Lucky Lao

DSC-VI : Practical-01

Family of Solutions: First Order Differential Equations

We'll plot the family of solutions of the following first order differential equations:

1 y' = x² where y(0)=k

1.1 Using the pre-defined function 'ode2()' (works for an O.D.E. of order upto 2)

```
-- ratprint : false $          /* suppresses error messages */
>  kill ( all ) $               /* clear all user-defined variables */
   de : ' diff ( y , x ) = x ^ 2 ;      /* the eqn. is y' = x^2 */
   sol : ode2 ( de , y , x ) ;       /* `sol` is assigned the General soln. of `de`*/
   sol1 : ic1 ( sol , x = 0 , y = k ) ;   /* `sol1` is a particular solution, w/ def. constt. %c being replaced by
   `k` */
   v1 : ev ( sol1 , k = − 2 ) ;        /* random values are given to `k` */
   v2 : ev ( sol1 , k = − 1 ) ;
   v3 : ev ( sol1 , k = 1 ) ;
   v4 : ev ( sol1 , k = 2 ) ;

   /* To plot the graphs */
   wxplot2d ( [ rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
         [ x , − 3 , 3 ] ,
         [ style , [ lines , 1 ] , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] ] ) $
```

$$\text{(de)}\ \frac{d}{dx}y = x^2$$

$$\text{(sol)}\ y = \frac{x^3}{3} + \%c$$

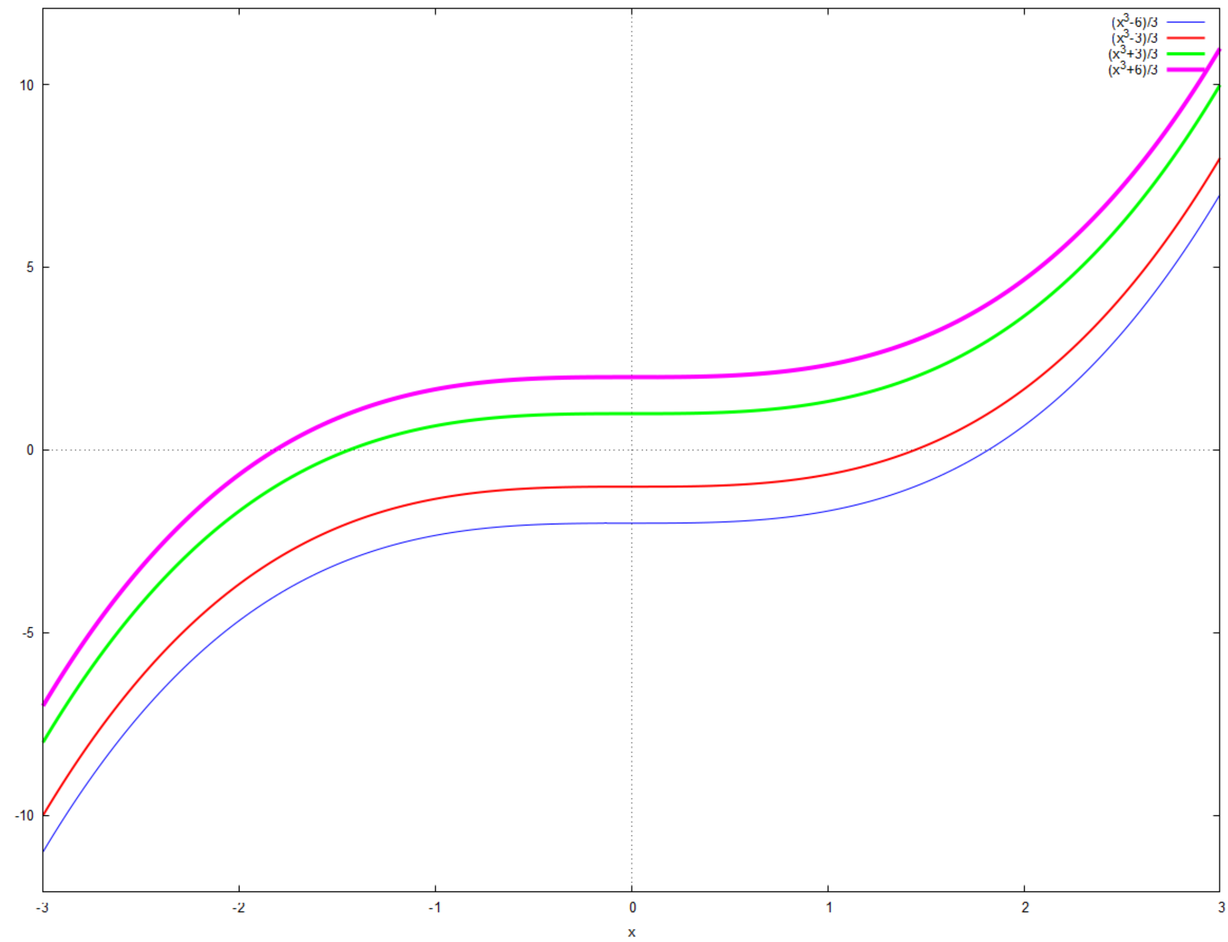
$$\text{(sol1)}\ y = \frac{x^3 + 3k}{3}$$

