Lucky Lao

DSC-VI : Practical-07

Logistic Growth Model

x(t): population at time t.
a: per capita death rate.
b: per capita birth rate.
r: b-a is the reproduction rate.
K: carrying capacity.
initial condition: x(0)=x0.

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--> r : 1 $ K : 1000 $
    eqn : ' diff ( x , t ) = r · x · ( 1 − x / K ) ;
    gs : ode2 ( eqn , x , t ) ;
    gs1 : logcontract ( gs ) ;
    gs2 : solve ( gs1 , x ) [ 1 ] ;
    ps : ic1 ( gs2 , t = 0 , x = x0 ) ;
    ps1 : ev ( ps , x0 = 10 ) $
    ps2 : ev ( ps , x0 = 200 ) $
    ps3 : ev ( ps , x0 = 500 ) $
    ps4 : ev ( ps , x0 = 800 ) $
    ps5 : ev ( ps , x0 = 1400 ) $
    wxplot2d ( [ rhs ( ps1 ) , rhs ( ps2 ) , rhs ( ps3 ) , rhs ( ps4 ) , rhs ( ps5 ) , K ] ,
        [ t , 0 , 10 ] ,
        [ legend , "x0=10" , "x0=200" , "x0=500" , "x0=800" , "x0=1400" , "K (=1000)" ] ,
        [ style , [ lines , 1 . 5 ] , [ lines , 2 ] , [ lines , 2 . 5 ] , [ lines , 3 ] , [ lines , 3 . 5 ] , [ lines , 1 ] ] ,
        [ xlabel , "t (in years)" ] , [ ylabel , "Population" ] ) $
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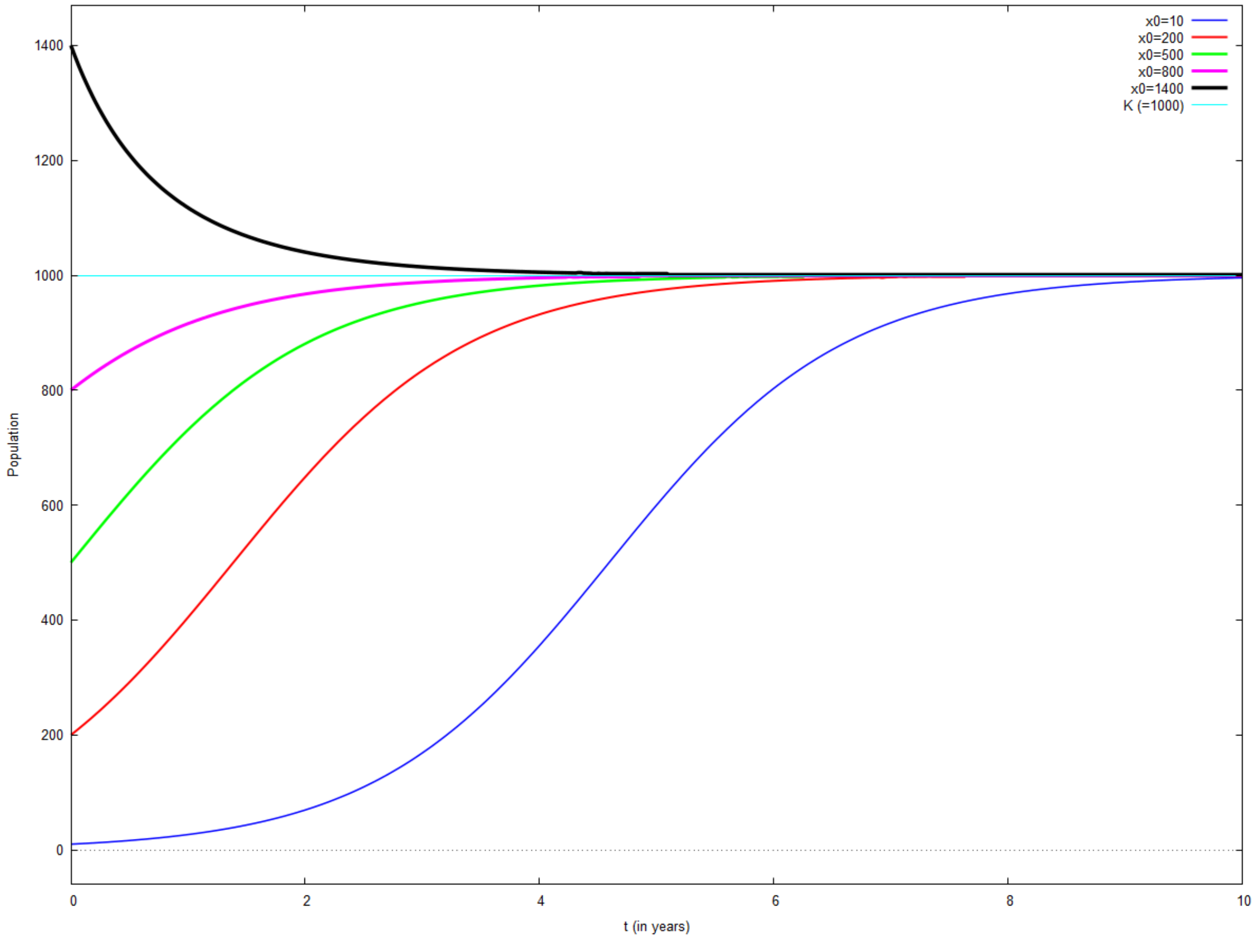
$$\frac{d}{dt}x = \left(1 - \frac{x}{1000}\right)x$$

$$\log\left(x\right) - \log\left(x - 1000\right) = t + \%\mathrm{c}$$

$$\log\left(\frac{x}{x - 1000}\right) = t + \%\mathrm{c}$$

$$x = \frac{1000\%e^{t + \%\mathrm{c}}}{\%e^{t + \%\mathrm{c}} - 1}$$

$$x = \frac{1000\%e^{t}\mathrm{x0}}{\left(\%e^{t} - 1\right)\mathrm{x0} + 1000}$$



Created with wxMaxima.
The source of this Maxima session can be downloaded here.