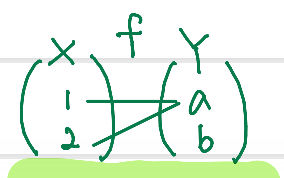
※3.1 Functions

- function
  - Def: assigns to each member of set X exactly one member of set Y.

ex)  $\{(1,a), (2,a)\}$

$X = \{1, 2\}$ $Y = \{a, b\}$

arrow diagram

- Terms
  - Let X and B be sets as $f: X \to Y$
    X: domain of f / Y: codomain of f
  - Modulus : remainder when x is dived by y
    ex) 2 mod 3 = 2
  - Floor : $\lfloor x \rfloor$ is the greatest integer less then or equal to x
    ex) $\lfloor 8.3 \rfloor = 8$ $\lfloor -8.7 \rfloor = -9$
  - One to one : If for all $x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Bijection

  - Onto Y : If for every $y \in Y$, there exists $x \in X$ such that (surjective) $f(x) = y$
  - Inverse : If f is bijection, $f^{-1} = \{(y, x) \mid (x, y) \in f\}$
  - Composition : $h(x) = f(g(x)) = (f \circ g)(x)$

※ 3.2 Sequence and Strings

- Sequence
  - Def: special type of function in which the domain consists of integers. Subset of Integers
    ① notation : S_n instead of $s(n)$
    ↳ Index of the sequence.
    ② Consecutive integer $\{k, k+1, \cdots, \}$ and the index of s is n, denote the sequence S as $\{S_n\}_{n=k}^{\infty}$
  - Type
    - finite sequence, infinite sequence
    - Increasing : for all i and j in the domain of s, if $i < j$, then $S_i < S_j$
    - non-decreasing : ", if $i < j$, $S_i \leq S_j$
    - Decreasing : ", if $i < j$, $S_i > S_j$
    - non-increasing : ", if $i < j$, $S_i \geq S_j$
  - Subsequence : To retain only certain terms of the original sequence, maintaining the order of terms in the given sequence.
    → denoted $\{S_{n_i}\}$
  - Terms
    - $\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_n$ " Sum notation
    - $\prod_{i=m}^{n} a_i = a_m \cdot a_{m+1} \cdots a_n$ " product notation
- Strings
  - def : finite sequence of characters, restricted to sequences composed of symbols drawn from a finite alphabet.
    → may be indexed from 0 or 1.
  - type
    - String over X : finite sequence of elements from X (X^*)
    - Null String : no elements strings (λ)
    - Non null string over X : X^+
  - Term
    - length : number of elements in string α. $|\alpha|$
    - Concatenation : consisting of α followed by b. αb
    - Substring : Obtained by selecting some or all elements

Study Title ______________________ Date __________

※ 3.3 Relations

- Relation
  - Def: be thought of as a table that lists which elements of the first set relate to which elements of the second set. R
    - □ Define a relation to be a set of ordered pairs.
    - □ Subset of the Cartesian product $X \times Y$

ex) Relation of students to courses

Student	*Course*
Bill	CompSci
Mary	Math
Bill	Art
Beth	History
Beth	CompSci
Dave	Math

Let $X = \{$ Bill. Mary. Beth. Dave $\}$, $Y = \{$ CompSci, Math. Art. History $\}$, then $R = \{$ (Bill. CompSci), (Mary. Math). (Bill. Art), … (Dave. Math $\}$

→ since (Beth. History) $\in R$, Beth R History

- Digraph
  - □ Informative way to picture a relation on a set is to draw
  - □ Draw dots or vertices to elements of X

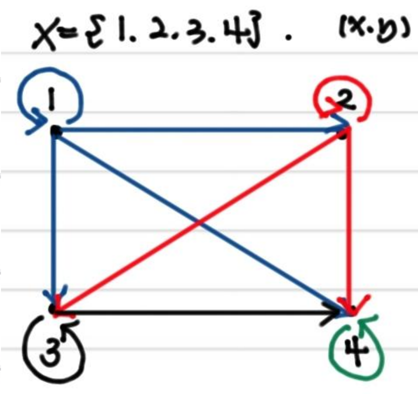
↓

If the element (x.y) is in the relation, we draw an arrow directed edge from x to y

