Lucky Lao

DSC-VI : Practical-09

Predator-Prey Model

1 Basic Lotka-Volterra Model

x(t): Number of prey per unit area.
y(t): Number of predators per unit area.
Initial condition: x(0)=200, y(0)=80.
The constant b1,c1,c2,a2 are all positive.

```
--> b1 : 1 . 0 $ a2 : 0 . 5 $ c1 : 0 . 01 $ c2 : 0 . 005 $
    eqn1 : ' diff ( x , t ) = b1 · x − c1 · x · y ;
    eqn2 : ' diff ( y , t ) = c2 · x · y − a2 · y ;
    pts : rk ( [ rhs ( eqn1 ) , rhs ( eqn2 ) ] , [ x , y ] , [ 200 , 80 ] , [ t , 0 , 20 , . 1 ] ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    x_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 2 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    y_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 3 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    wxplot2d ( [ [ discrete , x_pts ] , [ discrete , y_pts ] ] ,
        [ t , 0 , 20 ] , [ y , 0 , 250 ] ,
        [ style , [ lines , 2 ] , [ lines , 4 ] ] ,
        [ xlabel , "t (in years)" ] ,
        [ ylabel , "Population density" ] ,
        [ legend , "x(t): Preys" , "y(t): Predators" ] ) $
```

