HW1 Econometrics 3

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The Model:

$$y_t = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon_t \tag{1}$$

where,

ε_t is normally and independently distributed with $E[\varepsilon_t] = 0$, $E[\varepsilon_t^2] = \sigma_t^2$ and $\sigma_t^2 = exp(\alpha_0 + \alpha_1 x_1)$.

The parameter values used are $\beta_0 = 10$, $\beta_1 = \beta_2 = 1$, $\alpha_0 = -2$, and $\alpha_1 = 0.25$.

The design matrix X is given in a csv file.

Problem 1

The following questions are based on Monte Carlo experimental data. Generate 100 samples of y, each of size 20, using the model given in equation 1 (above).

(1.a)

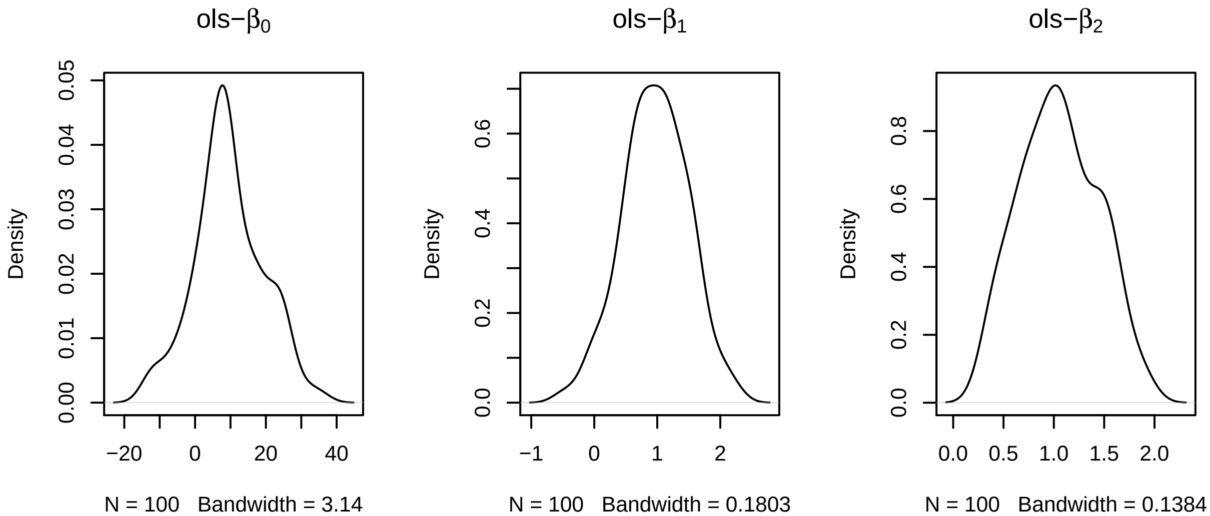
Estimate the parameters using the least squares principle and provide their covariance matrix. Compare your results with the true parameters. What can you conclude?

Table 1: Summary Statistics: Problem 1, part (a) and (b)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_ols	100	9.590	9.716	−13.569	35.337
beta_1_ols	100	0.971	0.520	−0.481	2.243
beta_2_ols	100	1.050	0.386	0.344	1.896
var_b0_ols	100	70.313	35.817	17.030	193.850
var_b1_ols	100	0.203	0.103	0.049	0.559
var_b2_ols	100	0.145	0.074	0.035	0.399
se_b0_ols	100	8.139	2.029	4.127	13.923
se_b1_ols	100	0.437	0.109	0.222	0.748
se_b2_ols	100	0.369	0.092	0.187	0.632
t_val_b0_ols	100	1.262	1.271	−1.565	5.098
t_val_b1_ols	100	2.335	1.219	−0.644	5.307
t_val_b2_ols	100	3.038	1.401	0.665	7.349
BP_testStat_ols	100	6.292	4.894	0.061	20.898
GV_HET_Test_ols	100	4.797	3.828	0.001	16.076

BP testStat ols = Breusch Pagan test against heteroskedasticity
GV HET Test ols = General test for LM assumptions
Chi Squared Crit df:2 at alpha:0.05 = 5.99

The results displayed in table 1 (above) show the summary for the 100 sample experiments preformed on the data using OLS. The means of the 100 samples are reported for various statistics. The average t-values for the beta parameters show the constant is insignificant while β_1 and β_2 are significant. To better understand the distribution of the parameters over the 100 experiments see density plots below.



