Lucky Lao

DSC-VI : Practical-02

Family of Solutions: Second Order Differential Equations

We'll now plot the family of solutions of the following second order differential equations:

1 y'' + 3y' + 2y = 0 where y'(0)=b, y(0)=1, varying between -3 and 3

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-- /* We'll use 'ode2()' */
> ratprint : false $

  de : ' diff ( y , x , 2 ) + 3 · ' diff ( y , x ) + 2 · y = 0 ;
  gsol : ode2 ( de , y , x ) ; /* general soln. */
  psol : ic2 ( gsol , x = 0 , y = 1 , ' diff ( y , x ) = b ) ;    /* particular soln. */

  /* Fixing values for 'b' */
  v0 : ev ( psol , b = 0 ) $ /* suppress o/p */
  v1 : ev ( psol , b = 1 ) $
  v2 : ev ( psol , b = 2 ) $
  v3 : ev ( psol , b = 3 ) $
  v4 : ev ( psol , b = − 1 ) $
  v5 : ev ( psol , b = − 2 ) $
  v6 : ev ( psol , b = − 3 ) $

  /* Using 'wxplot2d()' to plot the family of solutions */
  wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) , rhs ( v5 ) , rhs ( v6 ) ] ,
       [ x , − 1 , 4 ] ,
       [ style , [ lines , 1 ] , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] , [ lines , 3 ] , [ lines , 3 . 5 ] , [
  lines , 4 ] ] ,
       [ legend , "Curve-0" , "Curve-1" , "Curve-2" , "Curve-3" , "Curve-4" , "Curve-5" , "Curve-6" ,
  "Curve-7" ] ) ;
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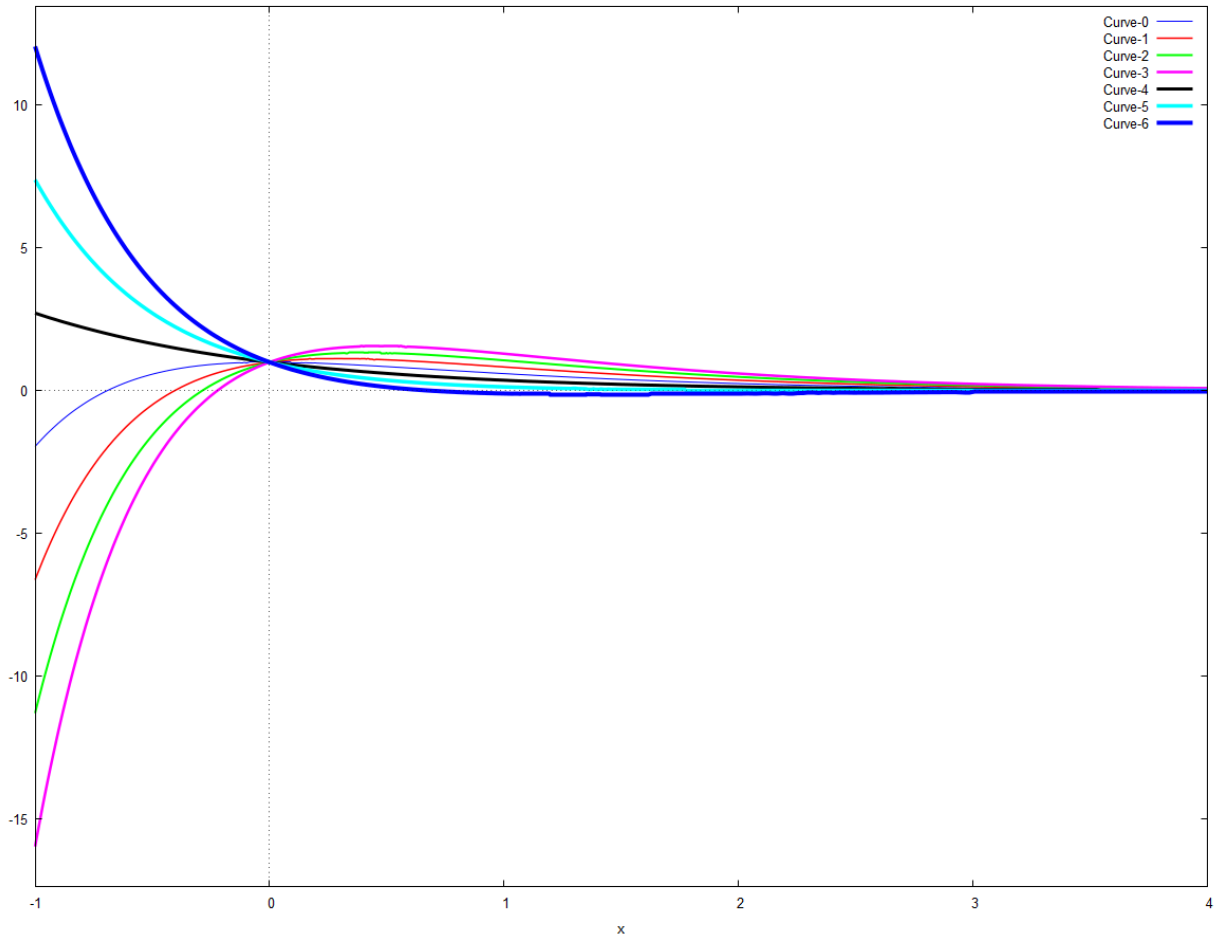
$$\frac{d^2}{dx^2}y + 3\left(\frac{d}{dx}y\right) + 2y = 0$$

