Table of Contents

Linearization of drone dynamics about hover & full-state feedback design 1
1.1) Simplified Dynamics 1
1.2) Linearizing Full Nonlinear Simulink Model (the model from Robotics Toolbox) 4
1.3) Comparing Results 5
2.1) Designing Full-state Feedback Controllers with Simplified Dynamics Model (1.1) via Pole Placement 6
Now place your own poles for the decoupled subsystems separately 9
Evaluate performance 9

Linearization of drone dynamics about hover & full-state feedback design

=============================== AUTHOR Fabian Riether CREATE DATE 2015/08/25 PURPOSE This code assists in linearizing drone dynamics and designing full-state feedback controls SPECIAL NOTES =============================== 2015/08/25 created ===============================

```
clear; close all

% Load drone parameters from RoboticsToolbox
mdl_quadrotor

% Load drone parameters from file
parameters_estimationcontrol
M_transform = controlHelperParams.Ts2Q;
```

1.1) Simplified Dynamics

```
% Symbolic variables
syms Pxw Pyw Pzw yaw pitch roll dpx dpy dpz p q r T tauy taup taur;
symsvector = [Pxw; Pyw;
 Pzw ;yaw ;pitch ;roll ;dpx ;dpy ;dpz ;p ;q ;r ;T ;tauy ;taup ;taur];

% Inertia
J = quad.J; %#ok<*DQUAD>

% Define rotation matrices

    Ryaw = [
        [ cos(yaw), -sin(yaw), 0]
        [ sin(yaw),  cos(yaw), 0]
        [        0,         0, 1]
    ];

    Rpitch = [
        [  cos(pitch), 0, sin(pitch)]
        [           0, 1,          0]
        [ -sin(pitch), 0, cos(pitch)]
```

```
    ];

    Rroll = [
        [ 1,          0,           0]
        [ 0, cos(roll), -sin(roll)]
        [ 0, sin(roll),  cos(roll)]
    ];

% Define rotation conversion matrices
Body2Global = Ryaw*Rpitch*Rroll; % Q_B/I
Global2Body = simplify(Body2Global^-1); % Q_I/B

% Transformation from body rates p-q-r to euler rates yaw-pitch-roll
iW = 1/cos(pitch)*...
    [0          sin(roll)            cos(roll);
     0          cos(roll)*cos(pitch) -sin(roll)*cos(pitch);
     cos(pitch) sin(roll)*sin(pitch) cos(roll)*sin(pitch)];

% Linearization point = hover
%-----------
state_equil = [0; 0; -1.5; 0; 0; 0; 0; 0; 0; 0; 0; 0]; % x_eq
input_equil = [-quad.g*quad.M; 0; 0; 0];               % u_eq (u_p)
% input_equil = M_transform\[-quad.g*quad.M; 0; 0; 0];   % u_eq (u_m)
equil       = [state_equil; input_equil];

% Dynamics
%----------
% P_dot [X_dot Y_dot Z_dot]
P_dot           = simplify(Body2Global*[dpx;dpy;dpz]);
P_dot_jacobian  = jacobian(P_dot,symsvector);
P_dot_jacobian_eql = subs(P_dot_jacobian,symsvector,equil);

% O_dot [yaw_dot pitch_dot roll_dot] (also [psi_dot theta_dot
 phi_dot])
O_dot           = iW*[p;q;r];
O_dot_jacobian  = jacobian(O_dot,symsvector);
O_dot_jacobian_eql = subs(O_dot_jacobian,symsvector,equil);

% p_ddot [vx_dot vy_dot vz_dot]
p_ddot          = Global2Body*[0;0;quad.g] + T/quad.M*[0;0;1] -
cross(transpose([p,q,r]),transpose([dpx,dpy,dpz]));
p_ddot_jacobian = jacobian(p_ddot,symsvector);
p_ddot_jacobian_eql = subs(p_ddot_jacobian,symsvector,equil);

% o_ddot [wx_dot wy_dot wz_dot]
o_ddot          = J\([taur; taup; tauy] - cross([p;q;r],J*[p;q;r]));
o_ddot_jacobian = jacobian(o_ddot,symsvector);
o_ddot_jacobian_eql = subs(o_ddot_jacobian,symsvector,equil);

%Dynamics matrix
%----------
matrixAB = [P_dot_jacobian_eql; O_dot_jacobian_eql;
 p_ddot_jacobian_eql; o_ddot_jacobian_eql];
A = double(matrixAB(1:12,1:12))
```

```
B = double(matrixAB(1:12,13:16))

%Note x_nonlinearSys = x_eq + x_linearizedSys! Thus, x0_linearizedSys
 = x0_nonlinear - x_eq;
%Note u_nonlinearSys = u_eq + x_linearizedSys!


A =

  Columns 1 through 7

         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0   -9.8100         0         0
         0         0         0         0         0    9.8100         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0

  Columns 8 through 12

         0         0         0         0         0
    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0         0         0    1.0000
         0         0         0    1.0000         0
         0         0    1.0000         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0


B =

   1.0e+04 *

         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
    0.0015         0         0         0
         0         0         0    1.4577
```

```
         0         0    1.0870         0
         0    0.7321         0         0
```