(1.b)

Test for the presence of heteroskedasticity? What do you conclude?

I have run the Breusch Pagan (BP) test for heteroskedasticity for the 100 samples and averaged over the experiments. The BP test statistic is distributed as Chi-Squared with k-1 degrees of freedom, where k is the total number of parameters in the model.

The null hypothesis of the Breusch-Pagan test is,

$$\sigma_i^2 = \sigma^2(\alpha_0 + \alpha' z_i)$$

where,

$H_0 : \alpha = 0$

σ_i^2 is the error variance for the ith observation and α_0 and α are regression coefficients.

The test statistic for the Breusch-Pagan test is:

$$bp = \frac{1}{v}(u - \bar{u}i)' Z(Z'Z)^{-1} Z'(u - \bar{u}i) \tag{2}$$

where $u = (e_1^2, e_2^2, ..., e_n^2)$, i is a $nx1$ vector of ones, and

$$v = \frac{1}{n}\sum_{i=1}^{n}\left(e_i^2 - \frac{e'e}{n}\right)^2$$

The result from any one of the 100 experiments may or may not indicate heteroskedasticity, however, when we average over the range of the 100 experiments we obtain a test statistic which would suggest, overall we have a heteroskedasticity problem.

This is a modified version of the Breusch-Pagan test, which is less sensitive to the assumption of normality than the original test (Greene 2012, 7th Ed.; p. 276).

(1.c)

Assuming multiplicative heteroskedasticity, estimate the parameters using GLS and provide their covariance matrix.

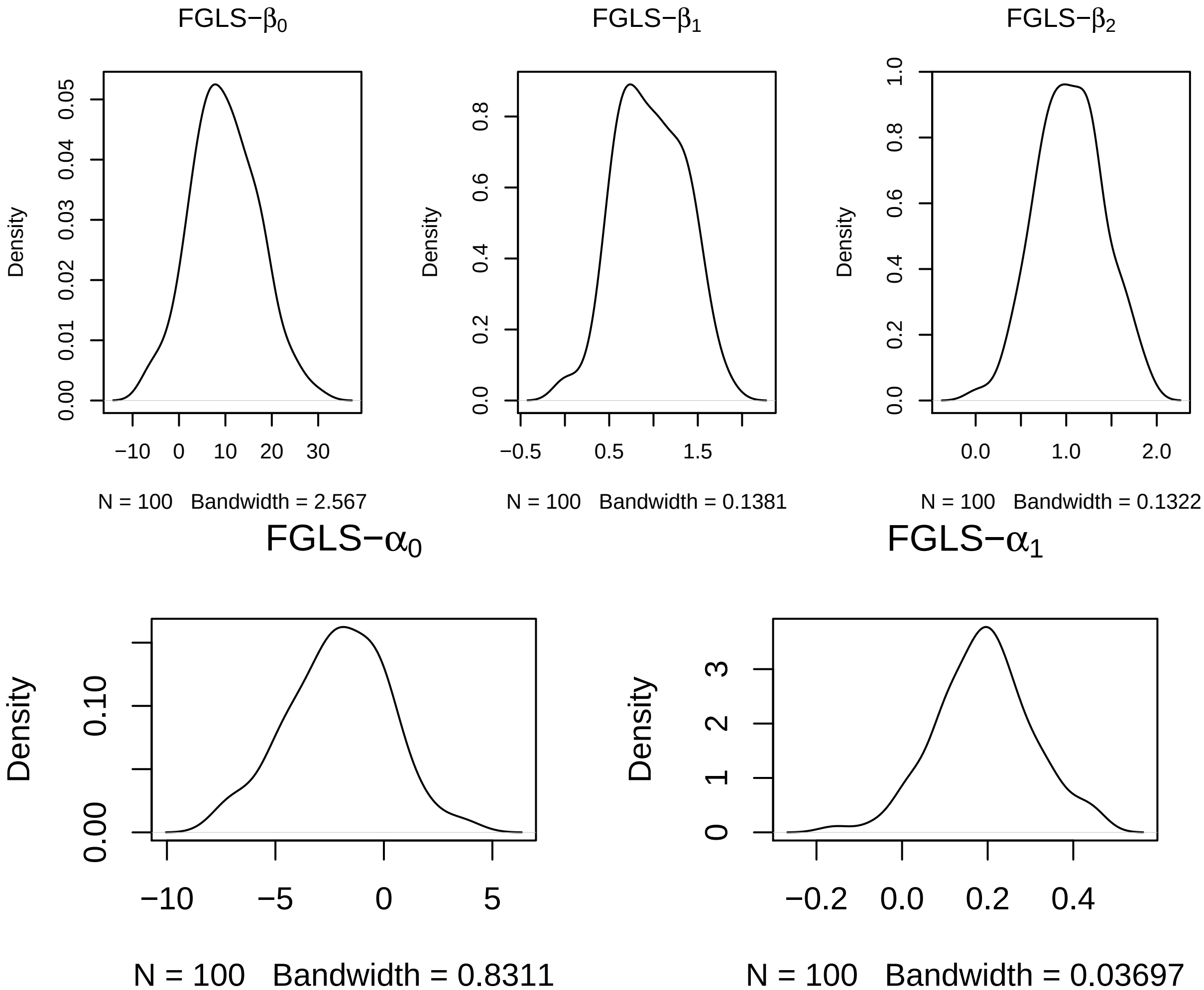
Table 2 (below) shows the summary of results obtained using Feasible generalized least squares (FGLS) to obtain an estimate of the variance-covariance structure. Once we have this structure ($\hat{\Omega}^{-1}$) in hand we can proceed with estimation of the Beta's and the correct variance-covariance.

