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Robust portfolio optimization for electricity planning: An application based on the Brazilian electricity mix

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ABSTRACT

One of the major challenges of today's policy makers and industry strategists is to achieve an electricity mix that presents a high level of energy security within a range of affordable costs and environmental constraints. Bearing in mind the planning of a more reliable electricity mix, the main contribution of this paper is to consider parameter uncertainties on the electricity portfolio optimization problem. We assume that the expected and the covariance matrix of the costs for the different energy technologies, such as gas, coal, nuclear, oil, biomass, wind, large and small hydropower, are not exactly known. We consider that these parameters belong to some uncertainty sets (box, ellipsoidal, lower and upper bounds, and convex polytopic). Three problems are analyzed: (i) finding a energy portfolio of minimum worst case volatility with guaranteed fixed maximum expected energy cost; (ii) finding an energy portfolio of minimum worst case expected cost with guaranteed fixed maximum volatility of the energy cost; (iii) finding a combination of the expected and variance of the cost, weighted by a risk aversion parameter. These problems are written as quadratic, second order cone programming (SOCP), and semidefinite programming (SDP), so that robust optimization tools can be applied. These results are illustrated by analyzing the efficient Brazilian electricity energy mix considered in Losekann et al. (2013) assuming possible uncertainties in the vector of expected costs and covariance matrix. The results suggest that the robust approach, being by nature more conservative, can be useful in providing a reasonable electricity energy mix conciliating CO_2 emission, risk and costs under uncertainties on the parameters of the model.

1. Introduction

Working to ensure either energy security as a whole, or electricity security in particular, is a major responsibility of national governments. One of the major challenges of today's policy makers and industry strategists is to achieve an electricity mix that presents a high level of energy security within a range of affordable costs, considering environmental and economic scenarios. There is no doubt that an electricity shortage can severely harm economies. This was, for instance, what happened in Brazil in 2001 when, due to rationing, the total Brazilian electricity consumption decreased by 7.89%, while the GDP variation was still positive, by +1.3%. However, according to SPE — Secretaria de Política Econômica do Ministério da Fazenda (2001), the growth rate for the year 2001 would be in a range of 2.4% and 3.6%, without the crisis of the electricity sector. At the same time, the local industry had to deal with the scarce supply associated with skyrocketing electricity prices in the short term market, that ultimately transformed positive margins of electric intensive companies into negative ones. From January 2001 up to May of that same year the spot market price in the Southeast submarket increased by twelvefold, jumping from R$ 56.92 to R$ 684.00 in Brazilian reals, during the rationing period. Since the required infrastructure to provide electricity takes time to be in place, good planning is always critical in this industry, especially in large populated developing countries such as China, India, Indonesia, Brazil, and many others which present high increasing rates for their electricity demand.

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Mean-variance optimization, originally introduced by Markowitz (1959), is one of the most important models in portfolio optimization and also the basis for asset allocation. However, as pointed out in Rustem et al. (2000), for the optimal mean-variance strategy to be useful the set of expected return of the component assets and the covariance matrix should be sufficiently precise. Indeed it was shown by Black and Litterman (1991) that small changes in the expected returns can produce large changes in asset allocation decisions. In practice this lack of robustness with respect to the inherent inaccuracy of the expected returns and covariance matrix estimates prevents the widespread use of mean-variance optimization by practitioners. Due to that several robust versions of portfolio optimization problems, including mean-variance optimization, have been proposed in the literature, considering uncertainties on the expected returns and covariance matrix (see, for instance, Rustem et al., 2000; Bertsimas et al., 2011; Costa and Paiva, 2002; Costa and Nabholz, 2002; El Ghaoui et al., 2003; Fabozzi et al., 2007a; Goldfarb and Iyengar, 2003; Kim et al., 2014; Lobo and Boyd, 2000; Tütüncü and Koenig, 2004).

Nowadays the mean-variance optimization tools have been widely applied in energy policy, considering the trade-off between the risks and costs of using different energy generation technologies (see, for instance, Awerbuch, 2006; Awerbuch and Berger, 2003; Bazilian and Roques, 2008; Delarue et al., 2011; Doherty et al., 2006; Favre-Perrod et al., 2010; Liu and Wu, 2006; Mari, 2014; Marrero and Ramos-Real, 2010; Marrero et al., 2015; Roques et al., 2010; Roques et al., 2008; Shakouri et al., 2015). Usually the analogy with the financial market is to consider the random price per MWh of each technology instead of the returns of the assets, so that it is desired to minimize the expected cost of the energy portfolio for a given level of uncertainty obtained from the covariance matrix of the costs. Frequently these expected values and covariance matrix of the different energy technology costs are obtained from Monte Carlo simulations using the levelized cost of electricity (LCOE), which naturally yields to imprecision on these parameters.

To deal with the challenge of fostering a more reliable electricity mix, the main contribution of this paper is to workout the application of some of the results from robust asset portfolio theory (see for instance Costa and Paiva, 2002; Fabozzi et al., 2007a; Goldfarb and Iyengar, 2003; Kim et al., 2014; Lobo and Boyd, 2000; Fabozzi et al., 2007b) for electricity planning and policy-making. Similarly as considered in the robust financial portfolio literature, we assume that the expected costs and the covariance matrix for the different energy technologies are not exactly known but, instead, belong to some uncertainty sets (box, ellipsoidal, componentwise lower and upper bounds, and convex polytope defined by some known vertices). The motivation for that is, as pointed out above, Monte Carlo simulations are usually used for obtaining these parameters, which naturally yields to imprecision on them. Besides that, this approach gives room for the possible inclusion of different future scenarios for the expected energy costs and covariance matrices.

Three problems will be analyzed in this paper: the first one is to find an energy portfolio of minimum worst case volatility with guaranteed fixed maximum expected energy cost. The second one is to find an energy portfolio of minimum worst case expected cost with guaranteed fixed maximum volatility of the energy cost. The third one is a combination of the expected and variance of the cost, weighted by a risk aversion parameter. As in the robust financial portfolio literature (see for instance El Ghaoui et al., 2003; Fabozzi et al., 2007a,b; Kim et al., 2014) these problems can be written as quadratic, second order cone programming (SOCP) or semidefinite programming (SDP) (see the Appendix), so that the robust optimization numerical packages nowadays available for this class of problems can be used (see, for instance, Boyd and Vandenberghe, 2004). For the case in which the model distinguishes the energy coming from already existing plants (denoted by "old" energy) of the energy that comes from the new ones (denoted by "new" energy) the problems mentioned above can be simplified. In this situation all the old energy will be used in the energy portfolio so that any increase in size of each technology, must be with "new plants" (see for instance, Losekann et al., 2013), yielding to a reduction on the number of variables in the optimization problems.

This paper is organized in the following way: Section 2 presents the notation, basic results, and problem formulation that will be considered throughout the work. Sections 3 and 4 introduce the robust electricity energy mix optimization problems, the considered uncertainty sets, and the formulation of the robust portfolio optimization problems in terms of quadratic, SOCP or SDP optimization problems. Section 5 considers the situation in which all the "old" energy will be used in the energy portfolio so that any increase in size of each technology must be with "new plants", which yields to a reduction on the number of variables in the optimization problems. In Section 6 we illustrate the robust technique by analyzing the efficient Brazilian electricity energy mix considered in Losekann et al. (2013) with 8 energy technologies, classified as "new" energy and "old" energy. The paper is concluded in Section 7 with some final comments. We recall in the Appendix some basic facts on SDP and SOCP.

2. Preliminaries

2.1. Notation

We denote by $\mathbb{R}^m$ the m-dimensional euclidian space ($\mathbb{R} = \mathbb{R}^1$ for simplicity) and by $\|.\|_2$ the usual euclidian norm. We define by $\mathbf{1}$ the vector of appropriate dimension formed by 1 in all positions, and $'$ denotes the transpose of a vector or matrix. For symmetric matrices Q and R, and a matrix S, we write for notational simplicity $\begin{pmatrix} Q & S \\ \star & R \end{pmatrix} := \begin{pmatrix} Q & S \\ S' & R \end{pmatrix}$. By $P \succ 0$ ($P \succeq 0$ respectively) we mean that the symmetric matrix P is positive definite (positive semidefinite), and $P^{1/2}$ represents the square root matrix of P. For two matrices P and S with the same dimension we write $P > S$ (respectively $P \geq S$) if for each element of P and S we have $P_{ij} > S_{ij}$ ($P_{ij} \geq S_{ij}$). For real number x_i, $i = 1, \ldots, n$ we denote by $\text{diag}(x_i)$ the $n \times n$ diagonal matrix with the element x_i on the entry (i, i), and zero elsewhere. Let $\mathbb{X}$ be a space of real vectors or matrices. For a collection of points $v^i \in \mathbb{X}$, $i = 1, \ldots, \kappa$, we define the convex polytope $Con\{v^1, \ldots v^\kappa\}$ as

$$Con\left\{v^1, \ldots v^\kappa\right\} := \left\{v \in \mathbb{X}; v = \sum_{i=1}^{\kappa} \lambda^i v^i, \sum_{i=1}^{\kappa} \lambda^i = 1, \lambda^i \geq 0\right\}.$$

Finally the expected value of a random vector V will be denoted by $E(V)$, its covariance matrix by $Cov(V)$, and for V, U random vectors we define $Cov(V, U) = E((V - E(V))(U - E(U))')$. If V is a scalar random variable we set $Var(V)$ as the variance of V.

