STAT 5370 – Decision Theory

Homework 2

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Problem 1

In this problem we will explore an alternate class of priors.

(a) Find a resource and write a basic description of what a Jeffrey's prior is. Be thorough, your exposition here needs to be such that someone who knows nothing of what a Jeffrey's prior is could learn the salient features. Identify why people like Jeffery's priors and discuss drawbacks.

(b) Derive the Jeffery's prior for the Bernoulli model (i.e., assuming $Y_i \stackrel{iid}{=} Bernoulli(p)$), and find the posterior distribution of p under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

(c) Derive the Jeffery's prior for the Poisson model (i.e., assuming $Y_i \stackrel{iid}{=} Poisson(\lambda)$), and find the posterior distribution of λ under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

(d) Derive the Jeffery's prior for the exponential model (i.e., assuming $Y_i \stackrel{iid}{=} Exponential(\beta)$), and find the posterior distribution of β under this prior. Is it a known distribution? Discuss how you would proceed to conduct posterior inference.

Problem 2

Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ such that X_1 and X_2 are independent. Define $Y = X_1 + X_2$. The goal of this problem is to determine the distribution of Y; i.e., the sum of two independent normal random variables.

(a) Obviously, this is trivial, so simply state the distribution of Y.

(b) Approach 1: Consider using Monte Carlo sampling to obtain a histogram and kernel density estimate (see the code that I have provided) of the pdf of Y by directly sampling both X_1 and X_2. Over plot the true density of Y and comment. Note, you should make use of large enough Monte Carlo sample that your results are reasonable.

(c) Approach 2: Note, the distribution of Y can also be obtained through the convolution of the probability distributions of X_1 and X_2. Sketch out theoretically how this would be done. Based on this idea, create a Monte Carlo sampling technique which can be used to approximate the pdf of Y evaluated at any point in the support. Use this function and add the approximation based on this technique to the Figure described in the part (b) above.