When comparing the FGLS results with OLS we notice the average of the parameter estimates are similar enough to be considered the same, as we expect. However, we observe the variance-covariance of the FGLS is much smaller, indicating a much better job at recovering the nature of the error structure and addressing the heteroskedasticity problem embedded in the data. Since we have generated the data with known structure and parameters we can verify the accuracy of our techniques ability to uncover the true parameters. The FGLS technique does an excellent job at recovering both the beta's of our data and the alpha parameters of our error structure. Using the FGLS technique we also notice all of our parameter estimates are significant at the 95% level, a change from the OLS procedure where only two of the three parameters were significant.

To allow us to understand the distribution of the FGLS parameter estimates over the range of the 100 experiments, I provide density plot's (below).

Table 2: Summary Statistics: Problem 1, part (c)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_ols	100	9.590	9.716	−13.569	35.337
beta_1_ols	100	0.971	0.520	−0.481	2.243
beta_2_ols	100	1.050	0.386	0.344	1.896
var_b0_ols	100	70.313	35.817	17.030	193.850
var_b1_ols	100	0.203	0.103	0.049	0.559
var_b2_ols	100	0.145	0.074	0.035	0.399
se_b0_ols	100	8.139	2.029	4.127	13.923
se_b1_ols	100	0.437	0.109	0.222	0.748
se_b2_ols	100	0.369	0.092	0.187	0.632
t_val_b0_ols	100	1.262	1.271	−1.565	5.098
t_val_b1_ols	100	2.335	1.219	−0.644	5.307
t_val_b2_ols	100	3.038	1.401	0.665	7.349
beta_0_fgls	100	9.836	7.165	−6.479	29.558
beta_1_fgls	100	0.965	0.385	−0.008	1.857
beta_2_fgls	100	1.044	0.369	0.022	1.865
alpha_0_fgls	100	−2.100	2.320	−7.556	3.861
alpha_1_fgls	100	0.192	0.113	−0.157	0.452
var_b0_fgls	100	21.369	20.497	4.819	100.744
var_b1_fgls	100	0.052	0.046	0.013	0.225
var_b2_fgls	100	0.022	0.013	0.008	0.096
se_b0_fgls	100	4.272	1.774	2.195	10.037
se_b1_fgls	100	0.214	0.082	0.115	0.474
se_b2_fgls	100	0.142	0.038	0.088	0.309
t_val_b0_fgls	100	2.573	2.236	−2.263	10.344
t_val_b1_fgls	100	4.911	2.391	−0.045	12.458
t_val_b2_fgls	100	7.864	3.571	0.168	18.313
var_b0_HCCM_0	100	73.048	62.861	8.697	357.686
var_b1_HCCM_0	100	0.233	0.196	0.031	1.094
var_b2_HCCM_0	100	0.085	0.049	0.024	0.280
var_b0_HCCM_3	100	126.622	115.448	12.261	645.598
var_b1_HCCM_3	100	0.395	0.358	0.048	2.010
var_b2_HCCM_3	100	0.164	0.123	0.038	0.704