$$\text{(v1)}\ y = \frac{x^3 - 6}{3}$$

$$\text{(v2)}\ y = \frac{x^3 - 3}{3}$$

$$\text{(v3)}\ y = \frac{x^3 + 3}{3}$$

$$\text{(v4)}\ y = \frac{x^3 + 6}{3}$$



1.2 Using the pre-defined function 'desolve()' (works for an O.D.E. of any order)

```
--> ratprint : false $
    kill ( all ) $                     /* clear all user-defined variables */
    de : diff ( y ( x ) , x ) = x ^ 2 ;       /* y is explicitly written as a function of x */
    sol : desolve ( de , y ( x ) ) ;       /* doesn't give constt.s explicitly but their values */
    sol1 : ev ( sol , y ( 0 ) = k ) ;
    v1 : ev ( sol1 , k = − 2 ) ;
    v2 : ev ( sol1 , k = − 1 ) ;
    v3 : ev ( sol1 , k = 1 ) ;
    v4 : ev ( sol1 , k = 2 ) ;

    /* To plot the graphs */
    wxplot2d ( [ rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
        [ x , − 3 , 3 ] ,
        [ style , [ lines , 1 ] , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] ] ) $
```

$$(\mathrm{de})\ \frac{d}{dx}\mathrm{y}(x)=x^2$$

$$(\mathrm{sol})\ \mathrm{y}(x)=\frac{x^3}{3}+\mathrm{y}(0)$$

