Lucky Lao

DSC-VI : Practical-06

Lake Pollution Model

1 Constant flow and constant pollution concentration inflow

c(t): concentration of pollutant in the lake at time t.
F: constant flow rate.
V: constant volume of the lake.
cin: constant concentration of pollutant in the flow entering the lake.
initial condition: c(0)=c0.(5 different initial conditions taken)

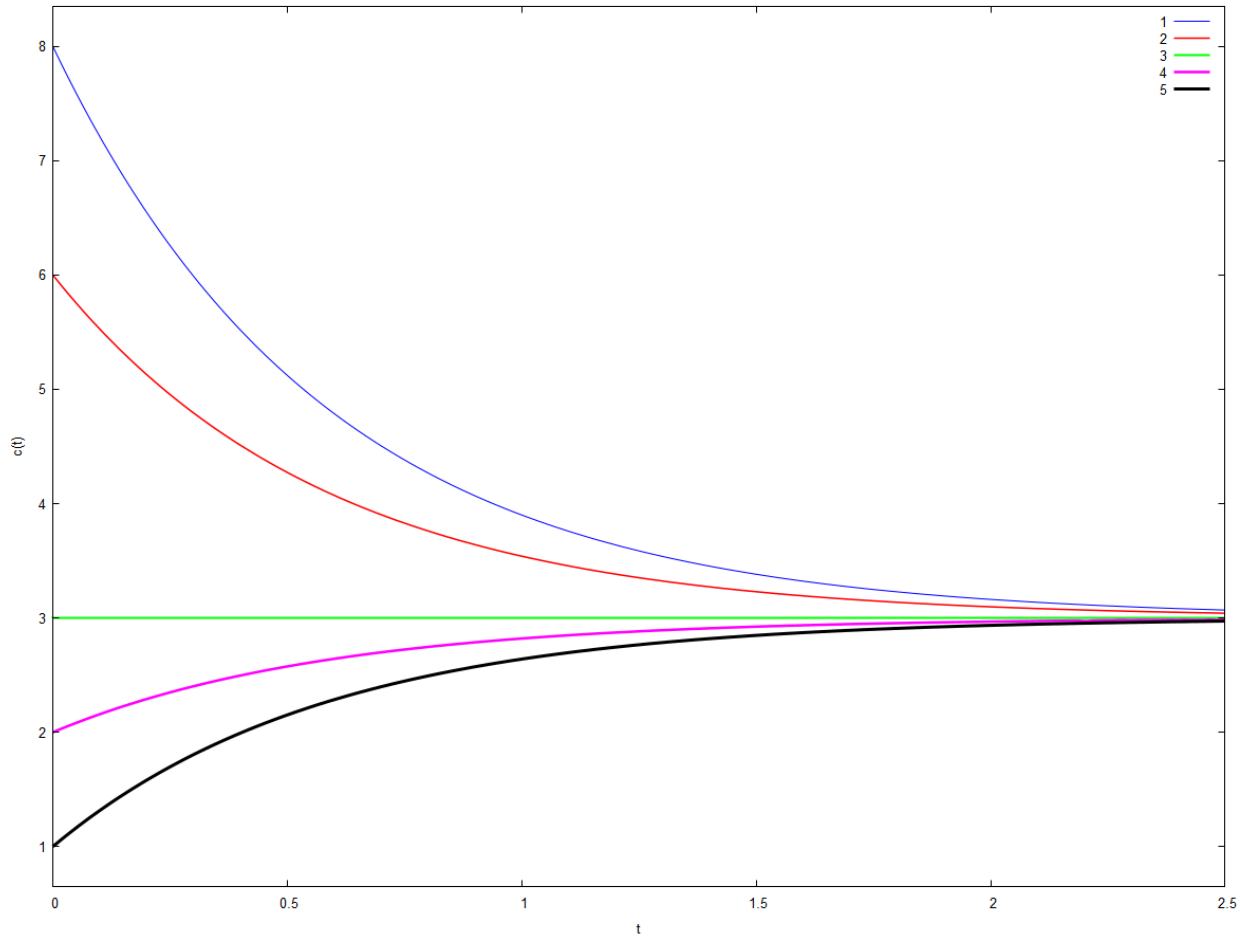
```
--> kill ( all ) $
    eqn1 : ' diff ( c , t ) = ( F / V ) · cin − ( F / V ) · c ;
    sol1 : ode2 ( eqn1 , c , t ) ;
    fsol1 : ic1 ( sol1 , c = c0 , t = 0 ) ;
    v : ev ( fsol1 , cin = 3 , V = 28 , F = 4 · 12 ) ;
    v1 : ev ( v , c0 = 8 ) $
    v2 : ev ( v , c0 = 6 ) $
    v3 : ev ( v , c0 = 3 ) $
    v4 : ev ( v , c0 = 2 ) $
    v5 : ev ( v , c0 = 1 ) $
    wxplot2d ( [ rhs ( v1 ) , rhs ( v2 ) , rhs ( v3 ) , rhs ( v4 ) , rhs ( v5 ) ] ,
          [ t , 0 , 2 . 5 ] ,
          [ legend , "1" , "2" , "3" , "4" , "5" ] ,
          [ style , [ lines , 1 ] , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] , [ lines , 3 ] ] ,
          [ ylabel , "c(t)" ] ) $
```