1.2) Linearizing Full Nonlinear Simulink Model (the model from Robotics Toolbox)

```
%use Simulation/controllers/controller_fullstate/Poleplacement/
linearizeDrone(...).slx and Simulink's ControlDesign/Linear Analysis

load('linearizeDrone_hover')
A_simulink = LinearAnalysisToolProject.Results.Data.Value.a
B_simulink = LinearAnalysisToolProject.Results.Data.Value.b
```

```
A_simulink =

  Columns 1 through 7

         0         0         0         0         0         0    1.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0   -9.8100         0   -0.0644
         0         0         0         0         0    9.8100    0.0000
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0    0.7558
         0         0         0         0         0         0         0

  Columns 8 through 12

         0         0         0         0         0
    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0         0         0    1.0000
         0         0         0    1.0000         0
         0         0    1.0000         0         0
         0         0         0    0.1390         0
   -0.0644         0   -0.1390   -0.0000   -0.0000
         0         0         0         0         0
   -1.0136         0   -2.1876         0         0
         0         0         0   -1.6312         0
         0         0    0.0000         0   -0.1248


B_simulink =

         0         0         0         0
         0         0         0         0
```

```
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
         0         0         0         0
   -0.0096    0.0096   -0.0096    0.0096
    0.4202    0.4202   -0.4202   -0.4202
    0.3133   -0.3133   -0.3133    0.3133
   -0.0115   -0.0115   -0.0115   -0.0115
```

1.3) Comparing Results

```
eigA = eig(A)
eigA_simulink = eig(A_simulink)
```

```
eigA =

     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
     0
     0


eigA_simulink =

   0.0000 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
  -2.7072 + 0.0000i
   0.5058 + 1.5757i
   0.5058 - 1.5757i
  -3.2144 + 0.0000i
   0.4812 + 1.6917i
   0.4812 - 1.6917i
  -0.1248 + 0.0000i
   0.0000 + 0.0000i
   0.0000 + 0.0000i
```