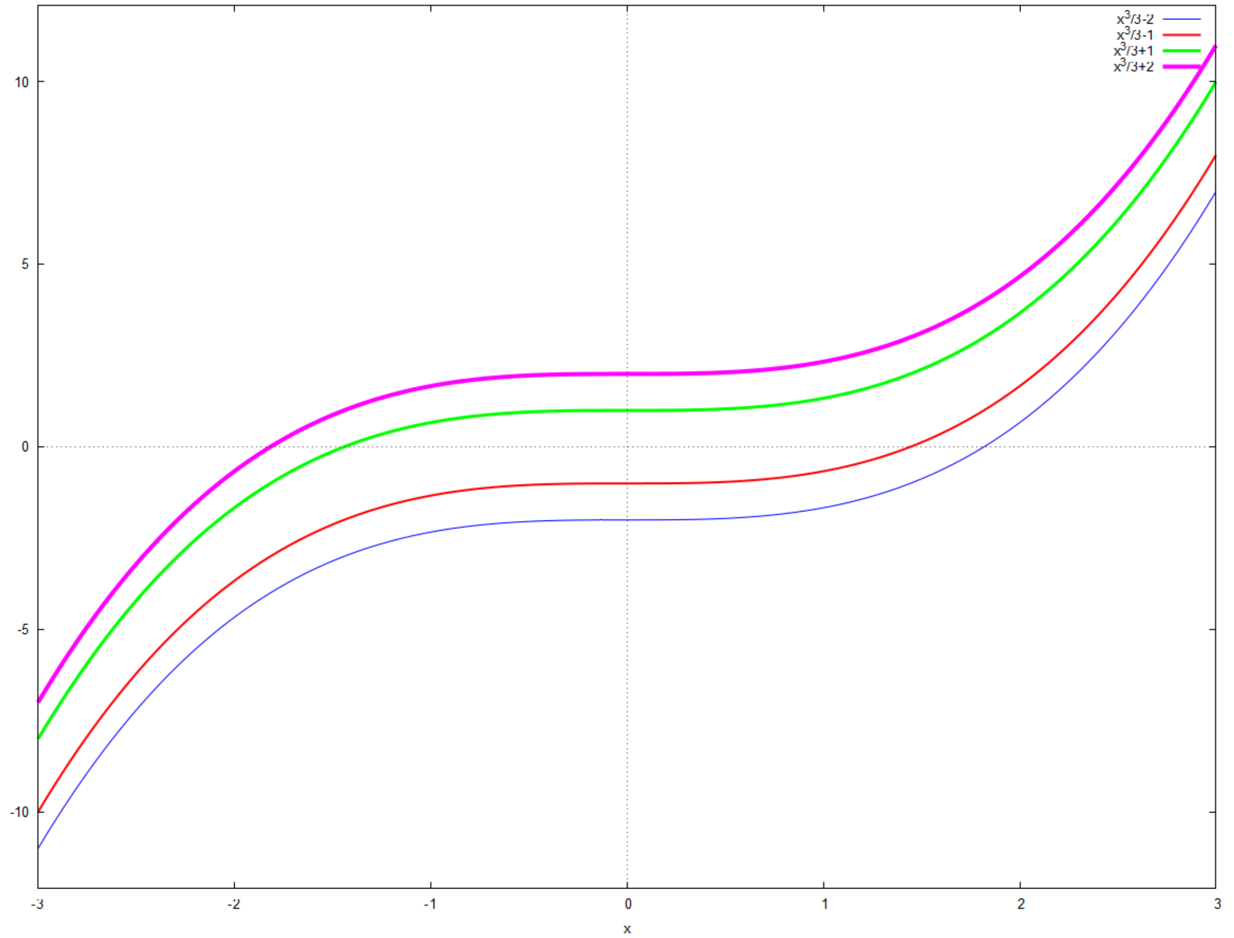
$$(\mathrm{sol1})\ \mathrm{y}(x)=\frac{x^3}{3}+k$$

$$(\mathrm{v1})\ \mathrm{y}(x)=\frac{x^3}{3}-2$$

$$(\mathrm{v2})\ \mathrm{y}(x)=\frac{x^3}{3}-1$$

$$(v3)\ y(x)=\frac{x^3}{3}+1$$

$$(v4)\ y(x)=\frac{x^3}{3}+2$$



2 y' = 9.8 - 0.196y

2.1 Using 'ode2()'

```
--> ratprint : false $
    kill ( all ) $
    de : ' diff ( y , x ) = 9 . 8 − 0 . 196 · y ;
    gsol : ode2 ( de , y , x ) ;
    psol : ic1 ( gsol , x = 0 , y = k ) ;
    v0 : ev ( psol , k = 0 ) ;
    v1 : ev ( psol , k = 1 ) ;
    v2 : ev ( psol , k = 2 ) ;
    v3 : ev ( psol , k = − 1 ) ;
    v4 : ev ( psol , k = − 2 ) ;
    wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
        [ x , − 1 , 1 ] ,
        [ style , [ lines , 1 ] , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] , [ lines , 5 ] ] ) $
```

$$(de)\ \frac{d}{dx}y=9.8\text{-}0.196y$$

$$(gsol)\ y=\%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}}+\%c\right)$$

$$\text{(psol)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} + k - 50\right)$$