$$\frac{d}{dt}c = \frac{F\,\mathrm{cin}}{V} - \frac{Fc}{V}$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} + \%\mathrm{c}\right)$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} - \mathrm{cin} + \mathrm{c0}\right)$$

$$c = \%e^{-\frac{12t}{7}}\left(3\%e^{\frac{12t}{7}} + \mathrm{c0} - 3\right)$$



2 Seasonal flow and constant pollution concentration inflow

c(t): concentration of pollutant in the lake at time t.
F: seasonal flow rate.
V: constant volume of the lake.
cin: constant concentration of pollutant in the flow entering the lake.
initial condition: c(0)=c0.

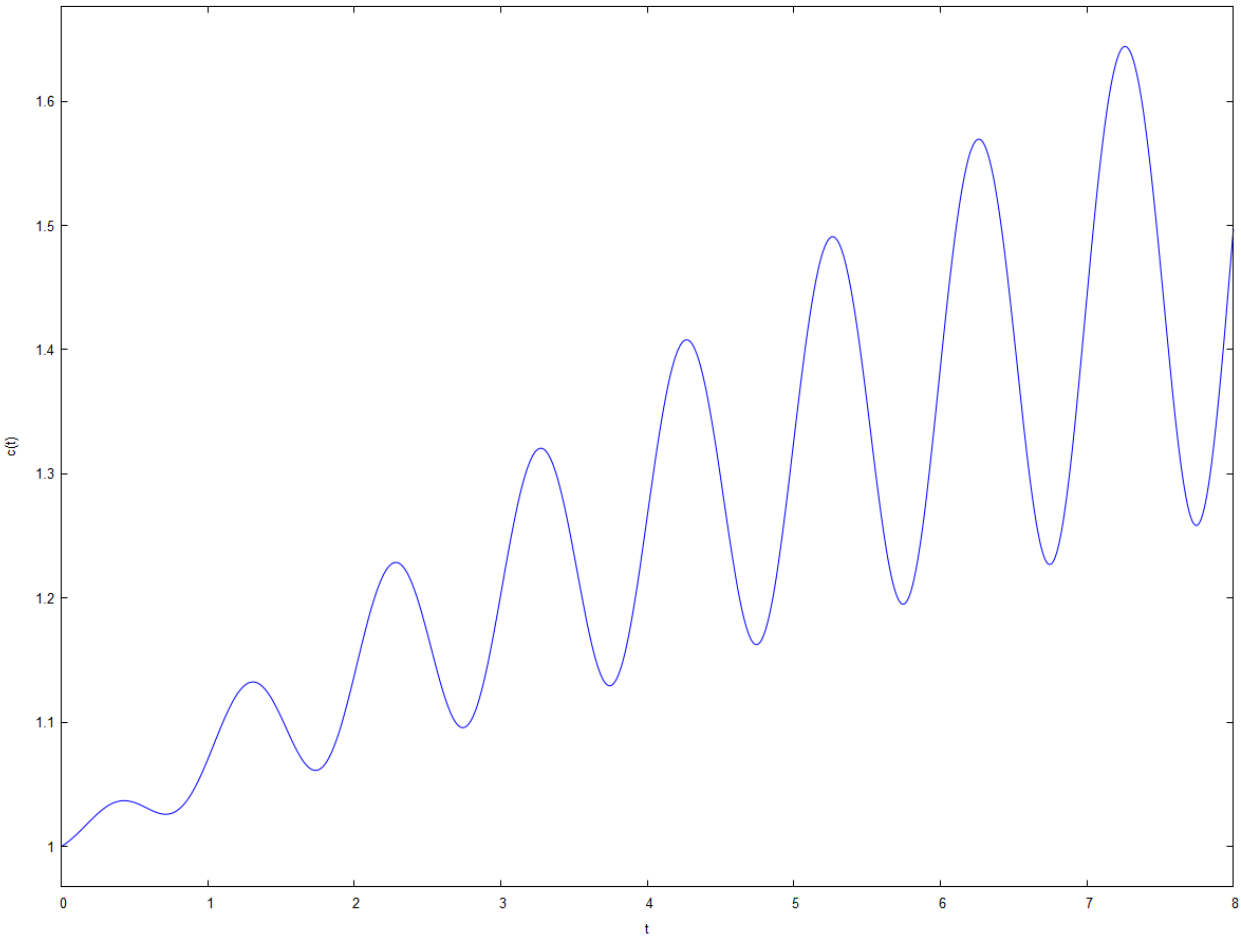
```
--> kill ( all ) $
    eqn1 : ' diff ( c , t ) = ( F / V ) · cin − ( F / V ) · c ;
    sol1 : ode2 ( eqn1 , c , t ) ;
    fsol1 : ic1 ( sol1 , c = c0 , t = 0 ) ;
    v : ev ( fsol1 , cin = 3 , V = 28 , F = 1 + 0 . 5 · sin ( 2 · π · t ) ) ;
    v1 : ev ( v , c0 = 1 ) $
    wxplot2d ( rhs ( v1 ) ,
          [ t , 0 , 8 ] ,
          [ legend , "" ] ,
          [ ylabel , "c(t)" ] ) $
```