$$y = \%k1\%e^{-x} + \%k2\%e^{-2x}$$

$$y = (b+2)\%e^{-x} + (-b-1)\%e^{-2x}$$



2 y'' + 3y' + 2y = 0 where y(0)=a, y'(0)=1

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--    /* We'll use 'ode2()' */
>     kill ( all ) $

      de : ' diff ( y , x , 2 ) + 3 · ' diff ( y , x ) + 2 · y = 0 ;
      gsol : ode2 ( de , y , x ) ; /* general soln. */
      psol : ic2 ( gsol , x = 0 , y = a , ' diff ( y , x ) = 1 ) ;    /* particular soln. */

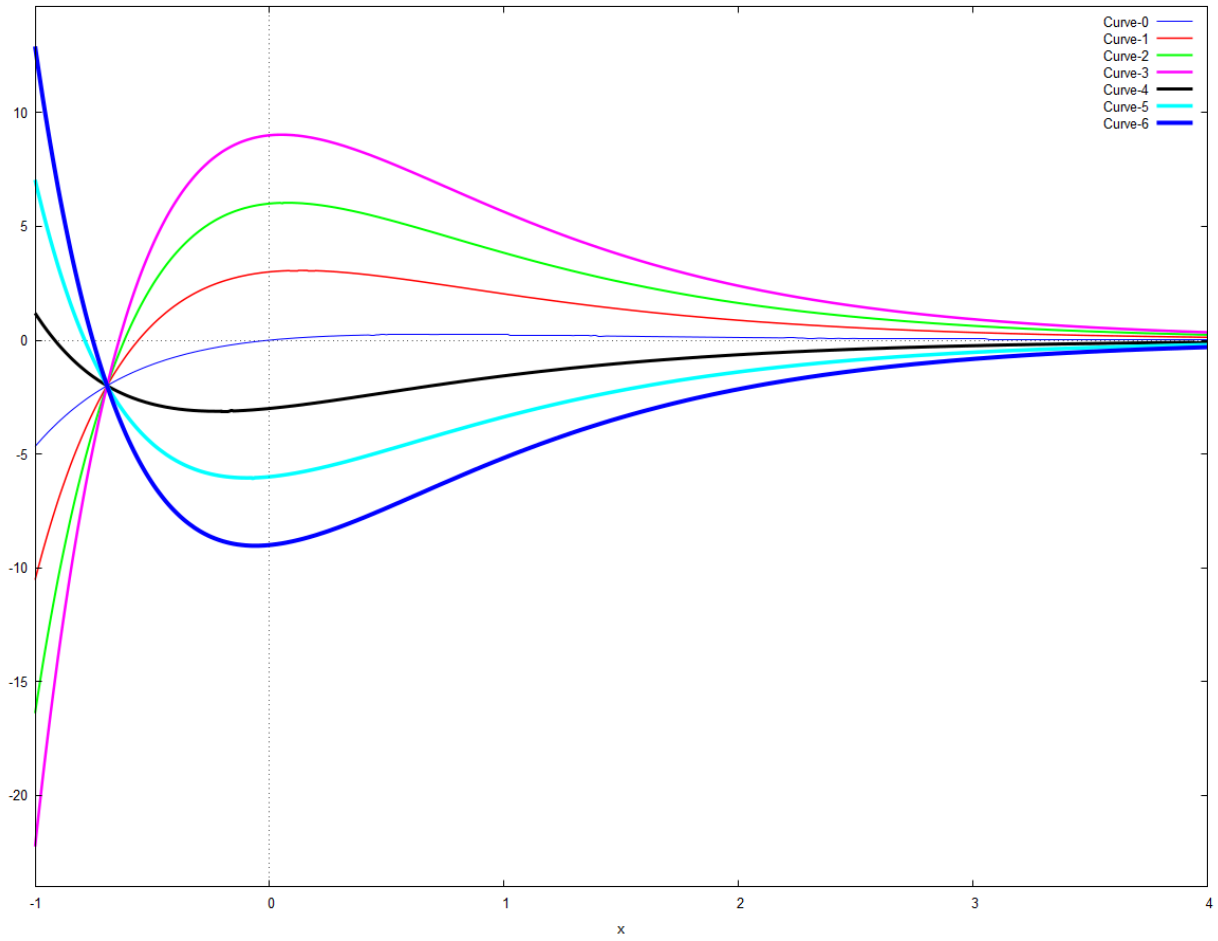
      /* Fixing values for 'a' */
      v0 : ev ( psol , a = 0 ) $ /* suppress o/p */
      v1 : ev ( psol , a = 3 ) $
      v2 : ev ( psol , a = 6 ) $
      v3 : ev ( psol , a = 9 ) $
      v4 : ev ( psol , a = − 3 ) $
      v5 : ev ( psol , a = − 6 ) $
      v6 : ev ( psol , a = − 9 ) $