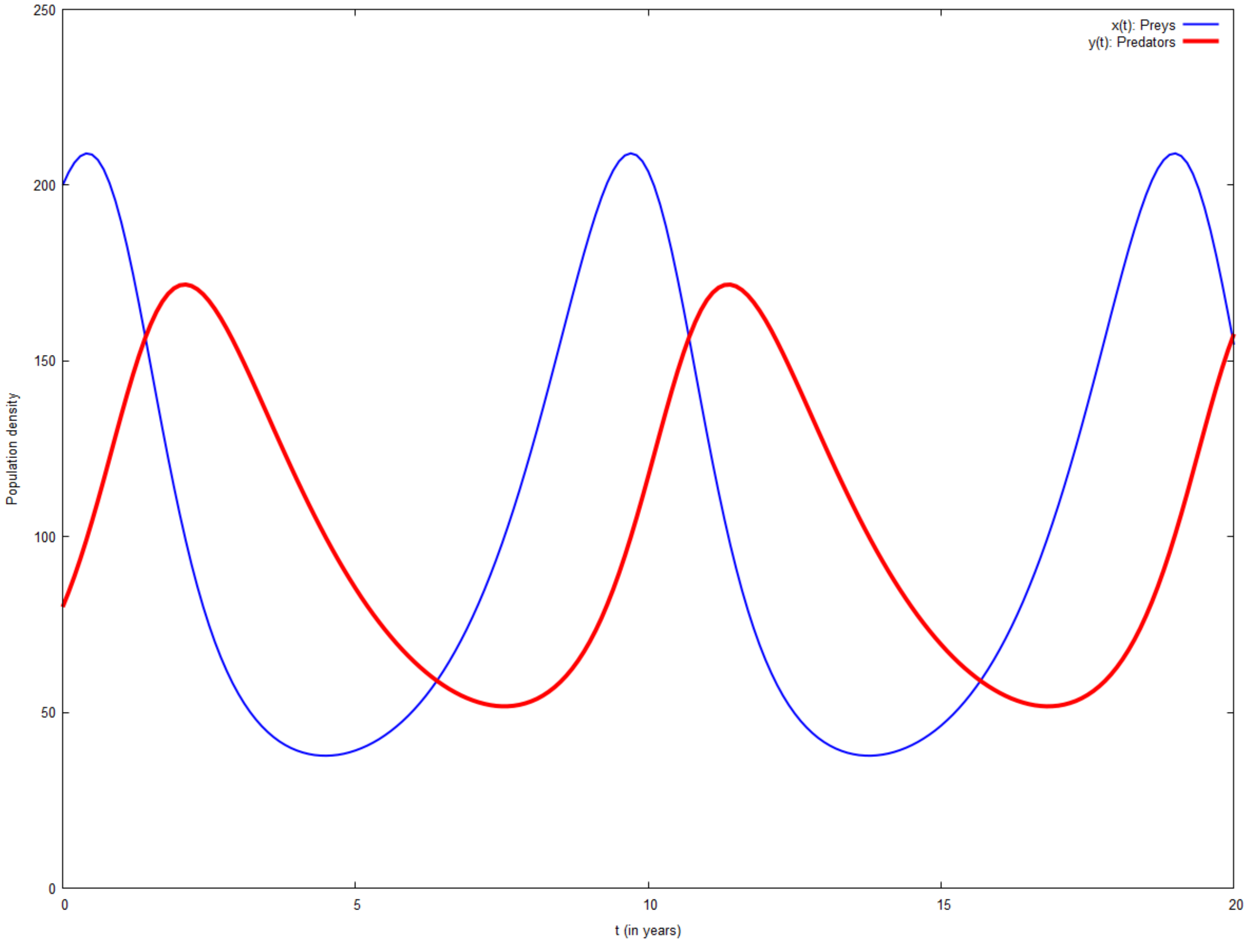
$$\frac{d}{dt}x = 1.0x - 0.01xy$$

$$\frac{d}{dt}y = 0.005xy - 0.5y$$

$$[[0.0\,,200.0\,,80.0]\,,[20.0\,,154.5921330991444\,,157.6845208218405]\,,201]$$

$$[[0.0\,,200.0]\,,[20.0\,,154.5921330991444]\,,201]$$

$$[[0.0\,,80.0]\,,[20.0\,,157.6845208218405]\,,201]$$



2 Density-Dependent Growth

x(t): Number of preys per unit area.
y(t): Number of predators per unit area.
Initial conditions: x(0)=120, y(0)=40.
The constants b1,c1,c2,a2,K are all positive.

```
--> kill ( all ) $
    b1 : 1 . 0 $ a2 : 0 . 5 $ c1 : 0 . 01 $ c2 : 0 . 005 $ K : 1000 $
    eqn1 : ' diff ( x , t ) = b1 · x · ( 1 − x / K ) − c1 · x · y ;
    eqn2 : ' diff ( y , t ) = c2 · x · y − a2 · y ;
    pts : rk ( [ rhs ( eqn1 ) , rhs ( eqn2 ) ] , [ x , y ] , [ 120 , 40 ] , [ t , 0 , 40 , . 1 ] ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    x_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 2 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    y_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 3 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    wxplot2d ( [ [ discrete , x_pts ] , [ discrete , y_pts ] ] ,
        [ t , 0 , 40 ] , [ y , 0 , 250 ] ,
        [ style , [ lines , 2 ] , [ lines , 4 ] ] ,
        [ xlabel , "t (in years)" ] ,
        [ ylabel , "Population density" ] ,
        [ legend , "x(t): Preys" , "y(t): Predators" ] ) $
```

