Problem 1

1a. Determine the amount of times for television advertisement and radio advertisement to maximize its revenue?

When the goal for our firm is to maximize revenue while spending a total of $10,000 on a mix of advertising with a given cost of $3000/min. for TV and $1000/min. for radio are as follows:

The optimal amount of television advertisement is 2.46 minutes and the optimal amount of radio advertising is 2.61 minutes.

1b. Determine the marginal rate of return on the company's additional advertising spending.

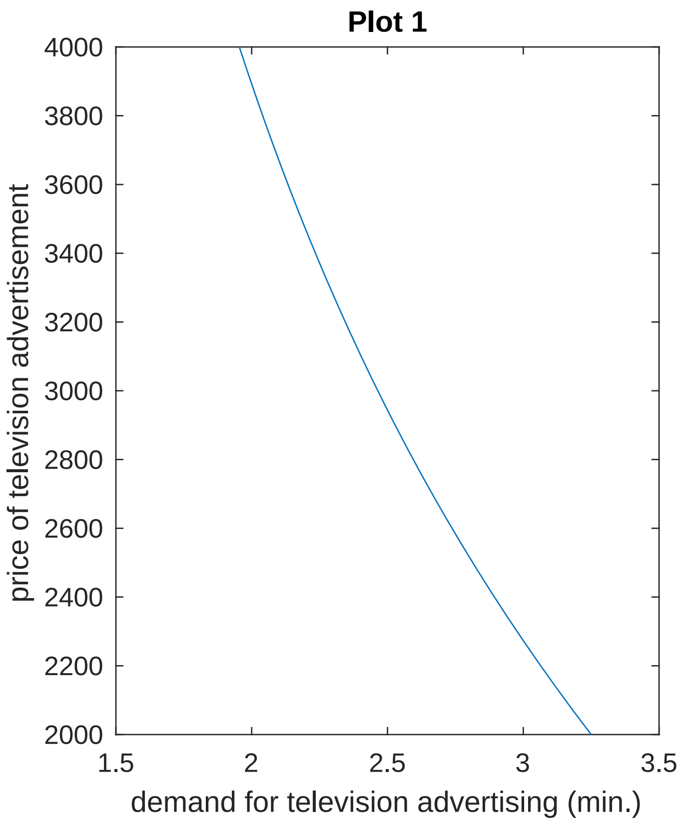
To determine the marginal rate of return we can consider the Lagrange multiplier, Lambda, which gives us the marginal rate of return for additional units of advertising spending. In this case we see that each additional dollar of spending will return 0.00025.

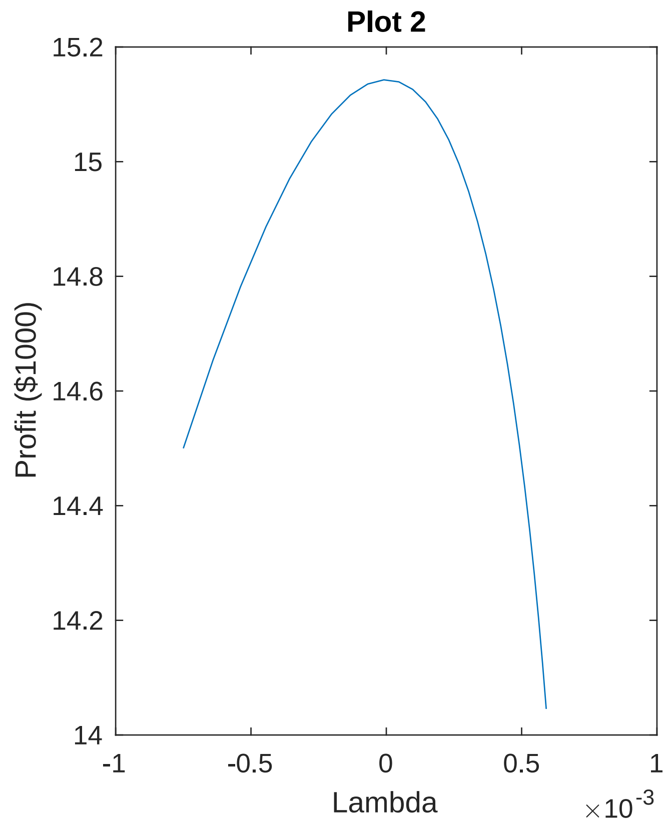
1c. Create a graph to show how the firm's demand for television advertising (measured in numbers of minutes) will change when the price of television advertisement varies from $2,000 to $4,000 per minute.

