STATISTICS FOR ENVIRONMENTAL SCIENCE

SPEA-E538

Fall 2023

10/26/2023

Intro to ANOVA

TODAY'S AGENDA

- ANOVA Concepts
- One-factor between-subjects example

One-Factor ANOVA

Between-subjects designs

2018/07/20 (updated: 2023-10-24)

Overview

1. ANOVA concepts
2. one-factor between subjects example

ANOVA

ANOVA stands for ANalysis Of VAriance

Purpose of ANOVA

1. Statistical inference test for multiple (2 or more) groups
2. The kind of ANOVA you run depends on the experimental design. This week we focus on **Between-Subjects designs**

Heads up

The end result of an ANOVA gives back similar information as a t-test

1. t(df) = t-value, p = p-value
2. F(df1, df2) = F-value, p = value

Reporting of F-tests also typically include one more thing:

3. F(df1, df2) = F-value, MSE = MS error value, p = value

Fast Example

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
IV <-
as.factor(rep(c("A","B","C"),each=3))
DV <- c(A,B,C)
df <- data.frame(IV,DV)
```

R: aov()

```
aov(DV~IV,df)
## Call:
##    aov(formula = DV ~ IV, data = df)
##
## Terms:
##                     IV Residuals
## Sum of Squares      72       230
## Deg. of Freedom      2         6
##
## Residual standard error: 6.191392
## Estimated effects may be unbalanced
```

R: summary()

The `summary()` function provides the ANOVA table

```
aov_results <- aov(DV~IV,df)
summary(aov_results)
##             Df Sum Sq Mean Sq F value
Pr(>F)
## IV           2     72   36.00   0.939
0.442
## Residuals    6    230   38.33
```

Things we need to understand

1. The logic of the ANOVA
2. What each part of the ANOVA table tells us

Let’s start by looking at the example for the next lab on ANOVA.

ANOVA lab example

Research Article

Psychological Science
2015, Vol. 26(8) 1201–1215

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Computer Game Play Reduces Intrusive Memories of Experimental Trauma via Reconsolidation-Update Mechanisms

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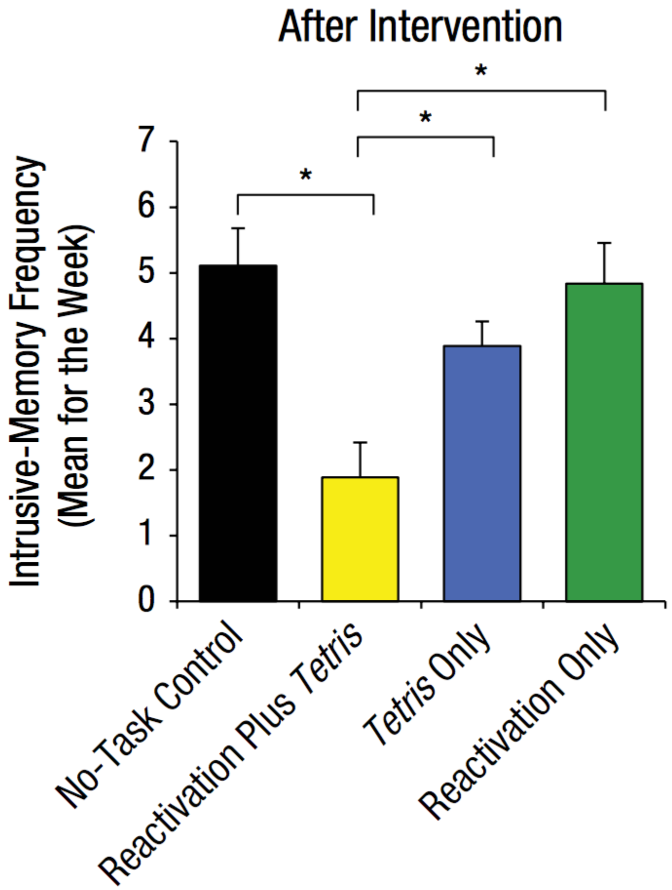
Design

class: center, middle, clear, nopad

Day 0	Day 1	Days 2-7
	Control	
Watch Scary Movie	Reactivation	Everyone keeps a journal keeping track of number of intrusive memories
	Tetris	
	Tetris + Reactivation	

Results



After Intervention
Intrusive-Memory Frequency
(Mean for the Week)
7
6
5
4
3
2
1
0
*
*
*
No-Task Control
Reactivation Plus *Tetris*
Tetris Only
Reactivation Only

Write-up

similar number of intrusive memories, $F(3, 68) = 0.16$, $p = .92$ (Fig. 4a).

Intrusive memories postintervention. Second, and critically, for the 7-day diary postintervention, there was a significant difference between groups in overall intrusion frequency in daily life, $F(3, 68) = 3.80$, $p = .01$, $\eta_p^2 = .14$ (Fig. 4b). Planned comparisons demonstrated that relative to the no-task control group, only those in the reactivation-plus-*Tetris* group, $t(22.63) = 2.99$, $p = .007$, $d = 1.00$, experienced significantly fewer intrusive memories; this finding replicated Experiment 1. Critically, as predicted by reconsolidation theory, the reactivation-plus-*Tetris* group had significantly fewer intrusive memories than the *Tetris*-only group, $t(27.96) = 2.52$, $p = .02$, $d = 0.84$, as well as the reactivation-only group, $t(25.68) =$

Things we need to understand

1. The logic of the ANOVA
2. What each part of the ANOVA table tells us

ANOVA is an omnibus test

- Omnibus definition: comprising many items
- ANOVAs can test for differences among many means (2 or more)
- Test question: Are there any differences among the means?
- If the answer is yes, then we still do not know which specific means are different from one another.

F-value

F is a ratio between two variances

```
##                Df Sum Sq Mean Sq F value
Pr(>F)
## IV              2     72   36.00   0.939
0.442
## Residuals       6    230   38.33
```

$$F = \frac{\text{MSE}_{\text{Effect}}}{\text{MSE}_{\text{Error}}} \quad F = \frac{\text{Mean Squared Error}_{\text{IV}}}{\text{Mean Squared Error}_{\text{Residuals}}}$$

```
36/38.33
## [1] 0.9392121
```

MSE: Mean squared Error (effect)

MSE (Mean Squared Error) is the SS (sums of sqaures) divided by the degrees of freedom

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## IV            2     72   36.00   0.939  0.442
## Residuals     6    230   38.33
```

$MSE_{Effect} = \frac{SS_{Effect}}{df_{Effect}}$ $MSE_{IV} = \frac{SS_{IV}}{df_{IV}}$

```
72/2
## [1] 36
```

MSE: Mean squared Error (error)

MSE (Mean Squared Error) is the SS (sums of sqaures) divided by the degrees of freedom

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## IV            2     72   36.00   0.939  0.442
## Residuals     6    230   38.33
```

$MSE_{Error} = \frac{SS_{Error}}{df_{Error}}$ $MSE_{Residuals} = \frac{SS_{Residuals}}{df_{Residuals}}$