2.2. Mean variance theory

Harry Markowitz (1959) developed the Theory of Portfolio Selection (TPS) in the 1950s to answer the question of how a risk averse investor should allocate resources among different investments. According to his theory, the investor should consider the tradeoff between risk and return with risk being measured through the variance of asset returns. This model became a new paradigm in finance. Based on TPS approach, it is possible to construct an efficient frontier describing the optimal return for each possible level of risk. According to the investor's risk preference — or the investor's utility function — he or she will choose a point in the efficient frontier, and will obtain a specific portfolio. Formally, the mean variance portfolio problem is modeled through optimization. Assume the investor will choose a portfolio within a universe of m risky assets. Considering a decision vector ω representing the percentage of each asset held in the portfolio, and supposing the assets have expected returns given by the m-dimensional vector r and a $m \times m$ positive definite covariance matrix Ψ, the investor problem is a constrained minimization problem defined as

$$\min_{\omega} \quad \omega'\Psi\omega$$
$$\text{subject to } \omega' r = \tau, \quad \omega \in \Gamma.$$

Parameter τ represents the expected return that it is desired to be obtained by the investor, and Γ represents the set of constraints on the portfolio components. An equivalent approach consists of finding a portfolio of maximum expected return constrained to a fixed level of risk, and a portfolio that maximizes a combination of the expected minus the variance of the cost, weighted by a risk aversion parameter. The analogy with the electricity energy mix optimization will be explained in Section 2.4, and has been considered by many authors (see for instance, Awerbuch, 2006; Awerbuch and Berger, 2003; Bazilian and Roques, 2008; Delarue et al., 2011; Doherty et al., 2006; Favre-Perrod et al., 2010; Liu and Wu, 2006; Mari, 2014; Marrero and Ramos-Real, 2010; Marrero et al., 2015; Roques et al., 2010; Roques et al., 2008; Shakouri et al., 2015). It is important to stress however that none of these papers has taken into account parameter uncertainties in the model, as it will be proposed in this work.

2.3. The levelized cost of energy (LCOE)

Measuring costs is the first step to the understanding of electricity mix. A common and useful measure to compare different electricity generating technologies is the Levelized Cost of Energy (LCOE) (see for instance, Awerbuch and Berger, 2003) representing the cost per kilowatt-hour of building and operating a generating plant. It considers different costs for an electricity generating technology over its operating life, including land, infrastructure, operation, maintenance, fuels and other costs over a span of time. There is no consensus about LCOE calculation (Losekann et al., 2013; Marrero et al., 2015), and it can additionally include costs (or benefits) from carbon abatement, intermittence and dispatch characteristics.

The calculation of the LCOE is based on the equivalence of the present value of the sum of discounted revenues and the present value of the sum of discounted costs (see for instance, Marrero and Ramos-Real, 2010, Appendix A). Based on forecasts about cost future values, LCOE values present high uncertainty and can vary according to the region in a country and across time due to technologic evolution and fuel prices movements. Two distinct environments sets of power generation shall be considered when analyzing the Brazilian electric energy supply in Section 6: (I) electric energy from existing power plants ("old energy") and (II) electric energy from new power plants yet to be built ("new energy"). Old power plants already in operation usually have low total costs, while new power plants usually have high costs since they are still in the process of amortizing payments, which are mainly charged on their first years of operations. Additionally, old power plants are mostly owned by the federal government while many of the new ones can either be government owned, partially owned, or can have only private partners.

To model the electricity portfolio mix, each different electricity generation technology is characterized by the expected LCOE, together with the standard deviation on this cost. Assuming that the expected value and the covariance between different costs are known, it is possible to obtain optimal portfolios and to find the efficient frontier by solving one of two optimization problems: minimize risk or minimize cost, adding a restriction on either a fixed cost or a fixed risk (see Section 2.4). The investment costs, costs of operation and maintenance and fuel costs that constitute LCOE are random variables with high variability. Although it is possible to obtain forecasts of expected costs and covariance matrix, there is a huge uncertainty in these parameters. Practitioners analyzing energy policies must be aware of the importance of considering this uncertainty in their decisions.

2.4. Mean-variance electricity generation mix problems

In this subsection we present the mean-variance energy planning portfolio problems that will be considered in the paper. For this we consider the availability of m energy technologies, with the random m dimensional LCOE vector for the technologies costs (in USD/KWh) denoted by

$$C = \begin{pmatrix} C_1 \\ \vdots \\ C_m \end{pmatrix}. \tag{1}$$

For $i = 1, \ldots, m$, the components of the vector $\omega \in \mathbb{R}^m$ represent the weights on each technology, that is, the ith entry $\omega_i \geq 0$ of ω is the energy portfolio's proportion generated by the technology i. We denote by Γ the set of admissible electricity generation mix so that we must have $\omega \in \Gamma$. The set Γ will represent constraints like the sum of the portfolio components ω being equal to 1, and minimum and maximum values for the contributions of each technology ($\omega_i^{\min}$ and $\omega_i^{\max}$ respectively), that is, constraints of the form

$$\omega'\mathbf{1} = 1, \omega_i^{\min} \leq \omega_i \leq \omega_i^{\max}, \; i = 1, \ldots, m.$$

In what follows we define $r := E(C)$ and $Cov(C) := \Psi$ (that is, the positive semidefinite $m \times m$ covariance matrix of C). The random energy cost associated to a portfolio $\omega \in \Gamma$, denoted by $\mathcal{C}(\omega)$, is given by

$$\mathcal{C}(\omega) = \sum_{i=1}^{m} \omega_i C_i = \omega' C. \tag{2}$$

From Eq. (2) it follows that the expected cost is given by $E(\mathcal{C}(\omega)) = \omega' r$ and the variance $Var(\mathcal{C}(\omega)) = \omega'\Psi\omega$. For the case in which the vector of expected costs r and the covariance matrix Ψ are known, three kinds of mean-variance problems are usually considered in the literature. The first one is to minimize the variance of the energy cost conditional on a target maximum expected cost τ. By target expected cost we mean a positive real number τ provided by the energy policy planner which represents the maximum allowable expected energy cost. More formally, the problem can be written as

$$\min_{\omega} \omega'\Psi\omega$$
$$\text{subject to} \quad \omega' r \leq \tau, \;\; \omega \in \Gamma. \tag{3}$$

Another problem is to minimize the expected energy cost conditional on a maximum value ϑ^2 for the variance of the energy cost. The value ϑ^2, provided by the energy policy planner, represents the maximum value that the variance of the energy cost could achieve. Mathematically, this problem can be written as

$$\min_{\omega} \quad \omega' r$$
$$\text{subject to} \;\; \omega'\Psi\omega \leq \vartheta^2, \;\; \omega \in \Gamma. \tag{4}$$

A third problem is the minimization of a combination of the expected and variance of the cost, weighted by a risk aversion parameter $\lambda > 0$, a higher value of λ indicates a greater risk aversion. Mathematically it can be written as

$$\min_{\omega} \ \omega' r + \lambda \omega' \Psi \omega$$
$$\text{subject to } \omega \in \Gamma. \tag{5}$$

By using solvers for quadratic programming or quadratically constrained quadratic program (QCQP), the above problems can be solved for the case in which the vector of expected costs r and covariance matrix Ψ are assumed to be known.

3. Robust energy mix optimization

Fabozzi et al. (2007a) highlights that the mean variance portfolio model can be unreliable in practice as it is very sensitive to changes in inputs (the covariance matrix of asset returns and expected returns of assets). Estimation errors notably affect the resulting portfolios and uncertainties in parameters of these models can lead to a bad performance in practical applications. Two well-known approaches in optimization theory deal with parameter uncertainty: Stochastic Programming and Robust Optimization. The first one considers scenarios of realization of random inputs but, in practice, it suffers from the curse of dimensionality: the model size grows exponentially with the number of scenarios. The robust approach, on its turn, incorporates explicitly the uncertainty about the inputs in the optimization model through the definition of deterministic "uncertainty sets" as it will be presented in Section 4. The first step in robust optimization is to describe a model — what will be called here the original model — based on nominal values of the parameters. The uncertain parameters are then assumed to belong to a set and the resulting model is called the robust counterpart of the original model.

Following this robust approach, we consider in this section the case in which the vector of expected costs r and covariance matrix Ψ are not assumed to be known. According to the worst case approach as, for instance, presented in Goldfarb and Iyengar (2003), Kim et al. (2014), Lobo and Boyd (2000) and Tütüncü and Koenig (2004), it is assumed that $(r, \Psi) \in \mathcal{U}$, where $\mathcal{U} = \mathcal{X} \times \mathcal{Y}$ and $\mathcal{X}$ represents the uncertainty set for the expected cost r and $\mathcal{Y}$ represents the uncertainty set for the covariance matrix Ψ. We present the following definitions regarding the robustness properties we shall consider:

Definition 3.1. We say that a portfolio ω is robust with respect to a maximum expected energy cost τ if $\omega \in \Gamma$ and $\max_{r\in\mathcal{X}} \ r'\omega \le \tau$. Similarly we say that a portfolio ω is robust with respect to a maximum variance ϑ^2 if $\max_{\Psi\in\mathcal{Y}} \ \omega'\Psi\omega \le \vartheta^2$.