(1.d)

Write a Matlab code to estimate the parameters using maximum likelihood method and provide their covariance matrix.

$$\ln L = -0.5n\log(2\pi) - 0.5\sum(\sigma^2) - 0.5\sum\left[\frac{(y - X'\beta)^2}{\sigma^2}\right] \tag{3}$$

where,

$\sigma^2 \simeq \exp(\alpha_0 + \alpha_1 x_1)$

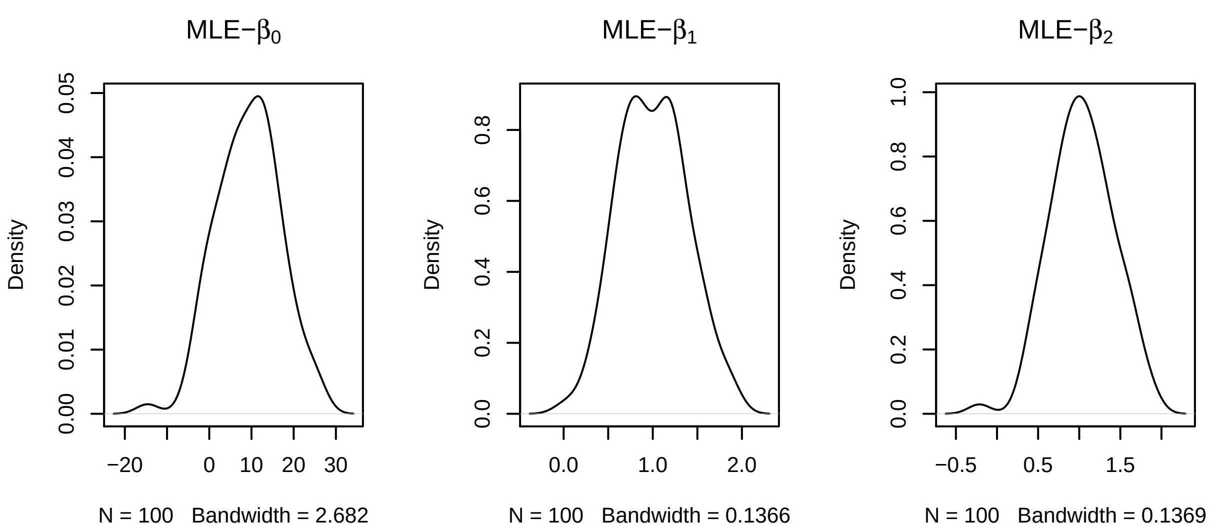
Table 3 (below) gives the summary statistics for the Maximum Likelihood Estimation method (MLE). Again we notice the MLE does a good job at uncovering the true parameters of our model. The MLE has the added benefit of obtaining all parameter estimates at one time, assuming the MLE assumptions are met.

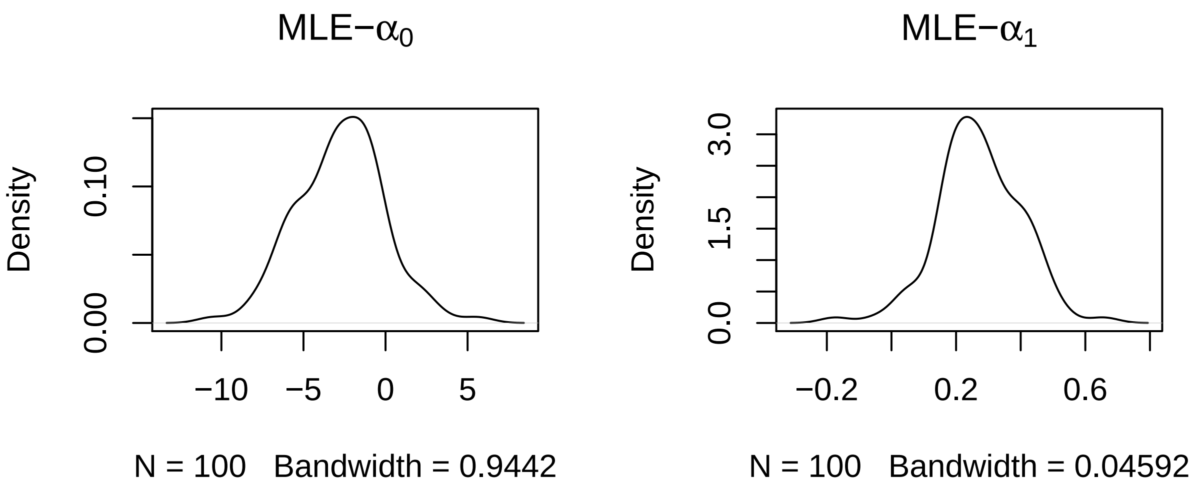
When we compare the results obtained using the MLE procedure we notice all parameter estimates are similar, however, when we view the variance-covariance matrix we see that the FGLS seems to do a better job over the MLE procedure. This may be due to the optimization routine used for the MLE or the convergence tolerance, or both. More experimenting with the optimization variables could yield better results. At any rate the MLE estimation strategy does a better job that OLS and just about as good a job as FGLS, in the current context.

