Développements limités usuels

1. **Exponentielle**
$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o\left(x^n\right)$$

2. **Inverses**

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o\left(x^n\right)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o\left(x^n\right)$$

3. **Logarithme**

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + o\left(x^n\right)$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots - \frac{x^n}{n} + o\left(x^n\right)$$

4. **Fonctions circulaires**

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+1}\right)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o\left(x^8\right)$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o\left(x^{2n+2}\right)$$

5. **Fonctions hyperboliques**

$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o\left(x^{2n+1}\right)$$

$$\operatorname{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o\left(x^{2n+1}\right)$$

6. **Puissances**
$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + o(x^n) \qquad (\alpha \in \mathbb{C})$$

Cas particuliers :

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots + (-1)^{n-1}\frac{1 \times 3 \times \cdots \times (2n-3)}{2^n n!}x^n + o\left(x^n\right)$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots + (-1)^n \frac{1 \times 3 \times \cdots \times (2n-1)}{2^n n!}x^n + o\left(x^n\right)$$