      /* Using 'wxplot2d()' to plot the family of solutions */
      wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) , rhs ( v5 ) , rhs ( v6 ) ] ,
            [ x , − 1 , 4 ] ,
            [ style , [ lines , 1 ] , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] , [ lines , 3 ] , [ lines , 3 . 5 ] , [
      lines , 4 ] ] ,
            [ legend , "Curve-0" , "Curve-1" , "Curve-2" , "Curve-3" , "Curve-4" , "Curve-5" , "Curve-6" ,
      "Curve-7" ] ) ;
```

$$\frac{d^2}{dx^2}y + 3\left(\frac{d}{dx}y\right) + 2y = 0$$

$$y = \%k1\%e^{-x} + \%k2\%e^{-2x}$$

$$y = (2a+1)\%e^{-x} + (-a-1)\%e^{-2x}$$



3 Solving (1) using desolve()
y'' + 3y' +2y = 0 where y'(0)=b, y(0)=1, varying between -3 and 3

```
--> kill ( all ) $

    de : diff ( y ( x ) , x , 2 ) + 3 · diff ( y ( x ) , x ) + 2 · y ( x ) = 0 ;
    gsol : desolve ( de , y ( x ) ) ;    /* general soln. */
    psol : ev ( gsol , y ( 0 ) = 1 , diff ( y ( x ) , x ) = b ) ; /* particular soln. */