Three kinds of problem are considered, which can be seen as robust versions of problems (3), (4) and (5) respectively:

Definition 3.2. The MVGCU problem (minimum worst case variance, guaranteed maximum expected cost under uncertainty): Find a portfolio ω_τ such that it is robust with respect to a maximum expected energy cost τ and for any other portfolio ω robust with respect to the maximum expected energy cost τ we have

$$\max_{\Psi\in\mathcal{Y}} \ \omega_\tau' \Psi \omega_\tau \le \max_{\Psi\in\mathcal{Y}} \ \omega'\Psi\omega. \tag{6}$$

It is easy to see that the MVGCU problem is equivalent to the following min–max problem:

$$\min_{\omega\in\Gamma} \ \max_{\Psi\in\mathcal{Y}} \ \omega'\Psi\omega$$
$$\text{subject to } \max_{r\in\mathcal{X}} \ \omega' r \le \tau. \tag{7}$$

Definition 3.3. The MCGVU problem (minimum worst case expected energy cost, guaranteed variance under uncertainty): Find a portfolio ω_ϑ such that it is robust with respect to a maximum variance ϑ^2, and for any other portfolio ω robust with respect to the maximum variance ϑ^2, we have

$$\max_{r\in\mathcal{X}} \ r'\omega_\vartheta \le \max_{r\in\mathcal{X}} \ r'\omega. \tag{8}$$

As before, it is easy to see that the MCGVU problem is equivalent to the following min–max problem:

$$\min_{\omega\in\Gamma} \ \max_{r\in\mathcal{X}} \ \omega' r$$
$$\text{subject to } \max_{\Psi\in\mathcal{Y}} \ \omega'\Psi\omega \le \vartheta^2. \tag{9}$$

Finally the robust version of problem (5) (minimization of a combination of the expected and variance of the cost, weighted by a risk aversion parameter $\lambda > 0$) is as follows:

$$\min_{\omega\in\Gamma} \ \max_{(r,\Psi)\in\mathcal{U}} \ (\omega' r + \lambda\omega'\Psi\omega). \tag{10}$$

The goal of the next section is to write these 3 problems as quadratic, SCOP or SDP, depending on the uncertainty sets that is being considered, and also consider the case in which the cost can be decomposed into p independent cost categories following the LCOE methodology (see Section 2.3).

4. Uncertainty sets and numerical formulations

We present in this section some possible uncertainty sets $\mathcal{X}$ and $\mathcal{Y}$ and the respective numerical formulations for problems (7), (9) and (10).

4.1. Box and ellipsoidal uncertainty for r

We start by considering uncertainty sets for the vector of expected cost r. The simplest one is the so-called box uncertainty set (see for instance Lobo and Boyd, 2000), which is written as $\mathcal{X} := \{r \in \mathbb{R}^m; \underline{r}_i \le r_i \le \bar{r}_i, \ i = 1, \ldots, m\}$ for some vectors $\underline{r}, \ \bar{r} \in \mathbb{R}^m, \underline{r} \ge 0, \bar{r} \ge 0$. Since all the weights ω_i in the electricity energy mix are positive we have that

$$\max_{r\in\mathcal{X}} \ \omega' r = \omega'\bar{r}$$

and problems (7), (9), and (10), considering Ψ fixed, can be rewritten appropriately and solved via quadratic programming.

Another more interesting uncertainty set for r is the so-called ellipsoidal constraint set (see again Lobo and Boyd, 2000). In this case $\mathcal{X} := \left\{r \in \mathbb{R}^m; (r-\bar{r})' S^{-1}(r-\bar{r}) \le 1\right\}$ for some $S \succ 0$ and $\bar{r} \in \mathbb{R}^m$, $\bar{r} \ge 0$. For this case we have, after making the change of variable $\tilde{r} = S^{-1/2}(r - \bar{r})$, that

$$\max_{r\in\mathcal{X}} \ \omega' r = \omega'\bar{r} + \max_{\|\tilde{r}\|\le 1} \ \omega' S^{1/2}\tilde{r}. \tag{11}$$

Since the optimal solution for Eq. (11) is $\tilde{r}^* = \frac{S^{1/2}\omega}{\left\|S^{1/2}\omega\right\|}$ we get that

$$\max_{r\in\mathcal{X}} \ \omega' r = \omega'\bar{r} + \sqrt{\omega' S\omega} = \omega'\bar{r} + \left\|S^{1/2}\omega\right\|. \tag{12}$$

Problems (7) and (9) considering Ψ fixed, can be re-written as SOCP as follows. For problem (7), we have

$$\begin{aligned} &\min_{\rho\geq 0,\ \ \omega\in\Gamma} \ \rho \\ &\text{subject to } \left\|\Psi^{1/2}\omega\right\| \leq \rho, \quad \omega'\bar{r} + \left\|S^{1/2}\omega\right\| \leq \tau. \end{aligned} \tag{13}$$

Similarly for problem (9) we have

$$\begin{aligned} &\min_{\rho\geq 0,\ \ \omega\in\Gamma} \ \rho \\ &\text{subject to } \left\|\Psi^{1/2}\omega\right\| \leq \vartheta, \quad \omega'\bar{r} + \left\|S^{1/2}\omega\right\| \leq \rho. \end{aligned}$$

Finally for problem (10), considering Ψ fixed, we can write it, using the Schur complement (see Proposition 8.1 in the Appendix), as a SDP as follows:

$$\begin{aligned} &\min_{\beta\geq 0,\ \rho\geq 0,\ \ \omega\in\Gamma} \ \rho+\beta \\ &\text{subject to } \begin{bmatrix} \lambda^{-1}\rho & (\Psi\omega)' \\ \bigstar & \Psi \end{bmatrix} \succeq 0, \ \begin{bmatrix} \beta-\bar{r}'\omega & (S^{1/2}\omega)' \\ \bigstar & (\beta-\bar{r}'\omega)I \end{bmatrix} \succeq 0. \end{aligned}$$

4.2. Uncertainty sets for r and Ψ

In this subsection we consider uncertainty sets for both the vector of expected cost r and covariance matrix Ψ. For the componentwise bound uncertainty set case, we have for r, as in the previous subsection, that $\mathcal{X} = \left\{r\in\mathbb{R}^m; \underline{r}\leq r\leq\bar{r}\right\}$ and for the covariance matrix Ψ that $\mathcal{Y} := \left\{\Psi; \underline{\Psi}\leq\Psi\leq\bar{\Psi}\right\}$. Since the electricity energy mix vector ω must be non-negative, and considering $\bar{\Psi}\succeq 0$ we have, as shown in Proposition 3 of Tütüncü and Koenig (2004), that

$$\max_{r\in\mathcal{X}} \ \omega' r = \omega'\bar{r}, \ \max_{\Psi\in\mathcal{Y}} \ \omega'\Psi\omega = \omega'\bar{\Psi}\omega.$$

Thus in this case problems (7), (9) and (10) can be solved via quadratic programming and QCQP replacing, in Eqs. (3)–(5), r and Ψ by, respectively, $\bar{r}$ and $\bar{\Psi}$.

For the independent convex polytopic uncertainty set case for r and Ψ, consider a set of covariance matrices $\Psi^i\succeq 0$, $i=1,\ldots,\kappa$ and vector of costs $r^\ell\geq 0$, $\ell=1,\ldots,\kappa$ (for simplicity we assume that the number of vertices κ is the same for the vector of costs and covariance matrices). We consider in this case that

$$\mathcal{Y} := Con\left\{\Psi^1,\ldots,\Psi^\kappa\right\}, \quad \mathcal{X} := Con\left\{r^1,\ldots,r^\kappa\right\}.$$

We have that problems (7) and (9) can be re-written as SOCP as follows. For problem (7), define the following SOCP:

$$\begin{aligned} &\min_{\rho\geq 0,\ \ \omega\in\Gamma} \ \rho \\ &\text{subject to } \left\|(\Psi^\ell)^{1/2}\omega\right\| \leq \rho, \quad \omega' r^\ell \leq \tau, \quad \ell=1,\ldots,\kappa, \end{aligned} \tag{14}$$

Proposition 4.1. *(a) Problem (14) has a solution $(\hat{\rho},\hat{\omega})$ if and only if (b) the MVGCU problem (see Definition 3.2) has a solution ω_τ. Moreover, if (a) holds then $\omega_\tau=\hat{\omega}$ is a solution to the MVGCU problem and similarly if (b) holds then $(\hat{\rho},\hat{\omega})$ is a solution to problem (14) where $\hat{\omega}=\omega_\tau$ and $\hat{\rho}=\max_{\ell=1,\ldots,\kappa}\left\|(\Psi^\ell)^{1/2}\omega_\tau\right\|$.*

Proof. Let (ρ,ω) be any feasible solution for problem (14). Without loss of generality we can consider that $\rho=\max_{\ell=1,\ldots,\kappa}\left\|(\Psi^\ell)^{1/2}\omega\right\|$. Suppose that $(\hat{\rho},\hat{\omega})$ is an optimal solution for problem (14). Then $\max_{\ell=1,\ldots,\kappa}\omega' r^\ell = \max_{r\in\mathcal{X}}\ \omega' r\leq\tau$, $\max_{\ell=1,\ldots,\kappa}\hat{\omega}' r^\ell = \max_{r\in\mathcal{X}}\ \hat{\omega}' r\leq\tau$, and from the optimality of $(\hat{\rho},\hat{\omega})$ we have that $\hat{\rho}^{\,2} = \max_{\ell=1,\ldots,\kappa}\ \hat{\omega}'\Psi^\ell\hat{\omega} = \max_{\Psi\in\mathcal{Y}}\ \hat{\omega}'\Psi\hat{\omega}\leq\rho^2 = \max_{\ell=1,\ldots,\kappa}\ \omega'\Psi^\ell\omega = \max_{\Psi\in\mathcal{Y}}\ \omega'\Psi\omega$, showing that $\hat{\omega}$ is a solution for problem MVGCU. On the other hand, if ω_τ is a solution for the MVGCU problem then from Definitions 3.1 and 3.2, $\max_{\ell=1,\ldots,\kappa}\omega_\tau' r^\ell = \max_{r\in\mathcal{X}}\ \omega_\tau' r\leq\tau$ and for any other portfolio ω robust with respect to the maximum energy cost τ we have that $\max_{\ell=1,\ldots,\kappa}\omega' r^\ell = \max_{r\in\mathcal{X}}\ \omega' r\leq\tau$ and from Eq. (6), $\hat{\rho}^{\,2} = \max_{\ell=1,\ldots,\kappa}\omega_\tau'\Psi^\ell\omega_\tau = \max_{\Psi\in\mathcal{Y}}\ \omega_\tau'\Psi\omega_\tau\leq\max_{\Psi\in\mathcal{Y}}\ \omega'\Psi\omega = \max_{\ell=1,\ldots,\kappa}\omega'\Psi^\ell\omega$, showing that $(\hat{\rho},\omega_\tau)$ is an optimal solution for problem (14). □

For problem (9) define the following SOCP:

$$\begin{aligned} &\min_{\rho\geq 0,\ \ \omega\in\Gamma} \ \rho \\ &\text{subject to } \left\|(\Psi^\ell)^{1/2}\omega\right\| \leq \vartheta, \quad \omega' r^\ell \leq \rho, \quad \ell=1,\ldots,\kappa. \end{aligned} \tag{15}$$

The proof of the following proposition follows the same lines as the proof of Proposition 4.1 and will be omitted.

