E.S.S.A.I	Janvier 2019

Examen Session Principale

Cours Séries Temporelles

Durée : 1h30	D. Malouche

- **Remplir la grille attaché au sujet en respectant les cases**
- **L'examen contient 3 exercices et 4 pages.**

Exercice 1

```
> library(sarima)
> x <- sim_sarima(n=100, model = list(ar=0.8,ma=.4))
```

$X_t = \frac{...(1)...}{...(2)...} Z_t$ tel que $Z_r \sim$ ...(3)...

```
> x <- sim_sarima(n=200,model=list(sar=0.8, nseasons=12, sigma2 = 1))
```

$X_t - ...(4)...X_{..(5)..} = Z_t$ tel que $Z_r \sim$ ...(6)...

```
> x <- sim_sarima(n=144, model = list(ar=c(1.2,-0.8), ma=0.4,
+                                     sar=0.3, sma=0.7, iorder=1,
+                                     siorder=2,
+                                     nseasons=12, sigma2 = 2))
```

$...(7)... \ ...(8)...X_t = \frac{...(9)...}{...(10)...} Z_t$

```
x <- sim_sarima(n=144, model = list(...(11)..., iorder=...(12)..., siorder=...(13)...,
+                                   nseasons=...(14)..., sigma2 = 1))
```

$(1 - B)^2(1 - B^4)^3 X_t = (1 + .8B)Z_t$

```
> library(urca)
> library(fpp2)
> ...(15)...
> t1<-h02%>%diff(12)%>%...(30)...(lags =...(16)...,type=...(17)...)
> ...(18)...
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression ...(19)...


Call:
lm(formula = z.diff ~ z.lag.1 + ...(20)...)

Residuals:
      Min        1Q    Median        3Q       Max
-0.192380 -0.032261  0.002563  0.029618  0.205681

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.01045    0.00536   1.950 0.052768 .
```

```
..(23)..     -0.36425    0.10213  -3.567 0.000465 ***
z.diff.lag1 -0.54432    ..(21)..  -4.906  2.1e-06 ***
z.diff.lag2 -0.21562    0.11675  -1.847 0.066444 .
z.diff.lag3  0.02955    0.11378   0.260 0.795362
z.diff.lag4 -0.11476    0.11145  -1.030 0.304557
z.diff.lag5 ..(22)..    0.10440   0.114 0.909241
z.diff.lag6 -0.01355    0.08031  -0.169 0.866160
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1  1

Residual standard error: 0.0584 on 177 degrees of freedom
Multiple R-squared:  0.5036,Adjusted R-squared:  0.4839
F-statistic: 25.65 on 7 and 177 DF,  p-value: < 2.2e-16


Value of test-statistic is: -3.5666 6.3869

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.46 -2.88 -2.57
..(24)..  6.52  4.63  3.81
```

On note par (X_t) le processus observé dans `h02`. On a effectué ci dessus le test d'hypothèse nulle

$$H_0 : ..(25)..., \text{ vs l'hypothèse alternative} H_1 : \pi...(26)...$$

On peut conclure que le processus ...(27)... est ...(28)...

```
> t2<-h02%>%diff(12)%>%...(29)...(type=...(33)...)
> ...(31)...

#######################
# ...(32)... Unit Root Test #
#######################

Test is of type: ...(34)... with 4 lags.

Value of test-statistic is: 0.432

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
> library(forecast)
> m1<-Arima(h02,order=c(..(35)..,1,..(36)..),include.drift = ..(37)..,
+           seasonal=list(order=..(38)..,period=4),
+           lambda=NULL)
> m1
Series: h02
ARIMA...(40)...(1,0,0)..(39).. with drift

Coefficients:
            ar1      ma1  ..(41).. ..(42)..
         0.5776  -1.0000  ..(48)..  0.0024
..(43).  0.0597   0.0154   0.0708  0.0003

sigma^2 estimated as 0.01933:  ...(44)...=112.3
AIC=..(45)..   AICc=-214.29   BIC=-198.03
> m1$aic+2*m1$loglik
[1] ..(46)..
```

```
> t_stat(m1)
             ar1      ma1       sar1     drift
t.stat ..(47).. -65.1087 -2.219095 ...(49)..
p.val  0.000000   0.0000  0.026480 0.000000
```

Le modèle estimé ci dessus sécrit alors

$$..(54)..X_t = ..(53)..+\frac{..(50)..}{..(51)..}Z_t$$

où $Z \sim$..(52)..

```
> m2<-sarima(h02,p = ..(55)..,d = 2,q = ..(56)..,P = ..(57)..,
+   D = 1,Q = ..(58)..,S = 4,details = F,no.constant = T)
> m2
$fit

Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), include.mean = !no.constant, optim.control = list(trace = trc,
    REPORT = 1, reltol = tol))

Coefficients:
          ar1    ..(65)..
      ..(59)..  -0.9999
s.e.   0.0716   0.0127

sigma^2 estimated as ..(66)..:  log likelihood = -0.97,  aic = 7.95

$degrees_of_freedom
[1] 196

$ttable
     Estimate     SE  t.value p.value
ar1  ..(61).. ..(60)..  -0.3254  0.7452
..(64)..  ..(63)..  0.0127  ..(62)..  0.0000

$AIC
[1] -1.875119

$AICc
[1] -1.864727

$BIC
[1] ...(67)...
```

Exercice 2

On considère le code ‘R‘ suivant

```
> polyroot(c(1,-.5,.3))
[1] 0.833333+1.624466i 0.833333-1.624466i
> polyroot(c(1,1,1.5))
[1] -0.3333333+0.745356i -0.3333333-0.745356i
> polyroot(c(1,.4,.5,.8))
[1]  0.259294+1.012827i -1.143588+0.000000i  0.259294-1.012827i
```

Soit $Z_t \sim$BB(0,1) un bruit blanc et X_t un processus ARMA vérifiant les équations ci dessous Indiquer si ces équations ARMA admettent des solutions stationnaires (Répondre par oui, si stationnaire et non, sinon)

— $(1.5B+.3B^2)X_t=(1+.8)Z_t$ ..(68)...
— $X_t=-.4X_{t-1}-.5X_{t-2}-.8X_{t-2}+Z_t+.4Z_{t-1}$ ..(69)..
— $(1+B+1.5B^2)X_t=Z_t$ ..(70)..