((x.x) is from x to x, loop)

ex) $X = \{1.2.3.4\}$ defined by $(x.y) \in R$ if $x \le y$. $x.y \in X$

$X = \{1.2.3.4\}$. $(x.y) \in R$ if $x \le y$. $x.y \in X$



- Properties
  - □ Reflective: If $(x.x) \in R$ for every $x \in X$

    ex) $X = \{1.2.3.4\}$ is reflective because (1.1) (2.2). (3.3) (4.4) $\in R$

  - □ Symmetric: If for all $x.y \in X$, if $(x.y) \in R$, then $(y.x) \in R$

    ex) $X = \{a.b.c.d\}$, $R = \{$ (a.a). (b.c). (c.b). (d.d) $\}$ is symmetric.

  - □ Antisymmetric: If for all $x.y \in X$, if $(x.y) \in R$ and $(y.x) \in R$, then $x = y$

    → so, If there is no symmetric other than xRx, it is an antisymmetric.

    ex) $X = \{1.2.3\}$. $R = \{(1.1). (2.3). (3.3)\}$ then. it is antisymmetric because there is only xRx of (3.3)

  - □ Transitive: If for all $x.y.z \in X$, if $(x.y)$ and $(y.z) \in R$, then $(x.z) \in R$

    ex) $X = \{1.2.3.4\}$. $(x.y) \in R$ if $x \le y$ is transitive

- Order
  - □ Partial order: R is reflexive, antisymmetric. transitive

    → R is partial order

  - □ Comparable: If $x.y \in X$ and either $x \le y$ or $y \le x$. then x and y are comparable

    ↔ Incomparable

  - □ Total order: If every pair of elements in X is comparable, then R is total order.

    → so, every (x.y) must be included in R, the order should not be reversed.

- other terms
  - □ Inverse: $R^{-1} = \{(y.x) \mid (x.y) \in R\}$
  - □ Composition: $R_2 \circ R_1 = \{(x.z) \mid (x.y) \in R_1$ and $(y.z) \in R_2$ for some $y \in Y\}$

Study Title Date

※ 3.4 Equivalence Relations

- Equivalence Relation

  - Def: Relations that are reflexive, symmetric, transitive

    → on Set A, $R \subset A \times A$ is "

    ex) $A = \{1, 2, 3\}$ on Relation R $(x, y \in R,\ x \leq y)$

    ① Reflexive : O (R have (1,1), (2,2), (3,3))

    ② symmetric : X (R doesn't have (2,1), (3,2))

    ③ transitive : O

    ⇒ R is not equivalence relation.

  - features

    - The whole is partioned into non-empty classes with no common-parts.

Subsets.

- Partion and equivalence class

  - Def : Set $\{S_1, S_2, \cdots, S_n\}$ of a set S that is not empty satisfied the following three conditions.

    ① Each elements of S belongs to only one S_m.

    ② $S = S_1 \cup S_2 \cup \cdots \cup S_m$

    ③ $i \neq j$, $S_i \cap S_j \neq 0$

    ⇒ Let R be an equivalence relation on a Set X.

    For each $a \in X$, let $[a] = \{x \in X \mid xRa\}$.

    Then $S = \{[a] \mid a \in X\}$ is a parion of X

Equivalence classes of X given by the Relation R

※ 3.5 Matrices of Relation

- Matrix of Relation
  - Def : Set the entry in row x and column y to 1 if xRy and to 0 otherwise.
    - Label the rows with the element of X, the columns with the element of Y (Relative to the orderings of X, Y)

  - Features
    - Always a square matrix
    - Convenient way to represent a relation.

  - Determine Property
    - Reflexive : If and only if matrix has 1's on the main diagonal
      - Main diagonal consists of the entries on a line from the upper left to lower right

ex) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow$ main diagonal

    - Symmetric : If and only if A is symmetric about the main diagonal

ex) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow$ symmetric

    - Transitive : If and only if whenever entry i, j in A^2 is non-zero, entry i, j in A is also non-zero

ex) $R = \{(a,a), (b,b), (c,c), (d,d), (b,c), (c,b)\}$

$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow A^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

non-zero　　non-zero