To help us better understand the distribution of the MLE parameter estimates over the range of the 100 experiments, I provide density plot's (below).

Table 3: Summary Statistics: Problem 1, part (d)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_mle	100	9.294	7.485	−14.598	26.088
beta_1_mle	100	1.001	0.381	0.029	1.898
beta_2_mle	100	1.037	0.382	−0.214	1.880
alpha_0_mle	100	−2.742	2.681	−10.509	5.598
alpha_1_mle	100	0.271	0.129	−0.174	0.656
var_b0_mle	100	36.725	17.487	11.604	85.507
var_b1_mle	100	0.113	0.052	0.038	0.291
var_b2_mle	100	0.082	0.032	0.026	0.176
var_alpha_0_mle	100	5.661	2.746	1.795	14.500
var_alpha_1_mle	100	0.013	0.006	0.004	0.033
se_b0_mle	100	5.900	1.391	3.406	9.247
se_b1_mle	100	0.329	0.073	0.195	0.539
se_b2_mle	100	0.281	0.056	0.162	0.419
se_alpha_0_mle	100	2.317	0.544	1.340	3.808
se_alpha_1_mle	100	0.110	0.026	0.062	0.182
t_val_b0_mle	100	1.712	1.489	−2.522	6.053
t_val_b1_mle	100	3.212	1.464	0.113	7.151
t_val_b2_mle	100	3.770	1.468	−0.607	8.509
t_val_alpha_0_mle	100	−1.258	1.180	−4.173	1.470
t_val_alpha_1_mle	100	2.617	1.312	−0.958	5.546





Problem 2

Now assume the model is homoskedastic. However, $\varepsilon_t = \rho\varepsilon_{t-1} + v_t$ and the v_t are independent normal random variables with zero mean and variance $E[v_t^2] = \sigma_v^2$. Generate 100 samples of y, each of size 20 with $\rho = 0.8$ and $\sigma_v^2 = 6.4$.

(2.a)

Estimate the parameters using the least squares principle and provide their covariance matrix. Compare your results with the true parameters. What can you conclude?

The OLS parameters are consistent with previous results, however, the vcov matrix is not correct due to autocorrelation. Without testing it would be difficult to know the nature of the problem. See table 4 for OLS parameter and vcov estimates.

(2.b)

Test for the presence of autocorrelation? What do you conclude?

We can use the Durbin-Watson Test:

The Hypotheses for the Durbin Watson test are:

$H_0 =$ no first order autocorrelation.

$H_1 =$ first order correlation exists.

Assumptions are:

That the errors are normally distributed with a mean of 0.

The errors are stationary.

$$DW = \frac{\sum_{t=2}^{T} (\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T} \varepsilon_t^2}$$

where,

ε_t are residuals from an ordinary least squares regression.

The Durbin Watson test reports a test statistic, with a value from 0 to 4, where:

if $DW = 2$ then no autocorrelation.

if $0 < DW < 2$ then positive autocorrelation (common in time series data).

if $2 < DW < 4$ then negative autocorrelation (less common in time series data).

Using the Durbin-Watson test (DW_Test) we obtain an average DW test statistic of 3.290 which indicate we do have autocorrelation that needs to be addressed. When we run the BP test (equation 2) we obtain a test statistic of 2.687 which indicates no problems with heteroskedasticity, as we already know since we constructed the data in this fashion.

