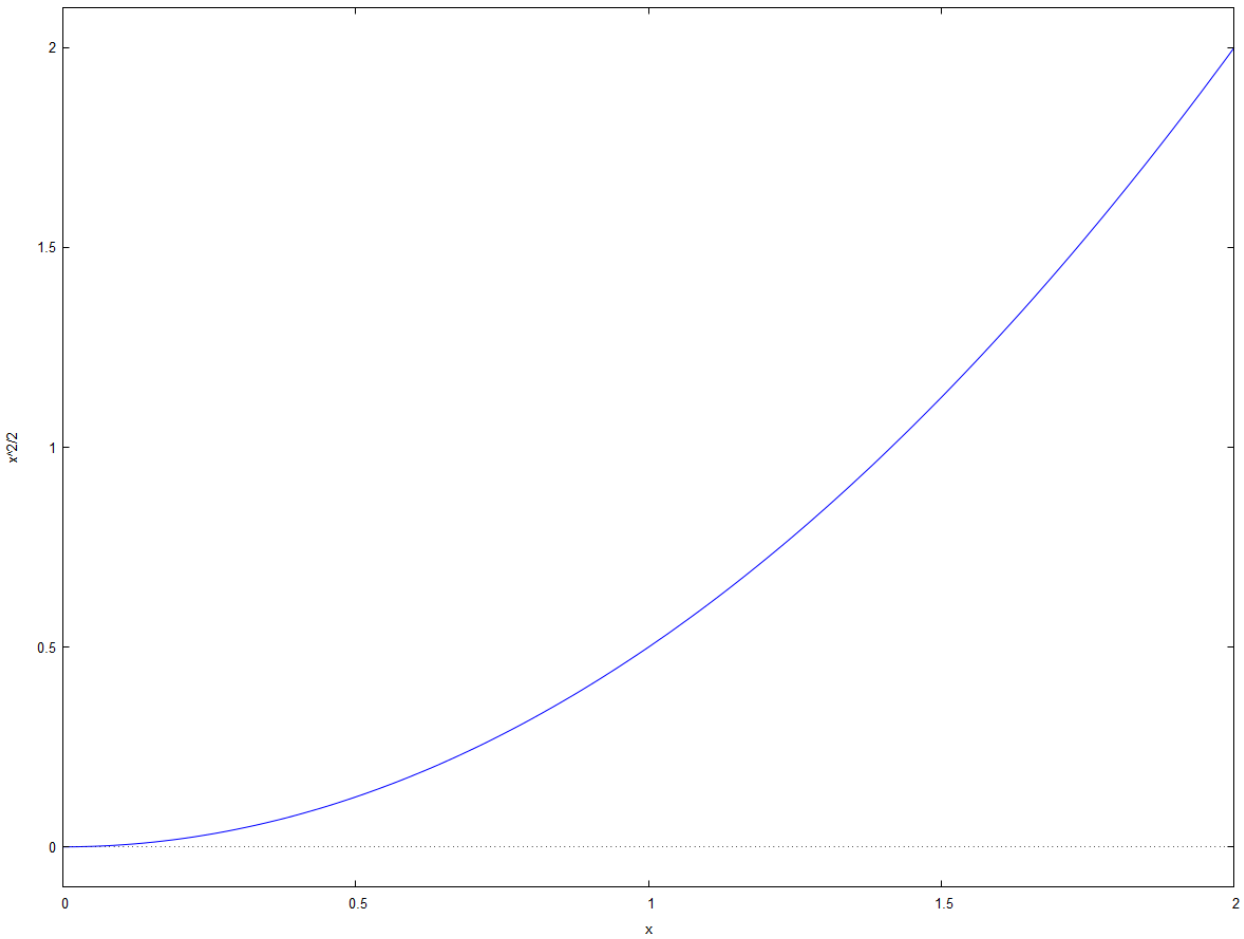
1.1 We solve the above differential equation using ode2():

```
--> eqn : ' diff ( y , x ) = x ;
    gs : ode2 ( eqn , y , x ) ;
    ps : ic1 ( gs , y = 0 , x = 0 ) ;
    wxplot2d ( rhs ( ps ) , [ x , 0 , 2 ] ) $
```