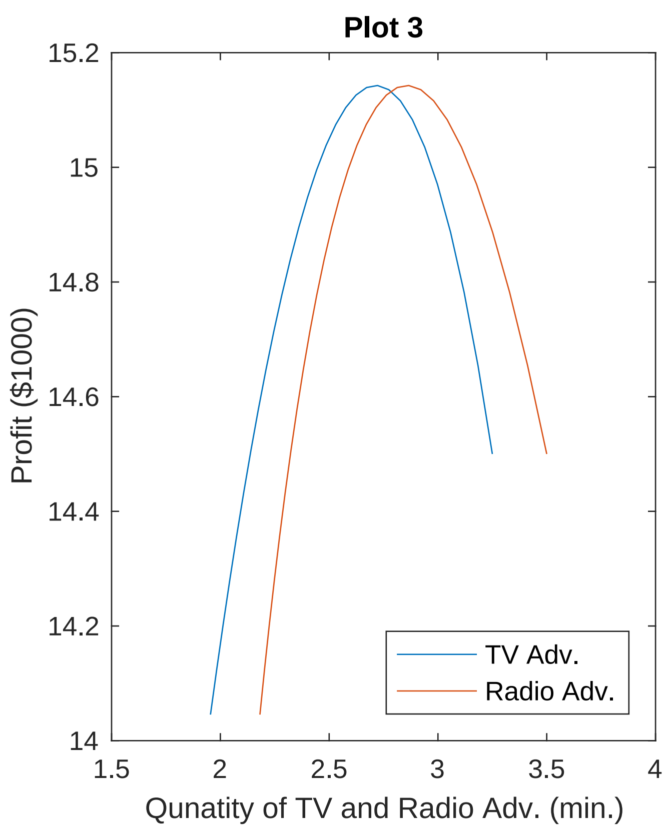
Please see attached graphs.

The first plot shows the change in how the demand changes with changes in price. As would be expected our demand curve is negatively sloped; an increase in price creates a decrease in the demand for advertising.

Also of interest is how lambda responds to changes in price. Plot 2 shows how our advertising problem is affected through changes in our constraints.


Plot 1
price of television advertisement
4000
3800
3600
3400
3200
3000
2800
2600
2400
2200
2000
1.5
2
2.5
3
3.5
demand for television advertising (min.)


Plot 2
Profit ($1000)
15.2
15
14.8
14.6
14.4
14.2
14
-1
-0.5
0
0.5
1
Lambda
×10-3


Plot 3
Profit ($1000)
15.2
15
14.8
14.6
14.4
14.2
14
1.5
2
2.5
3
3.5
4
Qunatity of TV and Radio Adv. (min.)
TV Adv.
Radio Adv.