(2.c)

Assuming the errors follow AR(1) process, estimate the parameters using GLS and provide their covariance matrix.

To obtain the parameters for the structure of the errors, in this case, AR(1), we can employ an FGLS method. In this situation we will use the technique developed by Cochrane and Orcutt (1949) which makes use of an iterative process to converge on the best fit for σ^2 and ρ. Once we have our error parameters we can construct Ω^{-1} and estimate our data parameters (beta's) and variance-covariance matrix, using FGLS. The results are given in table 4 (below). We see that using the Cochrane–Orcutt method gives relatively accurate estimates of our $\hat{\sigma}^2$ and $\hat{\rho}$ parameters. Our true values of rho and sigma squared are 0.8 and 6.4 and we obtain an estimated of $\hat{\rho} = 0.755$ and $\hat{\sigma}^2 = 6.055$. With a larger sample size I suspect our estimates would be more accurate but considering a sample size of 20, I feel that these estimates are impressive.

Our beta estimates, using FGLS are a bit on the low side, again a possible situation of the stochastic nature of multiple experiments and low sample size. But the estimates are reasonable. The variance-covariance matrix is much tighter that the OLS estimates so we can see the FGLS procedure does a good job at correcting for the autocorrelation embedded in our data. The t-values indicate that our parameters are significant.

To help us better understand the distribution of the MLE parameter estimates over the range of the 100 experiments, I provide density plot's (below).

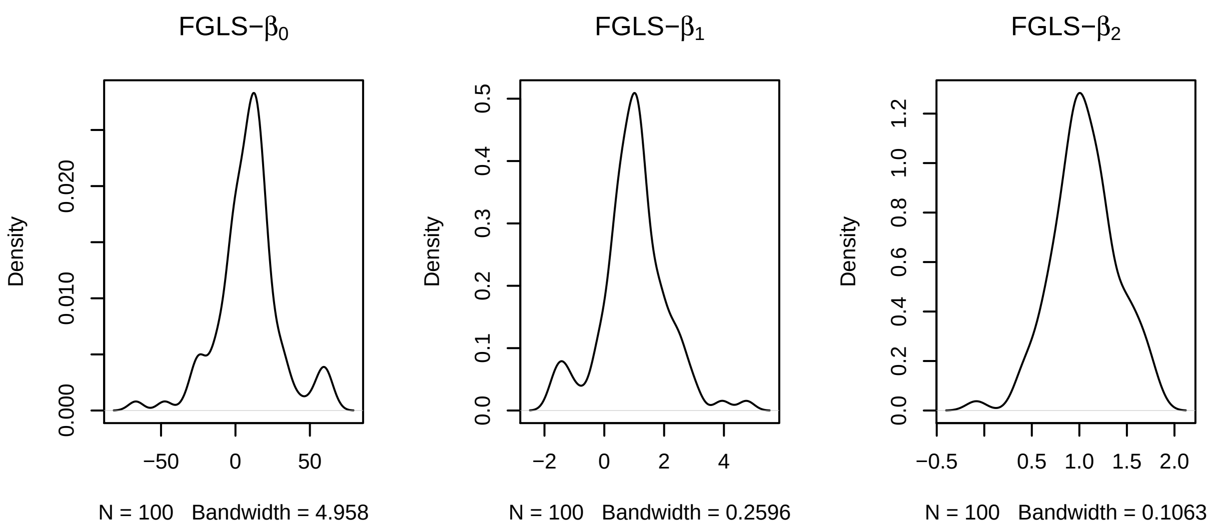
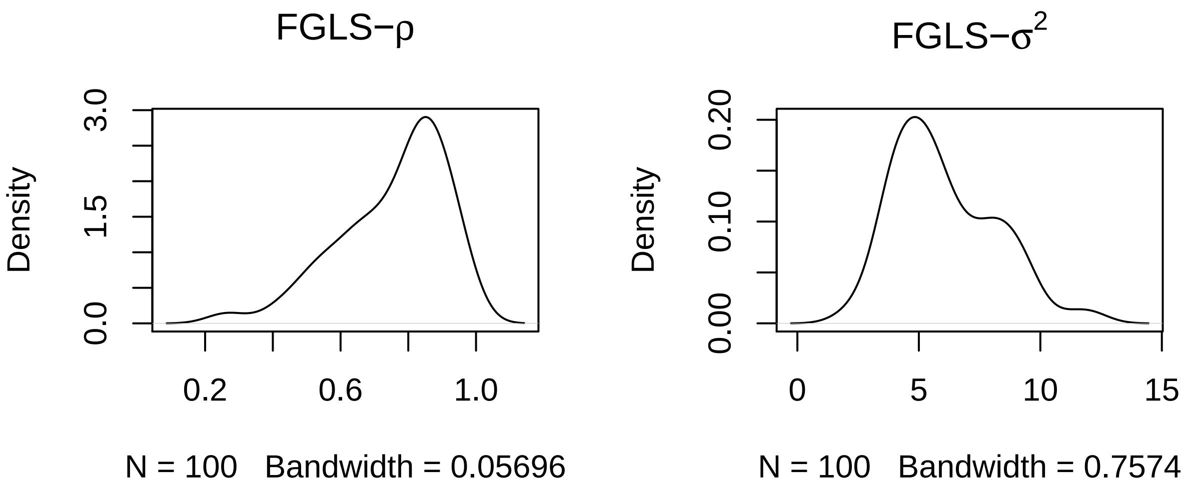