```
6/230
## [1] 0.02608696
```

ANOVA reference table

Source	df	SS	MSE	F	p
Effect	$k-1$	SS_{Effect}	$MS_{Effect} = \frac{SS_{Effect}}{k-1}$	$\frac{MS_{Effect}}{MS_{Error}}$	Calculated from F-distribution
Error	$n-k$	SS_{Error}	$MS_{Error} = \frac{SS_{Error}}{n-k}$		

k = number of groups; n = total sample-size

$$SS_{Effect} = \sum_{i=1}^{k} n_i(X_i - \bar{X})^2$$

$$SS_{Error} = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{X}_i)^2$$

The big idea

Partitioning the Variance

Idea is to split up the variance in the data into two parts:

1. The part due to the manipulation (IV)
2. The leftover part, due to random error

class: center, middle, clear, nopad

Partitioning data

Original data is split into two parts

Every score expressed as it's group mean

Every score minus it's group mean

	Original Data			=	Effect (Group Means)			+	Error (leftover bits)		
	A	**B**	**C**		**A**	**B**	**C**		**A**	**B**	**C**
	20	6	2		11	5	5		9	1	-3
	11	2	11		11	5	5		0	-3	6
	2	7	2		11	5	5		-9	2	-3
Means	11	5	5								
Grand Mean	7										

class: center, middle, clear, nopad

Example of partitioned data when the **effect of the IV** is the only cause of change

	Original Data			=	Effect (Group Means)			+	Error (leftover bits)		
	A	**B**	**C**		**A**	**B**	**C**		**A**	**B**	**C**
	11	5	5		11	5	5		0	0	0
	11	5	5		11	5	5		0	0	0
	11	5	5		11	5	5		0	0	0
Means	11	5	5								
Grand Mean	7										

class: center, middle, clear, nopad

Example of partitioned data when the **effect of the IV does NOT cause change**

	Original Data			=	Effect (Group Means)			+	Error (leftover bits)		
	A	B	C		A	B	C		A	B	C
	1	3	-1		5	5	5		-4	-2	-6
	4	10	7		5	5	5		-1	5	2
	10	2	9		5	5	5		5	-3	4
Means	5	5	5								
Grand Mean	5										

class: center, middle, clear, nopad

Example of partitioned data when there is a bit of both

	Original Data			=	Effect (Group Means)			+	Error (leftover bits)		
	A	B	C		A	B	C		A	B	C
	20	6	2		11	5	5		9	1	-3
	11	2	11		11	5	5		0	-3	6
	2	7	2		11	5	5		-9	2	-3
Means	11	5	5								
Grand Mean	7										

class: center, middle, clear, nopad

What the ANOVA does…

Original Data

	A	B	C
	20	6	2
	11	2	11
	2	7	2
Means	11	5	5
Grand Mean	7		

=

Estimate variation due to **effect**

Effect (Group Means)

A	B	C
11	5	5
11	5	5
11	5	5

+

Estimate variation due to **error**

Error (leftover bits)

A	B	C
9	1	-3
0	-3	6
-9	2	-3

$$\textbf{F ratio} = \frac{\textbf{Amount of change in data due to IV (Effect)}}{\textbf{Amount of change in data NOT due to IV (Error)}}$$

class: center, middle, clear, nopad

What is F telling us?

1. If F is greater than 1, then what?

2. If F is less than 1, then what?

class: center, middle, clear, nopad

Strategy for finding F

1. Find the total amount of change in the data (SS Total)

2. Split it up into two parts (SS Effect vs. SS Error)

3. Find the average amount of change for each (MS Effect vs. MS Error)

4. Divide the averages to get F

class: center, middle, clear, nopad

1. Partition data and get difference scores

	Original Data			=	Effect (Group Means)			+	Error (leftover bits)		
	A	B	C		A	B	C		A	B	C
	20	6	2		11	5	5		9	1	-3
	11	2	11		11	5	5		0	-3	6
	2	7	2		11	5	5		-9	2	-3
Means	11	5	5								
Grand Mean	7										

Difference Scores

Total Scores from Grand Mean			=	Effect Group means from Grand Mean			+	Error Scores from Group Means		
A	B	C		A	B	C		A	B	C
13	-1	-5		4	-2	-2		9	1	-3
4	-5	4		4	-2	-2		0	-3	6
-5	0	-5		4	-2	-2		-9	2	-3

2. Square the difference scores, and sum them up

class: center, middle, clear, nopad

	Squared Difference Scores										
	Total				Effect				Error		
	Scores from Grand Mean ^2			=	Group means from Grand Mean^2			+	Scores from Group Means^2		
	A	B	C		A	B	C		A	B	C
	169	1	25		16	4	4		81	1	9
	16	25	16		16	4	4		0	9	36
	25	0	25		16	4	4		81	4	9
Sums	210	26	66		48	12	12		162	14	54
	SS total =		**302**		**SS Effect =**		**72**		**SS Error =**		**230**

class: center, middle, clear, nopad

3. Divide SSes by Dfs to get MSEs (Variances)
4. Divide MSEs to get F

Source	Df	SS	MSE	F
Effect	k-1	72	?	?
Error	n-k	230	?	

Source	Df	SS	MSE	F
Effect	3-1 = 2	72	72/2 = 36	36/38.333 = .9391
Error	9-3 = 6	230	230/6 = 38.33	

Source	Df	SS	MSE	F
Effect	2	72	36	0.9391
Error	6	230	38.33	

SS total

$$SS_{\text{Total}} = SS_{\text{Effect}} + SS_{\text{Error}}$$
$$SS_{\text{Total}} = SS_{\text{Can Explain}} + SS_{\text{Can't Explain}}$$
$$SS_{\text{Total}} = SS_{\text{Change due to manipulation}} + SS_{\text{Change due to Chance}}$$
$$SS_{\text{Total}} = \sum_{i=1}^{n} (x_i - \bar{X})^2$$

- x_i = each score
- $\bar{X}$ = Grand Mean of all scores

SS total example

groups	scores	diff	diff_squared
A	20	13	169
A	11	4	16
A	2	-5	25
B	6	-1	1
B	2	-5	25
B	7	0	0
C	2	-5	25
C	11	4	16
C	2	-5	25
Sums	63	0	302
Means	7	0	33.5555555555556

R: SS total

$$SS_{\text{Total}} = \sum_{i=1}^{n} (x_i - \bar{X})^2$$

- the sum of the squared differences between every score and the grand mean

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
all_scores <- c(A,B,C)
grand_mean <- mean(all_scores)
SS_total <- sum((all_scores-grand_mean)^2)
print(SS_total)
```

```
## [1] 302
```

Remember what SS represents?

SS (the sums of squares) is a single number representing the sum of all of the **change** in the data.

- Specifically, the squared deviations from each score from the mean

Question

What are some **sources** of change that could cause SS_{Total} for a set of data to increase or decrease?

- what properties of the sample data could be changed that would increase or decrease SS_{Total}?

SS total and the Effect (IV)

A successful manipulation (IV) will cause an effect (e.g., cause differences between the sample means)

Question: How can the effect of an IV cause increases or decreases to SS_{Total}?

SS total and sampling error

Sampling error is due to existing variability in the data.

Question: How can the effect of sampling error cause increases or decreases to SS_{Total}?