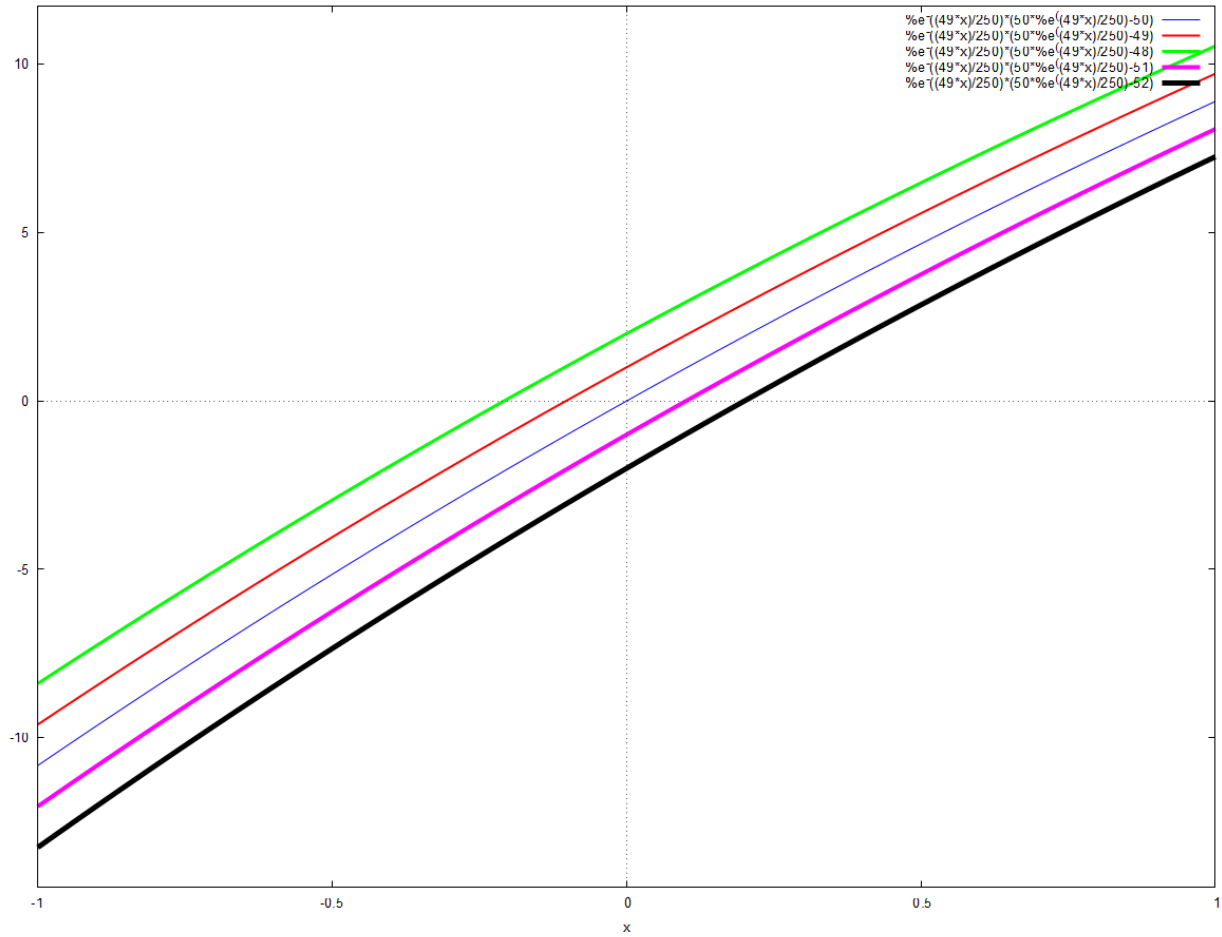
$$\text{(v0)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} - 50\right)$$

$$\text{(v1)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} - 49\right)$$

$$\text{(v2)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} - 48\right)$$

$$\text{(v3)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} - 51\right)$$

$$\text{(v4)}\ y = \%e^{-\left(\frac{49x}{250}\right)}\left(50\%e^{\frac{49x}{250}} - 52\right)$$



