HW1 Econometrics 3

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The Model:

$$y_t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_t \tag{1}$$

where,

ε_t is normally and independently distributed with $E[\varepsilon_t] = 0$, $E[\varepsilon_t^2] = \sigma_t^2$ and $\sigma_t^2 = exp(\alpha_0 + \alpha_1 x_1)$.

The paramater values used are $\beta_0 = 10$, $\beta_1 = \beta_2 = 1$, $\alpha_0 = -2$, and $\alpha_1 = 0.25$.

The design matrix X is given in a csv file.

Problem 1

The following questions are based on Monte Carlo experimental data. Generate 100 samples of y, each of size 20, using the model given in equation 1 (above).

(a)

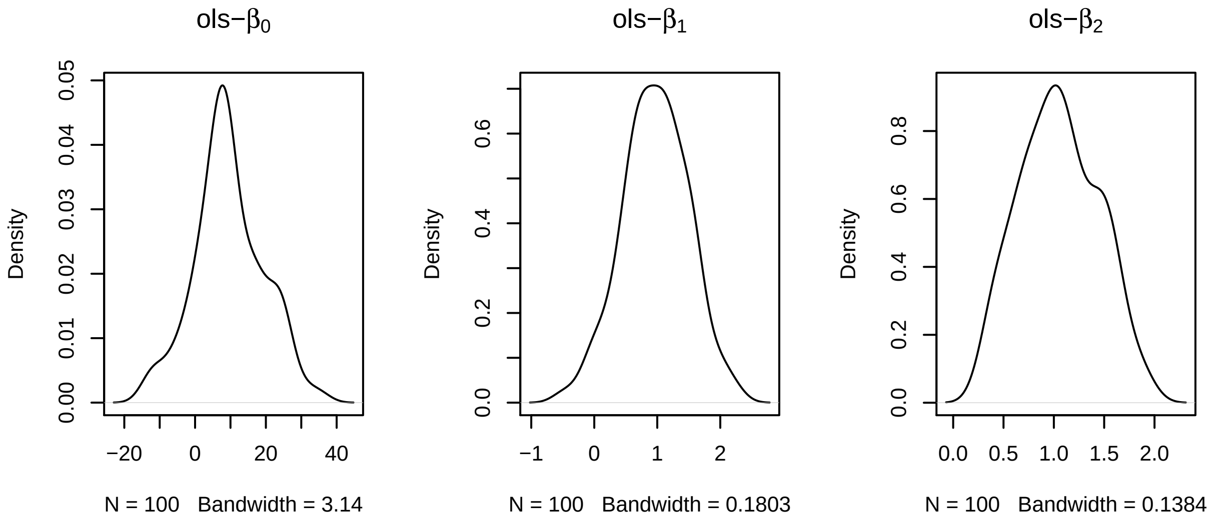
Estiamte the parameters using the least squares principle and provide their covariance matrix. Compare your results with the true parameters. What can you conclude?

Table 1: Summary Statistics: Problem 1, part (a) and (b)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_ols	100	9.590	9.716	−13.569	35.337
beta_1_ols	100	0.971	0.520	−0.481	2.243
beta_2_ols	100	1.050	0.386	0.344	1.896
var_b0_ols	100	70.313	35.817	17.030	193.850
var_b1_ols	100	0.203	0.103	0.049	0.559
var_b2_ols	100	0.145	0.074	0.035	0.399
se_b0_ols	100	8.139	2.029	4.127	13.923
se_b1_ols	100	0.437	0.109	0.222	0.748
se_b2_ols	100	0.369	0.092	0.187	0.632
t_val_b0_ols	100	1.262	1.271	−1.565	5.098
t_val_b1_ols	100	2.335	1.219	−0.644	5.307
t_val_b2_ols	100	3.038	1.401	0.665	7.349
BP_testStat_ols	100	6.292	4.894	0.061	20.898
GV_HET_Test_ols	100	4.797	3.828	0.001	16.076

BP testStat ols = Breusch Pagan test against heteroskedasticity
GV HET Test ols = General test for LM assumptions
Chi Squared Crit df:2 at alpha:0.05 = 5.99

The results displayed in table 1 (above) show the summary for the 100 sample experiments preformed on the data using OLS. The means of the 100 samples are reported for various statistics. The average t-values for the beta paramaters show the constant is insignificant while β_1 and β_2 are significant. To better understand the distribution of the paramaters over the 100 experiments see density plots below.



(b)

Test for the presence of heteroskedasticity? What do you conclude?

I have run the Breusch Pagan (BP) test for heteroskedasticity for the 100 samples and averaged over the experiments. The BP test staistic is distributed as Chi-Squared with k-1 degrees of freedom, where k is the total number of papamaters in the model.

The null hypothesis of the Breusch-Pagan test is,

$$\sigma_i^2 = \sigma^2(\alpha_0 + \alpha' z_i)$$

where,

$H_0 : \alpha = 0$

σ_i^2 is the error variance for the ith observation and α_0 and α are regression coefficients.

The test statistic for the Breusch-Pagan test is

$$bp = \frac{1}{v}(u - \bar{u}i)'Z(Z'Z)^{-1}Z'(u - \bar{u}i)$$

where $u = (e_1^2, e_2^2, ..., e_n^2)$, i is a $nx1$ vector of ones, and

$$v = \frac{1}{n}\sum_{i=1}^{n}\left(e_i^2 - \frac{e'e}{n}\right)^2$$

This is a modified version of the Breusch-Pagan test, which is less sensitive to the assumption of normality than the original test (Greene 2012, 7th Ed.; p. 276).

FGLS

Estimate the model using FGLS techniques Assume multiplicative hetero...