The F-distribution

Let’s examine some ANOVA concepts by simulation in R.

https://crumplab.github.io/psyc3400/Presentations/bw_ANOVA.html

The entire ANOVA table: Step 1

1. Put the data into a dataframe. One column should code the levels of the IV, and the other column should include the DV

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
IV <-
as.factor(rep(c("A","B","C"),each=3))
DV <- c(A,B,C)
df <- data.frame(IV,DV)
```

The entire ANOVA table: Step 2

```
summary(aov(DV~IV, df))
##             Df Sum Sq Mean Sq F value
Pr(>F)
## IV           2     72   36.00   0.939
0.442
## Residuals    6    230   38.33
```

SS total

The sum of the squared deviations between each score and the grand mean.

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
all_scores <- c(A,B,C)
grand_mean <- mean(all_scores)
SS_total <- sum((all_scores-grand_mean)^2)
print(SS_total)
```

```
## [1] 302
```

SS total using the ANOVA table

Add up both of the Sums of Squares...

```
summary(aov(DV~IV, df))
##             Df Sum Sq Mean Sq F value
Pr(>F)
## IV           2     72   36.00   0.939
0.442
## Residuals    6    230   38.33
```

SS effect

$$SS_{\text{Effect}} = \sum_{i=1}^{k} n_i \,(X_i - \bar{X})^2$$

Each score is treated as it's group mean. Then, all of these "group mean" scores for each subject is subtracted from the grand mean. The SS effect is the sum of these squared deviations.

R: SS effect

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
grand_mean <- mean(c(A,B,C))
scores_as_grp_means <- c(rep(mean(A),3),
                         rep(mean(B),3),
                         rep(mean(C),3))
SS_Effect <- sum((scores_as_grp_means-
grand_mean)^2)
print(SS_Effect)
```

```
## [1] 72
```

SS effect from ANOVA table

Or, just look at the Sums of squares in the first row of the ANOVA table (for IV in this case, which represents the effect).

```
summary(aov(DV~IV, df))
##             Df Sum Sq Mean Sq F value Pr(>F)
## IV           2     72   36.00   0.939  0.442
## Residuals    6    230   38.33
```

SS error

The sum of the squared deviations between each score and it’s group mean.

R: SS error

```
A <- c(20,11,2)
B <- c(6,2,7)
C <- c(2,11,2)
scores_as_grp_means <- c(rep(mean(A),3),
                          rep(mean(B),3),
                          rep(mean(C),3))
SS_Error <- sum((scores_as_grp_means-
c(A,B,C))^2)
print(SS_Error)
```

```
## [1] 230
```

SS error from ANOVA table

Or, just look at the Sums of squares in the second row of the ANOVA table (for residuals in this case, which represents the error).

```
summary(aov(DV~IV, df))
## Df Sum Sq Mean Sq F value
Pr(>F)
## IV 2 72 36.00 0.939
0.442
## Residuals 6 230 38.33
```

Getting F and p from ANOVA table

Put the ANOVA summary into a variable

```r
anova_variable <- summary(aov(DV~IV, df))
```

Get F and associated p

```r
anova_variable[[1]]$`F value`
## [1] 0.9391304        NA
anova_variable[[1]]$`Pr(>F)`
## [1] 0.4417359        NA
```

Getting Df, SS, and MS

