

# Nonparametric Demand Analysis

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*08/22/2017*

## **Abstract**

Nonparametric econometric analysis has been a growing field of study over the past several decades. Many new techniques have been developed within a theoretical framework. However, despite the rapid growth of theoretical results, nonparametric applied research has lagged considerably. This paper employs a nonparametric regression analysis within the context of demand theory. Data on prices and quantities of three commodities (meats, dairy products and beans) from the 2006 Ecuadorian consumer expenditure survey will be evaluated to derive Marshallian price and income elasticities. The nonparametric results will be compared with standard parametric demand analysis tools such as the Log-Log demand model and the Almost Ideal Demand system to gauge the effectiveness of using nonparametric techniques to estimate demand elasticities.

# 1 Introduction

Emperical demand analysis has been dominated by the use of parametric functional form models since the appearance of the Working-Lesser Model in the 1940's (Working 1943) (Leser 1963). A 2015 paper by Clements and Gao (Clements and Gao 2015) provide a citation count of journal articles related to four popular parametric demand models (Linear Expenditure, Rotterdam, Translog and Almost Ideal Demand). The authors considered multiple time periods over the date range of 1974-2013. Their results clearly show an upward trajectory for all four parametric demand models with the Linear Expenditure and Almost Ideal Demand (AIDS) models having the highest citation counts, both in the multiple thousands of citations. This result suggests that parametric demand modeling is still very much a hotly researched area.

While parametric form models have clearly dominated the research landscape there are other empirical techniques available to evalaute consumer expenditure datasets. The goal of empirical demand analysis is to answer basic questions about consumer behavior and in many cases, use these answers to design and implement policy (government) or imporove some aspect of business design to increase market exposure/profits (private industry). Beyond these practical objectives Hal Varian (Varian 1982) suggests that, given a consumers expenditure dataset the demand analyst should ask four basic questions concerning the consumers behavior,

- (1) Consistency? Is the observerd data consistent with the utility maximizing model?
- (2) Structure? Is the observed data consistent with a utility function with some special structure?
- (3) Recoverability? How can the underlying utility function be recovered?

#### (4) Extrapolation? How can we forecast behavior in other circumstances?

With these goals in mind economists are constantly refining their tools to improve their estimates of consumer behavior.

Nonparametric economic analysis has been a growing field of study over the past several decades. Many new techniques have been developed within a theoretical framework. However, despite the rapid growth of theoretical results, nonparametric applied research has lagged considerably. This may be due in part to the advanced mathematical and statistical exposition presented in many nonparametric research papers. It is often difficult for economist, while well trained in advanced mathematical techniques, to fully grasp the significance and translate from purely theoretical results into applied research. Theoretical researchers working within the field of mathematics and statistics often fail to consider applied economic problems and applied economists are not exploring these newly developed theoretical techniques and refining the theory once they have touched real world data. In addition, until recently, nonparametric techniques were substantially more computationally intensive compared to their parametric counterparts. While the previous statement would seem to imply that nonparametric techniques have now become computationally less intensive, in reality it is our computer hardware and programming techniques which have improved to the point where nonparametric analysis has become a viable alternative, especially where parametric techniques fall short. These improvements in computer hardware have effectively opened an area of research which, until recently, had remained closed.

Regression analysis is the workhorse technique employed in econometrics. However, in linear regression it is assumed the regressors enter the conditional mean in a linear fashion and each regressor is independent of the other which is often a violation of the data under study. Even when we use nonlinear regression techniques we often still

assume we know the functional form for the data generating process (DGP).

The single largest potential issue with using a parametric form model to evaluate consumer demand is the prior imposition of functional form on the demand model. If a demand system is well represented by a functional form and the econometric model is correctly specified with theoretical assumptions met then a parametric estimator is both consistent and efficient. In fact if the previous criteria are met parametric regression is superior in just about every way. However, these requirements are excessively difficult to satisfy when used in the wild to gain insight about real world data and thus the parametric approach can be seriously flawed and worst, can lead the researcher to faulty conclusions which can have serious repercussions if used to implement policy.

Nonparametric modelling affords many advantages over their parametric counterparts. Primary among them is the nonparametric models ability to help us uncover a more accurate representation of the unknown function, conditioned on the actual data in hand. In section 2 I will explore the methods and models used to estimate a nonparametric regression. In nonparametric analysis a critical step in achieving a reliable kernel estimator is choosing the correct bandwidth estimator. Some time will be spent exploring several different bandwidth estimators which are prominent in the current literature. I will also explore the two most popular kernel regression methods in current use, Local-Constant Least Squares (LCLS) and Local-Linear Least Squares (LLLS). I will summarize the data and quickly review the parametric models being used for comparison purposes. Section 3 explores the results obtained from the nonparametric kernel regression and compares these results with the parametric form models used on the same dataset. Section 4 concludes and details direction for additional studies.