Proposition 4.2. *(a) Problem (15) has a solution $(\hat{\rho},\hat{\omega})$ if and only if (b) the MCGVU problem has a solution ω_ϑ. Moreover, if (a) holds then $\omega_\vartheta=\hat{\omega}$ is a solution to the MCGVU problem and similarly if (b) holds then $(\hat{\rho},\hat{\omega})$ is a solution to problem (15) where $\hat{\omega}=\omega_\vartheta$ and $\hat{\rho}=\max_{\ell=1,\ldots,\kappa}\omega' r^\ell$.*

For problem (10) with the independent polytopic uncertainty case $\mathcal{U}=\mathcal{X}\times\mathcal{Y}$ considered above, we have that it can be written as the following QCQP

$$\begin{aligned} &\min_{\rho\geq 0,\ \ \beta\geq 0,\ \ \omega\in\Gamma} \ \rho+\beta \\ &\text{subject to } \omega'\Psi^\ell\omega \leq \frac{\rho}{\lambda}, \quad \omega' r^\ell \leq \beta, \quad \ell=1,\ldots,\kappa, \end{aligned}$$

and for the joint polytopic uncertainty case $\mathcal{U} := Con\left\{\left(\Psi^1\ \ r^1\right),\ldots,\left(\Psi^\kappa\ \ r^\kappa\right)\right\}$ it can be written as the following QCQP:

$$\begin{aligned} &\min_{\rho\geq 0,\omega\in\Gamma} \ \rho \\ &\text{subject to } \omega' r^\ell + \lambda\omega'\Psi^\ell\omega \leq \rho, \quad \ell=1,\ldots,\kappa, \end{aligned} \tag{16}$$

4.3. Decomposition of the costs

According to the LCOE method (see Section 2.3), the costs C_i can be written as the sum of p categories (usually fuel costs, operation and maintenance (OM) costs, investment costs, emission costs and intermittence costs), so that

$$C = \sum_{j=1}^{p} V^j, \ V^j = \begin{pmatrix} V_1^j \\ \vdots \\ V_m^j \end{pmatrix}, \ j=1,\ldots,p, \tag{17}$$

with $\left\{V^j; j=1,\ldots,p\right\}$ independent random vectors. Defining $v^j = E\left(V^j\right)$ and $\Psi^j = Cov\left(V^j\right)$ we have from Eq. (17) that $r=\sum_{j=1}^p v^j$ and, from independence of the random vectors V^j, that $\Psi=\sum_{j=1}^p\Psi^j$. For the box and ellipsoidal uncertainty for

v^j seen in Section 4.1 we would have, considering first $\mathcal{X} = \left\{r = \sum_{j=1}^{p} v^j; v^j \in \mathbb{R}^m, \underline{v}^j \leq v^j \leq \bar{v}^j,\ j = 1, \ldots, p\right\}$ for some vectors $\underline{v}^j,\ \bar{v}^j \in \mathbb{R}^m, \underline{v}^j \geq 0, \bar{v}^j \geq 0$, that

$$\max_{r \in \mathcal{X}} \ \omega' r = \omega' \bar{r}, \ \ \bar{r} = \sum_{j=1}^{p} \bar{v}^j$$

and again problems (7), (9), and (10) can be re-written appropriately and solved via quadratic programming or QCQP. For the ellipsoidal constraint set consider $\mathcal{X} = \left\{r = \sum_{j=1}^{p} v^j; v^j \in \mathbb{R}^m, \left(v^j - \bar{v}^j\right)' \left(S^j\right)^{-1} \left(v^j - \bar{v}^j\right) \leq 1\right\}$ for $S^j \succ 0$ and $\bar{v}^j \in \mathbb{R}^m, \bar{v}^j \geq 0$. For this case we have, after making the change of variable $\tilde{v}^j = (S^j)^{-1/2}(v^j - \bar{v}^j)$, that

$$\max_{r \in \mathcal{X}} \ \omega' r = \max_{r \in \mathcal{X}} \ \omega' \left(\sum_{j=1}^{p} v^j\right) = \omega' \bar{r} + \sum_{j=1}^{p} \max_{\|\tilde{v}^j\| \leq 1} \ \omega' (S^j)^{1/2} \tilde{v}^j, \ \ \bar{r} = \sum_{j=1}^{p} \bar{v}^j. \tag{18}$$

As in Eq. (11) we get that $\max_{\|\tilde{v}^j\| \leq 1} \ \omega' (S^j)^{1/2} \tilde{v}^j = \|(S^j)^{1/2}\omega\|$ and thus from Eq. (18),

$$\max_{r \in \mathcal{X}} \ \omega' r = \omega' \bar{r} + \sum_{j=1}^{p} \|(S^j)^{1/2}\omega\| \ . \tag{19}$$

Again problems (7) and (9) considering Ψ fixed, can be re-written as SOCP as follows. For problem (7), we have

$$\begin{aligned} &\min_{\rho \geq 0,\ \xi^j \geq 0,\ \ \omega \in \Gamma} \ \rho \\ &\text{subject to } \left\|\Psi^{1/2}\omega\right\| \leq \rho, \ \ \left\|(S^j)^{1/2}\omega\right\| \leq \xi^j, j = 1, \ldots, p, \ \ \omega' \bar{r} + \sum_{j=1}^{p} \xi^j \leq \tau, \end{aligned}$$

and for problem (9) we have

$$\begin{aligned} &\min_{\rho \geq 0,\ \xi^j \geq 0,\ \ \omega \in \Gamma} \ \rho \\ &\text{subject to } \left\|\Psi^{1/2}\omega\right\| \leq \vartheta, \ \ \left\|(S^j)^{1/2}\omega\right\| \leq \xi^j, j = 1, \ldots, p, \ \ \omega' \bar{r} + \sum_{j=1}^{p} \xi^j \leq \rho. \end{aligned}$$

Problem (10) can be re-written as a SDP as follows:

$$\begin{aligned} &\min_{\beta \geq 0, \rho \geq 0,\ \xi^j \geq 0,\ \omega \in \Gamma} \ \rho + \beta \\ &\text{subject to } \begin{bmatrix} \lambda^{-1}\rho & (\Psi\omega)' \\ \star & \Psi \end{bmatrix} \succeq 0, \ \begin{bmatrix} \xi^j & ((S^j)^{1/2}\omega)' \\ \star & \xi^j I \end{bmatrix} \succeq 0, \\ &j = 1, \ldots, p, \ \ \omega' \bar{r} + \sum_{j=1}^{p} \xi^j \leq \beta. \end{aligned}$$

For the componentwise bound uncertainty sets for r and Ψ as in Section 4.2, consider $\mathcal{X}$ as the box uncertainty set presented above, and $\mathcal{Y} := \left\{\Psi = \sum_{j=1}^{p} \Psi^j, \underline{\Psi}^j \leq \Psi^j \leq \bar{\Psi}^j\right\}$. Again recalling that the electricity energy mix vector ω must be non-negative, and considering $\bar{\Psi}^j \succeq 0$ for each $j = 1, \ldots, p$, we have from Proposition 3 of Tütüncü and Koenig (2004) that

$$\max_{r \in \mathcal{X}} \ \omega' r = \omega' \bar{r}, \ \ \bar{r} = \sum_{j=1}^{p} \bar{v}^j, \quad \max_{\Psi \in \mathcal{Y}} \ \omega' \Psi \omega = \omega' \bar{\Psi} \omega, \ \ \bar{\Psi} = \sum_{j=1}^{p} \bar{\Psi}^j.$$

As before, problems (7), (9) and (10) can be solved via quadratic programming or QCQP replacing, in Eqs. (3)–(5), r and Ψ by, respectively, $\bar{r}$ and $\bar{\Psi}$. For the convex polytopic uncertainty set case, consider a set of covariance matrices $\Psi^{\ell j} \succeq 0$ and vector of costs $v^{\ell j} \geq 0$, $\ell = 1, \ldots, \kappa$, $j = 1, \ldots, p$ (as before, for simplicity, we assume that the number of vertices κ is the same for the vector of costs and covariance matrices of all the p cost categories). We consider in this case that $\mathcal{U} = \mathcal{X} \times \mathcal{Y}$ with

$$\begin{aligned} \mathcal{Y} &:= \left\{\Psi = \sum_{j=1}^{p} \Psi^j; \Psi^j \in Con\left\{\Psi^{1j}, \ldots, \Psi^{\kappa j}\right\}\right\}, \\ \mathcal{X} &:= \left\{r = \sum_{j=1}^{p} v^j; v^j \in Con\left\{v^{1j}, \ldots, v^{\kappa j}\right\}\right\}. \end{aligned}$$

We have that problems (7) and (9) can be re-written as SOCP as follows. For problem (7) we have the following SOCP:

$$\begin{aligned} &\min_{\rho \geq 0,\ \ \omega \in \Gamma} \ \rho \\ &\text{subject to } \left\|\left(\sum_{j=1}^{p} \Psi^{\ell_j j}\right)^{1/2} \omega\right\| \leq \rho, \ \ \omega' \left(\sum_{j=1}^{p} v^{\ell_j j}\right) \leq \tau, \\ &j = 1, \ldots, p, \ell_j = 1, \ldots, \kappa. \end{aligned} \tag{20}$$