Problem 2

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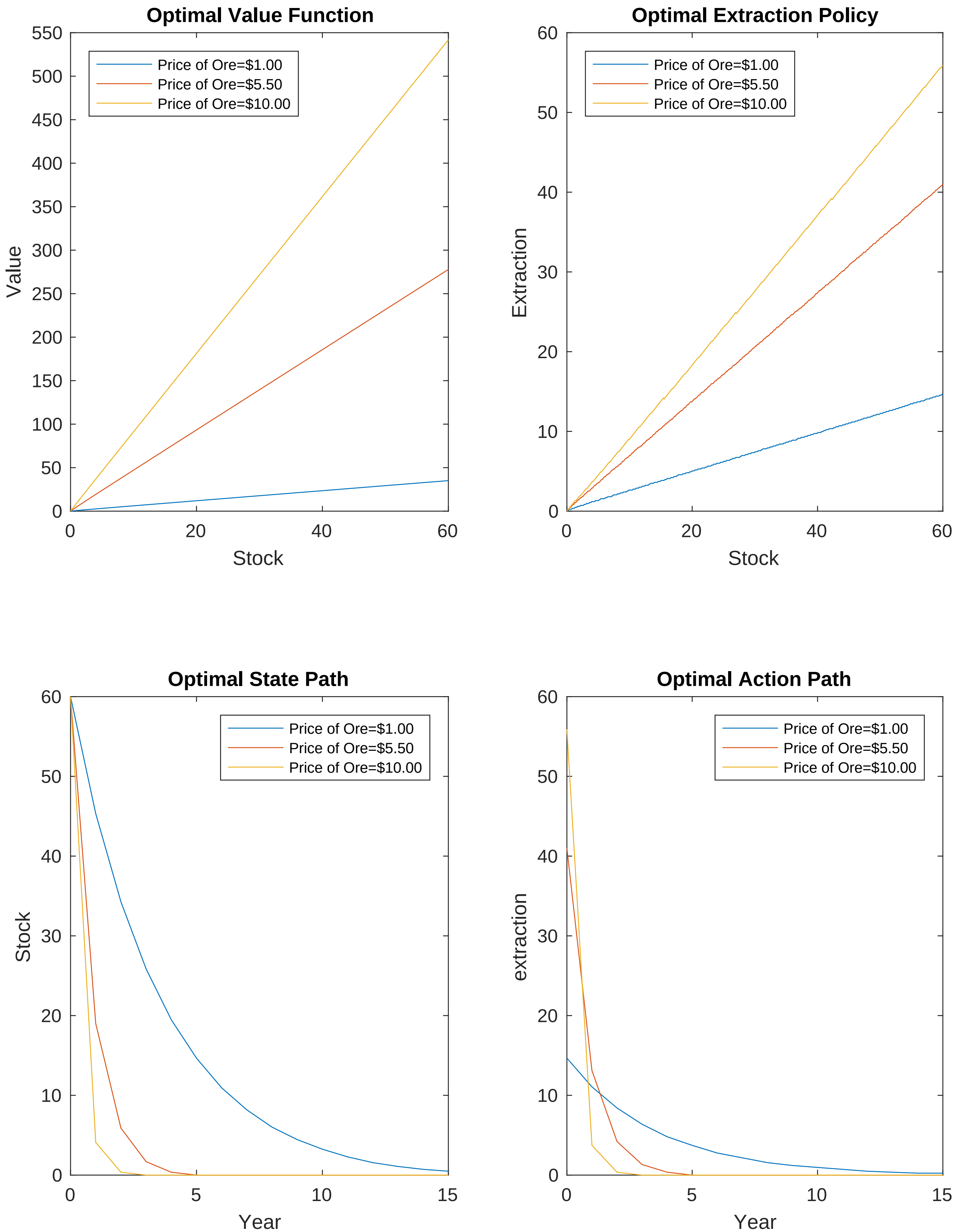
2 Modify the MATLAB script for the mine management problem to answer the following question. Modify the MATLAB script provided to you to conduct a sensitivity analysis in terms of the price of ore. Specifically, determine how an increase in the ore price will change the optimal extraction policy, the value of the mine, and lifetime of the mine. Show the MATLAB code and results you used to answer the questions.

Please see attached graphs.

In this problem I varied the price of ore from $1.00 - $5.50 - $10.00; holding all other variables constant.

The lower the price of ore the lower the extraction rate which increase the life of the mine but also means we have a lower value associated with the mine. In contrast, the higher the price of ore the higher our optimal extraction rate which drastically decreases the life of the mine but also increase the value of the mine substantially.