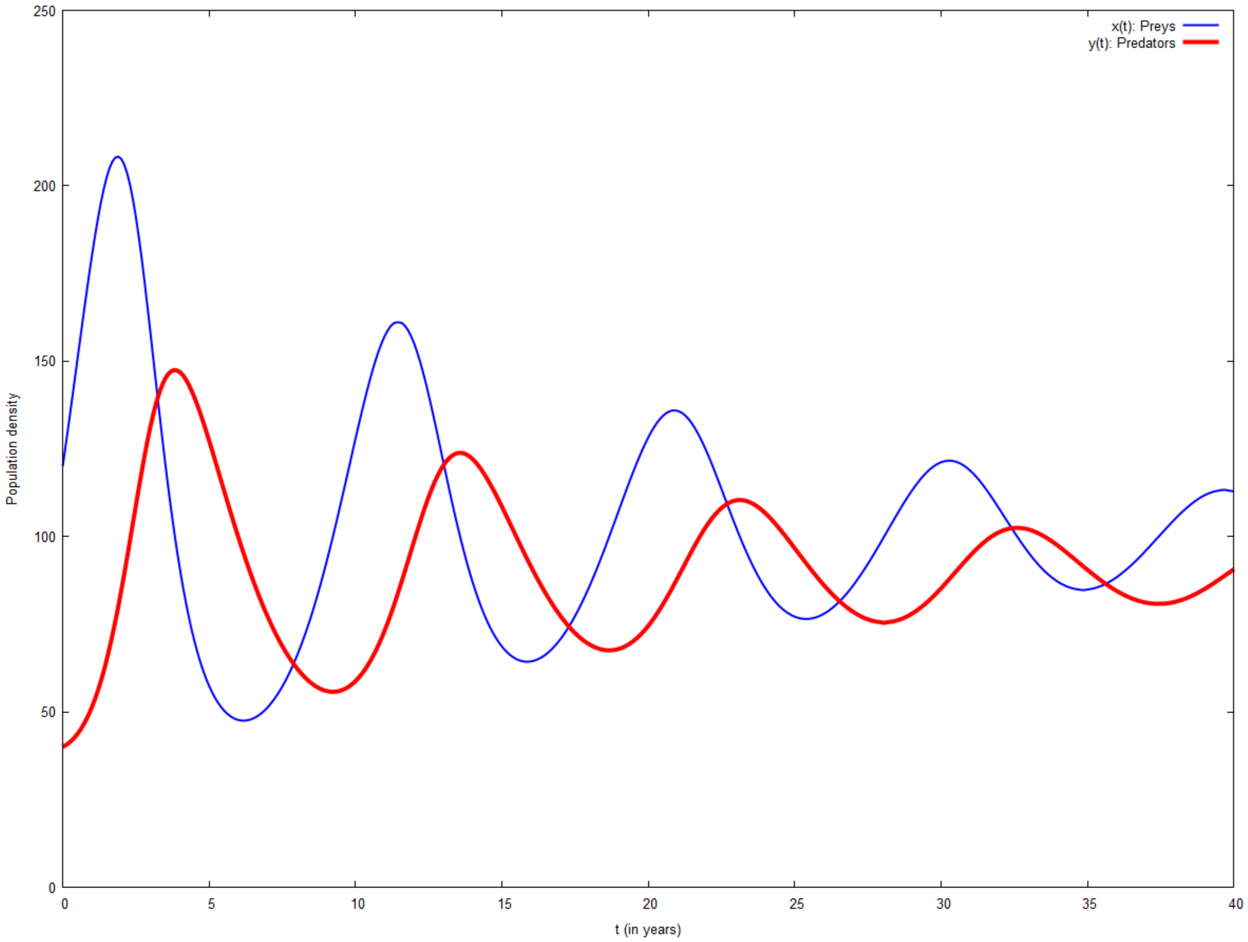
$$\frac{d}{dt}x = 1.0\left(1 - \frac{x}{1000}\right)x - 0.01xy$$

$$\frac{d}{dt}y = 0.005xy - 0.5y$$

$$[[0.0 , 120.0 , 40.0] , [40.0 , 112.8076936975965 , 90.67006550194145] , 401]$$