$$\frac{d}{dx} y = x$$

$$y = \frac{x^2}{2} + \%c$$

$$y = \frac{x^2}{2}$$



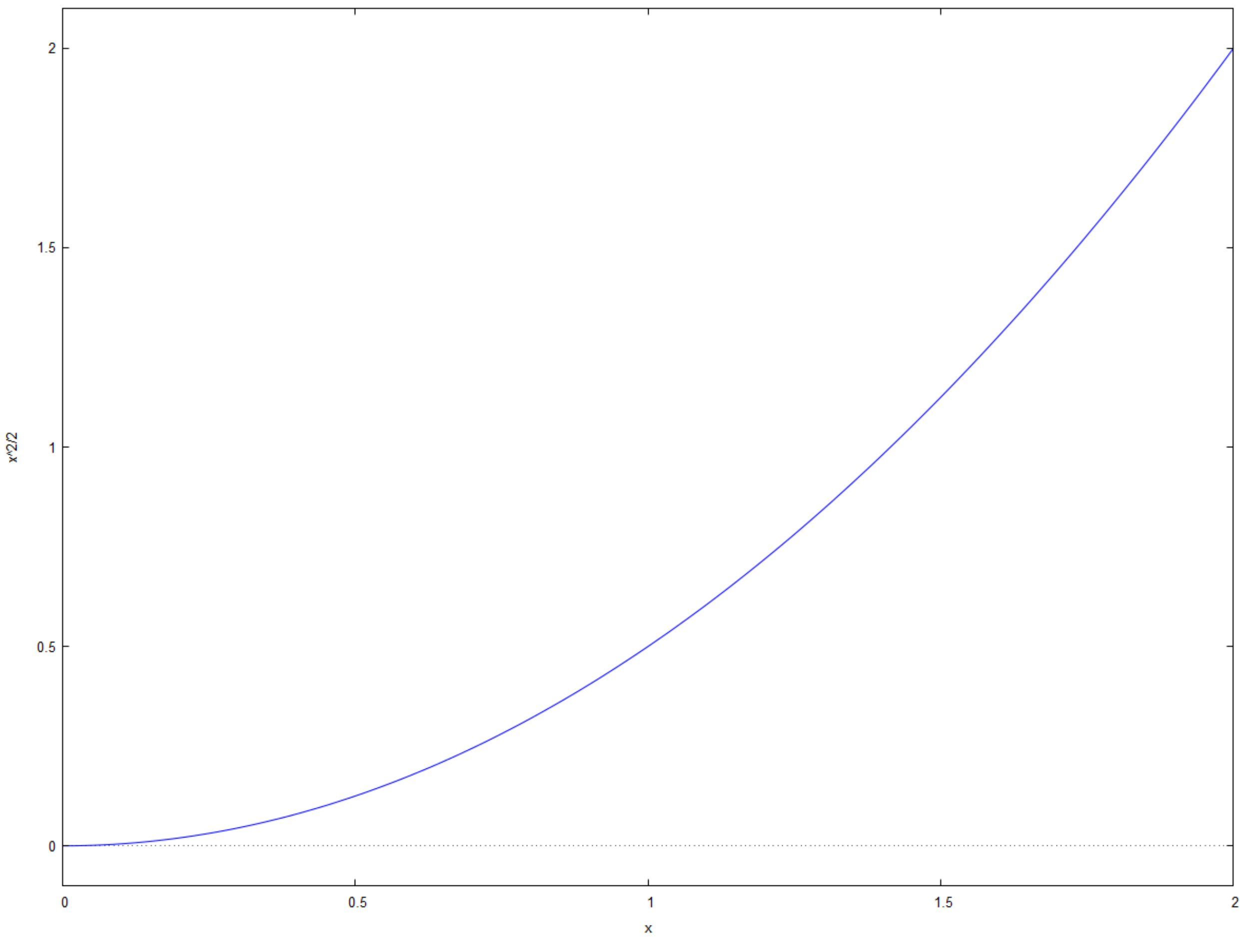
1.2 Now, we use desolve() to solve the same equation

```
--> kill ( all ) $
    eqn : diff ( y ( x ) , x ) = x ;
    gs : desolve ( eqn , y ( x ) ) ;
    ps : ev ( gs , y ( 0 ) = 0 ) ;
    wxplot2d ( [ rhs ( ps ) ] , [ x , 0 , 2 ] ) $
```

