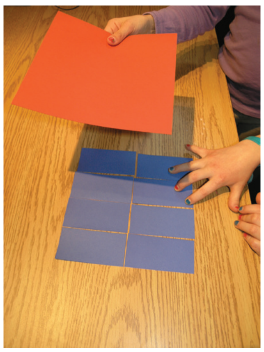
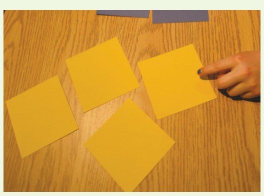
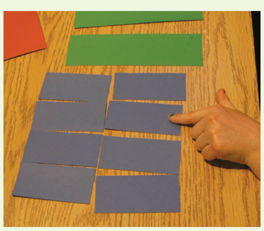
Being Responsive to Student Thinking

When educators work with a partner or in small groups to examine and annotate student work, it allows for student thinking to be unpacked – their understandings, strategies and transitional conceptions. Here, student thinking from a Grade 3 classroom, a Grade 7/8 classroom and a Grade 9 classroom is shared for educators to use in their own professional learning, individually or school-wide.

Grade 3

In this Grade 3 classroom, students were given sets of pre-partitioned fraction pieces and asked to identify relationships between the pieces. The following excerpts are transcripts of two students' dialogue captured on video.

Visual	Transcript	Student Assets
	[S1 places the red whole over the blue eighths] S1: These are eighths. Because we put eight together so they are eighths. And it is the same size as the whole. T: You have made the connection that *eight eighths* is a whole. So can you make any other connections between the pieces?	*The student had some understanding of the unit fraction language, such as half, fourth, third and eighth.*
 	S2: Oh yeah – this piece. [motions to a yellow fourth] T: OK. What is that called? S2: It is called a fourth. So, it won't really take as long to go together. [motions to a piece off screen] This piece would take the longest. [motions to the eighths] Well, actually this piece would take the longest. And, [picks up a fourth] this piece, all you've got to do is put the rest of the pieces together.	*This student had made an earlier prediction about how long it would take to build the whole by using eighths and continued to think of the fraction pieces in terms of how long it would take to build the whole.*

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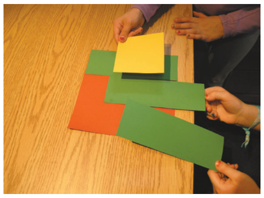
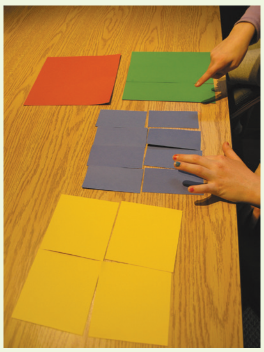
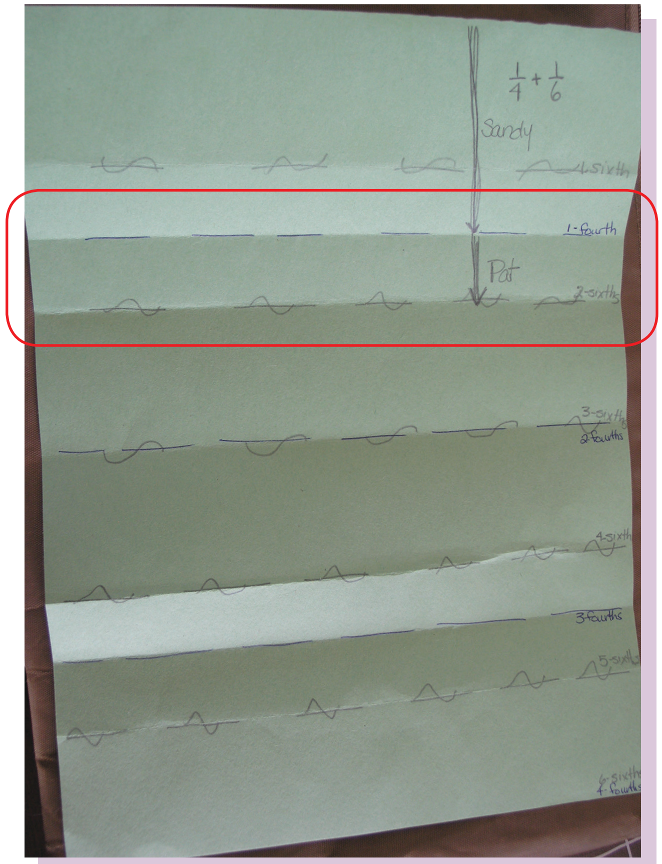
	*A bit later* S1: I think it is a … third. T: OK. And why do you think that? S1: Well, it looks like less than a half but more than a quarter. [places the yellow fourth on top to compare sizes as S2 is working to build the whole with the third pieces]	*This student used proportional reasoning and an understanding of benchmarks to develop an understanding of additional unit fractions.*
	S2: And you only have to put three together so then … [builds the whole] T: That was fast, wasn't it? So now you have … what would you call those? S2: Thirds. T: Three thirds. [points to three-thirds model] S1 and S2: Four fourths. Eight eighths. One whole.	*The students quickly adapted the language modelled to each of the other fractional representations.*