Table 4: Summary Statistics: Problem 2, part (a), (b), and (c)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_ols	100	9.530	2.594	3.907	16.808
beta_1_ols	100	0.990	0.192	0.477	1.481
beta_2_ols	100	1.032	0.175	0.550	1.489
var_b0_ols	100	36.308	24.120	7.328	148.470
var_b1_ols	100	0.105	0.070	0.021	0.428
var_b2_ols	100	0.075	0.050	0.015	0.306
se_b0_ols	100	5.767	1.754	2.707	12.185
se_b1_ols	100	0.310	0.094	0.145	0.654
se_b2_ols	100	0.262	0.080	0.123	0.553
t_val_b0_ols	100	1.803	0.711	0.462	3.630
t_val_b1_ols	100	3.479	1.272	1.367	7.728
t_val_b2_ols	100	4.290	1.449	1.426	9.489
DW_Test	100	3.290	0.357	2.252	3.871
BP_Test	100	2.687	1.977	0.052	8.801
rho_fgls	100	0.755	0.159	0.257	0.971
sigma2_fgls	100	6.055	2.114	2.008	12.177
beta_0_fgls	100	8.951	21.059	−66.922	64.418
beta_1_fgls	100	0.987	1.097	−1.709	4.750
beta_2_fgls	100	1.057	0.339	−0.083	1.801
var_b0_fgls	100	8.444	12.104	1.360	81.390
var_b1_fgls	100	0.057	0.090	0.005	0.604
var_b2_fgls	100	0.106	0.177	0.004	1.188
se_b0_fgls	100	2.497	1.494	1.166	9.022
se_b1_fgls	100	0.197	0.135	0.068	0.777
se_b2_fgls	100	0.262	0.194	0.065	1.090
t_val_b0_fgls	100	4.674	6.749	−14.133	17.114
t_val_b1_fgls	100	6.831	5.535	−4.868	17.325
t_val_b2_fgls	100	5.880	3.585	−0.165	16.919
var_b0_HCCM_0	100	27.372	15.930	4.489	81.518
var_b1_HCCM_0	100	0.079	0.046	0.010	0.262
var_b2_HCCM_0	100	0.056	0.057	0.007	0.413
var_b0_HCCM_3	100	43.248	25.773	6.306	143.734
var_b1_HCCM_3	100	0.121	0.071	0.015	0.418
var_b2_HCCM_3	100	0.108	0.118	0.012	0.851



(2.d)

Write a Matlab code to estimate the parameters using maximum likelihood method and provide their covariance matrix.

$$\ln L = \left\{ -0.5\log(2\pi) - 0.5\log({\sigma_u}^2) + 0.5\log(1-\rho^2) - \frac{(1-\rho^2)(y_{11} - (\beta_1 + \beta_2 x_{11} + \beta_3 x_{21}))^2}{2{\sigma_u}^2} \right\} +$$
$$\sum_{t=2}^{T} \left\{ -0.5\log(2\pi) - 0.5\log({\sigma_u}^2) - \frac{[(y_t - \rho y_{t-1}) - (\beta_1(1-\rho) + \beta_2(x_{1t} - \rho x_{1t-1}) + \beta_3(x_{2t} - \rho x_{2t-1})]^2}{2{\sigma_u}^2} \right\}$$

In the below tables are presented the MLE estimates for the betas the rho and sigma-squared parameters of the model with AR(1) innovations. The MLE method does a good job at estimating the parameters of our model, including the structure of the autocorrelated innovations. The average beta's, rho and sigma parameters are all in line with the true parameters and all t-values show the parameter estimates are significant. The variance-covariance is the best of all methods used and the ability of the MLE to estimate all parameters simultaneously is a great feature of this method of estimation.