$$\frac{d}{dx}\mathrm{y}(x) = x$$

$$\mathrm{y}(x) = \frac{x^2}{2} + \mathrm{y}(0)$$

$$\mathrm{y}(x) = \frac{x^2}{2}$$

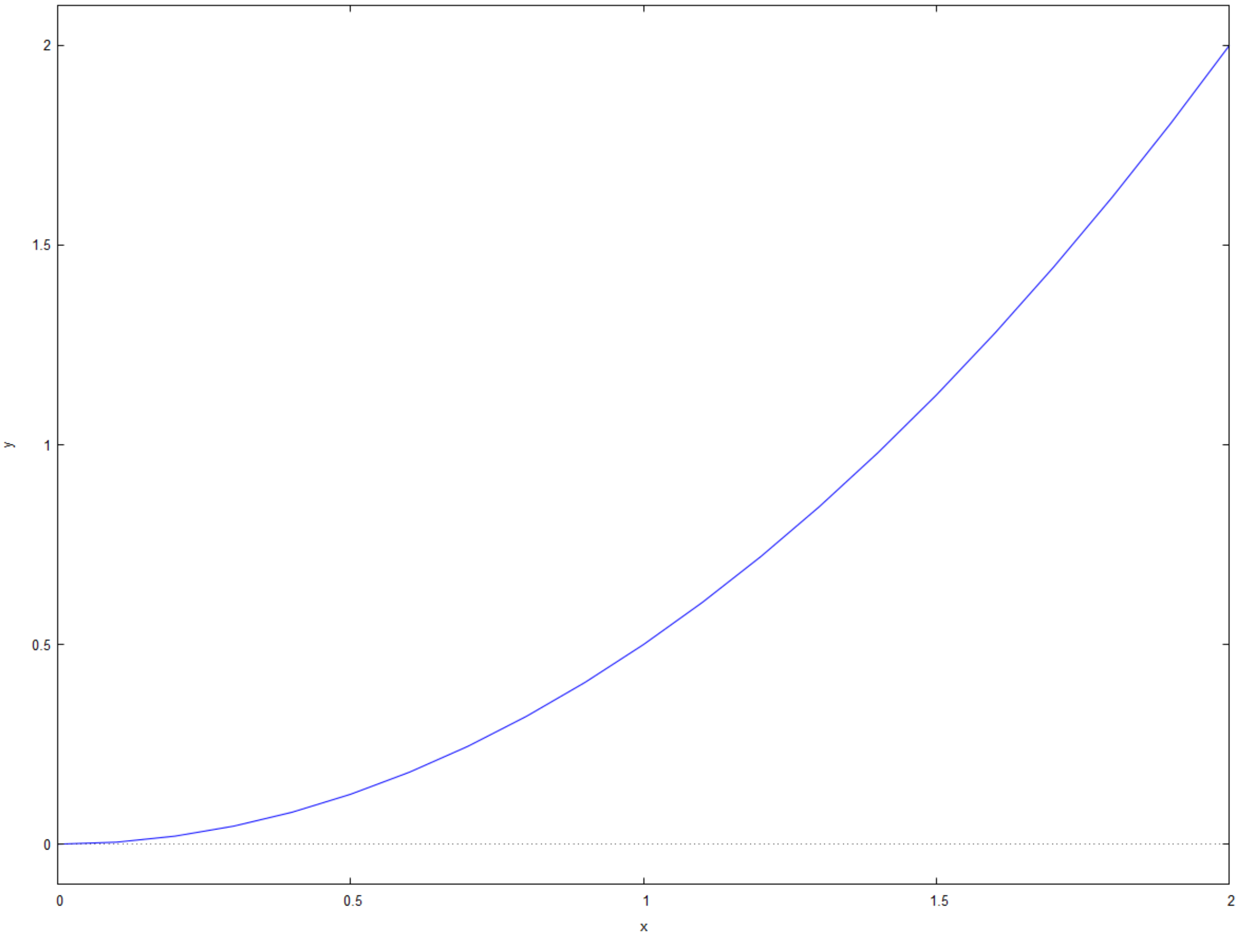


1.3 Finally, using the Runge-Kutta Method

Syntax:
rk([rhs(ODE_1),...,rhs(ODE_n)], [v1,...,vn], [init_1,...,init_n], domain)

```
--> kill ( all ) $
    eqn : ' diff ( y , x ) = x ;
    fsol : rk ( rhs ( eqn ) , [ y ] , [ 0 ] , [ x , 0 , 2 , 0 . 1 ] ) ;
    wxplot2d ( [ discrete , fsol ] ) $
```