$$[[0.0\,,120.0]\,,[40.0\,,112.8076936975965]\,,401]$$

$$[[0.0\,,40.0]\,,[40.0\,,90.67006550194145]\,,401]$$



3 Effect of DDT

x(t): Number of preys per unit area.
y(t): Number of predators per unit area.
Initial conditions: x(0)=200, y(0)=80.
The constants b1,c1,c2,a2,p1,p2 are all positive.

```
--> kill ( all ) $
    b1 : 1 . 0 $ a2 : 0 . 5 $ c1 : 0 . 01 $ c2 : 0 . 005 $ p1 : 0 . 1 $ p2 : 0 . 1 $
    eqn1 : ' diff ( x , t ) = b1 · x − c1 · x · y − p1 · x ;
    eqn2 : ' diff ( y , t ) = c2 · x · y − a2 · y − p2 · y ;
    pts : rk ( [ rhs ( eqn1 ) , rhs ( eqn2 ) ] , [ x , y ] , [ 200 , 80 ] , [ t , 0 , 20 , . 1 ] ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    x_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 2 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    y_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 3 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    wxplot2d ( [ [ discrete , x_pts ] , [ discrete , y_pts ] ] ,
        [ t , 0 , 20 ] , [ y , 0 , 250 ] ,
        [ style , [ lines , 2 ] , [ lines , 4 ] ] ,
        [ xlabel , "t (in years)" ] ,
        [ ylabel , "Population density" ] ,
        [ legend , "x(t): Preys" , "y(t): Predators" ] ) $
```

$$\frac{d}{dt}x = 0.9x - 0.01xy$$

$$\frac{d}{dt}y = 0.005xy - 0.6y$$

$$[[0.0, 200.0, 80.0], [20.0, 94.9961084048835, 134.7757919917374], 201]$$

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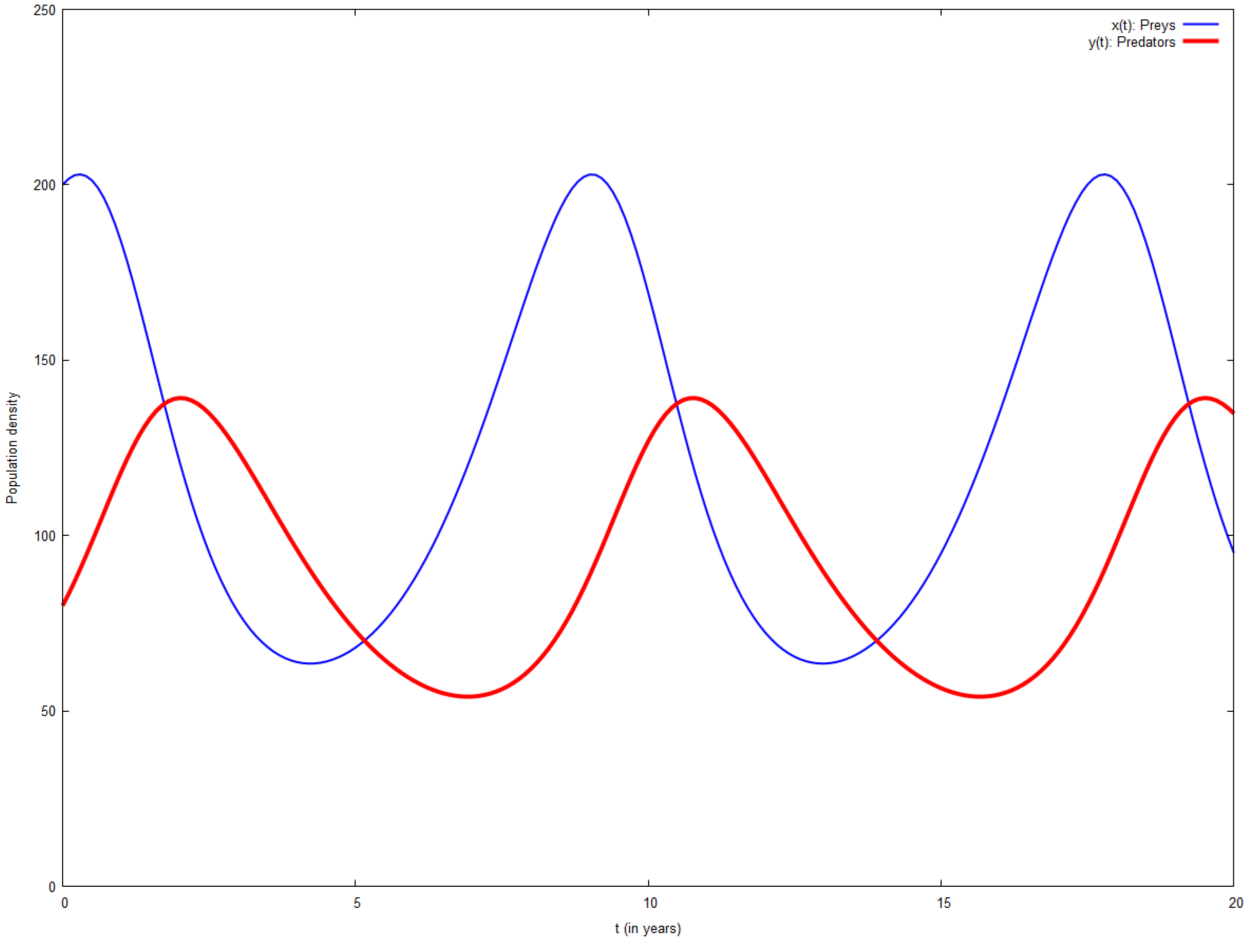
$$[[0.0, 80.0], [20.0, 134.7757919917374], 201]$$