As in Proposition 4.1, problem (20) has a solution $(\hat{\rho}, \hat{\omega})$ if and only if the MVGCU problem (see Definition 3.2) has a solution ω_τ. This follows from the same arguments as in the proof of Proposition 4.1, only noticing that now for any $\omega \in \Gamma$, $\max_{r \in \mathcal{X}} \ \omega' r = \sum_{j=1}^{p} \max_{\ell = 1, \ldots, \kappa} \omega' v^{\ell j} = \sum_{j=1}^{p} \omega' v^{\ell_j j}$ for some $\ell_j \in \{1, \ldots, \kappa\}$ and similarly $\max_{\Psi \in \mathcal{Y}} \ \omega' \Psi \omega = \sum_{j=1}^{p} \max_{\ell = 1, \ldots, \kappa} \omega' \Psi^{\ell j} \omega = \sum_{j=1}^{p} \omega' \Psi^{\ell_j j} \omega = \left\|\left(\sum_{j=1}^{p} \Psi^{\ell_j j}\right)^{1/2} \omega\right\|^2$ for some $\ell_j \in \{1, \ldots, \kappa\}$. For problem (9) define the following SOCP:

$$\begin{aligned} &\min_{\rho \geq 0,\ \ \omega \in \Gamma} \ \rho \\ &\text{subject to } \left\|\left(\sum_{j=1}^{p} \Psi^{\ell_j j}\right)^{1/2} \omega\right\| \leq \vartheta, \ \ \omega' \left(\sum_{j=1}^{p} v^{\ell_j j}\right) \leq \rho, \\ &j = 1, \ldots, p, \ \ell_j = 1, \ldots, \kappa. \end{aligned} \tag{21}$$

As in the Proposition 4.2 and the same arguments as above, problem (21) has a solution $(\hat{\rho}, \hat{\omega})$ if and only if the MCGVU problem has a solution ω_ϑ. Finally for problem (10) with the independent polytopic uncertainty case $\mathcal{U} = \mathcal{X} \times \mathcal{Y}$ considered above, we have that it can be written as the following QCQP:

$$\begin{aligned} &\min_{\rho \geq 0,\ \ \beta \geq 0,\ \ \omega \in \Gamma} \ \rho + \beta \\ &\text{subject to } \omega' \left(\sum_{j=1}^{p} \Psi^{\ell_j j}\right) \omega \leq \frac{\rho}{\lambda}, \ \ \omega' \left(\sum_{j=1}^{p} v^{\ell_j j}\right) \leq \beta, \\ &j = 1, \ldots, p, \ \ell_j = 1, \ldots, \kappa \end{aligned}$$

and for the joint polytopic uncertainty case $\mathcal{U}^j := Con\left\{\begin{pmatrix}\Psi^{1j} & r^{1j}\end{pmatrix}, \ldots, \begin{pmatrix}\Psi^{\kappa j} & r^{\kappa j}\end{pmatrix}\right\}$, $\mathcal{U} := \left\{\begin{pmatrix}\Psi & r\end{pmatrix} = \sum_{j=1}^{p} \begin{pmatrix}\Psi^j & r^j\end{pmatrix}; \begin{pmatrix}\Psi^j & r^j\end{pmatrix} \in \mathcal{U}^j\right\}$, it can be written as the following QCQP:

$$\begin{aligned} &\min_{\rho \geq 0, \omega \in \Gamma} \ \rho \\ &\text{subject to } \omega' \left(\sum_{j=1}^{p} v^{\ell_j j}\right) + \lambda \omega' \left(\sum_{j=1}^{p} \Psi^{\ell_j j}\right) \omega \leq \rho, \ \ j = 1, \ldots, p, \\ &\ell_j = 1, \ldots, \kappa. \end{aligned} \tag{22}$$

5. Old and new energy

In this section we consider as, for instance, in Losekann et al. (2013), and Awerbuch and Berger (2003), that the model distinguishes the energy coming from already existing plants (denoted by "old" energy) of the energy that comes from the projects to be constructed (denoted by "new" energy). It will be assumed, as in Losekann et al. (2013), that all the old energy will be used in the energy portfolio and thus, any increase in size of each technology, must be with "new plants". Under these assumptions the optimization problems considered in Eqs. (3)–(5) can be simplified. In Remark 5.1 we point out that this will not represent any loss of generality since we could consider in the optimization problem all the technologies as "new" energy and recast the same framework as in previous sections. More formally suppose that the random vector of costs C can be decomposed into the random vector for the technologies costs for the old energy C^o and the new energy C^n as follows:

$$C = \begin{pmatrix} C^o \\ C^n \end{pmatrix} \text{ with } C^o = \begin{pmatrix} C_1^o \\ \vdots \\ C_N^o \end{pmatrix} \text{ and } C^n = \begin{pmatrix} C_1^n \\ \vdots \\ C_N^n \end{pmatrix}, \tag{23}$$

and thus $m = 2N$. We decompose the vector ω as $\omega = \begin{pmatrix} \omega^o \\ \omega^n \end{pmatrix}$, where for $i = 1, \ldots, N$, the components of the vector ω^o represent the weights on each "old" technology, that is, the ith entry $\omega_i^o \geq 0$ of ω^o is the energy portfolio's proportion generated by the "old" technology i. Similarly the components of the vector ω^n represent the weights on each "new" technology, that is, the ith entry $\omega_i^n \geq 0$ of ω^n is the energy portfolio's proportion generated by the "new" technology i. As mentioned above, ω^o will be assumed to be fixed and the decision vector for the optimization problem will be given by ω^n. Clearly we must have $\sum_{i=1}^N (\omega_i^o + \omega_i^n) = 1$. Define $d = 1 - \sum_{i=1}^N \omega_i^o$, $r^o = E(C^o)$, $\zeta = \omega^{o\prime} r^o$, $r^n = E(C^n)$, $\Psi^o = Cov(C^o)$, $\Psi^n = Cov(C^n)$, $\Psi^{on} = Cov(C^o, C^n)$ and the matrix $\Psi = \begin{pmatrix} \Psi^o & \Psi^{on} \\ \star & \Psi^n \end{pmatrix}$. It is easy to see that $Cov(C) = \Psi$. As before, the set Γ represents the constraints for the sum of the portfolio components ω^n is equal to d and minimum and maximum values for the contributions of each new energy technology (ω_i^n min and ω_i^n max respectively), that is, constraints of the form $(\omega_i^n)'1 = d$, $\omega_i^n \min \leq \omega_i^n \leq \omega_i^n \max$, $i = 1, \ldots, N$. The random energy cost associated to a portfolio ω^n, denoted by $\mathcal{C}(\omega^n)$, is given by

$$\mathcal{C}(\omega^n) = \sum_{i=1}^N (\omega_i^o C_i^o + \omega_i^n C_i^n). \tag{24}$$

From Eq. (24) we have that

$$E(\mathcal{C}(\omega^n)) = \omega^{o\prime} r^o + \omega^{n\prime} r^n = \zeta + \omega^{n\prime} r^n, \tag{25}$$

and

$$\begin{aligned} Var(\mathcal{C}(\omega^n)) &= \omega' Cov(C)\omega = \begin{pmatrix} \omega^{o\prime} & \omega^{n\prime} \end{pmatrix} \Psi \begin{pmatrix} \omega^o \\ \omega^n \end{pmatrix} \\ &= \omega^{o\prime} Cov(C^o)\omega^o + 2\omega^{o\prime} Cov(C^o, C^n)(\omega^n) + \omega^{n\prime} Cov(C^n)\omega^n \\ &= \omega^{o\prime}\Psi^o\omega^o + 2\omega^{o\prime}\Psi^{on}\omega^n + (\omega^n)'\Psi^n\omega^n \\ &= c + b'\omega^n + \omega^{n\prime}\Psi^n\omega^n \end{aligned} \tag{26}$$

where $c = \omega^{o\prime}\Psi^o\omega^o$ and $b = 2\Psi^{on}\omega^o$. From Eqs. (25) and (26) we have that problem (3) can be written as

$$\begin{aligned} &\min_{\omega^n \in \Gamma} \ \omega^{n\prime}\Psi^n\omega^n + b'\omega^n + c \\ &\text{subject to} \quad \zeta + \omega^{n\prime} r^n \leq \tau, \end{aligned} \tag{27}$$

problem (4) as

$$\begin{aligned} &\min_{\omega^n \in \Gamma} \ \omega^{n\prime} r^n + \zeta \\ &\text{subject to} \quad \omega^{n\prime}\Psi^n\omega^n + b'\omega^n + c \leq \vartheta^2, \end{aligned} \tag{28}$$

and problem (5) as

$$\min_{\omega^n \in \Gamma} \ \omega^{n\prime} r^n + \zeta + \lambda(\omega^{n\prime}\Psi^n\omega^n + b'\omega^n + c). \tag{29}$$

Robust versions for problems (27)–(29) as presented in Sections 3 and 4 with respect to uncertainties on the expected value vectors r^o, r^n, and covariance matrix Ψ can be readily formulated.

Remark 5.1. Notice that the case in which there is no "old" energy we would have $\omega_i^o = 0$ so that $\zeta = 0$, $d = 1$, $b = 0$, $c = 0$, and problems (27)–(29) would recover problems (3)–(5), corresponding to the usual situation in which all the technologies could be changed in the portfolio optimization problems.