Optimal Value Function
Value
Stock
Price of Ore=$1.00
Price of Ore=$5.50
Price of Ore=$10.00
Optimal Extraction Policy
Extraction
Stock
Price of Ore=$1.00
Price of Ore=$5.50
Price of Ore=$10.00
Optimal State Path
Stock
Year
Price of Ore=$1.00
Price of Ore=$5.50
Price of Ore=$10.00
Optimal Action Path
extraction
Year
Price of Ore=$1.00
Price of Ore=$5.50
Price of Ore=$10.00

Problem 3

---

3 Modify the MATLAB script for the water management (irrigation vs. recreational use) problem to answer the following question. Modify the MATLAB script provided to you to conduct a sensitivity analysis in terms of the technological coefficient α *in* $F(x) = \alpha x^{\beta_1}$. Determine how an increase in the productivity of irrigation farmers affect the optimal water allocation policy, the value of the water resources in the reservoir, and the steady-state water level in the reservoir.

Please see attached graphs.

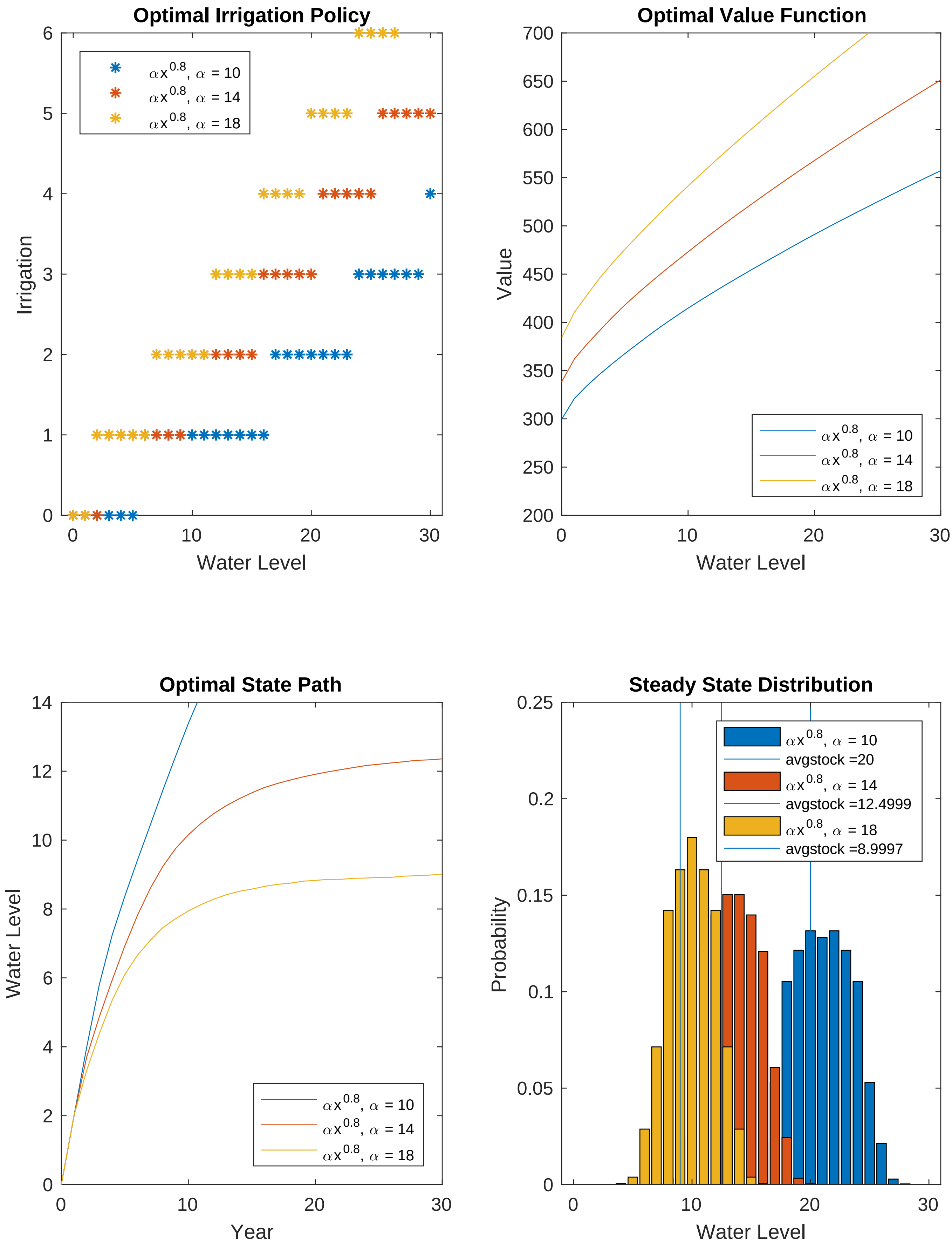
In this problem I varied my technological coefficient $\alpha 1$ from 10-14-18,; holding all other variables constant.