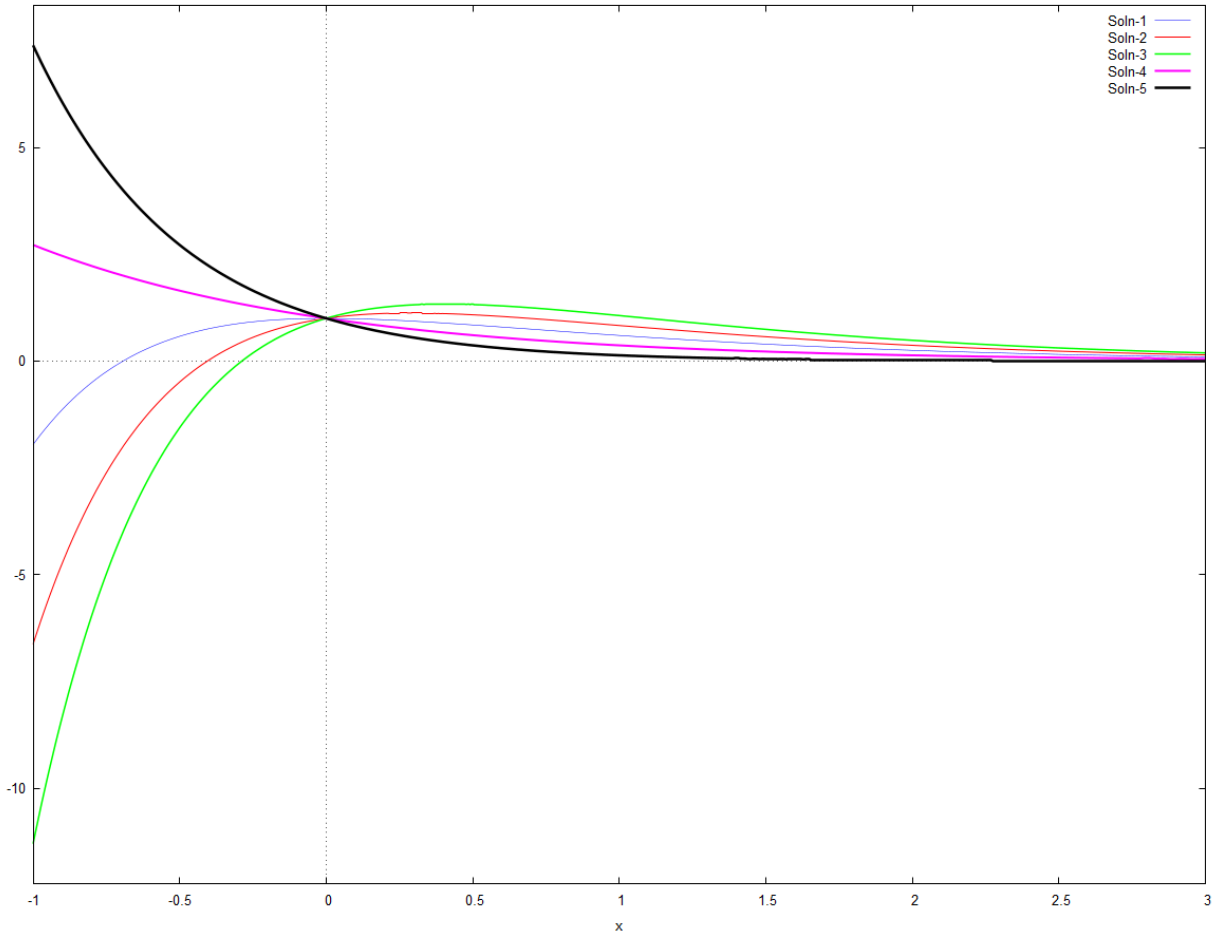
    /* Fixing values for 'b' */
    v0 : ev ( psol , b = 0 ) $
    v1 : ev ( psol , b = 1 ) $
    v2 : ev ( psol , b = 2 ) $
    v3 : ev ( psol , b = − 1 ) $
    v4 : ev ( psol , b = − 2 ) $

    /* Using 'wxplot2d()' to plot the family of solutions */
    wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
          [ x , − 1 , 3 ] ,
          [ style , [ lines , 0 . 5 ] , [ lines , 1 ] , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] ] ,
          [ legend , "Soln-1" , "Soln-2" , "Soln-3" , "Soln-4" , "Soln-5" ] ) $
```

$$\frac{d^2}{dx^2}\mathrm{y}(x) + 3\left(\frac{d}{dx}\mathrm{y}(x)\right) + 2\,\mathrm{y}(x) = 0$$

$$\mathrm{y}(x) = \%e^{-x}\left(\frac{d}{dx}\mathrm{y}(x)\Big|_{x=0} + 2\,\mathrm{y}(0)\right) + \%e^{-2x}\left(-\frac{d}{dx}\mathrm{y}(x)\Big|_{x=0} - \mathrm{y}(0)\right)$$

$$\mathrm{y}(x) = (b+2)\%e^{-x} + (-b-1)\%e^{-2x}$$



4 y'' + y' - 6y = 0 (No initial conditions given)

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--  /* We'll use 'ode2()' */
>   kill ( all ) $

    de : ' diff ( y , x , 2 ) + ' diff ( y , x ) − 6 · y = 0 ;
    gsol : ode2 ( de , y , x ) ; /* general soln. */
    psol : ic2 ( gsol , x = 0 , y = c , ' diff ( y , x ) = k ) ;    /* particular soln. */

    /* Fixing values for 'c' and 'k' */
    v0 : ev ( psol , c = 0 , k = 0 ) $
    v1 : ev ( psol , c = 1 , k = − 1 ) $
    v2 : ev ( psol , c = 2 , k = − 2 ) $
    v3 : ev ( psol , c = 3 , k = − 3 ) $
    v4 : ev ( psol , c = − 1 , k = 1 ) $
    v5 : ev ( psol , c = − 2 , k = 2 ) $
    v6 : ev ( psol , c = − 3 , k = 3 ) $

    /* Using wxplot2d() to plot the family of solutions */
    wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) , rhs ( v5 ) , rhs ( v6 ) ] ,
          [ x , − 1 , 2 ] ,
```