4 Two Prey and One Predator

x1(t): Number of prey1 per unit area.
x2(t): Number of prey2 per unit area.
y(t): Number of predators per unit area.
Initial condition: x1(0)=150, x2(0)=130, y(0)=80.
The constant b1,b2,c1,c2,c3,c4,a are all positive.

```
--  kill ( all ) $
>   b1 : 1 . 32 $ b2 : 1 . 3 $ a : 0 . 5 $ c1 : 0 . 01 $ c2 : 0 . 01 $ c3 : 0 . 003 $ c4 : 0 . 004 $
    eqn1 : ' diff ( x1 , t ) = b1 · x1 − c1 · x1 · y ;
    eqn2 : ' diff ( x2 , t ) = b2 · x2 − c2 · x2 · y ;
    eqn3 : ' diff ( y , t ) = c3 · x1 · y + c4 · x2 · y − a · y ;
    pts : rk ( [ rhs ( eqn1 ) , rhs ( eqn2 ) , rhs ( eqn3 ) ] , [ x1 , x2 , y ] , [ 150 , 130 , 80 ] , [ t , 0 , 40 , . 1 ] )
    $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    x1_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 2 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    x2_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 3 ] ] , i , 1 , length ( pts ) ) $
    [ % [ 1 ] , last ( % ) , length ( % ) ] ;
    y_pts : makelist ( [ pts [ i ] [ 1 ] , pts [ i ] [ 4 ] ] , i , 1 , length ( pts ) ) $
```

```
[ % [ 1 ] , last ( % ) , length ( % ) ] ;
wxplot2d ( [ [ discrete , x1_pts ] , [ discrete , x2_pts ] , [ discrete , y_pts ] ] ,
      [ t , 0 , 40 ] , [ y , 0 , 300 ] ,
      [ style , [ lines , 2 ] , [ lines , 3 ] , [ lines , 4 ] ] ,
      [ xlabel , "t (in years)" ] ,
      [ ylabel , "Population density" ] ,
      [ legend , "x1(t): Prey1" , "x2(t): Prey2" , "y(t): Predator" ] ) $
```