```
anova_variable[[1]]$Df
## [1] 2 6
anova_variable[[1]]$`Sum Sq`
## [1]  72 230
anova_variable[[1]]$`Mean Sq`
## [1] 36.00000 38.33333
```

Analysis of Variance (ANOVA)

- What happens when we have one continuous DV, and one categorical IV *with 2 or more levels*, and/or *more than one categorical IV with 2 or more levels each?*

  We use a *NEW test* **ysis of Variance (ANOVA)**

- **One-Way Analysis of Variance (One-Way ANOVA):** a statistical procedure used to test hypotheses about one categorical IV (called a "factor") with **two or more levels,** concerning the variance among group means of one continuous DV (Chapter 11)

➢ The "*One-way*" in the name indicates the number of categorical IV's being tested (<u>**NOT**</u> the number of levels in each IV!!)

➢ ***One-Way*** ANOVA tests hypotheses about ***one*** categorical IV with two or more levels (Chapter 11)

➢ ***Two-Way*** ANOVA tests hypotheses about ***two*** categorical IVs, each with two or more levels (Chapter 12)

➢ ***Three-Way*** ANOVA tests hypotheses about ***three*** categorical IVs, each with two or more levels (We won't cover these in PSYC2111)

2 Types of One-Way ANOVA

- **2 Types** of One-Way ANOVA (used for a continuous DV, and one categorical IV with 2 or more levels):

➢ One-Way Between-Subjects ANOVA

➢ One-Way Within-Subjects ANOVA

➢ **One-Way Between-Subjects ANOVA**: used when ***different participants*** are observed at each level of the one categorical IV.

✧ *Example*: A sample of patients with equivalent levels of chronic back pain are randomly assigned to one of 3 treatment conditions: 1) Drug A, 2) Drug B, 3) Placebo sugar pill. Pain severity measured on a 0-10 scale (continuous DV) is measured in each treatment group after 1 month of treatment

2 Types of One-Way ANOVA

- **2 Types** of One-Way ANOVA (used for a continuous DV, and one categorical IV with 2 or more levels):
- ➢ One-Way Between-Subjects ANOVA
- ➢ One-Way Within-Subjects ANOVA

- ➢ **One-Way Within-Subjects ANOVA**: used when the ***same participants*** are observed at each level of the one categorical IV.
- ✧ *Example*: A sample of professional athletes is recruited. Each athlete's speed is measured in meters per second (continuous DV) is recorded running the hundred-yard dash in each of 3 running conditions: 1) wearing Adidas shoes, 2) wearing Brooks shoes, 3) wearing Nike shoes.

What Type of ANOVA?

- In summary, the type of ANOVA test is dictated by **2 things:**
  1. The number of IV's (factors) being tested
     - In a **one-way ANOVA**, one IV is tested
     - In a **two-way ANOVA**, two IV's are tested, and so on…
  2. How the participants are observed across each level of the IV:
     - **Within-subjects design:** participants experience/are measured at every level of the IV
     - **Between-subjects design:** participants are measured only once at one level of the IV

ANOVA Terminology

- **n** = number of participants per group (**<u>not</u>** total number in a sample)
- **N** = total number of participants in in the whole study (**<u>not</u>** total number in a population)
- **k** = the number of levels in one categorical IV (i.e. the number of levels in one factor variable)

✧ *Example*: A sample of patients (N=90) with equivalent levels of chronic back pain are randomly assigned to one of 3 treatment conditions (k=3): 1) Drug A (n=30), 2) Drug B (n=30), 3) Placebo sugar pill (n=30).

N=90 patients in the whole study; n=30 patients

ıent groups (3 levels of the one IV called "treatment")

Learning Check!

- ◆*Example:* A sample of 75 CU Boulder undergraduate students are recruited directly after graduation and asked to respond to the statement "I am satisfied with my choice of undergraduate major" using a 7-point Likert scale (0="strongly disagree" to 7="strongly agree"). The students were evenly distributed across 3 majors: 1) Psychology, 2) Sociology, 3) Biology. There were no double-majors.
- ➢ What is the appropriate test to answer the question: did satisfaction with undergraduate major at graduation depend on which major was declared?
- ✧One-Way ANOVA
- ➢ Is this a between-subjects or within subjects design?
- ✧Between-subjects
- ➢ What are N, n, and k?
- ✧N=75; n=25 (1/3 in each major); k=3 (3 levels)

Learning Check!

- *Example:* A sample of 75 CU Boulder undergraduate students are asked whether they are satisfied with their choice of undergraduate major at each of 4 time points: 1) directly after declaring, 2) directly after graduation, 3) one year post-graduation, 4) 5-years post-graduation. Satisfaction the same 7-point Likert scale: “I am satisfied with my choice of undergraduate major” (0=“strongly disagree” to 7=“strongly agree”).

- What is the appropriate test to answer the question: did satisfaction with undergraduate major depend on when the question was asked?

- One-Way ANOVA

- Is this a between-subjects or within subjects design?

- Within-subjects

- What are N, n, and k?

- N=75; n=75 (everyone is in all 3 IV levels); k=4 (4 levels)

Sources of Variation

- ANOVA's evaluate if the means in each group significantly vary.

➢ Range of variance between group means: 0 to positive infinity

➢ If all means are equal ➔ variance of group means = 0

➢ The larger the difference between the group means, the larger the variance of group means

➢ In a **One-Way Between-Subjects ANOVA**, there are 2 ways in which the means can vary (a.k.a. **2 Sources of Variation**):

1. **Between-groups variation:** the variation attributed to mean difference between groups
2. **Within-groups variation:** the variation attributed to mean differences within each group. This source of variation cannot be attributed to differences between groups, and is therefore called "error variation" (attributed to error or chance).

Sources of Variation

1. **Between-groups variation:** variation attributed to the mean difference between groups
2. **Within-groups variation:** variation attributed to the mean differences within each group; called "error variation" (attributed to error or chance).

TABLE 11.1 Between-Groups and Within-Groups Variability for the One-Way Between-Subjects ANOVA

Type of Class		
Psychology	Sociology	Biology
5	3	4
2	2	6
4	6	5
3	5	4
1	4	6
$M_1 = 3$	$M_2 = 4$	$M_3 = 5$

Within-groups variability

Between-groups variability

Between-groups variability is attributed to differences between group means. Within-groups variability is attributed to variability that has nothing to do with having different groups.

ANOVA Test Statistic: "*F*"

- ANOVA's are also called "F Tests"
- Test Statistic for an ANOVA

$$F_{obt} = \frac{MS_{BG}}{MS_E} \quad \text{or} \quad \frac{\textit{variance between groups}}{\textit{variance within groups}}$$

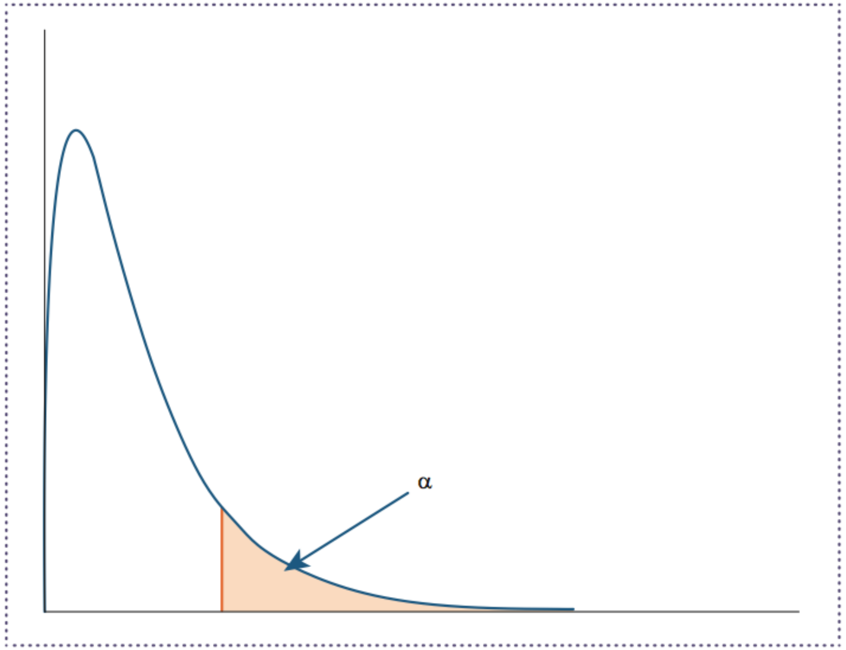
- *Formula in Words:* mean square (or variance) between groups divided by the mean square (or variance) within groups
- The distribution of all possible outcomes for the F test statistic is positively skewed (because variance cannot be negative!)
- The distribution, called the *F* distribution, is derived from a sampling distribution of *F* ratios

ANOVA Test Statistic: "*F*"