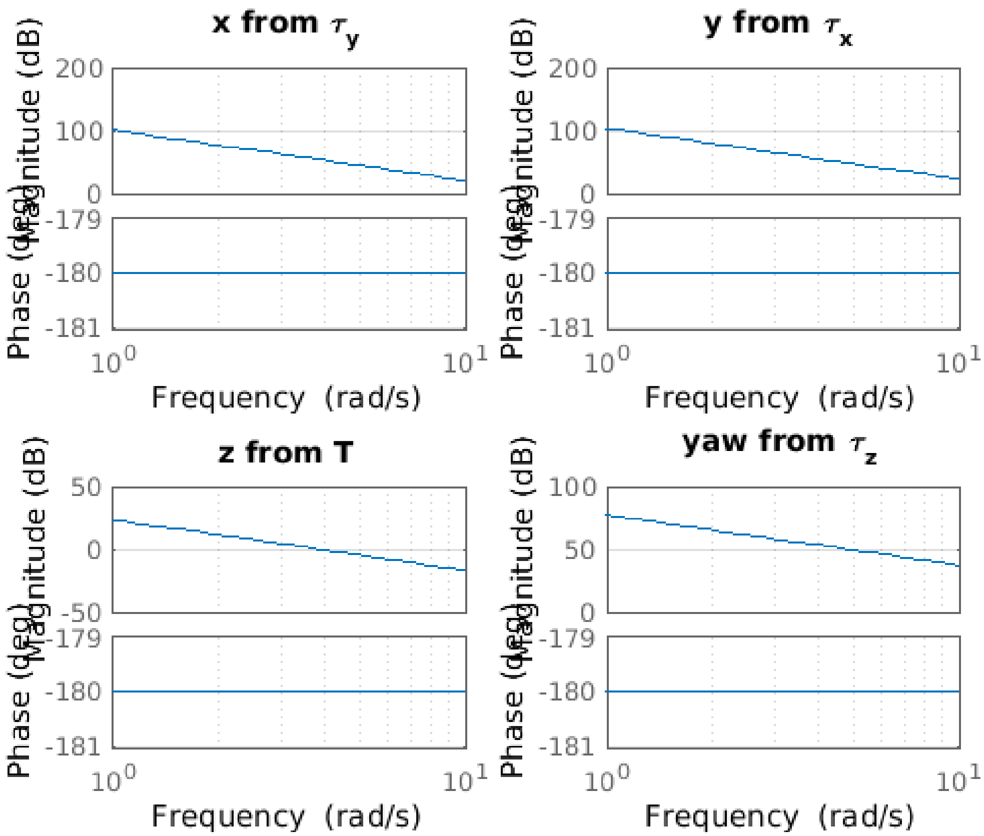
2.1) Designing Full-state Feedback Controllers with Simplified Dynamics Model (1.1) via Pole Placement

```
% Note: We linearized about hover. This also implies: The control
 "policy"
% to correct a position error was derived under a yaw-angle of zero!
% If your drone yaw-drifts 90 deg and runs into a world-X-error, it
 will
% still believe that pitch is the right answer to correct for this
 position
% error! You can compensate for this by rotating the X-Y-error by the
% current yaw angle.

% Find states to decouple
[V,J] = jordan(A);
V_eig_nrm = diag(1./sum(V,1))*V; % decoupled system will have a new
 state-vector x_dec = inv(V_eig_nrm)*x

% System matrices of decoupled system
A_dec   = inv(V_eig_nrm)*A*V_eig_nrm;
B_dec   = inv(V_eig_nrm)*B;

% Symbolic vector
syms x y z psi theta phi vx vy vz wx wy wz T tauz tauy taux
x = [x; y; z; psi; theta; phi; vx; vy; vz; wx; wy; wz];
u = [T; tauz; tauy; taux];
x_dec = inv(V_eig_nrm)*x

% Extract decoupled subsystems
A_dec_x   = A_dec(1:4,1:4);
B_dec_x   = B_dec(1:4,:);

A_dec_z   = A_dec(5:6,5:6);
B_dec_z   = B_dec(5:6,:);

A_dec_y   = A_dec(7:10,7:10);
B_dec_y   = B_dec(7:10,:);

A_dec_yaw = A_dec(11:12,11:12);
B_dec_yaw = B_dec(11:12,:);

% Compute decoupled subsystems Transfer Function (TF)
s = tf('s');

% TF from tau_y to x
G_X = tf(ss(A_dec_x,B_dec_x,eye(size(A_dec_x)),zeros(size(B_dec_x))));
G_x = G_X(1,3)

%
% TF from tau_x to y
```

```
G_Y = tf(ss(A_dec_y,B_dec_y,eye(size(A_dec_y)),zeros(size(B_dec_y))));
G_y = G_Y(1,4)

% TF from T to z
G_Z = tf(ss(A_dec_z,B_dec_z,eye(size(A_dec_z)),zeros(size(B_dec_z))));
G_z = G_Z(1,1)

% TF from tau_z to yaw
G_Yaw =
 tf(ss(A_dec_yaw,B_dec_yaw,eye(size(A_dec_yaw)),zeros(size(B_dec_yaw))));
G_yaw = G_Yaw(1,2)

% Plot bode plots
figure
subplot(2,2,1)
    bode(G_x)
    title('x from \tau_y')
    grid on
subplot(2,2,2)
    bode(G_y)
    title('y from \tau_x')
    grid on
subplot(2,2,3)
    bode(G_z)
    title('z from T')
    grid on
subplot(2,2,4)
    bode(G_yaw)
    title('yaw from \tau_z')
    grid on
```

```
x_dec =

     x
   -vx
 theta
    wy
     z
    vz
    -y
    vy
   phi
    wx
   psi
    wz


G_x =

  -1.066e05
  ---------
     s^4
```

```
Continuous-time transfer function.


G_y =

  -1.43e05
  --------
    s^4

Continuous-time transfer function.


G_z =

  14.71
  -----
   s^2

Continuous-time transfer function.


G_yaw =

  7321
  ----
  s^2

Continuous-time transfer function.
```