$$\frac{d}{dt}c = \frac{F\,\mathrm{cin}}{V} - \frac{Fc}{V}$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} + \%c\right)$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} - \mathrm{cin} + \mathrm{c0}\right)$$

$$c = \%e^{-\frac{t\,(0.5\sin(2\pi t)+1)}{28}}\left(3\%e^{\frac{t\,(0.5\sin(2\pi t)+1)}{28}} + \mathrm{c0} - 3\right)$$



3 Constant flow and seasonal pollution concentration inflow

c(t): concentration of pollutant in the lake at time t.
F: constant flow rate.
V: constant volume of the lake.
cin: seasonal concentration of pollutant in the flow entering the lake.
initial condition: c(0)=c0.

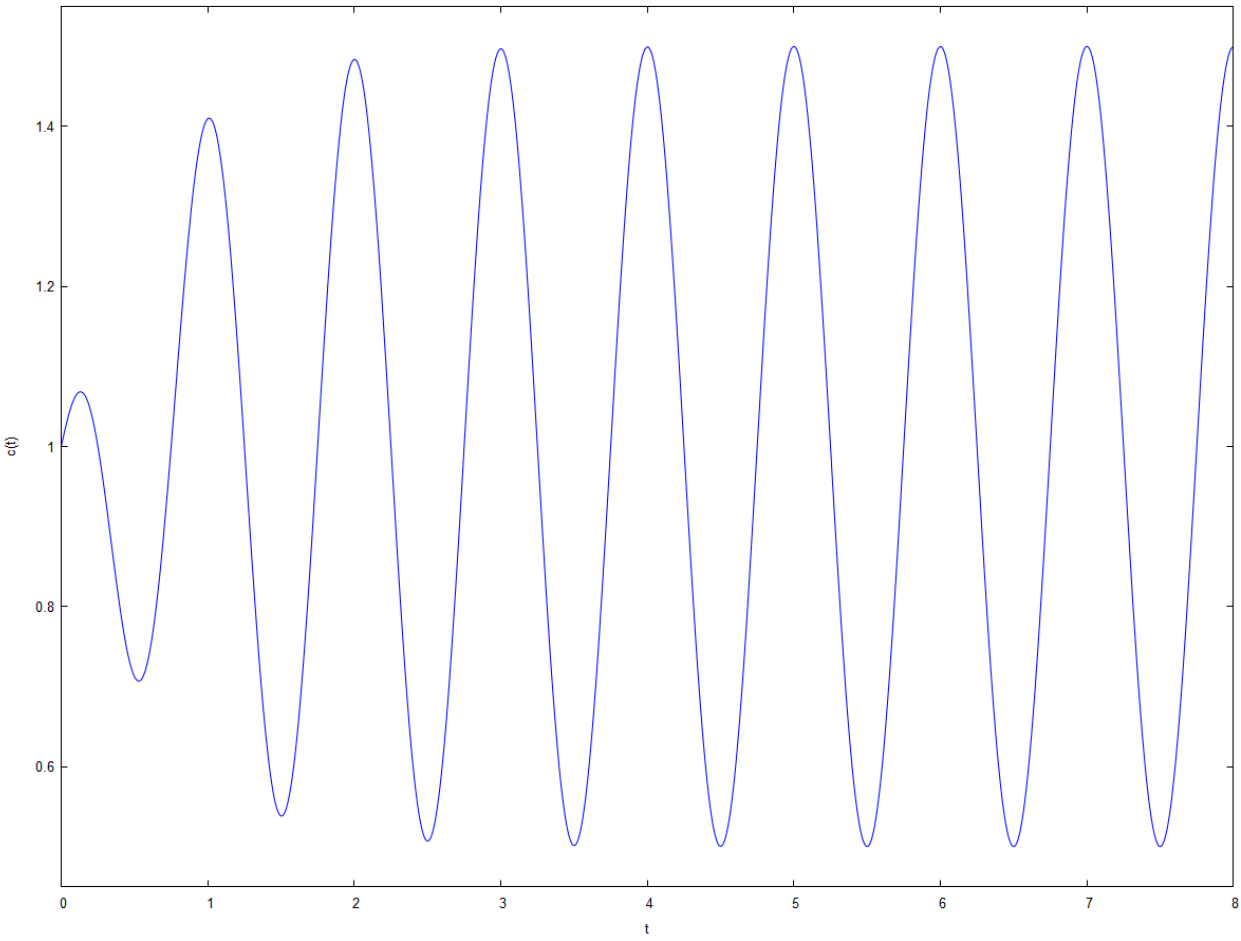
```
--> kill ( all ) $
    eqn1 : ' diff ( c , t ) = ( F / V ) · cin − ( F / V ) · c ;
    sol1 : ode2 ( eqn1 , c , t ) ;
    fsol1 : ic1 ( sol1 , c = c0 , t = 0 ) ;
    v : ev ( fsol1 , cin = 1 + 0 . 5 · cos ( 2 · π · t ) , V = 28 , F = 4 · 12 ) ;
    v1 : ev ( v , c0 = 1 ) $
    wxplot2d ( rhs ( v1 ) ,
          [ t , 0 , 8 ] ,
          [ legend , "" ] ,
          [ ylabel , "c(t)" ] ) $
```

$$\frac{d}{dt}c = \frac{F\,\mathrm{cin}}{V} - \frac{Fc}{V}$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} + \%c\right)$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} - \mathrm{cin} + \mathrm{c0}\right)$$

$$c = \%e^{-\frac{12t}{7}}\left(-0.5\cos\left(2\pi t\right) + \%e^{\frac{12t}{7}}\left(0.5\cos\left(2\pi t\right) + 1\right) + \mathrm{c0} - 1\right)$$



4 Seasonal flow and seasonal pollution concentration inflow

c(t): concentration of pollutant in the lake at time t.
F: seasonal flow rate.
V: constant volume of the lake.
cin: seasonal concentration of pollutant in the flow entering the lake.
initial condition: c(0)=c0.