2.2 Using 'desolve()'

```
--> ratprint : false $
    kill ( all ) $
    de : diff ( y ( x ) , x ) = 9 . 8 − 0 . 196 · y ( x ) ;
    gsol : desolve ( de , y ( x ) ) ;
    psol : ev ( gsol , y ( 0 ) = k ) ;
    v0 : ev ( psol , k = 0 ) ;
    v1 : ev ( psol , k = − 4 ) ;
    v2 : ev ( psol , k = − 2 ) ;
    v3 : ev ( psol , k = 2 ) ;
    v4 : ev ( psol , k = 4 ) ;
    wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
```

```
[ x , − 3 , 3 ] ,
[ style , [ lines , 1 ] , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] , [ lines , 5 ] ] ) $
```

$$\text{(de)}\ \frac{d}{dx}\mathrm{y}(x) = 9.8 - 0.196\,\mathrm{y}(x)$$

$$\text{(gsol)}\ \mathrm{y}(x) = \frac{(250\,\mathrm{y}(0) - 12500)\%e^{-\left(\frac{49x}{250}\right)}}{250} + 50$$

$$\text{(psol)}\ \mathrm{y}(x) = \frac{(250k - 12500)\%e^{-\left(\frac{49x}{250}\right)}}{250} + 50$$

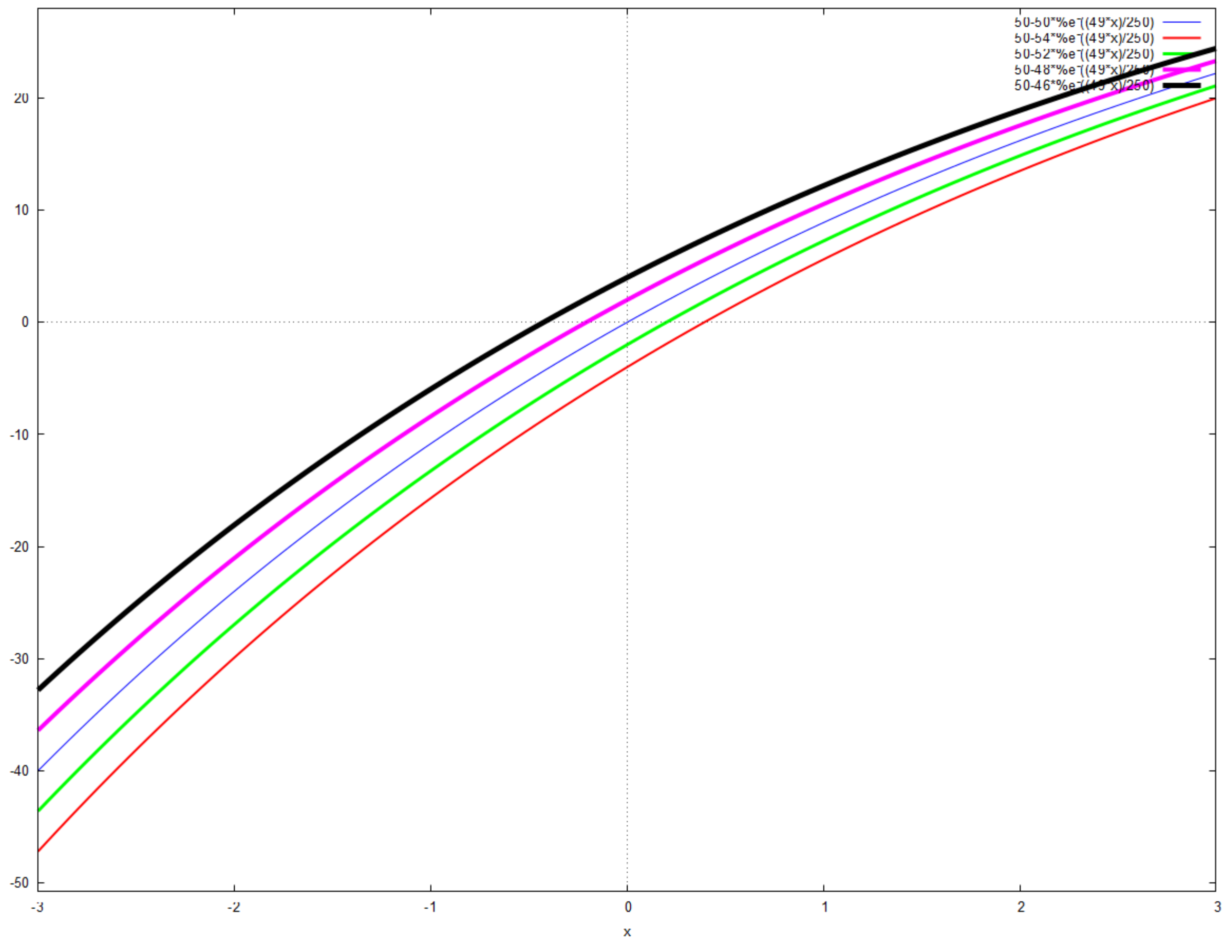
$$\text{(v0)}\ \mathrm{y}(x) = 50 - 50\%e^{-\left(\frac{49x}{250}\right)}$$