$$F_{obt} = \frac{MS_{BG}}{MS_E} \quad \text{or} \quad \frac{\textit{variance between groups}}{\textit{variance within groups}}$$

Alpha cut-off will always be in one tail of the *F* Distribution!

FIGURE 11.2 The *F* Distribution



Degrees of Freedom (*df*)

- The *df* for a one-way between-subjects ANOVA is $N - 1$
- You must split the total *df* ($N - 1$) into 2 parts: one for each source of variation:
  1. Degrees of freedom between groups (**df_{BG}**) or degrees of freedom numerator:
     - df_{BG} is the *df* associated with the variance for the group means in the numerator of the test statistic. It is equal to the number of groups (k) minus 1 ➔ **k-1**
  - Degrees of freedom error (**df_E**), degrees of freedom within groups, or degrees of freedom denominator:
     - df_E is the df associated with the error variance in the denominator. It is equal to the total sample size (*N*) minus the number of groups (k) ➔ **N-k**

$$\boxed{F_{obt} = \frac{\textit{variance between groups}}{\textit{variance within groups}} \xleftarrow{\text{degrees of freedom}} \frac{k-1}{N-k}}$$

4 Assumptions for One-Way Between-Subjects ANOVA

- There are four assumptions to compute the one-way between-subjects ANOVA:
    1. ***Normality***: assume that data in the population or populations being sampled from are normally distributed
    2. ***Random Sampling***: assume data measured were obtained from a sample that was selected using a random sampling procedure
    3. ***Independence***: assume that the probabilities of each measured outcome in a study are independent or equal
    4. ***Homogeneity of Variance***: assume that the variance in each population is equal to each other
- Sample size (*n*) must be equal in each group!

One-Way Between-Subjects ANOVA Example

- *Example:* Employee turnover is costly to businesses, and thus is a common research topic among Organizational Psychologists! Suppose an Organizational Psychologist were interested in whether stress levels in the workplace predict how long employees are willing to stay at a company. They recruit 30 employees at a major local company. Employees report their stress levels and are broken up evenly into 3 groups based on how stressful they rate the workplace: 1) **Low** stress, 2) **Moderate** stress, 3) **High** stress. Employees also report the time (in years) they predict they would remain at the company. Conduct an ANOVA to analyze significance of data at a .05 level of significance.
- What is the appropriate type of ANOVA for this example? Why?
- One-Way Between Subjects ANOVA –one IV, each person is only in one level of the 1V (either in Low, or Moderate, or High stress group)
- What are N, n, and k?
- N=30, n=10, k=3

One-Way Between-Subjects ANOVA Example

TABLE 11.3 The Times in Years That Employees Said They Would Stay With the Company Among Employees Who Rated the Workplace as Being a High-, Moderate-, or Low-Stress Environment

Perceived Stress Level of Workplace			
Low	Moderate	High	[$k = 3$]
3.4	3.5	2.9	
3.2	3.6	3.0	
3.0	2.7	2.6	
3.0	3.5	3.3	
3.5	3.8	3.7	
3.8	2.9	2.7	
3.6	3.4	2.4	
4.0	3.2	2.5	
3.9	3.3	3.3	
2.9	3.1	3.4	[$N = k \times n = 30$]
$n_1 = 10$	$n_2 = 10$	$n_3 = 10$	

One-Way Between-Subjects ANOVA Example

◆**STEP 1:** State the Hypotheses:

- ***H_0*****:** $\sigma^2_\mu = 0$ ➔ *In words:* group means (μ) do not vary (σ^2) in the company employee population
- ***H_1*****:** $\sigma^2_\mu > 0$ ➔ *In words*: group means in the company employee population do vary

- **STEP 2:** Set the criteria for a decision
  - Level of significance is .05
    - ***df_{BG}***= k-1: 3-1 = 2
    - ***df_E*** = N-k: 30-3 = 27
  - Critical value is 3.35 (Table B.3, Appendix B)

$$\boxed{F_{obt} = \frac{\textit{variance between groups}}{\textit{variance within groups}} \xleftarrow{\text{degrees of freedom}} \frac{k-1}{N-k}}$$

One-Way Between-Subjects ANOVA Example

- **Use the F Table in B.3 in Appendix B to find F_{cv}:**
  - df_{BG} (numerator)= k-1: 3-1 = 2
  - df_{E} (denominator)= N-k: 30-3 = 27
- **Fcv** = 3.35

Degrees of Freedom in the numerator

BOLD: α=.01
NOT Bold: α=.05

Degrees of Freedom in the denominator

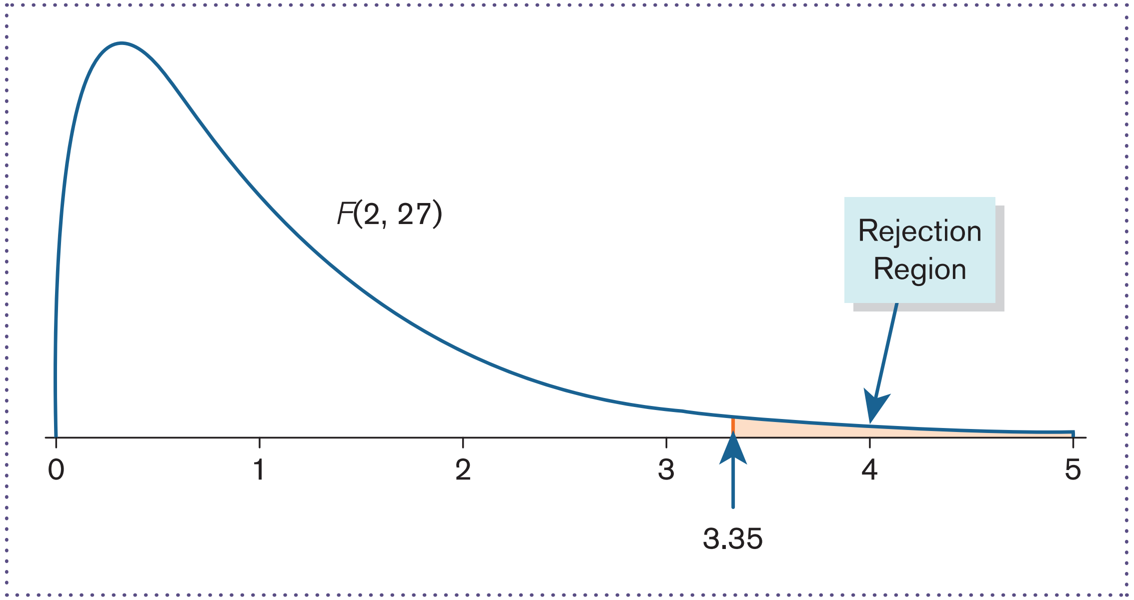
Degrees of freedom numera[illegible]

Degrees of freedom denominator	1	2	3	4	5	6	7	8	9	10	20
16	4.49 **8.53**	3.63 **6.23**	3.24 **5.29**	3.01 **4.77**	2.85 **4.44**	2.74 **4.20**	2.66 **4.03**	2.59 **3.89**	2.54 **3.78**	2.49 **3.69**	2.28 **3.25**
17	4.45 **8.40**	3.59 **6.11**	3.20 **5.18**	2.96 **4.67**	2.81 **4.34**	2.70 **4.10**	2.62 **3.93**	2.55 **3.79**	2.50 **3.68**	2.45 **3.59**	2.23 **3.16**
18	4.41 **8.28**	3.55 **6.01**	3.16 **5.09**	2.93 **4.58**	2.77 **4.25**	2.66 **4.01**	2.58 **3.85**	2.51 **3.71**	2.46 **3.60**	2.41 **3.51**	2.19 **3.07**
19	4.38 **8.18**	3.52 **5.93**	3.13 **5.01**	2.90 **4.50**	2.74 **4.17**	2.63 **3.94**	2.55 **3.77**	2.48 **3.63**	2.43 **3.52**	2.38 **3.43**	2.15 **3.00**
20	4.35 **8.10**	3.49 **5.85**	3.10 **4.94**	2.87 **4.43**	2.71 **4.10**	2.60 **3.87**	2.52 **3.71**	2.45 **3.56**	2.40 **3.45**	2.35 **3.37**	2.12 **2.94**
21	4.32 **8.02**	3.47 **5.78**	3.07 **4.87**	2.84 **4.37**	2.68 **4.04**	2.57 **3.81**	2.49 **3.65**	2.42 **3.51**	2.37 **3.40**	2.32 **3.31**	2.09 **2.88**
22	4.30 **7.94**	3.44 **5.72**	3.05 **4.82**	2.82 **4.31**	2.66 **3.99**	2.55 **3.76**	2.47 **3.59**	2.40 **3.45**	2.35 **3.35**	2.30 **3.26**	2.07 **2.83**
23	4.28 **7.88**	3.42 **5.66**	3.03 **4.76**	2.80 **4.26**	2.64 **3.94**	2.53 **3.71**	2.45 **3.54**	2.38 **3.41**	2.32 **3.30**	2.28 **3.21**	2.04 **2.78**
24	4.26 **7.82**	3.40 **5.61**	3.01 **4.72**	2.78 **4.22**	2.62 **3.90**	2.51 **3.67**	2.43 **3.50**	2.36 **3.36**	2.30 **3.25**	2.26 **3.17**	2.02 **2.74**
25	4.24 **7.77**	3.38 **5.57**	2.99 **4.68**	2.76 **4.18**	2.60 **3.86**	2.49 **3.63**	2.41 **3.46**	2.34 **3.32**	2.28 **3.21**	2.24 **3.13**	2.00 **2.70**
26	4.22 **7.72**	3.37 [illegible]	2.98 **4.64**	2.74 **4.14**	2.59 **3.82**	2.47 **3.59**	2.39 **3.42**	2.32 **3.29**	2.27 **3.17**	2.22 **3.09**	1.99 **2.66**
27	4.2[illegible] **7.68**	3.35 **5.49**	[illegible].96 **4.60**	2.73 **4.11**	2.57 **3.79**	2.46 **3.56**	2.37 **3.39**	2.30 **3.26**	2.25 **3.14**	2.20 **3.06**	1.97 **2.63**

One-Way Between-Subjects ANOVA Example

- **Use the F Table in B.3 in Appendix B to find F_{cv}:**
  - df_{BG} (numerator)= k-1: 3-1 = 2
  - df_{E} (denominator)= N-k: 30-3 = 27
  - **Fcv = 3.35**

FIGURE 11.3 The *F* Distribution With 2 and 27 Degrees of Freedom



The alpha level (α) is placed in the upper tail at or above 3.35.

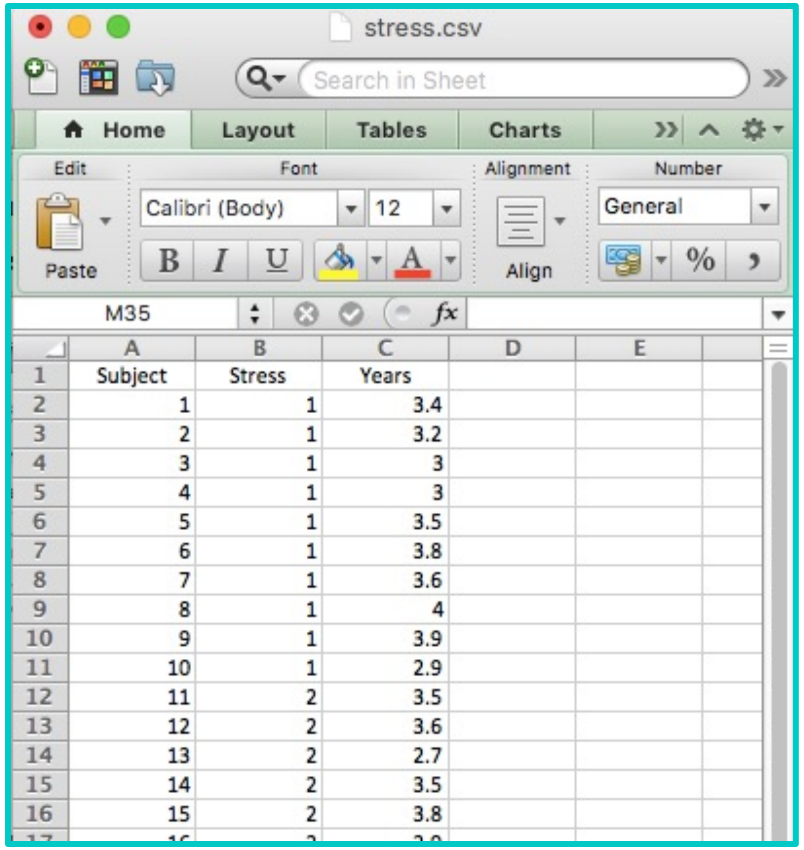
One-Way Between-Subjects ANOVA Example

- **STEP 3:** Compute the Test Statistic

$$F_{obt} = \frac{MS_{BG}}{MS_E}$$

- We are going to do this in R!

1. First enter the data into Excel: one row per subject, all levels of the IV in one column:
2. Then save as a ".csv" file
3. Import your dataset into R



	A	B	C	D	E
1	Subject	Stress	Years		
2	1	1	3.4		
3	2	1	3.2		
4	3	1	3		
5	4	1	3		
6	5	1	3.5		
7	6	1	3.8		
8	7	1	3.6		
9	8	1	4		
10	9	1	3.9		
11	10	1	2.9		
12	11	2	3.5		
13	12	2	3.6		
14	13	2	2.7		
15	14	2	3.5		
16	15	2	3.8		

One-Way Between-Subjects ANOVA Example

◆**STEP 3:** Compute the Test Statistic