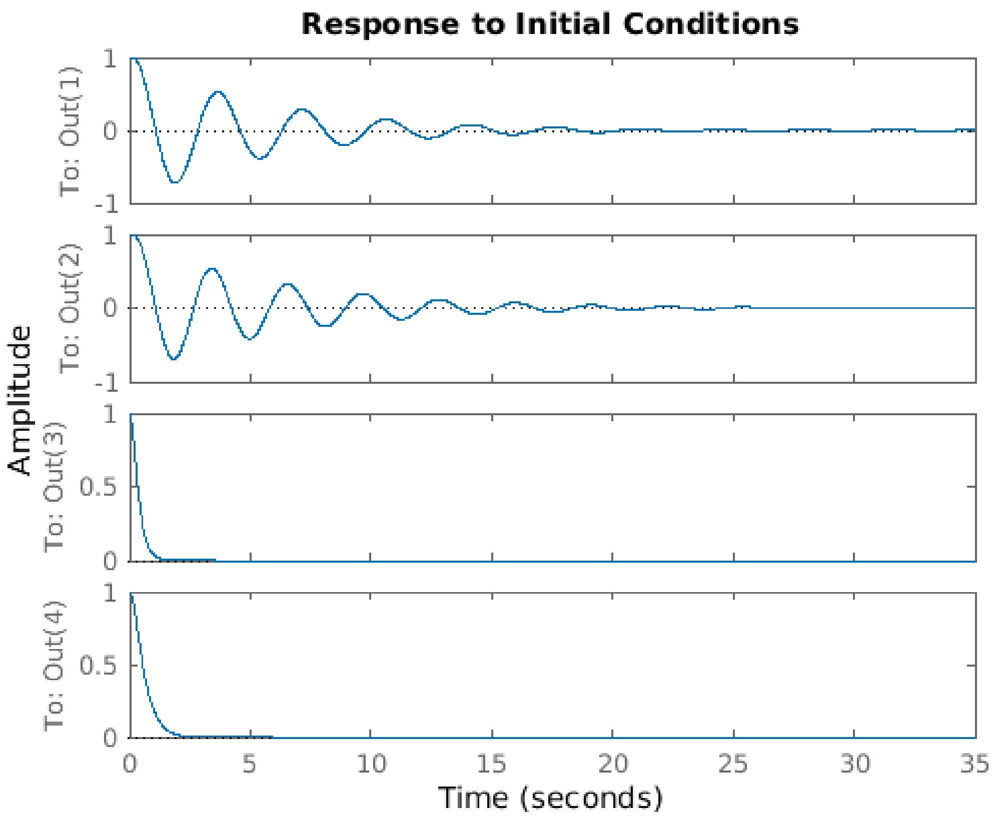
Now place your own poles for the decoupled subsystems separately

```
xpoles       = [-9+6i;-9-6i;-0.18+1.8i;-0.18-1.8i];
ypoles       = [-60;-4;-0.16+2i;-0.16-2i];
yawpoles     = [-3;-3.1];
%zpoles       = [-2;-2.1];              % Play around with poles here:
 Slow poles [-2;-2.1], Fast poles [-5;-5.1];
zpoles      = [-5;-5.1];               % Play around with poles here:
 Slow poles [-2;-2.1], Fast poles [-5;-5.1];

K_dec_x      = place(A_dec_x, B_dec_x, xpoles);
K_dec_y      = place(A_dec_y, B_dec_y, ypoles);
K_dec_z      = place(A_dec_z, B_dec_z, zpoles);
K_dec_yaw    = place(A_dec_yaw, B_dec_yaw, yawpoles);

% Compute full-state feedback for 'original' system
K_poleplace = [K_dec_x K_dec_z K_dec_y K_dec_yaw]*inv(V_eig_nrm);
K_poleplace(abs(K_poleplace)<1e-7)=0;
```

Evaluate performance

```
figure
```

```
Ahat = A-B*K_poleplace;
Chat = [eye(4) zeros(4,8)];
sys = ss(Ahat,[],Chat,[]);
x0 = [1; 1; 1; 1; zeros(8,1)];
initial(sys,x0)
```



Published with MATLAB® R2015b