$$\text{(v1)}\ \mathrm{y}(x) = 50 - 54\%e^{-\left(\frac{49x}{250}\right)}$$

$$\text{(v2)}\ \mathrm{y}(x) = 50 - 52\%e^{-\left(\frac{49x}{250}\right)}$$

$$\text{(v3)}\ \mathrm{y}(x) = 50 - 48\%e^{-\left(\frac{49x}{250}\right)}$$

$$\text{(v4)}\ \mathrm{y}(x) = 50 - 46\%e^{-\left(\frac{49x}{250}\right)}$$



3 y'cos(x) + ysin(x) = 2cos³(x)sin(x)-1

```
--> ratprint : false $
    kill ( all ) $
    de : ' diff ( y , x ) · cos ( x ) + y · sin ( x ) = 2 · ( cos ( x ) ) ^ 3 · sin ( x ) − 1 ;
    gsol : ode2 ( de , y , x ) ;
```

```
psol : ic1 ( gsol , x = 0 , y = k ) ;
v0 : ev ( psol , k = 0 ) ;
v1 : ev ( psol , k = − 1 ) ;
v2 : ev ( psol , k = − 2 ) ;
v3 : ev ( psol , k = 1 ) ;
v4 : ev ( psol , k = 2 ) ;
wxplot2d ( [ rhs ( v0 ) , rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) ] ,
    [ x , − 1 , 1 ] ,
    [ style , [ lines , 1 ] , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] , [ lines , 5 ] ] ) $
```

$$\text{(de)}\ \cos(x)\left(\frac{d}{dx}y\right)+\sin(x)y=2\cos(x)^3\sin(x)-1$$

$$\text{(gsol)}\ y=\cos(x)\left(-\left(\frac{1}{\tan(x)^2+1}\right)-\tan(x)+\%c\right)$$

$$\text{(psol)}\ y=-\left(\frac{\cos(x)\tan(x)^3+(-k-1)\cos(x)\tan(x)^2+\cos(x)\tan(x)-k\cos(x)}{\tan(x)^2+1}\right)$$