To help us better understand the distribution of the MLE parameter estimates over the range of the 100 experiments, I provide density plot's (below).

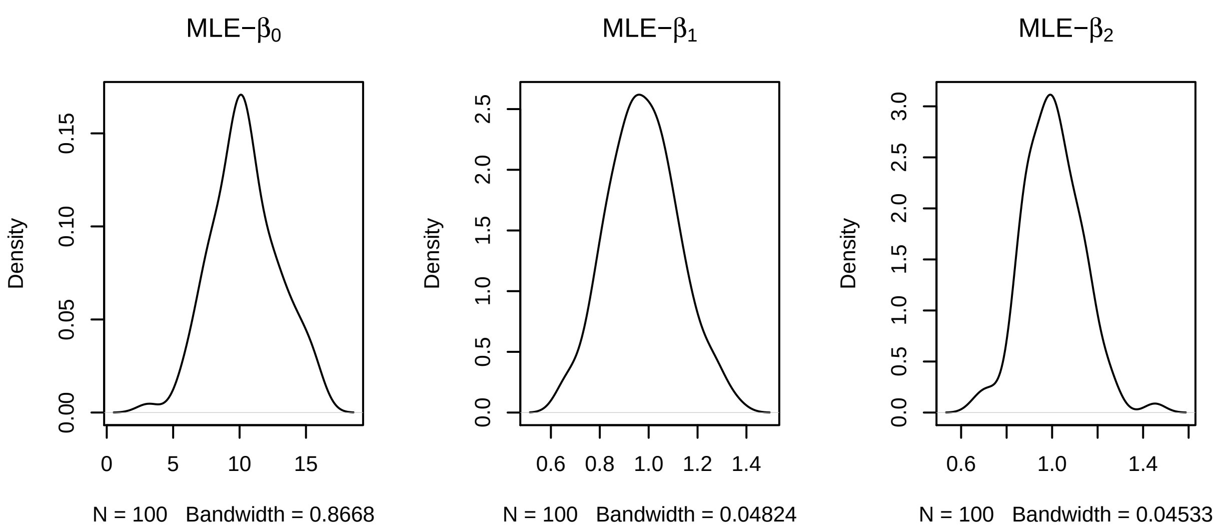


Table 5: Summary Statistics: Problem 2, part (d)

Statistic	N	Mean	St. Dev.	Min	Max
beta_0_mle	100	10.412	2.511	3.125	15.978
beta_1_mle	100	0.977	0.143	0.659	1.350
beta_2_mle	100	1.004	0.127	0.670	1.452
rho_mle	100	0.756	0.150	0.184	0.978
sigma_mle	100	2.484	0.490	1.520	4.609
var_b0_mle	100	6.127	2.752	1.871	21.048
var_b1_mle	100	0.019	0.008	0.006	0.064
var_b2_mle	100	0.014	0.007	0.004	0.051
var_rho_mle	100	0.020	0.013	0.001	0.069
var_sigma_mle	100	0.162	0.068	0.059	0.533
se_b0_mle	100	2.422	0.513	1.368	4.588
se_b1_mle	100	0.133	0.028	0.075	0.253
se_b2_mle	100	0.118	0.025	0.066	0.225
se_rho_mle	100	0.135	0.045	0.031	0.262
se_sigma_mle	100	0.395	0.078	0.243	0.730
t_val_b0_mle	100	4.499	1.599	1.590	11.682
t_val_b1_mle	100	7.677	2.019	2.924	14.594
t_val_b2_mle	100	8.904	2.223	4.559	16.320
t_val_rho_mle	100	6.826	4.269	0.740	31.071
t_val_sigma_mle	100	6.291	0.032	6.176	6.324
lnL_value	100	−46.736	3.785	−59.261	−37.667