Probing questions include the following:

- If students haven't expressed an understanding that the pieces are regions of an area model, try asking them, "How are the pieces, or regions, similar or different?" and "What connections have you made between the regions and the whole?"
- If students have used whole number counts to refer to the fraction pieces (i.e., one, two, three), ask, "How might we count those pieces, or regions, by using their fraction numbers?" It may be necessary to model this counting as "*one one-third*, *two one-thirds*, ..."
- If students haven't connected the denominator to the number of equi-partitions, write the fraction by using both words and symbols and ask, "What connections can you make between the different ways of showing this fraction?"

Grade 7/8



In a Grade 7/8 classroom, the following question was posed to students: "Out of one pan of brownies, Sandy ate $\frac{1}{4}$ and Pat ate $\frac{1}{6}$. What fraction of the brownies did Sandy and Pat eat altogether? Write a number sentence to explain." Students were provided with paper to fold to solve the task.

One student folded the paper as shown. Note the space between a *one-sixth* fold and a *one-fourth* fold (when folded as shown to the right) is one-twelfth of the area of the paper, which is the common unit. The student was unable to make this connection and wrote the number sentence as $\frac{1}{4} + \frac{1}{6} = \frac{2}{10}$.

Probing questions include the following:

- Explain your number sentence.
- If students don't realize that the regions should be the same size, ask, "Where is the $\frac{1}{4}$ in your model? Why is it named $\frac{1}{4}$?" This question is intended to draw students' attention to the equi-partitioning.
- If students understand that the regions should be the same size but are unsure of how to name the smaller fractional regions, ask, "Would it be possible to fold the other segments to create equal partitions?" This question is intended to support students in seeing that fractional regions can be split or partitioned into equivalent fractional segments.
- If students understand equi-partitioning and have folded their paper to create a common fractional unit of twelfths but are unsure how their answer compares with another response of $\frac{10}{24}$, ask, "Would it be possible to fold your model to create twenty-four equal partitions, or twenty-fourths? If you did that, how many would be comparable to your *five-twelfths*?"

Grade 9

A Grade 9 student was asked to solve for *x* in the equation $\frac{2}{3}x + 5 = 7 + \frac{4}{5}x$.

The student completed the following solution:

$$\frac{2}{3}x + 5 = 7 + \frac{4}{5}x$$
$$2x + 15 = 35 + 4x$$
$$-2x = 20$$
$$x = -10$$

Note that in the first line of the solution, the student multiplied the left side by 3 and the right side by 5.

Probing questions may include the following:

- Explain what procedure you used to create the first line in your solution.
- If the student doesn't realize that he or she is working with an equation and must perform the same calculation on all terms to preserve equality, ask, "Is this new equation equivalent to the original equation? How do you know?"
- If the student understands that the procedure should have been to multiply all terms by the same number but can't identify or justify the number, ask, "Would it be possible to write the equation so that each term is a fraction? If we wanted to have a common fraction unit for each term, what fraction unit (or denominator) could we use?" This question is intended to support students in seeing that multiplying by a common denominator to clear the fractions is really a shortcut. The mathematical justification is that if each term was first written in fraction form with a common denominator, it would be $\frac{10}{15}x + \frac{75}{15} = \frac{105}{15} + \frac{12}{15}x$. Then, since each term is based on the same fractional unit, we only need to equate the numerators. So the equation becomes $10x + 75 = 105 + 12x$.