Remark 5.2. Suppose, as in Section 4.3, that the random vector of costs C can be decomposed as in Eq. (17) and that, similarly, C^o, C^n can be written as

$$C^o = \sum_{j=1}^p V^{oj}, \ V^{oj} = \begin{pmatrix} V_1^{oj} \\ \vdots \\ V_N^{oj} \end{pmatrix}, \ C^n = \sum_{j=1}^p V^{nj}, \ V^{nj} = \begin{pmatrix} V_1^{nj} \\ \vdots \\ V_N^{nj} \end{pmatrix}, \ j = 1, \ldots, p. \tag{30}$$

Since V_i^{oj} and V_i^{nj} represent the same category of cost it would be reasonable to consider that the correlation factor between V_i^{oj} and V_i^{nj}, for $i = 1, \ldots, N$, $j = 1, \ldots, p$ would be equal to 1. Under this assumption we would have that with probability one (see Proposition 1.1.2 in Davis and Vinter, 1985),

$$V_i^{oj} = \frac{\sigma_i^{oj}}{\sigma_i^{nj}}\left(V_i^{nj} - r_i^{nj}\right) + r_i^{oj} \tag{31}$$

where σ_i^{oj} and σ_i^{nj} $\left(r_i^{oj} \text{ and } r_i^{nj}\right)$ denote the standard deviation (expected value) of the random variables V_i^{oj} and V_i^{nj} respectively. Eq. (31) essentially says that there is only one source of uncertainty between the costs V_i^{oj} and V_i^{nj}, that is, between the cost of the "old" and "new" energy for each cost category j. Define $\Psi^{oj} = Cov(V^{oj})$, $\Psi^{nj} = Cov(V^{nj})$, $\Psi^{onj} = Cov(V^{oj}, V^{nj})$ and the diagonal matrix $D^j := \text{diag}\left(\frac{\sigma_i^{oj}}{\sigma_i^{nj}}\right)$, $j = 1 \ldots, p$. From Eq. (31) we have that Ψ^o, Ψ^n and Ψ^{on} can be directly evaluated in terms of Ψ^{nj} and D^j as follows:

$$\Psi^o = \sum_{j=1}^p \Psi^{oj}, \quad \Psi^{oj} = D^j\Psi^{nj}D^j, \quad \Psi^{on} = \sum_{j=1}^p \Psi^{onj}, \quad \Psi^{onj} = D^j\Psi^{nj},$$
$$\Psi^n = \sum_{j=1}^p \Psi^{nj}.$$

6. Brazilian generation mix expansion

6.1. Mean variance data for energy portfolios in Brazil

Brazilian electric energy mix includes renewable sources, with the predominant role of hydroelectricity. The reservoirs in southeast

Brazil, which accounts for 70% of the water storage capacity of the country, depleted to 16% of its maximum capacity in November 2014, the worst historical value. Besides, the southeast reservoirs were, on average, at 30% of their maximum capacity over the years 2014 and 2015. So, the total contribution of thermal power plants that were around 14% on average in the last 16 years, sharply increased to 25% on average in the last 4 years. As new hydropower plants in Brazil do not have significant reservoirs to regulate capacity it seems that thermoelectric power plants will be a trend in the future. The adjustment capacity — measured by the ratio of the total storage capacity and system load — of the Brazilian reservoirs dropped from six to five months over the past 10 years and is expected to drop for four months by 2020. The smaller storage capacity of the reservoirs turns investment in other energy sources necessary, in order to attend the high growth of demand with safety and reliability. In this context, natural gas-fired power plant is far appropriate. Energy planners face the problem of how to achieve equilibrium between security, stability and price, i.e., how much electric energy should be required from each available source. As the variable cost of a thermo-power plant is essentially the cost of the fuel, when purchasing electric energy from a thermo-power plant the question of how to compare the different sources of fuel, i.e., a natural gas-fired power plant with oil-fired power plant, must also be addressed. To deal with the optimal Brazilian mix selection problem under cost risks, a quantitative approach based on the TPS was considered in Losekann et al. (2013) with 8 energy technologies, each one classified as “old” energy and “new” energy.

In this section we re-visit, from the robust optimization perspective, the efficient Brazilian mix generating portfolio analyzed in Losekann et al. (2013). The data considered in this subsection is based on the expected and variance of the costs presented in section 4 of Losekann et al. (2013), obtained from the LCOE method for three possible CO_2 emissions costs (none, intermediate and high emission costs) and parameters as in Table A1 in Losekann et al. (2013). It should be pointed out that, as in Losekann et al. (2013), it is not included in the present analysis the volatility of hydropower production. Due to the high volatile hydrology in Brazil, the hydropower “operating cost” should include the opportunity cost of water in the hydropower reservoir. As this requires a complex calculation, this cost was not considered in the analysis and, as aforementioned, we considered the same data as in Losekann et al. (2013). But it is worth mentioning that the inclusion of this cost would be an interesting point to be analyzed in the future as hydro generation volatility is one of the main characteristics of the Brazilian electricity system.

According to the “2024 Decennial Plan for Energy Expansion (DPEE2024)”, see Ministerio de Minas e Energia (MME) and Empresa de Pesquisa Energética (EPE) (2015), the weights of the “old” energy (vector ω^o) were updated as shown in Table 1. The fuel cost correlation matrix, denote by *CorrFuel*, and OM cost correlation matrix, denoted by *CorrOM*, are presented in Tables 2 and 3, following Awerbuch and Berger (2003).

The means and standard deviations for the technology costs in Table 4 are based on the data presented in section 4 of Losekann et al. (2013) for the zero emission CO_2 cost case (the values are in cents of USD/KWh). We will refer to these values as the nominal case. The following restrictions on the expansion of the generation capacity, due to technical or energy planning reasons, were considered: for the “new” nuclear energy it was set to 1% (total of 2%), wind to 11.54% (total of 14%), hydro to 33.05% (total of 80%), and small hydro to 3% (total of 5.2%). Solar energy was not included because it still represents less than 1% of the Brazilian electric energy mix.

Table 1
Technology mix for the old energy.

	Gas	Coal	Nuclear	Oil	Biomass	Hydro	Wind	Small hydro
$\omega^{o\prime}$	5.87%	1.53%	1.00%	2.42%	5.56%	44.95%	2.46%	2.73%

6.2. Scenarios for the convex polytopic uncertainty

In this subsection we present the scenarios that will be used for the convex polytopic uncertainty for r and Ψ discussed in Section 4.2, using the approach for old and new energy introduced in Section 5. Besides the nominal case described above, we considered two other scenarios regarding the expected costs and variance of the costs (thus $\kappa = 3$). They reflect some of the possible paths of the Brazilian energy sector according to the authors view. For the numerical values we follow a qualitative rather than quantitative approach, classifying the percentual changes with respect to the nominal values of the expected costs and volatilities into the categories “moderate” (variation of 5% or 10%), “intermediate” (variation of 20% or 25%) and “substantial” (variation of 30% or 40%). We would like to stress that the scenario generations choice was adopted by the authors bearing in mind the opinion of specialists in the field. It will be up to each analyst who will replicate the model to propose his own methodology for the scenario generation and thus obtain his own conclusions. The first scenario is related to a reduction on the costs and volatility for the natural gas and an increase on the costs and volatility for the large hydro, and the second one is related to an increase on the costs and volatility for the natural gas and coal, and moderate reduction on the costs and volatility of eolic and biomass energies.

Scenario 1:

1.1) A review of the Brazilian regulatory market for natural gas, with neither the obligation of the participation of the Brazilian petrol company (Petrobras) in new pre-salt projects nor local content obligation, would yield to a “substantial” reduction on the price and volatility of the natural gas, and an increase of supply. This regulatory review would be justified by the fact that the gas exploration blocks held by Petrobras would be of high risk and low profitability, in an environment of low petrol barrel prices. Since Petrobras would not have resources to increase investments, the review of the Brazilian regulatory market for natural gas would allow the participation of international companies, which would increase competition, burst investments, and so decrease production costs. Besides, through the import of goods and services, the competition would grow, and the costs would reduce as well. It is the authors view that this scenario would yield to a “substantial” fall in the Brazilian natural gas price, being approximately similar to the percentual difference between Petrobras 2016 natural gas average price to Brazilian market (Ministério de Minas e Energia, 2017) and the U.S. Natural Gas Electric Power Price (U.S. Energy Information Administration, 2017). In our simulations, under this scenario, we considered a 30% reduction of the LCOE for the natural gas (r_1^n). The volatility of natural gas prices in Brazil is high because there are cross subsidies, for example, the difference in values between beneficiary sectors (such as thermoelectric plants in the Thermoelectric Priority Program — PPT) and without subsidies (such as new thermoelectric plants) is up to 3 times. By opening of the market, the authors would expect that in the long term the subsidies would be eliminated, and the natural gas market for electricity would work with just one market price, and thus, with a similar market price volatility as the one in USA. Bearing this in mind, we considered for the numerical simulations a “substantial” 40% reduction for the volatility (σ_1^n).

Table 2
Fuel cost correlation matrix.

CorrFuel	Gas	Coal	Nuclear	Oil	Biomass	Hydro	Wind	Small hydro
Gas	1.00	0.47	0.06	0.49	−0.44	0	0	0
Coal	0.47	1.00	0.12	0.27	−0.38	0	0	0
Nuclear	0.06	0.12	1.00	0.08	−0.22	0	0	0
Oil	0.49	0.27	0.08	1.00	−0.17	0	0	0
Biomass	−0.44	−0.38	−0.22	−0.17	1.00	0	0	0
Hydro	0	0	0	0	0	1	0	0
Wind	0	0	0	0	0	0	1	0
Small hydro	0	0	0	0	0	0	0	1

Table 3
OM cost correlation matrix.