## 2 Methods, Models and Data

Consistent Model Specification Test (Hsiao et al. 2007)

A kernel based test which tests for correct specification of the parametric form

The null hypothesis is given by,

$$H_0 : P[E(y_i|x_i) = m(x_i, \beta)] = 1 \text{ for some } \beta \in \mathcal{B}$$

where,

$m(\cdot, \cdot)$  is a known function with  $\beta$  being a  $p \times 1$  vector of unknown parameters,

$\mathcal{B}$  is a compact subset in  $\mathbb{R}^p$ .

The alternative hypothesis is given by,

$$H_1 : P[E(y_i|x_i) = m(x_i, \beta)] < 1 \text{ for all } \beta \in \mathcal{B}$$

The test statistic was originally developed by Fan and Li (1996) and Zheng (1996).

The test statistic is given by,

$$I_n = \frac{1}{n^2} \sum_i \sum_{j \neq i} \hat{u}_i \hat{u}_j K_{\gamma_{ij}}$$

where,

$$K_{\gamma_{ij}} = W_{h,ij} L_{\lambda,ij}(\gamma = (h, \lambda)),$$

$\hat{u}_i = y_i - m(x_i, \hat{\beta})$  is the parametric null model's residual,

$\hat{\beta}$  is a  $\sqrt{(n)}$ -consistent estimator of  $\beta$  under  $H_0$ .

Hsiao et al. (2007) advocate for the use of cross-validation methods for selecting the kernel smoothing parameter vectors, which is the approach we took in this analysis.

Under assumptions (A1)-(A3) of thier paper we can obtain the CV-based test with the new test statistic  $\hat{J}_n$ :

$$n(\hat{h}_1, \dots, \hat{h}_q)^{1/2} \hat{I}_n \rightarrow N(0, \Omega) \text{ in distribution un } H_0,$$

where,

$$\Omega = 2E[\sigma^4(x_i)f(x_i)] [\int W^2(v)dv]$$

A consistent estimator of  $\Omega$  is given by,

$$\hat{\Omega} = n^{-2} 2(\hat{h}_1, \dots, \hat{h}_q) \sum_i \sum_{j \neq i} \hat{u}_i^2 \hat{u}_j^2 W_{\hat{h}, ij}^2 L_{\hat{\lambda}, ij}^2$$

Which gives the CV-based test statistic as,

$$\hat{J}_n = n(\hat{h}_1, \dots, \hat{h}_q)^{1/2} \hat{I}_n \hat{\Omega}^{-1/2} \rightarrow N(0, 1) \text{ in distribution under } H_0.$$

According to the authors, it can be easily shown that the  $\hat{J}_n$  test diverges to  $+\infty$  if  $H_0$  is false.

### 3 Results and Discussion

### 4 Conclusions

Table 1: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
obs	2,066	1,033.500	596.547	1	2,066
p1	2,066	2.610	0.820	0.365	11.351
p2	2,066	1.231	1.510	0.011	23.736
p3	2,066	1.218	0.444	0.010	4.386
q1	2,066	4.498	3.011	0.001	38.574
q2	2,066	6.244	5.635	0.001	63.762
q3	2,066	0.943	0.929	0.0004	14.074
p1q1	2,066	11.395	7.883	0.003	98.101
p2q2	2,066	4.619	4.215	0.001	51.451
p3q3	2,066	1.073	1.088	0.0005	15.250
X	2,066	17.087	10.679	0.199	128.602
w1	2,066	0.657	0.168	0.001	0.984
w2	2,066	0.269	0.159	0.0001	0.945
w3	2,066	0.074	0.070	0.00003	0.884

good 1 = meats  
good 2 = dairy  
good 3 = beans

Table 2: Parametric - Double Log Demand Model

	meats	dairy	pulses	income_elasticity	R-squared
meats_lnq1	-0.9073***	-0.07323***	-0.003844	1.164***	0.74226
dairy_lnq2	-0.05847	-0.8216***	-0.07056*	1.004***	0.64494
pulses_lnq3	-0.3804***	-0.05684*	-0.4394***	0.5658***	0.14743

\*\*\*Significant at the 1 percent level,  
\*\*Significant at the 5 percent level,  
\*Significant at the 10 percent level.

Table 3: Nonparametric Regression using Gaussian Kernel

	meats	dairy	pulses	income_elasticity	R-squared
meats_lnq1	-0.9674***	-0.06514*	-0.04756	1.124***	0.88725
dairy_lnq2	0.002597	-0.843***	-0.0752***	0.9875***	0.71745
pulses_lnq3	-0.3402***	-0.05632**	-0.7777***	0.4965***	0.27586

\*\*\*Significant at the 1 percent level,  
\*\*Significant at the 5 percent level,  
\*Significant at the 10 percent level.

Table 4: Full AIDS - Marshallian

	meats	dairy	pulses	income_elasticity	R-squared
meats_lnq1	-0.9813***	-0.06048***	-0.03157**	1.073***	0.067727
dairy_lnq2	-0.06908***	-0.8626***	-0.03394***	0.9657***	0.052564
pulses_lnq3	0.1063	0.006219	-0.5657***	0.4532***	0.13459

\*\*\*Significant at the 1 percent level,

\*\*Significant at the 5 percent level,

\*Significant at the 10 percent level.

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