A Replication Study

Bayesian Inference for Agreement Measures

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Goals of this Research

Replicate the results of:

- Bayesian Inference for Agreement Measures.
  - By: Ignacio Vidal and Mário de Castro.
  - Journal of biopharmaceutical statistics, 2016 p. 809-823.

The main ideas behind the paper:

Methods for assessing agreement between a set of paired observations has been well studied in the literature. Most of the studies have focused on using a frequentist approach to calculate several measures of measurement agreement.

A Motivating Example with Real World Data:

Renal lithiasis (Kidney Stones) can be defined as the consequence of an alteration of the normal crystallization conditions of urine in the urinary tract. In a healthy individual, during the residence time of urine in the urinary tract, crystals either do not form or are so small they are eliminated uneventfully. Clinicians often employ diagnostic imaging techniques to quantify the size, shape, and likelihood the stones will pass through the system without incident.

A Motivating Example with Real World Data:

There are two main imaging tools used by clinicians to assess renal lithiasis:

- High-speed or dual energy Computerized Tomography (CT).
- Intravenous Urography, which involves injecting dye into an arm vein and taking X-rays (intravenous pyelogram).

A Motivating Example with Real World Data:

- The imaging costs associated with Computerized Tomography far out weigh the costs of imaging with alternative methods, including Intravenous Urography.
- The question is do these two imaging modalities provide equivalently adequate results in terms of imaging quality, accuracy and precession.
- We need a way to assess how much agreement there is between measurements taken by Urography and Tomography.

A Motivating Example with Real World Data:

- We have data on the inferior pelvic infundibular angle (IPIA) for 52 kidneys, evaluated by means of computerized tomography and urography.
- We will use this data to determine the level of agreement between the two techniques.
- We will return to this example later. . .

Assessing Agreement Between Measurements:

- A way to assess agreement between two random variables X and Y is the mean squared deviation (MSD).

$$MSD = E\left[(X - Y)^2\right] = (\mu_x - \mu_y)^2 + \sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}$$

Assessing Agreement Between Measurements:

- Another measure of agreement between two random variables is the Concordance Correlation Coefficient (CCC).

$$\rho_c = 1 - \frac{MSD}{MSD|_{\sigma_{xy}=0}} = \frac{2\sigma_{XY}}{(\mu_X - \mu_Y)^2 + \sigma_X^2 + \sigma_Y^2}$$

Assessing Agreement Between Measurements:

- Two additional methods for assessing agreement that were explored by the authors are call the accuracy coefficient (χ_a) and the precision coefficient (ρ).

$$\chi_a = \frac{2}{\bar{\omega} + \frac{1}{\bar{\omega}} + v^2}, \bar{\omega} = \frac{\sigma^2 Y}{\sigma_X^2}, v^2 = \frac{(\mu_X - mu_Y)^2}{\sigma_X \sigma_Y}$$

$$\rho = \frac{\rho_c}{\chi_a}$$

where ρ is simply the Pearson correlation coefficient ($\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$).

The Bayesian Model

Likelihood

Prior

Jeffreys Prior

Independence Jefferys Prior

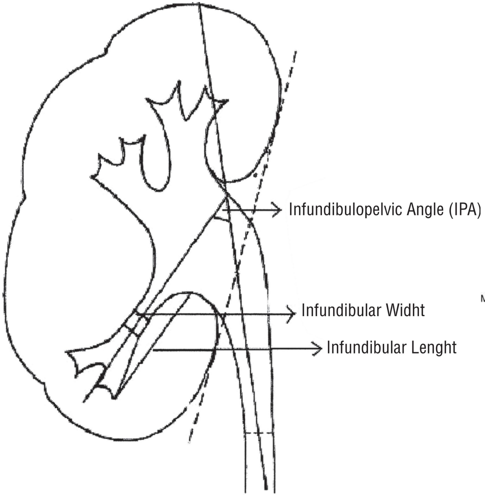
Reference Prior for ρ

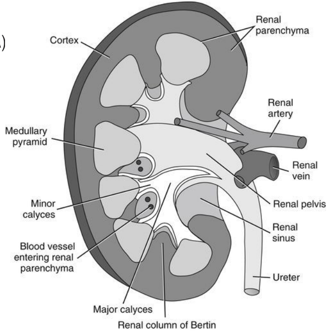
Posterior

Simulation Results

Inferior Pelvic Infundibular Angle Data

Kidney Anatomy



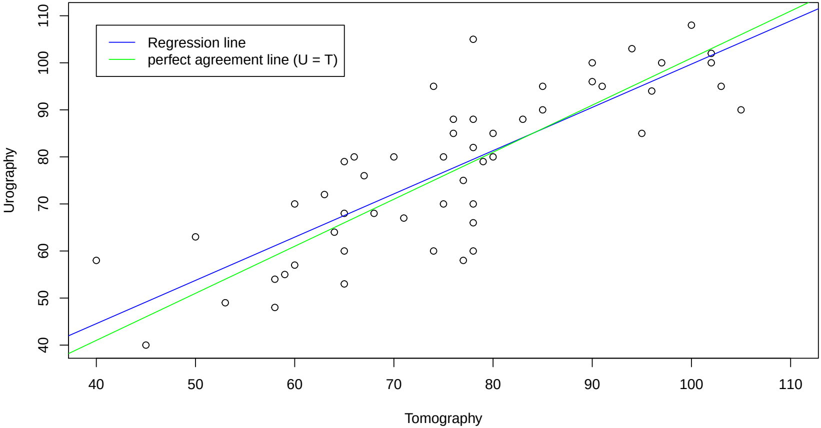


Data Summary

	Urography	Tomography
N.Valid	52.00	52.00
Mean	77.46	75.79
Q1	63.50	65.00
Median	79.50	76.50
Q3	92.00	85.00
Std.Dev	17.17	15.30
Min	40.00	40.00
Max	108.00	105.00

A visual assessment

- Regression line plot versus perfect agreement line (U = T).



Kidney Data - Bayesian Inference for:

- Mean Squared Deviation (MSD).
- Concordance Correlation Coefficient (CCC).
- Accuracy coefficient (χ_a).
- Precision coefficient (ρ).