```
#Input the Dataset "stress" into R
stress <-read.csv(file.choose())
stress

#STEP 1: Code the variable "Stress" as a factor
with three levels: "Low", "Moderate", "High"
stress$Stress_F <- factor(stress$Stress, labels=
c("Low", "Moderate", "High"))
```

```
> print(stress$Stress_F)
 [1] Low      Low      Low      Low      Low      Low      Low     
[10] Low      Moderate Moderate Moderate Moderate Moderate Moderate
[19] Moderate Moderate High     High     High     High     High    
[28] High     High     High    
Levels: Low Moderate High
```

One-Way Between-Subjects ANOVA Example

◆**STEP 3:** Compute the Test Statistic

```
#STEP 2: The "Omnibus Test" -- Run a One-Way
Between-Subjects ANOVA to test if there are any
differences between the groups!

#Generic code:

myanova <- aov(DV ~ IV, data=datasetname)

#Applied code:

myanova <- aov(Years ~ Stress_F, data=stress)

summary(myanova)
```

```
> myanova <- aov(Years ~ Stress_F, data=stress)
> summary(myanova)
            Df Sum Sq Mean Sq F value Pr(>F)
Stress_F     2  1.073  0.5363   3.517 0.0439 *
Residuals   27  4.117  0.1525
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

One-Way Between-Subjects ANOVA Example

"Df" is degrees of freedom: "Stress_F" row is the df_{BG}=2;
"Residuals" means error so this row is the df_E=27

F value is F_{obt}=3.52

Pr(>F) is the *p* value for the Omnibus test

```
> myanova <- aov(Years ~ Stress_F, data=stress)
> summary(myanova)
            Df Sum Sq Mean Sq F value Pr(>F)
Stress_F     2  1.073  0.5363   3.517 0.0439 *
Residuals   27  4.117  0.1525
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

"Mean Sq" is mean square (MS):
MS_{BG} .5363/MS_E .1525=3.517 (=F)

One-Way Between-Subjects ANOVA Example

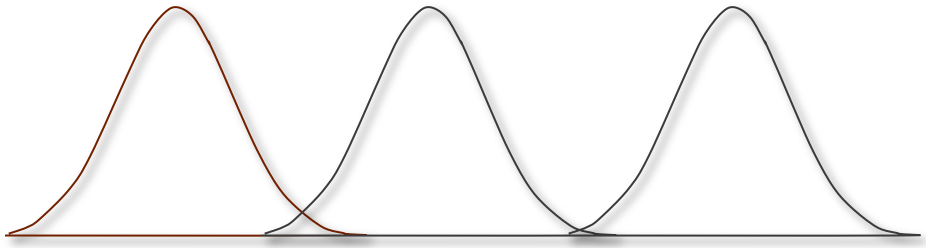
- **STEP 4:** Make a Decision

```
> myanova <- aov(Years ~ Stress_F, data=stress)
> summary(myanova)
            Df Sum Sq Mean Sq F value Pr(>F)
Stress_F     2  1.073  0.5363   3.517 0.0439 *
Residuals   27  4.117  0.1525
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

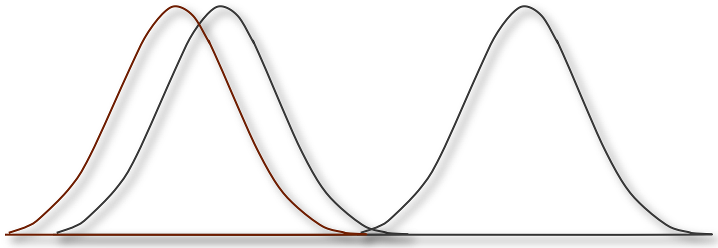
- $|F_{obt}=3.517| > |F_{cv}=3.35|$ → Reject the Null! There is at least one difference between the means!
- But ***WHERE*** is the difference?