CorrOM	Gas	Coal	Nuclear	Oil	Biomass	Hydro	Wind	Small hydro
Gas	1	0.2500	0.2400	0.0900	0.3200	−0.0400	0	0.0500
Coal	0.2500	1	0	−0.1800	0.1800	0.0300	−0.2200	−0.3900
Nuclear	0.2400	0	1	−0.1700	0.6500	−0.4100	−0.0700	0.3500
Oil	0.0900	−0.1800	−0.1700	1	0.0100	−0.2700	−0.5800	−0.0400
Biomass	0.3200	0.1800	0.6500	0.0100	1	−0.1800	−0.1800	0.2500
Hydro	−0.0400	0.0300	−0.4100	−0.2700	−0.1800	1	0.2900	0.3000
Wind	0	−0.2200	−0.0700	−0.5800	−0.1800	0.2900	1	0.0500
Small hydro	0.0500	−0.3900	0.3500	−0.0400	0.2500	0.3000	0.0500	1

1.2) The Brazilian government continues the expansion of new hydro power plants in the Amazon region, which may lead to delays on constructions (and capital expenditure overruns) caused by environmental, social and indigenous issues as capex overrun. Besides, the increasing environmental restrictions of hydraulic energy projects demand that their reservoirs hold small volumes in relation to the river flow, hence these power plants are of the run-of-the-river types, which increase the risk of non-attendances of supply contracts during the operational period. According to studies of the Brazilian government court of accounts (Tribunal de Contas da União, 2015), among the generation and transmission projects granted between 2005 and 2012, on average, 76% did not meet the schedule and the average delay in the construction of the hydroelectric plants is 8 months. The most critical cases of delay were hydroelectric projects located in the Amazon region, especially the Belo Monte Hydroelectric Power Plant, whose full energization had a delay of more than 2 years of the original schedule. Specialists from the electricity sector suggest that for each month of construction delay, the LCOE would need to be around 1% higher. According to our methodology this would correspond to a "moderate" increase of the costs, so that for our simulations we assumed an increase of 5% for the LCOE of large hydro (r_6^n). On the other hand the delay in the Belo Monte Hydroelectric Power Plant indicates that delays may cause a "substantial" increase on the volatility of the costs, so that in our simulations we adopted under this scenario a 30% increase of the volatility for the LCOE of large hydro (σ_6^n).

Table 4
Weights for the "old" energy, expected costs and standard deviation.

Fuel	Weight	Mean value (r_i^o and r_i^n)	Standard deviation (σ_i^o and σ_i^n)
Gas old	5.87%	9.9010	0.1500
Gas new	–	9.2770	0.1500
Coal old	1.53%	11.5560	0.1125
Coal new	–	11.1180	0.1187
Nuclear old	1.00%	10.1260	0.0625
Nuclear new	–	10.0110	0.1500
Fuel oil old	2.42%	19.0980	0.2250
Fuel oil new	–	16.4680	0.2188
Biomass old	5.56%	14.0390	0.0813
Biomass new	–	13.4560	0.0875
Hydro old	44.95%	4.1200	0.0313
Hydro new	–	5.0240	0.2062
Wind old	2.46%	10.9860	0.0250
Wind new	–	10.4440	0.1187
Small hydro old	2.73%	6.8850	0.0187
Small hydro new	–	6.9090	0.1187

Scenario 2:

2.1) Petrobras continues as the sole operator of the blocks of the pre-salt, and main investor due to regulatory barriers, so, without credit conditions to finance the necessary investments to achieve the growth demand, it is not possible to use all natural gas from the pre-salt plants. As a consequence, the expansion of gas-fired thermoelectric power plants would depend on the importation of LNG. Due to that the fuel cost will rely on international prices in foreign currency, which yields to an increase of the LCOE for the natural gas. To supply the growing demand, it will be also necessary to construct new coal-fired power plants but, however, as domestic coal is of poor quality, it will be necessary to import this fuel. Again, the fuel cost will depend on international prices in foreign currency, which means higher costs and greater volatility. From the authors point of view, this dependence on the fuel importation would yield to a "intermediate" increase on the LCOE for the natural gas and coal expected costs and volatility, reflecting the Brazilian exchange rate variation. For our numerical simulations and according to our methodology we adopted an increase of 20% for the LCOE expected cost for gas (r_1^n), 25% for coal (r_2^n), and of 25% for the gas and coal volatilities (σ_1^n and σ_2^n).

2.2) According to the 2014 IRENA report (International Renewable Energy Agency, 2014), the global weighted average LCOE of wind has fallen by 7% between 2010 and 2014. Bearing this data in mind, we considered a scenario that technological advances, fiscal incentives and government regulation for the renewable energies would yield to a "moderate" reduction on the LCOE expected costs and volatility for the eolic and biomass energies. For our numerical simulations and according to our methodology we adopted a reduction of 5% for the expected costs of the eolic (r_7^n) and biomass energies (r_5^n) and a reduction of 10% for their volatilities (σ_7^n and σ_5^n).

6.3. Convex polytopic uncertainty

In Fig. 1 we present the efficient frontier corresponding to the problem of minimizing the variance of the cost for the nominal data (problem (3) with just 1 scenario for the expected costs and variances as in Table 4)) in dashed line and the robust problems (14) and (16) with $\kappa = 3$, that is, 3 scenarios for the expected costs and 3 for the variances, one of them being the nominal case, the others as indicated in scenarios 1 and 2 above, in solid line for problem (14) and star line for problem (16). Notice that for problem (14) it was considered the independent convex polytopic uncertainty set case for r and Ψ, while for problem (16) it was considered the joint dependence for r and Ψ. In Fig. 1 and problem (16) we varied the risk trade-off parameter λ from 200 up to 1500.

As expected the robust efficient frontiers will be more conservative since it takes into account possible future scenarios for the parameters that are not considered for the nominal case. For lower volatilities we can see from Fig. 1 that the expected costs are very close for all the cases, while for higher volatilities the robust cases are around 6% higher than the nominal case. For problem (16) we plot in Fig. 1 the worst case standard deviation versus the worst case expected cost (maximum value among the 3 scenarios) and thus, as expected, the robust efficient frontier for problem (16) will be more conservative than the one for problem (14), but very close to each other.

6.4. Box and ellipsoidal uncertainty for r

In this subsection we consider the box and ellipsoidal uncertainty for r as presented in Section 4.1, using again the approach for old and new energy introduced in Section 5, and keeping the covariance matrix for the costs in their nominal values as in Section 6.3.

For the box uncertainty we consider in our example the vectors $\bar{r}^o$ and $\bar{r}^n$ as the costs related to the high CO_2 emission (50–60 USD/TM) cost case presented in Losekann et al. (2013) and reproduced in Table 5. As pointed out in Section 4.1, $\max_{r \in \mathcal{X}} \ \omega' r = \omega' \bar{r}$ and in this case the robust portfolio problem (7) is equivalent to solving problem

Table 5
Expected costs for the high CO_2 emission cost case.

Fuel	Mean value ($\bar{r}_i^o$ and $\bar{r}_i^n$)
Gas old	12.587
Gas new	11.973
Coal old	17.525
Coal new	16.985
Nuclear old	10.123
Nuclear new	10.025
Fuel oil old	22.854
Fuel oil new	20.214
Biomass old	14.063
Biomass new	13.463
Hydro old	4.132
Hydro new	5.006
Wind old	11.003
Wind new	10.469
Small hydro old	6.902
Small hydro new	6.909

(27) considering $\bar{r}^o$ and $\bar{r}^n$ instead of r^o and r^n. In Fig. 2 we show the efficient frontiers for the nominal and high CO_2 emission cost cases.

For the ellipsoidal uncertainty case we consider, for simplicity, that the expected costs for the old energy r^o is kept unchanged and the goal is to consider values of r^n such that the norm for the relative errors is less than a given upper bound ϵ. In other words, $\sum_{i=1}^{8} \left(\frac{r_i^n - \bar{r}_i^n}{\bar{r}_i^n} \right)^2 \le \epsilon^2$ for some pre-specified value $\epsilon > 0$, where $\bar{r}_i^n$ are the nominal values for the cost of the new energy as in Table 4 (that is, the sum of the square variations of the cost with respect to the nominal value is less than ϵ^2). It is easy to see that this problem is equivalent to considering the ellipsoidal uncertainty $\mathcal{X} = \left\{ r \in \mathbb{R}^m; (r - \bar{r})' S^{-1} (r - \bar{r}) \le 1 \right\}$ with $S^{1/2} = \text{diag}(\epsilon \ \bar{r}_i)$. In Fig. 3 we present again the efficient frontier corresponding to the problem of minimizing the variance of the cost for the nominal data (problem (3)) in dashed line and the robust problem (13) with $\epsilon = 0.2$ in solid line. Again, as expected, the robust efficient frontier will be more conservative as it is easy to see from the definition of the problem in Eq. (13).

6.5. Comparison with the DPEE2024 reference portfolio

As an example for the comparison among the portfolios, consider an expected cost of 7.155 (that is, $\tau = 7.155$), which would

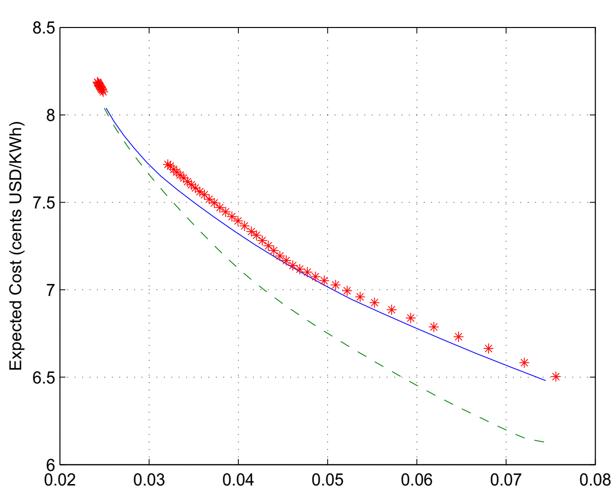

Fig. 1. Efficient frontier for the nominal data (dashed line), the robust data for problem (14) (solid line), and robust data for problem (16) (star line).

