Giapetto's Woodcarving Problem

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9/7/2017

1 Giapetto's Woodcarving, Inc. Example

1.1 Problem Paramaters:

Gaipetto's Woodcarving, Inc. manufactures two types of wooden toys: soldiers and trains.

A solider sells for \$27 and uses \$10 worth of raw materials. Each solider that is manufactured increases Giapetto's variable labor and overhead costs by \$14. A train sells for \$21 and uses \$9 worth of raw materials. Each train build increases Giapetto's variable labor and overhead costs by \$10.

The manufacturing of wooden soldiers and trains requires two types of skilled labor. A solider requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing labor and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw but only 100 finishing hours and 80 carpentry hours.

Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit (revenues - costs).

Formulate a mathematical model of Giapetto's situation that can be used to maximize Giapetto's weekly profit.

1.2 Proposed Solution:

Our goal is to maximize profits. We need to formulate a linear profit function (objective function) and identify our linear constraints.

$$\max_{x_i} f(x_i) \text{ s.t. constraints}$$

We know that $\pi = R - C$

$R = 27x_1 + 21x_2$,

$C = 10x_1 + 9x_2 + 14x_1 + 10x_2$

where,

$R =$ total revenue, $C =$ total cost, x_1 is quantity of soldiers, and x_2 is quantity of trains.

Then our objective profit function is given by,

$f(x_i) = \pi = 3x_1 + 2x_2$

Our constraints are as follows:

$$2x_s + x_t \leq 100$$

$$x_s + x_t \leq 80$$

$$-x_t \leq 0$$

$$x_s \leq 40$$

$$-x_s \leq 0$$

We can formulate in matrix form and solve with either Matlab's linprog algorithm or R's lp algorithm.

Construct A-Matrix of LHS constraints:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \end{bmatrix}$$

Construct b-matrix of RHS constraints:

$$b = \begin{bmatrix} 100 & 80 & 0 & 40 & 0 \end{bmatrix}$$

Objective function:

$$f = \begin{bmatrix} 3 & 2 \end{bmatrix}$$

Need to invert signs on objective function, ie. f(x) to -f(x) for a max. problem (if using Matlab).

Matlabs Code:

```
[x_lp, eval_lp]= linprog(-f,A,b)
A*x_lp % verify constraints are satisfied
```

The above code generates the following output:

x_lp = 20 60

ans = 100 80 -60 20 -20

So when $x_1 = 20$ and $x_2 = 60$ we are maximizing profits of 180 and all constraints are satisfied.

2 Dorian Auto Example

Dorian Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programs: comedy show and football games.

Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men.

A 1-minute comedy ad costs $50,000 and a 1-minute football ad costs $100,000. Dorian would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men.

Use linear programming to determine how Dorian Auto can meet its advertising requirements at minimum cost.