$$\frac{d}{dt}\mathrm{x1} = 1.32\mathrm{x1} - 0.01\mathrm{x1}y$$

$$\frac{d}{dt}\mathrm{x2} = 1.3\mathrm{x2} - 0.01\mathrm{x2}y$$

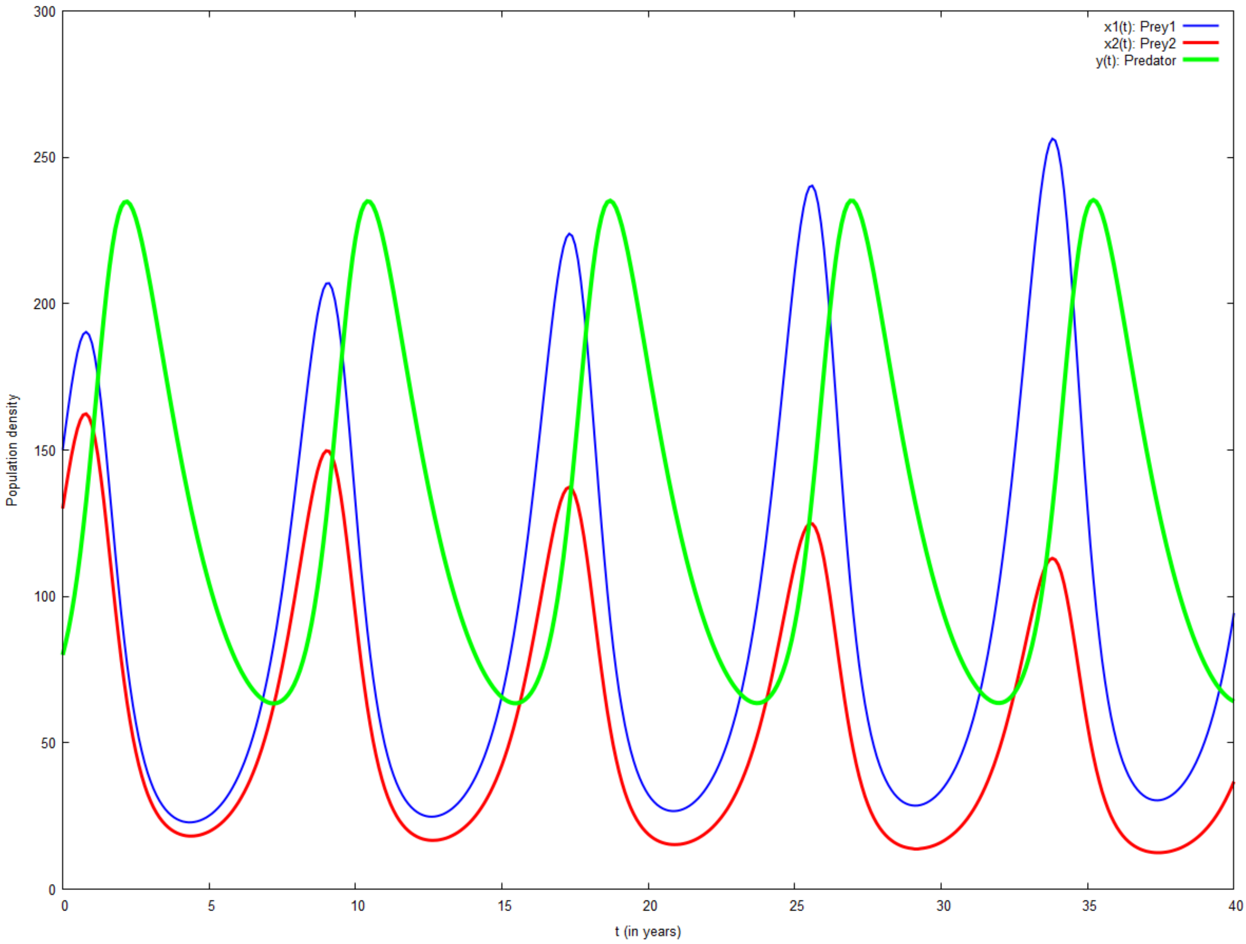
$$\frac{d}{dt}y = 0.004\mathrm{x2}y + 0.003\mathrm{x1}y - 0.5y$$

$$[[0.0, 150.0, 130.0, 80.0], [40.0, 94.32633644255458, 36.73244132377353, 64.13219452304699], 401]$$

$$[[0.0, 150.0], [40.0, 94.32633644255458], 401]$$

$$[[0.0, 130.0], [40.0, 36.73244132377353], 401]$$

$$[[0.0, 80.0], [40.0, 64.13219452304699], 401]$$



Created with wxMaxima.
The source of this Maxima session can be downloaded here.