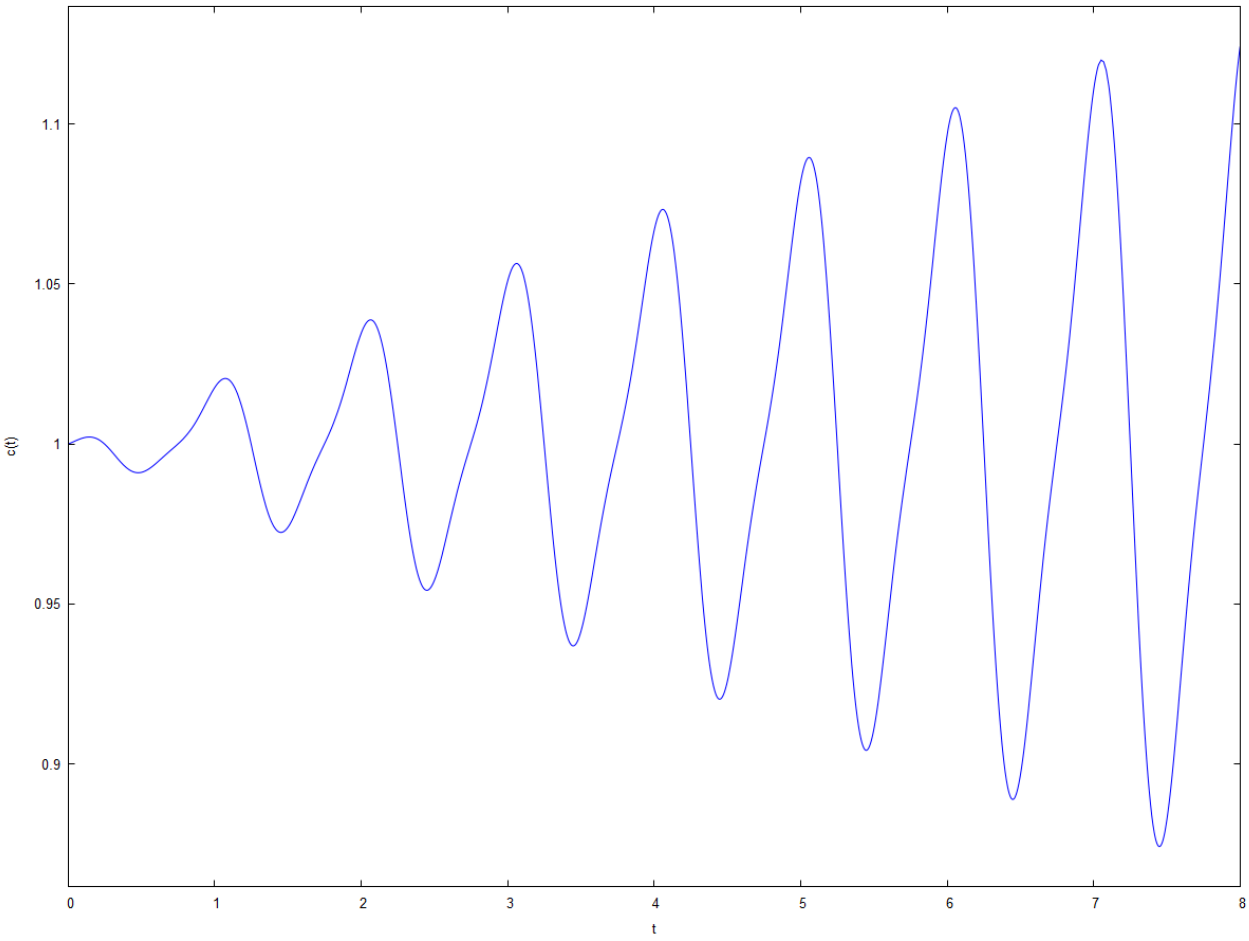
```
--> kill ( all ) $
    eqn1 : ' diff ( c , t ) = ( F / V ) · cin − ( F / V ) · c ;
    sol1 : ode2 ( eqn1 , c , t ) ;
    fsol1 : ic1 ( sol1 , c = c0 , t = 0 ) ;
    v : ev ( fsol1 , cin = 1 + 0 . 5 · cos ( 2 · π · t ) , V = 28 , F = 1 + 0 . 5 · sin ( 2 · π · t ) ) ;
    v1 : ev ( v , c0 = 1 ) $
    wxplot2d ( rhs ( v1 ) ,
          [ t , 0 , 8 ] ,
          [ legend , "" ] ,
          [ ylabel , "c(t)" ] ) $
```

$$\frac{d}{dt}c = \frac{F\,\mathrm{cin}}{V} - \frac{Fc}{V}$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} + \%c\right)$$

$$c = \%e^{-\frac{Ft}{V}}\left(\mathrm{cin}\%e^{\frac{Ft}{V}} - \mathrm{cin} + \mathrm{c0}\right)$$

$$c = \%e^{-\frac{t\,(0.5\sin(2\pi t)+1)}{28}}\left(-0.5\cos(2\pi t) + \%e^{\frac{t\,(0.5\sin(2\pi t)+1)}{28}}\,(0.5\cos(2\pi t)+1) + \mathrm{c0} - 1\right)$$



Created with wxMaxima.
The source of this Maxima session can be downloaded here.