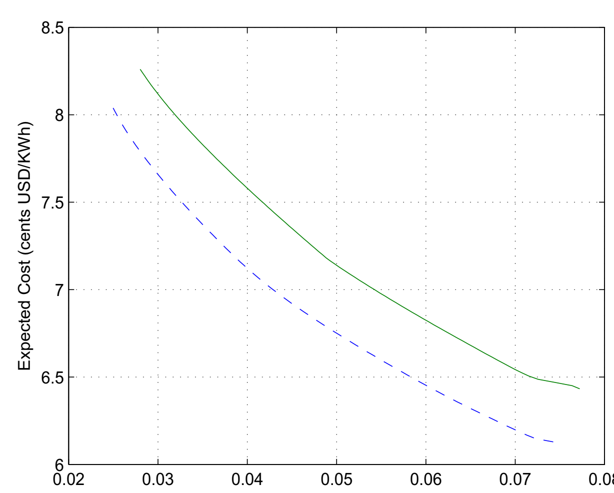

Fig. 2. Efficient frontier for the nominal data (dashed line) and the high CO_2 emission cost data (solid line).

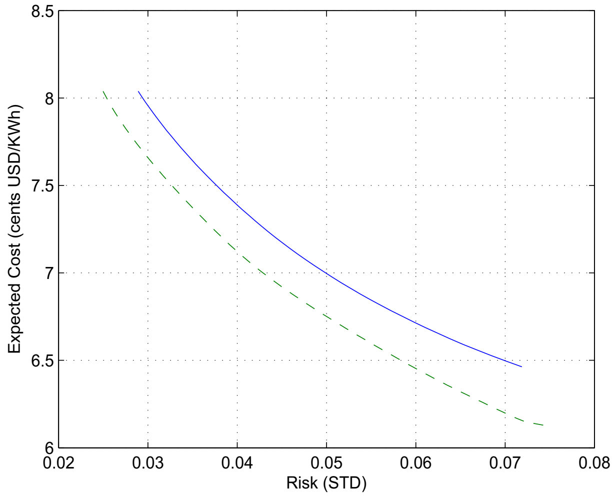

Fig. 3. Efficient frontier for the nominal data (dashed line) and the robust problem (13) with ellipsoidal uncertainty set for r (solid line).

correspond to the expected cost for the DPEE2024 reference portfolio, excluding the solar energy from the mix (see Ministerio de Minas e Energia (MME) and Empresa de Pesquisa Energética (EPE), 2015). In Table 6 we present the total weights (including the "old" and "new" energies) for the reference portfolio, the optimal one obtained from the nominal model (problem (3)), the optimal ones for the robust problems (14) and (16) considering the polytopic uncertainties, the optimal one for the problem (27) considering the box uncertainty set, and for the robust problem (13) considering the ellipsoidal uncertainty set.

As pointed out in Fabozzi et al. (2007b) the box and ellipsoidal uncertainty sets are related to regression techniques to estimate the uncertainty parameters. On the other hand the polytopic approach appears to be more appropriate when the modeler is more interested in creating economical scenarios *ad-hoc*. Thus it is important to notice that the box and ellipsoidal models cannot be directly compared with the polytopic model since, as aforementioned, they have different goals. We present below a comparison of the obtained results for each approach with the DPEE2024 reference portfolio (see Table 6).

a) For the box uncertainty model it was considered for the upper limit the costs related to the high CO_2 cost case presented in Losekann et al. (2013) and, as consequence, the results were similar to the ones obtained in this reference. As pointed out in Losekann et al. (2013), the effect of considering a higher CO_2 price yields to a substitution of the fossil fuels by renewable energies, even for the natural gas, which reduces its participation to around 5.9%, when compared to its share in the DPEE2024 reference portfolio, as shown in Table 6.
b) For the ellipsoidal uncertainty model the upper bound for the norm of the relative errors was arbitrarily chosen to be $\epsilon = 0.2$. Differently from the box uncertainty case, the ellipsoidal uncertainty model doesn't present a straight economical interpretation when compared with the DPEE2024 reference portfolio. As we can see from Table 6 the optimal portfolios for the nominal and robust case tend to be similar. We notice in Table 6 that the CO_2 emission for the portfolio obtained from the ellipsoidal uncertainty model is higher than the others, probably due to the fact that the choice of the ellipsoidal uncertainty for r^n is not selective among the renewable and non-renewable energies. Again we notice the increase on the weight on the gas energy when compared with the DPEE2024 reference mix, indicating a good risk mitigation contribution. In comparison with the DPEE2024 reference portfolio, and using the nominal values for the expected costs as shown in Table 4, we get that expected cost for the box uncertainty model is 6.8078, which corresponds to a reduction of 3.38% while, from Table 6, we see that the risk increased in 8.3%. This increase can be somehow seen as the price to be paid for considering the uncertainties on the expected cost parameters.
c) For the polytopic uncertainty model we notice that the robust cases (which consider 3 scenarios for the expected costs and 3 for the variances) has a higher worst case standard deviation (the maximum standard deviation among the 3 scenarios) than the standard deviation for the DPEE2024 reference and optimal portfolios, which is reasonable since the latter consider just 1 scenario for the expected values and variances. The worst case standard deviation for the robust portfolio from problem (14) is 7.38% higher than the reference mix, and 14.76% higher than the optimal mix. For the robust portfolio from problem (16) it is 8.57% higher than the reference mix, and 16.03% higher than the optimal mix. This increase of risk seems acceptable bearing in mind that we are considering the worst case volatility when we take into account the possible uncertainties for the parameters, not considered in the usual case. If we consider only the nominal values as in Table 4 the expected cost and standard deviation for the robust portfolio from problem (14) would be respectively 7.1 and 0.0406, a reduction of 0.8% and 3.4% with respect to the DPEE2024 reference portfolio. Similarly, considering only the nominal values, the expected cost and standard deviation for the robust portfolio from problem (16) would be respectively 7.08 and 0.041, a reduction of 1% and 2.6% with respect to the DPEE2024 reference portfolio. Thus, even considering the robust cases, the results suggest that there is room for improvement on the DPEE2024 portfolio expected costs and risk through diversification. Regarding the CO_2 emission we have that the robust mix from problems (14) and (16) have less emission than the optimal nominal one, which suggests that the robust approach can better conciliate the emission costs and price risk. We also notice the increase on the weight on the wind energy for both

Table 6
Weights, exp. costs (cents USD/KWh), STD, CO_2 emission (TM/KWh) for Ref. 2024, optimal and robust mixes.

Fuel	Ref. 2024	Optimal	Poly. (Eq. (14))	Poly. (Eq. (16))	Box Eq. (27))	Ellip. Eq. (13))
Gas	10.96%	11.69%	12.25%	12.15%	5.87%	11.94%
Coal	1.7%	2.88%	1.54%	1.82%	1.53%	3.13%
Nuclear	1.7%	2.00%	2.00%	2.00%	2.00%	2.00%
Fuel oil	2.16%	2.42%	2.41%	2.41%	2.42%	2.41%
Biomass	9%	5.56%	5.57%	5.59%	5.56%	5.57%
Hydro	58.56%	55.72%	56.51%	56.78%	63.21%	60.17%
Wind	12%	14.00%	14.00%	13.53%	13.68%	9.05%
Small hydro	3.92%	5.73%	5.72%	5.72%	5.73%	5.73%
Exp. cost	7.155	7.155	7.155	7.153	7.155	7.155
Stand. dev	0.0420	0.0393	0.0451	0.0456	0.0495	0.0455
CO_2 emis.	0.0728	0.0879	0.0780	0.0802	0.0518	0.0912

the optimal and robust cases in comparison with the reference mix, and the high participation of the gas, specially for the robust portfolios, indicating that these technologies have a good risk mitigation contribution.

7. Conclusions

In this paper we have considered the problem of optimal portfolio selection for electricity planning and policy-making when the vector of expected costs r as well as the covariance matrix Ψ for the different energy technologies belong to uncertainty sets. Tracing a parallel with the robust financial portfolio literature (see for instance, Fabozzi et al., 2007a) it was considered box and ellipsoidal uncertainty sets for r and componentwise bound and convex polytopic uncertainty sets for r and Ψ. It was shown that the problems of finding a portfolio of minimum worst case variance with guaranteed fixed maximum expected cost, minimum worst case expected cost with guaranteed fixed maximum variance, and minimum worst case combination of the expected and variance of the cost can be written as quadratic, SCOP and SDP problems. For the case that the model distinguishes the “old” energy from the “new” energy (as for instance in Awerbuch and Berger, 2003; Losekann et al., 2013) it was show that the optimization problems can be simplified.

A numerical example based on the data considered in Losekann et al. (2013) for the Brazilian generation mix expansion with 8 energy technologies, classified as “old” and “new” energies, was presented, considering the box, ellipsoidal and polytopic uncertainty sets. The efficient frontier for the problem of minimum worst case variance with guaranteed fixed maximum expected cost and the problem of minimizing the worst case combination of the expected and variance of the cost (for the polytopic case), weighted by a risk aversion parameter, were obtained and compared with the problem of minimum variance with fixed maximum expected cost for the case with just one scenario. As expected the robust efficient frontiers are more conservative but, on the other hand, it presented the advantage of taking into account possible future scenarios or uncertainty regions for the parameters that were not considered for the usual case. The results suggest that the robust approach, being by nature more conservative, can be useful in providing a reasonable electricity energy mix conciliating CO_2 emission, risk and costs under uncertainties on the parameters of the model.

We believe that the technique presented in this paper offers a useful computational tool in the direction of overcoming one of the main limitations of standard mean-variance optimization on energy planning, which is the uncertainty on the estimation of the expected and covariance matrix of the costs of the different energy technologies.

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Appendix A. Supplementary data

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