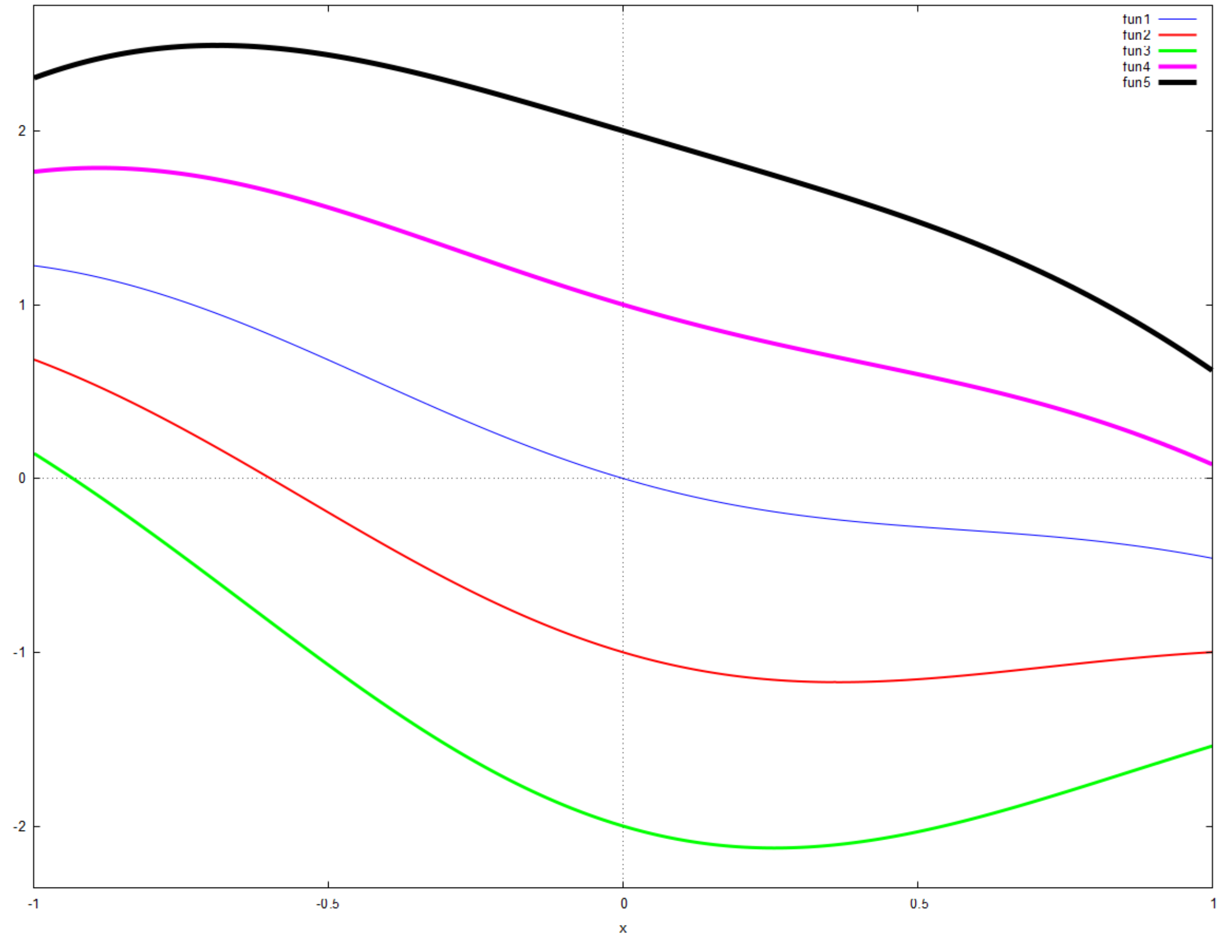
$$\text{(v0)}\ y=-\left(\frac{\cos(x)\tan(x)^3-\cos(x)\tan(x)^2+\cos(x)\tan(x)}{\tan(x)^2+1}\right)$$

$$\text{(v1)}\ y=-\left(\frac{\cos(x)\tan(x)^3+\cos(x)\tan(x)+\cos(x)}{\tan(x)^2+1}\right)$$

$$\text{(v2)}\ y=-\left(\frac{\cos(x)\tan(x)^3+\cos(x)\tan(x)^2+\cos(x)\tan(x)+2\cos(x)}{\tan(x)^2+1}\right)$$

$$\text{(v3)}\ y=-\left(\frac{\cos(x)\tan(x)^3-2\cos(x)\tan(x)^2+\cos(x)\tan(x)-\cos(x)}{\tan(x)^2+1}\right)$$

$$\text{(v4)}\ y=-\left(\frac{\cos(x)\tan(x)^3-3\cos(x)\tan(x)^2+\cos(x)\tan(x)-2\cos(x)}{\tan(x)^2+1}\right)$$



Created with wxMaxima.
The source of this Maxima session can be downloaded here.