$$\frac{d}{dx}y = x$$



2 We now have a coupled system of differential equations

dx/dt = 4 - x^2 - 4*y^2
dy/dt = y^2 - x^2 + 1

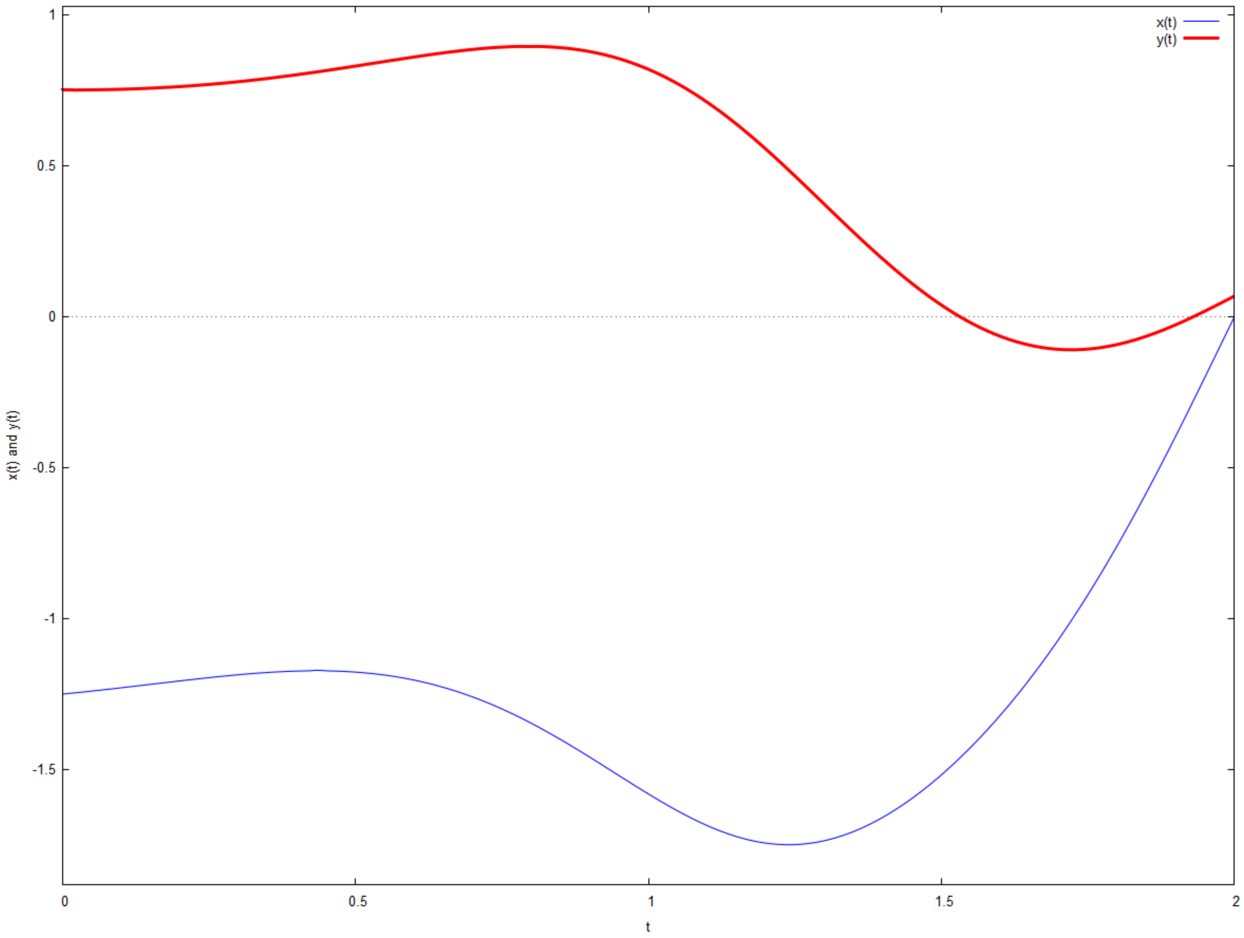
x(0) = -1.25, y(0) = 0.75

```
--> kill ( all ) $
    eqn1 : ' diff ( x , t ) = 4 − x ^ 2 − 4 · y ^ 2 ;
    eqn2 : ' diff ( y , t ) = y ^ 2 − x ^ 2 + 1 ;
    pts : rk ( [ rhs ( eqn1 ) , rhs ( eqn2 ) ] , [ x , y ] , [ − 1 . 25 , 0 . 75 ] , [ t , 0 , 2 , 0 . 01 ] ) $
```

$$\frac{d}{dt}x = -4y^2 - x^2 + 4$$

$$\frac{d}{dt}y = y^2 - x^2 + 1$$

```
--> curve_x : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 2 ] ] , i , 1 , length ( pts ) ) $
    /*wxplot2d([discrete, curve_x])$*/
    curve_y : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 3 ] ] , i , 1 , length ( pts ) ) $
    wxplot2d ( [ [ discrete , curve_x ] , [ discrete , curve_y ] ] ,
          [ legend , "x(t)" , "y(t)" ] ,
          [ style , [ lines , 1 ] , [ lines , 3 ] ] ,
          [ xlabel , "t" ] ,
          [ ylabel , "x(t) and y(t)" ] ) $
```



Created with wxMaxima.

The source of this Maxima session can be downloaded here.