One-Way Between-Subjects ANOVA Example

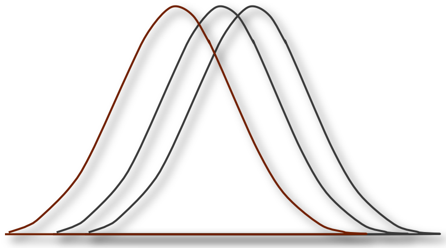
- But ***<u>WHERE</u>*** is the difference between the means? Options:



Option 1. All the means are different: $\mu_1 \neq \mu_2 \neq \mu_3$



Option 2. Two means are the same, one is different: $\mu_1 = \mu_2 \neq \mu_3$



Option 3. All means are the same: $\mu_1 = \mu_2 = \mu_3$

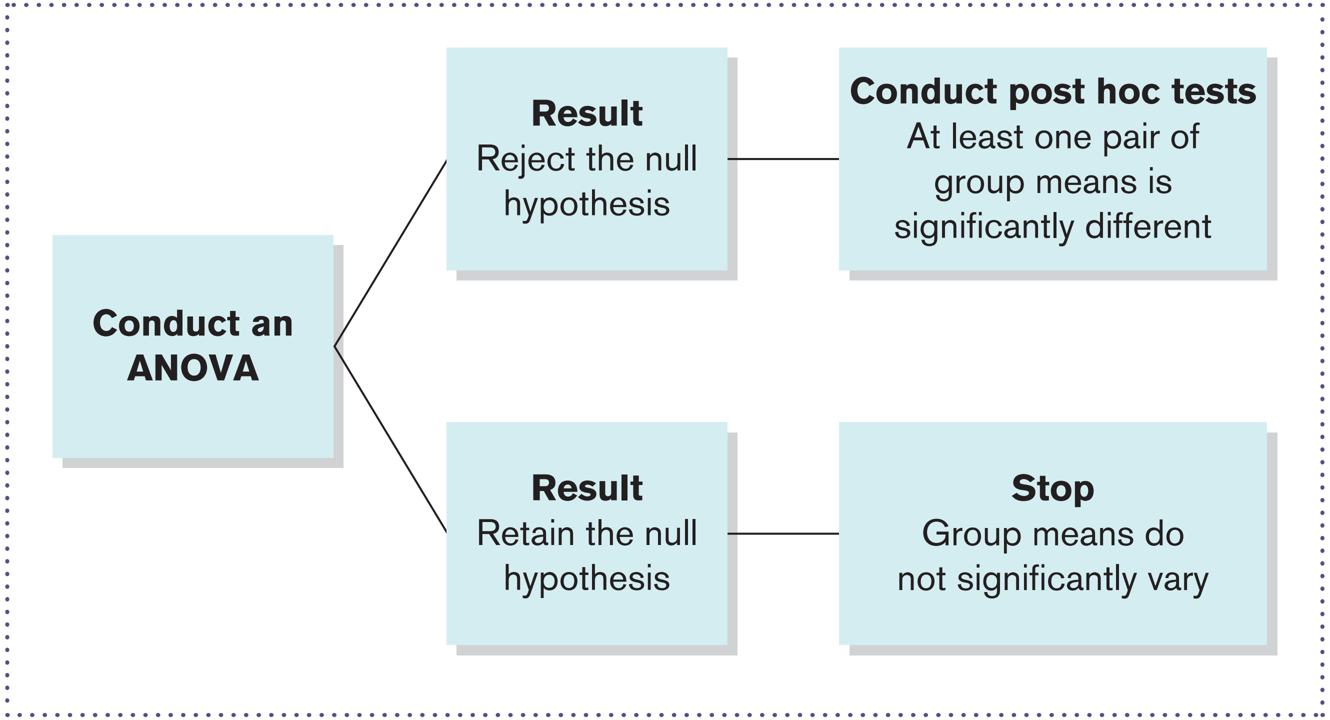
Post Hoc Tests

- **WHEN:** *How do we know where the significant difference is?* When the ANOVA "omnibus" (overall) test is significant, conduct Post Hoc Tests to determine which pair or pairs of group means significantly differ from one another!
- **Post Hoc Test:** statistical procedure computed following a significant ANOVA to determine which pair or pairs of group means significantly differ
    - These tests are necessary when $k > 2$ because multiple comparisons are needed
    - A Post Hoc Test evaluates all possible **pairwise comparisons** (i.e., comparisons between a pair of means) for an ANOVA with any number of groups
    - **Which Post Hoc Test?** We will use Tukey's Honestly Significant Difference (HSD)

Post Hoc Tests

- When do we compute a Post Hoc Test?

FIGURE 11.5 Following an ANOVA. A Decision Chart for When to Compute Post Hoc Tests



Post Hoc Tests

- **WHEN:** *How do we know where the significant difference is?* When the ANOVA "omnibus" (overall) test is significant, conduct Post Hoc Tests to determine which pair or pairs of group means significantly differ from one another!
- **HOW:**

```
#STEP 4: Run the Tukey's HSD Posthoc Test -- Compute the Pairwise Comparisons to find where the significant mean difference is!

TukeyHSD(myanova)
```

High vs. Low is the only significant difference (p=.04 <.05)

These are p values!

```
$Stress_F
                diff        lwr         upr     p adj
Moderate-Low  -0.13 -0.5629854  0.30298539 0.7395178
High-Low      -0.45 -0.8829854 -0.01701461 0.0404540
High-Moderate -0.32 -0.7529854  0.11298539 0.1782828
```

One-Way Between-Subjects ANOVA Example

◆**STEP 4:** Make a Decision & Write a Conclusion!

- We still don't know the mean and sd in each group!

```
#STEP 4: Use the "tapply" command to get the mean
and sd for each group

tapply(stress$Years, stress$Stress_F, mean)

tapply(stress$Years, stress$Stress_F, sd)
```

```
> tapply(stress$Years, stress$Stress_F, mean)
     Low Moderate      High
    3.43      3.30      2.98
> tapply(stress$Years, stress$Stress_F, sd)
      Low  Moderate      High
0.3973523 0.3333333 0.4341019
```

One-Way Between-Subjects ANOVA Example

◆**STEP 4:** Make a Decision & Write a Conclusion!

A. *Decision:* Reject the null ➔ there is at least one significant difference between the means!

B. *Post Hoc Test:* Run all Pairwise comparisons to find the significant difference(s) ➔ High vs. Low was the only significant difference!

C. *APA-Style Conclusion*: Results of the One-Way Between-Subjects ANOVA indicated that perceived workplace stress significantly predicted employee turnover, $F(2,27)=3.52$, $p<.05$. Specifically, employees who perceived the workplace environment as low stress predicted they would stay significantly longer ($M=3.43$, $SD=0.40$ years) than did employees who perceived the workplace environment as high stress ($M=2.98$, $SD=0.43$ years; Tukey's HSD, $p<.05$). Otherwise, no significant differences were evident (Tukey's HSD, $p>.05$).