Exercice 3

On sait que si une équation ARMA admet une solution, elle est de la forme

$$X_t=\sum_{k=0}^{\infty}\psi_k Z_{t-k}$$

où $Z_t \sim$BB$(0,1)$. La commande `ARMAtoMA` calcule les coefficients ψ_k étant donné les coefficients AR et MA.

1. Rappeler l'expression de $\mathbb{E}(X_t)$ et $\gamma_X(h)$ $\mathbb{E}(X_t)$ =.....(71)....... et $\gamma_X(h)$ =.....(72).......
2. Donner à partir du code ci dessous :
   — le modèle ARMA :....(73).....
   — $\gamma_X(0)$.....(74)....
   — $\gamma_X(2)$.....(75)....

```
> psi<-ARMAtoMA(ar = c(.5,.3),ma=.8, lag.max = 1000)
> psi1<-ARMAtoMA(ar = c(.5,.3),ma=.8, lag.max = 1001)
> psi2<-ARMAtoMA(ar = c(.5,.3),ma=.8, lag.max = 1002)
> sum(psi^2)
[1] 5.24359
> sum(psi*psi1[-1])
[1] 4.302564
> sum(psi*psi2[-c(1,2)])
[1] 3.724359
> psi[1:2]
[1] 1.30 ..(76)..
> sum(ARMAtoMA(ar = c(.5,.3),ma=.8, lag.max = 20))
[1] 7.678335
> library(FitAR)
> g=tacvfARMA(theta =  -.8,phi = c(.5,.3),sigma2 = 1,maxLag = 2)
> g
[1] ..(77).. ..(78).. 4.674359
> PacfDL(g,LinearPredictor = ...(79)..)
$Pacf
[1]  ..(80).. -0.2902338

$ARCoefficients
[1]  ..(82).. ..(81)..

$ResidualVariance
[1] ..(83)...
```

En déduire que

$$X_t = ..(84)... \times X_{t-1}+E^+_{t,1}$$

et

$$X_t = ...(85).. \times X_{t-1}+..(86).. \times X_{t-2}+E^+_{t,2}$$

où

$$\|E^+_{t,2}\|^2 = ..(87)...$$

Numéro	Réponse	Numéro	Réponses
1		26	
2		27	
3		28	
4		29	
5		30	
6		31	
7		32	
8		33	
9		34	
10		35	
11		36	
12		37	
13		38	
14		39	
15		40	
16		41	
17		42	
18		43	
19		44	
20		45	
21		46	
22		47	
23		48	
24		49	
25		50	

Numéro	Réponse	Numéro	Réponses
51		76	
52		77	
53		78	
54		79	
55		80	
56		81	
57		82	
58		83	
59		84	
60		85	
61		86	
62		87	
63		88	
64		89	
65		80	
66		91	
67		92	
68		93	
69		94	
70		95	
71		96	
72		97	
73		98	
74		99	
75		100	

Numéro	Réponse	Numéro	Réponses
1	$(1+.4B)$	26	< 0
2	$(1-.8B)$	27	$(1-B^{12})X_t$
3	BB(0,1)	28	stationnarité
4	-0.8	29	`ur.kpss`
5	X_{t-12}	30	`ur.df`
6	BB(0,2)	31	`summary(t2)`
7	$(1-B)$	32	`KPSS`
8	$(1-B^{12})^2$	33	`"mu"`
9	$(1+.4B)(1+.7B^{12})$	34	`mu`
10	$(1-1.2B+0.8B^2)(1-.3B^{12})$	35	1
11	`ma=.8`	36	1
12	2	37	`T`
13	3	38	`c(1,0,0)`
14	4	39	`[4]`
15	`data(h02)`	40	`(1,1,1)`
16	6	41	`sar1`
17	`"drift"`	42	`drift`
18	`summary(t1)`	43	`s.e.`
19	`drift`	44	`log likelihood`
20	`1 + z.diff.lag`	45	`-214.6`
21	`0.11095`	46	10
22	`0.01192`	47	9.676136
23	`z.lag.1`	48	-0.1570
24	`phi1`	49	7.139711
25	$\pi = 0$	50	$(1-B)$

Numéro	Réponse	Numéro	Réponses
51	$(1-0.58B)(1+0.16B^4)$	76	0.95
52	BB(0,0.02)	77	6.243590
53	0.0024	78	5.602564
54	(1-B)	79	T
55	1	80	0.8973306
56	1	81	-0.2902338
57	0	82	1.1577662
58	0	83	1.113787
59	-0.0233	84	0.8973306
60	0.0716	85	1.1577662
61	-0.0233	86	-0.2902338
62	-78.9945	87	1.113787
63	-0.9999	88	
64	ma1	89	
65	ma1	80	
66	0.05531	91	
67	-2.842589	92	
68	oui	93	
69	oui	94	
70	non	95	
71	0	96	
72	$\sigma^2 \sum_{j=0}^{k} \psi_j \psi_{h+j}$	97	
73	$(1-.5B+.3B^2)X_t = (1+.8B)Z_t$	98	
74	5.040553	99	
75	3.576947	100	