When our technological coefficient is low we see that our optimal irrigation policy tell us we will deplete our water resource while using 4 units of irrigation, however, when we improve our efficiency we can irrigate at a higher level while maintaining a higher water level.

The value of our water resources also increase when we increase our technological coefficient from 10 to 18. The overall benefit to society is improved when we improve the efficiency of our farmers irrigation systems.

The steady state water level is reached more quickly and at a lower water level with a higher level of technological efficiency to our farmers irrigation systems.

This problem shows clearly that with improved efficiency, it is possible to increase the overall benefit to society when measured against maintaining a viable water source for longer periods of time.



Optimal Irrigation Policy
Irrigation
Water Level
$\alpha x^{0.8}$, α = 10
$\alpha x^{0.8}$, α = 14
$\alpha x^{0.8}$, α = 18
Optimal Value Function
Value
Water Level
$\alpha x^{0.8}$, α = 10
$\alpha x^{0.8}$, α = 14
$\alpha x^{0.8}$, α = 18
Optimal State Path
Water Level
Year
$\alpha x^{0.8}$, α = 10
$\alpha x^{0.8}$, α = 14
$\alpha x^{0.8}$, α = 18
Steady State Distribution
Probability
Water Level
$\alpha x^{0.8}$, α = 10
avgstock =20
$\alpha x^{0.8}$, α = 14
avgstock =12.4999
$\alpha x^{0.8}$, α = 18
avgstock =8.9997

Problem 1 Matlab Code

```
clc
clear;

i= 1;       % price counter

P_x= linspace(2000, 4000, 30);

beq= [10000];
lb=[0 0];
x0 = [0; 0];

ans= zeros(numel(P_x), 5);

options=optimset('Algorithm','interior-point');

v = @(x) -1* (-2* x(1)^2- x(2)^2+ x(1)* x(2)+ 8* x(1)+ 3* x(2) );

figure('units','normalized','outerposition',[0 0 0.75 1]) % make plot window open to X-fraction
of screen
figure(1); hold on % allow plot build-up
tp= 0;

for i= 1:numel(P_x);

Aeq= [P_x(i) 1000;];
[x,fval,exitflag,output,lambda] = fmincon(v, x0, [], [], Aeq, beq, lb, [], [], options);

ans(i,1:2)= x;
ans(i,3)= P_x(i);
ans(i,4)= -fval;
ans(i,5)= lambda.eqlin;

i= i+1;

end

subplot(2,2,1);
plot(ans(:,1), ans(:,3));
title('Plot 1');
xlabel('demand for television advertising (min.)');
ylabel('price of television advertisement');

subplot(2,2,2);
plot(ans(:,5),ans(:,4));
title('Plot 2');
xlabel('Lambda');
ylabel('Profit ($1000)');

subplot(2,2,3);
plot(ans(:,1),ans(:,4))
```

```
hold on
plot(ans(:,2),ans(:,4))
title('Plot 3');
xlabel('Qunatity of TV and Radio Adv. (min.)');
ylabel('Profit ($1000)');
legend({...
   'TV Adv.',...
   'Radio Adv.'},...
   'FontSize', 10, 'Location', 'Southeast','Box','on');

print('mideterm_Q1_plot', '-dpdf', '-bestfit') % export plot to file
```