- *Stats Format:* $F(df_{BG}, df_E)=F_{obt}$, $p<>\alpha$
- *Order:* 1) State results of the Omnibus test, 2) if the Omnibus is significant, state the results of the Post Hoc tests

Relationship Between T_{obt} and F_{obt}

- *t* measures the distance between two means in terms of *standard deviations*!
- *F* measures the distance between two or more means in terms of *variance*!
- Standard Deviation squared = Variance! ➔ $\boldsymbol{t^2 = F}$!!

```
> t.test(GPA ~ Greek_F, data=GPA, alternative="two.sided", conf.level=.95, var.equal=TRUE)

        Two Sample t-test

data:  GPA by Greek_F
t = -0.5665, df = 8, p-value = 0.5866
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.1459367  0.6939367
sample estimates:
 mean in group No mean in group Yes
            2.800             3.026
```

```
> myanova1 <- aov(GPA ~ Greek_F, data=GPA)
> summary(myanova1)
            Df Sum Sq Mean Sq F value Pr(>F)
Greek_F      1  0.128  0.1277   0.321  0.587
Residuals    8  3.183  0.3979
```

- t_{obt}=-.5666^2 ➔ F_{obt}=-0.321
- Notice the p values also match! (*p*=.587)

One-Way Within-Subjects ANOVA

- **One-Way Within-Subjects ANOVA:** a statistical procedure used to test hypotheses for one categorical IV (factor) with two or more levels concerning the variance among group means. This test is used when the same participants are observed at each level of the IV and the variance in any one population is unknown.
- The term "*One-way*" means that you are testing one IV
- The term "*Within-Subjects*" means that the same participants are being observed in each group

4 Assumptions for One-Way Within-Subjects ANOVA

- There are four assumptions you must make to compute the One-Way Within-Subjects ANOVA
    1. **Normality:** assume that data in the population or populations being sampled from are normally distributed
    2. **Independence Within Groups:** assume that participants are independently observed within groups, but not between groups
    3. **Homogeneity of Variance:** assume that the variance in each population is equal to each other
    4. **Homogeneity of Covariance:** assume that participant scores in each group are related because the same participants are observed across or between groups

Sources of Variation for One-Way Within-Subjects ANOVA

- There are 3 Sources of Variation in One-Way Within-Subjects ANOVA:

TABLE 11.12 Three Sources of Variation in a Within-Subjects Design: Between Groups and Two Sources of Error Variation–Between Persons and Within Groups

Within-groups source of variation. This source of error variation is not attributed to having different groups, so it is placed in the denominator of the test statistic.

Between-persons source of variation. This is calculated by computing the average score of each person *(P)* in each row. This error variance is not attributed to having different groups and is removed from the denominator of the test statistic.

PARTICIPANT	GROUPS			
	A	B	C	
A	2	4	6	$P = 4$
B	1	3	5	$P = 3$
C	0	2	4	$P = 2$
D	5	7	9	$P = 7$
	$M = 2$	$M = 4$	$M = 6$	

Between-groups source of variation. This variance is attributed to having different groups and is placed in the numerator of the test statistic.

One-Way Within-Subjects ANOVA Example

- *Example:* Suppose we want to assess the effectiveness of three different ads on getting teens to quit smoking. A sample of teenagers (N=7) are asked to view each ad and to rate the ad's effectiveness on a scale from 1= "not at all effective" to 7="very effective".
  - One ad uses words only (a No-Cues condition). A second ad uses a generic abstract picture (Generic-Cues condition). A third ad shows a picture of a teenager smoking and coughing (Smoking-Related Cues condition).
  - Conduct the one-way within-subjects ANOVA at a .05 level of significance.

One-Way Within-Subjects ANOVA Example

TABLE 11.13 The Results of a Within-Subjects Study Design in Which Participants Rated the Effectiveness of Three Different Types of Antismoking Advertisements in Example 11.2

Person	Cues		
	No Cues	Generic Cues	Smoking-Related Cues
A	2	5	5
B	3	5	6
C	1	4	5
D	4	5	7
E	4	3	6
F	5	4	7
G	2	2	6

One-Way Within-Subjects ANOVA Example

- **STEP 1:** State the Hypotheses:
  - ***H_0*:** $\sigma^2_\mu = 0$ ➔ *In words:* Mean ratings for each advertisement do not vary in the population
  - ***H_1*:** $\sigma^2_\mu > 0$ ➔ *In words*: Mean ratings for each advertisement do vary in the population

- **STEP 2:** Set the criteria for a decision
  - Level of significance is .05
  - *Note*: we are going to skip calculating *df* and *Fcv* and go straight to R!

One-Way Within-Subjects ANOVA Example

◆**STEP 3:** Calculate the Test Statistic

A. Run the Omnibus One-Way Within-Subjects ANOVA test in R!

```
> my.rm.anova <- aov(effectiveness ~ ad_F + Error(subject/ad_F), data=smoke)
> summary(my.rm.anova)

Error: subject
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals  6  12.67   2.111

Error: subject:ad_F
          Df Sum Sq Mean Sq F value   Pr(>F)
ad_F       2  32.67  16.333   17.29 0.000292 ***
Residuals 12  11.33   0.944
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

F value is F_{obt}=17.29

This is your *p* value for the "omnibus" (overall) One-Way Within Subjects ANOVA test; p<.05 ➔ reject the null! Variance between the means > 0!

These are your degrees of freedom
APA Conclusion: F(2,12)

One-Way Within-Subjects ANOVA Example

◆**STEP 3:** Calculate the Test Statistic

A. Run the Omnibus One-Way Within-Subjects ANOVA test in R!

B. If significant (e.g., $p<.05$)➔ Reject the null ➔ Compute the Post Hoc Test to find where the mean differences are – here we will use a Bonferroni Post Hoc test!

```
> pairwise.t.test(smoke$effectiveness, smoke$ad_F, p.adj="bonferroni", paired=T)

        Pairwise comparisons using paired t tests

data:  smoke$effectiveness and smoke$ad_F

            NoCues GenericCues
GenericCues 0.5325 -
SmokeCues   0.0002 0.0288

P value adjustment method: bonferroni
```

These are your *p* values for each pairwise group comparison! Notice there is a significant difference between SmokeCues vs. NoCues (p=0.0002); and a significant difference between SmokeCues vs. GenericCues (p=0.0288); but the difference NoCues vs. GenericCues is NOT significant (p=0.5325).

One-Way Within-Subjects ANOVA Example

◆**STEP 3:** Calculate the Test Statistic

A. Run the Omnibus One-Way Within-Subjects ANOVA test in R!

B. If significant (e.g., $p<.05$)➔ Reject the null ➔ Compute the Post Hoc Test to find where the mean differences are – here we will use a Bonferroni Post Hoc test!

C. Use "tapply" to get the group means and sd's

```
> tapply(smoke$effectiveness, smoke$ad_F, mean)
    NoCues GenericCues   SmokeCues
         3           4           6
> tapply(smoke$effectiveness, smoke$ad_F, sd)
    NoCues GenericCues   SmokeCues
 1.4142136   1.1547005   0.8164966
```

One-Way Within-Subjects ANOVA Example

◆**STEP 4:** Make a Decision & Write a Conclusion!

A. *Decision:* Reject the null ➔ there is at least one significant difference between the means!

B. *Post Hoc Test:* Run all Pairwise comparisons to find the significant difference(s) ➔ High vs. Low was the only significant difference!

C. *APA-Style Conclusion*: Results of the One-Way Within-Subjects ANOVA indicated that the type of cue in the anti-smoking ads significantly predicted the perceived effectiveness of the ad, $F(2,12)=17.29$, $p<.05$. Specifically, ads that included smoking-related cues were perceived as significantly more effective ($M=6$, $SD=.82$) than were either the ads using generic cues ($M=4$, $SD=1.15$), or the ads using no cues ($M=3$, $SD=1.41$; Bonferroni $p<.05$). There was no significant difference between the perceived effectiveness of the ads using generic cues, and the perceived effectiveness of the ads using generic cues (Bonferroni $p>.05$).

➢ *Stats Format:* $F(df)=F_{obt}$, $p<>\alpha$

➢ *Order:* 1) State results of the Omnibus test, 2) if the Omnibus is significant, state the results of the Post Hoc tests