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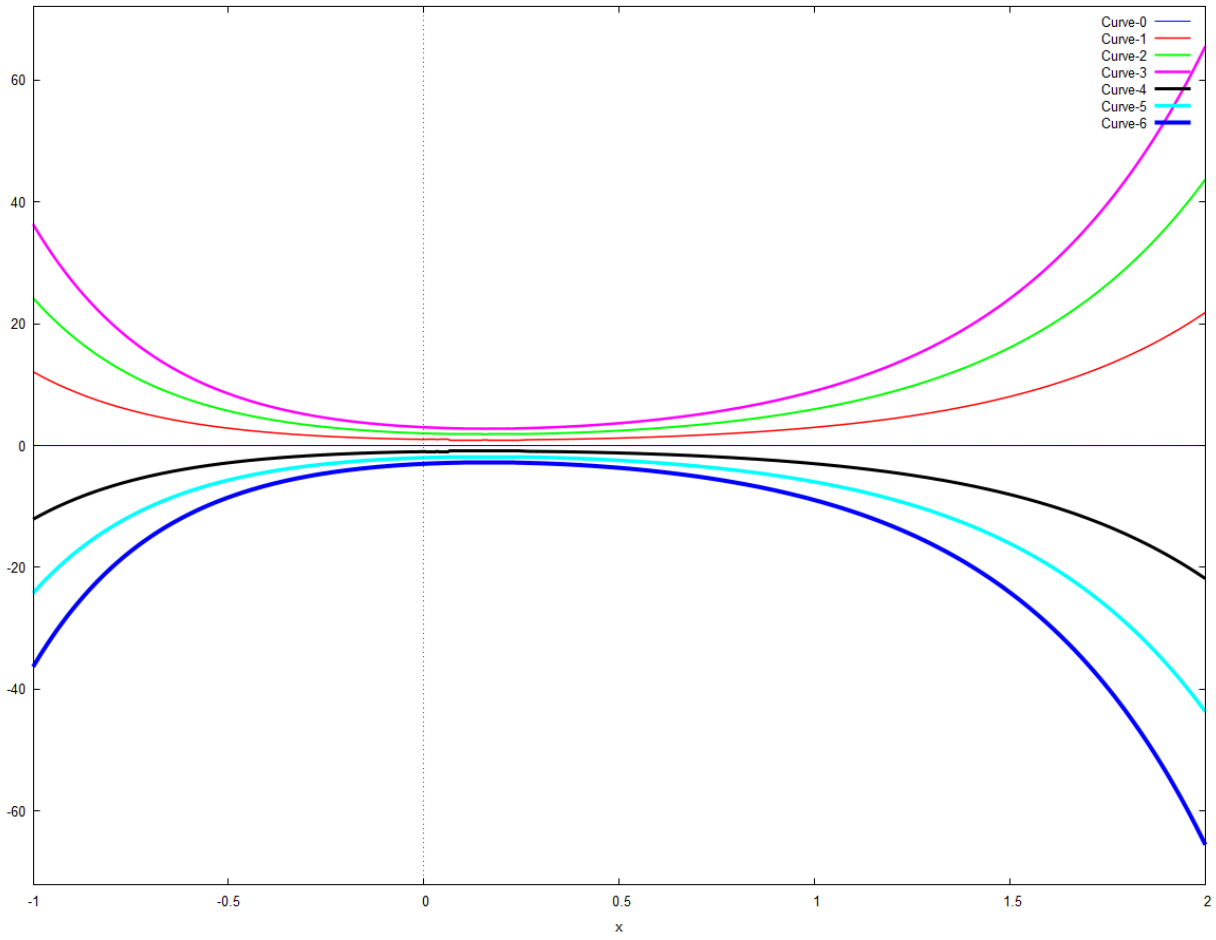
[ style , [ lines , 1 ] , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] , [ lines , 3 ] , [ lines , 3 . 5 ] , [ lines , 4 ] ] ,
[ legend , "Curve-0" , "Curve-1" , "Curve-2" , "Curve-3" , "Curve-4" , "Curve-5" , "Curve-6" , "Curve-7" ] ) ;
```

$$\frac{d^2}{dx^2}y+\frac{d}{dx}y-6y=0$$

$$y=\%\mathrm{k1}\%e^{2x}+\%\mathrm{k2}\%e^{-3x}$$

$$y=\frac{(k+3c)\%e^{2x}}{5}+\frac{(2c-k)\%e^{-3x}}{5}$$



Created with wxMaxima.
The source of this Maxima session can be downloaded here.