Problem 2 Matlab Code

```
% DEMDDP01 Mine Management Model
 fprintf('\nDEMDDP01 MINE MANAGEMENT MODEL\n')
 close all
 clear
 clc

% Enter model parameters
 price = linspace(1, 10, 3);   % price of ore
 z= 1;                        % price counter
 sbar  = 60;                    % initial ore stock
 delta = 0.9;                   % discount factor
 theta = 1.0;                   % coefficient of extraction efficiency

% Construct state and action spaces
 S= linspace(0, 60, 500)';       % vector of states
 X= linspace(0, 60, 500)';       % vector of actions
 n = length(S);                 % number of states
 m = length(X);                  % number of actions

figure('units','normalized','outerposition',[0 0 0.75 1]) % make plot window open to X-fraction
of screen
figure(1); hold on % allow plot build-up
tp= 0;

% Construct reward function (f) and state transition function (g)
% Non-vectorized version
for z=1:numel(price)

  f= zeros(n,m);

 for i=1:n
 for k=1:m
   if X(k)<=S(i)
     f(i,k) = price(z)*X(k)-(X(k)^2)./(1+ theta*S(i));
   else
     f(i,k) = -inf;
   end
```

```
  end
  end

  g = zeros(n,m);
  for i=1:n
  for k=1:m
    snext = S(i)-X(k);
    g(i,k) = getindex(snext,S);
  end
  end

% Pack model data
  clear model
  model.reward     = f;
  model.transfunc  = g;
  model.discount   = delta;

% Solve model using the function "ddpsolve"
  [v,x,pstar] = ddpsolve(model);

% Plot optimal value function
  subplot(2,2,1);
  plot(S,v);
  axis([0 60 0 550]);
  title('Optimal Value Function');
  xlabel('Stock');
  ylabel('Value');
  legend({...
    'Price of Ore=$1.00',...
    'Price of Ore=$5.50',...
    'Price of Ore=$10.00'},...
    'FontSize', 8, 'Location', 'Northwest','Box','on');
  hold on;

% Plot optimal policy function
  subplot(2,2,2)
  plot(S,X(x));
  title('Optimal Extraction Policy');
  xlabel('Stock');
  ylabel('Extraction');
   legend({...
    'Price of Ore=$1.00',...
    'Price of Ore=$5.50',...
    'Price of Ore=$10.00'},...
    'FontSize', 8, 'Location', 'Northwest','Box','on');
  hold on;

% Generate optimal state and action paths
  sinit = getindex(sbar,S); nyrs = 15;
  [spath, xpath] = ddpsimul(pstar,sinit,nyrs,x);

% Plot optimal state path
  subplot(2,2,3);
  plot(0:nyrs,S(spath));
```

```
  title('Optimal State Path');
  xlabel('Year');
  ylabel('Stock');
   legend({...
    'Price of Ore=$1.00',...
    'Price of Ore=$5.50',...
    'Price of Ore=$10.00'},...
    'FontSize', 8, 'Location', 'Northeast','Box','on');
  hold on;

% Plot optimal action path
  subplot(2,2,4);
  plot(0:nyrs,X(xpath));
  title('Optimal Action Path');
  xlabel('Year');
  ylabel('extraction');
  legend({...
    'Price of Ore=$1.00',...
    'Price of Ore=$5.50',...
    'Price of Ore=$10.00'},...
    'FontSize', 8, 'Location', 'Northeast','Box','on');
  hold on;

  pause(tp)
  z= z+1;

end

print('mideterm_Q2_plot', '-dpdf', '-bestfit') % export plot to file
```