Table 2: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_ols	100	9.590	9.716	−13.569	35.337
beta_1_ols	100	0.971	0.520	−0.481	2.243
beta_2_ols	100	1.050	0.386	0.344	1.896
var_b0_ols	100	70.313	35.817	17.030	193.850
var_b1_ols	100	0.203	0.103	0.049	0.559
var_b2_ols	100	0.145	0.074	0.035	0.399
se_b0_ols	100	8.139	2.029	4.127	13.923
se_b1_ols	100	0.437	0.109	0.222	0.748
se_b2_ols	100	0.369	0.092	0.187	0.632
t_val_b0_ols	100	1.262	1.271	−1.565	5.098
t_val_b1_ols	100	2.335	1.219	−0.644	5.307
t_val_b2_ols	100	3.038	1.401	0.665	7.349
beta_0_fgls	100	9.836	7.165	−6.479	29.558
beta_1_fgls	100	0.965	0.385	−0.008	1.857
beta_2_fgls	100	1.044	0.369	0.022	1.865
alpha_0_fgls	100	−2.100	2.320	−7.556	3.861
alpha_1_fgls	100	0.192	0.113	−0.157	0.452
var_b0_fgls	100	21.369	20.497	4.819	100.744
var_b1_fgls	100	0.052	0.046	0.013	0.225
var_b2_fgls	100	0.022	0.013	0.008	0.096
se_b0_fgls	100	4.272	1.774	2.195	10.037
se_b1_fgls	100	0.214	0.082	0.115	0.474
se_b2_fgls	100	0.142	0.038	0.088	0.309
t_val_b0_fgls	100	2.573	2.236	−2.263	10.344
t_val_b1_fgls	100	4.911	2.391	−0.045	12.458
t_val_b2_fgls	100	7.864	3.571	0.168	18.313
var_b0_HCCM_0	100	73.048	62.861	8.697	357.686
var_b1_HCCM_0	100	0.233	0.196	0.031	1.094
var_b2_HCCM_0	100	0.085	0.049	0.024	0.280
var_b0_HCCM_3	100	126.622	115.448	12.261	645.598
var_b1_HCCM_3	100	0.395	0.358	0.048	2.010
var_b2_HCCM_3	100	0.164	0.123	0.038	0.704

good 1 = meats
good 2 = dairy
good 3 = beans

MLE

$$\ln L = -0.5n\log(2\pi) - 0.5\sum(\sigma^2) - 0.5\sum\left[\frac{(y - X'\beta)^2}{\sigma^2}\right] \tag{2}$$

where,

$\sigma^2 \simeq \exp(\alpha_0 + \alpha_1 x_1)$

Table 3: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_mle	100	11.225	9.000	−10.431	37.272
beta_1_mle	100	0.930	0.398	−0.089	1.898
beta_2_mle	100	1.006	0.408	−0.213	1.884
alpha_0_mle	100	−2.151	2.604	−8.134	5.598
alpha_1_mle	100	0.245	0.122	−0.174	0.523
var_b0_mle	100	27.698	297.664	−2,815.444	593.279
var_b1_mle	100	0.094	0.515	−4.842	0.871
var_b2_mle	100	0.092	0.051	0.026	0.426
var_alpha_0_mle	100	5.972	6.287	−37.549	37.848
var_alpha_1_mle	100	0.013	0.011	−0.059	0.055

good 1 = meats
good 2 = dairy
good 3 = beans

Table 4: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Max
beta__0__ols	100	9.530	2.594	3.907	16.808
beta__1__ols	100	0.990	0.192	0.477	1.481
beta__2__ols	100	1.032	0.175	0.550	1.489
var__b0__ols	100	36.308	24.120	7.328	148.470
var__b1__ols	100	0.105	0.070	0.021	0.428
var__b2__ols	100	0.075	0.050	0.015	0.306
se__b0__ols	100	5.767	1.754	2.707	12.185
se__b1__ols	100	0.310	0.094	0.145	0.654
se__b2__ols	100	0.262	0.080	0.123	0.553
t__val__b0__ols	100	1.803	0.711	0.462	3.630
t__val__b1__ols	100	3.479	1.272	1.367	7.728
t__val__b2__ols	100	4.290	1.449	1.426	9.489
DW__Test	100	3.290	0.357	2.252	3.871
rho__fgls	100	0.755	0.159	0.257	0.971
sigma2__fgls	100	6.055	2.114	2.008	12.177
beta__0__fgls	100	8.951	21.059	−66.922	64.418
beta__1__fgls	100	0.987	1.097	−1.709	4.750
beta__2__fgls	100	1.057	0.339	−0.083	1.801
var__b0__fgls	100	8.444	12.104	1.360	81.390
var__b1__fgls	100	0.057	0.090	0.005	0.604
var__b2__fgls	100	0.106	0.177	0.004	1.188
se__b0__fgls	100	2.497	1.494	1.166	9.022
se__b1__fgls	100	0.197	0.135	0.068	0.777
se__b2__fgls	100	0.262	0.194	0.065	1.090
t__val__b0__fgls	100	4.674	6.749	−14.133	17.114
t__val__b1__fgls	100	6.831	5.535	−4.868	17.325
t__val__b2__fgls	100	5.880	3.585	−0.165	16.919
var__b0__HCCM__0	100	27.372	15.930	4.489	81.518
var__b1__HCCM__0	100	0.079	0.046	0.010	0.262
var__b2__HCCM__0	100	0.056	0.057	0.007	0.413
var__b0__HCCM__3	100	43.248	25.773	6.306	143.734
var__b1__HCCM__3	100	0.121	0.071	0.015	0.418
var__b2__HCCM__3	100	0.108	0.118	0.012	0.851

good 1 = meats
good 2 = dairy
good 3 = beans