Problem 3 Matlab Code

```
% DEMDDP05 Water Management Model
% This program solves the water management model under the high rainfall
% scenario.
  fprintf('\nDEMDDP05 WATER MANAGEMENT MODEL\n')
  warning ('off','all');
  close all
  clear
  clc

% Enter model parameters
  alpha1 = linspace(10, 18, 3);     % producer benefit function parameter
  beta1  = 0.8;                     % producer benefit function parameter
  alpha2 = 10;                      % recreational user benefit function parameter
  beta2  = 0.4;                     % recreational user benefit function parameter
  maxcap = 30;                      % maximum dam capacity
  r      = [0 1 2 3 4];             % rain levels
  p      = [0.1 0.2 0.4 0.2 0.1];   % rain probabilities
  delta  = 0.9;                     % discount factor
```

```
  z= 1;
  avgstock = zeros(1,3);

% Construct state space
  S = (0:maxcap)';              % vector of states
  n = length(S);                % number of states

% Construct action space
  X = (0:maxcap)';              % vector of actions
  m = length(X);                % number of actions

figure('units','normalized','outerposition',[0 0 0.75 1]) % make plot window open to X-fraction
of screen
figure(1); hold on  % allow plot build-up
tp= 1;              % time pause between plot iterations

% Construct reward function
for z= 1:numel(alpha1)

  f = zeros(n,m);
  for i=1:n
  for k=1:m
    if k>i
      f(i,k) = -inf;
    else
      f(i,k) = alpha1(z)*X(k).^beta1+ alpha2*(S(i)-X(k)).^beta2;

    end
  end
  end

% Construct state transition matrix
  P = zeros(m,n,n);
  for k=1:m
  for i=1:n
  for j=1:length(r)
    snext = min(S(i)-X(k)+r(j),maxcap);
    inext = getindex(snext,S);
    P(k,i,inext) = P(k,i,inext) + p(j);
  end
  end
  end

% Pack model structure
  clear model
  model.reward     = f;
  model.transprob  = P;
  model.discount   = delta;

% Solve infinite-horizon model using policy iteration
  [v,x,pstar] = ddpsolve(model);

% Plot optimal policy
  subplot(2,2,1);
```

```
  plot(S,X(x),'*');
  title('Optimal Irrigation Policy');
  xlabel('Water Level'); ylabel('Irrigation');
  xlim([-1 31]);
  ylim([0 6]);
  legend({...
    '\alphax^{0.8}, \alpha = 10',...
    '\alphax^{0.8}, \alpha = 14',...
    '\alphax^{0.8}, \alpha = 18'},...
    'FontSize', 8,'Location','Northwest','Box','on');
hold on;

% Plot optimal value function
  subplot(2,2,2);
  plot(S,v);
  title('Optimal Value Function');
  xlabel('Water Level');
  ylabel('Value');
  legend({...
    '\alphax^{0.8}, \alpha = 10',...
    '\alphax^{0.8}, \alpha = 14',...
    '\alphax^{0.8}, \alpha = 18'},...
    'FontSize', 8,'Location','Southeast','Box','on');
hold on;

% Generate random optimal paths, starting from zero water level
  sinit = ones(10000,1);
  nyrs  = 30;
  spath = ddpsimul(pstar,sinit,nyrs);
  ylim([200 700])

  % Plot expected water level over time, starting from zero water level
  subplot(2,2,3);
  plot(0:nyrs,mean(S(spath)));
  title('Optimal State Path');
  xlabel('Year');
  ylabel('Water Level');
  ylim([0 14]);
  legend({...
    '\alphax^{0.8}, \alpha = 10',...
    '\alphax^{0.8}, \alpha = 14',...
    '\alphax^{0.8}, \alpha = 18'},...
    'FontSize', 8,'Location','Southeast','Box','on');
hold on;

% Compute steady state distribution of water level
  subplot(2,2,4);
  pi = markov(pstar);
  avgstock(1,z) = pi'*S;
  h=bar(pi);
  line([avgstock(1,z) avgstock(1,z)], [0 0.25]);
  %set(h,'FaceColor',[.75 .75 .75])
  title('Steady State Distribution');
  xlabel('Water Level'); ylabel('Probability');
```

```
  xlim([-1 31])
  ylim([0 0.25])
  legend({...
    '\alphax^{0.8}, \alpha = 10',...
    ['avgstock =' num2str(avgstock(1,1) )],...
    '\alphax^{0.8}, \alpha = 14',...
    ['avgstock =' num2str(avgstock(1,2) )],...
    '\alphax^{0.8}, \alpha = 18',...
    ['avgstock =' num2str(avgstock(1,3) )]},...
    'FontSize', 8,'Location','Northeast','Box','on');
hold on;

  z= z+1;
  pause(tp)

end

print('mideterm_Q3_plot', '-dpdf', '-bestfit') % export plot to file
```